A tale of two growth engines: Interactive effects of monetary policy and intellectual property rights

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A Tale of Two Growth Engines:

Interactive Effects of Monetary Policy and Intellectual Property Rights

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Abstract

How do intellectual property rights that determine the market power of firms influence the growth and welfare effects of monetary policy? To analyze this question, we develop a monetary endogenous-growth model in which R&D and capital accumulation are both engines of long-run economic growth. We find that monetary expansion hurts economic growth and social welfare by reducing R&D and capital accumulation. Furthermore, a larger market power of firms strengthens these growth and welfare effects of monetary policy through the R&D channel but weakens these effects through the capital-accumulation channel. Therefore, whether the market power of firms amplifies or mitigates the welfare cost of inflation depends on the relative importance of the two growth engines. Finally, we calibrate the model using data in the United States and the Euro Area to quantitatively evaluate and compare the welfare cost of inflation in these two economies and find that the R&D channel dominates in both economies.

Keywords**:** economic growth, R&D, inflation, monetary policy, patent policy

JEL classification**:** O30, O40, E41

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1. Introduction

Since the seminal study by Tobin (1965), the relationship between inflation and economic growth has been a fundamental issue in monetary economics, and there is now an established literature on monetary policy and economic growth.[1] The present study relates to this literature by analyzing an unexplored implication that the market structure has an influence on the growth and welfare effects of monetary policy. Specifically, we analyze how intellectual property rights that determine the market power of firms influence the effects of monetary policy on economic growth and social welfare in a monetary endogenous-growth model in which R&D and capital accumulation are both engines of long-run economic growth. We find that monetary expansion that increases inflation raises the cost of consumption relative to leisure consequently reducing labor supply, which is an important factor input for R&D and capital accumulation. A reduction in this factor input in turn decreases economic growth and social welfare. Interestingly, the magnitude of these growth and welfare effects of monetary policy depends on the strength of patent protection. Specifically, a larger market power of firms strengthens the effects of monetary policy through the R&D channel but weakens these effects through the capital-accumulation channel. Thus, the market power of firms has drastically different implications on the welfare cost of inflation under the two growth engines. Whether it amplifies or mitigates the welfare cost of inflation depends on the relative importance of the two growth engines.

The above theoretical finding has an important implication on a recent policy reform. As a result of the World Trade Organization's Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS),[2] many countries have strengthened their protection for intellectual property rights. For example, according to the Ginarte-Park index of patent rights in Park (2008),

[1] See for example Gillman and Kejak (2005) for a survey of this literature.
[2] The WTO's TRIPS Agreement, which was initiated in the 1986-94 Uruguay Round, establishes a minimum level of intellectual property protection that must be provided by all member countries by 2006.

107 countries have experienced an increase in the strength of patent rights from 1995 to 2005.[3] In these 107 countries, the average increase in the Ginarte-Park index is 0.82.[4] Our theoretical result implies that the welfare cost of inflation would have increased in some of these countries. Given that innovation is likely to be the main engine of economic growth in developed countries, these countries would experience a larger welfare cost of inflation as a result of stronger patent protection. In contrast, for a developing country in which the main engine of growth is capital accumulation, our result implies that it should experience a smaller welfare cost of inflation as a result of stronger patent protection.

The reason why the strength of patent protection has different implications on the growth and welfare effects of monetary policy under the two growth engines is as follows. For a given supply of labor, increasing the market power of firms raises the incentives for innovation and the share of labor devoted to R&D. This increase in the R&D share of labor tends to *magnify* the growth and welfare effects of the decrease in labor supply driven by monetary expansion. In contrast, increasing the market power of firms reduces the income share of physical capital and the share of labor devoted to capital accumulation. This decrease in the capital share of labor tends to *mitigate* the growth and welfare effects of the decrease in labor supply driven by monetary expansion. Therefore, the market power of firms has drastically different implications on the growth and welfare effects of monetary policy as the relative importance of the two growth engines changes. In other words, the effects of monetary policy are influenced by an interaction between the growth engine and the market power of firms. To our knowledge, this interaction has never been explored in the literature.

[3] There are a total of 122 countries in the Ginarte-Park index. Of these 122 countries, 119 countries have available measure of patent rights from 1995 to 2005, and only one country, Iraq, has experienced a reduction in the strength of patent rights during this period.
[4] The index is a scale of 0 to 5, and a larger number indicates stronger patent rights. See Park (2008) for details.

In the quantitative analysis, we calibrate the model using data in the United States (US) and the Euro Area (EA) to quantitatively evaluate and compare the welfare cost of inflation in these two economies. We consider currency and M1 as alternative measures of money. In both economies, we find that the welfare cost of inflation is much higher under the M1 specification than under the currency specification as in Dotsey and Ireland (1996). We also find a significant difference in the welfare cost of inflation between the EA and the US when we use M1 as the measure of money but a negligible difference between the two economies when we consider currency as the measure of money. Under both money specifications, we find that increasing the markup magnifies the effects of inflation on economic growth and social welfare in both the US and the EA; in other words, the R&D channel dominates the capital-accumulation channel.

1.1. Literature review

Tobin (1965) argues that higher inflation stimulates the accumulation of physical capital via the substitution with money holding. In contrast to Tobin (1965), when money is required for purchasing capital goods (Stockman, 1981), higher anticipated inflation reduces real balances, capital investment and the level of output (i.e., the reversed Tobin effect). This theoretical result is also consistent with many subsequent studies in the literature that consider variants of the AK model with a cash-in-advance constraint on consumption goods and analyze the growth and welfare effects of inflation through elastic labor supply. For example, Gomme (1993) and Mino (1997) introduce money into the two-sector Lucas (1988) model via cash-in-advance constraints and emphasize how the money growth rate affects the consumption-leisure decision.[5] Our result

[5] In a recent study, Itaya and Mino (2007) use an endogenous-growth model with a cash-in-advance constraint to show an interesting result that the growth effect of money supply depends on the preference structure and production technology. Specifically, if the production technology exhibits strong non-convexity or if the utility function has a

of a negative effect of inflation on economic growth is driven by a similar mechanism as these studies.[6] Another branch of studies, such as Zhang (1996) and Jha *et al.* (2002), highlights the role of money in facilitating transactions for which a change in the inflation rate affects the consumption-leisure decision through transaction costs.[7] These studies in general support the negative relationship between inflation and economic growth regardless of whether the model is based on a cash-in-advance constraint or transaction costs.[8] In the present study, we explore a related growth-inflation relationship but introduce an additional growth engine that is R&D-driven innovation. Specifically, we incorporate a cash-in-advance constraint on consumption goods into a unified endogenous-growth model in which R&D and capital accumulation are both engines of long-run growth and allow for elastic labor supply.

In contrast to the well-established literature on monetary policy in the AK model, a small but growing number of studies, such as Marquis and Reffett (1994), Funk and Kromen (2006, 2010) and Chu and Lai (2012), has analyzed the effects of monetary policy on economic growth in the R&D-based growth model. The seminal study by Marquis and Reffett (1994) incorporates a transaction-service sector along with a cash-in-advance constraint into the Romer model. They show that higher inflation reduces growth through a reallocation of factor inputs from R&D and production to transaction services. Our model features a different mechanism from the Marquis-Reffett model by having a negative effect of inflation on economic growth through a reduction in labor supply. Chu and Lai (2012) incorporate money demand into a quality-ladder model similar to Grossman and Helpman (1991) with a money-in-utility specification and analyze how the

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high elasticity of intertemporal substitution, then there may be multiple balanced-growth paths that feature different growth effects of inflation.

[6] A recent study by Chu, Lai and Liao (2012) analyzes the effects of inflation on economic growth in the Lagos-Wright search model with AK endogenous growth and also finds a negative effect of inflation on growth.

[7] As for monetary growth models with money in utility, see for example Wang and Yip (1992) and Ho *et al.* (2007).

[8] In contrast, Itaya and Mino (2003) show that the Tobin effect (i.e., a positive growth effect of inflation) may emerge in an endogenous-growth model with transaction costs when labor externalities are sufficiently large.

elasticity of substitution between consumption and the real money balance affects the growth and welfare effects of inflation.[9] Funk and Kromen (2006, 2010) incorporate nominal price rigidity into a quality-ladder model to quantitatively evaluate the effects of inflation on economic growth,[10] and they analyze an interesting channel through which nominal price rigidity transmits the effects of inflation from the short run to the long run. The present paper differs from the abovementioned studies by (a) considering a unified endogenous-growth model in which both R&D and capital accumulation are engines of growth, (b) showing the different implications of firms' market power on the effects of monetary policy on R&D and capital investment, and (c) comparing the welfare cost of inflation between the US and the EA.

In an early study, Mansfield (1980) points out that higher inflation may reduce R&D by decreasing investment in the plant and equipment that are necessary for R&D and by increasing uncertainty on relative prices. Goel and Ram (2001) provide empirical evidence to confirm the latter effect by showing that inflation uncertainty has a negative effect on R&D. A recent study by Chu and Lai (2012) provides further empirical evidence that supports a negative relationship between R&D and the level of inflation using cross-country regressions. In addition to empirical studies, policy-oriented research also suggests that high inflation could potentially reduce R&D investment. For example, in Economic Development Indicators (chapter 8, 2005), "… high and volatile inflation also discourages investment, including human capital and R&D investment."

This study also relates to the literature on patent policy and economic growth. The seminal study in this literature is Judd (1985), who analyzes the effects of patent length on economic growth in a dynamic general-equilibrium framework; see also Iwaisako and Futagami

[9] In the monetary quality-ladder model in Chu and Lai (2012), a larger markup would also strengthen the effects of monetary policy through the R&D channel; however, their model does not feature the capital-accumulation channel.
[10] Vaona (2012) incorporates nominal rigidity into an AK-style model with learning by doing to analyze the growth effects of inflation, and he provides empirical evidence that shows a negative effect of inflation on economic growth.

(2003) and Futagami and Iwaisako (2007). Instead of patent length, we consider patent breadth against imitation;[11] see also Li (2001), Goh and Olivier (2002), Kwan and Lai (2003), Furukawa (2007) and Cysne and Turchick (2012). However, our model differs from these studies by modeling R&D and physical capital as two engines of long-run growth. Because patent breadth has asymmetric effects on R&D and capital accumulation, the overall effect of strengthening patent protection on economic growth is ambiguous due to a tradeoff between R&D and capital accumulation as in Iwaisako and Futagami (2012).[12] The present study relates to this literature by analyzing how patent policy interacts with monetary policy to affect growth and welfare.

The rest of the study is organized as follows. Section 2 sets up the monetary endogenous-growth model. Section 3 analyzes the effects of monetary policy on economic growth and social welfare. Section 4 calibrates the model to numerically evaluate the welfare cost of inflation in the EA and the US. Section 5 considers an extension of the model to examine the robustness of our results. The final section concludes.

2. A monetary endogenous-growth model

To analyze the interactive effects of monetary policy and patent policy, we modify the seminal R&D-based growth model in Romer (1990) by (a) introducing a cash-in-advance constraint on consumption goods to model money demand, (b) considering variable patent breadth as in Goh and Olivier (2002), (c) incorporating a capital-producing sector as in Iwaisako and Futagami (2012) so that capital accumulation is also an engine of long-run growth, and (d) allowing for

[11] Chu (2010) shows that at the current patent length of 20 years, extending the patent length would have negligible effects on R&D and social welfare. Therefore, we focus on the analysis of patent breadth in this study.

[12] See also Chu, Cozzi and Galli (2012), who analyze the asymmetric effects of blocking patents on variety expansion and quality improvement in an R&D-based growth model.

elastic labor supply. Given that the Romer model has been well-studied, the standard features of the model will be briefly described below to conserve space.

2.1. Households

There is a unit continuum of identical households, who have a lifetime utility function given by

(1) $$U = \int_0^\infty e^{-\rho t} u_t dt = \int_0^\infty e^{-\rho t} (\ln c_t - \psi l_t) dt.$$ [13]

Instantaneous utility u_t is increasing in consumption c_t and decreasing in the supply of labor l_t. As for the exogenous parameters, $\rho > 0$ is the discount rate, and $\psi > 0$ determines the disutility of labor supply. Households maximize utility subject to an asset-accumulation equation given by

(2) $$\dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t + \tau_t - c_t - \pi_t m_t.$$

a_t is the real value of assets owned by households, and these assets consist of tangible and intangible capital. r_t is the real interest rate. Households supply labor to earn a real wage w_t. τ_t is a real lump-sum transfer from the government. π_t is the inflation rate that determines the cost of holding money. m_t is the real money balance held by households to facilitate purchases of consumption goods that are subject to a cash-in-advance constraint given by $\xi c_t \le m_t$, where $0 < \xi \le 1$. The usual cash-in-advance constraint is captured by the special case of $\xi = 1$. Here we follow Dotsey and Ireland (1996) to consider a more general setup in which only a fraction of

[13] Our results are robust to a more general utility function given by $u_t = \ln c_t - \psi l_t^{1+\chi} /(1+\chi)$ for $\chi \ge 0$. Derivations are available upon request from the authors. However, when $\chi > 0$, the equilibrium allocations do not have closed-form solutions. Therefore, we focus on the special case of $\chi = 0$ for analytical tractability.

consumption expenditure is subject to a cash-in-advance constraint.[14] This generalization allows us to perform a more realistic quantitative investigation on the welfare cost of inflation.

Using standard dynamic optimization, the optimality condition for consumption is

(3) $$1/c_t = \lambda_t(1+\xi i_t),$$

where λ_t is the Hamiltonian co-state variable on (2), and $i_t = r_t + \pi_t$ is the nominal interest rate that captures the opportunity cost of holding money as opposed to accumulating tangible or intangible capital. The optimality condition for labor supply is

(4) $$w_t = \psi(1+\xi i_t)c_t.$$

The familiar intertemporal optimality condition is

(5) $$r_t = \rho - \dot{\lambda}_t / \lambda_t.$$

2.2. Final goods

Final goods y_t are produced by a standard CES aggregator using production labor $l_{y,t}$ and a continuum of differentiated intermediates goods $x_t(j)$ for $j \in [0, n_t]$ given by

(6) $$y_t = l_{y,t}^{1-\alpha}\int_0^{n_t} x_t^{\alpha}(j)dj,$$

where n_t is the number of intermediate goods available. This sector is perfectly competitive, and the producers take the output and input prices as given. The conditional demand functions for production labor and intermediate goods are respectively

(7) $$w_t = (1-\alpha)y_t / l_{y,t},$$

(8) $$p_t(j) = \alpha[l_{y,t} / x_t(j)]^{1-\alpha},$$

[14] See also Wu and Zhang (1998) who consider a generalized cash-in-advance constraint.

where $p_t(j)$ is the price of $x_t(j)$ relative to final goods.

2.3. Intermediate goods

There is a continuum of industries producing intermediate goods $x_t(j)$ for $j \in [0, n_t]$. Each industry is occupied by a monopolist who rents capital to produce intermediate goods in an one-to-one fashion; i.e., $x_t(j) = k_t(j)$. The monopolistic profit is

(9) $$\omega_{x,t}(j) = p_t(j)x_t(j) - q_t k_t(j),$$

where q_t is the rental price of capital.

The unconstrained optimization yields a profit-maximizing markup of $1/\alpha$. Here we follow Goh and Olivier (2002) to introduce patent breadth denoted by η as a policy variable by assuming that the unit cost of producing imitative products is increasing in patent breadth.[15] Thus, without sufficient strength of patent protection, the presence of monopolistic profits attracts imitation. Therefore, stronger patent protection allows monopolistic producers to charge a larger markup without the threat of imitation; see Li (2001) for a similar formulation of patent breadth in the quality-ladder model. This formulation is also consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price". In summary, the maximum markup is determined by η.[16] For the rest of this study, we assume $\eta < 1/\alpha$,[17] so that

(10) $$p_t(j) = \eta q_t$$

[15] In this study, we focus on patent breadth and make a standard assumption in the literature that the patent length is infinite for simplicity. See for example Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) for an analysis on finite optimal patent length in the Romer model. See also Palokangas (2011) for an analysis on optimal patent length and breadth in an R&D-based growth model.

[16] Alternatively, one can also view the limited markup as price regulation. For example, Evans *et al.* (2003) analyze price regulation in the Romer model without money demand.

[17] Given a capital share of about one-third, the unconstrained markup would be 200% (i.e., $1/\alpha - 1$) that is unrealistically large. Therefore, imposing an upper bound on the markup also helps to separate the effects of markup and capital share. See Jones and Williams (2000) for a discussion on this issue.

for $j \in [0, n_t]$. This formulation also serves to provide a simple way to separate capital share α and markup η. The amount of profit is symmetric across industries and given by

(11) $$\omega_{x,t}(j) = \left(\frac{\eta - 1}{\eta}\right) p_t(j) x_t(j) = (\eta - 1)\left(\frac{q_t k_t}{n_t}\right),$$

where the second equality of (11) uses the market-clearing condition for capital goods $x_t n_t = k_t$. Equation (11) shows that a larger markup η increases the amount of monopolistic profits, which in turn improves incentives for R&D investment; however, a larger η also decreases the capital share of income $q_t k_t / y_t = \alpha / \eta$,[18] which in turn worsens incentives for capital accumulation.

2.4. R&D

Denote the value of an invented variety as $v_{n,t}$. The familiar no-arbitrage condition for $v_{n,t}$ is

(12) $$r_t v_{n,t} = \omega_{x,t} + \dot{v}_{n,t}.$$

Intuitively, (12) equates the interest rate to the asset return per unit of asset, where the asset return is the sum of monopolistic profit $\omega_{x,t}$ and capital gain $\dot{v}_{n,t}$. In the R&D sector, there is a unit continuum of entrepreneurs who hire workers $l_{r,t}$ for R&D. The profit of R&D is

(13) $$\omega_{r,t} = v_{n,t}\dot{n}_t - w_t l_{r,t},$$

where $\dot{n}_t = \varphi n_t l_{r,t}$ is the mass of inventions created by the entrepreneur.[19] The parameter φ determines R&D productivity. The zero-profit condition in the R&D sector is

(14) $$\varphi v_{n,t} n_t = w_t.$$

[18] This condition can be derived by using (6), (8) and (10).

[19] Although we consider a deterministic R&D process as in the original Romer model, it is useful to note an interesting result by Li (1998) who shows that this deterministic R&D process can be derived from an underlying stochastic R&D process.

This condition determines the allocation of labor to R&D.

2.5. Capital production

Denote the value of one unit of capital as $v_{k,t}$. The no-arbitrage condition for $v_{k,t}$ is

(15) $$r_t v_{k,t} = q_t + \dot{v}_{k,t}.$$

Intuitively, (15) equates the interest rate to the asset return per unit of asset, where the asset return is the sum of capital rental price q_t and capital gain $\dot{v}_{k,t}$.[20] In the capital-producing sector, there is a unit continuum of firms that hire workers $l_{k,t}$ to produce capital. The profit is

(16) $$\omega_{k,t} = v_{k,t}\dot{k}_t - w_t l_{k,t},$$

where $\dot{k}_t = \phi A_t l_{k,t}$ is the amount of capital produced. ϕ is a parameter, and ϕA_t determines the productivity of capital accumulation. To introduce endogenous growth, we assume $A_t = k_t$ that captures the usual capital externality in the AK model.[21] The zero-profit condition is

(17) $$\phi v_{k,t} k_t = w_t.$$

This condition determines the allocation of labor to capital accumulation.

2.6. Monetary authority

The growth rate of money supply M_t is denoted by $\mu_t = \dot{M}_t / M_t$ that is exogenously set by the monetary authority. Given the definition of the real money balance $m_t \equiv M_t / P_t$ (where P_t is the price of final goods), the inflation rate π_t is endogenously determined by

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[20] Here we have made a simplifying assumption of zero capital depreciation as in Romer (1990) and Iwaisako and Futagami (2012).

[21] In Section 5, we consider an alternative specification for the laws of motion for capital and variety that allows for cross-sector spillovers.

(18) $$\pi_t = \mu_t - \dot{m}_t / m_t .$$

Any change in money supply is redistributed to households as a lump-sum transfer that has a real value of $\tau_t = \dot{M}_t / P_t = \mu_t m_t = \dot{m}_t + \pi_t m_t$, where the last equality follows from (18).

2.7. Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t, m_t, l_t, y_t, x_t(j), k_t, l_{y,t}, l_{r,t}, l_{k,t}\}_{t=0}^{\infty}$, a time path of prices $\{w_t, r_t, p_t(j), q_t, v_{k,t}, v_{n,t}\}_{t=0}^{\infty}$, and a time path of policies $\{\mu_t, \tau_t\}_{t=0}^{\infty}$. At each instant of time,

a. households choose $\{c_t, m_t, l_t\}$ to maximize (1) subject to (2) taking $\{w_t, r_t, \tau_t\}$ as given;

b. competitive final-goods firm produce $\{y_t\}$ to maximize profit taking $\{w_t, p_t(j)\}$ as given;

c. the monopolist in industry $j \in [0,1]$ produces $\{x_t(j)\}$ and chooses $\{p_t(j)\}$ subject to the level of patent breadth η to maximize profit taking $\{q_t\}$ as given;

d. R&D entrepreneurs maximize profit taking $\{w_t, v_{n,t}\}$ as given;

e. capital-producing firms maximize profit taking $\{w_t, v_{k,t}\}$ as given;

f. the market for final goods clears such that $y_t = c_t$;

g. the market for capital goods clears such that $k_t = x_t n_t$;

h. the labor market clears such that $l_t = l_{y,t} + l_{r,t} + l_{k,t}$;

i. the value of households' assets equals the total value of intangible and tangible capital in the economy such that $a_t = v_{n,t} n_t + v_{k,t} k_t$;

j. the monetary authority balances its budget such that $\tau_t = \mu_t m_t$.

2.8. Balanced growth path

In this subsection, we consider the dynamic properties of the model. Given that the monetary authority sets a stationary growth rate of money supply (i.e., $\mu_t = \mu$ for all *t*), the economy jumps to a unique and stable balanced growth path.[22] Lemma 1 summarizes this result, and the proof is relegated to Appendix A.

Lemma 1: *Given a stationary path of monetary policy (i.e., $\mu_t = \mu$ for all t), the economy jumps to a unique and stable balanced growth path.*

Proof: See Appendix A.□

On the balanced growth path, equilibrium labor allocations are stationary. Here we sketch out the derivations, and the detailed derivations are relegated to Appendix A. From (5) and (18), we obtain the steady-state nominal interest rate $i = r + \pi = \rho + \mu$. Substituting $i = \rho + \mu$ into (4) and equating the resulting condition with (7) yield the equilibrium allocation of production labor l_y. Combining (3), (4) and (14) yields $\varphi v_{n,t} n_t = \psi / \lambda_t$. Differentiating the log of this expression with respect to time yields $\dot{n}_t / n_t = -\dot{v}_{n,t} / v_{n,t} - \dot{\lambda}_t / \lambda_t$, where $\dot{n}_t / n_t = \varphi l_{r,t}$ and $\dot{\lambda}_t / \lambda_t = \rho - r_t$ from (5). Furthermore, we can substitute (7), (11) and (14) into (12) to solve for $\dot{v}_{n,t} / v_{n,t}$. From this procedure, we obtain the equilibrium allocation of R&D labor l_r. Similarly, combining (3), (4) and (17) yields $\phi v_{k,t} k_t = \psi / \lambda_t$. Differentiating the log of this expression with respect to time yields $\dot{k}_t / k_t = -\dot{v}_{k,t} / v_{k,t} - \dot{\lambda}_t / \lambda_t$, where $\dot{k}_t / k_t = \phi l_{k,t}$. Then, we can substitute (7), (17) and

[22] In an earlier version of this paper, we consider a monetary version of the canonical Romer model in which R&D is the only engine of long-run growth and derive the transition path of the economy from a change in monetary policy; see Chu, Lai and Liao (2010).

$q_t k_t / y_t = \alpha / \eta$ into (15) to solve for $\dot{v}_{k,t} / v_{k,t}$. From this procedure, we obtain the equilibrium allocation of capital-producing labor l_k. Finally, we compute labor supply using $l = l_y + l_r + l_k$.

Lemma 2: *The equilibrium labor allocations are given by*

(19) $$l_y = \frac{1-\alpha}{\psi[1+\xi(\mu+\rho)]},$$

(20) $$l_r = \frac{\eta-1}{\eta}\left(\frac{\alpha}{\psi[1+\xi(\mu+\rho)]}\right) - \frac{\rho}{\varphi},$$

(21) $$l_k = \frac{1}{\eta}\left(\frac{\alpha}{\psi[1+\xi(\mu+\rho)]}\right) - \frac{\rho}{\phi},$$

(22) $$l = \frac{1}{\psi[1+\xi(\mu+\rho)]} - \rho\left(\frac{1}{\varphi} + \frac{1}{\phi}\right).$$

Proof: See Appendix A.□

3. Growth and welfare effects of monetary policy

Applying $x_t(j) = x_t$ and $k_t = x_t n_t$ on (6) yields

(23) $$y_t = k_t^{\alpha}(n_t l_{y,t})^{1-\alpha}.$$

On the balanced-growth path, the growth rate of output is

(24) $$g_y \equiv \frac{\dot{y}_t}{y_t} = \alpha\frac{\dot{k}_t}{k_t} + (1-\alpha)\frac{\dot{n}_t}{n_t} = \alpha\phi l_k + (1-\alpha)\varphi l_r,$$

where $g_k = \phi l_k$ is the balanced growth rate of capital and $g_n = \varphi l_r$ is the balanced growth rate of varieties. To ensure that these growth rates are non-negative, we impose the following parameter restrictions. Condition R ensures that $l_r \geq 0$, whereas Condition K ensures that $l_k \geq 0$.

Condition R: $$\varphi \geq \tilde{\varphi} \equiv \frac{\rho\psi[1+\xi(\mu+\rho)]}{\alpha(\eta-1)/\eta}$$

Condition K: $$\phi \geq \tilde{\phi} \equiv \frac{\rho\psi[1+\xi(\mu+\rho)]}{\alpha/\eta}$$

Equation (20) shows that l_r is increasing in η. Intuitively, larger patent breadth increases monopolistic profits and the value of an invention providing more incentives for R&D. Equation (20) also shows that l_r is decreasing in μ. Intuitively, a larger μ increases inflation, which in turn raises the cost of holding money that is required for purchasing consumption goods. As a result, households consume more leisure reducing the supply of labor l as shown in (22), and they also decrease consumption reducing production labor l_y as shown in (19). The decrease in production labor l_y reduces the amount of profits $n_t \omega_{x,t} = \alpha y_t (\eta-1)/\eta$, which in turn decreases the incentives for R&D consequently reducing R&D labor l_r and the variety growth rate $g_n = \varphi l_r$. To see this, substituting (19) into (20) yields $l_r = \frac{\eta-1}{\eta}\left(\frac{\alpha}{1-\alpha}\right) l_y - \frac{\rho}{\varphi}$, where l_y is decreasing in μ. Interestingly, larger patent breadth strengthens the negative effect of inflation on R&D and variety growth. To see this, $\frac{\partial l_r}{\partial \mu} = \frac{\eta-1}{\eta}\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial l_y}{\partial \mu} < 0$; thus, a larger η magnifies the negative effect of $\partial l_y / \partial \mu$ on $\partial l_r / \partial \mu$.

Proposition 1: *The variety growth rate g_n is increasing in patent breadth η and decreasing in the money growth rate μ. Increasing patent breadth strengthens the negative effect of monetary policy on the variety growth rate.*

Proof: Recall that $g_n = \varphi l_r$ and note (20).□

Equation (21) shows that l_k is decreasing in η. Intuitively, larger patent breadth reduces the capital share of income $q_t k_t / y_t = \alpha / \eta$ and the value of capital resulting into less incentive for capital accumulation. Equation (21) also shows that l_k is decreasing in μ because the decrease in production labor l_y reduces capital income, which in turn decreases the incentives for capital accumulation consequently reducing capital-producing labor l_k and the capital growth rate $g_k = \phi l_k$. To see this, substituting (19) into (21) yields $l_k = \frac{1}{\eta}\left(\frac{\alpha}{1-\alpha}\right) l_y - \frac{\rho}{\phi}$, where l_y is decreasing in μ. Interestingly, increasing patent breadth weakens the negative effect of inflation on capital-producing labor l_k and the capital growth rate $g_k = \phi l_k$. To see this, $\frac{\partial l_k}{\partial \mu} = \frac{1}{\eta}\left(\frac{\alpha}{1-\alpha}\right)\frac{\partial l_y}{\partial \mu} < 0$; thus, a larger η mitigates the negative effect of $\partial l_y / \partial \mu$ on $\partial l_k / \partial \mu$.

Proposition 2: *The capital growth rate g_k is decreasing in patent breadth η and the money growth rate μ. Increasing patent breadth weakens the negative effect of monetary policy on the capital growth rate.*

Proof: Recall that $g_k = \phi l_k$ and note (21).□

From Propositions 1 and 2, we can infer the effects of inflation on economic growth in the R&D-based growth model and the AK model.[23] As $\phi \to \tilde{\phi}$, the capital-producing sector shuts down and the model reduces to a monetary R&D-based growth model. As $\varphi \to \tilde{\varphi}$, the R&D sector shuts down and the model reduces to a monetary AK model. The advantage of this

[23] In an earlier version of this study, we provide this analysis in the two models; see Chu, Lai and Liao (2010).

unified model is that we can analyze the effects of inflation when R&D and capital accumulation are both engines of long-run growth. Substituting (20) and (21) into (24) yields

(25)
$$g_y = \left(\alpha\phi\frac{1}{\eta} + (1-\alpha)\varphi\frac{\eta-1}{\eta}\right)\frac{\alpha}{\psi[1+\xi(\mu+\rho)]} - \rho .$$

In this case, the effect of patent breadth η on economic growth g_y is ambiguous. Specifically, $\partial g_y / \partial\eta > 0$ if and only if $(1-\alpha)\varphi > \alpha\phi$. Intuitively, $(1-\alpha)\varphi$ captures the importance of R&D on economic growth whereas $\alpha\phi$ captures the importance of capital accumulation. In other words, if and only if the R&D channel dominates the capital-accumulation channel, then patent breadth would have a positive growth effect. As for the effect of money growth, it continues to be negative as before (i.e., $\partial g_y / \partial\mu < 0$). Finally, whether patent breadth strengthens the growth effect of monetary policy depends on the same parameter condition $(1-\alpha)\varphi > \alpha\phi$ as $\partial g_y / \partial\eta > 0$. In other words, if $\partial g_y / \partial\eta > 0$ ($\partial g_y / \partial\eta < 0$), then increasing patent breadth would strengthen (weaken) the negative effect of monetary policy on economic growth.

Proposition 3: *Economic growth g_y can be increasing or decreasing in patent breadth η. If and only if $(1-\alpha)\varphi > \alpha\phi$, then g_y would be increasing in η. Economic growth is always decreasing in the money growth rate μ. Increasing patent breadth may strengthen or weaken the effect of monetary policy on economic growth. If and only if $(1-\alpha)\varphi > \alpha\phi$, then increasing patent breadth would strengthen the negative effect of monetary policy on economic growth.*

Proof: Apply simple differentiation to (25).□

Finally, we analyze the effect of monetary policy on social welfare. Imposing balanced growth on (1) yields

(26) $$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_y}{\rho} - \psi l\right),$$

where $c_0 = y_0 = k_0^{\alpha}(n_0 l_y)^{1-\alpha}$ in which the initial n_0 and k_0 are exogenous. Dropping $\alpha \ln k_0$ and $(1-\alpha)\ln n_0$ from (26) yields $\rho U = (1-\alpha)\ln l_y + g_y / \rho - \psi l$. Proposition 4 shows that the effect of patent breadth on social welfare is ambiguous as in the case of economic growth. Specifically, $\partial U / \partial \eta > 0$ if and only if $(1-\alpha)\varphi > \alpha\phi$, which is also the necessary and sufficient condition for $\partial g_y / \partial \eta > 0$. As for the effect of money growth on welfare, it is negative (i.e., $\partial U / \partial \mu < 0$) because the negative effects of μ on c_0 and g_y dominate the positive effect of increased leisure. Finally, whether patent breadth strengthens the welfare effect of monetary policy depends on the same parameter condition $(1-\alpha)\varphi > \alpha\phi$ as $\partial U / \partial \eta > 0$. In other words, if $\partial U / \partial \eta > 0$ ($\partial U / \partial \eta < 0$), then increasing patent breadth would strengthen (weaken) the negative effect of monetary policy on social welfare.

Proposition 4: *Social welfare U can be increasing or decreasing in patent breadth η. If and only if $(1-\alpha)\varphi > \alpha\phi$, then U would be increasing in η. Social welfare is always decreasing in the money growth rate μ. Increasing patent breadth may strengthen or weaken the effect of monetary policy on social welfare. If and only if $(1-\alpha)\varphi > \alpha\phi$, then increasing patent breadth would strengthen the negative effect of monetary policy on social welfare.*

Proof: Substitute (19), (22) and (25) into (26). Then, apply simple differentiation and $g_y \geq 0$.□

4. Quantitative analysis

In this section, we calibrate the model to provide a numerical analysis on the growth and welfare effects of inflation and quantitatively examine whether the markup magnifies or mitigates these effects. We consider two monetary aggregates, currency and M1, as alternative measures of money held by households for the purpose of facilitating transactions. On the one hand, currency holding by households is a subset of monetary assets that are subject to the cost of inflation. On the other hand, M1 includes interest-bearing assets, such as demand deposits, which are partly immune to the depreciation effect of inflation. Therefore, we report the welfare cost of inflation computed based on currency as a lower bound and the welfare cost computed based on M1 as an upper bound. For the EA, we set the cash-in-advance parameter $\xi = m/c$ to 0.10 when we match the average ratio of currency to households' final consumption expenditure from 1999 to 2010, and we set ξ to 0.66 when we match the ratio of M1 to consumption. For the US, we set ξ to 0.08 when we match the ratio of currency to consumption and ξ to 0.16 when we match the ratio of M1 to consumption.

For each of the other parameters, we either set it to a conventional value or calibrate its value using an empirical moment based on data from 1999 to 2010. For the money growth rate, we calibrate it using $\mu = g_y + \pi$, where g_y and π are taken from the data (to be reported in Table 1). For the R&D and capital productivity parameters ϕ and φ, we calibrate them using the output and capital growth rates g_y and g_k from the data. Given g_y and g_k, the variety growth rate can be computed as $g_n = (g_y - \alpha g_k)/(1-\alpha)$. For the markup η, we calibrate it using R&D as a percentage of GDP. Then, we use capital investment as a percentage of GDP to calibrate the value of ρ, and we use the labor share of income to calibrate the value of α. Finally, we calibrate the labor supply parameter ψ by setting the supply of labor l to 0.33. In Table 1, we report the values of all these variables and parameters.

[Insert Table 1 about here]

4.1. Numerical results

The policy experiment that we consider is to reduce the money growth rate to the level that achieves the Friedman rule (i.e., $i = 0$ which implies $\mu = -\rho$). Table 2 shows that under both the currency and M1 specifications, reducing money growth increases economic growth and social welfare in both the EA and the US.[24] However, the changes in the capital growth rate and the variety growth rate respond differently to the markup. Specifically, a larger markup *increases* the magnitude of the changes in the variety growth rate in response to lower inflation. In contrast, a larger markup *decreases* the magnitude of the changes in the capital growth rate in response to lower inflation. Overall, we find that the R&D channel dominates the capital-accumulation channel such that a larger markup tends to increase the magnitude of the changes in economic growth and social welfare in response to lower inflation. In other words, a larger market power of firms tends to magnify the welfare cost of inflation in these economies.

[Insert Table 2 about here]

In the EA, the welfare cost of inflation under the M1 specification is 5.79% whereas the welfare cost under the currency specification is 0.88%. In the US, the welfare costs of inflation under the M1 and currency specifications are 1.76% and 0.89% respectively. Therefore, in both economies, the welfare cost of inflation is much higher under the M1 specification than under the currency specification. Furthermore, when we use currency as the measure of money, we find a

[24] The welfare changes are expressed in terms of equivalent variation in annual consumption.

negligible difference in the welfare cost of inflation between the EA and the US; however, when we use M1 as the measure of money, we find a substantial difference in the welfare cost due to the much higher money-consumption ratio in the EA than in the US.

5. Extension: variety expansion and capital accumulation with cross-sector spillovers

In this section, we consider an alternative specification for the laws of motion for variety n_t and capital k_t. The following specification allows for cross-sector spillovers captured by $s \in (0,0.5)$.

(27) $$\dot{n}_t = \varphi n_t^{1-s} k_t^s l_{r,t},$$

(28) $$\dot{k}_t = \phi k_t^{1-s} n_t^s l_{k,t}.$$

Given these new laws of motion, the zero-profit conditions in (14) and (17) become

(29) $$\varphi v_{n,t} n_t^{1-s} k_t^s = w_t,$$

(30) $$\phi v_{k,t} k_t^{1-s} n_t^s = w_t.$$

The rest of the model is the same as before. In Lemma 3, we first discuss the dynamic properties of n_t / k_t in this extended model.

Lemma 3: *Given a stationary path of monetary policy (i.e., $\mu_t = \mu$ for all t), the dynamics of n_t / k_t is characterized by global stability such that it gradually converges to a unique and stable steady state.*

Proof: See Appendix A.□

On the balanced growth path, the growth rates of variety and capital are respectively

(31)
$$g_n = \left(\frac{k_t}{n_t}\right)^s \varphi l_r ,$$

(32)
$$g_k = \left(\frac{n_t}{k_t}\right)^s \phi l_k .$$

Because n_t / k_t is constant, it must be the case that $g_n = g_k$, which implies a steady-state ratio of

(33)
$$\frac{n_t}{k_t} = \left(\frac{\varphi l_r}{\phi l_k}\right)^{\frac{1}{2s}} .$$

Although this extended model has a fundamentally different property that the growth rates of variety and capital are the same on the balanced growth path, we will show that our main results are robust to this extension.

Here we first derive the equilibrium labor allocations. Following the same derivations as before, one can show that the nominal interest rate is $i = \rho + \mu$ and the equilibrium allocation of production labor l_y continues to be given by (19). Then, combining (3), (4) and (29) yields $\varphi v_{n,t} n_t^{1-s} k_t^s = \psi / \lambda_t$. Differentiating the log of this expression with respect to time yields

(34)
$$(1-s)\frac{\dot{n}_t}{n_t} + s\frac{\dot{k}_t}{k_t} = -\frac{\dot{v}_{n,t}}{v_{n,t}} - \frac{\dot{\lambda}_t}{\lambda_t} ,$$

where $\dot{\lambda}_t / \lambda_t = \rho - r_t$ from (5). Also, we can substitute (7), (11) and (29) into (12) to solve for

(35)
$$\frac{\dot{v}_{n,t}}{v_{n,t}} = r_t - \frac{\omega_{x,t}}{v_{n,t}} = r_t - \left(\frac{\eta-1}{\eta}\right)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{k_t}{n_t}\right)^s \varphi l_{y,t} .$$

Substituting (35) into (34) and imposing the balanced-growth condition $\dot{n}_t / n_t = \dot{k}_t / k_t$ yield

(36)
$$l_r = \left(\frac{\eta-1}{\eta}\right)\left(\frac{\alpha}{1-\alpha}\right) l_y - \frac{\rho}{\varphi}\left(\frac{n_t}{k_t}\right)^s ,$$

where we have used $g_n = (k_t / n_t)^s \varphi l_r$. Equation (36) shows that for a given n_t / k_t, R&D labor l_r is increasing in η as before.

Similarly, combining (3), (4) and (30) yields $\phi v_{k,t} k_t^{1-s} n_t^s = \psi / \lambda_t$. Differentiating the log of this expression with respect to time yields

$$(1-s)\frac{\dot{k}_t}{k_t} + s\frac{\dot{n}_t}{n_t} = -\frac{\dot{v}_{k,t}}{v_{k,t}} - \frac{\dot{\lambda}_t}{\lambda_t}. \tag{37}$$

Then, we can substitute (7), (30) and $q_t k_t / y_t = \alpha / \eta$ into (15) to solve for

$$\frac{\dot{v}_{k,t}}{v_{k,t}} = r_t - \frac{q_t}{v_{k,t}} = r_t - \frac{1}{\eta}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{n_t}{k_t}\right)^s \phi l_{y,t}. \tag{38}$$

Substituting (38) into (37) and imposing the balanced-growth condition $\dot{n}_t / n_t = \dot{k}_t / k_t$ yield

$$l_k = \frac{1}{\eta}\left(\frac{\alpha}{1-\alpha}\right) l_y - \frac{\rho}{\phi}\left(\frac{k_t}{n_t}\right)^s, \tag{39}$$

where we have used $g_k = (n_t / k_t)^s \phi l_k$. Equation (39) shows that for a given n_t / k_t, capital-producing labor l_k is decreasing in η. Therefore, this model also features the tradeoff of patent breadth on capital and R&D, and this tradeoff will be reflected in the common growth rate being a non-monotonic function in η.

Combining (19), (33), (36) and (39) yield the equilibrium allocations of l_r and l_k respectively given by

$$l_r = \frac{\eta-1}{\eta}\left(\frac{\alpha}{\psi[1+\xi(\mu+\rho)]}\right) - \rho\sqrt{\frac{\eta-1}{\varphi\phi}}, \tag{40}$$

$$l_k = \frac{1}{\eta}\left(\frac{\alpha}{\psi[1+\xi(\mu+\rho)]}\right) - \frac{\rho}{\sqrt{\varphi\phi(\eta-1)}}. \tag{41}$$

Finally, we can compute the supply of labor given by

(42) $$l = l_y + l_r + l_k = \frac{1}{\psi[1+\xi(\mu+\rho)]} - \rho\left(\frac{\eta}{\sqrt{\varphi\phi(\eta-1)}}\right).$$

Substituting (33), (40) and (41) into (31) or (32) yields

(43) $$g_n = g_k = \frac{\sqrt{\eta-1}}{\eta}\left(\frac{\alpha\sqrt{\varphi\phi}}{\psi[1+\xi(\mu+\rho)]}\right) - \rho,$$

which is also the growth rate of output because $g_y = g_n = g_k$ in this model. Equation (43) shows that the growth rate is an inverted-U function in patent breadth η due to the tradeoff between R&D and capital investment as in Iwaisako and Futagami (2012) and the growth rate reaches a maximum at $\eta = \bar{\eta} \equiv 2$. Also, the growth rate is always decreasing in the money growth rate μ as before. Whether increasing patent breadth strengthens or weakens this negative effect of monetary policy on economic growth depends on whether η is on the upward-sloping side (i.e., the R&D channel dominates) or the downward-sloping side (i.e., the capital-accumulation channel dominates) of the curve. Given that the empirical markup is often estimated to be less than 100% (i.e., $\eta < 2$),[25] the R&D channel is likely to dominate the capital-accumulation channel. Finally, one can also show that on the balanced growth path, welfare is monotonically decreasing in the money growth rate μ, and increasing patent breadth strengthens (weakens) this negative effect of monetary policy on welfare if η is less (greater) than $\bar{\eta}$.[26]

6. Conclusion

In this study, we have revisited a fundamental question in monetary economics originally raised by Tobin (1965) on the relationship between inflation and economic growth. The key departure from the literature is that we consider both innovation and capital accumulation as engines of

[25] See for example Jones and Williams (2000) for a discussion on the empirical range of the markup.
[26] Derivations are contained in an unpublished appendix (please see Appendix B) that is available upon request.

economic growth in the long run. In summary, we find that the growth and welfare effects of inflation and monetary policy are largely influenced by an unexplored interaction between the growth engine and the strength of patent protection. We believe that this interaction sheds some light on the importance of the growth engine and an interaction between monetary and patent policies that have been neglected in the growth-inflation literature.

Finally, it is well-known that the Romer model exhibits scale effects.[27] In this study, we normalize the size of population to unity, so that population size does not appear in the equilibrium growth rate. Instead, it is the supply of labor that affects growth; in other words, when R&D scientists and engineers devote more time to research, they generate more inventions. We believe that this implication is more plausible than the original version of scale effects based on population size. Nevertheless, it may be fruitful for future studies to further revisit the growth and welfare effects of monetary policy using other vintages of the R&D-based growth model.

[27] See Jones (1999) for an excellent discussion on scale effects in R&D-based growth models.

References

Chu, A., 2010. Effects of patent length on R&D: A quantitative DGE analysis. *Journal of Economics*, 99, 117-140.

Chu, A., Cozzi, G., and Galli, S., 2012. Does intellectual monopoly stimulate or stifle innovation? *European Economic Review*, forthcoming.

Chu, A., and Lai, C., 2012. Money and the welfare cost of inflation in an R&D growth model. *Journal of Money, Credit and Banking*, forthcoming.

Chu, A., Lai, C., and Liao, C., 2010. A tale of two growth engines: Interactive effects of monetary policy and intellectual property rights. *MPRA* Paper No. 30105.

Chu, A., Lai, C., and Liao, C., 2012. Search and endogenous growth: When Romer meets Lagos and Wright. *MPRA* Paper No. 36691.

Cysne, R., and Turchick, D., 2012. Intellectual property rights protection and endogenous economic growth revisited. *Journal of Economic Dynamics and Control*, forthcoming.

Dotsey, M., and Ireland, P., 1996. The welfare cost of inflation in general equilibrium. *Journal of Monetary Economics*, 37, 29-47.

Economic Development Indicators, 2005. Growth through innovation: Sustainable economic growth for all New Zealanders. Ministry of Economic Development and the Treasury.

Evans, L., Quigley, N., and Zhang, J., 2003. Optimal price regulation in a growth model with monopolistic suppliers of intermediate goods. *Canadian Journal of Economics*, 36, 463-474.

Funk, P., and Kromen, B., 2006. Short-term price rigidity in an endogenous growth model: Non-superneutrality and a non-vertical long-term Phillips-curve. *University of Cologne* Working Paper No. 29.

Funk, P., and Kromen, B., 2010. Inflation and innovation-driven growth. *The B.E. Journal of Macroeconomics* (Topics), vol. 10, article 23.

Furukawa, Y., 2007. The protection of intellectual property rights and endogenous growth: Is stronger always better? *Journal of Economic Dynamics and Control*, 31, 3644-3670.

Futagami, K., and Iwaisako, T., 2007. Dynamic analysis of patent policy in an endogenous growth model. *Journal of Economic Theory*, 132, 306-334.

Gilbert, R., and Shapiro, C., 1990. Optimal patent length and breadth. *RAND Journal of Economics*, 21, 106-112.

Gillman, M., and Kejak, M., 2005. Contrasting models of the effect of inflation on growth. *Journal of Economic Surveys*, 19, 113-136.

Goel, R., and Ram, R., 2001. Irreversibility of R&D investment and the adverse effect of uncertainty: Evidence from the OECD countries. *Economics Letters*, 71, 287-291.

Goh, A.-T., and Olivier, J., 2002. Optimal patent protection in a two-sector economy. *International Economic Review*, 43, 1191–1214.

Gomme, P., 1993. Money and growth revisited: Measuring the costs of inflation in an endogenous growth model. *Journal of Monetary Economics*, 32, 51-77.

Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies*, 58, 43-61.

Ho, W., Zeng, J., and Zhang, J., 2007. Inflation taxation and welfare with externalities and leisure. *Journal of Money, Credit and Banking*, 39, 105-131.

Itaya, J., and Mino, K., 2003. Inflation, transaction costs and indeterminacy in monetary economies with endogenous growth. *Economica*, 70, 451-470.

Itaya, J., and Mino, K., 2007. Technology, preference structure, and the growth effect of money supply. *Macroeconomic Dynamics*, 11, 589-612.

Iwaisako, T., and Futagami, K., 2003. Patent policy in an endogenous growth model. *Journal of Economics*, 78, 239-258.

Iwaisako, T., and Futagami, K., 2012. Patent protection, capital accumulation, and economic growth. *Economic Theory*, forthcoming.

Jha, S., Wang, P., and Yip, C., 2002. Dynamics in a transaction-based monetary growth model. *Journal of Economic Dynamics and Control*, 26, 611-635.

Jones, C., 1999. Growth: With or without scale effects. *American Economic Review*, 89, 139-144.

Jones, C., and Williams, J., 2000. Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth*, 5, 65-85.

Judd, K., 1985. On the performance of patents. *Econometrica*, 53, 567-585.

Kwan, Y., and Lai, E., 2003. Intellectual property rights protection and endogenous economic growth. *Journal of Economic Dynamics and Control*, 27, 853-873.

Li, C.-W., 1998. Stochastic variety innovation in a growth model. *University of Glasgow* Working Paper No. 9818.

Li, C.-W., 2001. On the policy implications of endogenous technological progress. *Economic Journal*, 111, C164-C179.

Lucas, R., 1988. On the mechanics of economic development. *Journal of Monetary Economics*, 22, 3-42.

Mansfield, E., 1980. Research and development, productivity, and inflation." *Science*, Sept., 1091-1093.

Marquis, M., and Reffett, K., 1994. New technology spillovers into the payment system. *Economic Journal*, 104, 1123-1138.

Mino, K., 1997. Long-run effects of monetary expansion in a two-sector model of endogenous growth. *Journal of Macroeconomics*, 19, 635-655.

Palokangas, T., 2011. Optimal patent length and breadth in an economy with creative destruction and non-diversifiable risk. *Journal of Economics*, 102, 1-27.

Park, W., 2008. International patent protection: 1960-2005. *Research Policy*, 37, 761-766.

Romer, P., 1990. Endogenous technological change. *Journal of Political Economy*, 98, S71-S102.

Stockman, A., 1981. Anticipated inflation and the capital stock in a cash-in-advance economy. *Journal of Monetary Economics*, 8, 387-93.

Tobin, J., 1965. Money and economic growth. *Econometrica*, 33, 671-684.

Vaona, A., 2012. Inflation and growth in the long run: A new Keynesian theory and further semiparametric evidence. *Macroeconomic Dynamics*, 16, 94-132.

Wang, P., and Yip, C., 1992. Alternative approaches to money and growth. *Journal of Money, Credit and Banking*, 24, 553-62.

Wu, Y., and Zhang, J., 1998. Endogenous growth and the welfare costs of inflation: A reconsideration. *Journal of Economic Dynamics and Control*, 22, 465-482.

Zhang, J., 1996. A simple pecuniary model of money and growth with transactions cost. *Journal of Macroeconomics*, 18, 127-137.

Appendix A

Proof of Lemma 1: In this proof, we first show that i_t is stationary given a constant μ. Using (5) and (18), we have

(A1) $$i_t = r_t + \pi_t = \rho - \frac{\dot{\lambda}_t}{\lambda_t} + \mu - \frac{\dot{m}_t}{m_t},$$

where $\dot{m}_t / m_t = \dot{c}_t / c_t$ because $m_t = \xi c_t$. Taking the log of (3) and differentiating with time yields

(A2) $$-\frac{\dot{\lambda}_t}{\lambda_t} - \frac{\dot{c}_t}{c_t} = \frac{\xi \dot{i}_t}{1+\xi i_t}.$$

Substituting (A2) into (A1) and then rearranging terms yield

(A3) $$\dot{i}_t = \left(\frac{1}{\xi} + i_t\right)(i_t - \rho - \mu).$$

Given the saddle-point stability of this dynamic system, i_t jumps to its steady state $i_t = \rho + \mu$.

In the rest of this proof, we show that given a constant i_t, equilibrium labor allocations are also stationary. Combining (4) and (7) yields

(A4) $$l_{y,t} = \frac{1-\alpha}{\psi(1+\xi i_t)},$$

where we have applied the resource constraint for final goods $c_t = y_t$. Equation (A4) shows that $l_{y,t}$ must be stationary given a constant i_t. Combining (3), (4) and (14) yields.

(A5) $$\varphi v_{n,t} n_t = \psi / \lambda_t.$$

Differentiating the log of (A5) with respect to time yields

(A6) $$\frac{\dot{n}_t}{n_t} = -\frac{\dot{v}_{n,t}}{v_{n,t}} - \frac{\dot{\lambda}_t}{\lambda_t}.$$

Substituting (6), (7), (8), (11) and (14) into (12), we obtain

(A7) $$\frac{\dot{v}_{n,t}}{v_{n,t}} = r_t - \frac{\omega_{x,t}}{v_{n,t}} = r_t - \varphi\left(\frac{\eta-1}{\eta}\right)\left(\frac{\alpha}{1-\alpha}\right)l_{y,t}.$$

Substituting (A7), (5) and $\dot{n}_t / n_t = \varphi l_{r,t}$ into (A6) yields

(A8) $$l_{r,t} = \frac{\eta-1}{\eta}\left(\frac{\alpha}{1-\alpha}\right)l_{y,t} - \frac{\rho}{\varphi},$$

which shows that $l_{r,t}$ is stationary given a constant $l_{y,t}$. Combining (3), (4) and (17) yields

(A9) $$\phi v_{k,t} k_t = \psi / \lambda_t .$$

Differentiating the log of (A9) with respect to time yields

(A10) $$\frac{\dot{k}_t}{k_t} = -\frac{\dot{v}_{k,t}}{v_{k,t}} - \frac{\dot{\lambda}_t}{\lambda_t}.$$

Substituting (6), (7), (8), (10) and (17) into (15), we obtain

(A11) $$\frac{\dot{v}_{k,t}}{v_{k,t}} = r_t - \frac{q_t}{v_{k,t}} = r_t - \frac{\phi}{\eta}\left(\frac{\alpha}{1-\alpha}\right)l_{y,t}.$$

Substituting (A11), (5) and $\dot{k}_t / k_t = \phi l_{k,t}$ into (A10) yields

(A12) $$l_{k,t} = \frac{1}{\eta}\left(\frac{\alpha}{1-\alpha}\right)l_{y,t} - \frac{\rho}{\phi},$$

which shows that $l_{k,t}$ is stationary given a constant $l_{y,t}$. Finally, the labor resource constraint is

(A13) $$l_t = l_{y,t} + l_{r,t} + l_{k,t},$$

which shows that l_t must be stationary given constant $l_{y,t}$, $l_{r,t}$ and $l_{k,t}$.□

Proof of Lemma 2: Substituting $i_t = \rho + \mu$ into (A4) yields (19). Substituting (19) into (A8) and (A12) yields (20) and (21). Finally, substituting (19), (20) and (21) into (A13) yields (22).□

Proof of Lemma 3: Following the same derivations as in the proof of Lemma 1, one can show that i_t has the same dynamics as in (A3) that is characterized by saddle-point stability such that it simply jumps to its steady state given by $i_t = \rho + \mu$. As a result, (A4) implies that $l_{y,t}$ also jumps to its steady state given by $l_{y,t} = (1-\alpha)/[\psi(1+\xi i_t)]$.

In the rest of this proof, we show that the dynamics of a transformed variable $z_t \equiv n_t / k_t$ is characterized by global stability such that z_t being a state variable gradually converges to a steady-state value. Taking the difference between (34) and (37) yields

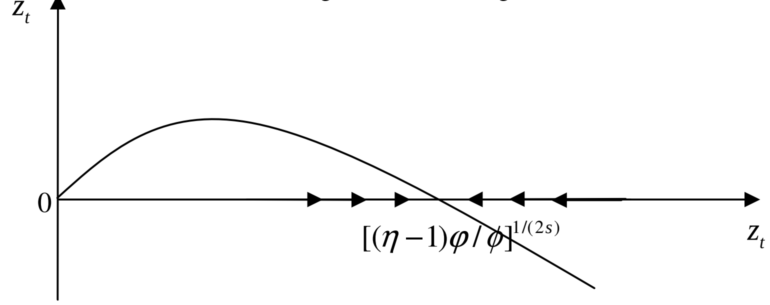
(A14) $$\frac{\dot{z}_t}{z_t} = \frac{\dot{n}_t}{n_t} - \frac{\dot{k}_t}{k_t} = \frac{1}{1-2s}\left(\frac{\dot{v}_{k,t}}{v_{k,t}} - \frac{\dot{v}_{n,t}}{v_{n,t}}\right),$$

where $1-2s>0$. Substituting (35) and (38) into (A14) yields

(A15) $$\dot{z}_t = \frac{z_t}{1-2s}\left(\frac{\alpha}{1-\alpha}\right)\left((\eta-1)\frac{\varphi}{z_t^s} - z_t^s\phi\right)\frac{l_y}{\eta},$$

where l_y is constant along the transition path of z_t. The interior steady-state value of z_t is given by $z_t = [(\eta-1)\varphi/\phi]^{1/(2s)}$. Figure 1 plots the dynamics of z_t characterized by global stability.□

Figure 1: Phase diagram



Appendix B (not for publication)

In this unpublished appendix, we provide a proof for the following results in the extended model of Section 5. On the balanced growth path, welfare is monotonically decreasing in the money growth rate μ, and patent breadth strengthens (weakens) this negative effect of monetary policy on welfare if η is less (greater) than $\overline{\eta} \equiv 2$.

From (26), we can obtain the balanced-growth level of social welfare given by

(B1) $$U = \frac{1}{\rho}\left(\alpha \ln k_0 + (1-\alpha)\ln n_0 + (1-\alpha)\ln l_y + \frac{g_y}{\rho} - \psi l\right).$$

From the proof of Lemma 3 in Appendix A, the steady-state ratio of $n_0 / k_0 = [(\eta - 1)\varphi / \phi]^{1/(2s)}$ is independent of μ. From (19), (42) and (43), we can show that the effects of monetary policy on production labor, labor supply and economic growth are respectively

(B2) $$\frac{\partial l_y}{\partial \mu} = -\frac{\xi(1-\alpha)}{\psi[1+\xi(\mu+\rho)]^2} < 0 ,$$

(B3) $$\frac{\partial l}{\partial \mu} = -\frac{\xi}{\psi[1+\xi(\mu+\rho)]^2} < 0 ,$$

(B4) $$\frac{\partial g_y}{\partial \mu} = -\frac{\sqrt{\eta-1}}{\eta}\left(\frac{\alpha\xi\sqrt{\varphi\phi}}{\psi[1+\xi(\mu+\rho)]^2}\right) < 0 .$$

Equations (B2)-(B4) show that an increase in the monetary target μ reduces production labor, labor supply and the growth rate. Overall, the welfare effect of monetary policy is

(B5) $$\rho\frac{\partial U}{\partial \mu} = \frac{1-\alpha}{l_y}\frac{\partial l_y}{\partial \mu} + \frac{1}{\rho}\frac{\partial g_y}{\partial \mu} - \psi\frac{\partial l}{\partial \mu}.$$

Substituting (B2)-(B4) into (B5) and then applying g_y from (43), (B5) becomes

(B6)
$$\rho\frac{\partial U}{\partial \mu}=-\frac{\xi}{[1+\xi(\mu+\rho)]}\underbrace{\left((1-\alpha)+\frac{g_y}{\rho}+\frac{\xi(\mu+\rho)}{[1+\xi(\mu+\rho)]}\right)}_{+}<0.$$

Equation (B6) shows that welfare is monotonically decreasing in the money growth rate μ.

Finally, taking the absolute value of (B6) and differentiating it with respect to η yields

(B7)
$$\frac{\partial}{\partial \eta}\left|\rho\frac{\partial U}{\partial \mu}\right|=\frac{\xi}{\rho[1+\xi(\mu+\rho)]}\frac{\partial g_y}{\partial \eta}=\left(\frac{\xi\alpha\sqrt{\varphi\phi/(\eta-1)}}{\rho\psi\eta^2[1+\xi(\mu+\rho)]^2}\right)\frac{2-\eta}{2}\gtrless 0;\text{ if } \quad \eta \lessgtr 2,$$

where we have used

(B8)
$$\frac{\partial g_y}{\partial \eta}=\left(\frac{\alpha\sqrt{\varphi\phi/(\eta-1)}}{\psi\eta^2[1+\xi(\mu+\rho)]}\right)\frac{2-\eta}{2}\gtrless 0;\text{ if } \quad \eta \lessgtr 2.$$

Therefore, patent breadth strengthens (weakens) the negative effect of monetary policy on social welfare if η is less (greater) than $\overline{\eta}\equiv 2$.□

Table 1a: Values of the variables and parameters in the EA

Variables	π	g_y	g_k	g_n	*R&D/GDP*	*I/GDP*	*wl/GDP*	*m/c*
Currency specification	0.0198	0.0153	0.0296	0.0052	0.0190	0.1823	0.6685	0.1042
M1 specification	0.0198	0.0153	0.0296	0.0052	0.0190	0.1823	0.6686	0.6609
Parameters	ρ	α	μ	ϕ	φ	η	ψ	ξ
Currency specification	0.0132	0.4150	0.0351	0.3289	0.5495	1.2569	2.5236	0.1042
M1 specification	0.0132	0.4150	0.0351	0.3289	0.5495	1.2569	2.4578	0.6609

Source: All data is obtained from Statistical Data Warehouse (ECB) except for (a) the data on R&D share of GDP and capital investment share of GDP sourced from Eurostat (European Commission), and (b) the data on labor compensation *wl* sourced from National Accounts of OECD Countries. The data set contains observations from 1999 to 2010. However, for labor compensation that does not have a complete data series from 1999 to 2010, we use available observations from 2003 to 2009.

Table 1b: Values of the variables and parameters in the US

Variables	π	g_y	g_k	g_n	*R&D/GDP*	*I/GDP*	*wl/GDP*	*m/c*
Currency specification	0.0246	0.0206	0.0307	0.0143	0.0268	0.1554	0.6871	0.0805
M1 specification	0.0246	0.0206	0.0307	0.0143	0.0268	0.1554	0.6871	0.1587
Parameters	ρ	α	μ	ϕ	φ	η	ψ	ξ
Currency specification	0.0189	0.3826	0.0452	0.4113	1.1142	1.2473	2.5329	0.0805
M1 specification	0.0189	0.3826	0.0452	0.4113	1.1142	1.2473	2.5204	0.1587

Source: All data is obtained from the Federal Reserve Economic Data database except for (a) the data on capital stock sourced from Bureau of Economic Analysis, (b) the data on R&D share of GDP sourced from the OECD database, and (c) the data on labor compensation sourced from National Accounts of OECD Countries. The data set contains observations from 1999 to 2010. However, for labor compensation and R&D share of GDP that do not have a complete data series from 1999 to 2010, we use available observations from 2003 to 2009 and from 1999 to 2009 respectively.

Table 2a: Growth and welfare effects of reducing μ to $-\rho$ in the EA

	η = 1.197	η = 1.217	η = 1.237	**η = 1.257**	η = 1.277	η = 1.297	η = 1.317
Currency specification							
Δg_n	0.0074%	0.0081%	0.0087%	**0.0093%**	0.0098%	0.0104%	0.0109%
Δg_k	0.0226%	0.0223%	0.0219%	**0.0216%**	0.0212%	0.0209%	0.0206%
Δg_y	0.0138%	0.0140%	0.0142%	**0.0144%**	0.0146%	0.0147%	0.0149%
ΔU	0.8364%	0.8524%	0.8678%	**0.8828%**	0.8973%	0.9113%	0.9249%
M1 specification							
Δg_n	0.0472%	0.0512%	0.0550%	**0.0587%**	0.0623%	0.0657%	0.0691%
Δg_k	0.1436%	0.1413%	0.1390%	**0.1368%**	0.1346%	0.1326%	0.1305%
Δg_y	0.0872%	0.0886%	0.0899%	**0.0911%**	0.0923%	0.0935%	0.0946%
ΔU	5.4867%	5.5926%	5.6952%	**5.7946%**	5.8909%	5.9844%	6.0752%

Table 2b: Growth and welfare effects of reducing μ to $-\rho$ in the US

	η = 1.187	η = 1.207	η = 1.227	**η = 1.247**	η = 1.267	η = 1.287	η = 1.307
Currency specification							
Δg_n	0.0136%	0.0148%	0.0160%	**0.0171%**	0.0182%	0.0193%	0.0203%
Δg_k	0.0268%	0.0264%	0.0260%	**0.0256%**	0.0252%	0.0248%	0.0244%
Δg_y	0.0187%	0.0193%	0.0198%	**0.0203%**	0.0209%	0.0214%	0.0219%
ΔU	0.7983%	0.8290%	0.8587%	**0.8874%**	0.9152%	0.9422%	0.9684%
M1 specification							
Δg_n	0.0269%	0.0292%	0.0315%	**0.0338%**	0.0359%	0.0380%	0.0400%
Δg_k	0.0529%	0.0521%	0.0512%	**0.0504%**	0.0496%	0.0488%	0.0481%
Δg_y	0.0368%	0.0380%	0.0391%	**0.0401%**	0.0411%	0.0421%	0.0431%
ΔU	1.5835%	1.6444%	1.7034%	**1.7605%**	1.8159%	1.8695%	1.9216%