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Abstract

This paper develops a DSGE model of sovereign default and contagion for small open economies that have common risk averse international investors. The financial links generated by these investors explain the endogenous determination of credit limits, capital flows, and the risk premium in sovereign bond prices. In equilibrium, these variables are a function of both an economy's own fundamentals and the fundamentals of other economies. The model is able to replicate both the Wealth and Portfolio Recomposition channels of contagion. Quantitatively, the model is consistent with the contagion of the Argentinean crisis to Uruguay and the Russian crisis to Brazil.

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1 Introduction

In the last decades, the world has witnessed several financial crises that have occurred simultaneously across countries. Examples include the Debt Crisis of 1982, the Mexican Crisis of 1994, the Asian Crises in 1997, the Russian Crisis of 1998, and more recently the Euro-debt Crisis in 2011. While the simultaneity of crises could be explained by the occurrence of a common shock to several economies, contagion is another plausible explanation, and the one this paper will focus on. Contagion corresponds to the transmission of a negative income or financial shock from one economy to other economies. The empirical literature that looks at the simultaneity of crisis is quite large, and evidence of contagion in sovereign bond markets is considerable.[1]

The current paper is concerned with advancing an endogenous theory of contagion of financial crises based on financial links between economies. Countries are linked financially when they have common investors. The emphasis on financial links is strongly supported in the empirical literature.[2]

The model in this paper studies financial market links across countries in a dynamic stochastic general equilibrium setting where the stochastic processes of the emerging economies' bond prices are endogenously determined. The model extends the literature in endogenous sovereign risk in order to consider sovereign bond markets in a multi-country framework.[3] This type of model allows for an endogenous determination of the price of one period non-contingent discount bonds as a function of the economy's default risk. Through the consideration of financial links across economies, the default risk of any economy becomes a function not only of the domestic fundamentals but also a function of the fundamentals of countries which share investors with the domestic country. The model is used to show quantitatively that contagion can explain co-movements in the price of emerging economy bonds, cap-

[1]See for example Valdes(1996), Baig and Goldfajn(1998), Edwards(2000), and Baig and Goldfajn(2000).

[2]See for example Kaminsky and Reinhart(1998), Van Rijckeghem and Weder(1999), Kaminsky and Reinhart(2000), Hernandez and Valdes(2001), Kaminsky et al.(2004), Broner et al.(2006), and Hau and Rey(2008).

[3]Some of the relevant papers considering a single country include Aguiar and Gopinath(2006), Arellano(2008), Cuadra and Sapriza(2008), Hatchondo, Martinez and Sapriza(2008), Mendoza and Yue(2008), Martinez and Hatchondo(2009), and Lizarazo(2012).

ital flows, output and consumption beyond the level explained by a country's own fundamentals.

Within the present model, the framework is one of a set of small open economies with stochastic endowments. These small open economies have access to an international credit market populated by international investors. International investors are assumed to be risk averse, with preferences that exhibit decreasing absolute risk aversion in wealth (DARA). There is a problem of enforcement in the sense that international investors cannot force the small open economies to repay their debts. If any economy defaults, it is temporarily excluded from the world credit market. This context forces international investors to consider the risk of default when choosing their portfolio. Any type of reallocation of the international investors portfolio has effects over several countries at the same time. Therefore, the risk of default is endogenously determined by the economy's own fundamentals, and by the fundamentals of other economies: income shocks to an emerging economy generate changes in the risk of default in that economy, and, through financial links, these changes in turn impact other emerging economies. Financial links generate contagion through two channels, the *Wealth* channel and the *Portfolio Recomposition* channel.

(i) The **Wealth Channel of Contagion**: When an income shock in a country forces that country into default, the shock generates losses for international investors. If the preferences of the investors exhibit DARA, the negative wealth effect of the shock reduces investors' tolerance for risk. A reduction in tolerance for risk makes investors shift away from risky investments (countries) toward riskless investments (T-Bills). Countries that initially neither default nor face an income shock would face a reduction in the amount of resources available to borrow from, leading to contagion.

(ii) The **Portfolio Recomposition Channel of Contagion:** When the risk of default is correlated across countries, an increase in the risk of default in one country modifies the optimal portfolio of international investors. As investors adjust their portfolios, some countries which did not face an income shock nonetheless face a reduction in the amount of resources available to borrow from, leading to contagion. Other countries, receiving capital inflows, experience flight to quality.

The wealth channel of contagion is analyzed in Kyle and Xiong (2001), Lagunoff and Schreft (2001), and Goldstein and Pauzner (2004). These papers show that if investors' preferences exhibit DARA, then as a consequence of the reduction on their tolerance toward risk at lower levels of wealth, the optimal response of the investors to financial losses is to reduce their risky investments. The portfolio recomposition channel of contagion is studied in the theoretical papers of Choueri (1999), Schinasi and Smith (1999), Kodres and Pritsker (2002), and Broner et al. (2006). Using a partial equilibrium approach where the determination of asset returns is exogenous to the model, these papers highlight the fact that contagion might be successfully explained by standard portfolio theory: in order to reestablish the optimal degree of risk exposure in their portfolio after a negative shock to the return of the assets of some economy, it is optimal for investors to liquidate holdings of assets with expected returns that exhibit some correlation with the expected return of the crisis country.

The results of the current paper are consistent with the empirical evidence regarding contagion as a consequence of financial links. First, since investors' preferences exhibit DARA, they are able to tolerate more default risk when they are wealthier. As a consequence, both capital flows to emerging economies and the equilibrium price of sovereign bonds are increasing functions of investors' wealth levels. Furthermore, the high correlation between investors' wealth and emerging economies' financing conditions can account for the simultaneity of crises because a default by any economy is equivalent to a negative wealth shock to the investors. This shock is transmitted to other countries via the wealth channel of contagion.

Second, because of the common investors, when the probability of default increases for some foreign country, other countries' financing conditions change. When the probability of default for some foreign country increases, two opposing forces affect the financing situation of other emerging economies: On the one hand, a decrease in the price of the sovereign bonds of the foreign country constitutes an expected future negative wealth shock to the investors due to the higher associated probability that this country will default. This effect increases the default risk of the other economies. On the other hand, an increase in default probabilities induces a substitution away from the assets of the economies whose risk responds more strongly towards the assets of the economies whose risk did not increase too much. This effect would tend to increase capital flows to some emerging economies. For any country different from the crisis country, if the first effect dominates contagion is observed: the correlation of capital flows across emerging economies is positive. On the other hand, if the second effect dominates, "flight to quality" is observed: emerging economies with robust fundamentals receive capital flows when other countries are affected by financial crises. In practice, whenever the economies fundamentals are *sufficiently* weak, the effect of the expected negative wealth shock will dominate the substitution effect.

Third, the likelihood of default in equilibrium for any emerging economy is a function also of other emerging economies' fundamentals. In the numerical simulations in the present paper, for economies with relatively high default probabilities, default is more likely to be an equilibrium outcome when the fundamentals of other economies deteriorate and sovereign spreads are positively correlated; for economies with relatively low probabilities of default, default is less likely to be an equilibrium outcome when the fundamentals of other economies deteriorate and sovereign spreads are negatively correlated.

The quantitative part of the paper studies the case of the contagion of the Argentinean crisis to Uruguay, the case of the contagion of the Russian crisis to Brazil, and finally looks at a counter-factual case that exhibits the occurrence of flight to quality for economies with relatively sound fundamentals.

The paper proceeds as follows: Section II develops the model; section III focuses on characterizing contagion, and presents a simpler version of the model in order to discuss the portfolio recomposition channel of contagion; section IV presents the numerical results of the paper; and section V concludes.

2 The Model

Definition 1 *The state of the world in the model,* $S = (s, W)$*, is given by the realization of the emerging economies' fundamentals,* $s = s_1 \times s_2 \times \ldots \times s_J$ *and the representative investor's asset position or wealth,* W*. In this model,* $s_j = (b_j, y_j, d_j)$*, where* b_j *is economy* j*'s asset position,* y_j *is economy* j*'s endowment, and* d_j *is a variable that describes if economy* j *is in default or repayment state.*

2.1 Sovereign Countries

There are $J < \infty$ identical small open economies each populated by risk averse households that maximize their discounted expected lifetime from consumption

$$\max_{\{c_{j,t}\}_{t=0}^{\infty}} \qquad E_{\tau} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{j,t}\right) \tag{1}$$

where $0 < \beta < 1$ is the discount factor and $c_{j,\ t}$ is emerging economy j's consumption at time t. The periodic utility of emerging economy j takes the functional form $u(c_j) = \frac{c_j^{1-\gamma}}{1-\gamma}$ where $\gamma > 0$ is the coefficient of relative risk aversion.

In each period, the households of each economy j receive a stochastic stream of consumption goods y_j. This income is independently distributed across emerging economies, and its realizations are assumed to have a compact support and to follow a Markov process with a transition function $f(y_j' \mid y_j)$. Households in each economy j also receive a lump-sum transfer from their government.

The government of each economy j is a benevolent government that aims to maximize the lifetime utility of the households in the economy. The governments have access to international financial markets, where they can trade one-period non-contingent bonds with international investors.[4] The governments use their access to financial markets to smooth the consumption path of the households in their economy.

In the international financial markets the governments borrow or save by buying one period bonds b_j' at price $q_j(b_j', S)$. Both the investors and each government j take as given the price of economy j's non-contingent discount bonds. In each period, the proceeds of these bonds are rebated back to the households in economy j.

The bonds of any emerging economy j, b_j', are risky assets because debt contracts between the government and the investors are not enforceable. At any time, government j can choose to default on its debt. If the government defaults, all its current debt is erased, and the government is temporarily excluded from international financial markets. Defaulting also entails a direct output cost for country j.

[4]Throughout the paper it is assumed that the governments of the economies are not able to trade financial assets between them.

Because international investors are risk averse, the bond prices of the emerging economies, $q_j(b'_j, S)$ for $j = 1, \ldots, J$, have two components: the price of the expected losses from default $q_j^{RN}(\delta_j(b'_j, S))$ that corresponds to the price of riskless bonds, q^f, (hereafter T-Bills) adjusted by the default probability $\delta_j(b'_j, S)$, and an "excess" premium or risk premium $\zeta_j^{RA}(b'_j, S)$.

For any emerging economy j, when $b'_j \geq 0$, the probability of default for the economy, $\delta_j(b'_j, S)$, is 0. Since the asset is riskless in this case, the risk premium, $\zeta_j^{RA}(b'_j, S)$, is also 0. Therefore, the price of economy j's bond is equal to the price of T-Bills which is $q^f = \frac{1}{1+r}$, where r is the constant international interest rate. Only when $b'_j \leq 0$ can $\delta_j(b'_j, S)$ and $\zeta_j^{RA}(b'_j, S)$ be different from 0.

For any economy j, when its government chooses to repay its debts, the resource constraint of the emerging economy is given by

$$c_j = y_j - q_j(b'_j, S)b'_j + b_j. \tag{2}$$

For the same economy, when the government chooses to default the resource constraint is given by

$$c_j = y_j^{def} \tag{3}$$

where $y_j^{def} = h(y_j)$ and $h(y_j)$ is an increasing function.

Define $V_j^0(S)$ as the value function of the government of economy j that has the option to default. The government starts the current period with assets b_j and income y_j; the other economies that share investors with country j start the current period with assets b_k and income y_k for $k = 1, \ldots, J$ and $k \neq j$; all these countries face a representative international investor that has wealth W. The government of any economy j decides whether to default or repay its debts to maximize the households' welfare. Each government takes as given the repayment/default decisions of the other governments.[5] Given the option of default for country j, $V_j^0(S)$ satisfies

$$V_j^0(S) = \max_{\{R,D\}} \left\{ V_j^R(S), V_j^D(S) \right\} \tag{4}$$

[5]Through the paper it is assumed that the governments of the economies make their repayment/default decision at the same time.

where $V_j^R(S)$ is the value to government j of repaying its debt and $V_j^D(S)$ is the value of defaulting in the current period.

If government j defaults, then the value of default is given by

$$V_j^D(S) \;=\; u(y_j^{def}) \;+$$
$$\beta \int_{y_1'} \cdots \int_{y_J'} [\theta V_j^0(0, y_j', \{s_k'\}_{k=1,k\neq j}^J, W') \;+\; (1-\theta) V_j^D(0, y_j', \{s_k'\}_{k=1,k\neq j}^J, W')] \prod_{h=1}^J f(y_h, y_h') dy_h'.$$

where θ is the probability that a defaulting economy regains access to credit markets.

If government j repays its debts, then the value of not defaulting is given by

$$V_j^R(S) = \max_{\{b_j'\}} \left\{ u(y_j - q_j(b_j', S) b_j' + b_j) + \beta \int_{y_1'} \cdots \int_{y_J'} V_j^0(S') \prod_{h=1}^J f(y_h, y_h') dy_h' \right\}.$$

For the government of emerging country j, the repayment/default decision depends on the comparison between the value of repaying its debt, $V_j^R(S)$, versus the value of opting for financial autarky, $V_j^D(S)$. The repayment/default decision is summarized by the indicator variable d_j which takes on a value of 1 when the government repays its debt and 0 when the government does not repay its debt.

For each economy j, and conditional on the representative investor's wealth level W and the other economies fundamentals $\{s_k\}_{k=1,k\neq j}^J$, emerging economy j's default policy can be characterized by its repayment and default sets:

Definition 2 *For a given level of wealth, W, and the fundamentals of other emerging economies in the investor's portfolio, $\{s_k\}_{k=1,k\neq j}^J$, the default set $D_j\left(b_j \mid \{s_k\}_{k=1,k\neq j}^J, W\right)$ consists of the equilibrium set of y_j for which default is optimal when the government's asset holdings are b_j:*

$$D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^J, W) = \left\{ y_j \in Y_j : V_j^R(S) \leq V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^J, W) \right\}.$$

The repayment set $A_j(b_j \mid \{s_k\}_{k=1,k\neq j}^J, W)$ is the complement of the default set. It corresponds to the equilibrium set of y_j for which repayment is optimal when the *government's asset holdings are* b_j*:*

$$A_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W) = \left\{ y_j \in Y_j : V_j^R(S) > V_j^D(y_j, \{s_k\}_{k=1,k\neq j}^{J}, W) \right\}.$$

Equilibrium default sets, $D_j(b_j' \mid \{s_k'(S)\}_{k=1,k\neq j}^{J}, W'(S))$, are related to equilibrium default probabilities, $\delta_j(S' \mid S)$, by the equation

$$\delta_j(b_j', S' \mid S) = 1 - E{d_j}'(b', S' \mid S) = \int\limits_{D_j(b_j' \mid \{s_k'(S)\}_{k=1,k\neq j}^{J}, W'(S))} f(y_j' \mid y_j) dy_j' \times \prod_{k=1,k\neq j}^{J} \int_{y_k'} f(y_k' \mid y_k) dy_k'. \quad (5)$$

In this model, conditional on other economies' fundamentals $\{s_k'\}_{k=1,k\neq j}^{J}$, and on the investors' wealth W, the well known results of comparative statics follow for the model of endogenous sovereign risk with risk neutral international investors (see for example Aguiar and Gopinath (2006) Arellano(2008)): First, default sets are shrinking in the economies' assets (i.e. if $b_{j,1} < b_{j,2}$ then $D_j\left(b_{j,2} \mid \{s_k'\}_{k=1,k\neq j}^{J}, W\right) \subseteq D_j\left(b_{j,1} \mid \{s_k'\}_{k=1,k\neq j}^{J}, W\right)$), and therefore the probability of default $\delta_j(b_j', S)$ is decreasing in b_j'. Second, the governments of the emerging economies only default when the economies are facing capital outflows, i.e. when $b_j - q_j\left(b_j'(S), S\right) b_j'(S) < 0$. Third, conditional on the persistence of the income process not being too high, the default risk of any economy j is larger for lower levels of income. Since the economic intuition of these results is identical to the intuition in the model of endogenous sovereign default risk with risk neutral investors, it will not be discussed in detail here.

On the other hand, as in models of endogenous sovereign risk and risk averse investors (see for example Lizarazo (2012)), the risk premium $\zeta_j^{RA}(b_j', S)$ is also decreasing in b_j'. Therefore bond prices $q_j(b_j', S)$ are increasing in b_j'.

2.2 International Investors

There are a large but finite number of identical competitive investors who will be represented by a representative investor. The representative investor is a risk averse agent whose preferences exhibit DARA. The investor has perfect information regarding the income processes of the emerging economies, and in each period the investor is able to observe the realizations of these incomes.

The representative investor maximizes her discounted expected lifetime utility from consumption

$$E_0 \sum_{t=0}^{\infty} \beta_L^t v\left(c_t^L\right) \tag{6}$$

where c^L is the investor's consumption and $v(c^L)$, her periodic utility, is given by $v(c^L) = \frac{\left(c^L\right)^{1-\gamma^L}}{1-\gamma^L}$, with $\gamma^L > 0$. The representative investor is endowed with some initial wealth, W_0, at time 0; in each period she receives an exogenous income X.

Because the representative investor is able to commit to honor her debt, she can borrow or lend from industrialized countries (which are not explicitly modeled here) by buying T-Bills at the deterministic risk free world price of q^f. The representative investor can also invest in non-contingent bonds of the emerging economies $j = 1, \ldots, J$ which have an endogenously determined stochastic price of q_j. As was mentioned in the sub-section on the emerging economies, this price is taken as given by both the investor and the governments of the emerging economies.

On the side of the investor, the timing of the decisions within each period is as follows: After the shocks to the economies' income are realized and the governments of these economies make their repayment/default decisions, the investor realizes her gains/losses and observes her actual wealth for the period, W. W is given by $W = \vartheta^{TB} + \sum_{j=1}^{J} \vartheta_j d_j$. After observing W, the investor chooses her next period portfolio allocation, investing in the economies whose governments have payed the debt from the previous period, ϑ_j', and in T-Bills, $\vartheta^{TB'}$. Finally, the representative international investor's consumption, c^L, takes place.

In each period the representative investor faces the budget constraint

$$c^L = X + W - q^f \vartheta^{TB\prime} - \sum_{j=1}^{J} q_j \vartheta_j' d_j. \tag{7}$$

To simplify the investor's optimization problem, it is assumed that the investor cannot go short in her investments with emerging economies. Therefore, whenever the governments of the emerging economies are saving, the representative international investor receives these savings and invests them completely in $\vartheta^{TB'}$. Therefore, for any economy j, $\vartheta_j' = -b_j'$ if the economy is borrowing, and is equal to 0 otherwise.

The law of motion of the representative investor's wealth is given by

$$W' = \sum_{j=1}^{J} \vartheta_j' d_j' + \vartheta^{TB\prime}. \tag{8}$$

For the representative investor that faces J governments, each with the possibility of defaulting and each with assets b_j and income y_j at the start of the period, define the value function, $V_L^0(S)$, as follows:

$$V_L^0(S) = \max_{\{\vartheta_j'\}_{j=1}^{J},\ \vartheta^{TB\prime}} \left\{ v(X + W - q^f \vartheta^{TB\prime} - \sum_{j=1}^{J} q_j \vartheta_j' d_j) + \beta_L \int_{y_1'} \cdots \int_{y_J'} V_L^0(S') \prod_{h=1}^{J} f(y_h, y_h') dy_h' \right\}.$$

Further, the representative investor faces a lower bound on her asset holdings $\underline{\underline{W}} < 0$ that prevents Ponzi schemes, $W' \geq \underline{\underline{W}}$. $\underline{\underline{\mathrm{W}}}$ corresponds to the "natural" debt limit discussed in Aiyagari (1994). Additionally, the investors asset position in bonds of the emerging economy is non-negative, i.e. $\vartheta_j \geq 0$ for $j = 1, \ldots, J$.

Because $v(c^L)$ satisfies the standard Inada conditions, and X is sufficiently large, $c_L > 0$ always. Because the representative investor is not credit constrained, when the government does not default in the current period the solution to the investor's optimization problem can be characterized by the following first order conditions:

$$\text{For } \vartheta^{TB\prime}: \quad q^f v_{c^L}\left(c^L\right) = \beta_L \int_{y_1'} \cdots \int_{y_J'} \left[v_{c^L}\left(c^{L'}\right) \right] \prod_{h=1}^{J} f(y_h, y_h') dy_h'. \tag{9a}$$

$$\text{For } \vartheta_j': \quad q_j v_{c^L}\left(c^L\right) = \beta_L \int_{y_1'} \cdots \int_{y_J'} \left[v_{c^L}\left(c^{L'}\right)\ d_j{'} \right] \prod_{h=1}^{J} f(y_h, y_h') dy_h'. \tag{9b}$$

The set of J equations (9) determine the allocation of the representative investor's resources to each one of the J emerging countries. It is possible to manipulate equations

(9) to get

$$
\begin{aligned}
q_j &= \beta_L \int_{y'_1} \cdots \int_{y'_J} \frac{\left[v_{c^L}\left(c^{L'}\right)\ d_j{}'\right]}{v_{c^L}\left(c^L\right)} \prod_{h=1}^{J} f(y_h, y'_h) dy'_h. \\
&= \beta_L \frac{Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'\right]}{v_{c^L}\left(c^L\right)} + q_j^{RN}. \\
&= \zeta_j^{RA} + q_j^{RN}. \qquad (10)
\end{aligned}
$$

where $q_j^{RN} = q^f\left(1-\delta_j\right)$. Equation (10) shows the two components of the bond prices of economies that trade financially with risk averse investors. The first component, q_j^{RN}, compensates investors for the expected loss from default. The second component, ζ_j^{RA}, corresponds to the risk premium that economy j's bonds must carry in order to induce risk averse investors to hold them. The main determinant of the "excess" risk premium ζ_j^{RA} is the covariance term in equation (10). This covariance term is non-positive: $Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'\right] \leq 0.$ [6]

Because c^L is a function of W, γ^L, and the investor's investments in other economies, it is possible to see from equation (10), that q_j for $j = 1, \cdots, J$ are also a function of those variables. Therefore, in this model, conditional on other economies' fundamentals $\{s_k\}_{k=1,k\neq j}^{J}$, and on the investors' realized wealth W, the comparative statics results of Lizarazo(2012) follow:

Proposition 1 *For any state of the world, S, as the risk aversion of the international investor increases, the governments' incentives to default increase.*

Proof. See Appendix. ■

[6] For b'_j with $\delta_j = 0$ or $\delta_j = 1$, $Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'\right] = 0$, and $q_j = q^f$ or $q_j = 0$ respectively. If $0 < \delta_j < 1$, then for the states of the word next period in which government j repays $\left[W' \mid_{d_j{}'=1}\right] = \vartheta_j{}' + \sum_{k=1,k\neq j}^{J} \vartheta_k{}' d_k{}' + \vartheta^{TB\prime}$; and for the states in which the government j defaults $\left[W' \mid_{d_j{}'=0}\right] = \sum_{k=1,k\neq j}^{J} \vartheta_k{}' d_k{}' + \vartheta^{TB\prime}$. Because $\left[W' \mid_{d_j{}'=1}\right] > \left[W' \mid_{d_j{}'=0}\right]$ then $\left[c^{L'} \mid_{d_j{}'=1}\right] > \left[c^{L'} \mid_{d_j{}'=0}\right]$ and by concavity of $v(\cdot)$, $\left[v_{c^L}\left(c^{L'}\right) \mid_{d_j{}'=1}\right] < \left[v_{c^L}\left(c^{L'}\right) \mid_{d_j{}'=0}\right]$. As a consequence, for $b_j{}'$ with more $d_j{}' = 1$, $v_{c^L}\left(c^{L'}\right)$ is lower. Clearly for this case $Cov\left[v_{c^L}\left(c^{L'}\right),\ d_j{}'\right] < 0$.

Proposition 2 *Default sets are shrinking in the assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b_j in some states y_j given W_2, then default will be optimal for b_j for the same states y_j given W_1 and therefore* $D_j\left(b_j \mid W_2, \{s_k\}_{k=1,k\neq j}^{J}\right) \subseteq D_j\left(b_j \mid W_1, \{s_k\}_{k=1,k\neq j}^{J}\right)$

Proof. See Appendix. ■

As discussed in Lizarazo (2012), γ^L is an important determinant of the emerging economies' access to credit markets and their risk of default. The more risk averse are international investors, the higher is the default risk and the tighter are the endogenous credit constraints faced by all emerging economies. The result of Proposition 2 is consistent with empirical findings which characterize the role of investor's risk aversion in the determination of country risk and sovereign yield.[7]

Also as in Lizarazo (2012), for the present model, the higher is W, the smaller is the default risk of any economy in the investor's portfolio, and hence the more relaxed is the economy's endogenous credit constraint. Several empirical papers are consistent with the results in Proposition 2.[8] The results in Proposition 2 are also consistent with the evidence regarding financial contagion across countries who share investors.[9]

Because both investors' wealth and the fundamentals of other emerging economies in the investors' portfolio have an effect on the determination of q_j, it is clear that sovereign bond prices across economies that share investors are jointly determined and therefore must be correlated. The discussion of this correlation will be postponed until the section on the characterization of contagion channels.

[7]See, for example, Arora and Cerisola(2001), FitzGerald and Krolzig(2003), Ferruci et al.(2004), Garcia-Herrero and Ortiz(2005), Gonzales and Levy(2006), and Longstaff et al.(2008). These papers find that changes in the risk appetite of international investors have an important impact on the determination of sovereign bond spreads of emerging economies.

[8]See, for example, FitzGerald and Krolzig(2003), Mody and Taylor(2003), Ferruci et al.(2004), Gonzales and Levy(2006), and Longstaff et al.(2008). These papers establish that proxies of international investors' wealth are important factors in the determination of sovereign bond spreads for emerging economies.

[9]See, for example, Kaminsky and Reinhart(1998), Van Rijckeghem and Weder(1999), Kaminsky and Reinhart(2000), Hernandez and Valdes(2001), Kaminsky et al.(2004), Broner et al.(2006), and Hau and Rey(2008).

2.3 Equilibrium

The recursive equilibrium for the model is defined as a set of policy functions for (i) the emerging economies' consumption $\{c_j(S)\}_{j=1}^J$, (ii) the governments' asset holdings $\left\{b_j'(S)\right\}_{j=1}^J$, (iii) the governments' default decisions $\{d_j(S)\}_{j=1}^J$ and the associated default sets $D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^J, W)$, (iv) the representative investor's consumption $c^L(S)$, (v) the representative investor's holdings of emerging economies' bonds $\left\{\theta_j'(S)\right\}_{j=1}^J$, (vi) the representative investor's holdings of T-Bills $\theta^{TB'}(S)$, and (vii) the emerging economies' bond price functions $\left\{q_j(S, b_j')\right\}_{j=1}^J$, such that:

(i) Taking as given the representative investor's policies and the bond price functions $\left\{q_j(S, b_j')\right\}_{j=1}^J$, the emerging economies' consumption $\{c_j(S)\}_{j=1}^J$ satisfies the economies' resource constraints. Additionally, the policy functions $\left\{b_j'(S)\right\}_{j=1}^J$, $\{d_j(S)\}_{j=1}^J$ and default sets $D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^J, W)$ satisfy the optimization problem of the governments of the emerging economies.

(ii) Taking as given the governments' policies and the bond price functions $\left\{q_j(S, b_j')\right\}_{j=1}^J$, the representative investor's consumption $c^L(s)$ satisfies her budget constraint. Also, the representative investor's policy functions $\left\{\vartheta_j'(S)\right\}_{j=1}^J$ and $\vartheta^{TB'}(S)$ satisfy her optimization problem and the law of motion of the investor's wealth.

(iii) Bond prices reflect the governments' probabilities of default and the risk premiums demanded by the representative international investor. These prices clear the market for all the emerging economies' bonds:

$$b_j\,'(S) = -\vartheta_j\,'(S) \quad \text{if } b_j\,'(S) < 0 \tag{11a}$$

$$0 = -\vartheta_j\,'(S) \quad \text{if } b_j\,'(s) \geq 0. \tag{11b}$$

3 Contagion

From equation (10) it is evident that in this model the bond prices of economy j depend on the income realizations of other emerging economies and the associated repayment/default decisions of those countries. Hence, considering a crisis in some foreign emerging economy k as a shock that changes the expected repayment/default decisions of the government of country k, and therefore δ_k and q_k, a crisis in the emerging economy k has a wealth and a substitution effect over the optimal investor's portfolio allocation to other emerging economies.

3.1 Wealth Channel of Contagion

First, the crisis in country k has a negative current or expected wealth effect. Because the investor's preferences exhibit DARA, she would move away from risky emerging economies' assets towards safer assets; this effect corresponds to the *wealth channel of contagion.*

Proposition 3 *There is a wealth channel of contagion. Proposition 2 implies that if economy k in the investor's portfolio defaults, then for economy j, which is also in the investor's portfolio, incentives to default increase.*

Proof. See appendix. ■

The intuition of Proposition 3 is straightforward: a default by some emerging economy in the investors' portfolio is equivalent to a negative wealth shock. Therefore, from Proposition 2, the probability of default for other economies in the investors' portfolio increases as a consequence of the default by economy k.

3.2 The Recomposition Channel of Contagion

Second, the crisis in country k generates substitution between different risky emerging economy assets in the investor's portfolio. The substitution effect of the crisis corresponds to the *portfolio recomposition channel of contagion.* This channel operates when δ_k increases in this period thereby reducing the expected wealth of the investor for the following period. This reduction in wealth makes risky countries less attractive to the investor and less risky countries more attractive. In this situation, countries with weak fundamentals, which are reflected in high default probabilities, experience contagion; countries with solid fundamentals, which are reflected in low default probabilities, experience flight to quality.[10]

[10] Kaminsky, Lyons, and Schmukler(2004) identify flight to quality in the following cases: during the first two quarters after the Mexican crisis when mutual fund flows to Malaysia, Colombia, Poland,

Previous literature on the subject of contagion in partial equilibrium models (see Kodres and Pritsker (1998)) has identified that the extent of the impact of the shock in one asset over another asset depends on the degree of correlation between the returns of those two assets. In the context of the current model, this result implies that the impact of a shock in economy k over a particular economy j depends on the strength of the positive correlation between q_k and q_j.

Quantitatively, the current model exhibits the following property: If the positive correlation between q_k and q_j is low, then the positive substitution effect of the crisis in country k might dominate its negative wealth effect, and there is flight to quality. This outcome is observed when the other economies in the investor's portfolio have relatively sound fundamentals. On the other hand, when the positive correlation between q_k and q_j is large, the negative expected wealth effect dominates, and contagion is observed due to the portfolio recomposition by investors. This outcome is observed if the other economies in the investor's portfolio have relatively weak fundamentals.

Because the complexity of the model does not allows us to unambiguously characterize the operation of the portfolio recomposition channel theoretically, in the following sub-section a simpler version of the model illustrates the portfolio recomposition channel of contagion.

3.3 Example: Contagion Channels - 3 Period Model

In this section we consider a model with only 3 periods and 3 countries (V, W, Z) that share a risk averse investor whose preferences exhibit DARA. A number of simplifying assumptions will be made in order to facilitate the solution of the model. These assumptions will not be discussed in detail here, except to say that the assumptions preserve the main character and results of the infinite-horizon model. The full set of assumptions is discussed in an appendix.

Investor's optimization problem At any point in time any country might default on its debts with the investor's fund. For any country which does not default, the

and the Czech Republic increased by more than 10%; during the first two quarters after the Thai crisis, when mutual fund flows to Venezuela, the Slovak Republic, and Sri Lanka increased by more than 5%; and during the two first quarters after the Russian crisis, when mutual fund flows to Mexico and Singapore increased by more than 5%.

fund receives a repayment of X. When an economy pays back its debt, the investor receives a fixed earning of hX from her fund with h being a constant. Therefore, in periods $t = 2, 3$, the investor receives earnings $G_t = z_t hX$, where z_t is the number of countries in period t that repay their loans. In addition to the portfolio earnings, in each period of her life, the investor receives a fixed endowment of $\overline{U}$. The budget constraints of the investor are given by $c_1^L = \overline{U} - S_1 q^f$, $c_2^{*L} = \overline{U} + S_1 + G_2$ (c_2^{*L} is the consumption that at $t = 1$ the investor thinks she will have in period $t = 2$[11]), $c_2^L = \overline{U} + S_1 + G_2 - S_2 q^f$ (c_2^L is the actual investor's consumption in the period 2), and $c_3^L = \overline{U} + S_2 + G_3$. S_t is the saving of the lender in period t; G_t are the earnings from the lending activities of the investor's fund that the investor receives in t.

Taking q^f and G_t as given, the investor chooses S_1 and S_2 to solve her maximization problems in periods $t = 1$ and $t = 2$: $\underset{S_1}{Max}\ v\left(c_1^L\right) + \beta E_1 v\left(c_2^{*L}\right)$, and $\underset{S_2}{Max}\ v\left(c_2^L\right) + \beta E_2 v\left(c_3^L\right)$.

Countries' problem On the other side of the market, the emerging economies $i = V, W, Z$ have a production technology given by $Y^i = \alpha\left\{q^i X\right\}$ if the country can borrow from the investor's fund, and $Y^i = \phi$ when the country is not able to borrow from the investor's fund. α corresponds to the productivity of the economy i. Every period, after the country has borrowed from the investor's fund and has produced its good, the country sets apart the repayment to the investor's fund X. However, with an exogenous probability δ_t^i these resources are destroyed by social upheaval at the beginning of period $t+1$. ($t+1$ is the period when the payment of the debt is due.) In period $t = 1$, all countries have the same productivity α and default probability δ_1. If country i is borrowing from the investor's fund, its consumption is given by $c^i = Y^i - X = (q^i\alpha - 1)X$, and if the country cannot borrow from the fund, its budget constraint implies $c^i = Y^i = \phi$.

Countries' equilibrium bond prices Given the methodology for the pricing of the bonds, for any country i, q_1^i and q_2^i are given by $q_1^i v_c(c_1^L) = \beta E v_c(c_2^{*L}) d_2^i$, and $q_2^i v_c(c_2^L) = \beta E v_c(c_3^L) d_3^i$, where $v_c(\cdot)$ is the marginal utility of consumption of the investor, and d_t^i takes the value of 1 when economy i repays its debt and takes the

[11]In order to simplify the characterization of the equilibrium, for this example it is assumed that at the beginning of her life the investor thinks she only will be living 2 periods instead of 3.

value of 0 otherwise.

Parameters The model is solved using the following parametrization: $v(c_t^L) = log(c_t^L)$, $\beta = 0.95$, $q^f = \beta$, $\alpha = 2/\beta$, $\phi = 0.075$, for $i = V, W, Z$, and $\overline{U} = 100$.

3.4 Solution

Given these parameters, in equilibrium q_t is decreasing in δ_t and hX, and S_t is increasing in those variables: larger δ_t implies a riskier sovereign bond, and a larger hX implies more volatile portfolio earnings.

3.4.1 Wealth Channel of Contagion

Given S_1, there are four different scenarios for the investor's maximization in $t = 2$. *Case A*: all countries paid their debt from period $t = 1$. *Case B:* two countries paid their debt from period $t = 1$. *Case C:* only one country paid its debt from period $t = 1$. *Case D:* no country paid its debt from period $t = 1$. Figure 1 shows q_2 for an economy that has not defaulted on its debts from the first period, conditional on the repayment of the other economies in the portfolio of the investor, and assuming $\delta_1^i = 0.1$ for $i = V, W, Z$, and $hX = 50$.

Comparing case A to the other cases, it can be observed that when some country in the investor's portfolio defaults the countries that do not default get a lower price for their bonds. Therefore there is contagion and all economies end up having a lower c_2^i than when no country defaults.[12]

Lower portfolio earnings as a consequence of a default in period $t = 2$ correspond to the situation in the infinite horizon model of an investor with lower W as consequence of a default at period t. These results illustrate the ***Wealth Channel of Contagion***.

[12]For example, assuming $\delta_2^i = 0.1$ for $i = V, W, Z$, if economy V pays back in $t = 2$ but some or all other countries do not, then instead of having $c_2^V = 0.7462X$ as in *Case A*, economy V gets a c_2^V that is 2.79% lower for *Case B*, and a c_2^V that is 8.74% lower for *Case C*.

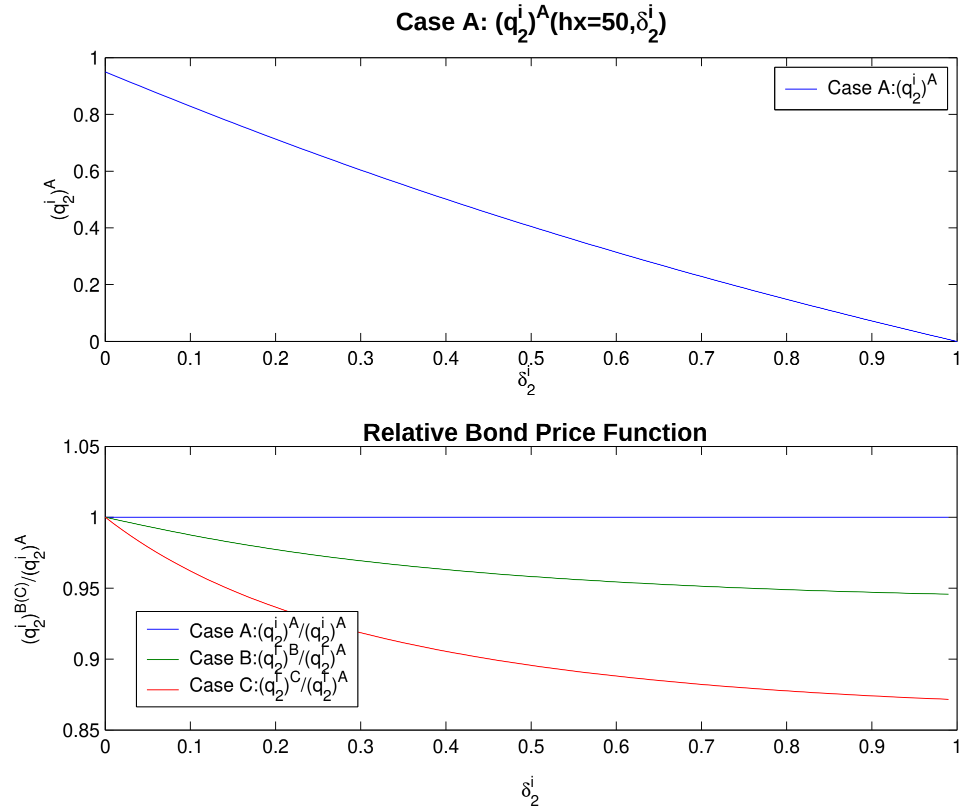

Figure 1: Wealth Channel of Contagion.

3.4.2 Portfolio Recomposition Channel

This subsubsection focuses on how an increase in the probability of default by one country effects the other countries. δ_1^i and hX are assumed to behave as in the previous subsection. It is further assumed that $\delta_2^i = 0.10$ for $i = V, W$ but that for country Z default probability is given by δ^*.

Again, there are several different scenarios for the lender's maximization in the second period. *Case A:* all countries paid their debt from period $t = 1$. *Case B1:* countries V and W paid their debt from period $t = 1$. *Case B2:* country Z and either country V or country W paid their debt from period $t = 1$. *Case C1:* only either country V or country W paid its debt from period $t = 1$. *Case C2:* only country Z paid its debt from period $t = 1$. *Case D:* no country paid its debt from period $t = 1$. For sake of brevity we focus on *Case A*, but the results apply also to *Cases B1* and

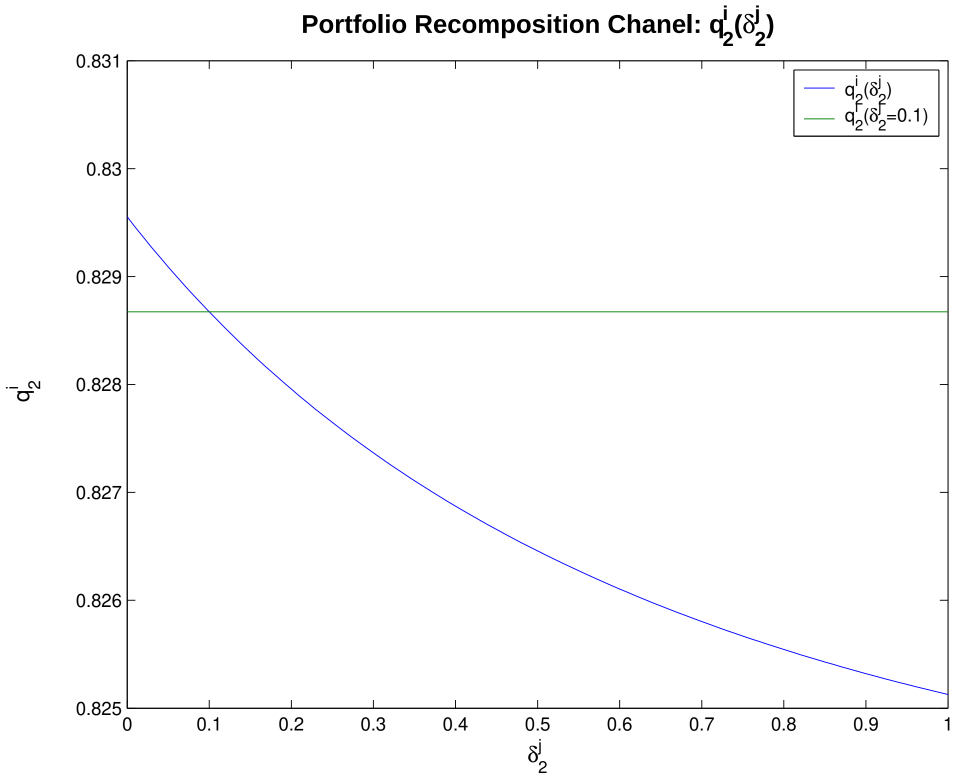

Figure 2: Portfolio Recomposition Channel.

$B2$, the other cases where is possible to observe portfolio recomposition.

Figure 2 illustrates that to have contagion it is sufficient to have the probability of default of a country increasing without the need for an actual default. This figure shows that in *Case A*, starting from a point in which $\delta^* = 0.10$, if δ^* increases, q_2^V and q_2^W fall.[13]

In the current example, a negative shock to δ^* triggers a recomposition of the investor's portfolio. This situation is observed in the infinite horizon model when one of the countries in the investor's portfolio faces a negative income shock that

[13]For example, if δ^* increases to 0.2, countries V and W consume 0.28% less than if δ^* had remained constant. The contraction in c_2^V in response to the increase in δ^* is even larger if the investor is suffering losses from a default by country W: in *Case B*2 if δ^* increases from 0.1 to 0.2, c_2^V falls by 0.63%. This contraction in c_2^V is almost 3 times larger than the contraction in c_2^V that occurs in *Case A*.

increases its default probability. As in this example, in the infinite horizon model the shock to the other country in the investor's portfolio results in a reallocation of the investor's portfolio away from risky investments causing contagion of the crisis to the other countries in the investor's portfolio. The results of this subsection illustrate the ***Portfolio Recomposition Channel of Contagion***.

Proposition 4 *There is a recomposition channel of contagion. Whenever the probability of default of some country k increases, there is a recomposition of the investor's portfolio away from other risky countries.*

3.4.3 Flight to quality

This subsection focuses on how the increase in δ^* effects the other economies in the investor's portfolio, taking into account that the default probability of the economies is endogenous to the financing conditions they face in international credit markets. In this exercise, in period $t = 2$ a country with a medium probability of default, (δ^*), receives an exogenous shock that increases its default probability. Assuming that δ_2^i increases when c^i falls, for $i = V, W, Z$, then a country with a relatively low risk (i.e., low δ_2) might experience capital inflows after the initial shock to δ^*. In the literature of financial flows this phenomena is identified as *flight to quality*. In this example it is assumed that after the initial exogenous shock to δ^*, the investment fund estimates the changes in δ_2^i of the other countries in the portfolio taking into account the comparative expected return of each of these countries for the fund after the initial shock to δ^*.[14]

We focus on *Case A* and continue to assume $\delta_1^i = 0.1$ and $hx = 50$. Additionally, it is assumed that $\delta_2^V = \delta_2^{Low} = 0.02$ initially, and $\delta_2^W = \delta^{High} = 0.2$.[15] For the revisions of δ_2^V and δ_2^W, the following functional form is assumed: $\delta_2^{i,g} = \delta_2^{i,g-1}\left(\frac{c_2^{i,g-2}-\phi}{c_2^{i,g-1}-\phi}\right)$, where $i = V, W$ and g corresponds to number of the round of the revision of δ_t^i for the economies in the investor's portfolio, and $c_t^i = \phi$ if an economy defaults.[16]

[14]The fund does this calculation taking as given the investor's savings decisions S_2.

[15]The results presented here have the fund making N rounds of sequential revisions to δ_2^V and δ_2^W (until the change in these probabilities is negligible). For simplicity, no revision to δ^* is done after the initial shock.

[16]The assumed relation between δ and c here is clearly consistent with the relation between these

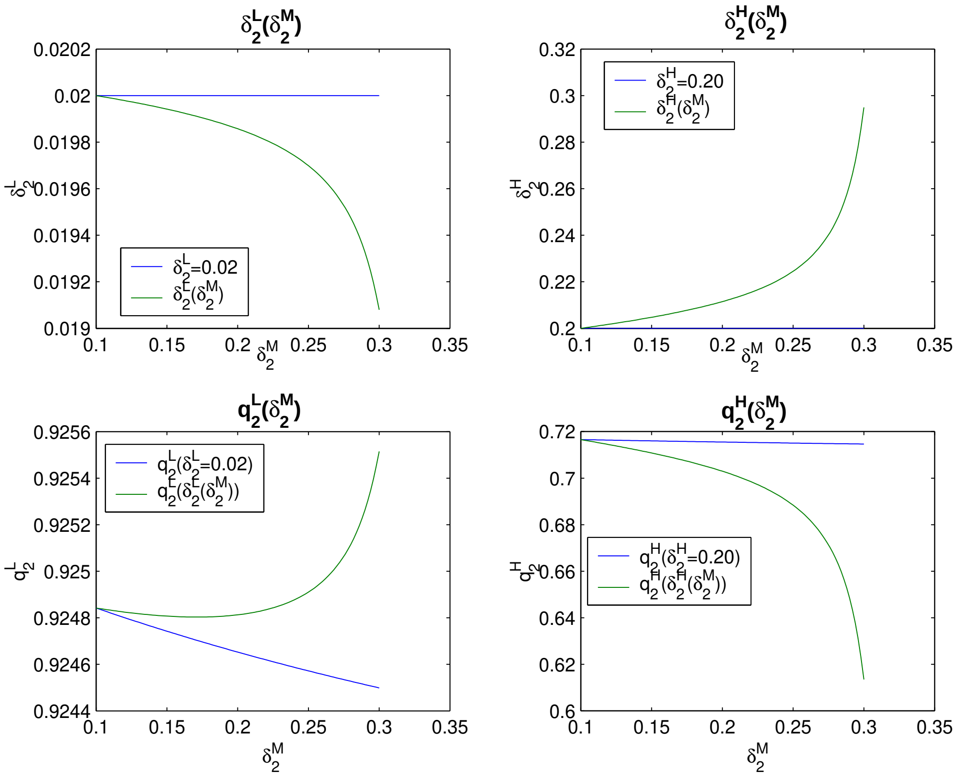

Figure 3: Flight to Quality.

Figure 3 shows q_2^V and q_2^W, and δ_2^V and δ_2^W as functions of δ^*. From this figure, it can be seen that when δ_2^i is endogenous, for $i = V, W$, and if δ^* is sufficiently large, and δ_2^V is sufficiently low, then economy V will experience flight to quality. Figure 3 also shows that if δ_2^i is constant, for $i = V, W$, then when δ^* increases, both countries suffer contagion after the shock to country Z.[17]

Because in this case capital flows to country V might increase as a consequence of the increase of the riskiness of economy Z (i.e., the increase in δ^*), this exercise illustrates the existence of ***flight to quality***.

two variables in the infinite horizon model.

[17]For example, if δ^* increases from 0.1 to 0.25, then q_2^V increases by 0.0074% instead of falling by 0.0292%, and c_2^V increases by 2.6196% instead of falling by 0.0634%; on the other hand, q_2^W falls by 3.9279% instead of falling by 0.2068%, and c_2^W falls by 9.3045% instead of falling by 0.6135%. Finally, δ_2^V falls from 2%, to 1.99997%, while δ_2^W increases from 20% to 23.166%.

Proposition 5 *For economies with relatively strong fundamentals, the effect of the crisis in some foreign economy k might be "flight to quality".*

3.4.4 Portfolio Recomposition: Contagion or Flight to quality

Figure 3 shows that the phenomena of flight to quality would occur as long as the initial change in δ^* were sufficiently large. This result is consistent with the previously mentioned results in Kodres and Pritsker (1998) that the extent of the impact of the shock in one asset over another asset depends on the degree of correlation between the assets' returns. In the context of the current example, if the shock to δ^* is not sufficiently large, the implied correlation between q_2^V and q_2^Z is positive enough so that V faces negative contagion as a result of the portfolio recomposition by the investor's fund. On the other hand, if the shock to δ^* is sufficiently large, the correlation between q_2^V and q_2^Z is negative enough to have country V experiencing capital inflows.

In the current example, to determine if flight to quality occurs, it is also important to consider the magnitude of the response of δ_2^i to changes in c_2^i. If δ_2^i responds too little or too much to changes in c_2^i, no flight to quality will be observed. Both too little and too much responsiveness of δ^i to changes in c^i fails to generate the negative correlation between q_2^V and q_2^Z. In the extreme, a very large response of δ^W to changes in c^W will have the overall riskiness of the fund's portfolio increasing 'too fast' in response to changes in δ^*. In this case, all countries will face negative contagion.

Therefore an additional conclusion can be drawn from the results of this example: Taking as given δ_2^W and δ_2^Z, there are some threshold values for δ^V that might determine if country V suffers contagion or benefits from flight to quality. Specifically, in the example here, if $\delta_2^V = 0.02$, $\delta_2^Z = 0.25$ and $\delta_2^W = 0.20$ initially, and if $\delta_2^W = 0.20$ subsequently increases to 0.25, country V receives capital inflows after the shock to δ_2^W if $\delta_2^V \leq 0.01984$; otherwise country V experiences capital outflows.[18]

This result illustrates the importance of a country's own fundamentals in determining if it is affected positively or negatively by a portfolio recomposition.

[18]Note that in the infinite horizon model, besides the financial links between the economies, the income shocks to the economies might modify δ_t in such a way as to strengthen the fundamentals of a country enough to experience flight to quality.

Corollary 3 *Given the riskiness of the other countries in the investor's portfolio, there is a critical value for the riskiness of the domestic economy that determines if the economy experiences flight to quality or negative contagion.*

3.4.5 From the Simpler Model to the Infinite Horizon Model

The model in the simple example presented here shares the most important elements of the infinite horizon model. Therefore, the richer set up of the infinite horizon model has the necessary elements to support the existence of the wealth and recomposition portfolio channels of contagion and might, under appropriate parameterizations, exhibit flight to quality. The quantitative results of the infinite horizon model confirm the results of the simpler example.

4 Quantitative Analysis

The simulation of the model in this paper looks at three different cases of transmission of crises through financial links. First, it analyzes the Argentinean default of 2001 and its contagion to Uruguay. Second, it looks at the Russian default of 1998 and its contagion to Brazil. Finally it illustrates the case of 'flight to quality' for the case of a fictional economy.

By considering the fundamentals of countries that share investors, the simulations presented here aim to replicate the following observed dynamics of sovereign yield spreads and capital flows to emerging economies: i) capital flows and domestic interest rates across emerging economies are positively correlated, and ii) default is more likely to be observed when the fundamentals of other emerging economies deteriorate.

4.1 Contagion of the Argentinean Default of 2001

During 2001 Argentina faced one of the worst economic crises of its history. The crisis forced the country to default on US$100 billion external government debt (which corresponded to nearly 37% of GDP) by the end of 2001, and had strong real effects that extended into 2002: according to estimates from the IMF, during 2001 Argentina's GDP fell by 4.4% and during 2002 it fell by an additional 10.9%.

For its part, Uruguay had been facing economic problems since 1998. These problems were aggravated in 2001 by the outburst of cow foot-and-mouth disease which negatively affected Uruguayan exports. Finally, the Argentinean crisis prompted caution in consumers and investors leading to a fall in the real demand for and a simultaneous exchange rate depreciation of the Uruguayan peso. As a result, there was a significant increase in the public debt to GDP ratio in Uruguay, reaching a level of 52%. According to the estimates of the IMF, during 2001 Uruguay's GDP fell by 3.5%, and during 2002 Uruguay's GDP fell by and additional 7.1%.

The fall in GDP in 2002 was due mainly to problems in Uruguay's financial sector which had strong financial links to Argentina: in early 2002, following the Argentina's default, Uruguay's financial sector experienced large dollar deposit outflows (these outflows exceeded US$100 million per day in the month of July 2002), as it faced a rapid decline in its international reserves (Uruguay's international reserves fell from 3 billion dollars at the end of 2001 to 650 million by August 2002). During 2002, Uruguay's debt was downgraded by investment rating agencies signaling the credit risk involved in Uruguay's external debt.

4.1.1 Simulation

Given the assumption of the model of identical economies that only differ in the realizations of their endowments, and in order to facilitate comparison with the previous literature on the subject, the parameters considered for the simulation are chosen to replicate the features of the Argentinean economy, and are taken from the calibration for this economy in Arellano (2008). The parameters related to international investors are taken from Lizarazo (2012) which presents a quantitative model with endogenous sovereign risk and risk averse international investors whose preferences exhibit DARA for the analysis of the Argentinean default.

Table 1 shows the parameters of the numerical analysis of the model. The coefficient of risk aversion of the economy is 2, a standard value considered in the business cycle literature. The free interest rate is set to 1.7%, to match the quarterly US interest rate of a bond with a maturity of 5 years during the period under study. GDP is assumed to follow a log-normal AR(1) process $log(y_t) = \rho log(y_{t-1}) + \varepsilon^y$ with $E[\varepsilon^y] = 0$ and $E[\varepsilon^{y2}] = \sigma_y^2$. The values estimated for the Argentinean economy are $\rho = 0.945$ and $\sigma_y = 0.025$. Following a default there is an asymmetrical function for

Table 1: Contagion: Parameter Values

Parameter	Value
Std. Dev. Emerging Economy's Income $std\,[y]$	0.025
Autocorr. Emerging Economy's Income Process	0.945
Emerging Economy's Discount Factor β	0.953
Emerging Economy's Risk Aversion γ	2
Probability of re-entry τ	0.282
Critical level of output for asymmetrical output cost	$\hat{y} = 0.969E(y)$
Representative investor's Income X	0.01
Representative Investor's Discount Factor β^L	0.98
Representative investor's Risk Aversion γ^L	2
Risk Free Interest Rate $r^f = \frac{1}{q^f}$	0.017

the output loss as follows:

$$\phi(y) = \left\{ \begin{array}{ll} \widehat{y} & \text{if } y > \widehat{y} \\ y & \text{if } y \leq \widehat{y} \end{array} \right\} \tag{12}$$

with $\widehat{y} = 0.969E(y)$ which in Arellano (2008) targets a value of 5.53% for the average debt service to GDP ratio. The probability of re-entry to credit markets after defaulting is set at 0.282, which is consistent with the empirical evidence regarding the exclusion from credit markets of defaulting countries (see Gelos et al. (2011)); in Arellano (2008) this value targets a volatility of 1.75 for the trade balance. The discount factor is set at 0.953 which in Arellano (2008) targets an annual default probability of 3%.

The parameters for the international investors are as follows: the representative investor's discount factor is set to 0.98. As in Lizarazo (2012), if there were no uncertainty, the discount factor of the investors would pin-down the international risk free interest rate (i.e., $\frac{\beta^L}{q^f} = 1$); however, with uncertainty, in order to have a well defined distribution for the investor's assets, it is necessary that the discount factor satisfies $\frac{\beta^L}{q^f} < 1$. The value of $\beta^L = 0.98$ is the highest value in the range commonly used in business cycle studies of industrialized countries such that for an international interest rate of 1.7% the asset distribution of the investors is well defined. The representative investor's coefficient of risk aversion is set at 2, and the criteria to choose this parameter is to generate a mean spread for model that is as close as possible to the mean spread in Argentina for the period of study, which corresponds to 12.67%.[19] The representative investor receives a deterministic income of $X = 1\%$ of the emerging economy's mean income in each period. As in Lizarazo (2012), this parameter is included to preclude the investors from not investing in the emerging economy in order to avoid a negative consumption level in the case of default. Therefore, the strategy for choosing X is to give it as little importance as possible by choosing a value that is close to 0 but that still allows for interior solutions regarding the investors' investments in the emerging economy's bonds.[20]

The model is simulated for two economies that are labeled as (A) and (U) respectively. For each economy the endowment shock is discretized into a 5 state Markov chain and the asset position of the economy is approximated by a 75 point grid. The investor's wealth level is approximated using a 10 point grid, over which the solution to the investor's problem is linearly interpolated. The business cycles statistics of the model are derived as follows: The model is simulated for $20,000$ periods. From these $20,000$ periods, sub-samples that have economy A staying in the credit market for 74 periods before going into a default are taken to compute the business cycles statistics of the two economies. This process is repeated $5,000$ times, and the cycle statistics are the average of the statistics derived from each of these repetitions.

4.1.2 Results

Table 2 describes the relevant business cycle statistics for Argentina and Uruguay for the periods under study for (i) the entire period for which data is available and (ii) for the year of the crisis. Additionally, in this table the results of the contagion model are compared with the results of a simulation of the same model with risk neutral investors. For comparison purposes, the risk neutral model has the same number of endowment shocks and the same economies' asset position as the contagion model.[21]

[19] Lizarazo(2012) also considers a value of 5 for γ^L which helps to attain a better match for the level of the spreads and their volatility, however this larger value for γ^L has important costs in terms of the computational time that it takes to solve the model. Therefore, given the larger dimension of the contagion model relative to model in Lizarazo(2012), the value of 2 is chosen for γ^L.

[20] Overall, the numerical analysis of the model shows that as long as X is not too large (i.e. $X < 100\%$ of the emerging economy's average income) the results of the model are not very sensitive to the value of X.

[21] The data for the business cycle statistics includes the period 1983:Q1-2001:Q4 for the all of the Argentinean series except the consumption series which is only available for the period 1993:Q1-

Table 2: **Business Cycle Statistics: The Model and the Data - Argentina.**

Statistics	Data		No-F.Links		F.Links	
	1983Q1-2001Q4	2001Q1-2001Q4	74Q B.D.	4Q B.D.	74Q B.D.	4Q B.D.
mean $(r^A - r^f)$ %	12.67	22.26	4.50	4.90	5.70	12.40
mean $(r^U - r^f)$ %	8.53	9.53	4.50	4.90	5.70	12.40
std $(r^A - r^f)$ %	5.42	13.59	0.60	1.00	1.11	0.81
std $(r^U - r^f)$ %	1.33	1.45	0.60	1.00	1.11	0.81
corr $(y^A, r^A - r^f)$	-0.60	-0.96	-0.13	-0.20	-0.06	-0.91
corr $(y^U, r^U - r^f)$	-0.30	-0.81	-0.13	-0.20	-0.06	-0.91
corr $(y^A, r^U - r^f)$	-0.24	-0.70	0.00	0.00	-0.03	-0.80
corr $(y^U, r^A - r^f)$	-0.44	-0.80	0.00	0.00	-0.03	-0.80
corr $(r^A - r^f, r^U - r^f)$	0.18	0.52	0.00	0.00	0.32	0.88
std $(tb/y)^A$ %	1.83	2.11	0.76	0.77	0.77	0.78
corr $(y^A, (tb/y)^A)$	-0.59	-0.85	-0.15	0.78	-0.16	0.60
std $(tb/y)^U$ %	4.27	5.62	0.76	0.77	0.77	0.78
corr $(y^U, (tb/y)^U)$	-0.48	0.26	-0.15	0.78	-0.16	0.60
corr (W, c^A)	0.35	0.21	0.00	0.00	0.31	0.84
corr (W, c^U)	0.35	0.21	0.00	0.00	0.31	0.84
corr $(W, r^A - r^f)$	-0.10	-0.71	0.00	0.00	-0.34	-0.89
corr $(W, r^U - r^f)$	-0.10	-0.71	0.00	0.00	-0.34	-0.89
Default Prob^A %	0.74		1.08		1.08	
Default Prob^B | D^A %			1.08		2.02	
mean $(-(b/y)^A)$ %	53.30		15.89	20.17	15.87	20.14
mean-Con $(-(b/y)^U)$ %			15.89	20.17	11.86	12.20

Table 2 shows that in general terms the contagion model fits the business cycle statistics of Argentina and Uruguay relatively better than the model without financial links (i.e. risk neutral investors). In the data, the spreads of Argentina are 12.67% for the whole period, and 22.26% during the crisis period, i.e. the year previous to

2001:Q4. For the Uruguayan series, the period for which the data are available corresponds to 1988:Q1-2001:Q4 for output, consumption, and trade balance, and to 1980:Q1-2001:Q4 for the interest rate. Therefore, the business cycle statistics for each variable correspond to the initial moment at which each of them is available until the fourth quarter of 2001. The correlations are taken for the common periods in which any pair of variables are available. Output and consumption for Argentina and Uruguay are seasonally adjusted and are in logs and filtered with the H-P filter. Argentina's and Uruguay's trade balances are reported as a percentage of their respective output. The interest spread is defined as the difference between the Argentinean and the Uruguayan interest rate and the yield of a 3 month U.S. T-Bill. For the Argentinean output, consumption, and trade balance, the source of the data is the IFS. For the interest rate of Argentina, the source is Neumeyer and Perri(2005). For Uruguay, the series for output, consumption, and trade balance are constructed using the Uruguayan Central Bank quarterly and annual data on indexes and volume for these variables. For Uruguay's interest rate, the source is the Uruguayan Central Bank. This rate corresponds to the domestic interest rate on loans, which is clearly not the interest rate on international loans, but should be positively correlated. Unfortunately, there is no EMBI for Uruguay. (While it would be possible to calculate an implicit interest rate from Uruguay's debt service data, such data is available only annually.)

a default episode. The contagion model generates a spread for the overall period of 5.7% and a spread of 12.4% for the crisis period, while the model without financial links only predicts a spread of 4.5% for the whole period and 4.9% for the crisis period. Both models under-predict the volatility of the spreads and they do so by nearly the same magnitude.

The better matching of the spreads by the contagion model is not the result of a higher probability of default vis-a-vis the risk-neutral model, since both models predict an annual probability of default of 4.4%. Also, the higher spreads of the contagion model do not imply a contraction in the mean debt level since both models have an unconditional mean debt level for the whole period of 15.9%, and in both models the unconditional mean debt level for the crisis period increases to 20.1%.[22]

It is also interesting to note that conditional that there is going to be a default episode by country A, the probability of default by country U is 0.94% larger per year in the contagion model than in the model without financial links. This result of the contagion model is consistent with the observed downgrading of the Uruguayan external debt by international credit rating agencies that occurred in light of the Argentinean crisis. Also, conditional that there is going to be a default episode by country A, the mean debt level for country U is 11.9% for the whole period and 12.2% for the year of the crisis.[23] This result affirms that what is going on with country A has important effects on country U's access to credit markets.

With respect to the counter-cyclical behavior of spreads and trade balances, the contagion model performs as well as the model without financial links and does better in some cases for the crisis period. For example, in the data for the period of the crisis, the correlation between Argentina's spread and its GDP is -0.96. For this same period, the correlation between the spread and the output predicted by the contagion

[22]The level of debt supported at equilibrium here is larger compared to the results in Arellano(2008). This difference might be explained by the results of Hatchondo and Martinez(2006) which show that the results of endogenous sovereign risk models are sensitive to the solution method employed as well as the dimension of the grid used to expand the endowment shocks and the asset position of emerging economies.

[23]The larger mean debt level that is observed during the periods of crisis reflects the debt dilution effect of the crisis: with lower bond prices the economies are forced to incur higher levels of debt during periods of economic distress.

model is -0.91 while the model without financial links predicts this correlation to be only -0.20. Regarding the correlation between Argentina's trade balance and Argentina's GDP for the whole period, in the data this correlation is -0.59 while both models predict the correlation to be negative and around -0.15. Unfortunately, for the period of the crisis, the correlations predicted by both models do not exhibit the counter-cyclical behavior observed in the data for Argentina. However, it is worth mentioning that for this period the correlation in the data for Uruguay is positive albeit smaller than the the one predicted by the model.

Regarding the correlations between the fundamentals of economies (A) and (U), the contagion model is clearly superior to the model without financial links: when the GDPs of the two countries are uncorrelated as assumed here, the model without financial links predicts no correlation between the two economies' fundamentals (corr(y^A,$r^U - r^f$), corr(y^U,$r^A - r^f$), and corr($r^A - r^f$,$r^U - r^f$) all equal to 0). On the other hand, the contagion model predicts the correct sign for the fundamentals' correlation for both the entire period and for the crisis period. In terms of the correlation between one country's endowment and the other country's spread, the correlation is present for the whole period but largely underestimated, as the model predicts -0.03, while the correlation in the data is -0.24. However for the period of crisis, the contagion model does a much better job at explaining this correlation: in the data the correlation is -0.70 while the model predicts it to be -0.80.

The contagion model also does very well with respect to the correlation between the spreads of the two economies. For the whole period and during the crisis period, the contagion model is consistent with the observed positive correlation of these two variables. The contagion model is also consistent with the pattern observed in the data of a significant increase in the correlation during the period of crisis. Both of these correlations are over-predicted for the case of the domestic interest rates of Argentina and Uruguay: in the data for the whole period, the correlation between the spreads is 0.18 while the model predicts it to be 0.32; for the period of crisis, the correlation is 0.52 in the data while the model predicts it to be 0.88.[24]

[24]While the correlation of the spreads is too high for the case studied here, Argentina and Uruguay, it is in line with the observed correlation of Argentina with other developing countries for the period 1994:Q3-2000:Q4. For example, the correlations predicted by the model are similar to the

The model is also able to reproduce relatively well the correlations between Argentina and Uruguay's fundamentals and the wealth of international investors as proxied by the GDP of the US. For example, in the data, there is a correlation of -0.10 and -0.71 between Argentina's spread and the GDP of the US during the whole period and the period of the crisis respectively, while the model predicts this correlation to be -0.34 and -0.89. Also, the correlations between investors' wealth and Argentina's consumption for the whole period and for the period of crisis are 0.35 and 0.21 respectively, while the contagion model predicts these correlations to be 0.31 and 0.84. The model with risk neutral investors cannot reproduce this behavior.

4.2 Contagion of the Russian Default of 1998

By mid-August of 1998 a severe crisis began in Russia. The crisis was brought on by fiscal imbalances, the deterioration of the capital account, the fall in international prices of Russian exports, and huge losses of international reserves. This crisis spread around the world to Argentina, Mexico, Venezuela, Brazil, Pakistan and South Africa.[25] For the remainder of this section, and in the numerical simulation, we will focus on Brazil. For Brazil, the impact of the Russian default was extreme: there was a substantial loss of reserves, leading to a devaluation of the Real against the Dollar of 66% by the end of January 1999; and there was an increase in the external debt-to-GDP ratio to 46%.[26] Baig and Goldfajn (2000) document that during the last quarter of 1999 there was a reduction in the international short term exposure to Brazilian banks such that out of US$6.6 billion that was maturing, only US$4.0 billion was rolled over. According to these authors, the exposure in Brazil decreased by around US$10 billion dollars from the first half of 1998 to the first half of 1999,

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ones observed for the EMBI+ pairs of Argentina-Brazil, Argentina-Mexico, Argentina-Morocco, Argentina-Nigeria, Argentina-South Africa, and Argentina-Venezuela. For 1994:Q3-2000:Q4, the average correlation for these pairs is 0.87.

[25]As noted in Kaminsky, Lyons and Schmukler(2004), following the Russian crisis total capital inflows for Latin America diminished 35%, and short term portfolio flows—bonds, equity and bank lending—fell by 60%. Dornbush, Park and Claessens(2000) argue that the sharp reversal in capital flows to emerging economies after this crisis triggered recessions in many developing countries, and that in 1999 two fifths of the world economy experienced recession, with most GDP declines concentrated in the developing world.

[26]During the year of the Russian default, capital flight from Brazil reached US$28 billion on top of US$10 billion of lost capital the year before.

Table 3: Contagion: Parameter Values - Russia

Parameter	Value
Std. Dev. Emerging Economy's Income $std[y]$	0.021
Autocorr. Emerging Economy's Income Process	0.912
Deterministic. Growth Rate of the Income Process	0.012
Emerging Economy's Risk Aversion γ	2.5
Critical level of output for asymmetrical output cost	$\hat{y} = 0.965E(y)$

while at the same time exposure in Russia decreased by almost US$15 billion dollars.

4.2.1 Simulation

Given the assumption in the model of identical economies that differ only in the realizations of their endowments, and since the default and contagion originated in Russia, the parameters for the productivity process for the simulation are chosen to replicate the Russian GDP process.[27] The parameters for the economies' risk aversion and the punishment function are set to best match the joint business cycle statistics of the Russian and Brazilian economies. The remaining parameters are the same as in the previous quantitative exercise.[28]

Table 3 shows the parameters of the numerical analysis of the model. The coefficient of risk aversion of the economy is 2.5, a value within the standard range of the business cycle literature. As before, the GDP is assumed to follow a log-normal AR(1) process $log(y_t) = \rho log(y_{t-1}) + \varepsilon^y$ with $E[\varepsilon^y] = 0$ and $E[\varepsilon^{y2}] = \sigma_y^2$. The values estimated for the Russian economy are $\rho = 0.912$ and $\sigma_y = 0.021$; additionally, an estimated deterministic growth rate of 1.21% for Russian GDP is taken into account in the the solution of the model. $\widehat{y} = 0.965E(y)$ targets a value of 11.76% for the average default probability of the Russian Economy.[29]

[27] Unlike the case of Argentina and Uruguay, the productivity process for Brazil is quite different from the productivity process for Russia, so this assumption of identical economies for the case of Russia and Brazil is not an innocuous one. However, as in the case of Argentina and Uruguay, the current exercise illustrates the role of financial links in international financial crises. Further research with a more quantitative objective in mind could provide a more realistic analysis of the Russian-Brazil contagion case by relaxing the assumption of identical economies.

[28] The model is simulated for two economies that are labeled as (R) and (B) respectively. The state space is discretized as before. For the business cycles statistics of the model, the period for the simulation is 35 quarters corresponding the period of time under study.

[29] Benjamin and Wright(2009) report that in the period $1989 - 2006$ Russia defaulted 2 times,

Table 4: **Business Cycle Statistics: The Model and the Data - Russia.**

Statistics	Data		No-F.Links		F.Links	
	1995Q1-2003Q3	1998Q3-1999Q2	35Q	4Q A.D.	35Q	4Q A.D.
mean $(r^R - r^f)$ %	12.37	43.80	1.20	2.15	6.91	13.23
mean $(r^B - r^f)$ %	4.78	6.97	1.20	2.15	6.91	13.23
std $(r^R - r^f)$ %	16.25	12.65	0.20	0.27	5.86	6.90
std $(r^B - r^f)$ %	4.87	1.76	0.20	0.27	5.86	6.90
corr $(y^R, r^R - r^f)$	-0.76	-0.84	-0.49	-0.28	-0.11	-0.45
corr $(y^B, r^B - r^f)$	-0.24	0.59	-0.49	-0.28	-0.11	-0.45
corr $(y^R, r^B - r^f)$	-0.13	-0.99	0.00	0.00	0.01	-0.42
corr $(y^B, r^R - r^f)$	-0.22	0.31	0.00	0.00	0.01	-0.42
corr $(r^R - r^f, r^B - r^f)$	0.06	0.83	0.00	0.00	0.04	0.61
std $(tb/y)^R$ %	4.99	5.21	1.15	0.85	1.79	2.36
corr $(y^R, (tb/y)^R)$	0.03	0.36	0.16	-0.24	-0.12	0.25
std $(tb/y)^B$ %	0.99	0.64	1.15	0.85	1.79	2.36
corr $(y^B, (tb/y)^B)$	-0.02	-0.37	0.16	-0.24	-0.12	0.25
corr (W, c^R)	-0.38	-0.75	0.00	0.00	-0.04	0.35
corr (W, c^B)	0.03	-0.53	0.00	0.00	-0.04	0.35
corr $(W, r^R - r^f)$	0.26	-0.87	0.00	0.00	-0.48	-0.68
corr $(W, r^B - r^f)$	-0.15	-0.53	0.00	0.00	-0.48	-0.68
Default Prob^R %	2.94		0.15		3.62	
Default Prob^B $\mid D^R$ %			0.15		3.95	
mean $(-(1+r)(b/y)^R)$ %	2.23		3.72	3.55	13.65	18.37
mean-Con $(-(1+r)(b/y)^B)$ %			3.72	3.55	8.04	8.23

4.2.2 Results

Table 4 describes the relevant business cycle for Russia and Brazil for the periods under study: 1995:Q1-2003:Q3—a period of high volatility in emerging sovereign bond markets—and the year of the Russian crisis, 1998:Q3-1999:Q2. Also in this table, the results of the model of contagion are compared with the results of a simulation of the model with risk neutral investors.[30] As before, this table shows that in general terms the contagion model fits the business cycle statistics of Russia and Brazil relatively better than the model without financial links (i.e. risk neutral investors).

In the data, the spreads of Russia are 12.37% for the whole period and 43.80%

implying an annual probability of default of 11.76%.

[30]The data for the business cycle statistics includes the period 1995:Q1-2003:Q3 for all the Russian and Brazilian series except the Russian EMBI+ series which is only available from 1997:Q4 onwards. As before, the correlations are taken for the common periods in which any pair of variables are available. Output and consumption for Russia and Brazil are seasonally adjusted and are in logs and filtered with the H-P filter. Russia and Brazil's trade balances are reported as a percentage of their respective output. The interest spread is defined as the difference between the Russian and the Brazilian EMBI+ and the yield of a 3 month U.S. T-Bill. The source of all series except the EMBI+ is the IFS. For the EMBI+ the source is JP Morgan.

during the crisis period. (In this case the crisis period refers to the year of the default episode).[31] The contagion model generates a spread for the overall period of 6.9% and 13.2% for the crisis period, while the model without financial links only predicts a spread of 1.2% for the whole period and 2.2% for the crisis period. Regarding the volatility of the spreads, both models under-predict this volatility, but the model without financial links under-predicts this volatility by much more—quantitatively, the contagion model predicts a volatility that is approximately 25 times larger than the volatility predicted by the model without financial links.

For Russia and Brazil, unlike Argentina and Uruguay, the better matching of the spreads by the contagion model is at least in part a result of the higher probability of default predicted by this model compared to the model without financial links. (Compared to the model without financial links, the contagion model predicts a probability of default for Russia much closer to the one reported in Benjamin and Wright(2009).) However, as in the case of Argentina and Uruguay, the higher spreads for the contagion model do not imply a contraction in the mean debt level supported by the model in comparison to the mean debt level supported the model without financial links. Also, as in the case of Argentina and Uruguay, for Russia and Brazil, conditional that there is going to be a default episode by country R, the probability of default for country B is larger—by 0.33% per year.

As before, both the contagion model and the model without financial links reproduce the counter-cyclical behavior of the spreads, but the contagion model seems to fit the data better for Russia's trade balance.[32] Also the contagion model is able to explain the correlation between the investor's wealth and the spreads, while the model without financial links is silent about this correlation.

As in the case of Argentina and Uruguay, for Russia and Brazil the contagion

[31]The effects of the Russian crisis on other economies were not felt before the default, but after it. For this reason, in the description of the data, the crisis period refers to the year of the default episode and not the prior year as in the case of the Argentinean Crisis. However, for consistency, the model itself treats the year prior to default as the crisis period.

[32]Russia's trade balance behaves differently than Brazil's trade balance. While Brazil's trade balance is counter-cyclical, Russia's is pro-cyclical. Since the model here assumes an income process that replicates Russia's, the model should better match Russia, which is precisely what the contagion model does in comparison to the model without financial links.

Table 5: Contagion: Parameter Values - Fictional Economy

Parameter	Value
Std. Dev. Emerging Economy's Income $std[y]$	0.057
Autocorr. Emerging Economy's Income Process	0.784

model explains relatively well the correlations between the fundamentals of economy (R) and economy (B). For correlations between the GDP of the crisis country (R) and the other country (B), the contagion model is not able to replicate the magnitude of the correlation; however, the contagion model is consistent with an important increase in the correlation during the crisis period in relation to the whole period: in the data the correlation is -0.99 while the model predicts it to be -0.42. Regarding the correlation between the spreads of the two countries, the contagion model does a very good job: in the data, for the whole period the correlation between the spreads of these rates is 0.06 and the model predicts it to be 0.04; for the period of crisis the correlation is 0.83 in the data while the model predicts it to be 0.61.

4.3 Quantitative Example of Flight to Quality

The previous quantitative results seem to suggest that the contagion model will always tend to predict a positive correlation between the spreads of economies that share common lenders. What follows is an example for a fictional economy that differs from Argentina and Russia in that its endowment process exhibits a lower autocorrelation and a larger variance for the shock, and has a zero growth rate. Previous literature in endogenous sovereign default models has shown that small persistence in the endowment process or a relatively large variance in the shock to the process can be associated with relatively lower equilibrium default probabilities. In the context of the model in this paper, smaller equilibrium default probabilities are equivalent to more sound fundamentals. In what follows, we see that for this case the contagion model can predict flight to quality.

4.3.1 Simulation

The parameters for this simulation will be identical to the ones in the Argentinean-Uruguay exercise with the exception of the parameters for the productivity process of the economy.

Table 6: **Business Cycle Statistics: The Model - Fictional Economy.**

Statistics	No-F.Links		F.Links	
	74 B.DQ	4Q B.D.	74 B.DQ	4Q B.D.
mean $(r^{(1)} - r^f)$ %	2.70	4.63	3.24	5.35
mean $(r^{(2)} - r^f)$ %	2.70	4.63	3.24	5.35
std $(r^{(1)} - r^f)$ %	1.72	3.01	2.08	2.63
std $(r^{(2)} - r^f)$ %	1.72	3.01	2.08	2.63
corr $(y^{(1)}, r^{(1)} - r^f)$	-0.63	-0.60	-0.62	-0.61
corr $(y^{(2)}, r^{(2)} - r^f)$	-0.63	-0.60	-0.62	-0.61
corr $(y^{(1)}, r^{(2)} - r^f)$	0.00	0.00	0.04	0.25
corr $(y^{(2)}, r^{(1)} - r^f)$	0.00	0.00	0.04	0.25
corr $(r^{(1)} - r^f, r^{(2)} - r^f)$	0.00	0.00	-0.05	-0.38
std $(tb/y)^{(1)}$ %	0.96	0.59	1.17	0.91
corr $(y^{(1)}, (tb/y)^{(1)})$	0.32	-0.37	0.39	-0.25
std $(tb/y)^{(2)}$ %	0.96	0.59	1.17	0.91
corr $(y^{(2)}, (tb/y)^{(2)})$	0.32	-0.37	0.39	-0.25
corr $(W, c^{(1)})$	0.00	0.00	-0.02	-0.05
corr $(W, c^{(2)})$	0.00	0.00	-0.02	-0.05
corr $(W, r^{(1)} - r^f)$	0.00	0.00	0.02	0.05
corr $(W, r^{(2)} - r^f)$	0.00	0.00	0.02	0.05
Default Prob$^{(1)}$ %	0.93		0.91	
Default Prob$^{(2)}$ $\mid D^{(1)}$ %	0.93		0.87	
mean $(-(1+r)(b/y)^{(1)})$ %	4.10	5.57	6.03	8.28
mean-Con $(-(1+r)(b/y)^{(2)})$ %	4.10	5.57	5.82	5.91

Table 5 shows the parameters for the productivity process of the economy; as before the GDP is assumed to follow a log-normal AR(1) process $log(y_t) = \rho log(y_{t-1}) + \varepsilon^y$ with $E[\varepsilon^y] = 0$ and $E[\varepsilon^{y2}] = \sigma_y^2$. The values for the fictional economy are $\rho = 0.784$ and $\sigma_y = 0.057$.[33]

4.3.2 Results

Table 6 describes the predictions for the business cycle statistics of the fictional economy. The table shows that the contagion model explains larger average spreads than the model without financial links and that the spreads exhibit larger volatility. However, the simulation also shows that for economy under study the correlation between the spreads of the bonds is negative instead of positive, showing that flight to quality occurs in equilibrium.

This flight to quality is the last piece to the contagion puzzle. In general, the

[33]The model is simulated for two economies that are labeled as (1) and (2) respectively. The state space is discretized as before, and the business cycles statistics of the model are derived as in the analysis of the Argentinean default.

possibility of flight to quality shows the robustness and flexibility of the model to apply to real-world situations. In the case of two "riskier" countries, as with Argentina and Uruguay, or Russia and Brazil, crisis in one country leads to crisis in the other country. However, as the last simulation shows, under the appropriate conditions, crisis in one country could lead to more investment in other countries.

5 Conclusions

The empirical literature in international finance presents evidence that points to a very relevant role for the fundamentals of other emerging countries in the determination of sovereign credit spreads and capital flows to emerging economies. The model in this paper is the first model that endogenously determines sovereign bond prices and at the same time endogenously accounts for contagion of crises.

The endogenization of bond prices and contagion occurs in two ways. First, the consideration of enforcement problems in sovereign debt contracts allows default risk and default incentives to be endogenized; therefore sovereign bond prices can be determined endogenously by the model. Second, the assumption of DARA for investors allows for endogenous financial links across economies that share investors. Together, these two elements build a framework that explains the contagion of crisis. The intuition for contagion is as follows: whenever a negative shock occurs in one country, this shock increases the risk associated with that country; this increase in risk implies expected future negative wealth shocks for investors. Given DARA, investors' tolerance toward risk decreases following the wealth shock, leading to a portfolio recomposition. Investors shift away from risky investments towards less risky ones.

Qualitatively the results of the model are consistent with the empirical evidence of contagion from Argentina to Uruguay and Russia to Brazil: First, sovereign spreads and capital flows to emerging economies are positively correlated across economies. Second, the fundamentals of foreign emerging economies affect the determination of domestic sovereign spreads and capital flows. Third, the average financing conditions of an economy are less favorable after other countries have defaulted.

Quantitatively, implementation of the current model faces two primary hurdles, first, the discontinuity of the default decisions, and second, the high dimension of the state space of the model. The problem of the high dimensionality of the state might be overcome if the steepness of the pricing function could be reduced: currently, prices respond too strongly to changes in the economy's debt level. (Eventhough the price function is made less steep by endogenizing financial links.) Therefore it is necessary to have very fine grids for the asset position of the economies in order to capture a great deal of the default action. This need, if satisfied, has an explosive effect on the dimensionality of the state space of the model.

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Appendix A - Proofs

The following proofs assume permanent exclusion of credit markets after a default. Under this assumption the value function of default is independent of γ^L, and W. The quantitative analysis of the model generalizes the results to the case of temporary exclusion. We focus in $b'_j < 0$ (only in this situation is default possible), and the equilibrium of the credit market implies $\theta'_j = -b'_j > 0$. More borrowing implies a more negative b'_j.

Proposition 2 *For any state of the world, S, as the risk aversion of the international investor increases, the emerging economies' incentives to default increase.*

Proof. Considering the case in which the government has not defaulted and assuming an interior solution for the allocation to the emerging economy j's asset the first order condition of the investor's problem is

$$\phi\left(\vartheta'_j\right) = E\left\{-q_j v_c\left(c_L\left(\vartheta'_j\right)\right) + \beta v_c\left(c'_L\left(\vartheta'_j\right)\right) d'_j\right\} = 0.$$

Because the periodic utility of the international investor is of the CRRA type and $\gamma_1^L < \gamma_2^L$, then there exists a concave function $\psi(\cdot)$ such that $v_2\left(c;\gamma_2^L\right) = \psi\left(v_1\left(c;\gamma_2^L\right)\right)$. If $\vartheta'_{j,1}$ is the optimal allocation when $\gamma^L = \gamma_1^L$, and $\vartheta'_{j,2}$ is the optimal allocation when $\gamma^L = \gamma_2^L$ then it holds that

$$\phi_1\left(\vartheta'_{j,1}\right) = E\left\{-q_j v_{1,c}\left(c_L\left(\vartheta'_{j,1}\right)\right) + \beta v_{1,c}\left(c'_L\left(\vartheta'_{j,1}\right)\right) d'_j\right\} = 0.$$
$$\phi_2\left(\vartheta'_{j,2}\right) = E\left\{-q_j v_{2,c}\left(c_L\left(\vartheta'_{j,2}\right)\right) + \beta v_{2,c}\left(c'_L\left(\vartheta'_{j,2}\right)\right) d'_j\right\} = 0.$$

Using $v_2\left(c;\gamma_2^L\right) = \psi\left(v_1\left(c;\gamma_2^L\right)\right)$ it is possible to define

$$\phi_2\left(\vartheta'_{j,1}\right) = E\psi'\left[v_1\left(c_L(\vartheta'_{j,1})\right)\right]\left\{-q v_{1,c}\left(c_L\left(\vartheta'_{j,1}\right)\right) + \beta\Upsilon(\vartheta'_{j,1}) v_{1,c}\left(c'_L\left(\vartheta'_{j,1}\right)\right) d'\right\} < 0.$$

where

$$\Upsilon(\vartheta_j') = \frac{\psi'\left[v\left(c_L'(\vartheta_j')\right)\right]}{\psi'\left[v\left(c_L(\vartheta_j')\right)\right]}, \;\; \Upsilon(\vartheta_j') > 0 \;\text{ and }\; \Upsilon'(\vartheta_j') < 0.^{34}$$

The last inequality comes from the fact that both $\Upsilon(\vartheta_j')$ and $\psi'\left(\vartheta_j'\right)$ are positive and decreasing. The inclusion of these functions in the previous equation implies that $\phi_2\left(\vartheta_{j,1}'\right)$ is lower than $\phi_2\left(\vartheta_{j,2}'\right)$ because $\Upsilon'(\vartheta_j')$ and $\psi'\left(\vartheta_j'\right)$ give little weight to the realizations of $d_j{}' = 1$, and high weight to the realizations of $d_j{}' = 0$. Therefore $\phi_2\left(\theta_{j,2}'\right) > \phi_2\left(\theta_{j,1}'\right)$.

The concavity of $V^L(\cdot)$ implies that given q_j and the risk of default δ_j, $\phi\left(\theta_j'\right)$ is a decreasing function and as a consequence $\theta_{j,2}' < \theta_{j,1}'$ which in equilibrium implies $b_{j,2}' > b_{j,1}'$.

Then for any state of the world S, taking as given q_j and (δ_j), a higher γ^L would result in the investor allocating a lower proportion of her portfolio to each of the economies' sovereign bonds. Therefore, when the investor is less risk averse there are financial contracts that are available to each emerging economy which are not available when the investor is more risk averse. Consequently given q_j and δ_j then $V_{j,1}^C\left(S;\gamma_1^L\right) \geq V_{j,2}^C\left(S;\gamma_2^L\right)$

Because the utility of autarky for the emerging economy does not depend on γ^L, it is clear that if for some S, default is optimal for economy j when $\gamma^L = \gamma_1^L$, then for the same S default would be optimal when $\gamma^L = \gamma_2^L$. Additionally, because incentives to default would be higher whenever $\gamma^L = \gamma_2^L$, than when $\gamma^L = \gamma_1^L$ at equilibrium $\delta_j\left(S, b_j{}'; \gamma_2^L\right) > \delta_j\left(S, b_j{}'; \gamma_1^L\right)$, and therefore $q_j\left(S, b_j{}'; \gamma_2^L\right) < q_j\left(S, b_j{}'; \gamma_1^L\right)$. ■

Proposition 3 *Default sets are shrinking in the assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b_j in some states y_j, given W_2 then default will be optimal for b_j for the same states y_j, given W_1 therefore*

[34] Taking as given the portfolio allocations of the investor to other emerging economies the derivative of $\Upsilon(\vartheta_j')$ is given by

$$\Upsilon'(\vartheta_j') = \frac{\psi''[v(c_L'(\vartheta_j'))]v_c(c_L'(\vartheta_j'))\frac{\partial c_L'(\vartheta_j')}{\partial \vartheta_j'} \; - \; \psi''[v(c_L(\vartheta_j'))]v_c(c_L(\vartheta_j'))\frac{\partial c_L(\vartheta_j')}{\partial \vartheta_j'}\Upsilon(\vartheta_j')}{\psi'\left[v\left(c_L(\vartheta_j')\right)\right]} < 0.$$

$$D_j\left(b_j \mid W_2,\psi,\{s_k\}_{k=1,k\neq j}^{J}\right) \subseteq D_j\left(b_j \mid W_1,\psi,\{s_k\}_{k=1,k\neq j}^{J}\right)$$

Proof. Because the periodic utility of the international investor exhibit DARA $v\left(W_1,\vartheta_j'\right)$ is a concave transformation of $v\left(W_2,\vartheta_j'\right)$ so if $\vartheta_{j,1}'$ is the optimal allocation when $W=W_1$, and $\vartheta_{j,2}'$ is the optimal allocation when $W=W_2$, it is possible to define $v_1\left(\vartheta_{j,1}'\right)=v\left(W_1,\vartheta_{j,1}'\right)$ and $v_2\left(\vartheta_{j,2}'\right)=v\left(W_2,\vartheta_{j,2}'\right)$, where $v_1\left(\vartheta_j'\right)=\psi\left(v_2\left(\vartheta_j'\right)\right)$. The first order conditions of the investor are

$$\begin{aligned}\phi_1\left(\vartheta_{j,1}'\right) &= E\left\{-q_j v_{1,c}\left(c_L\left(\vartheta_{j,1}'\right)\right)+\beta v_{1,c}\left(c_L'\left(\vartheta_{j,1}'\right)\right)d_j{}'\right\}=0,\\ \phi_2\left(\vartheta_{j,2}'\right) &= E\left\{-q_j v_{2,c}\left(c_L\left(\vartheta_{j,2}'\right)\right)+\beta v_{2,c}\left(c_L'\left(\vartheta_{j,2}'\right)\right)d_j{}'\right\}=0,\end{aligned}$$

and therefore

$$\phi_1\left(\vartheta_{j,2}'\right)=E\psi'\left[v_2\left(\vartheta_{j,2}'\right)\right]\left\{-q_j v_{2,c}\left(c_L\left(\vartheta_{j,2}'\right)\right)+\beta\Upsilon(\vartheta_{j,2}')v_{2,c}\left(c_L'\left(\vartheta_{j,2}'\right)\right)d_j{}'\right\}<0.$$

$\Upsilon(\vartheta')$ is defined as before, and as before the inequality comes from the fact that $\Upsilon(\vartheta_j')$ and $\psi'(\vartheta_j')$ are both positive and decreasing. Therefore $\phi_1\left(\vartheta_{j,2}'\right)<\phi_1\left(\vartheta_{j,1}'\right)$.

Again the concavity of $V^L(\cdot)$ implies that given q_j and δ_j, $\phi\left(\vartheta_j'\right)$ is a decreasing function, and as consequence $\vartheta_{j,2}'>\vartheta_{j,1}'$ which in equilibrium implies $b_{j,2}'<b_{j,1}'$.

Then for any S and taking as given q_j and δ_j, a lower level of W would result in this agent allocating a lower proportion of her portfolio to the emerging economies' sovereign bonds. Therefore, when the investor is more wealthy there are financial contracts that are available to the emerging economy that are not available when the investor is less wealthy. Consequently, given q_j and δ_j then $V_{j,1}^C(S_{-W};W_2)\geq V_{j,2}^C(S_{-W};W_1)$. [35]

Because the utility of autarky for the emerging economy does not depend on W, it is clear that if for some b_j in some states y_j, default is optimal when $W=W_2$, then for the same states y_j default would be optimal when $W=W_1$. Because incentives to default would be higher whenever $W=W_1$, than when $W=W_2$ at equilibrium $\delta_j\left(S,b_j{}';W_1'\right)>\delta\left(S,b_j{}';W_2'\right)$, and therefore $q\left(s,b_j{}';W_1'\right)<q\left(s,b_j{}';W_2'\right)$. ∎

[35] S_{-W} corresponds to all the state variables of the world except for the wealth level of the investors.

Proposition 6 *There is a wealth channel of contagion. Proposition 2 implies that if economy k which is in the investor's portfolio defaults in her debts, incentives to default for economy j which is also in the investor's portfolio increase.*

Proof. If economy k defaults in her debts with the investor, the wealth of this agent will be $(W \mid d_k = 0) = \theta^{TB'} + \sum_{m=1,m\neq k}^{J} \theta'_m$, which is lower than the wealth for economy k if she decides not to default, which is $(W \mid d_k = 1) = \theta^{TB'} + \sum_{m=1,m\neq k}^{J} \theta'_m + \theta'_k$. Therefore

$$V_j^C\left(y_j, b_j, \{y_m\}_{m=1}^J, \{b_m\}_{m=1}^J, (W \mid d_k{=}1)\right) > V_j^C\left(y_j, b_j, \{y_m\}_{m=1}^J, \{b_m\}_{m=1}^J, (W \mid d_k{=}0)\right)$$

which implies that emerging economy $j's$ incentives to default are larger when some economy k which shares investors defaults. ■

Appendix B - 3 Period Contagion Model

Assumptions for 3 Period Contagion Model In this section we discuss a number of simplifying assumptions made to facilitate the analytical solution of the three period model used in the paper. The model assumptions discussed here preserve the main character and results of the infinite-horizon model.

(i) Initially, the default probability of the countries, δ^i, for $i = V, W, Z$ is assumed to be exogenous. Subsequently, to study the phenomena of flight to quality, it will be assumed that δ^i is a decreasing function of the level of consumption of the economies.[36]

(ii) The investor's portfolio allocation to T-Bills (S_t) is decided independently of the investor's portfolio allocation to the emerging economies: The investor owns an investing fund through which she invests in the countries. The fund has an initial amount of resources that it can use only to invest in the economies; any resources left in the fund at the end of the third period go to pay for the fund's operating expenses. On the other hand, taking into account only the earnings from the investments made by the fund on her behalf, the investor decides her borrowing or saving in riskless bonds, S_t, that she trades in international credit markets at a risk free bond price of q^f.[37]

(iii) The portfolio allocations of the investor's fund to the emerging economies are fixed, and identical across countries: Each period the countries borrow a fixed amount $-b^{i\prime} = X$, at a price q^i that is endogenous: q^i depends on the default probability of the economy δ^i and on the endogenous risk premium that risk averse investors demand in order to invest in risky assets ζ^i.[38]

(iv) To simplify the model even further, it is assumed that at the beginning of the initial period $t = 1$, the investor believes that she will only be alive during two

[36] In the infinite horizon model δ^i is endogenous and depends on the level of consumption of economy i. The functional form for δ^i in the infinite horizon model is more complex than the one assumed here for the simpler model.

[37] In the infinite horizon model the allocation between emerging economies' bonds and T-Bills is done jointly.

[38] In the infinite horizon model countries' borrowing is endogenous.

periods; only at the beginning of period $t = 2$ does the investor realize that she is going to be around for an additional period.[39]

(v) The pricing of the bonds of the economies q^i is done as in the infinite horizon model using the utility function of the investor. In other words, it is assumed that the profit function of the investor's fund is identical to the investor's utility function.[40]

Model First Order Conditions In what follows the first order condtions for the investor and the investing fund are presented:

(i) The first order conditions for the investor who chooses S_1 and S_2 are given by:

$$S_1 \quad : \qquad q^f u_c(c_1^L) = \beta E_1 u_c(c_2^{*L}). \tag{A-1}$$

$$S_2 \quad : \qquad q^f u_c(c_2^L) = \beta E_2 u_c(c_3^L). \tag{A-2}$$

(ii) The first order conditions for the investor's fund which determined the equilibrium bond prices of the economies q_1^i and q_2^i are given by:

$$q_1^i v_c(c_1^L) \quad = \quad \beta E v_c(c_2^{*L}) d_2^i. \tag{A-3}$$

$$q_2^i v_c(c_2^L) \quad = \quad \beta E v_c(c_3^L) d_3^i. \tag{A-4}$$

where $v_c(\cdot)$ is the marginal utility of consumption of the investor, and d_t^i is a variable that takes the value of 1 when economy i repays its debt and takes the value of 0 otherwise.

Note on the Solution of the Model

[39] In the infinite horizon model the investor is fully aware of the duration of her life. In the 3 period model this assumption is made in order to simplify the determination of the bond prices of the economy by having S_1 depending only on G_1 and G_2 (which are exogenous), and q_1 depending only on G_1, G_2, and S_1. Otherwise q_1 would depend also on G_3 and S_2 and S_2 would in turn depend on the repayment decisions of period $t = 1$ and the expected repayment decisions in periods $t = 2$, and $t = 3$.

[40] In the three period model the pricing of the bonds should be computed using the investing fund's objective function which could be quite different from the investor's utility function. However, the goal here is to facilitate comparison with the infinite horizon model.

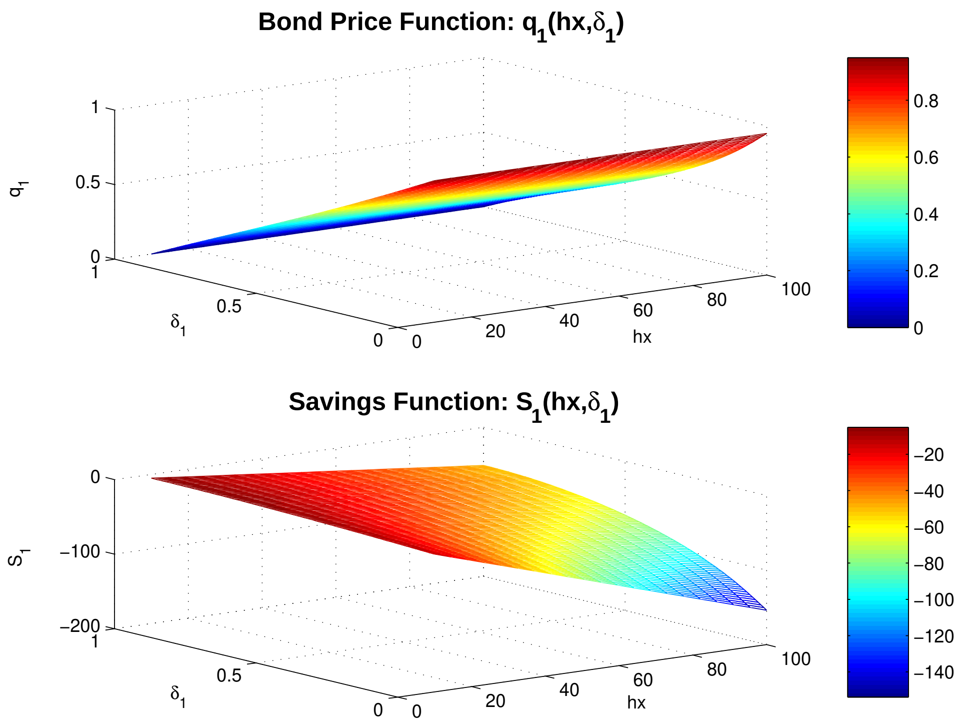

Figure 4: Bond Price Function and Savings Function.

(i) Given the parameterization of the model, Figure 4 presents q_1^i and S_1 as functions of δ_1^i and hX.[41] From Figure 4, it is possible to observe that q_t is decreasing in δ_t and hX, and S_t is increasing in those variables: larger δ_t implies a riskier sovereign bond, and a larger hX implies more volatile portfolio earnings.

(ii) In the discussion of the wealth channel of contagion for the simpler model, q_2 is given by the following FOC for each of the cases (A, B, C):

$$\begin{aligned}
q_2^{\{A\}} \frac{1}{U + S_1 + 3xh - S_2 q^f} &= \beta \left\{ (1-\delta_2)^3 \frac{1}{U + S_2 + 3xh} + 3(1-\delta_2)^2 \delta_2 \frac{1}{U + S_2 + 2xh} \right. \\
&\qquad \left. + 3(1-\delta_2){\delta_2}^2 \frac{1}{U + S_2 + xh} \right\}, \\
q_2^{\{B\}} \frac{1}{U + S_1 + 2xh - S_2 q^f} &= \beta \left\{ (1-\delta_2)^2 \frac{1}{U + S_2 + 2xh} + (1-\delta_2) \frac{1}{U + S_2 + xh} \right\}, \\
q_2^{\{C\}} \frac{1}{U + S_1 + xh - S_2 q^f} &= \beta \left\{ (1-\delta_2) \frac{1}{U + S_2 + xh} \right\}.
\end{aligned}$$

[41] Results for the second period are analogous.