A simple model of the commercial lobbying industry

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Abstract

In this paper we present a model of the behavior of commercial lobbying firms (such as the so-called K-Street lobbyists of Washington, D.C.). In contrast to classical special interest groups, commercial lobbying firms represent a variety of clients and are not directly affected by policy outcomes. They are hired by citizens, or groups of citizens, to act as intermediaries on their behalf with policymakers. In our analysis we address two basic questions; why do commercial lobbying firms exist, and what are the implications of their existence for social welfare? We answer the first part of this question by proposing that commercial lobbying firms possess a verification technology that allows them to improve the quality of information concerning the social desirability of policy proposals. This gives policymakers the incentive to allocate their scarce time to lobbying firms. Essentially it is this access to policymakers that lobbying firms sell to their clients. To address the question of social welfare we construct a simple general equilibrium model that includes commercial lobbying firms, and compare the equilibrium obtained under market provision of lobbying services to the first best optimum. We find that the market level of lobbying services can be socially either too large or too small, and characterize when each will be the case.

Keywords: Lobbying, Influence Activities, Information Acquisition, Financial Contributions, Commercial Lobbying Firms, Political Access

JEL classification: D72, D82.
1 Introduction

The economic literature on lobbying focuses almost entirely on the activities of special interest groups such as trade and occupational associations. These groups are motivated to try to influence the political process because they are directly affected, either ideologically or via financial gain, by the policy outcomes in question.\(^1\) They are typically concerned only with the subset of policies relevant to the organizing principle around which they have coalesced.\(^2\) It is surprising then to discover that by some measures only 40% of lobbying in the US is performed by representatives of special interest groups, and that they are not responsible for the observed rapid growth in lobbying expenditures over the 1999-2008 period.\(^3\) The preponderance of lobbying activity in the US, and a great deal of lobbying activity in Europe, is performed by for-profit commercial lobbying firms that are typically neither directly affected by the policies they lobby over, nor have ideological preferences over policy outcomes.

Commercial lobbying firms sell their services as intermediaries between citizens or special interest groups and policymakers. These intermediation services include direct advocacy to branches of government, legal and political consulting, advice about the political feasibility of clients’ objectives, facilitating the formation of coalitions and grass root organizations, legislative drafting, legislative witness hearing preparation, and public relations. To our knowledge the behavior of commercial lobbying firms and their economic implications have not been analyzed in the theoretical economics literature, and it is our intent that this paper begins to fill this void.\(^4\)

The commercial lobbying industry is both large and influential.\(^5\) For example, in the

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\(^1\)See Persson and Tabellini (2000) and Grossman and Helpman (2001) for a detailed review of special interest group activities.

\(^2\)See Olson’s (1965) seminal work for the formation of special interest groups.

\(^3\)Bertrand, Bombardini, and Trebbi (2011) show that there has been a rapid growth of commercial lobbying. They show that, the share of special interest group lobbyists amongst all lobbyists has fallen from 60 percent in 1999 to 40 percent in 2008. Also, total lobbying expenses have increased by 30 percent in that time period but a constant growth for commercial lobbying revenues doubled commercial lobbying revenues. Commercial lobbying revenues accounted for 60 percent of all expenses in 2008.

\(^4\)The only empirical paper in this area is the recent working paper by Bertrand, Bombardini, and Trebbi (2011). This was written concurrently with the analysis presented here, and represents an out of sample test which confirms the bulk of our theoretical reasoning.

\(^5\)See Bertrand, Bombardini, and Trebbi (2011) for both measures of the scale of commercial lobbying and empirical analysis of their activities. Their empirical findings support many of the assumptions made in our model.
U.S. lobbying expenditures in 2010 were $3.47 billion at the federal level and there were 12,951 registered lobbyists. The majority of whom were commercial lobbyists. The top 25 commercial lobbying contracts in 2010 ranged between $1 and $10 million. These expenditures are related to the political representation of lobbying firms’ clients and exclude their clients’ political campaign contributions. Commercial lobbying firms explicitly advertise their political experience, expertise in specific political and legal areas, and political contacts. That they advertise their experience and contacts suggests that most relationships between lobbyists and policymakers are personal and typically closed to outsiders. These are features we shall incorporate into our analysis.

That lobbying is an important phenomenon in almost all Western democracies is reflected in the increase in public concern over the influence of lobbying on the political process. Recent parliamentary discussions and reforms in the United Kingdom and the European Union serve only to emphasize this point. Pressure to regulate lobbying activities has grown in most democratic countries. This regulation has taken the form of public registers, codes of conduct, and activity reports, such as those adopted recently in the United States, Canada, and Australia. Also, policy contingent payments to policymakers experience or the number of years their employees held public offices.

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6 This compares to $1.9 billion in campaign contributions reported by the US Federal Election Committee for the 2009-10 congressional electoral cycle.

7 The identities and revenues are from www.opensecret.org; a website from the Center for Responsive Politics. Their data are collected from the individual lobbying registrations and reports provided by the US Senate Office of Public Records. The Lobbying Disclosure Act (1995) requires professional lobbyists to register and report their activities on the federal level.

8 Numerous commercial lobbying firms possess their own Political Action Committees (PAC), and commercial lobbyists make campaign contributions to politicians with whom they share engagements in political issues. See the website of the Center for Responsive Politics for the identities of lobbying firms with PACs and Bertrand, Bombardini, and Trebbi (2011) for an analysis of lobbyists’ campaign contributions.

9 Many lobbying firms advertise on their websites the number of employees with Congress or House staff experience or the number of years their employees held public offices.

10 This observation was confirmed by University of Oregon Associate Vice President for Public & Government Affairs Betsy Boyd, who is in charge of the university’s political representation and registered as a lobbyist under the Lobbying Disclosure Act. In recent empirical work Blanes i Vidal, Draza and Fons-Rosen (2011) and Eggers (2010) focus on the close personal relationships of politicians and lobbyists. They use lobbyists’ former staff experience and party affiliation as an indicator for personal networks and analyze how changes in political offices affect the revenues of lobbyists.

11 The Public Administration Select Committee of the British parliament reported in 2009 about the practice of lobbying in the United Kingdom and public mistrust. The committee focused initially on lobbyists “for-hire” and emphasized the growing appearance and mistrust of commercial lobbyists in the United Kingdom. The European Parliament implemented a mandatory registration; the European Commission changed their rules in 2008 from self-regulation to a voluntary register for lobbyists. Unfortunately, data is unavailable for the United Kingdom and the European.

12 The U.S. Lobbying Disclosure Act of 1995 (LDA) regulates lobbying activities on the federal level in the United States. The regulation includes registration, frequent reports, and penalties for potential violations.
ers are widely illegal, and the amounts of campaign contributions are frequently limited. Additionally, there is a recent trend towards demands for increased transparency over policymakers’ personal income and other financial records. This increased transparency is intended to provide the public with valuable information about policymakers’ behavior. The public has also expressed concerns that professional lobbying may crowd citizens out of the political process. However, it remains the case that most Western democracies have very limited regulation of lobbying activities.\textsuperscript{13}

The conventional wisdom of lobbying is that special interest groups or citizens may have valuable information that an imperfectly informed policymaker wishes to learn. Generally as a society we care about the quality of policy decisions and benefit from this provision of information. Unfortunately, private incentives to misrepresent information may limit what a policymaker may learn from the signals sent by citizens or special interest groups, and hence the quality of their policy decisions and social welfare may suffer. It is clear that politicians are cognizant of these issues; Tony Wright MP, Chairman of the Public Administration Select Committee (PASC) of the British parliament in 2009, stated:

\begin{quote}
“Lobbying enhances democracy, but it can also subvert it. Government has accepted that it should be more open to outside interests and ideas, and this has brought benefits. But there are risks around influence and public mistrust of government, and these risks have not been managed closely enough.”\textsuperscript{14}
\end{quote}

We argue that commercial lobbying firms engage in information verification activities and exist to (at least partially) mitigate this problem. That they maximize profit and respond to standard market incentives may allow them to credibly transmit information that cannot be transmitted by special interest groups. These market incentives, which we shall formalize as an agency relationship between policymakers and lobby firms, allow

\textsuperscript{13}See Chari, Hogan, and Murphy (2010) for an extensive overview of lobbying regulations across countries.

\textsuperscript{14}Tony Wright’s MP statement is taken from a press notice by the Public Administration Select Committee (2009).
for the credible transmission of information of a given “quality” where we shall make the notion of quality precise shortly.

Our commercial lobbyists are different from the “experts” and “advocates” that have been discussed in the economic literature. Experts, advocates and commercial lobbyists all possess some form of private information, but differ in the compensation schemes that determine their payoffs and hence in their incentives. An expert usually possesses some form of private information that a policymaker wishes to learn and receives a payoff directly affected by the policymaker’s decision.\textsuperscript{15} An advocate is usually an agent who argues on behalf of a client and is rewarded on the basis of the results achieved or possibly the process by which those results are achieved.\textsuperscript{16} In contrast a commercial lobbyist is not rewarded (at least directly) for results, neither are they directly effected by the policymakers decision; a commercial lobbyist acts as an intermediary between clients and policymakers, they offer their clients access to policymakers and provide resources to policymakers in exchange for this political access.

Commercial lobbying firms typically have many clients (and many more potential clients) but have relatively fewer political contacts. It seems natural then to assume the relationship between commercial lobbyists and their clients is mediated via a competitive market on which there is a market clearing price for intermediation services, however for the relationship between the commercial lobbyists and policymakers it seems more natural to assume this is mediated via agency contracts. This again contrasts with the experts and advocates literatures where the agency relationships are between the expert or advocate and their client, here the agency relationship is between the lobbyist and the policymaker.

In addition to analyzing the consequences of special interest groups supplying information, the established literature also considers financial contributions designed either to

\textsuperscript{15} Exemplary studies are Crawford and Sobel (1982), Gilligan and Krebhiel (1989), Austen-Smith (1994), and Krishna and Morgan (2001) who study the behavior of a single or multiple experts who advise an imperfectly informed decision-maker. A biased expert has an incentive to misrepresent information, and a decision-maker may want to consult competing experts for improved decision-making. Krishna and Morgan (2001) provide a review of this literature.

\textsuperscript{16} Dewatriport and Tirole (1999) study the behavior of advocates. They point out two compensation schemes for advocates: Decision-based rewards and information-based rewards. Decision-based rewards are based on the advocates’ achievements for the client; information-based rewards are based on how outcomes were achieved. In their analysis Dewatriport and Tirole (1999) focus exclusively on decision-based rewards and show that informational benefits are maximized when each advocate argues for a specific cause.
influence electoral outcomes or to influence policy choices. We also shall allow commercial lobbyists to make financial contributions to policymakers, and, as in the existing literature these expenditures are typically distortionary. However they are distortionary for different reasons. In the model to be presented below information quality and financial contributions are substitutes. Policymakers incent commercial lobbyists to supply their preferred combination of information quality and financial contributions by offering them agency contracts that link these elements to political access.

In order to focus on the implications of commercial lobbying we shall develop a simple general equilibrium model in which there are only three types of agents: citizens, lobbying firms, and policymakers. All three agents types are assumed rational and self interested, there is no benevolent social planner (except a fictitious one used to generate a benchmark for making welfare comparisons). The number of policymakers will be determined by a constitution. Those not chosen as policymakers may decide to be citizens or lobbyists, and this choice will satisfy a simple arbitrage condition. Our model will be constructed such that there is no rationale for the formation of classical special interest groups. Singleton citizens pursuing their own private interests via the political process could in principle decline the intermediation services of lobbying firms and make direct representations to policymakers, that is be their own special interest group. However we show that this is typically not an equilibrium outcome in our model, and frequently it is not socially desirable. Policy decisions will be assumed to have private benefits to the citizens that propose them, but will also have spillover effects for the other agents in the model. These spillovers may be socially desirable or undesirable and are not perfectly observed ex ante. The role of commercial lobbying firms is to observe a signal correlated with the social desirability of any proposal, that is they “verify” them; they thus have the potential to pass along to policymakers a portfolio of proposals some of which have been verified and some not, and amongst those that have been verified a mix of proposals that they observed to be associated with positive and negative signals. That is they may pass along a portfolio of a given expected quality in the sense of the associated expected value of spillovers. Generally in our model more policy proposals is a good thing, but more verification is also desirable. For a given number of
agents in the economy there will be an optimal mix of citizens, lobbyists and policymakers. This optimal mix will typically not be achieved as a market outcome, which may involve too much or too little lobbying, furthermore given that resource allocations are market driven the second best constitution (number of policymakers) may be greater of less than at the first best optimum.\textsuperscript{17}

2 The Basic Model

We assume an economy with three types of agents: citizens, \(c\), lobbyists, \(l\), and policymakers, \(p\). The total population is of size \(T\) consisting of \(C\) citizens, \(L\) lobbyists, and \(P\) policymakers. Naturally \(C + L + P = T\). Each agent in the economy may be any of these three types. The number of agents who are policymakers will be assumed to be determined by a constitution, the residual \(T - P = L + C\) agents may choose their own type.

Each citizen is endowed with a policy proposal which if enacted by a policymaker will yield a private benefit of \(\pi^c > 0\) and a spillover to the rest of society of \(e^c\). These spillovers may be either positive or negative and are assumed to be absolutely large enough such that \(\pi^c + e^c > 0\) if \(e^c > 0\) and \(\pi^c + e^c < 0\) if \(e^c < 0\). That is proposals with positive spillovers are socially desirable, while those with negative spillovers are not. There are a number of possible interpretations for these spillovers; they may be externalities or associated with impure public goods in the traditional sense, or they may represent the public provision of capital or education etc. Total output of the economy is simply the sum of all realized private benefits plus spillovers.

The ex ante exogenous probability of a positive spillover is \(\rho(e^+)\), and the complementary probability of a negative spillover is \(\rho(e^-) = 1 - \rho(e^+)\). Overall, the expected social value of a randomly drawn proposal is assumed positive; this ensures potential progress through political decisions.

Policy proposals may be either presented directly by citizens to policymakers, or indirectly via commercial lobbying firms. We shall assume that all policy proposals that are

\textsuperscript{17} Our analysis can also explain the empirically relevant case of the simultaneous provision of information and financial contributions by competing lobbyists in equilibrium, a feature which is not present in the analysis of direct lobbying.
presented to policymakers, and are not known to be socially undesirable, are enacted and thus yield their private benefits and spillovers. In this sense policymakers are assumed to act only as “gatekeepers” whose role is to decide how to allocate their scarce time by choosing which agents proposals listen to. The solution to the policymakers problem then specifies political access rules.

2.1 Citizens

Citizens act as rational, self-interested agents.\(^{18}\) Each faces two basic decisions; whether or not to become a lobbyist, and if they decide to remain a citizen what to do with their policy proposal endowment. They have three possible choices for their endowment, attempt to gain direct access to a policymaker so as to realize its payoffs, attempt to gain access for their proposal indirectly by employing a commercial lobbying firm to act as an intermediary with a policymaker on their behalf, or do not attempt to gain access for their proposal. Direct access to a policymaker, if achieved, is costless. Employing a commercial lobbying firm to act as an intermediary requires they pay the fee of \(k\).\(^{19}\) We assume that a citizen can hire only one lobbyist. The citizen’s payoffs associated with each outcome are then

\[
\Pi^c = \begin{cases} 
\pi^c + \frac{1}{T} \sum_{c=1}^{A} e^c & \text{if } c \text{ gains access directly}, \\
\frac{1}{T} \sum_{c=1}^{A} e^c & \text{if } c \text{ gains no access}, \\
\pi^c - k + \frac{1}{T} \sum_{c=1}^{A} e^c & \text{if } c \text{ chooses lobbying firm } l \text{ and the proposal is presented}, \\
-k + \frac{1}{T} \sum_{c=1}^{A} e^c & \text{if } c \text{ chooses lobbying firm } l \text{ and the proposal is not presented}.
\end{cases}
\]

\(^{18}\)There is a large literature on lobbying as a form of rent-seeking going back to Tullock (1967), Krueger (1974), and Buchanan (1980). A recent extensive survey is supplied by Congleton, Hillman, and Konrad (2008). In our approach lobbying is not pure rent-seeking.

\(^{19}\)We assume that lobbyists are compensated for their overall services they provide to clients and not just rewarded for success. The use of “lobbying success fees” - where the lobbyist’s compensation from the client depends on the lobbyist’s success - is sensitive, since such fees are often illegal or restricted. Lobbying success fees are illegal in connection to U.S. federal government contracts - see 10 U.S.C. 2306(b) - but exceptions apply for lobbying Congress members - see the Lobbying Disclosure Act Guidance (2011) for further details. Also, 43 U.S. states prohibit the use of lobbying success fees and 3 states restrict them - see the Center for Ethics in Government’s (2010) “50 State Chart: Contingency Fees” for an overview.
Notice that we assume each member of the economy shares in the sum of total spillovers from all enacted policy proposals, \( A \); we may interpret this as supplying a public good which is subject to crowding. It follows that for \( T \) large enough we may assume that each individual neglects the effects of their own decisions on their share of spillovers.

### 2.2 Lobbying Firms

There are \( L \) commercial lobbyists each of whom constitutes a lobbying firm. Each firm accepts proposals from \( n^l \) clients and charges a lobbying service fee of \( k \) per proposal. Each lobbyist \( l \) receives political access of \( \tilde{a}^{lp} \) from policymaker \( p \). Overall, lobbyist \( l \) receives political access of \( \tilde{a}^l = \sum_{p=1}^{l} \tilde{a}^{lp} \) from their \( p^l \) political contacts, and it is this access which they sell to their citizen clients.

In return for access the lobby firms supply policymakers with the proposals of their clients and potentially also financial contributions of \( f^{lp} \) to each political contact.\(^{20,21}\) We assume that lobbying firms have expertise which allows them to investigate the potential spillovers of a policy proposal.\(^{22}\) This expertise takes the form of a verification technology which returns a signal \( x, x \in \{x^+, x^-\} \), which is positively correlated with the sign of the spillover. Formally we have, \( \rho(e^+|x^+) > \rho(e^+) \) and \( \rho(e^-|x^-) > \rho(e^-) \). Investigated proposals with a positive signal have greater expected social value than unverified proposals; verified proposals with a negative signal have negative expected social value. The expected

\(^{20}\)Financial contributions are not linked to policy outcomes here, they exist as part of the price of access paid by commercial lobbyists; for the implications of policy contingent financial contributions see Bernheim and Whinston (1986), Grossman and Helpman (1994), and Besley and Coate (2001). In general, a financial contribution is any lobbyist’s expense that benefits a policymaker but does not improve available information. This can be a monetary transfer, campaign contribution, gift, entertainment, travel, etc, etc.

\(^{21}\)The assumption that only commercial lobbyists make financial contributions is a simplification. However, Bertrand, Bombardini, and Trebbi (2011) provide empirical evidence that lobbyists make larger campaign contributions than their clients and that out-of-house lobbyists make larger ones than in-house lobbyists. They also show that lobbyists’ campaign contributions are a standard practice and can be linked to politicians with whom they overlap in lobbying and political issues.

\(^{22}\)This assumption was confirmed in an interview with a professional lobbyist, and is also supported by the conclusions of Bertrand, Bombardini, and Trebbi (2011) who construct a measure of lobbyists’ concentration in specific issues. They distinguish between “specialists” who focus on a few issues and “generalists” who are involved with a larger range of issues. They find that out-of-house lobbyists are more likely to be specialized than in-house lobbyists.
spillover of a policy proposal can be summarized by

\[
\rho(e^+|x^+)(\pi^c + s) + \rho(e^-|x^+)(\pi^c - s) > \rho(e^+)(\pi^c + s) + \rho(e^-)(\pi^c - s) \\
> 0 > \rho(e^+|x^-)(\pi^c + s) + \rho(e^-|x^-)(\pi^c - s)
\]

where \( \pm s \) is the magnitude of the spillover \( e^c \).

Verification is costly, and is represented by the increasing convex cost function \( F(m^l) \), where \( m^l \) is the number of proposals verified. In addition each proposal, whether verified or not, incurs the lobbyist a processing cost, represented by the increasing convex cost function \( G(n^l) \), where \( n^l \) is the number of proposals processed.\(^{23}\) Additionally, the verification and processing costs have the property \( F'(0) = G'(0) = 0 \).

Lobbyists also enjoy a share of aggregate spillovers, hence their payoffs are

\[
\Pi^l = kn^l - G(n^l) - F(m^l) - \sum_{p=1}^{p^l} f^{lp} + \frac{1}{T} \sum_{c=1}^{A} e^c.
\]

Notice that the lobbyists’ payoffs imply that their only direct interest in the information they pass along to policymakers operates through their contributions to total spillovers \( \frac{1}{T} \sum_{c=1}^{A} e^c \), so contrary to the conventional literature that examines information transmission, they have no incentive to misrepresent the signals they receive.\(^{24}\)

### 2.3 Policymakers

Each of the \( P \) policymakers has a given endowment of time that allows them to approve a maximum of \( A^p \) proposals. Hence policymakers in aggregate can approve a total of \( A \leq PA^p \) proposals. Each policymaker has to decide how to allocate political access across citizens and lobbyists. Policymakers do not have an independent verification technology. Nevertheless, each policymaker \( p \) can design rules \( \tilde{a}^{cp}(.) \) and \( \tilde{a}^{lp}(.) \) that determine the conditions of access for citizens and lobbyists. Financial contributions by lobbyists may be

\(^{23}\)Processing costs are any lobbyist’s expenses that do not involve the verification technology, clients must be met and paperwork filed even if the quality of a proposal is not investigated. These costs can be related to internal organizational costs or external networking costs.

a part of these access rules.\textsuperscript{25}

Policymakers receive a dollar denominated ego rent from holding office of $\theta$, potentially receive financial contributions, $f^{lp}$, from their $lp$ lobbying contacts, and enjoy a share of aggregate spillovers.\textsuperscript{26} The valuation of financial contributions is parameterized by $\alpha$ with $\alpha \in [0, 1]$; this may be interpreted in several ways, if these financial contributions are illegal or considered unethical then $\alpha$ may be interpreted as the degree of dishonesty or corruption, alternatively if the transfers are in-kind then $\alpha$ represents the efficiency of transfers. The payoff for policymaker $p$ is then

$$
\Pi^p = \theta + \alpha \sum_{l=1}^{lp} f^{lp} + \frac{1}{T} \sum_{c=1}^{A} e^c.
$$

The objective function gives a hint as to the nature of the policymaker’s problem in designing access rules; if there is a trade off between financial contributions and spillovers, as indeed we shall argue, then this will be reflected in the optimal access rules.

3 Social Welfare

The analysis begins with a characterization of the social welfare optimum, which acts as the benchmark for our analysis. The social planner maximizes ex ante social welfare by choosing the allocation of all resources in society; this includes choosing the number

\textsuperscript{25}There is a well developed literature on the purchase of access via financial contributions by special interest groups, see for example Austen-Smith (1995) and Lohmann (1995), however in each of these contributions access is purchased with the aim of transmitting information so as to influence the policymaker, here access is purchased by commercial lobbyists so as to sell this access to their clients. Another literature focuses on the strategic interaction of observable information acquisition and campaign contributions. Bennedsen and Feldmann (2006) discuss an information externality that arises when several interest groups compete via information acquisition and financial contributions. This information externality reduces interest groups’ incentives to provide policy relevant information to policymakers and results in complete separation where those interest groups with better information technologies provide only information and other interest groups provide only financial contributions. Here, information is policy specific and the empirically relevant case of a simultaneous provision of information and financial contributions by lobbyists can be explained in equilibrium. Dahm and Porteiro (2008) consider the possibility that the acquisition of informations can benefit or harm an interest group. The uncertainty of information makes financial contributions more likely or serve as “damage control” for undesirable information. Here, policymakers announce political access rules, and their preferences determine the effectiveness of each means to gain political access. The access rules reward commercial lobbyists’ effort, and induce lobbyists to provide the desired mix of information and financial contributions.

\textsuperscript{26}The only role this ego rent plays in our analysis is to motivate agents to accept the role of policymaker in the degenerate case where $\alpha = 0$. 

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of policymakers and level of lobbying activities. There are two potential social welfare optima, with either a positive or zero level of commercial lobbying. This obviously speaks to the question: Should lobbying be illegal?

3.1 The Social Welfare Function

We assume that the social planner is a utilitarian social welfare maximizer that attaches equal weights to the payoffs of all members of society. Social welfare is thus

$$\Pi^s = \sum_{c=1}^{C} \Pi^c + \sum_{l=1}^{L} \Pi^l + \sum_{p=1}^{P} \Pi^p.$$  \hspace{1cm} (3.1)

This may be written as:

$$\Pi^s = A^l \left[ \pi^c - k + \frac{1}{T} \sum_{c=1}^{A} e^c \right] + (N - A^l) \left[ -k + \frac{1}{T} \sum_{c=1}^{A} e^c \right] + A^c \left[ \pi^c + \frac{1}{T} \sum_{c=1}^{A} e^c \right]$$

$$+ (C - N - A^c) \left[ \frac{1}{T} \sum_{c=1}^{A} e^c \right] + Nk - \sum_{l=1}^{L} F(m^l) - \sum_{l=1}^{L} G(n^l) - \sum_{l=1}^{L} f^l + \frac{L}{T} \sum_{c=1}^{A} e^c$$

$$+ P\theta + \alpha \sum_{p=1}^{P} f^p + \frac{P}{T} \sum_{c=1}^{A} e^c,$$  \hspace{1cm} (3.2)

where $N$ is the total number of proposals passed to lobbying firms for potential political representation; $A^l$ is the number of policy proposals presented by lobbyists; the remaining $N - A^l$ proposals are passed to lobbying firms but are not presented to policymakers. Citizens may access policymakers directly. There are $A^c$ proposals presented by citizens; the remaining proposals are neither presented directly or indirectly by citizens. The financial contributions $f^l$ are transfers from lobbyists to policymakers.

In expected terms, (3.2) reduces to

$$E[\Pi^s] = A^c \pi^c - \sum_{l=1}^{L} F(m^l) - \sum_{l=1}^{L} G(n^l) + P\theta + (\alpha - 1) \sum_{l=1}^{L} f^l + E \left[ \sum_{c=1}^{A} e^c \right].$$  \hspace{1cm} (3.3)
The details of expected spillovers are given by

\[
E \left[ \sum_{c=1}^{A} e^c \right] = \left( A^c + \sum_{l=1}^{L} u^l \right) s \left[ \rho(e^+ - \rho(e^-) \right] + \rho(x^+) \sum_{l=1}^{L} m^l \left[ \rho(e^+ | x^+) - \rho(e^- | x^+) \right],
\]

where \( u^l \) is the number of proposals presented by lobbyist \( l \) that have not been verified.

We break our analysis of the social planner’s problem into several stages. First, we consider the social planner’s problem in the absence of commercial lobbying. With no commercial lobbying there is no verification and the problem collapses to specifying an optimal division of society’s population between citizens and policymakers. Second, we explore the social planner’s problem when commercial lobbying exists; in this eventuality the problem also includes determining the optimal number of lobbyists, their verification activities, and the financial contributions to be made to policymakers by lobbying firms. Having completed these two stages we then compare expected social welfare with and without lobbying to determine when lobbying is socially desirable.

### 3.2 Social Optimum without Commercial Lobbying

In the absence of commercial lobbying the social planner’s problem reduces to choosing the numbers of policymakers and citizens so as to maximize expected social welfare. Since there is no verification technology and the expected social value of an enacted unverified policy proposal is positive, it follows that the social planner wishes to maximize the number of policy proposals enacted.\(^{27}\) Since each agent in the economy may be either a citizen or a policymaker, it follows that if there are fewer policy proposals (citizens) than the total time of policymakers, it would be socially improving to convert a policymaker into a citizen (ignoring indivisibility issues); alternatively if the total time endowment of policymakers exceeds the number of policy proposals, then it is socially improving to convert a policymaker into a citizen. It immediately follows that the social welfare optimum in the absence of commercial lobbying may be summarized by

\(^{27}\)We are of course assuming that ego rents from holding political office are not so large such that the social welfare optimum involves the entire population consisting of unproductive policymakers!
Proposition 1. The socially optimal solution in the absence of commercial lobbying requires that the government resources equal the number of available policy proposals and citizens - \( PA^p = A \) and \( PA^p = C \).

Proof. The proof of this and all subsequent propositions and lemmas may be found in the appendix.

Given \( T = C + P \) and proposition 1, the socially optimal number of policymakers and citizens in the absence of commercial lobbying is simply

\[
P^* = \frac{T}{A^p + 1} \quad \text{and} \quad C^* = \frac{TA^p}{A^p + 1}.
\]

Maximal expected social welfare in the absence of commercial lobbying is then

\[
E[\Pi^s] = \frac{T (A^p \pi^c + s [\rho(e^+) - \rho(e^-)]) + \theta)}{A^p + 1},
\]

where \( E[e^c] = s [\rho(e^+) - \rho(e^-)] \) represents expected spillovers.

3.3 Social Optimum with Commercial Lobbying

When there are commercial lobbying firms in the economy the social planner’s problem is considerably more complex. The social planner must choose the optimal allocation of the population between the three roles of citizen, lobbyist and policymaker, the optimal level of verification of proposals by lobbing firms, the portfolios of proposals passed from lobbying firms to policymakers, the number of proposals passed to policymakers directly by citizens, and the financial contributions to be made to policymakers by lobbying firms.

As a first step we take the distribution of the population across the three roles as given and examine optimal portfolios passed from commercial lobbying firms to policymakers and their financial contributions, we then employ these results to characterize the rest of the solution to the social planner’s problem.
3.3.1 Optimal Portfolios

Suppose initially that the social planner has identified the optimal distribution of the population between policymakers, citizens and lobbyists, we then take this allocation as given and derive the optimal portfolios of policy proposals consisting of verified and unverified proposals. The maximization problem then involves choosing $m^l$ and $f^l$, as well as the allocation of political access between citizens and lobbyists, so as to maximize the objective function (3.3), or

$$
\max_{n^l, m^l, u^l, r^l, f^l, A^e} E [\Pi^s] = \left( P A^p \pi^e - \sum_{l=1}^L F(m^l) \right) - \left( \sum_{l=1}^L G(n^l) + P \theta + (\alpha - 1) \sum_{l=1}^L f^l \right)
$$

$$
+ \left( P A^p - \rho(x^+) \sum_{l=1}^L m^l \right) s \left[ \rho(e^+) - \rho(e^-) \right]
$$

$$
+ \rho(x^+) \sum_{l=1}^L m^l s \left[ \rho(e^+|x^+) - \rho(e^-|x^+) \right]
$$

(3.7)

$$
s.t. \quad n^l = m^l + u^l + r^l,
$$

(3.8)

where $u^l$ is the number of unverified but presented proposals, and $r^l$ the number of proposals that are accepted from citizens but not presented to any policymaker. (Perhaps obviously, at the social welfare optimum, $u^l = r^l = 0$.)

After substituting (3.8) into (3.7) and differentiating we obtain the first-order conditions

$$
\frac{\partial E [\Pi^s]}{\partial m^l} = - \frac{\partial F(m^l)}{\partial m^l} - \frac{\partial G(m^l + u^l + r^l)}{\partial m^l} + \rho(x^+) s \left[ \rho(e^+|x^+) - \rho(e^-|x^+) - \rho(e^+) + \rho(e^-) \right] \forall l
$$

(3.9)

with $m^l > 0$ because of $F'(0) = G'(0) = 0$,

$$
\frac{\partial E [\Pi^s]}{\partial u^l} = - \frac{\partial G(m^l + u^l + r^l)}{\partial n^l} + s \left[ \rho(e^+) - \rho(e^-) \right] \leq 0 \forall l,
$$

(3.10)

$$
\frac{\partial E [\Pi^s]}{\partial r^l} = - \frac{\partial G(m^l + u^l + r^l)}{\partial n^l} \leq 0 \forall l
$$

(3.11)
with \( r^l = 0 \) because of \( m^l > 0 \) and \( G'(\cdot) > 0 \),

\[
\frac{\partial E[\Pi^s]}{\partial f^l} = \alpha - 1 \leq 0 \forall l,
\]

\[
\frac{\partial E[\Pi^s]}{\partial A^c} = s \left[ \rho(e^+) - \rho(e^-) \right] > 0 \forall c.
\]

It is straightforward to show that the second-order conditions with respect to verification are satisfied because \( F(\cdot) \) and \( G(\cdot) \) are increasing and convex.

Given the described optimization problem and first-order conditions, we can state the following.

**Proposition 2.** If the solution to (3.7) is at a corner, then all proposals are verified by lobbyists and all access is granted to lobbyists, who present only those proposals with positive verification signals. Each lobbyist presents \( m^l^* = \frac{PA^p}{\rho e^+} \) proposals.

If there is an interior solution, then lobbyists verify \( m^* \) proposals and present only those with positive verification signals. The remaining government resources are employed to approve unverified proposals presented by citizens.

Whether or not the solution for \( m^* \) is interior has an interesting implication. At the corner solution all proposals presented are verified, that is all policymakers’ time is allocated to lobbyists with no access for citizens. There is in this sense complete, but socially desirable, political capture. At the interior solution some unverified proposals are presented to policymakers, it follows that as any proposal that passes through a lobby firm incurs a processing cost, it is socially desirable that all unverified proposals come from citizens. It follows that it is never socially desirable for lobbyists to act as pure intermediaries for any proposals, which would amount to pure distortionary rent-seeking.\(^\text{28}\)

If there is an interior solution, then the optimal number of verified proposals, \( m^* \), is a function of the spillovers’ magnitudes, the quality of the verification technology, as given by the improvement in information about spillovers, and the costs of commercial lobbying.

\(^{28}\text{For all unverified proposals it is true that } \frac{\partial E[\Pi^s]}{\partial u^l} < \frac{\partial E[\Pi^s]}{\partial A^c}.\)
as below

\[
m^* = m \begin{pmatrix}
\mathcal{m} \mathcal{s}, \rho(x^+), \rho(e^+ | x^+), \rho(e^- | x^+), \rho(e^+), \rho(e^-)
\end{pmatrix}.
\] (3.14)

Where the signs under the variables indicate the direction of the comparative statics effects.

If the magnitude of spillovers, \(s\), increases, then it is more valuable to distinguish between proposals with positive and negative spillovers and it is hence optimal to invest more resources in verification. The same holds for the verification technology. If the technology is more effective at distinguishing between proposals, then it is more valuable to invest resources in verification. Notice that at an interior solution the optimal investigation efforts \(m^*\) are invariant with respect to the number of lobbyists, \(L\), and policymakers, \(P\).

The results differ at a corner solution, where the number of policymakers and lobbyists determine the amount of verification. The amount of verification at a corner solution depends positively on the number of policymakers, \(P\), and individual political resources \(A^p\), and is decreasing in the number of lobbyists, \(L\), and the likelihood of a positive verification signal \(\rho(x^+))\).

Also note that in both the interior and corner solutions, if policymakers discount financial contributions, then these payments are not pure transfers, are therefore socially wasteful, and are set to zero.\(^{29}\) However, if financial contributions are pure transfers, then these payments are lump sum transfers and do not affect the social optimum.

3.3.2 The Socially Optimal Division of the Population

Maintaining the assumption that it is socially desirable to have a strictly positive number of commercial lobbyists we may then employ our solution for \(m^*\) to derive the following

\textbf{Proposition 3.} If commercial lobbying is socially desirable, then the optimum requires that all government resources are employed to approve proposals, \(PA^p = A\). Lobbying firms verify all policy proposals from citizens, \(m^*L = C\). Only policy proposals with a positive verification signal are passed to policymakers, \(\rho(x^+)m^* = \frac{PA^p}{L}\).

\(^{29}\)The financial contributions are discounted by the degree of dishonesty. A policymaker with a lower degree of dishonesty, low \(a\), discounts financial contributions more than a more dishonest policymaker.
Here the social planner requires that all policy proposals are verified. They then allocate the population across the roles of citizen, lobbyist and policymaker such that there are just sufficient policymakers to enact all proposals that are expected to receive a positive verification signal from lobbyists, there are sufficient citizens to pass along to lobbyists the number of proposals to give the required number with expected positive verification signals, and there are sufficient lobbyists such that if each receives and verifies \( m^* \) then the expected sum of those proposals receiving positive verification signals equals the sum of all policymakers’ time endowments.

It follows that given \( T = C + L + P \) and proposition 3, the socially optimal number of policymakers, lobbyists, and citizens in the presence of commercial lobbying is

\[
P^{**} = \frac{\rho(x^+) T m^*}{Z}, \quad L^{**} = \frac{T A^p}{Z}, \quad C^{**} = \frac{T A^p m^*}{Z},\tag{3.15}
\]

where \( Z = \rho(x^+) m^* + A^p + A^p m^* \).

It then follows that optimal expected social welfare with commercial lobbying is

\[
E[\Pi^{**}] = \frac{T (\rho(x^+) m^* (\theta + A^p (\pi^c + E [e^c|x^+])) - A^p (F(m^*) + G(m^*)))}{Z}\tag{3.16}
\]

with an expected spillover of \( E[e^c|x^+] = s [\rho(e^+|x^+) - \rho(e^-|x^+)] \).

3.4 The Social Planner’s Question: Should there be Commercial Lobbying?

Intuitively the trade-off the social planner faces in deciding whether commercial lobbying should be permissible or not is straightforward. If members of the population are lobbyists they cannot be citizens or policymakers, hence the social cost of lobbyists stems from a reduction in the number of proposals that can be presented and enacted. The benefit from lobbying follows from the improvement in information arising from verification activities and the subsequent avoidance of enacting some socially undesirable proposals. To help in making these statements more precise we first state a simple lemma.

**Lemma 1.** *The optimal size of government is larger in the absence of commercial lobbying,*
that is $P^* > P^{**}$.

Lemma 1 isn’t quite as obvious as it might first appear. In the absence of commercial lobbying policymakers approve all available proposals, and total proposals equal the sum of all policymakers’ time endowments. When there is commercial lobbying not only are some agents lobbyists, thus reducing the number of agents available to be policymakers, but also since all proposals are verified and only those receiving a positive signal passed on to policymakers, the number of citizens with proposals required to generate the proposal received for any given number of policymakers must therefore also be greater; again reducing the optimal number of policymakers.

We may now compare the payoffs in the two potential social optima, employing (3.6) and (3.16) gives

$$E[\Pi^{*}] - E[\Pi^{**}] \geq 0 \iff \left( P^* - P^{**} \right) \left( \theta + A^p \pi_c \right) \geq 0 \quad (\text{+})$$

$$+ A^p \left( P^* E[c^c] - P^{**} E[c^c|x^+] \right) + L^{**} \left( F(m^*) + G(m^*) \right) \geq 0. \quad (3.17)$$

The first term on the right hand side of (3.17) represents the pure private gains from additional policymakers, that is the ego rents earned by the policymakers themselves, and the additional private benefits realized because more policymakers can approve more proposals. The second term gives the benefits from improved information when lobbyists exist, but these are moderated by the fact that more lobbyists imply fewer policymakers. The third term simply recognizes that commercial lobbying is costly. We conclude that if the second term is absolutely larger than the sum of the first and the third, that is if lobbying improves information sufficiently, then commercial lobbying is socially desirable.

We conclude that commercial lobbying tends to be socially desirable if: the verification technology significantly improves information, ego rents and private benefits from proposals are smaller, or the lobbyists cost of processing and verifying proposals are smaller.\footnote{One may argue that policymakers’ ego rents are of secondary interest in the welfare comparison. Nonetheless, the welfare comparison can be summarized as a trade-off between the foregone private benefits from direct political access and a larger government in the absence of commercial lobbying and the social benefits from information improvement and better spillovers in the presence of commercial lobbying.}
4 The Market Outcome

In the previous section we characterized the social optimum. Here, we characterize the market equilibrium and ask if this equilibrium is socially efficient. The basic structure of preferences and technologies is the same as in the previous section. However, resource allocations are now determined by market mechanisms. There are three markets in the economy: A perfectly competitive intermediation services market on which citizens and lobbyists trade intermediation services at the equilibrium price \( k \), a political access market on which lobbyists and policymakers trade access according to rules designed by policymakers (simple agency contracts), and a labor market on which agents are allocated costlessly between the roles of citizens and lobbyists according to an arbitrage condition.\(^{31}\)

To complete the market environment we need to specify an information structure.

4.1 Information Structure

The information structure adopted is deliberately simple and somewhat artificial; but is designed to focus attention on the questions we wish to address. Here we want to explore how market incentives in the presence of commercial lobbying distort social welfare from the first best optimum. That is we are interested in how the constraints implied by the market provision of these intermediation services effect welfare. We make our informational assumptions accordingly. We assume that ex ante no agent observes the spillovers. However, all society’s members know the exogenous probabilities \( \rho(e^+) \), \( \rho(e^-) \), \( \rho(e^+|x^+) \), \( \rho(e^-|x^+) \), \( \rho(e^+|x^-) \), and \( \rho(e^-|x^-) \). Further, citizens do not observe the lobbyists’ actions or the interactions between lobbyists and policymakers. However, they can observe realized political access \( \tilde{a}^l \) and the number of clients \( n^l \) of each lobbying firm \( l \). Policymakers can

\(^{31}\)The structure of the market environment with free access to the verification technology for lobbying industry entrants but agency characteristics with access rules is supported by Bertrand, Bombardini, and Trebbi (2011) as well as Blanes i Vidal, Draca, and Fons-Rosen (2011). Both highlight the importance of personal relationships and the barriers to entry for political access rather than limited access to expertise.
observe both the verification efforts of lobbyists and the signals generated.\textsuperscript{32,33} That the number of policymakers is determined by a constitution is common knowledge.

### 4.2 Citizens

Citizens are assumed to know the structure of the economy outlined above and take the distribution of political access as given. A citizen can approach a policymaker directly at no cost. If access is granted the proposal is approved. However, some approaches may be unsuccessful, hence the citizens must calculate the probability of gaining access in computing their expected payoffs. This probability is simply, $\tilde{A}^c/(C - N)$, where $\tilde{A}^c$, is total access granted by all policymakers to citizens, and $C - N$ is the number of citizens competing for this access.

As an alternative citizens may hire commercial lobbying firms to present their policy proposals. These citizens cannot observe the lobbying firm’s activities and have to form expectations about the likelihood that their proposals will be presented. Citizens observe the amount of political access a lobbyist enjoys, $\tilde{a}^l$, and number of a lobbyist’s clients, $n^l$, hence they correctly compute the probability that their proposal will be presented to a policymaker to be $\tilde{a}^l/n^l$.

A citizen can always decide to be politically inactive, in which case the policy proposal expires, and the citizen receives only a share of spillovers. If all government resources are employed to approve policy proposals and if citizens must make their choices of whether to present their proposals prior to lobbying firms engaging in any verification activity, then citizens enjoy the same share of expected spillovers independent of their individual choices.

The citizen’s decision reduces to a maximizing the incremental payoff from this decision,

\textsuperscript{32}This may be given the interpretation that policymakers are competent and ask lobbyists for “hard facts,” which consist of research reports, from which they may deduce lobbyists’ verification activities. Bennedsen and Feldmann (2006) adopt a set up in which a policymaker asks interest groups to provide independent information from a reputable third-party. They cite in its support Milgrom and Roberts (1986), Laffont and Tirole (1990), and Bennedsen and Feldmann (2002a, b).

\textsuperscript{33}In a companion piece, Groll and Ellis (2012), we allow the verification efforts of lobbyists to be private information and explore repeated agency relationships between policymakers and lobbyists.
\( \Delta E[\Pi^c] \), where

\[
\Delta E[\Pi^c] = \begin{cases} 
\tilde{A}^c_c N \pi^c & \text{if } c \text{ chooses the direct approach}, \\
\frac{\tilde{a}^l}{n^l} \pi^c - k & \text{if } c \text{ passes the proposal to lobbyist } l, \\
0 & \text{if } c \text{ does not participate}.
\end{cases}
\]  

If citizens expect political capture by lobbyists, that is all political access goes to the lobbying firms, then the choice alternatives reduce to hiring a lobbying firm or being politically disenfranchised. Hence the demand for lobbying services satisfies

\[
\frac{\tilde{a}^l}{n^l} \pi^c - k \geq 0 \text{ for all } l.
\]  

Given the market structure for commercial lobbying, each citizen also has the opportunity to become a lobbyist.\(^{34}\) The decision, whether to enter the industry or not, depends on the citizens’ and lobbyists’ expected payoffs. More specific whether \( E[\Pi^c] \gtrless E[\Pi^l] \). We shall assume that this market is perfectly arbitrated and will present this condition after defining the payoffs to being a lobbyist.

### 4.3 Lobbyists

Commercial lobbying firms are assumed to be profit maximizers. They also receive a share of total spillovers, but we assume the contributions of their own activities to aggregate spillovers are sufficiently small such that they are neglected in their choice problems. Each faces access rules devised by policymakers that specify a portfolio of verified and unverified proposals to be presented by any given lobbying firm to any given policymaker. Policymakers may also demand financial contributions from lobbyists as part of these access rules. We assume that any lobbying firm that fails to deliver on the requirements of the access rule

\footnote{\(^{34}\) Citizens have free access to the verification technology. However, policymakers control political access. This asymmetry is supported by Bertrand, Bombardini, and Trebbi (2011) who argue that political access is the scarce resource rather than expertise. To provide some further support for a perfectly competitive lobbying industry notice that total U.S. lobbying expenditures in 2010 were $3.47 billion at the federal level and there were 12,951 registered lobbyists, yet the top 10 U.S. lobbying firms only reported revenues from lobbying services of around $252 million. This seems to suggest that the industry is not very concentrated.}
is immediately denied all access to the policymaker in question.\footnote{This is consistent with our informational assumptions here, and can be supported in a more complex informational environment via a repeated agency relationship between policymakers and commercial lobbying firms (see Groll and Ellis, 2012).} Firm $l$’s optimization problem is characterized by

$$
\max_{n^l} E[\Pi^l] = kn^l - G(n^l) - F(\bar{m}^l) - \bar{f}^l.
$$

(4.3)

Each firm $l$ must provide verification effort of $\bar{m}^l = \sum_{p=1}^{p^l} \bar{m}_{lp}$ and financial contributions of $\bar{f}^l = \sum_{p=1}^{p^l} \bar{f}_{lp}$ to its $p^l$ political contacts according to the terms of the access rules. Each firm then chooses the number of clients to take on so as to maximize profits. The first-order condition with respect to the number of clients is then

$$
k = \frac{\partial G(n^l)}{\partial n^l} 
$$

(4.4)

with $n^l > 0$ because of $G'(0) = 0$.

For the agent to choose to be a lobbyist rather than become a citizen requires $E[\Pi^l] \geq E[\Pi^c]$.

### 4.4 Policymakers

Each policymaker takes the lobbying service fee, $k$, the size of each firm, $n^l$, and the number of lobbyists, $L$, as given and determines the distribution of their political resources, $A^p$, and the access rule for lobbyists. The access rule consists of a required level of verification effort, $m_{lp}$, the number of policy proposals to be presented, and a given financial contribution, $f_{lp}$, for each lobbying firm.

A policymaker has to respect the lobbyist’s participation constraint and cannot force his $l^p$ lobbying contacts to realize economic losses. Further, the policymakers play a Nash game between themselves, that is each simply takes the access rules and behaviors of the other, $A^{-p}$, policymakers as given. We assume that policymakers value transfers at less then their dollar-for-dollar value, so $\alpha \in (0, 1]$.\footnote{The behavior of perfectly honest policymakers is discussed in the Appendix 8.2.} This may be because these payments are illegal and accepting them requires the use of resources to conceal their receipt, or they
may be in the form of in-kind transfers, or, perhaps, the policymaker is dishonest but has some conscience about receiving transfers. The expected payoff for a policymaker is

$$E[\Pi^p] = \theta + \alpha \sum_{l=1}^{I^p} f^{lp} + \frac{1}{T} E \left[ \sum_{c \in A^{A-p}} e^c \right] + \frac{1}{T} E \left[ \sum_{c \in A^c} e^c \right].$$

(4.5)

Given that all the policy proposals that will be presented have a positive expected spillover effect, then each policymaker always exhausts political access. Any unverified or positively verified proposal increases his share of expected spillovers. All proposals with negative verification signals are ignored. Further, each policymaker may ignore some unverified proposals, \( r^{lp} \), because of time constraints. Political access is granted by each policymaker as long as lobbying firms provide the requested verification efforts, financial contributions, and policy proposals. A policymaker knows that a citizen would provide only a single unverified proposal, but a lobbyist could provide verification efforts and financial contributions. Therefore, a policymaker has no incentive to allocate access to citizens as long as lobbyists provide enough proposals.

The policymaker’s optimization problem is characterized by

$$\max_{m^{lp}, f^{lp}, \tilde{a}^{lp}, r^{lp}} E[\Pi^p] = \theta + \alpha \sum_{l=1}^{I^p} f^{lp} + \frac{1}{T} E \left[ \sum_{c \in A^{A-p}} e^c \right]$$

$$+ \frac{1}{T} \left[ \rho(x^+) \sum_{l=1}^{I^p} m^{lp} \right] s \left[ \rho(e^+|x^+) - \rho(e^-|x^+) \right]$$

$$+ \frac{1}{T} \left[ \sum_{l=1}^{I^p} (\tilde{a}^{lp} - \rho(x^+)m^{lp}) \right] s \left[ \rho(e^+) - \rho(e^-) \right]$$

(4.6)

subject to lobbying firms’ proposal constraint (with Lagrange multiplier \( \omega^{lp} \))

$$n^l = \tilde{a}^{lp} + \rho(x^-)m^{lp} + r^{lp} + \sum_{h \neq p} m^{lh} + \sum_{h \neq p} u^{lh} + \sum_{h \neq p} r^{lh} \text{ for every } l^p$$

(4.7)

37 If there is political capture by lobbyists, then \( A^p = \sum_{l=1}^{I^p} \tilde{a}^{lp} \).

38 The number of unverified presented proposals can be written as \( u^{lp} = \tilde{a}^{lp} - \rho(x^+)m^{lp} \).
and the lobbying firms’ participation constraint (with multiplier $\lambda_{lp}$)

$$n^l k - f^{lp} - \sum_{h \neq p} f^{lh} - F(m^{lp} + \sum_{h \neq p} m^{lh}) - G(n^l) \geq E[\Pi^p] \text{ for every } l^p. \quad (4.8)$$

The first-order and associated complementary slackness conditions are

$$\frac{\partial E[\Pi^p]}{\partial m^{lp}} = \rho(x^+) \frac{s}{T} [\rho(e^+|x^+) - \rho(e^-|x^+)] - \rho(e^-) + \rho(e^+) \lambda_{lp} \frac{\partial F(.)}{\partial m^{lp}} - \rho(x^-) \omega^{lp} \leq 0 \text{ for every } l^p$$

and $\frac{\partial L}{\partial m^{lp}} m^{lp} = 0$ and $m^{lp} \geq 0$,

$$\frac{\partial E[\Pi^p]}{\partial f^{lp}} = \alpha - \lambda_{lp} \leq 0 \text{ for every } l^p \quad (4.9)$$

and $\frac{\partial E[\Pi^p]}{\partial f^{lp}} f^{lp} = 0$ and $f^{lp} \geq 0$,

$$\frac{\partial E[\Pi^p]}{\partial \tilde{a}^{lp}} = \frac{s}{T} \left[ \rho(e^+) - \rho(e^-) \right] + \frac{\partial}{\partial \tilde{a}^{lp}} \sum_{h \neq l} \tilde{a}^{hp} \left[ \rho(e^+) - \rho(e^-) \right] - \omega^{lp} \leq 0 \text{ for every } l^p, \quad (4.11)$$

and $\frac{\partial E[\Pi^p]}{\partial \tilde{a}^{lp}} \tilde{a}^{lp} = 0$ and $\tilde{a}^{lp} \geq 0$, and

$$\frac{\partial E[\Pi^p]}{\partial r^{lp}} = -\omega^{lp} \leq 0 \text{ for every } l^p,$$ \quad (4.12)

and $\frac{\partial E[\Pi^p]}{\partial r^{lp}} r^{lp} = 0$ and $r^{lp} \geq 0$.

**Lemma 2.** Each policymaker with $\alpha \neq 0$ extracts all potential resources up to the point that each lobbyist with whom he has contact is indifferent between staying in and leaving the industry.

This result is standard in the classical principal-agent literature, the only twist being that the policymakers do not compensate their agents directly but rather transfer to them an asset, access, which they sell to their citizen-clients.\(^{40}\)

\(^{39}\)Citizens and lobbyists realize the same expected share of spillovers independent of their choices, and therefore take only private benefits into account.

Proposition 4. The solution to (4.6) may take one of four possible forms dependent on parameter values:

1. If the solution is at a corner with respect to verified proposals, then all approved policy proposals received positive verification signals. All remaining rents are extracted by policymakers via financial contributions.

2. If the solution is at a corner with respect to verified and unverified proposals, then the solution to the policymaker’s problem involves lobbyists verifying $m^{co}$ proposals, which exhaust a lobbyist’s financial resources, and presenting those proposals which received a positive verification signal together with sufficient unverified proposals to exhaust access. No rents are extracted via financial contributions because of a sufficiently small $\alpha$. The amount of verification at the firm-level is determined by

$$F \left( m^{co} + \sum_{h \neq p} m^{th} \right) = n^l k - \sum_{h \neq p} f^{th} - G \left( n^l \right) - E [\Pi^c | \text{private ben.}] - E \left[ \Pi^c \right].$$

(4.13)

3. If the solution is interior with respect to verification and financial contributions, then the policymaker’s problem involves lobbyists verifying $m^#$ proposals, and presenting those proposals which received a positive verification signal together with sufficient unverified proposals to exhaust access. All remaining rents are extracted by policymakers via financial contributions. The amount of verification at the firm-level is determined by

$$\frac{\partial F(m^l)}{\partial m^l p} = \rho(x^+) \frac{s}{\alpha T} \left[ \rho(e^+|x^+) - \rho(e^-|x^+) - \rho(e^+) + \rho(e^-) \right].$$

4. If the solution is at a corner with respect to financial contributions and the following holds

$$\alpha > \rho(x^+) \frac{s}{T} \left( \rho(e^+|x^+) - \rho(e^-|x^+) - \rho(e^+) + \rho(e^-) \right),$$

then all approved proposals are unverified. All rents are extracted by policymakers via financial contributions.
Proposition 4 allows us to understand what determines the quality of information demanded of lobbyists by policymakers. Policymakers value both information quality (because of expected spillovers) and financial contributions. The lobbying firms participation and proposal constraints face the policymakers with a trade off between these two objectives. At two of the potential corner solutions the policymaker chooses either to have all proposals verified and extract any remaining rents via financial contributions, or to verify no proposals so as to maximize financial contributions. The two key elements in this choice are the valuation policymakers place on financial contributions relative to the valuation they place on spillovers (characterized by $\alpha$) and the efficacy of the verification technology in terms of how great an improvement in information is obtained from the lobbyists’ signals. At the other corner solution policymakers do not value financial contributions sufficiently and choose to exhaust lobbyists’ scarce financial resources via verification efforts. At the interior solution the policymakers equates the marginal value of extra verification, which equals the marginal opportunity cost in terms of lost financial contributions, to the marginal value of their share of additional spillovers arising from an improvement in information quality.

4.5 Market Equilibrium

The preceding sections characterize the behaviors of citizens, lobbyists and policymakers faced by market incentives and the information structure described above. We may now employ these results to derive the overall market equilibrium. This is characterized by supply equals demand in the lobbying service market, a Nash equilibrium between policymakers in selecting agency contracts to offer lobbyists, and perfect arbitrage in the market for allocating labor between lobbyists and citizens. We attain this equilibrium under the assumption of a given constitution which specifies the number of policymakers, $\bar{P}$.

4.5.1 The Commercial Lobbying and Labor Markets

On the commercial lobbying market citizens demand commercial lobbying services up to the point where their expected benefit equals the price, while each lobbying firm supplies
lobbying services up to the point where the marginal processing cost of another proposal just equals that same price, $k$; assuming symmetric lobbying firms, this leads to the equilibrium condition

$$\frac{\bar{P}Ap}{Ln_l}π^c = k = \frac{∂G(n^l)}{∂n^l} \text{ for every } l. \quad (4.14)$$

The market clearing lobbying service fee depends on the number of clients, lobbyists, government resources, private benefits from an approved proposal, and technology for processing proposals.

On the labor market, agents can choose to be citizens or lobbyists, perfect arbitrage and free entry into the lobbying market implies $E[Π^c] = E[Π^l]$. It now follows that

Lemma 3. In an equilibrium with a perfectly competitive lobbying market and a perfectly arbitrated labor market, all citizens are clients of commercial lobbying firms, that is $C = n^L$.

This is an immediate implication from the assumption that the market for lobbying services is perfectly contestable. If a citizen exists who is not a client of a lobbying firm, then he realizes no expected private benefits. Given that the costs of lobbying activities are increasing and convex that citizen can always enter the lobbying industry at a lower cost per client than pre-existing firms, hence all citizens must either be clients of lobbying firms or become lobbyists themselves.

4.5.2 The Market for Political Access

On the market for political access policymakers make take-it-or-leave-it offers to lobbyists. They require a given number or proposals be submitted, a certain percentage of which must have been verified and have received positive signals, the remainder being unverified. They also demand financial contributions. The commercial lobbying market equilibrium conditions determine the number of proposals available from lobbying firms, while proposition 4 determines the number of proposals that each lobbying firm will be required to verify by policymakers. Given these pieces of information what remains is to apply some adding up constraints to characterize equilibrium in the market for political access.
We know that provided they can supply sufficient proposals policymakers allocate all political access to lobbyists, so given that the number of policymakers is constitutionally determined, it follows that in the symmetric case the supply of access (or demand for proposals) is simply
\[ \tilde{a}^l = \frac{\bar{P}A^p}{L} \text{ for every } l. \] (4.15)

The number of clients per lobbying firm is given by
\[ n^l = m^l + u^l + r^l \text{ for every } l. \] (4.16)

However, recall that proposals that receive a negative verification signal are not presented to policymakers, so the number of proposals actually supplied (alternatively the demand for access) is given by
\[ \tilde{a}^l = \rho(x^+)m^l + u^l \text{ for every } l. \] (4.17)

Equating (4.16) to (4.18) gives the symmetric equilibrium condition for the political access market expressed in per lobby firm terms, that is
\[ \rho(x^+)m^l + u^l = \frac{\bar{P}A^p}{L} \text{ for every } l. \] (4.18)

Now applying the fact that policymakers extract all residual rents from lobby firms via financial contributions, we have that in a symmetric political access market equilibrium the sum of financial contributions paid to lobbyists by firm \( l \) is given by
\[ \bar{f}^l = n^l k^l - F(m^l) - G(n^l) \geq 0 \text{ for all } l. \] (4.19)

We may now specify the per policymaker per lobbyist contract that clears the market for political access, this consists of the triple \( \{m^l, \rho(x^+)m^l + u^l, \frac{\bar{f}^l}{P^l}\} \), where \( m^l \) ensures that given the demands of other policymakers each lobbying firm verifies \( m^\# \), \( u^l \) ensures each policymaker has no unused time, and \( \frac{\bar{f}^l}{P^l} \) extracts all residual rents.
4.5.3 Full Equilibrium

The full market equilibrium is characterized by the market equilibrium conditions discussed above and the population constraint

$$T = C + L + \bar{P},$$

(4.20)

where \(\bar{P}\) is the constitutionally determined number of policymakers. Employing (4.15), (4.21) and lemma 3 (twice) provides us with a three equations in three unknowns which may be rearranged to give the implicit solutions

$$\frac{n^\#}{1 + n^\#} \frac{\partial G(n^l)}{\partial n^l} \bigg|_{n^l = n^\#} = \frac{P A^p \pi^c}{T - \bar{P}}, \quad L^\# = \frac{T - \bar{P}}{1 + n^\#}, \text{ and } C^\# = n^\# L^\#.$$  

(4.21)

The equilibrium number of clients per firm, \(n^\#\), may be shown to be unique and positive.\(^{41}\)

Implying unique positive solutions for each of the variables in equilibrium. Exploiting these results and using (4.15) and (4.16) we may obtain the equilibrium lobbying service fee and the equilibrium level of political access per lobbying firm

$$k^\# = \frac{\partial G(n^l)}{\partial n^l} \bigg|_{n^l = n^\#} \quad \text{and} \quad \bar{\alpha}^\# = \frac{P A^p}{L^\#}.$$  

(4.22)

Political access is granted by policymakers in exchange for the presentation of portfolios of proposals with requisite expected social value and financial contributions. For the interior solution, the presented proposals consist of verified proposals with a positive verification signal, \(\rho(x^+)m^\#\), such that

$$\frac{\partial F(m^l)}{\partial m^l} \bigg|_{m^l = m^\#} = \rho(x^+) \frac{s}{\alpha T} [\rho(e^+|x^+) - \rho(e^-|x^+) - \rho(e^+) + \rho(e^-)]$$  

(4.23)

and unverified proposals described by

$$u^\# = \bar{\alpha}^\# - \rho(x^+)m^\# \geq 0.$$  

(4.24)

\(^{41}\)See the Appendix 8.1.7.\(^{42}\)If there is a corner solution to the policymaker’s problem with respect to verification, then the equilibrium verification at the firm-level is \(m^\# = \frac{\bar{\alpha}^\#}{\rho(x^+)}\) and \(u^\# = 0\). However, if there is a corner solution with
The equilibrium number of unverified unpresented proposals is therefore

\[ r^\# = n^\# - m^\# - u^\# \geq 0. \]  

(4.25)

The equilibrium amount of financial contributions per firm is

\[ f^\# = n^\# k^\# - F(m^\#) - G(n^\#) \geq 0. \]  

(4.26)

Notice that here each lobbying firm may both supply information and make financial contributions despite being engaged in competition for political access. This contrasts with Bennedsen and Feldmann (2006) where some special interest groups make financial contributions and others supply information. Which special interest group completes each task depends on which has the superior information gathering technology. It also contrasts with Dahms and Porteiro (2008) where a single special interest group supplies information and financial contributions to a single policymaker.

This completes our description of the market equilibrium with commercial lobbying, we next turn our attention to its welfare properties.

5 Comparison of the Market and Socially Optimal Outcomes

In the section 3.4 we described when the full information social welfare optimum is characterized by positive levels of commercial lobbying, that is when the solution to the social planner’s problem is interior or at the corner, which arises when the information improvement that arises from the verification technology is large. In this section we maintain this as an assumption, essentially we are assuming that the benefits from employing the lobby firms’ verification technology outweigh the costs. We begin by explaining the differences between the full information social welfare optimum and market solutions, then we investigate some policy options.

}\footnote{Respect to financial contributions, then }\ m^\#  = 0 \text{ and } u^\# = a^\#.
5.1 The Socially Optimal and Market Levels of Verification and Financial Contributions

Comparing the requests for verification and financial contributions made by a policymaker in a market environment with their socially optimal levels allows us to identify the distortions associated with an economy in which there is commercial lobbying. A social planner takes all costs and benefits of commercial lobbying into account, but each policymaker in a market environment neglects the value of spillovers to others and all costs that do not impose direct constraints on their choices. Further, the value placed on financial contributions by a policymaker provides them with an incentive to substitute financial contributions for information quality.

Using (3.9) and (4.11), the verification effort levels determined by the social planner and the verification efforts requested by policymakers relate such that

$$\frac{\partial F(m^l)}{\partial m^l} \bigg|_{m^l=m^#} = \frac{1}{\alpha T} \left( \frac{\partial F(m^l)}{\partial m^l} \bigg|_{m^l=m^*} + \frac{\partial G(n^l)}{\partial n^l} \bigg|_{n^l=m^*} \right).$$

(5.1)

As a result, we can state the following.

**Proposition 5.** Comparing the verification effort levels for the full information social welfare optimum and the requests by policymakers under the full information market outcome, we have

$$\frac{\partial F(m^l)}{\partial m^l} \bigg|_{m^l=m^*} \geq \frac{1}{\alpha T - 1} \frac{\partial G(n^l)}{\partial n^l} \bigg|_{n^l=m^*} \Rightarrow m^# \geq m^*.$$

Proposition 5 is intuitive once we recognize that there are several distortions in operation. First, each policymaker receives only a share of aggregate spillovers and therefore does not fully internalize all benefits from improved political decisions. The smaller is the policymaker’s share of expected aggregate spillovers, through a larger population $T$, the more likely is underverification at the firm level. Second, a policymaker recognizes and responds to a trade-off between verification efforts and financial contributions constrained by lobbying firms’ participation constraints. In contrast, a social planner does not respect this trade-off, and indeed when $\alpha < 1$ financial contributions are banned. The greater is the weight the policymaker places on financial contributions the more likely is underverifica-
ration at the firm-level. Finally, a policymaker ignores processing costs which are taken into account by a social planner. As a consequence, higher marginal processing costs increase the likelihood of oververification at the firm level.

Using lemma 2 and proposition 4, we can state the following.

**Lemma 4.** If \( \alpha < 1 \), then policymakers may request financial contributions that are socially inefficient.

Lemma 4 is trivially obvious since if a transfer from one agent to another is valued more by the giver than the recipient there must be a welfare loss relative to the first best, however in a second best sense inefficient transfers may be socially desirable. We know that policymakers in this structure do not fully take into account lobbyists’ costs, or fully internalize spillovers, such that they may request under or over verification of proposals as described by proposition 5 above. The value placed by policy makers on financial contributions determines, in part, the trade off that they face and hence may offset their incentive to engage in inefficient levels of verification.

### 5.2 The Social Optimum and Market Aggregates

In the preceding section we compared the market equilibrium contract offered by policymakers to citizens to the socially optimal levels of the variables determined in the contract. Here we compare the levels of market aggregates to their socially optimal equivalents.

#### 5.2.1 The Number of Lobbyists and Citizens

Recall that we are maintaining the assumption that a constitution fixes the number of policymakers at the first best optimal level. Given this, this section ask if there is too much or too little lobbying in the market equilibrium. We are not here asking if the number of lobbyists is second best optimal, but are rather seeking to characterize and understand the differences between the market outcome and the first best.

Employing proposition 5 and lemma 4, we may obtain the following

**Proposition 6.** *Relative to the full information social welfare optimum*
• If there is oververification at the firm level, then the lobbying industry is more concentrated; with fewer lobbying firms processing more total proposals.

• If there is underverification at the firm level this leads to:
  
  – a less concentrated lobbying industry with more lobbying firms processing fewer proposals in total if \( \rho(x^+)^-m^* > \rho(x^+)^-m^# + u^# \),

  – a more concentrated lobbying industry with fewer lobbying firms processing more proposals in total if \( \rho(x^+)^-m^* < \rho(x^+)^-m^# + u^# \).

If policymakers request more verification per firm than is socially optimal, then each lobbyist receives more political access than is socially optimal. More political access per firm increases the willingness to pay for a firm’s lobbying services and increases the firm’s number of clients. More clients per firm and verification efforts increases processing and verification costs, therefore the industry is more concentrated than at the socially efficient level.

Alternatively if policymakers’ requests involve underverification at the firm level then there are two possibilities The first case occurs when the policymakers’ requests for verification efforts cause lower firm verification efforts and each lobbyist receives less political access than is socially efficient, then citizens are willing to pay less for a firm’s lobbying services and the number of clients per firm decreases. Fewer clients per firm decrease verification and processing costs. As a consequence, the lobbying service fee is lower and there are more lobbyists and fewer citizens than is socially efficient. The other case involves each lobbyist receives more political access. This circumstance leads to fewer lobbyists but larger firms than is socially efficient.

It is tempting to interpret the sum of policymakers and lobbyists as the “political establishment” in which case the preceding results tell an intuitive tale. When policymakers care very little about financial contributions relative to expected spillovers, then there tend to be over verification of proposals at the firm level (proposition 5), and the political establishment will be inefficiently small (proposition 6). However, when policymakers care a lot about financial contributions relative to expected spillovers, then there will tend to
be under verification at the firm level, and if this leads to to a less concentrated lobbying industry, then the political establishment will be inefficiently large.

5.3 Social Welfare at the Market Equilibrium

Expected social welfare evaluated at the market equilibrium with self-interested policymakers and a perfectly competitive commercial lobbying market may be written as

$$E[\Pi^\#] = \bar{P} \left( \theta + \alpha f^\# L^\# \right) + \bar{PE}[e^\alpha].$$ (5.2)

Notice that this expression utilizes the fact that the policymakers extract all rents from the lobby firms, and arbitrage between the roles of lobbyist and citizen ensures that they also earn zero rents in equilibrium. What then remains are spillovers and the rents captured by policymakers. The expected spillovers are defined by

$$E[e^\alpha] = \left( \frac{m^\# \rho(x^+)}{m^\# \rho(x^+)+u^\#} \right) s[\rho(e^+|x^+)-\rho(e^-|x^+)] + \left( \frac{u^\#}{m^\# \rho(x^+)+u^\#} \right) s[\rho(e^+)-\rho(e^-)].$$

Expected spillovers, $E[e^\alpha]$, depend on the weight the policymaker places on financial contributions via the effect of $\alpha$ on $m^\#$, see (4.24).

Optimal social welfare when commercial lobbying is socially desirable, is

$$E[\Pi^{***}] = P^{**} (\theta + Ap^c) - L^{**} (F(m^*) + G(m^*)) + P^{**} E[e^c|x^+],$$ (5.3)

where all proposals are verified and only those with positive verification signals are enacted.

Continuing to maintain the assumption that the constitution fixes the number of policymakers at their full information socially optimal number, $\bar{P} = P^{**}$, allows the excess of social welfare at the full information social welfare optimum over social welfare at the market equilibrium, $E[\Pi^{***}] - E[\Pi^\#] \geq 0$ to be summarized by

$$\underbrace{\bar{P}Ap^c - L^{**} (F(m^*) + G(m^*))}_{(?)} - \alpha \underbrace{\bar{P}f^\# L^\#}_{(+) \geq 0} + \bar{P} \left( E[e^c|x^+] - E[e^\alpha] \right) \geq 0,$$ (5.4)

which can be broken down into the three terms indicated. The first term is the potential
pure private gains for citizens and lobbyists in the social welfare optimum with commercial lobbying. The second term gives the private benefits for self-interested policymakers from financial contributions in a market environment. The difference between the first and second terms has an immediate and intuitive interpretation. In the market equilibrium policymakers set contracts so as to capture all private rents from lobbyists, while arbitrage in the labor market ensured the rents enjoyed by lobbyists and citizens are equal, hence policymakers capture all private rents. It follows that the difference between the first and second terms in expression (5.4) is the difference between private rents in the first best optimum and market outcomes, and captures the effects of the distortions on these private rents. Notice that it need not be the case that private rents are greater in the first best, as the socially planner trades off private rents against the benefits from spillovers via the selection of the level of verification $m^*$. The third term in (5.4) identifies the change in aggregate spillovers due to the distortions that arises because self-interested policymakers may substitute unverified proposals with verified proposals to realize higher financial contributions. This substitution cannot increase the expected quality of information and hence the level of expected spillovers.

5.4 Welfare Improvements in the Political Structure at the Market Equilibrium

To this point we have taken the number of policymakers as fixed at their first best level, and the characteristics of these policymakers, in terms of their taste for financial contributions, as exogenous. Suppose instead that the number and nature of policymakers could vary. We might then ask; given resource allocations are market determined, how should the number of policymakers be adjusted from their first best level and what taste for financial contributions would we like these policymakers to have?

To investigate this question consider social welfare evaluated at the full market equilibrium.

\footnote{The more ambitious question asking how the market allocation compares to the second best social welfare optimum is difficult to analyze at this level of abstraction, however it is amenable to numerical analysis if functional forms are specified.}
librium given there are $\bar{P}$ policymakers

$$E[\Pi^\#] = \bar{P} \left( \theta + A^p s (\rho(e^+) - \rho(e^-)) \right) + \alpha L^\# f^\#$$

$$+ \rho(x^+) L^\# m^\# s \left( \rho(e^+ | x^+) - \rho(e^- | x^+) - \rho(e^+ + \rho(e^-) \right), \quad (5.5)$$

and suppose that here is a marginal increase for the number of policymakers, utilizing that
the verification efforts at the firm level are independent to the number of policymakers we then obtain

$$\frac{\partial E[\Pi^\#]}{\partial \bar{P}} = \left( \frac{\partial}{\partial \bar{P}} \right) \theta + A^p s (\rho(e^+) - \rho(e^-)) + \alpha L^\# \frac{\partial f^\#}{\partial \bar{P}} + \rho(x^+) L^\# m^\# s \psi, \quad (5.6)$$

where $\psi = \rho(e^+ | x^+) - \rho(e^- | x^+) - \rho(e^+ + \rho(e^-) \psi$ and $s\psi$ measures the additional expected benefit from verifying an additional policy proposal.

Expression (5.6) is perhaps best understood by considering the following: Suppose that
there is an increment to the number of policymakers, ceteris paribus, each lobbying firm
will be granted more access. More access for a given number of proposals in each lobbying
firm raises the probability that each will be presented, hence the price each citizen is willing to pay to the lobbying firm, $k$, rises. Each lobbying firm then demands more clients/proposals, however if this to not violate the adding up constraint for the total population this must imply that there are fewer (but larger) lobbying firms. From this little story we can then back out the various welfare effects. The first term on the right
hand side of (5.6) represents the direct effects of more policymakers, that is more ego rents
are realized and more, unverified, proposals can be enacted unambiguously raising social
welfare. The second term is the effect of more policymakers and therefore more access on
the per-firm rents that can be realized by lobbyists and extracted by policymakers, since
the firms receive a higher fee per proposal from citizens and then re-optimize by demanding
more clients it follows that this term is positive. The third term on the right hand side
of (5.6) is the effect of fewer lobbyists, first this tends to reduce the total rents that can
be extracted by policymakers, second as each lobbying firm continues to verify the same
number of proposals and as there are fewer firms and more access it must be the case
that the quality of information and the expected value of spillovers goes down, reducing
social welfare. We may conclude that it is welfare improving to increase the number of
policymakers above the first best if the increment to concentration of the lobbying industry
is small, if the gain in information from verification is small or if the increase in per-firm
rents from an increment to access is large.

Finally, we ask how a change of the policymakers’ quality, measured by the their taste
for financial contributions \( \alpha \), would affect social welfare at the full market equilibrium. We
have given a number of interpretation for \( \alpha \) and it follows that a change in this variable
is subject to similar interpretations; for example if it represents dishonesty then a change
in \( \alpha \) may be interpreted as a change in motoring activity or in social norms. It maybe be
that if there is heterogeneity in the population in terms of \( \alpha \) then a change in this variable
may reflect choosing different individuals to be policymakers.

Differentiating of (5.5) with respect to \( \alpha \), gives

\[
\frac{\partial E[\Pi^s#]}{\partial \alpha} = L^# \left( f^# + \alpha \frac{\partial f^#}{\partial \alpha} + \rho(x^+)s\psi \frac{\partial m^#}{\partial \alpha} \right),
\]

which can be re-written using the equilibrium conditions as

\[
\frac{\partial E[\Pi^s#]}{\partial \alpha} = L^# \left( f^# + \left( \frac{\rho(x^+)s\psi}{\alpha^2 T \frac{\partial^2 F(m^#)}{\partial m^2}} \right) \left( \frac{1}{T} - 1 \right) \right) \gg 0.
\]

We might anticipate that an increase in \( \alpha \) would be unambiguously welfare improving
as it brings closer the value to a recipient and donor of any dollar transferred. However
a change in \( \alpha \) also changes the contracts policymakers offer commercial lobbying firms.
As \( \alpha \) increases they demand greater financial contributions and require lower levels of
verification.\(^{44}\) We know that because the policymaker fails to fully internalize spillovers
and does not fully take into account lobbying firms costs such that verification at the market

\(^{44}\)See the Appendix 8.3.3 for calculations for the two effects. The result of the comparison is \( \alpha \frac{\partial f^#}{\partial m^#} < \rho(x^+)s\psi \frac{\partial m^#}{\partial \alpha} \).
outcome can be too low or high relative to the first best, it is thus unsurprising that the welfare effects of changes in $\alpha$ are ambiguous. We can however note that the general implications of (5.8) are quite intuitive, an increase in $\alpha$ tends to raise social welfare when; 

$\rho(x^+)s\psi$ is small, that is the expected cost of reducing verification is small; when $T$ is small, that is when policymakers tend to take spillovers more in to account simply because of self-interest; and, when $\frac{\partial^2 F(m#)}{\partial m^{\rho^2}}$ is large, that is the costs of verification are rapidly increasing such that there is little change in verification associated with a change in $\alpha$.

Notice that if we interpret $\alpha$ as representing the policymakers honesty or integrity, then perfectly honest policymakers may not be socially desirable.

6 Conclusion

This paper provides an analysis of the hither-to-fore neglected commercial lobbying industry. It proposes an explanation for the existence of this industry, and supplies an analysis of some of its potential impacts on social welfare.

We argue that commercial lobbying firms exist to provide an intermediation service between citizens and policymakers. Citizens are endowed with policy proposals that if enacted by policymakers have private benefits and social spillovers. The lobbying firms are argued to possess a verification technology that allows them to observe a private signal correlated with the social value of a policy proposal. This information has value to policymakers who allocate their scarce time to lobbyists in return for a portfolio that includes policy proposals of a given quality and financial contributions. This political access the lobbyists then sell to citizens by acting as their intermediaries with policymakers.

The introduction of commercial lobbyists as intermediaries between citizens and policymakers provides several new insights into lobbying and political influence. We derive conditions under which the first best optimum involves commercial lobbying and when it involves its prohibition. This depends on whether the benefits from implied improvements in policy information outweigh the foregone private benefits from direct political access for citizens plus the costs of commercial lobbying. We also demonstrate how commercial lobbying arises endogenously in a simple general equilibrium market model, and identify
potential sources of inefficiencies. We find that, relative to the first best optimum when commercial lobbying is socially desirable, the market equilibrium may involve inefficient levels of verification activity by commercial lobbying firms, and an industry that may be either too concentrated or insufficiently concentrated. We are able to provide intuitive explanations of each of these potential biases.

Throughout most of our analysis we take the number of policymakers as fixed at the first best optimal level and the preferences of these policymakers as exogenously given. In our final section we explore the possibility that a deviation from first best institutions can be welfare improving when allocations are market determined. The conclusion is affirmative but the directions of the implied institutional changes are ambiguous, which is not undesirable, as this allows us to suggest why we might anticipate seeing variations in the number of policymakers and their characteristics in different countries and to anticipate the welfare consequences of such.

Since this is, to our knowledge, the first formal economic model of the commercial lobbying industry our approach has been quite straightforward and there are many interesting aspects of the industry not explored here that remain for future research. The informational assumptions made are deliberately simple. It would be interesting to examine the same set of problems in an information structure where lobbyists' actions are not directly observable. Further there is no political competition in our analysis since all policymakers, lobbyists and citizens are homogeneous (except for their given roles). It would seem that the analysis can be extended in this direction by allowing heterogeneity amongst policymakers or citizens. Political competition based on either distributional conflict of efficiency concerns then seems possible within the structure developed above.
7 References


8 Appendix

8.1 Proofs

8.1.1 Proof of Proposition 1

For the first statement, suppose $PA^p > A$. In case of an excess of political resources, decreasing political resources by civilizing a policymaker would increase the number of available proposals, which are in expected terms welfare increasing. This holds iff $\pi^c + E[e^c] > \theta$. If this condition would not hold, then it would be optimal to have a population only consisting of policymakers. Now suppose $PA^p < A$. It is not feasible that the number of approved proposals exceeds the available government resources.

For the second statement, suppose $PA^p > C$. In case of an excess of political resources, decreasing political resources by civilizing a policymaker would increase the number of available proposals, which are accepted because of $PA^p = A$. This holds iff $\pi^c + E[e^c] > \theta$.

Now suppose $PA^p < C$. Assigning a disenfranchised citizen, who receives only a common share of aggregate spillovers, to a government office would increase the expected social welfare by $A^p (\pi^c + E[e^c]) + \theta$.

Therefore, $PA^p = A$ and $PA^p = C$.

8.1.2 Proof of Proposition 2

For the first statement, $PA^p = P$, see the proof for Proposition 1 in 8.1.1.

For the third statement, suppose $\rho(x^+)m^* L < PA^p$. This would describe an excess of political resources and approving proposals with negative verification signal would decrease expected social welfare. Social welfare could be increased by decreasing government resources and increasing the number of verified proposals supplied through more lobbyists.

Now suppose $\rho(x^+)m^* L > PA^p$. This excess of verification is socially wasteful and less verification efforts would increase welfare.

For the second statement, suppose $m^* L < C$. Increasing the number of lobbyists would increase the number of verified policy proposals. By assumption, commercial lobbying is socially efficient and outweighs the costs of citizens now being a lobbyist. Now suppose
m^*L > C. It is not feasible that there are more clients than citizens.
To summarize, it follows that PA = A, m^*L = C, and \( \rho(x^+)m^*L = PA^* \).

### 8.1.3 Proof of Lemma 1

The first statement relates to the comparison of \( P^* \) and \( P^{**} \). It follows that

\[
\begin{align*}
\frac{P^*}{T} &\quad \rho(x^+)m^*T \\
\frac{A^p}{A^p + 1} &\quad \rho(x^+)m^* + A^p + A^pm^* \\
\rho(x^+)m^* + A^p + A^pm^* &\quad A^p\rho(x^+)m^* + \rho(x^+)m^* \\
1 + m^* &\quad (\rho(x^+) - 1)m^* \\
1 &\quad \rho(x^*) \\
\end{align*}
\]

where \( \rho(x^+) \leq 1 \) and \( m^* > 0 \).

### 8.1.4 Proof of Lemma 2

If \( \alpha \in (0, 1] \), then the first term in (4.10) is positive. Suppose \( \lambda^{lp} = 0 \), then (4.10) is positive and a policymaker can increase his payoff by increasing \( f^{lp} \) because of the lobbyist’s financial resources. This contradicts \( \lambda^{lp} = 0 \).

Therefore, the lobbyists’ participation constraint has to bind for \( \alpha \neq 0 \).

### 8.1.5 Proof of Proposition 4

An auxiliary result used for the subsequent statements. Suppose a policymaker’s time resource is exhausted - \( A^p = \sum_{l=1}^{l^p} \tilde{a}^{lp} \). It follows that

\[
\frac{s}{T}[\rho(e^+) - \rho(e^-)] + \frac{s}{T} \sum_{\tilde{h} \neq l} \frac{\partial}{\partial \tilde{a}^{lp}} \left[ \rho(e^+) - \rho(e^-) \right] = 0,
\]

since one proposal crowds out another. Using (4.11) and (8.2), this can be written as

\[
\frac{\partial E[\Pi^p]}{\partial \tilde{a}^{lp}} = -\omega^{lp} \leq 0 \text{ for every } l^p.
\]
If $\omega^{lp} > 0$, then $\tilde{a}^{lp} = 0$. Because of symmetry across lobbyists and exhausted political resources it has to be true that $\omega^{lp} = 0$. Now suppose a policymaker’s resources are not exhausted - $A^p > \sum_{l=1}^{lp} \tilde{a}^{lp}$. It follows that

$$\frac{\partial E[\Pi^p]}{\partial \tilde{a}^{lp}} = \frac{s}{T} [\rho(e^+) - \rho(e^-)] - \omega^{lp} \leq 0 \text{ for every } l^p.$$  \hfill (8.4)

If $\omega^{lp} = 0$, then a policymaker would allocate more political access to lobbyists until $A^p = \sum_{l=1}^{lp} \tilde{a}^{lp}$. On the other hand, if $\omega^{lp} > 0$ and $A^p$ is not exhausted, then a policymaker would grant further access to firm $j$ with $\omega^{jp} = 0$.

For the first statement with $\omega^{lp} = 0$. If $\frac{\partial E[\Pi^p]}{\partial m^{lp}} > 0$ and no unverified proposal is approved because of an exhausted policymaker’s time constraint, then there is a corner solution with respect to verified proposals. Using lemma 2, remaining resources are extracted via financial contributions.

For the second statement with $\omega^{lp} = 0$. If $\frac{\partial E[\Pi^p]}{\partial m^{lp}} = 0$ and $\alpha$ is sufficiently small such that

$$\lambda^{lp} = \rho(x^+) \frac{s}{T} [\rho(e^+ | x^+) - \rho(e^- | x^+) - \rho(e^+) + \rho(e^-)] > \alpha,$$  \hfill (8.5)

then there is a corner solution with respect to verified and unverified proposals with $f^{lp} = 0$.

The number of verified proposals follows from

$$n^{lk} - \sum_{h \neq p} f^{lh} - F \left( m^{co} + \sum_{h \neq p} m^{lh} \right) - G \left( n^{l} \right) = E[\Pi^c | \text{private ben.}]$$  \hfill (8.6)

with $u^{lp} > 0$.

For the third statement with $\omega^{lp} = 0$. If $\frac{\partial E[\Pi^p]}{\partial m^{lp}} = \frac{\partial E[\Pi^p]}{\partial f^{lp}} = 0$, then there is an interior solution with respect to verified and unverified proposals as well as financial contributions such that

$$\frac{\partial F(\cdot)}{\partial m^{lp}} = \rho(x^+) \frac{s}{\alpha T} [\rho(e^+ | x^+) - \rho(e^- | x^+) - \rho(e^+) + \rho(e^-)]$$  \hfill (8.7)

with $u^{lp} > 0$ and $f^{lp} > 0$.

For the fourth statement. Suppose $\alpha > \rho(x^+) \frac{s}{T} [\rho(e^+ | x^+) - \rho(e^- | x^+) - \rho(e^+) + \rho(e^-)]$. In this case, the marginal benefit from a financial contribution outweighs the marginal
benefit from a verified proposal independent of the amount of verification. The policymaker extracts all resources via financial contributions and approves only unverified proposals.

8.1.6 Proof of Lemma 3

Suppose \( C > n^L \) for a symmetric market equilibrium. A discouraged realizes a private payoff of zero. Entering the lobbying industry given \( k \), he could represent another discouraged citizen and contest the lobbying market equilibrium. Operating at lower marginal processing costs, \( G'(.) \), the entrant has more resources to provide in exchange to political access.

Now suppose \( C < n^L \) for a symmetric market equilibrium. It is not feasible that there are more clients than citizens.

Therefore, \( C = n^L \).

8.1.7 Uniqueness of Equilibrium Number of Clients per Firm

Define

\[
H(n) = \frac{n}{1 + n} \frac{\partial G(n)}{\partial n}.
\]  

(8.8)

If \( H(n) \) is monotonically increasing or decreasing, then \( H(n) \) equal to a constant determines a unique \( n \). Taking the derivative of \( H(n) \) with respect to \( n \), we have

\[
\frac{\partial H(n)}{\partial n} = \frac{1}{1 + n} \frac{\partial G(n)}{\partial n} - \frac{n}{(1 + n)^2} \frac{\partial G(n)}{\partial n} + \frac{n}{1 + n} \frac{\partial^2 G(n)}{\partial n^2}.
\]  

(8.9)

The first two terms can be reduced to

\[
\frac{1}{1 + n} > \frac{n}{(1 + n)^2}
\]  

(8.10)

and therefore

\[
\frac{\partial H(n)}{\partial n} > 0
\]  

(8.11)

since \( \frac{\partial^2 G(n)}{\partial n^2} > 0 \).

Therefore the number of clients per firm is unique.
8.1.8 Proof of Proposition 5

The policymaker’s first-order condition with respect to verification can be written for the interior solution as

$$\frac{\partial F(\cdot)}{\partial ml^p} = \rho(x^+) \frac{s}{\alpha T} \left[ \rho(e^+ | x^+) - \rho(e^- | x^+) - \rho(e^+) + \rho(e^-) \right].$$  \hspace{1cm} (8.12)

To compare this to the social planner’s first-order condition with respect to verification we can replace the right-hand side with the social planner’s first-order condition. We get

$$\frac{\partial F(\cdot)}{\partial ml^p} \bigg|_{ml^p = m^\#} = \frac{1}{\alpha T} \left( \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*} + \frac{\partial G(\cdot)}{\partial nl^l} \bigg|_{nl^l = m^*} \right).$$  \hspace{1cm} (8.13)

When is $m^\# \gtrless m^*$ - i.e., $\frac{\partial F(\cdot)}{\partial ml^p} \bigg|_{ml^p = m^\#} \gtrless \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*}$ ?

Suppose $\frac{\partial G(\cdot)}{\partial nl^l} \bigg|_{nl^l = m^*} = (\alpha_1 T_1 - 1) \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*}$. Then (8.13) can be written as

$$\frac{\partial F(\cdot)}{\partial ml^p} \bigg|_{ml^p = m^\#} = \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*}. \hspace{1cm} (8.14)$$

Therefore, $m^\# = m^*$.

Suppose $\frac{\partial G(\cdot)}{\partial nl^l} \bigg|_{nl^l = m^*} < (\alpha_2 T_2 - 1) \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*}$. This can be written as $\frac{\partial G(\cdot)}{\partial nl^l} \bigg|_{nl^l = m^*} = (\alpha_2 T_2 - 1) \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*} - \epsilon$ with $\epsilon > 0$. Then (8.13) can be written as

$$\frac{\partial F(\cdot)}{\partial ml^p} \bigg|_{ml^p = m^\#} = \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*} - \frac{\epsilon}{\alpha_2 T_2}. \hspace{1cm} (8.15)$$

Therefore, $m^\# < m^*$.

Suppose $\frac{\partial G(\cdot)}{\partial nl^l} \bigg|_{nl^l = m^*} > (\alpha_3 T_3 - 1) \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*}$. This can be written as $\frac{\partial G(\cdot)}{\partial nl^l} \bigg|_{nl^l = m^*} = (\alpha_3 T_3 - 1) \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*} + \epsilon$ with $\epsilon > 0$. Then (8.13) can be written as

$$\frac{\partial F(\cdot)}{\partial ml^p} \bigg|_{ml^p = m^\#} = \frac{\partial F(\cdot)}{\partial ml^l} \bigg|_{ml^l = m^*} + \frac{\epsilon}{\alpha_3 T_3}. \hspace{1cm} (8.16)$$

Therefore, $m^\# > m^*$. 

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8.1.9 Proof of Proposition 6

Suppose $\bar{P} = P^{**}$. All political resources are exhausted and it follows that

$$\rho(x^+)L^{**}m^* = \bar{P}A^* = L^# \left( \rho(x^+)m^# + u^# \right).$$  \hspace{1cm} (8.17)

For the first statement. If $m^# > m^*$, then $L^# < L^{**}$. This is independent of $u^#$. Given the number of policymakers and the population size, it follows that $C^# > C^{**}$. Using lemma 3, we can conclude that the industry is larger, $N^# > N^{**}$, and with symmetry, firms are larger, $n^# > n^{**}$.

Therefore, the industry is more concentrated with fewer but larger firms.

For the second statement. Suppose $m^# < m^*$, then the following analysis depends on the level of $u^#$. If $\rho(x^+)m^* > \rho(x^+)m^# + u^#$, then $L^{**} < L^#$. Given the number of policymakers and the population size, it follows that $C^# < C^{**}$. Using lemma 3, we can state that the industry is smaller, $N^# < N^{**}$, and by symmetry, firms are smaller, $n^# < n^{**}$.

If $\rho(x^+)m^* < \rho(x^+)m^# + u^#$, then $L^{**} > L^#$. Given the number of policymakers and the population size, it follows that $C^# > C^{**}$. Using lemma 3, we can conclude that the industry is larger, $N^# > N^{**}$, and by symmetry, firms are larger, $n^# > n^{**}$.

Therefore, if the commercial lobbying industry is more concentrated or less concentrated than socially efficient depends on $\rho(x^+)m^* \lesssim \rho(x^+)m^# + u^#$. 
8.2 Perfectly Honest Policymakers

For the special case of perfectly honest policymakers, $\alpha = 0$, who do not value financial contributions, we can state the following.

**Proposition 7.** If lobbying contacts have sufficient lobbying resources, then each perfectly honest policymaker approves only proposals with positive verification signals.

**Proof.** Take (4.9) and suppose $\lambda^{lp} = 0$. So we can write

$$\frac{\partial E[\Pi^p]}{\partial m^{lp}} = \rho(x^+) \frac{s}{T} \left[ \rho(e^+|x^+) - \rho(e^-|x^+) - \rho(e^+) + \rho(e^-) \right] - \rho(x^-) \omega^{lp} \leq 0. \quad (8.18)$$

Further suppose $\omega^{lp} = 0$. It follows that there is a corner solution with only approved proposals with positive verification signals and exhausted political resources.

Now suppose $\omega^{lp} > 0$. In this case all available policy proposals are verified and political resources are not exhausted since proposals with negative verification signals are ignored. Therefore, only proposals with positive verification signals are approved if there are enough lobbying resources.

Each perfectly honest policymaker maximizes expected spillovers given firms’ resources. But it is not necessarily the case that this is socially efficient. A perfectly honest, but self-interested, policymaker does not internalize the costs of lobbying. Furthermore because other agents in the economy do not fully internalize spillovers, the policymaker may face a resource constraint in terms of resources to finance verification. Notice that these two distortions with respect to verification work in opposite directions creating the possibility that verification will be at the socially optimal level.

**Lemma 5.** If there are sufficient financial resources in each lobbying firm to verify all presented proposals, then a perfectly honest policymaker does not dissipate all private economic rents. However, if there are insufficient financial resources to verify all presented proposals, then perfectly honest policymakers’ demands for verification exhaust all private rents.
If revenues earned from lobbying equal or exceed the processing and requested verification costs, $\lambda_{lp} = 0$, and enough citizens hire lobbyists, $\omega_{lp} = 0$, then lobbyists realize a positive economic profit. In comparison to the case of a dishonest policymaker, a perfectly honest policymaker requests exclusively proposals with positive verification signals and does not request financial contributions to extract remaining economic lobbying rents. However, a perfectly honest policymaker’s verification demands may exhaust all lobbying rents and cause that lobbying firms break even.
8.3 Comparative Statics of Market Equilibrium

8.3.1 Jacobian

Population:

\[ g_1 = T - \bar{P} - C - L = 0. \]  \hspace{1cm} (8.19)

Political access:

\[ g_2 = \bar{P}A^p - L \left( \rho(x^+)m^l + u^l \right) = 0. \]  \hspace{1cm} (8.20)

Free entry:

\[ g_3 = n^l k - F(m^l) - G(n^l) - f^l - \frac{\rho(x^+)m^l + u^l}{n^l} \pi^c + k = 0. \]  \hspace{1cm} (8.21)
with symmetry for \( \forall \ l. \)

Demand for commercial lobbying services:

\[ g_4 = \frac{\bar{P}A^p}{LN^c} \pi^c - k = 0. \]  \hspace{1cm} (8.22)

Supply of commercial lobbying services:

\[ g_5 = \frac{\partial G(n^l)}{\partial n^l} - k = 0. \]  \hspace{1cm} (8.23)

Total number of clients:

\[ g_6 = C - n^l L = 0. \]  \hspace{1cm} (8.24)

Verification effort per firm:

\[ g_7 = \frac{\partial F(m^l)}{\partial m^lp} - \rho(x^+) \frac{s}{\alpha T} \left[ \rho(e^+|x^+) - \rho(e^-|x^+) - \rho(e^+) + \rho(e^-) \right] = 0 \]  \hspace{1cm} (8.25)
with \( m^l = \bar{P}m^{lp}. \)

Firm’s proposal portfolio:

\[ g_8 = n^l - m^l - u^l - r^l = 0. \]  \hspace{1cm} (8.26)
The general Jacobian, \( J \), of the system of equilibrium equations can be written as

\[
J = \begin{bmatrix}
\frac{\partial g_1}{\partial L} & \frac{\partial g_1}{\partial C} & \frac{\partial g_1}{\partial n^l} & \frac{\partial g_1}{\partial \pi^c} & \frac{\partial g_1}{\partial u} & \frac{\partial g_1}{\partial \rho} & \frac{\partial g_1}{\partial m^l} & \frac{\partial g_1}{\partial \gamma} & \frac{\partial g_1}{\partial t^f} \\
\frac{\partial g_2}{\partial L} & \frac{\partial g_2}{\partial C} & \frac{\partial g_2}{\partial n^l} & \frac{\partial g_2}{\partial \pi^c} & \frac{\partial g_2}{\partial u} & \frac{\partial g_2}{\partial \rho} & \frac{\partial g_2}{\partial m^l} & \frac{\partial g_2}{\partial \gamma} & \frac{\partial g_2}{\partial t^f} \\
\frac{\partial g_3}{\partial L} & \frac{\partial g_3}{\partial C} & \frac{\partial g_3}{\partial n^l} & \frac{\partial g_3}{\partial \pi^c} & \frac{\partial g_3}{\partial u} & \frac{\partial g_3}{\partial \rho} & \frac{\partial g_3}{\partial m^l} & \frac{\partial g_3}{\partial \gamma} & \frac{\partial g_3}{\partial t^f} \\
\frac{\partial g_4}{\partial L} & \frac{\partial g_4}{\partial C} & \frac{\partial g_4}{\partial n^l} & \frac{\partial g_4}{\partial \pi^c} & \frac{\partial g_4}{\partial u} & \frac{\partial g_4}{\partial \rho} & \frac{\partial g_4}{\partial m^l} & \frac{\partial g_4}{\partial \gamma} & \frac{\partial g_4}{\partial t^f} \\
\frac{\partial g_5}{\partial L} & \frac{\partial g_5}{\partial C} & \frac{\partial g_5}{\partial n^l} & \frac{\partial g_5}{\partial \pi^c} & \frac{\partial g_5}{\partial u} & \frac{\partial g_5}{\partial \rho} & \frac{\partial g_5}{\partial m^l} & \frac{\partial g_5}{\partial \gamma} & \frac{\partial g_5}{\partial t^f} \\
\frac{\partial g_6}{\partial L} & \frac{\partial g_6}{\partial C} & \frac{\partial g_6}{\partial n^l} & \frac{\partial g_6}{\partial \pi^c} & \frac{\partial g_6}{\partial u} & \frac{\partial g_6}{\partial \rho} & \frac{\partial g_6}{\partial m^l} & \frac{\partial g_6}{\partial \gamma} & \frac{\partial g_6}{\partial t^f} \\
\frac{\partial g_7}{\partial L} & \frac{\partial g_7}{\partial C} & \frac{\partial g_7}{\partial n^l} & \frac{\partial g_7}{\partial \pi^c} & \frac{\partial g_7}{\partial u} & \frac{\partial g_7}{\partial \rho} & \frac{\partial g_7}{\partial m^l} & \frac{\partial g_7}{\partial \gamma} & \frac{\partial g_7}{\partial t^f} \\
\frac{\partial g_8}{\partial L} & \frac{\partial g_8}{\partial C} & \frac{\partial g_8}{\partial n^l} & \frac{\partial g_8}{\partial \pi^c} & \frac{\partial g_8}{\partial u} & \frac{\partial g_8}{\partial \rho} & \frac{\partial g_8}{\partial m^l} & \frac{\partial g_8}{\partial \gamma} & \frac{\partial g_8}{\partial t^f} \\
\frac{\partial g_9}{\partial L} & \frac{\partial g_9}{\partial C} & \frac{\partial g_9}{\partial n^l} & \frac{\partial g_9}{\partial \pi^c} & \frac{\partial g_9}{\partial u} & \frac{\partial g_9}{\partial \rho} & \frac{\partial g_9}{\partial m^l} & \frac{\partial g_9}{\partial \gamma} & \frac{\partial g_9}{\partial t^f} \\
\frac{\partial g_{10}}{\partial L} & \frac{\partial g_{10}}{\partial C} & \frac{\partial g_{10}}{\partial n^l} & \frac{\partial g_{10}}{\partial \pi^c} & \frac{\partial g_{10}}{\partial u} & \frac{\partial g_{10}}{\partial \rho} & \frac{\partial g_{10}}{\partial m^l} & \frac{\partial g_{10}}{\partial \gamma} & \frac{\partial g_{10}}{\partial t^f}
\end{bmatrix}.
\] 

(8.27)

Given the equations above, the Jacobian can be written as

\[
J = \begin{bmatrix}
-1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\rho(x^+)m^l - u^l & 0 & 0 & 0 & -\rho(x^+)L & -L & 0 & 0 & 0 \\
0 & 0 & k - \frac{\partial G(n^l)}{\partial n^l} + \frac{\rho(x^+)m^l + u^l}{n^l} & n^l + 1 & \frac{\partial^2 F(m^l)}{\partial m^l} & \rho(x^+)\pi^c & n^l & 0 & -1 \\
-\frac{\rho AP\pi^c}{L^2 n^l} & 0 & -\frac{\rho AP\pi^c}{Ln^l} & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\partial^2 G(n^l)}{\partial m^l^2} & -1 & 0 & 0 & 0 & 0 & 0 \\
-n^l & 1 & -L & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\partial^2 F(m^l)}{\partial m^l^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 & -1 & 0 & 0
\end{bmatrix}.
\] 

(8.28)

8.3.2 Determinant of Jacobian

The determinant of the Jacobian is

\[
|J| = -\frac{\partial^2 F(m^l)}{\partial m^l^2} \left( \frac{AP\pi^c + Ln^l^2 (1 + n^l) \frac{\partial^2 G(n^l)}{\partial m^l^2}}{n^l^2} \right) < 0.
\] 

(8.29)
8.3.3 Comparative Statics

Cramer’s Rule  For the comparative statics we can use Cramer’s Rule defined by

\[
\frac{\partial y}{\partial x} = \frac{|J_y|}{|J|},
\]

where \( y \) is an endogenous parameter and \( x \) an exogenous variable.

Comparative Statics of Market Equilibrium: Government Size  Verification per firm

\[
\frac{\partial m^\#}{\partial P} = \frac{|J_m|}{|J|} = 0. \tag{8.31}
\]

Lobbyists

\[
\frac{\partial L^\#}{\partial P} = \frac{|J_L|}{|J|} = -\frac{A^p \pi c (L \ln l + \bar{P}) + L \ln^2 l \frac{\partial^2 G(n)}{\partial n^2}}{L P A^p \pi c + L \ln^2 l (1 + n) \frac{\partial^2 G(n)}{\partial n^2}} < 0. \tag{8.32}
\]

Citizens

\[
\frac{\partial C^\#}{\partial P} = \frac{|J_C|}{|J|} = \frac{L \ln l \left( A^p \pi c - n^2 \frac{\partial^2 G(n)}{\partial n^2} \right)}{L P A^p \pi c + L \ln^2 l (1 + n) \frac{\partial^2 G(n)}{\partial n^2}} \leq 0. \tag{8.33}
\]

The sign depends on \( A^p \pi c - n^2 \frac{\partial^2 G(n)}{\partial n^2} \geq 0 \).

Clients per firm

\[
\frac{\partial n^\#}{\partial P} = \frac{|J_n|}{|J|} = \frac{A^p \pi c n^l (L + L \ln l + \bar{P})}{L \left( L P A^p \pi c + L \ln^2 l (1 + n) \frac{\partial^2 G(n)}{\partial n^2} \right)} > 0. \tag{8.34}
\]

Lobbying service fee

\[
\frac{\partial k^\#}{\partial P} = \frac{|J_k|}{|J|} = \frac{A^p \pi c n^l (L + L \ln l + \bar{P}) \frac{\partial^2 G(n)}{\partial n^2}}{L \left( L P A^p \pi c + L \ln^2 l (1 + n) \frac{\partial^2 G(n)}{\partial n^2} \right)} > 0. \tag{8.35}
\]

Unverified proposals per firm

\[
\frac{\partial u^\#}{\partial P} = \frac{|J_u|}{|J|} = \frac{A^p \pi c \left( L P A^p + (L \ln l + \bar{P})(\rho(x^+ m^l + u^l)) + L \ln^2 l (A^p + A^p n^l + \rho(x^+ m^l + u^l) \frac{\partial^2 G(n)}{\partial n^2}) \right)}{L \left( L P A^p \pi c + L \ln^2 l (1 + n) \frac{\partial^2 G(n)}{\partial n^2} \right)} > 0. \tag{8.36}
\]
Financial contributions per firm

\[
\frac{\partial f^\#}{\partial \bar{P}} = \frac{\pi_t^{n_t^2} A_{t}^{p} (L + Ln_t + \bar{P}) \frac{\partial^2 G(n_t^1)}{\partial n_t^2}}{L \left( \bar{P} A_t^{p} \pi_t^{n_t^2} (1 + n_t^1) \frac{\partial^2 G(n_t^1)}{\partial n_t^2} \right)} > 0. \tag{8.37}
\]

Comparative Statics of Market Equilibrium: Dishonesty

Verification per firm

\[
\frac{\partial m^\#}{\partial \alpha} = \frac{|J_m|}{|J|} = -\rho(x^+) \frac{s}{\alpha^2 T} \frac{\psi}{\partial^2 F(m^t)} < 0. \tag{8.38}
\]

Lobbyists

\[
\frac{\partial L^\#}{\partial \alpha} = \frac{|J_L|}{|J|} = 0. \tag{8.39}
\]

Citizens

\[
\frac{\partial C^\#}{\partial \alpha} = \frac{|J_C|}{|J|} = 0. \tag{8.40}
\]

Clients per firm

\[
\frac{\partial n^\#}{\partial \alpha} = \frac{|J_n|}{|J|} = 0. \tag{8.41}
\]

Lobbying service fee

\[
\frac{\partial k^\#}{\partial \alpha} = \frac{|J_k|}{|J|} = 0. \tag{8.42}
\]

Unverified proposals per firm

\[
\frac{\partial u^\#}{\partial \alpha} = \frac{|J_u|}{|J|} = \rho(x^+) \frac{s}{\alpha^2 T} \frac{\psi}{\partial^2 F(m^t)} > 0. \tag{8.43}
\]

Financial contributions per firm

\[
\frac{\partial f^\#}{\partial \alpha} = \frac{|J_f|}{|J|} = \rho(x^+) \frac{s}{\alpha^2 T} \frac{\psi}{\partial^2 F(m^t)} > 0. \tag{8.44}
\]

Comparative Statics of Market Equilibrium: Spillovers

Verification per firm

\[
\frac{\partial m^\#}{\partial s} = \frac{|J_m|}{|J|} = \rho(x^+) \frac{1}{\alpha T} \frac{\psi}{\partial^2 F(m^t)} > 0. \tag{8.45}
\]

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Lobbyists

\[ \frac{\partial L^\#}{\partial s} = \frac{|J_L|}{|J|} = 0. \quad (8.46) \]

Citizens

\[ \frac{\partial C^\#}{\partial s} = \frac{|J_C|}{|J|} = 0. \quad (8.47) \]

Clients per firm

\[ \frac{\partial n^\#}{\partial s} = \frac{|J_n|}{|J|} = 0. \quad (8.48) \]

Lobbying service fee

\[ \frac{\partial k^\#}{\partial s} = \frac{|J_k|}{|J|} = 0. \quad (8.49) \]

Unverified proposals per firm

\[ \frac{\partial u^\#}{\partial s} = \frac{|J_u|}{|J|} = -\rho(x^+)^2 \frac{1}{\alpha T} \frac{\psi}{\partial^2 F(m^l)} \partial m^l > 0. \quad (8.50) \]

Financial contributions per firm

\[ \frac{\partial f^\#}{\partial s} = \frac{|J_f|}{|J|} = -\rho(x^+) \frac{1}{\alpha T} \frac{\partial F(m^l)}{\partial m^l} \partial F(m^l) \partial m^l < 0. \quad (8.51) \]

Comparative Statics of Market Equilibrium: Private Benefits

Verification per firm

\[ \frac{\partial m^\#}{\partial \pi^c} = \frac{|J_m|}{|J|} = 0. \quad (8.52) \]

Lobbyists

\[ \frac{\partial L^\#}{\partial \pi^c} = \frac{|J_L|}{|J|} = -\frac{\tilde{P} A^p L n^l}{\tilde{P} A^p \pi^c + L n^l (1 + n^l) \partial^2 G(n^l)} < 0. \quad (8.53) \]

Citizens

\[ \frac{\partial C^\#}{\partial \pi^c} = \frac{|J_C|}{|J|} = \frac{\tilde{P} A^p L n^l}{\tilde{P} A^p \pi^c + L n^l (1 + n^l) \partial^2 G(n^l)} > 0. \quad (8.54) \]

Clients per firm

\[ \frac{\partial n^\#}{\partial \pi^c} = \frac{|J_n|}{|J|} = \frac{\tilde{P} A^p n^l (1 + n^l)}{\tilde{P} A^p \pi^c + L n^l (1 + n^l) \partial^2 G(n^l)} > 0. \quad (8.55) \]
Lobbying service fee

\[ \frac{\partial k^t}{\partial \pi c} = \frac{|J_k|}{|J|} = \frac{\bar{P}A^p n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2} + L n^l}{\bar{P}A^p \pi c + L n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2}} > 0. \tag{8.56} \]

Unverified proposals per firm

\[ \frac{\partial u^t}{\partial \pi c} = \frac{|J_u|}{|J|} = \frac{\bar{P}A^p n^l (\rho(x^+)) m^l + u^l}{\bar{P}A^p \pi c + L n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2}} > 0. \tag{8.57} \]

Financial contributions per firm

\[ \frac{\partial f^t}{\partial \pi c} = \frac{|J_f|}{|J|} = \frac{n^l t^2 (1 + n^l) \bar{P}A^p \frac{\partial^2 G(n^l)}{\partial n^2} + L m^l}{\bar{P}A^p \pi c + L n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2}} > 0. \tag{8.58} \]

Comparative Statics of Market Equilibrium: Population

Verification per firm

\[ \frac{\partial m^t}{\partial T} = \frac{|J_m|}{|J|} = -\rho(x^+) \frac{s}{\alpha T^2} \frac{\psi}{\frac{\partial^2 F(m^l)}{\partial m^2}} < 0. \tag{8.59} \]

Lobbyists

\[ \frac{\partial L^t}{\partial T} = \frac{|J_L|}{|J|} = \frac{\bar{P}A^p \pi c + L n^l \frac{\partial^2 G(n^l)}{\partial n^2}}{\bar{P}A^p \pi c + L n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2}} > 0. \tag{8.60} \]

Citizens

\[ \frac{\partial C^t}{\partial T} = \frac{|J_C|}{|J|} = \frac{L n^l \frac{\partial^2 G(n^l)}{\partial n^2}}{\bar{P}A^p \pi c + L n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2}} > 0. \tag{8.61} \]

Clients per firm

\[ \frac{\partial n^t}{\partial T} = \frac{|J_n|}{|J|} = \frac{\bar{P}A^p \pi c n^l}{L \left( \bar{P}A^p \pi c + L n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2} \right)} < 0. \tag{8.62} \]

Lobbying service fee

\[ \frac{\partial k^t}{\partial T} = \frac{|J_k|}{|J|} = \frac{\bar{P}A^p \pi c n^l \frac{\partial^2 G(n^l)}{\partial n^2}}{L \left( \bar{P}A^p \pi c + L n^l (1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^2} \right)} < 0. \tag{8.63} \]
Unverified proposals per firm

\[
\frac{\partial u^\#}{\partial T} = \frac{|J_u|}{|J|} = -\alpha T^2 \left( \rho(x^+) m^l + u^l \right) \frac{\partial^2 F(m^l)}{\partial m^l^2} \left( \bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2} \right) \\
+ \rho(x^+) s L \psi \left( \bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2} \right) \\
\alpha L T^2 \frac{\partial^2 F(m^l)}{\partial m^l^2} \left( \bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2} \right) \leq 0. \tag{8.64}
\]

Financial contributions per firm

\[
\frac{\partial f^\#}{\partial T} = \frac{|J_f|}{|J|} = \rho(x^+) s L \psi \frac{\partial F(m^l)}{\partial m^l_i} \left( \bar{P} A^l_i \pi^c_i + Ln^l(1 + n^l_i) \frac{\partial^2 G(n^l_i)}{\partial n^l_i^2} \right) \\
- \alpha \pi^c_i n^l_i \bar{P} A^l_i T^2 \frac{\partial^2 F(m^l_i)}{\partial m^l_i^2} \frac{\partial^2 G(n^l_i)}{\partial n^l_i^2} \alpha L T^2 \frac{\partial^2 F(m^l)}{\partial m^l_i} \left( \bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2} \right) \tag{8.65}
\]

Comparative Statics of Market Equilibrium: Political Resources Verification per firm

\[
\frac{\partial m^\#}{\partial A^l} = \frac{|J_m|}{|J|} = 0. \tag{8.66}
\]

Lobbyists

\[
\frac{\partial L^\#}{\partial A^l} = \frac{|J_L|}{|J|} = - \frac{\bar{P} \pi^c Ln^l}{\bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2}} < 0. \tag{8.67}
\]

Citizens

\[
\frac{\partial C^\#}{\partial A^l} = \frac{|J_C|}{|J|} = \frac{\bar{P} \pi^c Ln^l}{\bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2}} > 0. \tag{8.68}
\]

Clients per firm

\[
\frac{\partial n^\#}{\partial A^l} = \frac{|J_n|}{|J|} = \frac{\bar{P} \pi^c n^l(1 + n^l)}{\bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2}} > 0. \tag{8.69}
\]

Lobbying service fee

\[
\frac{\partial k^\#}{\partial A^l} = \frac{|J_k|}{|J|} = \frac{\bar{P} \pi^c n^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2}}{\bar{P} A^l \pi^c + Ln^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l^2}} > 0. \tag{8.70}
\]
Unverified proposals per firm

\[
\frac{\partial u^\#}{\partial A^p} = \frac{|J_u|}{|J|} = \frac{\mathcal{P} \left( \pi^c \left( \mathcal{P} A^p + \text{Ln}^l \left( \rho(x^+)m^l + u^l \right) \right) + \text{Ln}^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l \partial n^l} \right)}{L \left( \mathcal{P} A^p \pi^c + \text{Ln}^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l \partial n^l} \right)} > 0. \tag{8.71}
\]

Financial contributions per firm

\[
\frac{\partial f^\#}{\partial A^p} = \frac{|J_f|}{|J|} = \frac{\mathcal{P} \pi^c n^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l \partial n^l}}{\mathcal{P} A^p \pi^c + \text{Ln}^l(1 + n^l) \frac{\partial^2 G(n^l)}{\partial n^l \partial n^l}} > 0. \tag{8.72}
\]