A tale of two growth engines: Interactive effects of monetary policy and intellectual property rights

Angus C. Chu and Ching-Chong Lai and Chih-Hsing Liao

Durham University, Academia Sinica, National Chengchi University

October 2010

Online at https://mpra.ub.uni-muenchen.de/40372/
A Tale of Two Growth Engines:
Interactive Effects of Monetary Policy and Intellectual Property Rights

Angus C. Chu\textsuperscript{a}, Ching-Chong Lai\textsuperscript{b}, and Chih-Hsing Liao\textsuperscript{c}

Revised Version: May 2012

Abstract

How do intellectual property rights that determine the market power of firms influence the growth and welfare effects of monetary policy? To analyze this question, we develop a monetary endogenous-growth model in which R&D and capital accumulation are both engines of long-run economic growth. We find that monetary expansion hurts economic growth and social welfare by reducing R&D and capital accumulation. Furthermore, a larger market power of firms strengthens these growth and welfare effects of monetary policy through the R&D channel but weakens these effects through the capital-accumulation channel. Therefore, whether the market power of firms amplifies or mitigates the welfare cost of inflation depends on the relative importance of the two growth engines. Finally, we calibrate the model using data in the United States and the Euro Area to quantitatively evaluate and compare the welfare cost of inflation in these two economies and find that the R&D channel dominates in both economies.

Keywords: economic growth, R&D, inflation, monetary policy, patent policy

JEL classification: O30, O40, E41

\textsuperscript{a} Durham Business School, Durham University, Durham, UK. E-mail: angusccc@gmail.com.
\textsuperscript{b} Corresponding author: Institute of Economics, Academia Sinica, Taipei, Taiwan. E-mail: cclai@econ.sinica.edu.tw.
The authors would like to thank Juin-Jen Chang, Chi-Ting Chin, Fu-Sheng Hung, Yiting Li, Ming-Fu Shaw, and two anonymous referees for their insightful comments. The usual disclaimer applies.
\textsuperscript{c} Department of Economics, National Chengchi University, Taipei, Taiwan. Email: 93258507@nccu.edu.tw.
1. Introduction

Since the seminal study by Tobin (1965), the relationship between inflation and economic growth has been a fundamental issue in monetary economics, and there is now an established literature on monetary policy and economic growth.¹ The present study relates to this literature by analyzing an unexplored implication that the market structure has an influence on the growth and welfare effects of monetary policy. Specifically, we analyze how intellectual property rights that determine the market power of firms influence the effects of monetary policy on economic growth and social welfare in a monetary endogenous-growth model in which R&D and capital accumulation are both engines of long-run economic growth. We find that monetary expansion that increases inflation raises the cost of consumption relative to leisure consequently reducing labor supply, which is an important factor input for R&D and capital accumulation. A reduction in this factor input in turn decreases economic growth and social welfare. Interestingly, the magnitude of these growth and welfare effects of monetary policy depends on the strength of patent protection. Specifically, a larger market power of firms strengthens the effects of monetary policy through the R&D channel but weakens these effects through the capital-accumulation channel. Thus, the market power of firms has drastically different implications on the welfare cost of inflation under the two growth engines. Whether it amplifies or mitigates the welfare cost of inflation depends on the relative importance of the two growth engines.

The above theoretical finding has an important implication on a recent policy reform. As a result of the World Trade Organization’s Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS),² many countries have strengthened their protection for intellectual property rights. For example, according to the Ginarte-Park index of patent rights in Park (2008),

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¹ See for example Gillman and Kejak (2005) for a survey of this literature.
² The WTO’s TRIPS Agreement, which was initiated in the 1986-94 Uruguay Round, establishes a minimum level of intellectual property protection that must be provided by all member countries by 2006.
107 countries have experienced an increase in the strength of patent rights from 1995 to 2005.\textsuperscript{3} In these 107 countries, the average increase in the Ginarte-Park index is 0.82.\textsuperscript{4} Our theoretical result implies that the welfare cost of inflation would have increased in some of these countries. Given that innovation is likely to be the main engine of economic growth in developed countries, these countries would experience a larger welfare cost of inflation as a result of stronger patent protection. In contrast, for a developing country in which the main engine of growth is capital accumulation, our result implies that it should experience a smaller welfare cost of inflation as a result of stronger patent protection.

The reason why the strength of patent protection has different implications on the growth and welfare effects of monetary policy under the two growth engines is as follows. For a given supply of labor, increasing the market power of firms raises the incentives for innovation and the share of labor devoted to R&D. This increase in the R&D share of labor tends to \textit{magnify} the growth and welfare effects of the decrease in labor supply driven by monetary expansion. In contrast, increasing the market power of firms reduces the income share of physical capital and the share of labor devoted to capital accumulation. This decrease in the capital share of labor tends to \textit{mitigate} the growth and welfare effects of the decrease in labor supply driven by monetary expansion. Therefore, the market power of firms has drastically different implications on the growth and welfare effects of monetary policy as the relative importance of the two growth engines changes. In other words, the effects of monetary policy are influenced by an interaction between the growth engine and the market power of firms. To our knowledge, this interaction has never been explored in the literature.

\textsuperscript{3} There are a total of 122 countries in the Ginarte-Park index. Of these 122 countries, 119 countries have available measure of patent rights from 1995 to 2005, and only one country, Iraq, has experienced a reduction in the strength of patent rights during this period.

\textsuperscript{4} The index is a scale of 0 to 5, and a larger number indicates stronger patent rights. See Park (2008) for details.
In the quantitative analysis, we calibrate the model using data in the United States (US) and the Euro Area (EA) to quantitatively evaluate and compare the welfare cost of inflation in these two economies. We consider currency and M1 as alternative measures of money. In both economies, we find that the welfare cost of inflation is much higher under the M1 specification than under the currency specification as in Dotsey and Ireland (1996). We also find a significant difference in the welfare cost of inflation between the EA and the US when we use M1 as the measure of money but a negligible difference between the two economies when we consider currency as the measure of money. Under both money specifications, we find that increasing the markup magnifies the effects of inflation on economic growth and social welfare in both the US and the EA; in other words, the R&D channel dominates the capital-accumulation channel.

1.1. Literature review

Tobin (1965) argues that higher inflation stimulates the accumulation of physical capital via the substitution with money holding. In contrast to Tobin (1965), when money is required for purchasing capital goods (Stockman, 1981), higher anticipated inflation reduces real balances, capital investment and the level of output (i.e., the reversed Tobin effect). This theoretical result is also consistent with many subsequent studies in the literature that consider variants of the AK model with a cash-in-advance constraint on consumption goods and analyze the growth and welfare effects of inflation through elastic labor supply. For example, Gomme (1993) and Mino (1997) introduce money into the two-sector Lucas (1988) model via cash-in-advance constraints and emphasize how the money growth rate affects the consumption-leisure decision.5 Our result

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5 In a recent study, Itaya and Mino (2007) use an endogenous-growth model with a cash-in-advance constraint to show an interesting result that the growth effect of money supply depends on the preference structure and production technology. Specifically, if the production technology exhibits strong non-convexity or if the utility function has a
of a negative effect of inflation on economic growth is driven by a similar mechanism as these studies. Another branch of studies, such as Zhang (1996) and Jha et al. (2002), highlights the role of money in facilitating transactions for which a change in the inflation rate affects the consumption-leisure decision through transaction costs. These studies in general support the negative relationship between inflation and economic growth regardless of whether the model is based on a cash-in-advance constraint or transaction costs. In the present study, we explore a related growth-inflation relationship but introduce an additional growth engine that is R&D-driven innovation. Specifically, we incorporate a cash-in-advance constraint on consumption goods into a unified endogenous-growth model in which R&D and capital accumulation are both engines of long-run growth and allow for elastic labor supply.

In contrast to the well-established literature on monetary policy in the AK model, a small but growing number of studies, such as Marquis and Reffett (1994), Funk and Kromen (2006, 2010) and Chu and Lai (2012), has analyzed the effects of monetary policy on economic growth in the R&D-based growth model. The seminal study by Marquis and Reffett (1994) incorporates a transaction-service sector along with a cash-in-advance constraint into the Romer model. They show that higher inflation reduces growth through a reallocation of factor inputs from R&D and production to transaction services. Our model features a different mechanism from the Marquis-Reffett model by having a negative effect of inflation on economic growth through a reduction in labor supply. Chu and Lai (2012) incorporate money demand into a quality-ladder model similar to Grossman and Helpman (1991) with a money-in-utility specification and analyze how the high elasticity of intertemporal substitution, then there may be multiple balanced-growth paths that feature different growth effects of inflation.

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6 A recent study by Chu, Lai and Liao (2012) analyzes the effects of inflation on economic growth in the Lagos-Wright search model with AK endogenous growth and also finds a negative effect of inflation on growth.

7 As for monetary growth models with money in utility, see for example Wang and Yip (1992) and Ho et al. (2007).

8 In contrast, Itaya and Mino (2003) show that the Tobin effect (i.e., a positive growth effect of inflation) may emerge in an endogenous-growth model with transaction costs when labor externalities are sufficiently large.
elasticity of substitution between consumption and the real money balance affects the growth and welfare effects of inflation.\textsuperscript{9} Funk and Kromen (2006, 2010) incorporate nominal price rigidity into a quality-ladder model to quantitatively evaluate the effects of inflation on economic growth,\textsuperscript{10} and they analyze an interesting channel through which nominal price rigidity transmits the effects of inflation from the short run to the long run. The present paper differs from the abovementioned studies by (a) considering a unified endogenous-growth model in which both R&D and capital accumulation are engines of growth, (b) showing the different implications of firms’ market power on the effects of monetary policy on R&D and capital investment, and (c) comparing the welfare cost of inflation between the US and the EA.

In an early study, Mansfield (1980) points out that higher inflation may reduce R&D by decreasing investment in the plant and equipment that are necessary for R&D and by increasing uncertainty on relative prices. Goel and Ram (2001) provide empirical evidence to confirm the latter effect by showing that inflation uncertainty has a negative effect on R&D. A recent study by Chu and Lai (2012) provides further empirical evidence that supports a negative relationship between R&D and the level of inflation using cross-country regressions. In addition to empirical studies, policy-oriented research also suggests that high inflation could potentially reduce R&D investment. For example, in Economic Development Indicators (chapter 8, 2005), “… high and volatile inflation also discourages investment, including human capital and R&D investment.”

This study also relates to the literature on patent policy and economic growth. The seminal study in this literature is Judd (1985), who analyzes the effects of patent length on economic growth in a dynamic general-equilibrium framework; see also Iwaisako and Futagami

\textsuperscript{9} In the monetary quality-ladder model in Chu and Lai (2012), a larger markup would also strengthen the effects of monetary policy through the R&D channel; however, their model does not feature the capital-accumulation channel.\textsuperscript{10} Vaona (2012) incorporates nominal rigidity into an AK-style model with learning by doing to analyze the growth effects of inflation, and he provides empirical evidence that shows a negative effect of inflation on economic growth.
(2003) and Futagami and Iwaisako (2007). Instead of patent length, we consider patent breadth against imitation;\textsuperscript{11} see also Li (2001), Goh and Olivier (2002), Kwan and Lai (2003), Furukawa (2007) and Cysne and Turchick (2012). However, our model differs from these studies by modeling R&D and physical capital as two engines of long-run growth. Because patent breadth has asymmetric effects on R&D and capital accumulation, the overall effect of strengthening patent protection on economic growth is ambiguous due to a tradeoff between R&D and capital accumulation as in Iwaisako and Futagami (2012).\textsuperscript{12} The present study relates to this literature by analyzing how patent policy interacts with monetary policy to affect growth and welfare.

The rest of the study is organized as follows. Section 2 sets up the monetary endogenous-growth model. Section 3 analyzes the effects of monetary policy on economic growth and social welfare. Section 4 calibrates the model to numerically evaluate the welfare cost of inflation in the EA and the US. Section 5 considers an extension of the model to examine the robustness of our results. The final section concludes.

2. A monetary endogenous-growth model

To analyze the interactive effects of monetary policy and patent policy, we modify the seminal R&D-based growth model in Romer (1990) by (a) introducing a cash-in-advance constraint on consumption goods to model money demand, (b) considering variable patent breadth as in Goh and Olivier (2002), (c) incorporating a capital-producing sector as in Iwaisako and Futagami (2012) so that capital accumulation is also an engine of long-run growth, and (d) allowing for

\textsuperscript{11} Chu (2010) shows that at the current patent length of 20 years, extending the patent length would have negligible effects on R&D and social welfare. Therefore, we focus on the analysis of patent breadth in this study.

\textsuperscript{12} See also Chu, Cozzi and Galli (2012), who analyze the asymmetric effects of blocking patents on variety expansion and quality improvement in an R&D-based growth model.
elastic labor supply. Given that the Romer model has been well-studied, the standard features of
the model will be briefly described below to conserve space.

2.1. Households
There is a unit continuum of identical households, who have a lifetime utility function given by

\[ U = \int_{0}^{\infty} e^{-\rho t} u_c dt = \int_{0}^{\infty} e^{-\rho t} (\ln c_t - \psi l_t) dt. \]

Instantaneous utility \( u_t \) is increasing in consumption \( c_t \) and decreasing in the supply of labor \( l_t \).

As for the exogenous parameters, \( \rho > 0 \) is the discount rate, and \( \psi > 0 \) determines the disutility
of labor supply. Households maximize utility subject to an asset-accumulation equation given by

\[ \dot{a}_t + \dot{m}_t = r_t a_t + w_t l_t + \tau_t - c_t - \pi_t m_t. \]

\( a_t \) is the real value of assets owned by households, and these assets consist of tangible and
intangible capital. \( r_t \) is the real interest rate. Households supply labor to earn a real wage \( w_t \). \( \tau_t \)
is a real lump-sum transfer from the government. \( \pi_t \) is the inflation rate that determines the cost
of holding money. \( m_t \) is the real money balance held by households to facilitate purchases of
consumption goods that are subject to a cash-in-advance constraint given by \( \xi c_t \leq m_t \), where
\( 0 < \xi \leq 1 \). The usual cash-in-advance constraint is captured by the special case of \( \xi = 1 \). Here we
follow Dotsey and Ireland (1996) to consider a more general setup in which only a fraction of

\[ u_t = \ln c_t - \psi_t l_t^{1+\chi} / (1 + \chi) \] for \( \chi \geq 0 \). Derivations
are available upon request from the authors. However, when \( \chi > 0 \), the equilibrium allocations do not have closed-form solutions. Therefore, we focus on the special case of \( \chi = 0 \) for analytical tractability.
consumption expenditure is subject to a cash-in-advance constraint.\textsuperscript{14} This generalization allows us to perform a more realistic quantitative investigation on the welfare cost of inflation.

Using standard dynamic optimization, the optimality condition for consumption is

\begin{equation}
1 / c_t = \lambda_t (1 + \xi i_t),
\end{equation}

where $\lambda_t$ is the Hamiltonian co-state variable on (2), and $i_t = r_t + \pi_t$ is the nominal interest rate that captures the opportunity cost of holding money as opposed to accumulating tangible or intangible capital. The optimality condition for labor supply is

\begin{equation}
w_t = \psi (1 + \xi i_t) c_t.
\end{equation}

The familiar intertemporal optimality condition is

\begin{equation}
r_t = \rho - \dot{\lambda} / \lambda_t.
\end{equation}

### 2.2. Final goods

Final goods $y_t$ are produced by a standard CES aggregator using production labor $l_{y,t}$ and a continuum of differentiated intermediates goods $x_t(j)$ for $j \in [0, n_t]$ given by

\begin{equation}
y_t = \int_0^n l_{y,t}^{1-\alpha} x_t(\alpha) dj,
\end{equation}

where $n_t$ is the number of intermediate goods available. This sector is perfectly competitive, and the producers take the output and input prices as given. The conditional demand functions for production labor and intermediate goods are respectively

\begin{equation}
w_t = (1 - \alpha) y_t / l_{y,t},
\end{equation}

\begin{equation}
p_t(j) = \alpha [l_{y,t} / x_t(\alpha)]^{1-\alpha},
\end{equation}

\textsuperscript{14} See also Wu and Zhang (1998) who consider a generalized cash-in-advance constraint.
where \( p_x(j) \) is the price of \( x_i(j) \) relative to final goods.

### 2.3. Intermediate goods

There is a continuum of industries producing intermediate goods \( x_i(j) \) for \( j \in [0, n_i] \). Each industry is occupied by a monopolist who rents capital to produce intermediate goods in an one-to-one fashion; i.e., \( x_i(j) = k_i(j) \). The monopolistic profit is

\[
\omega_{x_i}(j) = p_x(j)x_i(j) - q_i k_i(j),
\]

where \( q_i \) is the rental price of capital.

The unconstrained optimization yields a profit-maximizing markup of \( 1/\alpha \). Here we follow Goh and Olivier (2002) to introduce patent breadth denoted by \( \eta \) as a policy variable by assuming that the unit cost of producing imitative products is increasing in patent breadth.\(^{15}\) Thus, without sufficient strength of patent protection, the presence of monopolistic profits attracts imitation. Therefore, stronger patent protection allows monopolistic producers to charge a larger markup without the threat of imitation; see Li (2001) for a similar formulation of patent breadth in the quality-ladder model. This formulation is also consistent with Gilbert and Shapiro’s (1990) seminal insight on “breadth as the ability of the patentee to raise price”. In summary, the maximum markup is determined by \( \eta \).\(^{16}\) For the rest of this study, we assume \( \eta < 1/\alpha \),\(^{17}\) so that

\[
p_x(j) = \eta q_i,
\]

\(^{15}\) In this study, we focus on patent breadth and make a standard assumption in the literature that the patent length is infinite for simplicity. See for example Iwaisako and Futagami (2003) and Futagami and Iwaisako (2007) for an analysis on finite optimal patent length in the Romer model. See also Palokangas (2011) for an analysis on optimal patent length and breadth in an R&D-based growth model.

\(^{16}\) Alternatively, one can also view the limited markup as price regulation. For example, Evans et al. (2003) analyze price regulation in the Romer model without money demand.

\(^{17}\) Given a capital share of about one-third, the unconstrained markup would be 200% (i.e., \( 1/\alpha - 1 \)) that is unrealistically large. Therefore, imposing an upper bound on the markup also helps to separate the effects of markup and capital share. See Jones and Williams (2000) for a discussion on this issue.
for \( j \in [0, n_j] \). This formulation also serves to provide a simple way to separate capital share \( \alpha \) and markup \( \eta \). The amount of profit is symmetric across industries and given by

\[
(11) \quad \omega_{x,t}(j) = \left( \frac{\eta - 1}{\eta} \right) p_t(j)x_t(j) = (\eta - 1) \left( \frac{q_t k_t}{n_t} \right),
\]

where the second equality of (11) uses the market-clearing condition for capital goods \( x_t n_t = k_t \).

Equation (11) shows that a larger markup \( \eta \) increases the amount of monopolistic profits, which in turn improves incentives for R&D investment; however, a larger \( \eta \) also decreases the capital share of income \( q_t k_t / y_t = \alpha / \eta \),\(^{18}\) which in turn worsens incentives for capital accumulation.

### 2.4. R&D

Denote the value of an invented variety as \( v_{n,t} \). The familiar no-arbitrage condition for \( v_{n,t} \) is

\[
(12) \quad r_t v_{n,t} = \omega_{x,t} + \dot{v}_{n,t}.
\]

Intuitively, (12) equates the interest rate to the asset return per unit of asset, where the asset return is the sum of monopolistic profit \( \omega_{x,t} \) and capital gain \( \dot{v}_{n,t} \). In the R&D sector, there is a unit continuum of entrepreneurs who hire workers \( l_{r,t} \) for R&D. The profit of R&D is

\[
(13) \quad \omega_{r,t} = v_{n,t} \dot{n}_t - w_l l_{r,t},
\]

where \( \dot{n}_t = \varphi n_t l_{r,t} \) is the mass of inventions created by the entrepreneur.\(^{19}\) The parameter \( \varphi \) determines R&D productivity. The zero-profit condition in the R&D sector is

\[
(14) \quad \varphi v_{n,t} n_t = w_l.
\]

\(^{18}\) This condition can be derived by using (6), (8) and (10).

\(^{19}\) Although we consider a deterministic R&D process as in the original Romer model, it is useful to note an interesting result by Li (1998) who shows that this deterministic R&D process can be derived from an underlying stochastic R&D process.
This condition determines the allocation of labor to R&D.

### 2.5. Capital production

Denote the value of one unit of capital as \( v_{k,t} \). The no-arbitrage condition for \( v_{k,t} \) is

\[
rv_{k,t} = q_t + \dot{v}_{k,t}.
\]

Intuitively, (15) equates the interest rate to the asset return per unit of asset, where the asset return is the sum of capital rental price \( q_t \) and capital gain \( \dot{v}_{k,t} \).\(^{20}\) In the capital-producing sector, there is a unit continuum of firms that hire workers \( l_{k,t} \) to produce capital. The profit is

\[
\omega_{k,t} = v_{k,t} \dot{k}_t - w_l l_{k,t},
\]

where \( \dot{k}_t = \phi A_t l_{k,t} \) is the amount of capital produced, \( \phi \) is a parameter, and \( \phi A_t \) determines the productivity of capital accumulation. To introduce endogenous growth, we assume \( A_t = k_t \) that captures the usual capital externality in the AK model.\(^{21}\) The zero-profit condition is

\[
\phi v_{k,t} k_t = w_t.
\]

This condition determines the allocation of labor to capital accumulation.

### 2.6. Monetary authority

The growth rate of money supply \( M_t \) is denoted by \( \mu_t = \dot{M}_t / M_t \) that is exogenously set by the monetary authority. Given the definition of the real money balance \( m_t = M_t / P_t \) (where \( P_t \) is the price of final goods), the inflation rate \( \pi_t \) is endogenously determined by

\[\ldots\]
\[ \pi_i = \mu_i - \dot{m}_i / m_i. \]

Any change in money supply is redistributed to households as a lump-sum transfer that has a real value of \[ \tau_i = \frac{\dot{M}_i}{P_i} = \mu m_i = \dot{m}_i + \pi_i m_i, \]
where the last equality follows from (18).

### 2.7. Decentralized equilibrium

The equilibrium is a time path of allocations \( \{c_i, m_i, l_i, y_i, x_i(j), k_i, l_{y,i}, l_{r,i}, l_{k,i}\}_{t=0}^{\infty} \), a time path of prices \( \{w_i, r_i, p_i(j), q_i, v_{k,i}, v_{n,i}\}_{t=0}^{\infty} \), and a time path of policies \( \{\mu_i, \tau_i\}_{t=0}^{\infty} \). At each instant of time,

a. households choose \( \{c_i, m_i, l_i\} \) to maximize (1) subject to (2) taking \( \{w_i, r_i, \tau_i\} \) as given;

b. competitive final-goods firm produce \( \{y_i\} \) to maximize profit taking \( \{w_i, p_i(j)\} \) as given;

c. the monopolist in industry \( j \in [0,1] \) produces \( \{x_i(j)\} \) and chooses \( \{p_i(j)\} \) subject to the level of patent breadth \( \eta \) to maximize profit taking \( \{q_i\} \) as given;

d. R&D entrepreneurs maximize profit taking \( \{w_i, v_{n,i}\} \) as given;

e. capital-producing firms maximize profit taking \( \{w_i, v_{k,i}\} \) as given;

f. the market for final goods clears such that \( y_i = c_i \);

g. the market for capital goods clears such that \( k_i = x_i n_i \);

h. the labor market clears such that \( l_i = l_{y,i} + l_{r,i} + l_{k,i} \);

i. the value of households’ assets equals the total value of intangible and tangible capital in the economy such that \( a_i = v_{a,i} n_i + v_{k,i} k_i \);

j. the monetary authority balances its budget such that \( \tau_i = \mu m_i. \)
2.8. Balanced growth path

In this subsection, we consider the dynamic properties of the model. Given that the monetary authority sets a stationary growth rate of money supply (i.e., \( \mu_t = \mu \) for all \( t \)), the economy jumps to a unique and stable balanced growth path.\(^{22}\) Lemma 1 summarizes this result, and the proof is relegated to Appendix A.

**Lemma 1:** Given a stationary path of monetary policy (i.e., \( \mu_t = \mu \) for all \( t \)), the economy jumps to a unique and stable balanced growth path.

**Proof:** See Appendix A. □

On the balanced growth path, equilibrium labor allocations are stationary. Here we sketch out the derivations, and the detailed derivations are relegated to Appendix A. From (5) and (18), we obtain the steady-state nominal interest rate \( i = r + \pi = \rho + \mu \). Substituting \( i = \rho + \mu \) into (4) and equating the resulting condition with (7) yield the equilibrium allocation of production labor \( l_y \). Combining (3), (4) and (14) yields \( \phi v_{n,y} n_y = \psi / \lambda_y \). Differentiating the log of this expression with respect to time yields \( \dot{n}_y / n_y = -\dot{v}_{n,y} / v_{n,y} - \dot{\lambda}_y / \lambda_y \), where \( \dot{n}_y / n_y = \phi l, \) and \( \dot{\lambda}_y / \lambda_y = \rho - r \) from (5). Furthermore, we can substitute (7), (11) and (14) into (12) to solve for \( \dot{v}_{n,y} / v_{n,y} \). From this procedure, we obtain the equilibrium allocation of R&D labor \( l_r \). Similarly, combining (3), (4) and (17) yields \( \phi v_{k,r} k_r = \psi / \lambda_r \). Differentiating the log of this expression with respect to time yields \( \dot{k}_r / k_r = -\dot{v}_{k,r} / v_{k,r} - \dot{\lambda}_r / \lambda_r \), where \( \dot{k}_r / k_r = \phi l_{k,r} \). Then, we can substitute (7), (17) and

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\(^{22}\) In an earlier version of this paper, we consider a monetary version of the canonical Romer model in which R&D is the only engine of long-run growth and derive the transition path of the economy from a change in monetary policy; see Chu, Lai and Liao (2010).
Applying $x_j(j) = x_j$ and $k_j = x_jn_j$ on (6) yields

\[ y_j = k_j^n (n_j l_j)^{1-\sigma}. \]

On the balanced-growth path, the growth rate of output is

\[ g_y \equiv \frac{\dot{y}_j}{y_j} = \alpha \frac{\dot{k}_j}{k_j} + (1-\alpha) \frac{\dot{n}_j}{n_j} = \alpha \phi l_k + (1-\alpha) \phi l_r, \]

where $g_k = \phi l_k$ is the balanced growth rate of capital and $g_n = \phi l_r$ is the balanced growth rate of varieties. To ensure that these growth rates are non-negative, we impose the following parameter restrictions. Condition R ensures that $l_k \geq 0$, whereas Condition K ensures that $l_k \geq 0$. 

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Lemma 2: The equilibrium labor allocations are given by

(19) \[ l_y = \frac{1-\alpha}{\psi[1+\xi(\mu+\rho)]}, \]

(20) \[ l_r = \frac{\eta - 1}{\eta} \left( \frac{\alpha}{\psi[1+\xi(\mu+\rho)]} \right) - \frac{\rho}{\phi}, \]

(21) \[ l_k = \frac{1}{\eta} \left( \frac{\alpha}{\psi[1+\xi(\mu+\rho)]} \right) - \frac{\rho}{\phi}, \]

(22) \[ l = \frac{1}{\psi[1+\xi(\mu+\rho)]} - \rho \left( \frac{1}{\phi} + \frac{1}{\phi} \right). \]

Proof: See Appendix A. □

3. Growth and welfare effects of monetary policy

$q, k, y, = \alpha/\eta$ into (15) to solve for $\dot{v}_{k,j} / v_{k,j}$. From this procedure, we obtain the equilibrium allocation of capital-producing labor $l_k$. Finally, we compute labor supply using $l = l_y + l_r + l_k$. 

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Condition R: \[ \varphi \geq \tilde{\varphi} = \frac{\rho \psi[1 + \xi(\mu + \rho)]}{\alpha(\eta - 1)/\eta} \]

Condition K: \[ \varphi \geq \tilde{\varphi} = \frac{\rho \psi[1 + \xi(\mu + \rho)]}{\alpha/\eta} \]

Equation (20) shows that \( l_r \) is increasing in \( \eta \). Intuitively, larger patent breadth increases monopolistic profits and the value of an invention providing more incentives for R&D. Equation (20) also shows that \( l_r \) is decreasing in \( \mu \). Intuitively, a larger \( \mu \) increases inflation, which in turn raises the cost of holding money that is required for purchasing consumption goods. As a result, households consume more leisure reducing the supply of labor \( l \) as shown in (22), and they also decrease consumption reducing production labor \( l_y \) as shown in (19). The decrease in production labor \( l_y \) reduces the amount of profits \( n_y \omega_{y,t} = \alpha y_t(\eta - 1)/\eta \), which in turn decreases the incentives for R&D consequently reducing R&D labor \( l_r \) and the variety growth rate \( g_n = \varphi l_r \). To see this, substituting (19) into (20) yields \( l_r = \frac{\eta - 1}{\eta} \left( \frac{\alpha}{1 - \alpha} \right) l_y - \frac{\rho}{\varphi} \), where \( l_y \) is decreasing in \( \mu \). Interestingly, larger patent breadth strengthens the negative effect of inflation on R&D and variety growth. To see this, \[ \frac{\partial l_y}{\partial \mu} = \frac{\eta - 1}{\eta} \left( \frac{\alpha}{1 - \alpha} \right) \frac{\partial l_y}{\partial \mu} < 0; \] thus, a larger \( \eta \) magnifies the negative effect of \( \partial l_y / \partial \mu \) on \( \partial l_r / \partial \mu \).

**Proposition 1:** The variety growth rate \( g_n \) is increasing in patent breadth \( \eta \) and decreasing in the money growth rate \( \mu \). Increasing patent breadth strengthens the negative effect of monetary policy on the variety growth rate.

**Proof:** Recall that \( g_n = \varphi l_r \) and note (20). \( \square \)
Equation (21) shows that $l_k$ is decreasing in $\eta$. Intuitively, larger patent breadth reduces the capital share of income $q_k/y = \alpha/\eta$ and the value of capital resulting into less incentive for capital accumulation. Equation (21) also shows that $l_k$ is decreasing in $\mu$ because the decrease in production labor $y_l$ reduces capital income, which in turn decreases the incentives for capital accumulation consequently reducing capital-producing labor $l_k$ and the capital growth rate $g_k = \phi l_k$. To see this, substituting (19) into (21) yields $l_k = \frac{1}{\eta(1-\alpha)} y_l - \frac{\rho}{\phi}$, where $y_l$ is decreasing in $\mu$. Interestingly, increasing patent breadth weakens the negative effect of inflation on capital-producing labor $l_k$ and the capital growth rate $g_k = \phi l_k$. To see this, 

$$\frac{\partial l_k}{\partial \mu} = \frac{1}{\eta(1-\alpha)} \frac{\partial y_l}{\partial \mu} < 0;$$

thus, a larger $\eta$ mitigates the negative effect of $\partial y_l/\partial \mu$ on $\partial l_k/\partial \mu$.

**Proposition 2:** The capital growth rate $g_k$ is decreasing in patent breadth $\eta$ and the money growth rate $\mu$. Increasing patent breadth weakens the negative effect of monetary policy on the capital growth rate.

**Proof:** Recall that $g_k = \phi l_k$ and note (21). $\square$

From Propositions 1 and 2, we can infer the effects of inflation on economic growth in the R&D-based growth model and the AK model. As $\phi \rightarrow \tilde{\phi}$, the capital-producing sector shuts down and the model reduces to a monetary R&D-based growth model. As $\varphi \rightarrow \tilde{\varphi}$, the R&D sector shuts down and the model reduces to a monetary AK model. The advantage of this

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23 In an earlier version of this study, we provide this analysis in the two models; see Chu, Lai and Liao (2010).
unified model is that we can analyze the effects of inflation when R&D and capital accumulation are both engines of long-run growth. Substituting (20) and (21) into (24) yields

\[
g_y = \left( \frac{\alpha \phi}{\eta} + \frac{1 - \alpha \phi}{\eta} \eta - 1 \right) \frac{\alpha}{\psi(1 + \xi(\mu + \rho))} - \rho.
\]

In this case, the effect of patent breadth \( \eta \) on economic growth \( g_y \) is ambiguous. Specifically, \( \partial g_y / \partial \eta > 0 \) if and only if \((1 - \alpha)\phi > \alpha \phi \). Intuitively, \((1 - \alpha)\phi \) captures the importance of R&D on economic growth whereas \( \alpha \phi \) captures the importance of capital accumulation. In other words, if and only if the R&D channel dominates the capital-accumulation channel, then patent breadth would have a positive growth effect. As for the effect of money growth, it continues to be negative as before (i.e., \( \partial g_y / \partial \mu < 0 \)). Finally, whether patent breadth strengthens the growth effect of monetary policy depends on the same parameter condition \((1 - \alpha)\phi > \alpha \phi \) as \( \partial g_y / \partial \eta > 0 \). In other words, if \( \partial g_y / \partial \eta > 0 \) \( \partial g_y / \partial \eta < 0 \), then increasing patent breadth would strengthen (weaken) the negative effect of monetary policy on economic growth.

**Proposition 3:** *Economic growth* \( g_y \) *can be increasing or decreasing in patent breadth* \( \eta \). *If and only if* \((1 - \alpha)\phi > \alpha \phi \), *then* \( g_y \) *would be increasing in* \( \eta \). *Economic growth is always decreasing in the money growth rate* \( \mu \). *Increasing patent breadth may strengthen or weaken the effect of monetary policy on economic growth.* *If and only if* \((1 - \alpha)\phi > \alpha \phi \), *then increasing patent breadth would strengthen the negative effect of monetary policy on economic growth.*

**Proof:** Apply simple differentiation to (25). \( \square \)
Finally, we analyze the effect of monetary policy on social welfare. Imposing balanced growth on (1) yields

\[ U = \frac{1}{\rho} \left( \ln c_0 + \frac{g_y}{\rho} - \psi l \right), \]

where \( c_0 = y_0 = k_0^{a}(n_0l)^{1-a} \) in which the initial \( n_0 \) and \( k_0 \) are exogenous. Dropping \( \alpha \ln k_0 \) and \((1-\alpha)\ln n_0 \) from (26) yields \( \rho U = (1-\alpha)\ln l_y + g_y / \rho - \psi l \). Proposition 4 shows that the effect of patent breadth on social welfare is ambiguous as in the case of economic growth. Specifically, \( \partial U / \partial \eta > 0 \) if and only if \((1-\alpha)\varphi > \alpha \varphi \), which is also the necessary and sufficient condition for \( \partial g_y / \partial \eta > 0 \). As for the effect of money growth on welfare, it is negative (i.e., \( \partial U / \partial \mu < 0 \)) because the negative effects of \( \mu \) on \( c_0 \) and \( g_y \) dominate the positive effect of increased leisure.

Finally, whether patent breadth strengthens the welfare effect of monetary policy depends on the same parameter condition \((1-\alpha)\varphi > \alpha \varphi \) as \( \partial U / \partial \eta > 0 \). In other words, if \( \partial U / \partial \eta > 0 \) \((\partial U / \partial \eta < 0)\), then increasing patent breadth would strengthen (weaken) the negative effect of monetary policy on social welfare.

**Proposition 4:** Social welfare \( U \) can be increasing or decreasing in patent breadth \( \eta \). If and only if \((1-\alpha)\varphi > \alpha \varphi \), then \( U \) would be increasing in \( \eta \). Social welfare is always decreasing in the money growth rate \( \mu \). Increasing patent breadth may strengthen or weaken the effect of monetary policy on social welfare. If and only if \((1-\alpha)\varphi > \alpha \varphi \), then increasing patent breadth would strengthen the negative effect of monetary policy on social welfare.

**Proof:** Substitute (19), (22) and (25) into (26). Then, apply simple differentiation and \( g_y \geq 0 \).
4. Quantitative analysis

In this section, we calibrate the model to provide a numerical analysis on the growth and welfare effects of inflation and quantitatively examine whether the markup magnifies or mitigates these effects. We consider two monetary aggregates, currency and M1, as alternative measures of money held by households for the purpose of facilitating transactions. On the one hand, currency holding by households is a subset of monetary assets that are subject to the cost of inflation. On the other hand, M1 includes interest-bearing assets, such as demand deposits, which are partly immune to the depreciation effect of inflation. Therefore, we report the welfare cost of inflation computed based on currency as a lower bound and the welfare cost computed based on M1 as an upper bound. For the EA, we set the cash-in-advance parameter $\zeta = m/c$ to 0.10 when we match the average ratio of currency to households’ final consumption expenditure from 1999 to 2010, and we set $\zeta$ to 0.66 when we match the ratio of M1 to consumption. For the US, we set $\zeta$ to 0.08 when we match the ratio of currency to consumption and $\zeta$ to 0.16 when we match the ratio of M1 to consumption.

For each of the other parameters, we either set it to a conventional value or calibrate its value using an empirical moment based on data from 1999 to 2010. For the money growth rate, we calibrate it using $\mu = g_y + \pi$, where $g_y$ and $\pi$ are taken from the data (to be reported in Table 1). For the R&D and capital productivity parameters $\phi$ and $\rho$, we calibrate them using the output and capital growth rates $g_y$ and $g_k$ from the data. Given $g_y$ and $g_k$, the variety growth rate can be computed as $g_
u = (g_y - \alpha g_k)/(1-\alpha)$. For the markup $\eta$, we calibrate it using R&D as a percentage of GDP. Then, we use capital investment as a percentage of GDP to calibrate the value of $\rho$, and we use the labor share of income to calibrate the value of $\alpha$. Finally, we
calibrate the labor supply parameter $\psi$ by setting the supply of labor $l$ to 0.33. In Table 1, we report the values of all these variables and parameters.

[Insert Table 1 about here]

4.1. Numerical results

The policy experiment that we consider is to reduce the money growth rate to the level that achieves the Friedman rule (i.e., $i = 0$ which implies $\mu = -\rho$). Table 2 shows that under both the currency and M1 specifications, reducing money growth increases economic growth and social welfare in both the EA and the US.\(^{24}\) However, the changes in the capital growth rate and the variety growth rate respond differently to the markup. Specifically, a larger markup *increases* the magnitude of the changes in the variety growth rate in response to lower inflation. In contrast, a larger markup *decreases* the magnitude of the changes in the capital growth rate in response to lower inflation. Overall, we find that the R&D channel dominates the capital-accumulation channel such that a larger markup tends to increase the magnitude of the changes in economic growth and social welfare in response to lower inflation. In other words, a larger market power of firms tends to magnify the welfare cost of inflation in these economies.

[Insert Table 2 about here]

In the EA, the welfare cost of inflation under the M1 specification is 5.79% whereas the welfare cost under the currency specification is 0.88%. In the US, the welfare costs of inflation under the M1 and currency specifications are 1.76% and 0.89% respectively. Therefore, in both economies, the welfare cost of inflation is much higher under the M1 specification than under the currency specification. Furthermore, when we use currency as the measure of money, we find a

\(^{24}\) The welfare changes are expressed in terms of equivalent variation in annual consumption.
negligible difference in the welfare cost of inflation between the EA and the US; however, when we use M1 as the measure of money, we find a substantial difference in the welfare cost due to the much higher money-consumption ratio in the EA than in the US.

5. Extension: variety expansion and capital accumulation with cross-sector spillovers

In this section, we consider an alternative specification for the laws of motion for variety \( n_t \) and capital \( k_t \). The following specification allows for cross-sector spillovers captured by \( s \in (0,0.5) \).

\[
\dot{n}_t = \phi m_t^{1-s} k_t^{s} l_{r,t},
\]

\[
\dot{k}_t = \phi k_t^{1-s} n_t^{s} l_{k,t}.
\]

Given these new laws of motion, the zero-profit conditions in (14) and (17) become

\[
\varphi v_{n,t} n_t^{1-s} k_t^s = w_t,
\]

\[
\varphi v_{k,t} k_t^{1-s} n_t^s = w_t.
\]

The rest of the model is the same as before. In Lemma 3, we first discuss the dynamic properties of \( n_t / k_t \) in this extended model.

**Lemma 3:** Given a stationary path of monetary policy (i.e., \( \mu_t = \mu \) for all \( t \)), the dynamics of \( n_t / k_t \) is characterized by global stability such that it gradually converges to a unique and stable steady state.

**Proof:** See Appendix A. □

On the balanced growth path, the growth rates of variety and capital are respectively
(31) \[ g_n = \left( \frac{k_t}{n_t} \right)^{\phi l_r}. \]

(32) \[ g_k = \left( \frac{n_t}{k_t} \right)^{\phi l_k}. \]

Because \( n_t / k_t \) is constant, it must be the case that \( g_n = g_k \), which implies a steady-state ratio of

(33) \[ \frac{n_t}{k_t} = \left( \frac{\phi l_r}{\phi l_k} \right)^{\frac{1}{\gamma}}. \]

Although this extended model has a fundamentally different property that the growth rates of variety and capital are the same on the balanced growth path, we will show that our main results are robust to this extension.

Here we first derive the equilibrium labor allocations. Following the same derivations as before, one can show that the nominal interest rate is \( i = \rho + \mu \) and the equilibrium allocation of production labor \( l_t \) continues to be given by (19). Then, combining (3), (4) and (29) yields

\[ \phi v_{n,t} n_t^{1-s} k_t^s = \psi / \lambda_t. \]

Differentiating the log of this expression with respect to time yields

(34) \[ (1-s) \frac{\dot{n}_t}{n_t} + s \frac{\dot{k}_t}{k_t} = -\frac{\dot{v}_{n,t}}{v_{n,t}} - \frac{\dot{\lambda}_t}{\lambda_t}, \]

where \( \dot{\lambda}_t / \lambda_t = \rho - r_t \) from (5). Also, we can substitute (7), (11) and (29) into (12) to solve for

(35) \[ \frac{\dot{v}_{n,t}}{v_{n,t}} = r_t - \omega_{n,t} = r_t - \left( \frac{\eta - 1}{\eta} \right) \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{k_t}{n_t} \right)^{\phi l_{r,t}}. \]

Substituting (35) into (34) and imposing the balanced-growth condition \( \dot{n}_t / n_t = \dot{k}_t / k_t \) yield

(36) \[ l_t = \left( \frac{\eta - 1}{\eta} \right) \left( \frac{\alpha}{1 - \alpha} \right) l_r - \rho \left( \frac{n_t}{k_t} \right)^{\phi l_r}, \]

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where we have used \( g_n = (k_i / n_i)' \varphi l_r \). Equation (36) shows that for a given \( n_i / k_i \), R&D labor \( l_r \) is increasing in \( \eta \) as before.

Similarly, combining (3), (4) and (30) yields \( \varphi v_{k,j} k_i^{1-\alpha} n_i' = \psi / \lambda_i \). Differentiating the log of this expression with respect to time yields

\[
(37) \quad (1-s)\frac{k_i'}{k_i} + s \frac{n_i'}{n_i} = -\frac{\dot{v}_{k,j}}{v_{k,j}} - \frac{\dot{\lambda}_i}{\lambda_i}.
\]

Then, we can substitute (7), (30) and \( q_j k_i / y_i = \alpha / \eta \) into (15) to solve for

\[
(38) \quad \frac{\dot{v}_{k,j}}{v_{k,j}} = r_r - \frac{\dot{q}_j}{v_{k,j}} = r_r - \frac{1}{\eta} \left( \frac{\alpha}{1-\alpha} \right) n_i \left( k_i \right) \psi l_{j,y}.
\]

Substituting (38) into (37) and imposing the balanced-growth condition \( \dot{n}_i / n_i = \dot{k}_i / k_i \) yield

\[
(39) \quad l_k = \frac{1}{\eta} \left( \frac{\alpha}{1-\alpha} \right) n_i \left( k_i \right) \phi l_{k,y}.
\]

where we have used \( g_k = (n_i / k_i)' \varphi l_k \). Equation (39) shows that for a given \( n_i / k_i \), capital-producing labor \( l_k \) is decreasing in \( \eta \). Therefore, this model also features the tradeoff of patent breadth on capital and R&D, and this tradeoff will be reflected in the common growth rate being a non-monotonic function in \( \eta \).

Combining (19), (33), (36) and (39) yield the equilibrium allocations of \( l_r \) and \( l_k \) respectively given by

\[
(40) \quad l_r = \frac{\eta - 1}{\eta} \left( \frac{\alpha}{\psi[1+\xi(\mu+\rho)]} \right) - \rho \frac{\sqrt{\eta - 1}}{\phi \phi},
\]

\[
(41) \quad l_k = \frac{\eta - 1}{\eta} \left( \frac{\alpha}{\psi[1+\xi(\mu+\rho)]} \right) - \rho \frac{\sqrt{\phi \phi(\eta - 1)}}{\phi \phi(\eta - 1)}.
\]

Finally, we can compute the supply of labor given by
Substituting (33), (40) and (41) into (31) or (32) yields

\[
g_n = g_k = \frac{\sqrt{\eta - 1}}{\eta} \left( \frac{\alpha \sqrt{\phi \phi}}{\psi[1 + \xi(\mu + \rho)]} \right) - \rho,
\]

which is also the growth rate of output because \( g_y = g_n = g_k \) in this model. Equation (43) shows that the growth rate is an inverted-U function in patent breadth \( \eta \) due to the tradeoff between R&D and capital investment as in Iwaisako and Futagami (2012) and the growth rate reaches a maximum at \( \eta = \bar{\eta} = 2 \). Also, the growth rate is always decreasing in the money growth rate \( \mu \) as before. Whether increasing patent breadth strengthens or weakens this negative effect of monetary policy on economic growth depends on whether \( \eta \) is on the upward-sloping side (i.e., the R&D channel dominates) or the downward-sloping side (i.e., the capital-accumulation channel dominates) of the curve. Given that the empirical markup is often estimated to be less than 100\% (i.e., \( \eta < 2 \)),\(^{25}\) the R&D channel is likely to dominate the capital-accumulation channel. Finally, one can also show that on the balanced growth path, welfare is monotonically decreasing in the money growth rate \( \mu \), and increasing patent breadth strengthens (weakens) this negative effect of monetary policy on welfare if \( \eta \) is less (greater) than \( \bar{\eta} \).\(^{26}\)

6. Conclusion

In this study, we have revisited a fundamental question in monetary economics originally raised by Tobin (1965) on the relationship between inflation and economic growth. The key departure from the literature is that we consider both innovation and capital accumulation as engines of

\(^{25}\) See for example Jones and Williams (2000) for a discussion on the empirical range of the markup.

\(^{26}\) Derivations are contained in an unpublished appendix (please see Appendix B) that is available upon request.
economic growth in the long run. In summary, we find that the growth and welfare effects of inflation and monetary policy are largely influenced by an unexplored interaction between the growth engine and the strength of patent protection. We believe that this interaction sheds some light on the importance of the growth engine and an interaction between monetary and patent policies that have been neglected in the growth-inflation literature.

Finally, it is well-known that the Romer model exhibits scale effects. In this study, we normalize the size of population to unity, so that population size does not appear in the equilibrium growth rate. Instead, it is the supply of labor that affects growth; in other words, when R&D scientists and engineers devote more time to research, they generate more inventions. We believe that this implication is more plausible than the original version of scale effects based on population size. Nevertheless, it may be fruitful for future studies to further revisit the growth and welfare effects of monetary policy using other vintages of the R&D-based growth model.

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27 See Jones (1999) for an excellent discussion on scale effects in R&D-based growth models.
References


Appendix A

Proof of Lemma 1: In this proof, we first show that \( i_t \) is stationary given a constant \( \mu \). Using (5) and (18), we have

\[
(A1) \quad i_t = r_t + \pi_t = \rho - \frac{\hat{\lambda}_t}{\lambda_t} + \mu - \frac{\dot{m}_t}{m_t},
\]

where \( \dot{m}_t / m_t = \dot{c}_t / c_t \) because \( m_t = \xi c_t \). Taking the log of (3) and differentiating with time yields

\[
(A2) \quad -\frac{\hat{\lambda}_t}{\lambda_t} - \frac{\dot{c}_t}{c_t} = \frac{\xi \dot{i}_t}{1 + \xi i_t}.
\]

Substituting (A2) into (A1) and then rearranging terms yield

\[
(A3) \quad i_t = \left( \frac{1}{\xi} + i_t \right) (i_t - \rho - \mu).
\]

Given the saddle-point stability of this dynamic system, \( i_t \) jumps to its steady state \( i_t = \rho + \mu \).

In the rest of this proof, we show that given a constant \( i_t \), equilibrium labor allocations are also stationary. Combining (4) and (7) yields

\[
(A4) \quad l_{y,t} = \frac{1 - \alpha}{\psi(1 + \xi i_t)},
\]

where we have applied the resource constraint for final goods \( c_t = y_t \). Equation (A4) shows that \( l_{y,t} \) must be stationary given a constant \( i_t \). Combining (3), (4) and (14) yields

\[
(A5) \quad \varphi_{v_{n,t}} n_t = \psi / \hat{\lambda}_t.
\]

Differentiating the log of (A5) with respect to time yields

\[
(A6) \quad \frac{\dot{n}_t}{n_t} = \frac{\dot{v}_{n,t}}{v_{n,t}} - \frac{\hat{\lambda}_t}{\lambda_t}.
\]
Substituting (6), (7), (8), (11) and (14) into (12), we obtain

\begin{equation}
\frac{\dot{v}_{n,t}}{v_{n,t}} = r - \frac{\alpha}{v_{n,t}} = r - \frac{\eta - 1}{\eta} \left( \frac{1}{1 - \alpha} \right) l_{y,t}.
\end{equation}

Substituting (A7), (5) and \( \dot{n}_t / n_t = \phi l_{r,t} \) into (A6) yields

\begin{equation}
l_{r,t} = \frac{\eta - 1}{\eta} \left( \frac{1}{1 - \alpha} \right) l_{y,t} - \frac{\rho}{\phi},
\end{equation}

which shows that \( l_{r,t} \) is stationary given a constant \( l_{y,t} \). Combining (3), (4) and (17) yields

\begin{equation}
\phi \dot{v}_{k,t} k_i = \psi / \lambda_i.
\end{equation}

Differentiating the log of (A9) with respect to time yields

\begin{equation}
\frac{\dot{k}_i}{k_i} = - \frac{\dot{v}_{k,t}}{v_{k,t}} - \frac{\dot{\lambda}_i}{\lambda_i}.
\end{equation}

Substituting (6), (7), (8), (10) and (17) into (15), we obtain

\begin{equation}
\frac{\dot{v}_{k,t}}{v_{k,t}} = r_i - \frac{q_i}{v_{k,t}} = r_i - \frac{\phi}{\eta} \left( \frac{\alpha}{1 - \alpha} \right) l_{y,t}.
\end{equation}

Substituting (A11), (5) and \( \dot{k}_i / k_i = \phi l_{r,t} \) into (A10) yields

\begin{equation}
l_{r,t} = \frac{1}{\eta} \left( \frac{\alpha}{1 - \alpha} \right) l_{y,t} - \frac{\rho}{\phi},
\end{equation}

which shows that \( l_{r,t} \) is stationary given a constant \( l_{y,t} \). Finally, the labor resource constraint is

\begin{equation}
l_i = l_{y,t} + l_{r,t} + l_{k,t},
\end{equation}

which shows that \( l_i \) must be stationary given constant \( l_{y,t} \), \( l_{r,t} \) and \( l_{k,t} \). □

**Proof of Lemma 2:** Substituting \( i_i = \rho + \mu \) into (A4) yields (19). Substituting (19) into (A8) and (A12) yields (20) and (21). Finally, substituting (19), (20) and (21) into (A13) yields (22). □
Proof of Lemma 3: Following the same derivations as in the proof of Lemma 1, one can show that $i_t$ has the same dynamics as in (A3) that is characterized by saddle-point stability such that it simply jumps to its steady state given by $i_t = \rho + \mu$. As a result, (A4) implies that $l_{y,t}$ also jumps to its steady state given by $l_{y,t} = (1 - \alpha)/(\psi(1 + \xi i_t))$.

In the rest of this proof, we show that the dynamics of a transformed variable $z_t = n_t/k_t$ is characterized by global stability such that $z_t$ being a state variable gradually converges to a steady-state value. Taking the difference between (34) and (37) yields

\[ \frac{\dot{z}_t}{z_t} = \frac{\dot{n}_t}{n_t} - \frac{\dot{k}_t}{k_t} = \frac{1}{1 - 2s} \left( \frac{\dot{v}_{k,t}}{v_{k,t}} - \frac{\dot{v}_{n,t}}{v_{n,t}} \right), \]

where $1 - 2s > 0$. Substituting (35) and (38) into (A14) yields

\[ \dot{z}_t = \frac{z_t}{1 - 2s} \left( \frac{\alpha}{1 - \alpha} \right) \left( (\eta - 1) \frac{\phi}{z_t^*} - z_t^* \phi \right) \frac{l_y}{\eta}, \]

where $l_y$ is constant along the transition path of $z_t$. The interior steady-state value of $z_t$ is given by $z_t^* = [(\eta - 1)\phi/\phi]^{1/(2s)}$. Figure 1 plots the dynamics of $z_t$ characterized by global stability. □

![Figure 1: Phase diagram](image)
Appendix B (not for publication)

In this unpublished appendix, we provide a proof for the following results in the extended model of Section 5. On the balanced growth path, welfare is monotonically decreasing in the money growth rate $\mu$, and patent breadth strengthens (weakens) this negative effect of monetary policy on welfare if $\eta$ is less (greater) than $\bar{\eta} \equiv 2$.

From (26), we can obtain the balanced-growth level of social welfare given by

\[
U = \frac{1}{\rho} \left( \alpha \ln k_0 + (1-\alpha) \ln n_0 + (1-\alpha) \ln l + \frac{g_y}{\rho} - \psi l \right).
\]

From the proof of Lemma 3 in Appendix A, the steady-state ratio of $n_0/k_0 = [(\eta - 1)\phi/\phi]^{1/(2s)}$ is independent of $\mu$. From (19), (42) and (43), we can show that the effects of monetary policy on production labor, labor supply and economic growth are respectively

\[
\frac{\partial l_y}{\partial \mu} = -\frac{\xi(1-\alpha)}{\psi(1+\xi(\mu + \rho))^2} < 0 ,
\]

\[
\frac{\partial l}{\partial \mu} = -\frac{\xi}{\psi(1+\xi(\mu + \rho))^2} < 0 ,
\]

\[
\frac{\partial g_y}{\partial \mu} = -\frac{\sqrt{\eta-1}}{\eta} \left( \frac{\alpha \xi \sqrt{\phi \phi}}{\psi(1+\xi(\mu + \rho))^2} \right) < 0 .
\]

Equations (B2)-(B4) show that an increase in the monetary target $\mu$ reduces production labor, labor supply and the growth rate. Overall, the welfare effect of monetary policy is

\[
\frac{\rho}{\rho} \frac{\partial U}{\partial \mu} = \frac{1-\alpha}{l_y} \frac{\partial l_y}{\partial \mu} + \frac{1}{\rho} \frac{\partial g_y}{\partial \mu} - \psi \frac{\partial l}{\partial \mu}.
\]

Substituting (B2)-(B4) into (B5) and then applying $g_y$ from (43), (B5) becomes
Equation (B6) shows that welfare is monotonically decreasing in the money growth rate $\mu$.

Finally, taking the absolute value of (B6) and differentiating it with respect to $\eta$ yields

$$
\frac{\partial}{\partial \eta} \left[ \rho \frac{\partial U}{\partial \mu} \right] = \frac{\xi}{\rho \left[ 1 + \xi (\mu + \rho) \right]} \frac{\partial g_s}{\partial \eta} = \left( \frac{\xi \alpha \sqrt{\psi / (\eta - 1)}}{\rho \psi \eta^2 \left[ 1 + \xi (\mu + \rho) \right]^2} \right) \frac{2 - \eta}{2} > 0; \text{ if } \eta \leq 2,
$$

where we have used

$$
\frac{\partial g_s}{\partial \eta} = \left( \frac{\alpha \sqrt{\psi / (\eta - 1)}}{\psi \eta^2 \left[ 1 + \xi (\mu + \rho) \right]} \right) \frac{2 - \eta}{2} < 0; \text{ if } \eta \geq 2.
$$

Therefore, patent breadth strengthens (weakens) the negative effect of monetary policy on social welfare if $\eta$ is less (greater) than $\bar{\eta} = 2$. $\square$
Table 1a: Values of the variables and parameters in the EA

<table>
<thead>
<tr>
<th>Variables</th>
<th>π</th>
<th>g₀</th>
<th>g₁</th>
<th>g₂</th>
<th>g₃</th>
<th>R&amp;D/GDP</th>
<th>I/GDP</th>
<th>wl/GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency specification</td>
<td>0.0198</td>
<td>0.0153</td>
<td>0.0296</td>
<td>0.0052</td>
<td>0.0190</td>
<td>0.1823</td>
<td>0.6685</td>
<td>0.1042</td>
</tr>
<tr>
<td>M1 specification</td>
<td>0.0198</td>
<td>0.0153</td>
<td>0.0296</td>
<td>0.0052</td>
<td>0.0190</td>
<td>0.1823</td>
<td>0.6686</td>
<td>0.6609</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>ρ</th>
<th>α</th>
<th>µ</th>
<th>φ</th>
<th>ψ</th>
<th>η</th>
<th>η</th>
<th>ι</th>
<th>ζ</th>
</tr>
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<tbody>
<tr>
<td>Currency specification</td>
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<td>0.4150</td>
<td>0.0351</td>
<td>0.3289</td>
<td>0.5495</td>
<td>1.2569</td>
<td>2.5236</td>
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<tr>
<td>M1 specification</td>
<td>0.0132</td>
<td>0.4150</td>
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<td>0.3289</td>
<td>0.5495</td>
<td>1.2569</td>
<td>2.4578</td>
<td>0.6609</td>
<td></td>
</tr>
</tbody>
</table>

Source: All data is obtained from Statistical Data Warehouse (ECB) except for (a) the data on R&D share of GDP and capital investment share of GDP sourced from Eurostat (European Commission), and (b) the data on labor compensation wl/GDP sourced from National Accounts of OECD Countries. The data set contains observations from 1999 to 2010. However, for labor compensation that does not have a complete data series from 1999 to 2010, we use available observations from 2003 to 2009.

Table 1b: Values of the variables and parameters in the US

<table>
<thead>
<tr>
<th>Variables</th>
<th>π</th>
<th>g₀</th>
<th>g₁</th>
<th>g₂</th>
<th>g₃</th>
<th>R&amp;D/GDP</th>
<th>I/GDP</th>
<th>wl/GDP</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0206</td>
<td>0.0307</td>
<td>0.0143</td>
<td>0.0268</td>
<td>0.1554</td>
<td>0.6871</td>
<td>0.0805</td>
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<tr>
<td>M1 specification</td>
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<td>0.0206</td>
<td>0.0307</td>
<td>0.0143</td>
<td>0.0268</td>
<td>0.1554</td>
<td>0.1587</td>
<td>0.1587</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>ρ</th>
<th>α</th>
<th>µ</th>
<th>φ</th>
<th>ψ</th>
<th>η</th>
<th>η</th>
<th>ι</th>
<th>ζ</th>
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<tr>
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<td>0.0452</td>
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<td>1.1142</td>
<td>1.2473</td>
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<td>M1 specification</td>
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<td>0.3826</td>
<td>0.0452</td>
<td>0.4113</td>
<td>1.1142</td>
<td>1.2473</td>
<td>2.5204</td>
<td>0.1587</td>
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</tbody>
</table>

Source: All data is obtained from the Federal Reserve Economic Data database except for (a) the data on capital stock sourced from Bureau of Economic Analysis, (b) the data on R&D share of GDP sourced from the OECD database, and (c) the data on labor compensation sourced from National Accounts of OECD Countries. The data set contains observations from 1999 to 2010. However, for labor compensation and R&D share of GDP that do not have a complete data series from 1999 to 2010, we use available observations from 2003 to 2009 respectively.

Table 2a: Growth and welfare effects of reducing µ to –ρ in the EA

<table>
<thead>
<tr>
<th>η</th>
<th>Δg₀</th>
<th>Δg₁</th>
<th>Δg₂</th>
<th>ΔU</th>
</tr>
</thead>
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<td>1.197</td>
<td>0.0074%</td>
<td>0.0081%</td>
<td>0.0087%</td>
<td>0.0093%</td>
</tr>
<tr>
<td>1.217</td>
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<td>0.0081%</td>
<td>0.0087%</td>
<td>0.0093%</td>
</tr>
<tr>
<td>1.237</td>
<td>0.0074%</td>
<td>0.0081%</td>
<td>0.0087%</td>
<td>0.0093%</td>
</tr>
<tr>
<td>1.257</td>
<td>0.0074%</td>
<td>0.0081%</td>
<td>0.0087%</td>
<td>0.0093%</td>
</tr>
<tr>
<td>1.277</td>
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<td>0.0081%</td>
<td>0.0087%</td>
<td>0.0093%</td>
</tr>
<tr>
<td>1.297</td>
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<td>0.0081%</td>
<td>0.0087%</td>
<td>0.0093%</td>
</tr>
<tr>
<td>1.317</td>
<td>0.0074%</td>
<td>0.0081%</td>
<td>0.0087%</td>
<td>0.0093%</td>
</tr>
</tbody>
</table>

Table 2b: Growth and welfare effects of reducing µ to –ρ in the US

<table>
<thead>
<tr>
<th>η</th>
<th>Δg₀</th>
<th>Δg₁</th>
<th>Δg₂</th>
<th>ΔU</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.187</td>
<td>0.0472%</td>
<td>0.0512%</td>
<td>0.0550%</td>
<td>0.0587%</td>
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<td>1.247</td>
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<td>0.0512%</td>
<td>0.0550%</td>
<td>0.0587%</td>
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<tr>
<td>1.267</td>
<td>0.0472%</td>
<td>0.0512%</td>
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<td>0.0512%</td>
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<tr>
<td>1.307</td>
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<td>0.0512%</td>
<td>0.0550%</td>
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</table>

Source: All data is obtained from the Federal Reserve Economic Data database except for (a) the data on R&D share of GDP sourced from the OECD database, and (c) the data on labor compensation sourced from National Accounts of OECD Countries. The data set contains observations from 1999 to 2010. However, for labor compensation and R&D share of GDP that do not have a complete data series from 1999 to 2010, we use available observations from 2003 to 2009 respectively.