Outsourcing versus technology transfer: Hotelling meets Stackelberg

Pierce, Andrea and Sen, Debapriya

Ryerson University

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Andrea Pierce† Debapriya Sen‡

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Abstract

We consider a Hotelling duopoly with two firms A and B in the final good market. Both can produce the required intermediate good, firm B having a lower cost due to a superior technology. We compare two contracts: outsourcing (A orders the intermediate good from B) and technology transfer (B transfers its technology to A). An outsourcing order is equivalent to building an endogenous capacity and it generates a Stackelberg leadership effect for firm A which is absent in technology transfer. We show that compared to the situation of no contracts there are always Pareto improving outsourcing contracts (making both firms better off and all consumers at least weakly better off), but no Pareto improving technology transfer contracts. It is also shown that if firm B has a relatively large bargaining power in its negotiations with A, then both firms prefer technology transfer while all consumers prefer outsourcing.

Keywords: Outsourcing, Technology transfer, Hotelling duopoly, Endogenous capacity, Stackelberg effect, Pareto improving contracts

JEL Classification: D43, L11, L13

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†Department of Economics, Ryerson University, 380 Victoria Street, Toronto, Ontario M5B 2K3, Canada. Email: akpierce@ryerson.ca

‡Corresponding author. Department of Economics, Ryerson University, 380 Victoria Street, Toronto, Ontario M5B 2K3, Canada. Email: dsen@economics.ryerson.ca

1 Introduction

In this era of globalization, it has become increasingly common for firms to outsource their required inputs rather than produce them in-house. While many factors influence a firm's decision to outsource, it can be argued that outsourcing is primarily driven by cost considerations.[1] A firm will choose to outsource if the input supplier can offer a price that is lower than the firm's in-house cost. This will be the case if the supplier has a cost advantage in one or more factors of production. Such advantages can be interpreted broadly as the supplier having a superior production technology. For example, the supplier may be able to hire skilled labor at a relatively low wage or it may possess advanced machineries. It is therefore plausible that as a mode of production, transfer of technology across firms could be an alternative to outsourcing. In fact, like outsourcing, technology transfers have also grown substantially in recent years.[2] As outsourcing and technology transfer broadly serve the same purpose of enabling one firm to use the cost-efficient production process of another firm, a natural question is, what would make firms choose one of these contracts over another? A closely related question is, what are the relative effects of these contracts on the consumers? This paper seeks to address these questions in the context of imperfectly competitive markets.

There could be different possible reasons for firms to prefer outsourcing over technology transfer. For instance, the superior technology may be labor intensive and difficult to transfer due to imperfect mobility of labor. Additionally, transfer of technology may involve other barriers such as intellectual property rights laws, or large initial investments that firms may want to avoid. On the other hand, under technology transfer, a firm can produce its inputs in-house using the superior technology which gives it complete control over its production. Therefore a firm will prefer technology transfer over outsourcing if it wants to maintain a higher quality standard or if it wants to avoid the risk of relying on another firm for its inputs.

Apart from these reasons, strategic considerations play an important role in determining the nature of input production decisions of a firm. The strategic motive will be particularly dominant when the input-seeking firm competes with the supplier in the final good market. This paper aims to shed light on these strategic aspects in a model of price competition. Specifically, we consider a Hotelling duopoly[3] with two firms A and B who are located at two different end points of the unit interval in the final good market φ. Consumers are uniformly distributed in this interval and incur linear transportation costs for traveling to the end points. Any consumer buys at most one unit of φ from either A or B. We consider a production process where an intermediate good η is required to manufacture φ. Each firm can convert one unit of good η into one unit of good φ at zero cost. Both A and B can produce η, but firm B has a lower cost due to a superior technology. Outsourcing (firm

[1] See, e.g., Hummels et al. (2001) and Amiti and Wei (2005) for evidence of the growth of international outsourcing. For a discussion of diverse factors that drive outsourcing, see, e.g., Jarillo (1993), Vidal and Goetschalckx (1997), Domberger (1998) and Vagadia (2007).

[2] For some recent empirical evidence on the growth of technology transfer across firms, see, e.g., Mendi (2005), Nagaoka (2005), Branstetter et al. (2006) and Wakasugi and Ito (2009).

[3] The location in the Hotelling model can literally stand for geographic location or it could correspond to product characteristic. See, e.g., Anderson et al., (1992) and Gabszewicz and Thisse (1992) for different interpretations of the Hotelling model.

A orders η from firm B) and technology transfer (firm B transfers its technology to firm A) are two contracts that naturally arise in this situation. We study these contracts by considering unit-based pricing policies for both cases, where the unit price is determined through negotiations between firms A and B. Under an outsourcing contract, firm A can place any order with firm B at the agreed upon price. Under a technology transfer contract, firm A uses the superior technology of firm B by paying a price for each unit of production, i.e., the technology transfer contract is based on a unit royalty.[4] We denote ω to be the *effective* unit cost of firm A in any contract and compare these contracts by fixing ω. We show that these two contracts generate different strategic interaction between firms and consequently result in different prices in the final good market.

Specifically we show that compared to the case of no contracts, prices in the final good market never rise under outsourcing (Proposition 2) while this is not necessarily the case under technology transfer. There are always weakly Pareto improving outsourcing contracts that make both firms better off and no consumers worse off compared to the situation of no contracts. Moreover if the cost difference of firms is relatively large (i.e., firm B's technology is sufficiently superior), there are strictly Pareto improving outsourcing contracts that make both firms as well as all consumers better off compared to the case of no contracts (Prop 3). On the other hand, there are no Pareto improving technology transfer contracts: whenever both firms prefer technology transfer over no contracts, there are always some consumers who pay a price in the final good market that is higher than what they paid in the case of no contracts (Prop 4).

Comparing outsourcing and technology transfer contracts by fixing ω (the effective unit cost of η for firm A), it is shown that due to the difference in strategic interaction between these two contracts, there is a conflict between the incentives of firms and the interest of consumers. For any ω, (i) if both firms prefer outsourcing over technology transfer, there are always some consumers who prefer technology transfer over outsourcing and (ii) if both firms prefer technology transfer over outsourcing, there are always some consumers who prefer outsourcing over technology transfer (Prop 5). It is also shown that if the supplier firm B has a relatively large bargaining power in negotiating the terms of a contract, then both firms prefer technology transfer while all consumers prefer outsourcing (Prop 6).

An outsourcing order from A to B is equivalent to capacity building by A prior to competition in the final good market. Thus, starting from the basic framework of traditional price competition with no capacity constraints in a Hotelling duopoly, *endogenous capacity* building emerges through the outsourcing contract. The volume of this endogenous capacity plays an interesting role of information transmission because it credibly informs B that A is committed to maintain a specific market share. It provides a strategic advantage to firm A that has the effect of establishing A as the Stackelberg leader.[5] This is where *Hotelling meets Stackelberg*.

Two points are worth noting in this regard. First, capacity is not exogenously given in our model; it occurs endogenously through an outsourcing contract. Second, firm A's capacity is

[4]We consider unit pricing policies for outsourcing and unit royalty policies for technology transfer because they are most frequently observed in practice. See Robinson and Kalakota (2004) and Vagadia (2007) for evidence on outsourcing and Mendi (2005) and Nagaoka (2005) for technology transfer.

[5]When firms A and B compete in quantities as Cournot duopolists, A's outsourcing order corresponds to the Stackelberg leader output (see Baake et al., 1995; Chen et al., 2011). See Section 2 for a more detailed discussion.

in effect built by its rival firm B which compels B to know A's capacity size in advance. This is the novel feature of our model that generates the Stackelberg leadership effect. Wauthy (1996) and Boccard and Wauthy (2005) also consider the role of capacities in the Hotelling location model, but they have exogenous capacities. Capacity is endogenous in the well-known model of Kreps and Scheinkman (1983) [KS]. However, firms choose their capacities independently in KS, so unlike our model there is no prior information transmission. As capacities are either exogenous or independently chosen by firms, the Stackelberg leadership effect is absent in these papers.

To see the impact of the Stackelberg leadership effect, observe that when the contracted unit price ω of the intermediate good η is small, the Stackelberg leader's market share is larger than A's market share under no contracts. This large share is sustained in equilibrium by a lower price of firm A in the final good market φ. Given that A's commitment of Stackelberg market share is credible, B's equilibrium price is also set lower and prices of both firms fall. This results in lower profit in the market φ for firm B compared to the case of no contracts. So B would accept such an outsourcing contract only if it can obtain a large supplier profit from η to compensate for its losses in the market φ. This happens when firm B is sufficiently more efficient compared to firm A. The upshot is that when the cost difference of two firms is sufficiently large, there are strictly Pareto improving outsourcing contracts, i.e., both firms prefer outsourcing over no contracts and both set a lower price for φ that makes all consumers better off (Prop 3(I)(a)).

Under technology transfer, firm A acquires B's superior technology and produces η itself using this technology. As a result, firm B knows the quantity of η produced by A only when it receives its payments for technology transfer. As these payments are received *after* profits are realized in the final good market, the informational aspect in outsourcing is completely absent under technology transfer. Moreover since firm B's payments from the transfer depends on the demand of A, it has an incentive to ensure that A's demand is not too small. This creates a distortion that raises the effective cost of B which in turn adversely affects prices in the market φ. Due to this distortion, technology transfer contracts that are preferred by both firms necessarily make some consumers worse off. Consequently, unlike the case of outsourcing, there are no Pareto improving technology transfer contracts in relation to the situation of no contracts (Prop 4(III)(c)). It should be mentioned that the distortive effect of royalty contracts is well known in the literature. Our analysis closely follows Matsumura et al. (2010) who consider technology transfer among firms in a Hotelling model. However, they focus on the optimal contract from the viewpoint of the efficient firm, while our objective is to characterize the market outcomes for all values of ω and compare these with the corresponding outcomes under outsourcing.

To sum up, the difference between outsourcing and technology transfer in our model is driven by two factors: first, the information transmission and the subsequent Stackelberg leadership effect that leads to lower prices under outsourcing is absent under technology transfer, and second, outsourcing orders are obtained upfront before firms set their prices, so outsourcing has no distortive effect.

The paper is organized as follows. In Section 2, we discuss our model and results in relation to the existing literature. We present the model in Section 3. Three contractual settings—no contracts, outsourcing and technology transfer, are studied in Section 4. Comparison of outsourcing and technology transfer contracts is carried out in Section 5. We conclude in Section 6. Most proofs are relegated to the Appendix.

2 Related literature

As we consider outsourcing contracts between rival firms who compete in prices in the final good market, this paper is most closely related to the literature on *horizontal outsourcing* (i.e., a firm outsources to a supplier which is its competing rival in the final good market) under price competition. The existing literature has considered horizontal outsourcing under different models of price competition such as a Hotelling duopoly (Shy and Stenbacka, 2003),[6] a duopoly with differentiated products (Chen et al., 2004) and a Bertrand duopoly (Arya et al., 2008). All these papers arrive at the same conclusion: horizontal outsourcing is inefficient and leads to higher prices in the final good market. To a certain extent, we obtain similar implications under technology transfer where we show that there are no Pareto improving contracts: for any contract that makes both firms better off, there are always some consumers who are worse off. In contrast, we show that under outsourcing, prices never rise and there always exist Pareto improving contracts that make both firms better off and no consumers worse off (and sometimes all consumers better off as well). Thus our result on outsourcing sharply differs with the conclusion of the existing literature.

This difference arises because the existing literature generally treats outsourcing and technology transfer equivalently. Specifically, it overlooks the informational aspect of outsourcing which is the main focus of this paper. The information transmission and the subsequent Stackelberg leadership effect in outsourcing is driven by our presumed sequence of events. In our model, firm A places its outsourcing order of η first and then prices are set in the market φ. The papers mentioned before implicitly assume an alternative sequence where the input-seeking firm places its outsourcing order with its rival after firms set their prices in the final good market. Under this sequence, outsourcing does not transmit any information to the supplier firm prior to price competition. Since outsourcing order is not received upfront, to obtain higher profit from outsourcing the supplier has to ensure that the input-seeking firm's demand is not too small. As a result, the distortive effect of technology transfer is present in outsourcing which explains why outsourcing contracts are inefficient under the sequence assumed in the existing literature.

Having outsourcing and technology transfer in the same model enables us to identify the difference in the strategic aspects of these two industry practices, which are not generally recognized in the existing literature of horizontal outsourcing under price competition. In so doing, we present a more complete analysis of production choices of firms. In particular, this paper brings to light the Stackelberg leadership effect in this context, whose efficiency implications are very different than those obtained in the literature. As we noted, our distinct modeling aspect is the presumed sequence: firms place outsourcing orders before rather than after the product market meets. It can be justified on the ground that outsourcing orders take time to process due to logistics, transportation and stages of inspection. These factors are particularly dominant if the supplier is located offshore. If a firm places its outsourcing order after receiving its demand, it may not be able to meet its demand on time. For this reason it is natural to assume that firms negotiate and sign an outsourcing contract well in advance before the final goods market meets. However, if the nature of the input is such that it can be supplied in relatively short notice, then the presumed sequence of the existing

[6] The primary focus of Shy and Stenbacka (2003) is *vertical outsourcing* (i.e. firms outsource to an outside supplier), although they consider horizontal outsourcing as well.

literature may apply. Empirical research points out the growth of international outsourcing in manufacturing as well as service sectors (see, e.g., Egger and Stehrer, 2003; Amiti and Wei, 2009). Our modeling sequence more readily fits the manufacturing sector where input production and supply arguably involve more time compared to service-based inputs.

It should be mentioned that our approach is also consistent with the literature of outsourcing under quantity competition. In these models, the input-seeking firm chooses its outsourcing order first and then firms choose quantities in the final good market. The Stackelberg leadership effect is direct under quantity competition: by placing an outsourcing order with a rival firm in a Cournot duopoly, the input-seeking firm can establish itself as the Stackelberg leader (see Baake et al., 1995; Chen et al., 2011). The leadership effect is indirect in our model. Following the outsourcing order by firm A, equilibrium prices are formed in a way so that A's market share exactly equals the quantity of intermediate good it has ordered from B. By showing the presence of the leadership effect under price competition, this paper bridges the gap in the existing literature between the outsourcing models of quantity and price competitions.

This paper also relates to the literature of organizational choices.[7] Since in our model an outsourcing order is equivalent to endogenous capacity, it can be viewed as building of inventory. Milgrom and Roberts (1988) argue that inventory building and information on future demand are substitutes, as more information on future demand reduces the need to build a large inventory. By showing that inventory building in the form of outsourcing order may itself convey information on the intended future demand, this paper brings out an alternative interrelation between inventory and information. Our framework also closely resembles Nickerson and Vanden Bergh (1999) [NVB] who consider a two-stage Cournot duopoly game where firms choose one of the two organizational structures (market and hierarchy) before they compete. NVB show that depending on the magnitude of asset specificity, "organizational heterogeneity" (i.e., firms choosing different organizational structures) may emerge. In NVB, firms independently make their organizational choices. In our model an organizational structure is a specific form of contract (outsourcing or technology transfer) between the rival firms. So the organizational choices of firms are necessarily dependent in our model, which is the key difference with NVB. To sum up, although both papers have a duopoly model where production modes are chosen before the competition stage, the main focus of NVB is to study the role of asset specificity, while we seek to understand the strategic aspects of production modes that result from contracts between rival firms.

We conclude this section by noting that the duopoly structure of this paper applies to various situations. One important application of the model is in retailing. Our model can correspond to a retail market where firm A is primarily a retailer with an inefficient production while its competing retailer B is an efficient manufacturer. In that case, outsourcing and technology transfer will correspond to two different forms of retail contracts. Under outsourcing, only the efficient retailer is active as manufacturer while both are active under technology transfer. In our paper these contracts are based on unit prices but our basic framework can be used to consider more general retail contracts. Our model also applies to an international setting where one of the firms could be a foreign firm, or both could be multinationals competing in a third country.

[7] This literature is large (see, e.g., Riordan and Williamson, 1985; Milgrom and Roberts, 1990) and we do not attempt to summarize it here. See Williamson (2002) for a comprehensive overview.

3 The model

The final good market: The market for the final good φ is a linear city Hotelling duopoly with two firms A and B. Firm A is located at point 0 and firm B at point 1 of the unit interval $[0,1]$. Firms compete in prices.

Consumers are uniformly distributed in $[0,1]$. Any consumer buys either one unit of good φ, or buys nothing. Consumers receive utility $V > 0$ from good φ and utility 0 from not buying.

The unit cost of transportation is $\tau > 0$. For a consumer at location $x \in [0,1]$, the transportation cost to travel to firm A is τx, while the cost to travel to firm B is $\tau(1-x)$. A consumer who buys one unit of good φ from either A or B gets utility V, pays the price and incurs the cost of transportation.

Let $p_A, p_B \geq 0$ be the prices set by firms A, B and denote $p \equiv (p_A, p_B)$. Given any p, let $u_x^p(i)$ be the net utility of the consumer at location $x \in [0,1]$ from purchasing good φ from firm i, so that

$$u_x^p(i) = \begin{cases} V - p_A - \tau x & \text{if } i = A \\ V - p_B - \tau(1-x) & \text{if } i = B \end{cases} \tag{1}$$

We assume that V is a sufficiently large positive number, i.e., consumers receive a large utility from purchasing good φ, so that not buying the good is never an optimal choice. Under this assumption, any consumer buys the good from either firm A or firm B and consequently the market φ is *covered*.[8] A consumer at location x determines her optimal purchasing choice by comparing $u_x^p(A)$ and $u_x^p(B)$ from (1).

Demand of firms: Let $D_A(p)$ and $D_B(p)$ be the demand received by firms A, B when they set prices p_A, p_B. It follows from (1) that $u_x^p(A) \gtreqqless u_x^p(B) \Leftrightarrow x \lesseqqgtr \widetilde{x}(p)$ where

$$\widetilde{x}(p) = (p_B - p_A + \tau)/2\tau \tag{2}$$

So a consumer at location $x \in [0,1]$ buys from A if $x \leq \widetilde{x}(p)$ and from B if $x > \widetilde{x}(p)$. Observe that if $p_A \geq p_B + \tau$, then $\widetilde{x}(p) \leq 0$ and all consumers buy from B. On the other hand, if $p_B \geq p_A + \tau$, then $\widetilde{x}(p) \geq 1$ and all consumers buy from A. If $p_A < p_B + \tau$ and $p_B < p_A + \tau$, then $0 < \widetilde{x}(p) < 1$. In that case, consumers at location $x \in [0, \widetilde{x}(p)]$ buy from A and $x \in (\widetilde{x}(p), 1]$ from B. Hence we conclude that

$$(D_A(p), D_B(p)) = \begin{cases} (0,1) & \text{if } p_A \geq p_B + \tau \\ (1,0) & \text{if } p_B \geq p_A + \tau \\ (\widetilde{x}(p), 1 - \widetilde{x}(p)) & \text{if } p_A < p_B + \tau \text{ and } p_B < p_A + \tau \end{cases} \tag{3}$$

The intermediate good: An intermediate good η is required to produce φ. Both firms can convert one unit of good η into one unit of good φ at the same constant marginal cost, which we normalize to zero.

The constant marginal cost of production of good η is $\overline{c} > 0$ for A and $\underline{c} > 0$ for B. Firm B has a superior technology for producing η, so its cost is lower, i.e., $\underline{c} < \overline{c}$. We also assume that the costs are sufficiently small. Specifically, it is assumed that

$$0 < \underline{c} < \overline{c} < \tau \tag{4}$$

[8]When the market is covered, linear and quadratic transportation cost generates the same demands and profits.

The *effective* unit cost of η for a firm will depend on the nature of contracts that A and B have in the intermediate good market. We consider the following possibilities:

(i) No contract between A and B;

(ii) Outsourcing contract between A and B: A orders η from B;

(iii) Technology transfer from B to A: firm B transfers its superior technology of producing η to firm A.

Before formally describing the three contractual situations above, it will be useful for our analysis to introduce the Hotelling duopoly game $\mathbb{H}(c_A, c_B)$.

The Hotelling duopoly game $\mathbb{H}(c_A, c_B)$: This is the standard Hotelling duopoly game played between firms A and B in the final good market φ, where the constant unit cost of producing the intermediate good η is $c_A \geq 0$ for firm A and $c_B \geq 0$ for firm B and each firm can transform one unit of η to one unit of φ at zero cost. This game has the following stages.

Stage 1: Firms A and B simultaneously set prices $p_A, p_B \geq 0$. For any $p \equiv (p_A, p_B)$, firm i receives the demand $D_i(p)$, which is given by (3).

Stage 2: Observing $D_i(p)$, firms A and B simultaneously choose $q_A, q_B \geq 0$ where

$$q_i = \text{ the quantity of } \eta \text{ that firm } i \text{ produces in order to fulfill its demand of } \varphi$$

The *demand fulfilling* constraints are $q_A \geq D_A(p)$ and $q_B \geq D_B(p)$. If the cost of producing η is positive for firm i, optimality requires that it produces $q_i = D_i(p)$ units of η and transforms these $D_i(p)$ units to good φ to fulfill its demand. If the cost is zero, it is optimal for i to produce any $q_i \geq D_i(p)$ units of η and transform $D_i(p)$ units to good φ to fulfill its demand. In either case, the payoff (profit) functions of A and B in $\mathbb{H}(c_A, c_B)$ are $\Phi_A(p) = (p_A - c_A)D_A(p)$ and $\Phi_B(p) = (p_B - c_B)D_B(p)$.

Lemma 1 characterizes Subgame Perfect Nash Equilibrium (SPNE) of $\mathbb{H}(c_A, c_B)$.

Lemma 1 *Let $c_A, c_B < \tau$. The game $\mathbb{H}(c_A, c_B)$ has a unique SPNE. For $i \in \{A, B\}$, let $p_i(c_A, c_B)$, $D_i(c_A, c_B)$ and $\Phi_i(c_A, c_B)$ be the SPNE price, market share and profit of firm i:*

(i) $p_A = \tau + (2c_A + c_B)/3$, $p_B = \tau + (c_A + 2c_B)/3$.

(ii) $D_A = 1/2 - (c_A - c_B)/6\tau$, $D_B = 1 - D_A = 1/2 + (c_A - c_B)/6\tau$.

(iii) $\Phi_A = (3\tau - c_A + c_B)^2/18\tau$, $\Phi_B = (3\tau + c_A - c_B)^2/18\tau$.

Proof See the Appendix. ∎

4 Three contractual settings

4.1 No contracts between A and B

When there are no contracts between firms A and B in the intermediate good market η, the unit cost of producing η is $\overline{c}$ for firm A and $\underline{c}$ for firm B. Accordingly, the Hotelling duopoly game $\mathbb{H}(\overline{c}, \underline{c})$ is played between A and B in the market φ. Denote

$$\underline{\theta} \equiv (2\underline{c} + \overline{c})/3 \text{ and } \overline{\theta} \equiv (\underline{c} + 2\overline{c})/3$$

Proposition 1 *When there are no contracts between firms A and B, the Hotelling duopoly game $\mathbb{H}(\overline{c}, \underline{c})$ is played. This game has a unique SPNE. For $i \in \{A, B\}$, let p_i^0, D_i^0 and Φ_i^0 be the SPNE price, demand and profit of firm i. Then*

(i) $p_A^0 = \tau + \overline{\theta}$, $p_B^0 = \tau + \underline{\theta}$;

(ii) $D_A^0 = 1/2 - (\overline{c} - \underline{c})/6\tau$, $D_B^0 = 1 - D_A^0 = 1/2 + (\overline{c} - \underline{c})/6\tau$;

(iii) $\Phi_A^0 = (3\tau - \overline{c} + \underline{c})^2/18\tau$, $\Phi_B^0 = (3\tau + \overline{c} - \underline{c})^2/18\tau$.

Proof Follows from Lemma 1 by taking $c_A = \overline{c}$ and $c_B = \underline{c}$. ■

4.2 Outsourcing contract between A and B

When there is an outsourcing contract between A and B, firm A has two options of acquiring the intermediate good η: (i) it can order η from firm B or (ii) it can produce it in-house at unit cost $\overline{c}$. We do not impose any exclusivity restriction on outsourcing contracts. That is, firm A can order η from firm B *as well as* produce it in-house. Firm B produces its required η entirely by itself at unit cost $\underline{c}$.

We consider linear unit pricing contracts: firm B charges a constant price for each unit of η that it supplies to firm A. The unit price is determined through negotiations between firms A and B. Under outsourcing contracts, the strategic interaction between A and B is described as follows.

Negotiation stage: In the beginning, firms A and B negotiate[9] on the unit price ω at which B can supply η to A. As firm A can produce η itself at unit cost $\overline{c}$, an outsourcing contract can lower its cost of production only if $\omega < \overline{c}$. Since firm B's unit cost of η is $\overline{c}$, it obtains a positive profit as a supplier only if $\omega > \overline{c}$. For this reason, we restrict $\omega \in (\underline{c}, \overline{c})$.

If firms do not agree on a price, firm A produces the required η entirely by itself at cost $\overline{c}$ and the game $\mathbb{H}(\overline{c}, \underline{c})$ is played in the final good market. If firms agree on a price $\omega \in (\underline{c}, \overline{c})$, the game $\Gamma^S(\omega)$ is played between A and B.

The game $\Gamma^S(\omega)$: It is an extensive form game that has the following stages.

[9] Instead of explicitly modeling the bargaining process through which A and B determine ω, we completely characterize the outcomes for all possible values of ω. The solution of a particular bargaining process with specific bargaining powers of A and B can be immediately obtained from our conclusions. See Section 5.2.

Stage I: Firm A chooses the amount $K \in [0, 1]$ of η to order[10] from firm B. Firm A receives K units of η by paying ωK to firm B and B obtains the supplier profit $(\omega - \underline{c})K$.

Stage II: Firms A, B play the Hotelling duopoly game $\mathbb{H}^K(\bar{c}, \underline{c})$ that has the following stages.

Stage 1: Firms A, B simultaneously announce prices p_A, p_B for the final good market φ. For any $p \equiv (p_A, p_B)$, the demand received by firm $i \in \{A, B\}$ is $D_i(p)$, given by (3).

Stage 2: Observing $D_i(p)$, firms A, B simultaneously choose $q_A, q_B \geq 0$ where

$$q_i = \text{ the quantity of } \eta \text{ that firm } i \text{ produces in order to fulfill its demand of } \varphi$$

As A already has K units of η from stage 1, its demand fulfilling constraint is

$$K + q_A \geq D_A(p) \tag{5}$$

As firm B produces η entirely by itself, the corresponding constraint for B is

$$q_B \geq D_B(p) \tag{6}$$

Each firm fulfills its demand, profits are realized and the game terminates.

Since the unit cost of producing η is positive for each firm, by (6), optimality requires that firm B produces $q_B = D_B(p)$ units of η and transforms $D_B(p)$ units to good φ to fulfill its demand.

By (5), if $D_A(p) \leq K$ (firm A's demand does not exceed the amount of η it has ordered from B), then it is optimal for A to choose $q_A = 0$, (i.e., it does not produce η in-house) and transform $D_A(p)$ units of η to φ. If $D_A(p) > K$, it is optimal to choose $q_A = D_A(p) - K$ (i.e., firm A produces exactly the additional amount of η that it needs to meet its demand) and transform $D_A(p)$ units of η to φ. Therefore, $q_A = \max\{0, D_A(p) - K\}$.

Payoffs of firms in $\Gamma^S(\omega)$: We can write the payoffs of firms by using the optimal values of q_A, q_B. Firm B's payoff has three components: (i) revenue from market φ, (ii) cost of producing q_B units of η to fulfill its demand and (iii) profit from supplying K units of η to A at price ω. Since firm B's unit cost of η is $\underline{c}$ and $q_B = D_B(p)$, its payoff is

$$\pi_B^\omega(K, p) = p_B D_B(p) - \underline{c} q_B + (\omega - \underline{c})K = (p_B - \underline{c})D_B(p) + (\omega - \underline{c})K \tag{7}$$

Firm A's payoff also has three components: (i) revenue from market φ, (ii) cost of producing q_A units of η in-house to fulfill its demand and (iii) its payment to firm B for acquiring K units of η at price ω. Since firm A's unit cost of η is $\bar{c}$ and $q_A = \max\{0, D_A(p) - K\}$, its payoff is

$$\pi_A^\omega(K, p) = p_A D_A(p) - \bar{c} q_A - \omega K = \begin{cases} p_A D_A(p) - \omega K \text{ if } \ D_A(p) \leq K \\ p_A D_A(p) - \bar{c}(D_A(p) - K) - \omega K \text{ if } \ D_A(p) > K \end{cases} \tag{8}$$

We determine SPNE of $\Gamma^S(\omega)$ by backward induction. So we begin from stage II.

[10]Since the maximum demand that a firm can have is 1, there is no loss of generality in restricting $K \leq 1$. In our model firms A and B negotiate on the price ω and then A chooses the outsourcing order K. Alternatively, one can allow A and B to negotiate on both ω and K. Our qualitative conclusions remain unaltered under this alternative.

4.2.1 Stage II of $\Gamma^S(\omega)$: The Hotelling duopoly game $\mathbb{H}^K(\overline{c}, \underline{c})$

The game $\mathbb{H}^K(\overline{c}, \underline{c})$ can be viewed as a Hotelling duopoly game in which firm A has built a "capacity" of K prior to the game.[11] Specifically, in this game:

(i) Firm B has constant unit cost $\underline{c}$.

(ii) Firm A has capacity K which is commonly known between A and B. If the demand received by A does not exceed K, it can fulfill the demand at zero unit cost. However, if its demand exceeds K, A has to incur the cost $\overline{c}$ for every additional unit.

Observe from (8) that for firm A, ωK is the cost of capacity K that it pays upfront to firm B before stage II, so ωK plays no role from stage II onwards. From (8), firm A's profit in $\mathbb{H}^K(\overline{c}, \underline{c})$ is

$$\Phi_A^K(p) = \begin{cases} p_A D_A(p) \text{ if } D_A(p) \leq K \\ p_A D_A(p) - \overline{c}(D_A(p) - K) = (p_A - \overline{c}) D_A(p) + \overline{c}K \text{ if } D_A(p) > K \end{cases} \tag{9}$$

Therefore, firm A has unit cost zero if $D_A(p) \leq K$, while its effective unit cost is $\overline{c} > 0$ if $D_A(p) > K$. It follows from (7) that $(\omega - \underline{c})K$ is the profit that firm B obtains upfront before stage II and it plays no role thereafter. Ignoring these terms, from (7), firm B's profit in $\mathbb{H}^K(\overline{c}, \underline{c})$ is

$$\Phi_B^K(p) = (p_B - \underline{c}) D_B(p) \tag{10}$$

Lemma 2 characterizes SPNE of the Hotelling duopoly game $\mathbb{H}^K(\overline{c}, \underline{c})$. Recall that for $i \in \{A, B\}$, $p_i(c_A, c_B)$, $D_i(c_A, c_B)$ and $\Phi_i(c_A, c_B)$ denote the SPNE price, market share and profit of firm i in the standard Hotelling game $\mathbb{H}(c_A, c_B)$.

Lemma 2 *$\mathbb{H}^K(\overline{c}, \underline{c})$ has a unique SPNE that has the following properties, where p_A^K, p_B^K denote the prices and Φ_A^K, Φ_B^K the profits of firms A, B in the SPNE.*

(i) *If $K < D_A(\overline{c}, \underline{c})$, the prices and market shares of firms are the same as in the SPNE of $\mathbb{H}(\overline{c}, \underline{c})$, $\Phi_A^K = \Phi_A(\overline{c}, \underline{c}) + \overline{c}K$ and $\Phi_B^K = \Phi_B(\overline{c}, \underline{c})$. Firm A fully utilizes its capacity K and in addition produces $D_A(\overline{c}, \underline{c}) - K$ units of η in-house to fulfill its demand.*

(ii) *If $K > D_A(0, \underline{c})$, the prices, market shares and profits of firms are the same as in the SPNE of $\mathbb{H}(0, \underline{c})$. Firm A does not utilize $K - D_A(0, \underline{c})$ units of its capacity and does not produce η in-house.*

(iii) *If $K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})]$,*

$$p_A^K = 3\tau + \underline{c} - 4\tau K \text{ and } p_B^K = 2\tau + \underline{c} - 2\tau K \tag{11}$$

The market share of firm A is K, that of firm B is $1 - K$, $\Phi_A^K = p_A^K K$ and $\Phi_B^K = \left(p_B^K - \underline{c}\right)(1 - K)$. Firm A fully utilizes its capacity K and does not produce η in-house.

(iv) *The prices p_A^K, p_B^K and the profits Φ_A^K, Φ_B^K are continuous in K.*

[11] When $K = 0$, $\mathbb{H}^K(\overline{c}, \underline{c})$ becomes the standard Hotelling duopoly game $\mathbb{H}(\overline{c}, \underline{c})$.

Proof See the Appendix. ■

Observe that in the game $\mathbb{H}^K(\bar{c}, \underline{c})$, firm B's unit cost is always $\bar{c}$. For firm A, the minimum possible unit cost is 0 while the maximum possible unit cost is $\bar{c}$. Therefore, in an SPNE of $\mathbb{H}^K(\bar{c}, \underline{c})$ the maximum market share that firm A can have is $D_A(0, \underline{c})$ (its SPNE market share in the standard Hotelling game $\mathbb{H}(0, \underline{c})$), while the minimum market share that it can have is $D_A(\bar{c}, \underline{c})$ (the corresponding market share in $\mathbb{H}(\bar{c}, \underline{c})$).

Lemma 2 shows that if the capacity of firm A is too small $[K < D_A(\bar{c}, \underline{c})]$, building such a capacity gives A no strategic advantage in $\mathbb{H}^K(\bar{c}, \underline{c})$ and the game yields the same SPNE outcome as $\mathbb{H}(\bar{c}, \underline{c})$. On the other hand, if the capacity is too large $[K > D_A(0, \underline{c})]$, the game results in the same SPNE outcome as $\mathbb{H}(0, \underline{c})$ where part of the capacity remains unutilized (given positive cost of capacity, it is clear that building such large capacity cannot be optimal for firm A). Intermediate capacities $[D_A(\bar{c}, \underline{c}) \leq K \leq D_A(0, \underline{c})]$ have a commitment value in that for these values of K, the SPNE prices are such that the market share of A in $\mathbb{H}^K(\bar{c}, \underline{c})$ exactly equals K. As a result, the capacity is fully utilized and A does not produce η in-house. Such intermediate capacities have the effect of establishing firm A as the Stackelberg leader in the final good market.

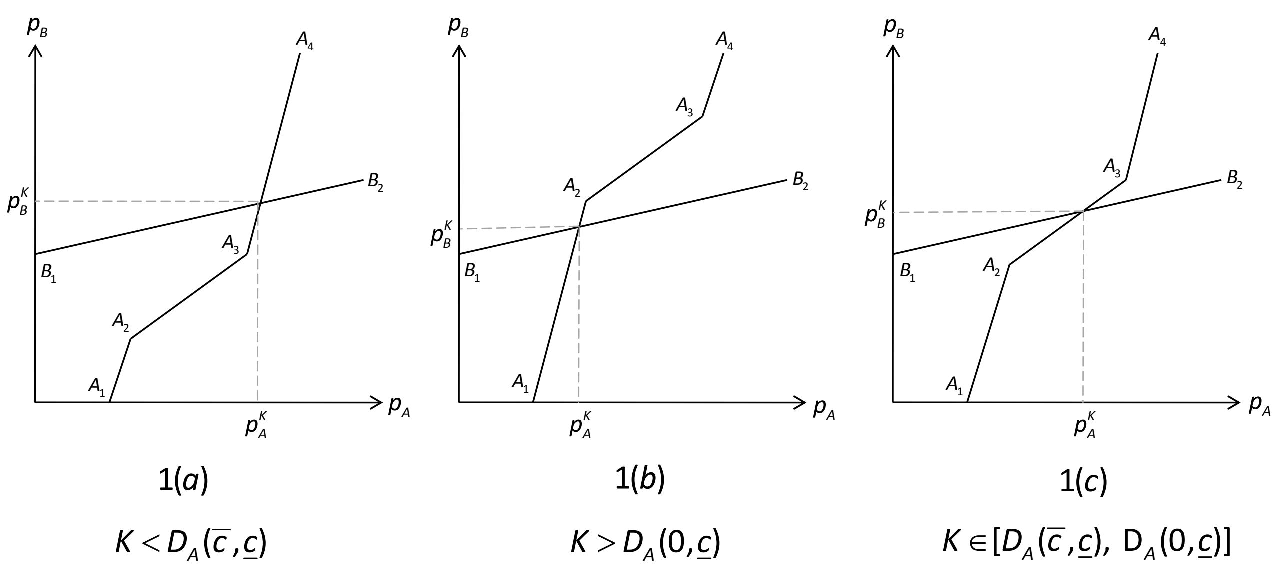

Figure 1: Hotelling Meets Stackelberg

Figure 1 illustrates this Stackelberg effect. It identifies the SPNE (p_A^K, p_B^K) of $\mathbb{H}^K(\bar{c}, \underline{c})$ for different values of K. Since firm B's unit cost is always $\underline{c}$, its best response is the same as in a standard Hotelling game, given by the line B_1B_2. Firm A's unit cost depends on its demand and its best response is a piecewise linear function that has three segments. If B's price p_B is relatively small, A's best response is the same as in the standard Hotelling game $\mathbb{H}(0, \underline{c})$, given by A_1A_2. On the other hand, if p_B is relatively large, A's best response is the same as in $\mathbb{H}(\overline{c}, \underline{c})$, given by A_3A_4. For intermediate values of p_B, its best response A_2A_3 is such that the demand it receives is exactly equal to its capacity K.

Figure 1(a) corresponds to the case $K < D_A(\overline{c}, \underline{c})$. For this case, B_1B_2 intersects the best response of A at the segment A_3A_4. The SPNE is the same as in $\mathbb{H}(\overline{c}, \underline{c})$. Figure 1(b) corresponds to the case $K > D_A(0, \overline{c})$ where B_1B_2 intersects the best response of A at the segment A_1A_2 and the SPNE is the same as in $\mathbb{H}(0, \underline{c})$. Figure 1(c) corresponds to $K \in [D_A(\underline{c}, \overline{c}), D_A(0, \overline{c})]$. For this case B_1B_2 intersects the best response of A at the intermediate segment A_2A_3. The SPNE (p_A^K, p_B^K) is such that firm A's market share exactly equals its capacity K, effectively establishing firm A as the Stackelberg leader in the Hotelling duopoly. This is where *Hotelling meets Stackelberg*.

4.2.2 Stage I of $\Gamma^S(\omega)$

Now we move back to stage I of $\Gamma^S(\omega)$ where firm A chooses its outsourcing order $K \in [0, 1]$ of η. For any such K, the game $\mathbb{H}^K(\overline{c}, \underline{c})$ is played in stage II whose unique SPNE is given in Lemma 2. Let Φ_A^K be the SPNE profit of firm A in $\mathbb{H}^K(\overline{c}, \underline{c})$. In any SPNE play of $\Gamma^S(\omega)$, when firm A orders K units of η from firm B in stage I, its payoff is $\Phi_A^K - \omega K$ (its SPNE profit in $\mathbb{H}^K(\overline{c}, \underline{c})$ net of its payment ωK that it makes to firm B). By Lemma 2, the payoff of A is

$$\pi_A^{\omega}(K) = \begin{cases} \Phi_A(\overline{c}, \underline{c}) + \overline{c}K - \omega K & \text{if } K < D_A(\overline{c}, \underline{c}) \\ p_A^K K - \omega K & \text{if } K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})] \\ \Phi_A(0, \underline{c}) - \omega K & \text{if } K > D_A(0, \underline{c}) \end{cases} \tag{12}$$

The payoff of firm B is its SPNE profit in $\mathbb{H}^K(\overline{c}, \underline{c})$ plus its input supplier profit $(\omega - \underline{c})\, K$. By Lemma 2, this payoff is

$$\pi_B^{\omega}(K) = \begin{cases} \Phi_B(\overline{c}, \underline{c}) + (\omega - \underline{c})K & \text{if } K < D_A(\overline{c}, \underline{c}) \\ \left(p_B^K - \underline{c}\right)(1 - K) + (\omega - \underline{c})K & \text{if } K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})] \\ \Phi_B(0, \underline{c}) + (\omega - \underline{c})K & \text{if } K > D_A(0, \underline{c}) \end{cases} \tag{13}$$

Observe that both functions above are continuous in K. To determine SPNE of $\Gamma^S(\omega)$, in stage I, we solve the single-person decision problem of firm A which is to choose $K \in [0, 1]$ to maximize $\pi_A^{\omega}(K)$ given by (12). In the next proposition we show the existence of SPNE of game $\Gamma^S(\omega)$ and two useful general properties of any SPNE.

Proposition 2 *For any $\omega \in (\underline{c}, \overline{c})$, SPNE of $\Gamma^S(\omega)$ exists. The following hold at any SPNE.*

(I) *The market share of firm A in the final good market φ exactly equals the quantity of η that it orders from firm B. Consequently firm A fully utilizes the amount of η that it orders from B and does not produce η in-house.*

(II) *Compared to no contracts, firm A is better off and no consumer is worse off.*

Proof Since $\omega < \overline{c}$ and $\omega > 0$, note from (12) that $\pi^{\omega}(K)$ is increasing for $K \leq D_A(\overline{c}, \underline{c})$ and decreasing for $K \geq D_A(0, \underline{c})$. So firm A's problem in stage 1 reduces to choosing $K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})]$ to maximize the continuous function $\pi^{\omega}(K)$. As this problem has a solution, SPNE of $\Gamma^S(\omega)$ exists. By Lemma 2(iii) it follows that in any SPNE, the market share of firm A equals K, which proves (I).

Observe from (12) that by choosing $K = D_A(\overline{c}, \underline{c})$ in stage 1, firm A obtains more than $\Phi_A(\overline{c}, \underline{c})$, which is its payoff under no contracts. As firm A chooses K optimally in any SPNE, it must obtain more than its no-contract payoff.

Recall from Proposition 1 that the prices p_A^0, p_B^0 under no contracts coincide with the corresponding SPNE prices of $\mathbb{H}(\overline{c}, \underline{c})$. By Lemma 2((i)&(iv)), if $K = D_A(\overline{c}, \underline{c})$ then $p_A^K = p_A^0$ and $p_B^K = p_B^0$. As p_A^K, p_B^K are both decreasing for $K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})]$ (Lemma 2(ii)), it follows that $P_A^K \leq p_A^0$ and $p_B^K \leq p_B^0$ in any SPNE, proving that no consumer is worse off. ∎

Remark 2.1 Note that in any SPNE, the prices in the final good market never rise above their corresponding no-contract levels. This result is in sharp contrast with the conclusion of the literature of horizontal outsourcing under price competition (e.g., Shy and Stenbacka, 2003; Chen et al., 2004; Arya et al., 2008).

Remark 2.2 As the equilibrium market share of firm A exactly equals the volume of A's outsourcing order, there is no unutilized capacity. This implies that even if the outsourcing contract includes the option of returning any unused good, this option will not be exercised in equilibrium. This result is particularly useful when our model is applied to a retail market duopoly, as input contracts in such markets are likely to include a return policy for unused goods.

Proposition 3 completely characterizes SPNE of $\Gamma^S(\omega)$ and gives more specific properties.

Proposition 3 *For any $\omega \in (\underline{c}, \overline{c})$, $\Gamma^S(\omega)$ has a unique SPNE. Let $K(\omega)$ be the amount of η that firm A orders from firm B and for $i \in \{A, B\}$, let $p_i^S(\omega)$, $D_i^S(\omega)$ and $\Pi_i^S(\omega)$ be the price, market share and payoff of firm i in the SPNE. The SPNE has the following properties.*

(I) *Let $\overline{c} - \underline{c} > (3/4)\tau$. There is $\widehat{c} \in (\underline{c}, \overline{c})$ such that*

(a) *If $\omega \in (\underline{c}, \widehat{c})$, then $K(\omega) = \Theta(\omega) := 3/8 - (\omega - \underline{c})/8\tau \in (D_A(\overline{c}, \underline{c}), D_A(0, \underline{c}))$. The market share of firm A is $\Theta(\omega)$ and that of firm B is $1 - \Theta(\omega)$. The prices are increasing in ω, given by*

$$p_A^S(\omega) = (3\tau + \underline{c} + \omega)/2 < p_A^0 \text{ and } p_B^S(\omega) = (5\tau + 3\underline{c} + \omega)/4 < p_B^0 \tag{14}$$

Consequently all consumers are better off compared to no contracts. The payoffs are

$$\Pi_A^S(\omega) = p_A^S(\omega)\Theta(\omega) - \omega\Theta(\omega) \text{ and } \Pi_B^S(\omega) = \left(p_A^S(\omega) - \underline{c}\right)(1 - \Theta(\omega)) + (\omega - \underline{c})\,\Theta(\omega)$$

There is $\widetilde{c} \in (\underline{c}, \widehat{c})$ such that $\Pi_B^S(\omega) \gtreqless \Phi_B^0 \Leftrightarrow \omega \gtreqless \widetilde{c}$, i.e., compared to no contracts, firm B is better off only if and only if $\omega \in (\widetilde{c}, \widehat{c})$.

(b) *If $\omega \in [\widehat{c}, \overline{c})$, then $K(\omega) = D_A(\overline{c}, \underline{c}) = D_A^0$. The prices and market shares of firms are exactly the same as in the case of no contracts. The payoffs are*

$$\Pi_A^S(\omega) = \Phi_A^0 + (\overline{c} - \omega)D_A^0 \text{ and } \Pi_B^S(\omega) = \Phi_B^0 + (\omega - \underline{c})D_A^0$$

Compared to no contracts, both firms are better off and consumers are neither better off nor worse off.

(c) *An outsourcing contract is strictly Pareto improving (both firms and all consumers are better off) if* $\omega \in (\widetilde{c}, \widehat{c})$ *and weakly Pareto improving (both firms are better off and no consumer is worse off) if* $\omega \in [\widehat{c}, \overline{c})$.

(II) *Let* $\overline{c} - \underline{c} \leq (3/4)\tau$. *Then for any* $\omega \in (\underline{c}, \overline{c})$, $K(\omega) = D_A(\overline{c}, \underline{c})$ *and the conclusion is the same as in* (I)(b). *Consequently an outsourcing contract is weakly Pareto improving for all* $\omega \in (\underline{c}, \overline{c})$.

(III) $\Pi_A^S(\omega)$ *is decreasing and* $\Pi_B^S(\omega)$ *is increasing in* ω. *Define*

$$c^* := \begin{cases} \widehat{c} \text{ if } \overline{c} - \underline{c} > (3/4)\tau \\ \underline{c} \text{ if } \overline{c} - \underline{c} \leq (3/4)\tau \end{cases} \tag{15}$$

Compared to no contracts, both firms are better off if and only if $\omega \in (c^*, \overline{c})$.

Proof See the Appendix. ■

To see the intuition for Proposition 3, recall that firm A's outsourcing order of η is equivalent to an endogenous capacity built by A prior to price competition which establishes firm A as the Stackelberg leader in the market φ. When ω (the unit cost of building capacity) is relatively large, the Stackelberg leader market share coincides with A's market share under no contracts. Then firm A does not utilize its leadership advantage since capacity building is relatively costly (Prop 3(I)(b)). On the other hand, when ω is relatively small, the Stackelberg leader market share is larger than A's market share under no contracts. In such a case, capacity building is relatively less expensive which enables firm A to utilize its leadership advantage (Prop 3(I)(a)). The Stackelberg leader market share is sustained in equilibrium by a lower price of firm A. Equilibrium reasoning implies that that B's price is also set lower. This results in lower profit for firm B in the market φ compared to the case of no contracts. It is acceptable to B only if it can recover its losses in the market φ from its supplier profit in the market η, which could be the case only when B is significantly more efficient compared to A in the production of η. Consequently, if the cost difference of two firms is sufficiently large [specifically, $\overline{c} - \underline{c} > (3/4)\tau$], there are strictly Pareto improving outsourcing contracts: consumers are better off since prices of both firms fall, firm A is better off due to its Stackelberg leadership advantage and firm B is also better off since its supplier profit from market η more than offsets its losses from market φ.

4.3 Technology transfer contract between A and B

When firm B transfers its superior technology to firm A, both A and B can produce the intermediate good η at lower cost $\underline{c}$. As in the case of outsourcing contract, we consider linear unit pricing contracts. The unit pricing contract for technology transfer is the *unit royalty* contract where the rate of royalty is denoted by r. The strategic interaction between A and B under technology transfer is described as follows.[12]

[12] The analysis of this section closely follows Matsumura et al. (2010) who consider the problem of technology transfer via royalty licensing between firms that compete in a Hotelling duopoly with endogenous locations. While their primary objective is to determine the optimal royalty for the firm with superior technology (and then resolve the equilibrium existence problem), here we characterize the market outcomes for all ω, which are then compared with the corresponding outcomes under outsourcing. For this reason, it is useful to present the analysis.

Negotiation stage: In the beginning, firms A and B negotiate on the unit royalty r. Under the unit royalty contract, firm A uses the superior technology of firm B. In return, A pays B the royalty r for each unit of η that it produces using the superior technology. So, firm A's *effective unit cost* of η is $\underline{c}+r$. As A can produce η itself at unit cost $\overline{c}$, a royalty contract can lower its cost of production only if $\underline{c}+r<\overline{c}$ or $r<\overline{c}-\underline{c}$. On the other hand, firm B can obtain a positive revenue from technology transfer only if $r>0$. So we restrict $r \in (0, \overline{c}-\underline{c})$.

To compare royalty contracts with outsourcing contracts, it will be convenient to denote $\omega \equiv \underline{c}+r$. Then ω represents the *effective unit cost* of η for firm A under the superior technology, while $r=\omega-\underline{c}$ represents the *unit profit* of firm B from technology transfer. Since $r \in (0, \overline{c}-\underline{c})$, we have $\omega \in (\underline{c}, \overline{c})$.

If firms do not agree on a price, firm A produces the required η entirely by itself at cost $\overline{c}$ and the game $\mathbb{H}(\overline{c}, \underline{c})$ is played in the final good market. If firms agree on a price $\omega \in (\underline{c}, \overline{c})$, firm B transfers its superior technology to firm A and the game $\Gamma^T(\omega)$ is played between firms A and B.

Remark 1 Observe that the interpretation of ω is the same as in outsourcing contracts. For firm A, ω is the unit cost of obtaining η from firm B. For firm B, $(\omega-\underline{c})$ is the unit profit from supplying η to A. The difference between outsourcing and technology transfer is that under outsourcing, A chooses the quantity of η and places its order with B *before* firms set their prices for the final good market φ. Firm B produces η using its superior technology and supplies η to A at price ω. In contrast, under technology transfer, A uses the superior technology to produce η itself *after* prices are set and its demand is known. This difference, which is generally overlooked in the existing literature, alters the strategic interaction and affects the prices of the final good φ.

The game $\Gamma^T(\omega)$: It is an extensive form game that has the following stages.

Stage I: Firms A and B simultaneously announce prices p_A, p_B for the final good market φ. For any $p \equiv (p_A, p_B)$, the demand received by firm $i \in \{A, B\}$ is $D_i(p)$, given by (3).

Stage II: Observing $D_i(p)$, firms A and B simultaneously choose $q_A, q_B \geq 0$ where

$$q_i = \text{ the quantity of } \eta \text{ that firm } i \text{ produces in order to fulfill its demand of } \varphi$$

The demand fulfilling constraints for firms A, B are

$$q_A \geq D_A(p) \text{ and } q_B \geq D_B(p) \tag{16}$$

Each firm fulfills its demand, profits are realized, firm A makes its royalty payments to firm B and the game terminates.

If firm A produces η using its pre-contract inferior technology, its unit cost is $\overline{c}$. If it produces η using the superior technology, its unit cost is $\omega<\overline{c}$. So it is optimal for firm A to produce η entirely using the superior technology. Firm B's unit cost of producing η is $\underline{c}>0$. Since both ω and $\overline{c}$ are positive, by (16), optimality requires that for $i \in \{A, B\}$, firm i produces $q_i=D_i(p)$ units of η and transforms $D_i(p)$ units to good φ to fulfill its demand.

Payoffs of firms in $\Gamma^T(\omega)$: Using optimal values of q_A, q_B, we can write the payoff of each firm. Firm A's payoff has two components: (i) revenue from market φ, (ii) total effective cost of producing q_A units of η to fulfill its demand. Observe that this total effective cost is $(\underline{c}+r)q_A = \omega q_A$, so it includes firm A's royalty payments to firm B. As $q_A = D_A(p)$, the payoff of firm A is

$$\pi_A^\omega(p) = p_A D_A(p) - \omega q_A = (p_A - \omega) D_A(p) \tag{17}$$

Firm B's payoff has three components: (i) revenue from market φ, (ii) cost of producing q_B units of η to fulfill its demand and (iii) profit from technology transfer $rq_A = (\omega - \underline{c})q_A$. Since firm B's unit cost of η is $\underline{c}$ and $q_i = D_i(p)$, its payoff is

$$\pi_B^\omega(p) = p_B D_B(p) - \underline{c} q_B + (\omega - \underline{c}) q_A = (p_B - \underline{c}) D_B(p) + (\omega - \underline{c}) D_A(p)$$

From (3), $D_A(p) + D_B(p) = 1$. Using this in the expression above, the payoff of firm B is

$$\pi_B^\omega(p) = (p_B - \underline{c}) D_B(p) + (\omega - \underline{c})(1 - D_B(p)) = (p_B - \omega) D_B(p) + (\omega - \underline{c}) \tag{18}$$

Since $(\omega - \underline{c})$ is a constant, from (18) it follows that, firm B in effect solves the problem of a firm that has unit cost ω. By (17), firm A has unit cost ω. Therefore, firms A and B effectively play the Hotelling duopoly game $\mathbb{H}(\omega, \omega)$ and SPNE of $\Gamma^T(\omega)$ coincides with SPNE of $\mathbb{H}(\omega, \omega)$ with the only modification that firm B's payoff has an additional constant $(\omega - \underline{c})$.

Proposition 4 *For any $\omega \in (\underline{c}, \overline{c})$, $\Gamma^T(\omega)$ has a unique SPNE. For $i \in \{A, B\}$, let $p_i^T(\omega)$, $D_i^T(\omega)$ and $\Pi_i^T(\omega)$ be the price, market share and payoff of firm i in the SPNE $\Gamma^T(\omega)$. The SPNE has the following properties.*

(I) *The prices and market shares of firms are the same as in the SPNE of $\mathbb{H}(\omega, \omega)$. Each firm sets the same price $\tau + \omega$ and obtains the same market share $1/2$. The payoffs are $\Pi_A^T(\omega) = \tau/2$ and $\Pi_B^T(\omega) = \tau/2 + (\omega - \underline{c})$, i.e., $\Pi_A^T(\omega)$ is a constant and $\Pi_B^T(\omega)$ is increasing in ω.*

(II) $\Pi_A^T(\omega) > \Phi_A^0$, *i.e., compared to no contracts, firm A is better off.*

(III) *There are constants $\underline{c} < \underline{\theta} < \widehat{\theta} < \overline{\theta} < \overline{c}$ such that*

(a) *Compared to no contracts, all consumers are better off if $\omega \in (\underline{c}, \underline{\theta})$ and all consumers are worse off if $\omega \in (\overline{\theta}, \overline{c})$. If $\omega \in [\underline{\theta}, \overline{\theta}]$, then $\exists\ \lambda(\omega) \in (0, 1/2]$ such that consumers at location $x \in [0, \lambda]$ are better off and $x \in (\lambda, 1]$ are worse off.*

(b) $\Pi_B^T(\omega) \gtreqqless \Phi_B^0 \Leftrightarrow \omega \gtreqqless \widehat{\theta}$, *i.e., compared to no contracts firm B is better off (and consequently both firms are better off) if and only if $\omega \in (\widehat{\theta}, \overline{c})$.*

(c) *Whenever both firms prefer technology transfer over no contracts [i.e., if $\omega \in (\widehat{\theta}, \overline{c})$], there are always some consumers who prefer no contracts over technology transfer. Consequently, there is no technology transfer contract that is Pareto improving (i.e., making both firms as well as all consumers better off).*

Proof Using the conclusion from the paragraph preceding the proposition and taking $c_A = c_B = \omega$ in Lemma 1, part (I) follows. Part (II) follows directly from part (I) and Proposition 1(iii). See the Appendix for the proof of part (III). ■

To see the intuition for Proposition 4, first observe that in contrast to the case of outsourcing, firm B does not receive its revenue from technology transfer upfront. It is received *after* the price competition stage in the form of royalty payments. To obtain relatively large royalty payments from technology transfer, B has an incentive to ensure that A's share in the market φ is not too small. This has a distortive effect which causes B's effective unit cost to rise to $\omega > \underline{c}$ [see (18)]. As A's unit cost falls to $\omega < \overline{c}$, firm A has an efficiency gain while B has an efficiency loss. The resulting effect on consumers depends on which one of these opposing factors dominates. When ω is sufficiently small ($\omega < \underline{\theta}$), the efficiency gain of A dominates, prices of both firms fall and all consumers are better off. When ω is sufficiently large ($\omega > \overline{\theta}$), the efficiency loss of B dominates, prices of both firms rise and all consumers are worse off. For intermediate values of ω ($\omega \in [\underline{\theta}, \overline{\theta}]$), the effect on consumers is ambiguous and it depends on their location. Consumers who are close to A ($x < \lambda$) benefit from the efficiency gain of A and therefore are better off. In contrast, consumers who are close to B ($x \geq \lambda$) are adversely affected by the efficiency loss of B and are worse off (Prop 4(III)(a)).

Observe that all consumers are better off under technology transfer compared to no contract only if ω is sufficiently small ($\omega < \underline{\theta}$). However, when ω is small, B obtains a lower revenue from royalty. For this reason, firm B prefers technology transfer over no contract only if ω is relatively large, in which case there are always some consumers who are worse off. This explains why there is no Pareto improving technology transfer contract (Prop 4(III)(c)).

5 Outsourcing versus technology transfer

Having characterized the outcomes of outsourcing and technology transfer games, in this section we compare these two contracts from the points of view of the two firms as well as the consumers.

5.1 Comparison of contracts with same ω

Recall that under both outsourcing and technology transfer, ω is firm A's effective unit cost of obtaining η and $(\omega - \underline{c})$ is firm B's unit profit from the market η. Proposition 5 compares these two contracts by keeping ω fixed across contracts, so that the effects of cost efficiency (for firm A) and supplier profits (for firm B) are the same across contracts. Therefore this proposition identifies the differences between these two contracts that are purely driven by the salient strategic aspects of these contracts: the Stackelberg leadership effect for outsourcing and the distortive effect for technology transfer.

Proposition 5 *Let $\omega \in (\underline{c}, \overline{c})$. There are constants $\underline{c} < \alpha < \beta < \overline{c}$ and $\underline{c} < \underline{\theta} < \overline{\theta} < \overline{c}$ such that the following hold.*

(I) *If $\omega \in (\underline{c}, \alpha)$, both firms prefer outsourcing and if $\omega \in (\beta, \overline{c})$, both firms prefer technology transfer. If $\omega \in [\alpha, \beta]$, then firm A prefers technology transfer while firm B prefers outsourcing.*

(II) *If* $\omega \in (\underline{c}, \underline{\theta})$*, all consumers prefer technology transfer and if* $\omega \in (\overline{\theta}, \overline{c})$*, all consumers prefer outsourcing. If* $\omega \in [\underline{\theta}, \overline{\theta}]$*, then* $\exists\ \lambda(\omega) \in (0, 1/2]$ *such that consumers at location* $x \in [0, \lambda]$ *prefer technology transfer while consumers at* $x \in (\lambda, 1]$ *prefer outsourcing.*

(III) *The following inequalities hold:* $\underline{c} < \alpha < \underline{\theta} < \beta < \overline{\theta}$*. Consequently whenever both firms prefer a specific contract, there always exist some consumers who prefer the other contract. Specifically*

(a) *Both firms prefer outsourcing if and only if* $\omega < \alpha$*. In that case all consumers prefer technology transfer.*

(b) *Both firms prefer technology transfer if and only if* $\omega > \beta$*. In that case there always exist some consumers who prefer outsourcing. Moreover if* $\omega > \overline{\theta}$*, then all consumers prefer outsourcing.*

Proof See the Appendix. ■

Firm A's payoff under technology transfer is a constant $\tau/2$, while its payoff under outsourcing is decreasing in ω. This is why A prefers outsourcing for relatively small values of ω and technology transfer otherwise. Firm B's payoff has profits from two markets: the final good φ and the intermediate good η. We know that under technology transfer, there is a distortive effect that raises the effective cost of B. As a result, the profit of B in φ is lower under technology transfer than outsourcing. Therefore, if B is solely interested in the profits from the market φ, it would prefer outsourcing. On the other hand, if B is only interested in the profits from the market η, it would prefer technology transfer since its supplier profit from η increases with the market share of A, which is higher under technology transfer. This trade-off is settled by the magnitude of ω. When ω is relatively small, the profit from η does not contribute significantly to B's payoff. As a result, the effect of the market φ dominates and B prefers outsourcing. On the other hand, when ω is relatively large, the profit from η contributes significantly to B's payoff, so it prefers technology transfer.

For relatively large values of ω, prices under outsourcing are the same as in the case of no contracts, but prices under technology transfer exceed the no contract levels. Accordingly, all consumers prefer outsourcing for relatively large values of ω. On the other hand, for relatively small values of ω, prices under outsourcing may fall, but prices under technology transfer fall significantly below the no contract levels. Consequently all consumers prefer technology transfer for relatively small values of ω. For intermediate values of ω, the price of firm A falls while the price of B rises under technology transfer. As a result, the preference of consumers depends on their location as in Proposition 4.

Finally it is shown in Proposition 5 that the interest of consumers and incentives of firms conflict each other. Whenever both firms prefer one of the two contracts, there always exist some consumers who prefer the other one.

5.2 Comparison of contracts under bargaining

In the last section we compared the outcomes of outsourcing and technology transfer by keeping the price ω same across the two contracts. However, if for each form of contract, firms A, B bargain to choose ω, the chosen value may not be the same for outsourcing and technology transfer. In this section we see if our conclusions are robust to the case where the choice of ω is decided through bargaining between the two firms. Instead of explicitly specifying the bargaining process, we take a more general approach by imposing certain natural efficiency requirements on the contracts. The starting point is the notion of *undominated* contracts which is defined as follows.

Definition For any form of contract, ω is *undominated* if
(i) no firm is worse off at ω compared to no contracts,
(ii) there is no ω' such that (a) no firm is worse off and (b) at least one firm is better off at ω' compared to ω.

We assume that for any form of contract, firms bargain over only those ω that are undominated. To identify this set first observe that (1) for both outsourcing and technology transfer, if $\omega > \bar{c}$, then firm A is better off not accepting the contract, (2) there is a constant $c^* \in [\underline{c}, \bar{c})$ such that both firms prefer outsourcing over no contracts if and only if $\omega \in (c^*, \bar{c})$ (Prop 3(III)) and (3) there is a constant $\widehat{\theta} \in (\underline{c}, \bar{c})$ such that both firms prefer technology transfer over no contracts if and only if $\omega \in (\widehat{\theta}, \bar{c})$ (Prop 4(III)(b)).

Let U^S be the set of all undominated contracts for outsourcing and U^T be the corresponding set for technology transfer. Denote the bargaining power of firm B by $\gamma \in (0,1)$, so that the bargaining power of firm A is $1-\gamma$. Let $\omega^S(\gamma)$ be the value of ω chosen under outsourcing when B has bargaining power γ and $\omega^T(\gamma)$ be the corresponding value under technology transfer.

- Outsourcing: From (1)-(2) above, $U^S \subseteq [c^*, \bar{c}]$. Since $\Pi_A^S(\omega)$ is decreasing and $\Pi_B^S(\omega)$ is increasing in ω (Prop 3), it follows that any $\omega \in [c^*, \bar{c}]$ is undominated, so that $U^S = [c^*, \bar{c}]$. We assume that $\omega^S(\gamma) : (0,1) \to [c^*, \bar{c}]$ is a continuous and increasing function with $\lim_{\gamma\to 0} \omega^S(\gamma) = c^*$ and $\lim_{\gamma \to 1} \omega^S(\gamma) = \bar{c}$. This specification implies that a firm obtains a higher payoff as its bargaining power increases and as a firm's bargaining power becomes close to 1, its payoff approaches the maximum possible payoff in the set U^S.

- Technology transfer: From (1) and (3) above, $U^T \subseteq [\widehat{\theta}, \bar{c}]$. Since $\Pi_A^T(\omega) = \tau/2$ is a constant and $\Pi_B^T(\omega) = \tau/2 + (\omega - \underline{c})$ is increasing in ω (Prop 4), it follows that $\omega = \bar{c}$ is the only undominated contract,[13] i.e., $U^T = \{\bar{c}\}$. Therefore $\omega^T(\gamma) = \bar{c}$ for all $\gamma \in (0,1)$.

For $\gamma \in (0,1)$ and $i \in \{A, B\}$, let $\Psi_i^S(\gamma)$ be the payoff and $\rho_i^S(\gamma)$ be the price of firm i under the outsourcing contract that is chosen when firm B's bargaining power is γ, i.e.,

$$\Psi_i^S(\gamma) = \Pi_i^S(\omega^S(\gamma)) \text{ and } \rho_i^S(\gamma) = p_i^S(\omega^S(\gamma)) \tag{19}$$

Let $\Psi_i^T(\gamma)$ and $\rho_i^T(\gamma)$ be the corresponding expressions under technology transfer. Noting that $\omega^T(\gamma) = \bar{c}$ for all $\gamma \in (0,1)$ and $p_i^T(\bar{c}) = \tau + \bar{c}$ (Prop 4), we have

$$\Psi_i^T(\gamma) = \Pi_i^T(\omega^T(\gamma)) = \Pi_i^T(\bar{c}) \text{ and } \rho_i^T(\gamma) = p_i^T(\omega^T(\gamma)) = p_i^T(\bar{c}) = \tau + \bar{c} \tag{20}$$

Proposition 6

(I) $\Psi_B^T(\gamma) > \Psi_B^S(\gamma)$ *for all* $\gamma \in (0,1)$, *i.e., firm B always prefers technology transfer over outsourcing.*

[13]Note that when $\omega = \bar{c}$, firm A is indifferent between using its own technology and the superior technology of firm B. If it uses its own technology, it obtains Π_A^0 while by accepting the technology transfer contract, it obtains $\tau/2 > \Pi_A^0$. So firm A will accept a technology transfer contract with $\omega = \bar{c}$.

(II) *For $i \in \{A, B\}$, $\rho_i^T(\gamma) > \rho_i^S(\gamma)$ for all $\gamma \in (0,1)$. Consequently all consumers prefer outsourcing over technology transfer.*

(III) *$\exists\ \gamma^* \in (0,1)$ such that $\Psi_A^S(\gamma) \gtreqless \Psi_A^T(\gamma)$ if and only if $\gamma \lesseqgtr \gamma^*$, i.e., firm A prefers technology transfer if B's bargaining power is relatively large ($\gamma > \gamma^*$) and it prefers outsourcing if B's bargaining power is relatively small ($\gamma < \gamma^*$).*

(IV) *If $\gamma > \gamma^*$ then both firms prefer technology transfer while all consumers prefer outsourcing. Otherwise all consumers and firm A prefer outsourcing while firm B prefers technology transfer.*

Proof (I) Since $\Pi_B^S(\omega)$ is increasing in ω (Prop 3) and $\omega^S(\gamma) \leq \bar{c}$, it follows from (19) that $\Psi_B^S(\gamma) \leq \Pi_B^S(\bar{c})$. By (20), for all $\gamma \in (0,1)$, we have $\Psi_B^T(\gamma) = \Pi_B^T(\bar{c})$. Since $\Pi_B^T(\bar{c}) > \Pi_B^S(\bar{c})$ (by Prop 5(I)), it follows that $\Psi_B^T(\gamma) > \Psi_B^S(\gamma)$.

(II) Since $p_i^S(\omega) \leq p_i^0$ for all $\omega \in [c^*, \bar{c}]$ (Prop 3) and $\omega^S(\gamma) \in [c^*, \bar{c}]$, it follows from (19) that $\rho_i^S(\gamma) \leq p_i^0$ for all $\gamma \in (0,1)$. By (20), $\rho_i^T(\gamma) = p_i^T(\bar{c}) = \tau + \bar{c}$. As p_A^0, p_B^0 are both less than $\tau + \bar{c}$ (Prop 1), the result follows.

(III) Note from (19) and Prop 5(I) that $\Psi_A^S(\gamma) = \Pi_A^S(\omega^S(\gamma)) \gtreqless \tau/2 = \Psi_A^T(\gamma)$ iff $\omega^S(\gamma) \lesseqgtr \alpha$. Since $c^* < \alpha < \bar{c}$ (see Lemma A4 in the Appendix) and $\omega^S(\gamma) : (0,1) \to [c^*, \bar{c}]$ is increasing with $\lim_{\gamma \to 0} \omega^S(\gamma) = c^*$ and $\lim_{\gamma \to 1} \omega^S(\gamma) = \bar{c}$, $\exists\ \gamma^* \in (0,1)$ such that $\omega^S(\gamma) \gtreqless \alpha$ iff $\gamma \gtreqless \gamma^*$. Hence $\Psi_A^S(\gamma) \gtreqless \Psi_A^T(\gamma)$ iff $\gamma \lesseqgtr \gamma^*$.

(IV) Follows from (I)-(III). ■

As the only undominated technology transfer contract results in maximum possible prices for both firms, consumers always prefer outsourcing. If the bargaining power of firm B is relatively large ($\gamma > \gamma^*$), both firms prefer technology transfer, resulting in conflict with the interest of all consumers. Recall that with same ω across contracts, whenever both firms prefer one form of contract, there always exist some consumers who prefer the other one (Prop 5). Under bargaining, we obtain a stronger result: whenever both firms prefer one form of contract, all consumers prefer the other contract. Two remarks are in order.

Remark 6.1 Note that the unique undominated technology transfer contract $\omega = \bar{c}$ corresponds to the maximum possible royalty $r = \bar{c} - \underline{c}$ that firm B can charge firm A. It is shown in Matsumura et al. (2010) that the optimal royalty policy of technology transfer for the efficient firm is to charge the maximum royalty from its rival. Proposition 6 strengthens their result by showing that if firm B's bargaining power is sufficiently large, then the maximum royalty contract also dominates outsourcing contracts for both firms.

Remark 6.2 Suppose firms A, B are competing retailers. Then under an outsourcing contract, the efficient retailer (firm B) manufactures for both the retailers, while under technology transfer, both retailers are active in manufacturing. The result of Proposition 6 implies that if the efficient retailer has sufficiently large bargaining power in determining the terms of each contract, then it is better for both retailers that the inefficient retailer stays active in manufacturing. Although both retailers then use the efficient process of B, there is a distortive effect in the form of a high transfer price $\omega = \bar{c}$ in the manufacturing stage. This enables the retailers to sustain a high retail price in the downstream market.

Let us now look at the case $\gamma < \gamma^*$. In this case firm A, like all consumers, also prefer outsourcing, but firm B prefers technology transfer. A natural question is, which contract will be chosen in such a case? The model as it is does not immediately answer this question. However, this ambiguity can be addressed if one extends the model by providing more decision choices to firms. Here we provide an example of one such possible extension. Consider an extended model where one of the firms moves first to propose a specific form of contract (outsourcing or technology transfer) which the other firm can accept or reject. In the case of acceptance firms implement the proposed contract where the price ω is determined through bargaining and in case of rejection, both firms obtain their no-contract payoffs. In this set-up, if $\gamma > \gamma^*$, then outsourcing will be proposed and accepted if firm A moves first, while technology transfer will be chosen if firm B moves first. Thus, if firm A is given sufficiently large bargaining power ($\gamma < \gamma^*$) and further strategic advantage as the proposer of the contract form, then the conflict between incentives of firms and interest of consumers can be resolved in that firms will choose a contract (i.e. outsourcing) that is preferred by all consumers.

6 Concluding remarks

This paper has compared two contracts that are frequently observed in industry practices: outsourcing and technology transfer. Departing from the existing literature we have shown that these two contracts generate different strategic interactions that alter incentives of firms and affect prices. Identifying the Stackelberg leadership effect in a Hotelling duopoly model, we have shown that there are always Pareto improving outsourcing contracts. In contrast, there are no Pareto improving technology transfer contracts. We have also shown that there is generally a conflict between the incentives of firms and interest of consumers.

In this paper the locations of firms are exogenously given at the two end points of the Hotelling linear city. A natural extension would be to endogenize the location choices. This will raise some interesting theoretical issues in view of three results of the existing literature: (i) pure strategy equilibrium fail to exist if there is sufficient cost asymmetry between firms (Ziss, 1993), (ii) there is a mixed strategy equilibrium where firms randomly choose to locate at two end points (Matsumura and Matsushima, 2009) and (iii) technology transfer through royalty can restore existence of pure strategy equilibrium (Matsumura et al., 2010).

The choice of endogenous locations is also of interest when the firms are retailers and a location corresponds to a specific product characteristic. Our existing model applies to retail markets where retailers, located at two end points, have exogenous product characteristic that are very different. Making the location choices endogenous will enable us to see if competing retailers who interact in the input market have incentive to choose similar or different features for their products. In particular, if the retailers choose the same characteristic it will imply that the two retailers effectively become two franchisees of the same product. Thus, endogenizing location choices may be useful to understand some aspects of franchising. These questions are left for future research.

Appendix

We begin with Lemma A1 which will be used to prove Lemma 1.

Lemma A1 *Let $c_A, c_B < \tau$. The following hold in the game $\mathbb{H}(c_A, c_B)$:*
(i) *Let $i, j \in \{A, B\}$ and $i \neq j$. The best response of firm i to firm j's price p_j is*

$$b_{c_i}(p_j) = \begin{cases} (p_j + \tau + c_i)/2 & \text{if } p_j \leq 3\tau + c_i \\ p_j - \tau & \text{if } p_j > 3\tau + c_i \end{cases} \tag{21}$$

(ii) *If (p_A, p_B) is an SPNE of $\mathbb{H}(c_A, c_B)$, then $p_A \leq 3\tau + c_B$ and $p_B \leq 3\tau + c_A$.*

Proof (i) By (3), if $p_i \leq p_j - \tau$, then $D_i = 1$ and i's payoff is $p_i - c_i$, which is increasing in p_i. If $p_i \geq p_j + \tau$, then $D_i = 0$ and i's payoff is zero. Therefore, to determine best response of i, it is sufficient to consider $p_i \in [p_j - \tau, p_j + \tau]$. In that case, by (2) and (3), $D_i = (p_j - p_i + \tau)/2\tau$ and i's payoff is $\Phi_i = (p_i - c_i)(p_j - p_i + \tau)/2\tau$. The unconstrained maximum of Φ_i with respect to p_i is attained at $b(p_j) = (p_j + \tau + c_i)/2$. As $c_i < \tau$, we have $b(p_j) < p_j + \tau$. Noting that $b(p_j) \geq p_j - \tau$ iff $p_j \leq 3\tau + c_j$ and $p_j - \tau > 0$ for $p_j > 3\tau + c_i$, the result in (21) follows.

(ii) Suppose (p_A, p_B) is an SPNE and $p_j > 3\tau + c_i$ for some $i, j \in \{A, B\}$ and $i \neq j$. Then by (21), $p_i = b_{c_i}(p_j) = p_j - \tau$. In that case, $D_j = 0$ and firm j obtains zero payoff. Let j deviate to set the price $p'_j = p_i = p_j - \tau$. Following this deviation, by (2) and (3), firm j will receive demand 1/2 and payoff $(p_j - c_j)/2$. Since $p_j > 3\tau + c_j > c_j$, firm j's post-deviation payoff is positive, showing that j has improved its payoff following the deviation, a contradiction. ∎

Proof of Lemma 1 By Lemma A1(ii), to find SPNE of $\mathbb{H}(c_A, c_B)$, it is sufficient to consider $p_A \leq 3\tau + c_B$ and $p_B \leq 3\tau + c_A$. Then by (21), the best response of A is to set $p_A = (p_B + \tau + c_A)/2$ and that of B is to set $p_B = (p_A + \tau + c_B)/2$. The system of best response equations has a unique solution: $p_A = \tau + (2c_A + c_B)/3$ and $p_B = \tau + (c_A + 2c_B)/3$. This proves (i). Parts (ii)-(iii) follow directly from (i). ∎

Lemma A2 will be used to prove Lemma 2.

Lemma A2 *Denote $p \equiv (p_A, p_B)$, $g(p_B) := p_B + \tau - 2\tau K$, $\underline{p}(K) := 4\tau K - \tau$ and $\overline{p}(K) := 4\tau K - \tau + \overline{c}$. In the game $\mathbb{H}^K(\underline{c}, \overline{c})$:*
(i) $D_A(p) \substack{\leq \\ = \\ >} K \Leftrightarrow p_A \substack{\geq \\ = \\ <} g(p_B)$.
(ii) *The profit of firm A is*

$$\Phi_A^K(p) = \begin{cases} p_A - \overline{c} + \overline{c}K & \text{if } \; p_A < p_B - \tau \\ (p_A - \overline{c})(p_B - p_A + \tau)/2\tau + \overline{c}K & \text{if } \; p_B - \tau \leq p_A < g(p_B) \\ p_A(p_B - p_A + \tau)/2\tau & \text{if } \; g(p_B) \leq p_A \leq p_B + \tau \\ 0 & \text{if } \; p_A > p_B + \tau \end{cases} \tag{22}$$

(iii) *The best response of A to B's price p_B is*

$$b_A^K(p_B) = \begin{cases} b_0(p_B) = (p_B + \tau)/2 & \text{if } \; p_B < \underline{p} \\ g(p_B) & \text{if } \; \underline{p} \leq p_B \leq \overline{p} \\ b_{\overline{c}}(p_B) = (p_B + \tau + \overline{c})/2 & \text{if } \; \overline{p} < p_B \leq 3\tau + \overline{c} \\ b_{\overline{c}}(p_B) = p_B - \tau & \text{if } \; p_B > 3\tau + \overline{c} \end{cases} \tag{23}$$

(iv) *Consider the demand that firm A receives when it sets price $p_A = b_A^K(p_B)$. This demand is less than K if $p_B < \underline{p}$, more than K if $p_B > \overline{p}$ and exactly equals K if $\underline{p} \leq p_B \leq \overline{p}$.*

(v) *The profit of firm B is*

$$\Phi_B^K(p) = \begin{cases} p_B - \underline{c} & \text{if } \ p_B < p_A - \tau \\ (p_B - \underline{c})(p_A - p_B + \tau)/2\tau & \text{if } \ p_A - \tau \leq p_B \leq p_A + \tau \\ 0 & \text{if } \ p_B > p_A + \tau \end{cases} \tag{24}$$

(vi) *The best response of B to A's price p_A is*

$$b_B^K(p_A) = b_{\underline{c}}(p_A) = \begin{cases} (p_A + \tau + \underline{c})/2 & \text{if } \ p_A \leq 3\tau + \underline{c} \\ p_A - \tau & \text{if } \ p_A > 3\tau + \underline{c} \end{cases} \tag{25}$$

(vii) *If (p_A, p_B) is an SPNE of $\mathbb{H}^K(\underline{c}, \overline{c})$, then $p_A \leq 3\tau + \underline{c}$ and $p_B \leq 3\tau + \overline{c}$.*

Proof (i) Observe that since $K \in [0, 1]$, we have $p_B - \tau \leq g(p_B) \leq p_B + \tau$. It follows from (3) that if $p_A \leq p_B - \tau$, then $D_A(p) = 1 \geq K$ and if $p_A \geq p_B + \tau$, then $D_A(p) = 0 \leq K$. Now consider $p_A \in [p_B - \tau, p_B + \tau]$. Then from (2) and (3), we have $D_A(p) = (p_B - p_A + \tau)/2\tau \lesseqqgtr K \Leftrightarrow p_A \gtreqqless g(p_B)$. This completes the proof of (i).

(ii) Observe from (9) that $\Phi_A^K(p) = (p_A - \overline{c})D_A(p) + \overline{c}K$ if $D_A(p) \geq K$. The first expression of (22) follows by noting that $D_A(p) = 1 \geq K$ for $p_A < p_B - \tau$. Since $D_A(p) = (p_B - p_A + \tau)/2\tau \geq K$ for $p_A \in [p_B - \tau, g(p_B)]$ (by part (i)), the second expression follows.

Again from (9), $\Phi_A^K(p) = p_A D_A^p$ if $D_A(p) \leq K$. Since $D_A(p) = (p_B - p_A + \tau)/2\tau \leq K$ for $p_A \in [g(p_B), p_B + \tau]$ (by part (i)), the third expression of (22) follows. The last expression follows by noting that $D_A(p) = 0 \leq K$ for $p_A > p_B + \tau$.

(iii) It follows from (22) that $\Phi_A^K(p)$ is increasing for $p_A \leq p_B - \tau$ and it equals zero for $p_A \geq p_B + \tau$. Therefore, to determine best response of A, it is sufficient to consider $p_A \in [p_B - \tau, p_B + \tau]$.

Let $E_1 = [p_B - \tau, g(p_B)]$ and $E_2 = [g(p_B), p_B + \tau]$. Observe from (22) that for $p_A \in E_1$, firm A's effective unit cost is $\overline{c}$ and its problem is the same as in the standard Hotelling game $\mathbb{H}(\overline{c}, c_B)$. Taking $i = A$ and $c_A = \overline{c}$ in (21) of Lemma A1, the unconstrained maximum of $\Phi_A^K(p)$ over $p_A \in E_1$ is attained at $p_A = b_{\overline{c}}(p_B)$. Note from (21) that if $p_B > 3\tau + \overline{c}$, then $b_{\overline{c}}(p_B) = p_B - \tau$. If $p_B \leq 3\tau + \overline{c}$, then $b_{\overline{c}}(p_B) = (p_B + \tau + \overline{c})/2 \lesseqqgtr g(p_B) \Leftrightarrow p_B \gtreqqless \overline{p}$ where $\overline{p} := 4\tau K - \tau + \overline{c} \leq 3\tau + \overline{c}$. Hence we conclude that

$$\arg\max_{p_A \in E_1} \Phi_A^K(p) = \begin{cases} g(p_B) & \text{if } \ p_B < \overline{p} \\ (p_B + \tau + \overline{c})/2 & \text{if } \ \overline{p} \leq p_B \leq 3\tau + \overline{c} \\ p_B - \tau & \text{if } \ p_B > 3\tau + \overline{c} \end{cases} \tag{26}$$

Observe from (22) that for $p_A \in E_2$, firm A's effective unit cost is 0 and its problem is the same as in the standard Hotelling game $\mathbb{H}(0, c_B)$. Taking $i = A$ and $c_A = 0$ in (21) of Lemma A1, the unconstrained maximum of $\Phi_A^K(p)$ over $p_A \in E_2$ is attained at $p_A = b_0(p_B)$. Note from (21) that if $p_B > 3\tau$, then $b_0(p_B) = p_B - \tau \leq g(p_B)$, so the maximum is attained at $p_A = g(p_B)$. If $p_B \leq 3\tau$, then $b_0(p_B) = (p_B + \tau)/2 \gtreqqless g(p_B) \Leftrightarrow p_B \lesseqqgtr \underline{p}$ where $\underline{p} := 4\tau K - \tau \leq 3\tau$. Hence we conclude that

$$\arg\max_{p_A \in E_2} \Phi_A^K(p) = \begin{cases} (p_B + \tau)/2 & \text{if } \ p_B \leq \underline{p} \\ g(p_B) & \text{if } \ p_B > \underline{p} \end{cases} \tag{27}$$

As $g(p_B) \in E_1 \cap E_2$, choosing $p_A = g(p_B)$ is feasible for both E_1 and E_2. Using this fact, the result in (23) follows from (26)-(27).

(iv) It follows from (iii) that $b_A^K(p_B) > g(p_B)$ if $p_B < \underline{p}$, $b_A^K(p_B) < g(p_B)$ if $p_B > \overline{p}$ and $b_A^K(p_B) = g(p_B)$ if $\underline{p} \le p_B \le \overline{p}$. Using this fact, the result follows from (i).

(v)-(vi) Noting that firm B's constant unit cost of η is $\underline{c}$, (24) follows from (2) and (3), and (25) follows from (21) by taking $i = B$ and $c_B = \underline{c}$.

(vii) Follows from (23) and (25) by the same reasoning as the proof of Lemma A1(ii). ■

Proof of Lemma 2 Using Lemma A2(vii), to find SPNE of $\mathbb{H}^K(\underline{c}, \overline{c})$, consider $p_A \le 3\tau + \underline{c}$ and $p_B \le 3\tau + \overline{c}$. From (23), firm A's best response $b_A^K(p_B)$ is piecewise linear in p_B with three segments: $b_{\overline{c}}(p_B)$ (if $p_B > \overline{p}$), $b_0(p_B)$ (if $p_B < \underline{p}$) and $g(p_B)$ (if $p_B \in [\underline{p}, \overline{p}]$). From (25), firm B's best response is linear, given by $b_{\underline{c}}(p_B)$. Hence any segment of $b_A^K(p_B)$ can intersect $b_{\underline{c}}(p_B)$ at most once. It will be useful to recall that for $i = 1, 2$, the SPNE price and market share of firm i in $\mathbb{H}(c_A, c_B)$ are denoted by $p_i(c_A, c_B)$ and $D_i(c_A, c_B)$.

Note that $b_{\overline{c}}(p_B)$ is the best response of A in the standard Hotelling game $\mathbb{H}(\overline{c}, \underline{c})$. Firm B's best response in this game is $b_{\underline{c}}(p_A)$. By Lemma 1(II), the unique solution of the system $(p_A = b_{\overline{c}}(p_B), p_B = b_{\underline{c}}(p_A))$ has $p_A = p_A(\overline{c}, \underline{c})$ and $p_B = p_B(\overline{c}, \underline{c})$. We note that $p_B(\overline{c}, \underline{c}) \gtreqless \overline{p} \Leftrightarrow K \lesseqgtr D_A(\overline{c}, \underline{c})$. Hence we have an SPNE with $p_B > \overline{p}$ iff $p_B = p_B(\overline{c}, \underline{c}) > \overline{p}$, which holds iff $K < D_A(\overline{c}, \underline{c})$ (see Figure 1(a)). For this case, firm A fully utilizes its capacity K and moreover produces $D_A(\overline{c}, \underline{c}) - K$ units of η in-house to meet its demand.

Next observe that $b_0(p_B)$ is the best response of A in the standard Hotelling game $\mathbb{H}(0, \underline{c})$. Firm B's best response in this game is $b_{\underline{c}}(p_A)$. By Lemma 1(II), the unique solution of the system $(p_A = b_0(p_B), p_B = b_{\underline{c}}(p_A))$ has $p_A = p_A(0, \underline{c})$ and $p_B = p_B(0, \underline{c})$. We note that $p_B(0, \underline{c}) \lesseqgtr \underline{p} \Leftrightarrow K \gtreqless D_A(0, \underline{c})$. Hence we have an SPNE with $p_B < \underline{p}$ iff $p_B = p_B(0, \underline{c}) < \underline{p}$, which holds iff $K > D_A(0, \underline{c})$ (see Figure 1(b)). For this case, firm A does not utilize $K - D_A(0, \underline{c})$ units of its capacity and does not produce η in-house.

Finally observe that the unique solution of $(p_A = g(p_B), p_B = b_{\underline{c}}(p_A))$ has $p_A = 3\tau + \underline{c} - 4\tau K$ and $p_B = 2\tau + \underline{c} - 2\tau K$. Note that $2\tau + \underline{c} - 2\tau \gtreqless \underline{p} \Leftrightarrow K \lesseqgtr D_A(0, \underline{c})$ and $2\tau + \underline{c} - 2\tau K \lesseqgtr \overline{p} \Leftrightarrow K \gtreqless D_A(\overline{c}, \underline{c})$. Hence we have an SPNE with $p_B \in [\underline{p}, \overline{p}]$ iff $p_B = 2\tau + \underline{c} - 2\tau K \in [\underline{p}, \overline{p}]$, which holds iff $K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})]$ (see Figure 1(c)). For this case, firm A's SPNE market share exactly equals its capacity K. It fully utilizes its capacity and does not produce η in-house.

The results (i)-(iii) of Lemma 2 follow from the conclusions of the last three paragraphs. ■

Proof of Proposition 3 From the proof of Prop 2 in the main text, we know that in stage 1 of $\Gamma^S(\omega)$, firm A's problem is to choose $K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})]$ to maximize $\pi^\omega(K)$. By (11) and (12),

$$\pi_A^\omega(K) = p_A^K K - \omega K = (3\tau + \underline{c} - \omega - 4\tau K)\, K,$$

whose unconstrained maximum is attained at

$$\Theta(\omega) := 3/8 - (\omega - \underline{c})/8\tau \tag{28}$$

As $\Theta(\omega) < D_A(0, \underline{c})$, over $K \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})]$, the unique maximizer of $\pi_A^\omega(K)$ is

$$K(\omega) = \max\{\Theta(\omega), D_A(\overline{c}, \underline{c})\} \tag{29}$$

Comparing $\Theta(\omega)$ with $D_A(\overline{c}, \underline{c}) = 1/2 - (\overline{c} - \underline{c})/6\tau$, we have

$$\Theta(\omega) \gtreqless D_A(\overline{c}, \underline{c}) \Leftrightarrow \omega \lesseqgtr \widehat{c} \text{ where } \widehat{c} \equiv 4\overline{c}/3 - \underline{c}/3 - \tau \tag{30}$$

First note that $\widehat{c} < \overline{c}$, since $\overline{c} - \widehat{c} = \tau - (\overline{c} - \underline{c})/3 > \tau - \overline{c} > 0$. Next observe that $\widehat{c} - \underline{c} = (4/3)\left[\overline{c} - \underline{c} - (3/4)\tau\right]$. Hence

$$\widehat{c} \gtreqless \underline{c} \Leftrightarrow \overline{c} - \underline{c} \gtreqless (3/4)\tau \tag{31}$$

From (29), (30) and (31), we conclude that

Observation 1 If $\overline{c} - \underline{c} > (3/4)\tau$, then (i) $K(\omega) = \Theta(\omega)$ for $\omega \in (\underline{c}, \widehat{c})$ and (ii) $K(\omega) = D_A(\overline{c}, \underline{c})$ for $\omega \in [\widehat{c}, \overline{c})$.

Observation 2 If $\overline{c} - \underline{c} \leq (3/4)\tau$, then $K(\omega) = D_A(\overline{c}, \underline{c})$ for all $\omega \in (\underline{c}, \overline{c})$.

Noting that $K(\omega) \in [D_A(\overline{c}, \underline{c}), D_A(0, \underline{c})]$ for all cases, by Lemma 2(iii) it follows that the market share of firm A is $K(\omega)$ and that of firm B is $1 - K(\omega)$.

Part I(a) The first two statements of I(a) are immediate from Observation 1(i). The prices p_A^S, p_B^S in (14) follow by taking $K = \Theta(\omega)$ in (11). Recall from Prop 1(II) that the prices under no contracts are

$$p_A^0 = \tau + \overline{\theta} \text{ and } p_B^0 = \tau + \underline{\theta} \text{ where } \underline{\theta} \equiv (2\underline{c} + \overline{c})/3 \text{ and } \overline{\theta} \equiv (\underline{c} + 2\overline{c})/3$$

From (14), $p_A^0 - p_A^S(\omega) = (\widehat{c} - \omega)/2 > 0$ and $p_B^0 - p_B^S(\omega) = (\widehat{c} - \omega)/4 > 0$ where $\widehat{c}$ is given by (30). This proves that all consumers are better off compared to no contracts.

The payoffs of the firms A, B are obtained by taking $K = \Theta(\omega)$ in (12) and (13). Using the values of p_A^S, p_B^S and $\theta(\omega)$, it follows that

$$\Pi_A^S(\omega) = (3\tau - \omega + \underline{c})^2/16\tau \tag{32}$$

$$\Pi_B^S(\omega) = (5\tau + \omega - \underline{c})^2/32\tau + (\omega - \underline{c})(3\tau + \underline{c} - \omega)/8\tau \tag{33}$$

By standard computations, it follows that $\pi_A^S(\omega)$ is decreasing and $\pi_B^S(\omega)$ is increasing in ω.

Recall from Prop 1 that under no contracts, B obtains $\Pi_B^0 = (3\tau + \overline{c} - \underline{c})^2/18\tau$. Note that since $\tau > \overline{c}$ and $\overline{c} - \underline{c} > (3/4)\tau$, we have

$$\Pi_B^S(\underline{c}) - \Pi_B^0 = (27\tau - 4\overline{c} + 4\underline{c})[(3/4)\tau - \overline{c} + \underline{c}]/72\tau < 0 \text{ and}$$

$$\Pi_B^S(\widehat{c}) - \Pi_B^0 = 2(3\tau - \overline{c} + \underline{c})[\overline{c} - \underline{c} - (3/4)\tau]/9\tau > 0.$$

By the monotonicity of $\Pi_B^S(\omega)$, $\exists\ \widetilde{c} \in (0, \widehat{c})$ such that $\Pi_B^S(\omega) \gtreqless \Pi_B^0 \Leftrightarrow \omega \gtreqless \widetilde{c}$. Standard computations show that

$$\widetilde{c} := (11/3)\tau + \underline{c} - (2/9)\sqrt{333\tau^2 - 72\sigma\tau - \sigma^2} \tag{34}$$

This completes the proof of I(a).

Part (I)(b) From Observation 1(ii) it follows that $K(\omega) = D_A(\overline{c}, \underline{c}) = D_A^0$ for $\omega \in [\widehat{c}, \overline{c})$. As a result, prices and market shares are same as in the case of no contracts (Lemma 2(i)&(iv)) which shows that consumers are neither better off nor worse off. The payoffs follow from (12) and (13). It is immediate that both firms are better off compared to no contracts.

Parts (II)-(III) It follows by Observations 1(ii) and 2 that the conclusion of part (II) is the same as (I)(b). Part (III) follows from parts (I)-(II). ■

Proof of Proposition 4 Parts (I) and (II) have been proved in the main text. Here we prove part (III).

Part (III)(a) Recall from Prop 1 that when there are no contracts, the SPNE prices are $p_A^0 = \tau + \overline{\theta}$ and $p_B^0 = \tau + \underline{\theta}$ where $\underline{c} < \underline{\theta} < \overline{\theta} < \overline{c}$ with

$$\underline{\theta} \equiv (2\underline{c} + \overline{c})/3 \text{ and } \overline{\theta} \equiv (\underline{c} + 2\overline{c})/3 \tag{35}$$

The SPNE prices in $\Gamma^T(\omega)$ are $p_A^T(\omega) = p_B^T(\omega) = \tau + \omega$. Therefore, if $\omega \in (\underline{c}, \underline{\theta})$, then $p_i^T(\omega) < p_i^0$ for $i \in \{A, B\}$ and all consumers prefer technology transfer over no contracts. If $\omega \in (\overline{\theta}, \overline{c})$, $p_i^T(\omega) > p_i^0$ for $i \in \{A, B\}$ and all consumers prefer no contracts over technology transfer.

If $\omega \in [\underline{\theta}, \overline{\theta}]$, then $p_A^T(\omega) \leq p_A^0$ and $p_B^T(\omega) \geq p_B^0$. Note that in $\Gamma^T(\omega)$, firm A's SPNE market share is $1/2 > D_A^0$ where $D_A^0 = D_A(\overline{c}, \underline{c})$ is the SPNE market share of firm A under no contracts. Consider the consumers at $x \in [0, D_A^0]$. In both cases (i.e., no contracts and $\Gamma^T(\omega)$), they buy from firm A. As $p_A^T(\omega) \leq p_A^0$, these consumers are better off in $\Gamma^T(\omega)$. Next consider the consumers at $x \in [1/2, 1]$. In both cases they buy from firm B. As $p_B^T(\omega) \geq p_B^0$, they are worse off in $\Gamma^T(\omega)$. Finally consider any consumer at $x \in [D_A^0, 1/2)$. When there is no contract, such a consumer buys from firm B to obtain the net utility

$$U_x^0 = V - p_B^0 - \tau(1 - x) = V - (\tau + \underline{\theta}) - \tau(1 - x)$$

In $\Gamma^T(\omega)$, this consumer buys from A to obtain the net utility

$$U_x^T = V - p_A^S - \tau x = V - (\tau + \omega) - \tau x$$

Hence $U_x^T - U_x^0 \gtreqless 0 \Leftrightarrow x \lesseqgtr \lambda(\omega) := 1/2 - (\omega - \underline{\theta})/2\tau$. Since $\omega \geq \underline{\theta}$, we have $\lambda(\omega) \leq 1/2$. Since $D_A^0 = 1/2 - (\overline{c} - \underline{c})/6\tau$, from (35) we have $\lambda(\omega) - D_A^0 = (\overline{\theta} - \omega)/2\tau \geq 0$ (since $\omega \leq \overline{\theta}$). Thus $\lambda(\omega) \in [D_A^0, 1/2]$. We conclude that consumers at $x \in [D_A^0, \lambda]$ prefer technology transfer while consumers at $x \in (\lambda, 1/2)$ prefer no contracts. Since consumers at $x \in [0, D_A^0)$ prefer technology transfer and $x \in [1/2, 1]$ prefer no contracts, the proof of (III)(a) is complete.

Part (III)(b) Note that $\Pi^T(\omega) = \tau/2 + (\omega - \underline{c})$ and $\Phi_B^0 = (3\tau + \overline{c} - \underline{c})^2/18\tau$. Denoting $f(\omega) := \Pi^T(\omega) - \Phi_B^0$, note that $f(\omega)$ is increasing, $f(\underline{c}) = -(\overline{c} - \underline{c})(6\tau + \overline{c} - \underline{c})/18\tau < 0$ and $f(\overline{c}) = (\overline{c} - \underline{c})(12\tau - \overline{c} + \underline{c})/18\tau > 0$. Hence $\exists\ \widehat{\theta} \in (\underline{c}, \overline{c})$ such that $\Pi^T(\omega) \gtreqless \Phi_B^0 \Leftrightarrow \omega \gtreqless \widehat{\theta}$. Standard computations show that $\widehat{\theta} \equiv \underline{\theta} + (\overline{c} - \underline{c})^2/18\tau > \underline{\theta}$. Comparing $\widehat{\theta}$ with $\overline{\theta}$ from (35), we have $\overline{\theta} - \widehat{\theta} = (\overline{c} - \underline{c})(6\tau + \underline{c} - \overline{c})/18\tau > 0$ proving that $\underline{\theta} < \widehat{\theta} < \overline{\theta}$.

Part (III)(c) Follows from parts (a) and (b). ■

Lemma A3 will be used to prove Proposition 5.

Lemma A3 *Let $\omega \in (\underline{c}, \overline{c})$. There are constants $\alpha, \beta \in (\underline{c}, \overline{c})$ such that*

(I) $\Pi_A^T(\omega) \gtreqless \Pi_A^S(\omega) \Leftrightarrow \omega \gtreqless \alpha$.

(II) $\Pi_B^T(\omega) \gtreqless \Pi_B^S(\omega) \Leftrightarrow \omega \gtreqless \beta$.

Proof Denote $\sigma := \overline{c} - \underline{c} < \tau$. For $i \in \{A, B\}$ let $\Delta_i(\omega) := \Pi_i^T(\omega) - \Pi_i^S(\omega)$. We prove the lemma by showing that there are constants $\alpha, \beta \in (\underline{c}, \overline{c})$ such that (I) $\Delta_A(\omega) \gtreqless 0 \Leftrightarrow \omega \gtreqless \alpha$ and (II) $\Delta_B(\omega) \gtreqless 0 \Leftrightarrow \omega \gtreqless \beta$.

(I) Recall that $\Pi_A^T(\omega) = \tau/2$ for all $\omega \in (\underline{c}, \overline{c})$ (Prop 3) and $\Pi_A^S(\omega)$ is decreasing in ω (Prop 3). Hence $\Delta_A(\omega)$ is increasing in ω. To determine $\Pi_A^S(\omega)$, we consider the following possible cases where $\widehat{c} \equiv 4\overline{c}/3 - \underline{c}/3 - \tau$.

Case 1 $\sigma > (3/4)\tau$:

Subcase 1(a) $\omega \in (\underline{c}, \widehat{c})$: For this case, by Prop II(a) and (32), we have

$$\Pi_A^S(\omega) = (3\tau + \underline{c} - \omega)^2/16\tau, \text{ so that } \Delta_A(\omega) = \tau/2 - (3\tau + \underline{c} - \omega)^2/16\tau$$

Observe that $\Delta_A(\underline{c}) = -\tau/16 < 0$ and $\Delta_A(\widehat{c}) = [2\sigma^2 - (3\tau - 2\sigma)^2]/18\tau$. Hence $\Delta_A(\widehat{c}) \gtreqless 0 \Leftrightarrow \sigma \gtreqless 3(2 - \sqrt{2})\tau/2$. We have the following two possibilities.

(i) If $3(2 - \sqrt{2})\tau/2 < \sigma < \tau$, then $\Delta_A(\widehat{c}) > 0$. Since $\Delta_A(\underline{c}) < 0$, $\exists\ \widehat{\alpha} \in (\underline{c}, \widehat{c})$ such that $\Delta_A(\omega) \gtreqless 0 \Leftrightarrow \omega \gtreqless \widehat{\alpha}$. Standard computations show that

$$\widehat{\alpha} \equiv (3 - 2\sqrt{2})\tau + \underline{c} \tag{36}$$

(ii) If $(3/4)\tau < \sigma \leq 3(2 - \sqrt{2})\tau/2$, then $\Delta_A(\widehat{c}) \leq 0$. Hence $\Delta_A(\omega) < 0$ for all $\omega \in (\underline{c}, \widehat{c})$.

Subcase 1(b) $\omega \in [\widehat{c}, \overline{c})$: Using the value of $\Pi_A^S(\omega)$ from Prop 3(I)(b) and Prop 1, for this case

$$\Delta_A(\omega) = \tau/2 - \Pi_A^S(\omega) = \tau/2 - (3\tau - \sigma)^2/18\tau - (\overline{c} - \omega)(3\tau - \sigma)/6\tau$$

Note that $\Delta_A(\overline{c}) = \sigma(6\tau - \sigma)/18\tau > 0$. Noting that $\Delta_A(\omega)$ is continuous, from the last case we know that $\Delta_A(\widehat{c}) \gtreqless 0 \Leftrightarrow \sigma \gtreqless 3(2 - \sqrt{2})\tau/2$. Again we consider two possibilities.

(i) If $3(2 - \sqrt{2})\tau/2 < \sigma < \tau$, then $\Delta_A(\widehat{c}) > 0$. Hence $\Delta_A(\omega) > 0$ for all $\omega \in [\widehat{c}, \overline{c})$.

(ii) If $(3/4)\tau < \sigma \leq 3(2 - \sqrt{2})\tau/2$, then $\Delta_A(\widehat{c}) \leq 0$. Since $\Delta_A(\overline{c}) > 0$, $\exists\ \widetilde{\alpha} \in (\widehat{c}, \overline{c})$ such that $\Delta_A(\omega) \gtreqless 0 \Leftrightarrow \omega \gtreqless \widetilde{\alpha}$. Standard computations show that

$$\widetilde{\alpha} \equiv 2\underline{c}\tau/(3\tau - \sigma) - \sigma(\underline{c} + \overline{c})/3(3\tau - \sigma) + \overline{c}/3. \tag{37}$$

Case 2 $\sigma \leq (3/4)\tau$: For this case, by Prop 3(II), $\Pi_A^S(\omega)$ and $\Delta_A(\omega)$ are the same as in Subcase 1(b) for all $\omega \in (\underline{c}, \overline{c})$. We know from Subcase 1(b) that $\Delta_A(\overline{c}) > 0$. As $\Delta_A(\underline{c}) = -\sigma(3\tau - \sigma)/18\tau < 0$, we conclude that $\exists\ \widetilde{\alpha} \in (\underline{c}, \overline{c})$ [given in (37)] such that $\Delta_A(\omega) \gtreqless 0 \Leftrightarrow \omega \gtreqless \widetilde{\alpha}$.

Define

$$\alpha := \begin{cases} \widehat{\alpha} & \text{if } 3(2 - \sqrt{2})\tau/2 < \sigma < \tau \\ \widetilde{\alpha} & \text{if } \sigma \leq 3(2 - \sqrt{2})\tau/2 \end{cases} \tag{38}$$

Using (38), for $3(2-\sqrt{2})\tau/2 < \sigma < \tau$, the result follows from Subcases [1(a)(i)]-[1(b)(i)], for $(3/4)\tau < \sigma \leq 3(2-\sqrt{2})\tau/2$, it follows from Subcases [1(a)(ii)]-[1(b)(ii)] and for $\sigma \leq (3/4)\tau$, from Case 2.

(II) Recall from Prop 4 that $\Pi_B^T(\omega) = \tau/2 + (\omega - \underline{c})$ for all $\omega \in (\underline{c}, \overline{c})$ so that $\Delta_B(\omega) = \tau/2 + (\omega - \underline{c}) - \Pi_B^S(\omega)$. We consider the following possible cases.

Case 1 $\sigma > (3/4)\tau$:

Subcase 1(a) $\omega \in (\underline{c}, \widehat{c})$: For this case, using the value of $\Pi_B^S(\omega)$ from Prop 3(II) and (33), we have

$$\Delta_B(\omega) = \tau/2 + (\omega - \underline{c})[5/8 + (\omega - \underline{c})/8\tau] - (5\tau - \underline{c} + \omega)^2/32\tau$$

Note that $\Delta_B(\omega)$ is increasing in ω. Now observe that $\Delta_B(\widehat{c}) = [(\tau + 2\sigma)^2 - 13\tau^2]/24\tau < (9\tau^2 - 13\tau^2)/24\tau < 0$ (since $\sigma < \tau$). Hence $\Delta_B(\omega) < 0$ for all $\omega \in (\underline{c}, \widehat{c})$.

Subcase 1(b) $\omega \in [\widehat{c}, \underline{c})$: Using the value of $\Pi_B^S(\omega)$ from Prop 3(I)(b) and Prop 1, for this case

$$\Delta_B(\omega) = \tau/2 + (\omega - \underline{c})(1/2 + \sigma/6\tau) - (3\tau + \sigma)^2/18\tau$$

Note that $\Delta_B(\omega)$ is increasing in ω. We know from the last case that $\Delta_B(\widehat{c}) < 0$. Observing that $\Delta_B(\overline{c}) = \sigma(3\tau + 2\sigma)/18\tau > 0$, we conclude that $\exists\ \beta \in (\widehat{c}, \overline{c})$ such that $\Delta_B(\omega) \gtreqless 0 \Leftrightarrow \omega \gtreqless \beta$. Standard computations show that

$$\beta \equiv 2\overline{c}\tau/(3\tau + \sigma) + \sigma(\underline{c} + \overline{c})/3(3\tau + \sigma) + \underline{c}/3 \tag{39}$$

Case 2 $\sigma \leq (3/4)\tau$: For this case, by Prop 3(II), $\Pi_B^S(\omega)$ and $\Delta_B(\omega)$ are the same as in Subcase 1(b) for all $\omega \in (\underline{c}, \overline{c})$. From Subcase 1(b), we know that $\Delta_B(\overline{c}) > 0$. Noting that $\Delta_B(\underline{c}) = -\sigma(6\tau + \sigma)/18\tau < 0$, we conclude that $\exists\ \beta \in (\underline{c}, \overline{c})$ [given in (39)] such that $\Delta_B(\omega) \gtreqless 0 \Leftrightarrow \omega \gtreqless \beta$.

The result for $\sigma > (3/4)\tau$ follows from Subcases 1(a)-(b) and for $\sigma \leq (3/4)\tau$, it follows from Case 2. ∎

Proof of Proposition 5: **Part (I)** We prove (I) from Lemma A3 by showing that $\alpha < \beta$. Recall the notation $\sigma := \overline{c} - \underline{c}$. First let $\sigma > 3(2-\sqrt{2})\tau/2 > (3/4)\tau$. Then by Case 1 of the proof Lemma A3(II), $\beta > \widehat{c}$ and by subcases [1(a)(ii)]-[1(b)(ii)] and (38) of the proof of Lemma A3(I), $\alpha = \widehat{\alpha} < \widehat{c}$. Hence $\beta > \alpha$. Next consider $\sigma \leq 3(2-\sqrt{2})\tau/2$. Then by (38), $\alpha = \widetilde{\alpha}$. By (37) and (39) we have $\beta - \widetilde{\alpha} = \sigma(9\tau^2 + \sigma^2)/3(9\tau^2 - \sigma^2) > 0$.

Part (II) Recall from Prop 4 that SPNE prices in $\Gamma^T(\omega)$ are $p_A^T(\omega) = p_B^T(\omega) = \tau + \omega$. We consider the following possible cases.

Case 1 $\sigma > (3/4)\tau$:

Subcase 1(a) $\omega \in (\underline{c}, \widehat{c})$: For this case, by (14) of Prop 3, the SPNE prices in $\Gamma^S(\omega)$ are

$$p_A^S(\omega) = (3\tau + \underline{c} + \omega)/2 \text{ and } p_B^S(\omega) = (5\tau + 3\underline{c} + \omega)/4$$

Hence $p_A^S(\omega) - p_A^T(\omega) = (\tau + \underline{c} - \omega)/2 > 0$ (since $\tau > \overline{c} > \omega$) and $p_B^S(\omega) - p_B^T(\omega) = (\tau + 3\underline{c} - 3\omega)/4 \gtreqless 0 \Leftrightarrow \omega \lesseqgtr \tau/3 + \underline{c}$. Note from (30) that $\widehat{c} \equiv 4\overline{c}/3 - \underline{c}/3 - \tau$. Hence $(\tau/3 + \underline{c}) - \widehat{c} = (4/3)(\tau + \underline{c} - \overline{c}) > 0$. Therefore, for all $\omega \in (\underline{c}, \widehat{c})$, we have $\omega < (\tau/3 + \underline{c})$ so that $p_B^S(\omega) > p_B^T(\omega)$. Consequently for this case all consumers prefer technology transfer over outsourcing.

Subcase 1(b) $\omega \in [\widehat{c}, \overline{c})$: Note from Prop 3(I)(b) that for this case the SPNE prices in $\Gamma^S(\omega)$ are the same as in the case of no contracts. Therefore, comparing outsourcing and technology transfer for consumers is the same as comparing technology transfer with no contracts and we can use the results of Prop 4(III)(a).

Since $\underline{\theta} = (2\underline{c} + \overline{c})/3$ and $\widehat{c} = 4\overline{c}/3 - \underline{c}/3 - \tau$, we have $\underline{\theta} - \widehat{c} = \tau + \underline{c} - \overline{c} > 0$, i.e., $\underline{\theta} > \widehat{c}$. Using the partition $[\widehat{c}, \overline{c}) = [\widehat{c}, \underline{\theta}) \cup [\underline{\theta}, \overline{\theta}] \cup (\overline{\theta}, \overline{c})$, the conclusion for this case is immediate from Prop 4(III)(a). Combining the conclusions of Subcases 1(a)-(b), the proof for $\sigma > (3/4)\tau$ is complete.

Case 2 $\sigma \leq (3/4)\tau$: For this case, for all $\omega \in (\underline{c}, \overline{c})$, the SPNE prices in $\Gamma^S(\omega)$ are the same as the case of no contracts and the result is again direct from Prop 4(III)(a).

Part (III) First we prove (i) $\alpha < \underline{\theta}$ and (ii) $\underline{\theta} < \beta < \overline{\theta}$.

To prove inequality (i), note from (38) that if $\sigma > 3(2-\sqrt{2})\tau/2 > (3/4)\tau$, then $\alpha = \widehat{\alpha} < \widehat{c} < \underline{\theta}$. If $\sigma \leq 3(2-\sqrt{2})\tau/2$, then $\alpha = \widetilde{\alpha}$ and by (37) we have $\underline{\theta} - \widetilde{\alpha} = \sigma^2/3(3\tau - \sigma) > 0$. Therefore $\alpha < \underline{\theta}$ in all cases. Using inequality (i), part (III)(a) follows from (I)-(II).

Inequality (ii) follows from (39) and (35) by noting that $\beta - \underline{\theta} = \tau\sigma/(3\tau + \sigma) > 0$ and $\overline{\theta} - \beta = \sigma^2/3(3\tau + \sigma) > 0$. Hence $\underline{\theta} < \beta < \overline{\theta}$. Using inequality (ii), part (III)(b) follows from (I)-(II). ■

Lemma A4 *$c^* < \alpha$ where c^* is given in (15) and α is given in (38).*

Proof Recall that we denote $\sigma := \overline{c} - \underline{c}$. First note from (15) that if $\sigma \leq (3/4)\tau$, then $c^* = \underline{c} < \alpha$. If $\sigma > (3/4)\tau$, then $c^* = \widehat{c}$. Note from (38) that if $(3/4)\tau < \sigma \leq 3(2-\sqrt{2})\tau/2$, then $\alpha = \widetilde{\alpha}$. From the conclusion just preceding (37), we have $\widetilde{\alpha} > \widehat{c} = c^*$. Finally observe from (38) that if $\sigma > 3(2-\sqrt{2})\tau/2$, then $\alpha = \widehat{\alpha}$. From (34) and (36), standard computations yield

$$\widehat{\alpha} > \widehat{c} \Leftrightarrow h(\sigma) > 0 \text{ where } h(\sigma) := -12\sigma^2 - 72\tau\sigma + 3(110 - 3\sqrt{2})\tau^2 > 0$$

Noting that $h(\sigma)$ is an inverse u-shaped quadratic function and $h(3(2-\sqrt{2})\tau/2)$ and $h(\tau)$ are both positive, it follows that $h(\sigma) > 0$ and hence $\widehat{\alpha} > \widehat{c}$ for all $\sigma \in (3(2-\sqrt{2})\tau/2, \tau)$. This completes the proof of the result. ■

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