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Inter-generational Effect of Parental Time and Its Policy Implications*

Guozhong Zhu† and Gulfer Vural‡

Abstract

Why do parents with more human capital spend more time teaching and taking care of their children, in spite of the higher opportunity cost? How does this affect inter-generational mobility and wage inequality? Does this have any implications on the policy that provides public schooling through income taxation? We develop and estimate a theoretical model to answer these questions, in the light that parental time investment is a powerful means of transmitting human capital inter-generationally.

JEL Classification: E20, R20, R30.

Keywords: Human capital production, Parental time investment, Wage inequality, Earnings persistence, Public schooling.

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1 Introduction

Empirical studies reveal that parental time with children is strongly positively correlated with parents’ human capital, proxied by either wage rate or educational attainment. Using the 2003-2006 waves of the American Time Use Survey, Guryan et al. (2008) document a positive wage/education gradient in child care time which holds true for different categories of child care, including basic, educational, recreational and travel-related child care. Similar patterns have been documented in many earlier studies, although in a less comprehensive way. Positive wage gradient of parental time is found in Hill and Stafford (1974), Kimmel and Connelly (2007) and others; while positive education gradient is seen in Leibowitz (1974b), Leibowitz (1974a), DeSimone and Dills (2006), Ramey and Ramey (2010), etc. The positive correlation is also found in countries other than the U.S., including both developed and developing countries.\footnote{See Gauthier et al. (2004), Guryan et al. (2008).}

Why should higher wage/education parents spend more time with their children despite the higher opportunity cost? We answer this question based on a simple idea – altruistic parents make both time investment and goods investment to produce their next generation’s human capital. If the two types of investment have low substitutability, then higher wage/education parents make more time investment in optimality to complement goods investment.

We formalize this idea in a model featuring inter-generational transmission of human capital. We show analytically that the wage gradients of parental investment are positive if time-goods substitutability is low. Further, the gradients are reduced by the public policy that levies a proportional
tax on labor income and provides public schooling.

Ramey and Ramey (2010) provide another channel that leads to the positive wage/education gradient of time investment. Using a model in which parental time is the only input in human capital production, Ramey and Ramey (2010) show that more educated parents make more time investment on the premise that their time investment is more efficient than that of their less educated counterparts. This channel is admitted in an extended version of our model. When human capital production needs both goods and time inputs, we show that whether higher productivity of time investment leads to more time investment also depends on the substitutability between time and goods investment. When the substitutability is high, parents with more human capital should make less time investment and work more to provide more goods investment, unless their advantage in parenting outweighs the advantage on the labor market.\(^2\)

Our model is developed for studying the inter-generational effects of parental investment. Through time and goods investment, parents partially transmit their human capital to the next generation, thus earnings must exhibit inter-generational persistence. In addition, because richer parents make more investment, parental investment is also a source of long-run wage inequality. In order to assess these inter-generational effects, we estimate the model parameters through the simulated method of moments, then use these parameters to decompose wage inequality and earnings persistence quantitatively.

\(^2\)We are grateful to a referee for pointing out the importance of this additional mechanism.
We also quantitatively analyze the policy effects of public schooling. Since the public policy triggers a reinforcing mechanism among time investment, goods investment and human capital accumulation, it effectively reduces wage inequality and earnings persistence, and increases wage level, leisure and consumption. In an otherwise similar model that assumes exogenous time investment, the policy effects are significantly weaker. Given the strong empirical evidence of positive wage gradient, the endogenization of time investment is critical in policy analysis. We also show that, if parents with more human capital are more efficient in time investment, the public policy is less effective in reducing inequality and earnings persistence, but more effective in promoting human capital accumulation.

The paper that is closest to ours is Restuccia and Urrutia (2004) which considers a model of inter-generational human capital transmission featuring two types of goods investment: early education, and college education. Many important traits exist both in that paper and ours. Both papers assume that altruistic parents make all the decisions for children. Individuals are heterogeneous in their own human capital and their children’s innate ability which is persistent across generations. Innate ability interacts multiplicatively with parental investment in the production of human capital. In addition, both

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3 Two stylized data facts are consistent with the view that parental time with children is a type of investment. First, in cross sectional data, parental time with children follows a different pattern than either leisure or home production time – the amount of time allocated to home production and to leisure falls sharply as income and educational attainment rise (Aguiar and Hurst 2007), Kimmel and Connelly (2007), Guryan et al. (2008)). Second, parental time exhibits positive effects on children’s outcome (Leibowitz 1974b), Cooksey and Fondell (1996), Li et al. (2005)).
papers find that innate ability accounts for the majority of inter-generational persistence. The distinctive feature of our model is the role of parental time investment. We compare our model and a variant of their model to show that policy effects are significantly different when parents can respond to the policy with changing time investment. Our paper is also closely related to Glomm and Ravikumar (1992) and Glomm and Ravikumar (2003). Both papers use dynamic models to explore the interactions among parental investment, inter-generational earnings persistence and long-run wage inequality.

The paper is also related to the vast literature that studies the inter-generational correlation of earnings and educational attainment. Empirically, earnings exhibit a significant inter-generational correlation. For example, Solon (1992) regresses children’s log earnings when adults against parents’ log earnings, and obtains the slope coefficient that is around 0.45. Aaronson and Mazumder (2008) also report high inter-generational correlation. It is debated whether the inter-generational correlation is largely due to the “nature effect” or the “nurture effect”. A large number of studies find that the “nature effect” is the key determinant. Using a life cycle model, Huggett et al. (2011) also find that learning ability differences constitute an important part of the rise in earnings dispersion over the lifetime. Our structural esti-

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4Parental income (or parents’ educational performance) is also correlated with children’s educational outcomes. See Acemoglu and Pischke (2001), Dahl and Lochner (2008) and Tomainy (2009), Oreopoulos et al. (2003), Chevalier (2004), Black et al. (2005) and others.

5See Behrman and Taubman (1989), Shea (2000), Sacerdote (2002), Plug and Vijverberg (2003), Maurin (2002). A number of papers explicitly study the causality between parental income (education) and children’s educational attainment. See, for example, Blau (1999), Black et al. (2005), and Loken (2010).
mation results are consistent with these findings, showing that the “nature effect” accounts for a large fraction of inter-generational earnings persistence.

The rest of the paper is organized as follows: Section 2 introduces the model and presents the analytical results regarding positive wage gradient of time investment. Some analytical results related to policy effects are also obtained. In section 3, we estimate the parameter values of the model and use them to show how public policy affects resource allocation and wage structure. Quantitative decomposition of wage inequality and earnings persistence are also carried out in this section. Section 4 emphasizes the role played by time investment by comparing results between our model and a model in which time investment is exogenous. Section 5 includes further discussion of (i) the extended model in which parents with more human capital are more efficient in time investment, (ii) substitutability between time and goods investment, (iii) cross country comparison of parental time and public spending on education, (iv) other forms of parental altruism. Section 6 concludes. Description of the data and model solution strategy are left to the Appendix.

2 The baseline model and analytical results

In this section we lay out the baseline model and present its properties. We consider an overlapping generations model. The economy is populated by a continuum of individuals who live for two periods. In the first period, an individual is a child, receiving time and goods investment from the adults (parents) to form her own human capital. In the second period she is an
adult, making decisions regarding her time and goods allocation.

2.1 Human capital production

Let $h_i$ be the human capital of the $i^{th}$ adult, $e_i$ and $g_i$ be her time investment and goods investment, then the human capital of her child is

$$h'_i = z_i A[\alpha e_i^\sigma + (1 - \alpha)g_i^\sigma]^{\frac{1}{\sigma}}$$

(1)

where $z_i$ is the child’s learning ability, $A > 0$ is the aggregate technology level.

Since our paper focuses on the role of time investment in determining earnings persistence and wage inequality, we simply view $A$ as a scaling parameter. The model can easily be extended to encompass growth by including a time trend in $A$. The coefficient $\sigma \in (-\infty, 1]$ governs the substitutability between $e_i$ and $g_i$. Define $\eta = \frac{1}{1-\sigma}$, then $\eta$ is the elasticity of substitution between $e_i$ and $g_i$. We say that $e_i$ and $g_i$ are gross complements if $\eta < 1$ ($\sigma < 0$), and gross substitutes if $\eta > 1$ ($\sigma > 0$). The coefficient $\alpha \in (0, 1)$ determines the relative share of time investment and goods investment in the production of human capital.

Learning ability $z_i$ is random and exogenous. It evolves according to

$$\ln z'_i = \rho \ln z_i + \epsilon_i$$

(2)

where $\epsilon_i$ is an i.i.d. random shock drawn from normal distribution with mean $-\frac{1}{2} \frac{\nu^2}{\nu^2 \rho}$ and variance $\nu^2$. It is easy to show that the unconditional mean of learning ability is $E[z_i] = 1$.\(^6\)

\(^6\)From the distribution of $\epsilon_i$, $\ln z_i$ is also normally distributed with mean $\mu_{\ln z} = -\frac{1}{2} \frac{\nu^2}{\nu^2 \rho}$.

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The formulation in equation (1) assumes that the human capital of a child is entirely chosen by the adult. This distinguishes our model from those that assume a child can choose her own human capital stock. For example, Glomm and Ravikumar (1992) assume that a child allocates her time between leisure and human capital production. In this paper we emphasize the role played by parental time in human capital formation and its effect on wage inequality and inter-generational earnings persistence. Therefore we do not take child’s own time allocation decision into account.

2.2 Individual’s optimization behavior

An adult is endowed with one unit of time, and allocates it among work, leisure and time investment to maximize lifetime utility.

2.2.1 The optimization problem

The \( i^{th} \) adult solves the following optimization problem.

\[
\max_{c_i, n_i, e_i, g_i} \ln c_i + \gamma \ln n_i + \beta \ln h_i'
\]

subject to equation (1) and

\[
c_i + g_i = (1 - e_i - n_i)w_i \tag{3}
\]

\[
w_i = h_i \tag{4}
\]

where \( c_i \) and \( n_i \) are consumption and leisure respectively. The relative importance of leisure is governed by \( \gamma \), and the strength of parental altruism is and variance \( \nu_{lnz}^2 = \frac{\nu^2}{1-\rho^2} \). Therefore \( z_i \) follows log-normal distribution with mean \( \exp(\mu_{lnz} + \frac{1}{2} \nu_{lnz}^2) = 1 \).
determined by $\beta$. In equation (3), $1 - e_i - n_i$ is the adult’s work hour and $w_i$ is the wage rate which equals the stock of human capital ($h_i$). Shocks to learning ability are revealed before the adult makes decisions, hence there is no uncertainty about the outcome of these decisions.

It is assumed that the adult cares only about the child’s human capital stock. This modeling strategy, following Glomm and Ravikumar (1992), is very common in the literature of inter-generational transmission of human capital. In our framework, it enables us to derive closed-form solutions to individual’s optimization problem and analytical results regarding the effects of public policy.

2.2.2 Optimal decisions

Why do parents with more human capital make more time investment? The question is answered in the solution to the individual’s optimization problem. For simplicity we omit subscript $i$. The following equations present the optimal investment decisions as functions of the adult’s human capital stock $h$.

\begin{align*}
g &= \frac{h}{\left(\frac{1+\gamma}{\beta} + 1\right) \left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma - 1}} h^{\frac{\sigma}{\sigma - 1}} + 1\right]} \tag{5} \\
e &= \frac{\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma - 1}} h^{\frac{\sigma}{\sigma - 1}}}{\left(\frac{1+\gamma}{\beta} + 1\right) \left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma - 1}} h^{\frac{\sigma}{\sigma - 1}} + 1\right]} \tag{6}
\end{align*}

Following Becker (1981), a strand of literature emerged discussing whether parents choose to invest more in the human capital of abler children or not. From equation (5) and equation (6) below, it can be seen that our model implicitly assumes that parental investment is independent of children’s ability.
The derivatives of parental investment with respect to human capital stock are given in the following two equations.

\[
\begin{align*}
\frac{dg}{dh} &= \frac{\left(\frac{1}{1-\sigma}\right) \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h^{\frac{1}{\sigma-1}} + 1}{\left(\frac{1+\gamma}{\beta} + 1\right) \left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h^{\frac{1}{\sigma-1}} + 1\right]^2} \\
\frac{de}{dh} &= -\frac{\left(\frac{\sigma}{1-\sigma}\right) \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h^{\frac{1}{\sigma-1}}}{\left(\frac{1+\gamma}{\beta} + 1\right) \left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h^{\frac{1}{\sigma-1}} + 1\right]^2}
\end{align*}
\] (7) (8)

Equations (7) and (8) are the wage gradients of parental investment. Both are positive as long as time and goods investment are complements \((\sigma < 0)\).

Therefore, our model can explain the stylized data facts we introduced in the very beginning. These results are delivered by two model assumptions. First, altruistic parents equate their own marginal utility with the marginal product of goods investment, thus richer parents make more goods investment. Second, the two types of investment have low substitutability, thus richer parents also make more time investment to complement goods investment.

### 2.2.3 Evolution of human capital

Plugging the solutions for \(g\) and \(e\) into equation (1), we express the child’s human capital \(h'\) as a function of her own learning ability \(z\) and her parents’ human capital \(h\).

\[
h' = z D^{\frac{1}{\sigma}} \left[\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\sigma}} + h^{\frac{1}{1-\sigma}}\right]^{\frac{1-\sigma}{\sigma}}
\] (9)

where \(D\) is a constant defined as \(D = \left(\frac{\beta A}{1+\gamma+\beta}\right)^{\frac{\sigma}{1-\sigma}} (1-\alpha)^{\frac{1}{1-\sigma}}\).
Clearly $h'$ increases with $h$ and $z$. Therefore in our model wage is persistent inter-generationally for two reasons: inter-generational transmission via parental time and goods investment, and the persistence of learning ability $z$. Recall that we assumed that $z$ follows an AR(1) process in equation (2).

From equation (9), $h$ is convergent as long as $D < 1$. To see this, it is sufficient to show that $h^* = h^{\frac{\sigma}{1-\sigma}}$ is convergent. The law of motion of $h^*$ is

$$h^* = z^{1-\sigma} D \left( \frac{\alpha}{1-\alpha} \right)^{\frac{1}{1-\sigma}} + z^{\frac{\sigma}{1-\sigma}} Dh^*$$

Therefore $h^*$ is a linear function of $h^*$. Recall that $z$ is a random number with mean one. When $z = 1$, as long as $D < 1$, the linear function can be represented by a straight line that crosses the 45-degree line in a two-dimensional space, so $h^*$ is convergent. When $z$ is random, the function can be represented by perturbations around the straight line, and $h^*$ is still convergent. The convergent $h$ ensures the existence of a stationary equilibrium in which we study inter-generational persistence, wage inequality and policy effects.

### 2.3 The public policy

We introduce into our model the public policy that levies proportional tax and uses the tax revenue to provide public schooling. With the policy, the wage gradients of parental investment are reduced, but remain positive until the non-negativity constraint for goods investment becomes binding.
2.3.1 Description of policy

The public policy considered here can be fully characterized by the sequence \( \{ \tau, P_t \}_{t=1}^{\infty} \) where \( \tau \) is the tax rate and \( P_t \) is total public investment in human capital in period \( t \). Notice that we assume tax rate \( \tau \) to be time invariant. Given any \( \tau, P_t \) changes over time until the economy reaches the stationary equilibrium defined below. For simplicity, in the individual’s optimization problem below, we drop time subscripts.

2.3.2 Policy effects

In the regime with public schooling, an individual maximizes the same utility function, but subject to different constraints. Specifically, the budget constraint becomes
\[
c + g = (1 - \tau)(1 - e - n)w
\]
where \( w \) is before-tax wage rate which again equals human capital stock \( h \). With public schooling, \( h \) evolves according to
\[
h' = zA[\alpha e^{\sigma} + (1 - \alpha)(g + P)^{\sigma}]^{\frac{1}{\sigma}} \tag{10}
\]
where \( g + P \) is the total goods investment. Equation (10) implicitly assumes that public investment and private investment are perfectly substitutable.\(^8\)

When the government makes excessive investment in education, the individual prefers to make negative private investment. We preclude that and assume \( g \geq 0 \). When the nonnegativity constraint of goods investment is not binding, we have the following.

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\(^8\)See Restuccia and Urrutia (2004) for a discussion about this assumption.
Proposition I When time and goods investments are gross complements ($\sigma < 0$), the model with public schooling has the following properties.

1. $\frac{\partial y}{\partial h} > 0$, and $\frac{\partial e}{\partial h} > 0$ as long as the two types of investment have low substitutability (a sufficient condition is $\sigma < -\frac{\beta}{1+\gamma}$).

2. Private goods investment decreases with $P$, while time investment and human capital accumulation increase with $P$.

3. Both $\frac{\partial e}{\partial h}$ and $\frac{\partial y}{\partial h}$ decrease with $P$.

Proof of the proposition is given in the Appendix. The first property states that wage gradients are still positive in the presence of the policy. Next, the proposition states that public investment in education crowds out private goods investment, but crowds in time investment. Overall it induces more human capital accumulation. Public investment leads to more equalized time and goods investment, thus the policy leads to reduced wage inequality and persistence.

When the nonnegativity constraint of goods investment is binding, private goods investment is zero. In this case, it is easy to show that wage gradient of time investment is zero. In addition, time investment no longer increases with public investment $P$ rather, it decreases with $P$.\footnote{The proof is also given in the Appendix.} This is because, as the government makes excessive public investment, the marginal value of the next generation’s human capital is too low and marginal utility of consumption is high. Therefore the optimal strategy for individuals is to reduce time investment, and increase consumption and leisure.
Next, we consider the case of $\sigma > 0$. Proposition II summarizes the results.

**Proposition II** When time and goods investments are gross substitutes ($\sigma > 0$), the model with public schooling has the following properties.

1. $\frac{\partial g}{\partial h} > 0$, but $\frac{\partial e}{\partial h} < 0$.

2. Private goods investment decreases with $P$, while time investment and human capital accumulation increase with $P$.

3. $\frac{\partial e}{\partial h}$ decreases with $P$, but $\frac{\partial g}{\partial h}$ increases with $P$.

Proof of the proposition II is also given in the Appendix. Notice that the second property in Proposition II is exactly the same as in Proposition I. When $\sigma > 0$, however, the wage gradient of time investment is negative, which is inconsistent with the data. Furthermore, public schooling now leads to more unequal private investment, implying that public schooling is less effective in reducing wage inequality and persistence compared with the case of $\sigma < 0$.

In the model with public investment in education, the explicit solution for $\tilde{h}$ is not obtained, but $\tilde{h}$ satisfies:

$$
\tilde{h} = D^{\frac{1-\sigma}{\sigma}} \left[ \left( \frac{1 - \alpha}{\alpha} \right)^{\frac{1}{\sigma - 1}} \left[ (1 - \tau)\tilde{h} \right]^\frac{\sigma}{\sigma - 1} + 1 \right]^{\frac{1-\sigma}{\sigma}} \left[ (1 - \tau)\tilde{h} + P \right] \quad (11)
$$

where $D$ is defined the same way as in equation (9).
The figure illustrates the reinforcing mechanism among goods investment, time investment and human capital accumulation.

### 2.3.3 The Reinforcing Mechanism

In the presence of parental time investment, income taxation and public schooling crowd in time investment. Increased time input, along with public goods input, leads to more human capital accumulation. More human capital results in more tax revenue and public investment. This in turn induces increased time investment due to the complementarity. This reinforcing mechanism, as illustrated in Figure 1, implies significantly stronger policy in our model than in a model without endogenous time investment. We return to this point with quantitative analysis in section 4.2.
2.4 Distribution and Stationary Equilibrium

We are interested in the inter-generational earnings persistence and wage inequality implied in our model. In this subsection we discuss the distribution of individuals and provide definitions of persistence, inequality and equilibrium. These definitions prepare us for the quantitative study that follows.

2.4.1 Distribution of Individuals

Adult individuals in the economy are heterogeneous in two aspects: own human capital stock \((h)\) and the child’s learning ability \((z)\). Individuals are distributed across the state space \(H \times Z\) where \(H \subset \mathbb{R}^+\) is the space for adult’s human capital stock, and \(Z \subset \mathbb{R}^+\) is the space for child’s learning ability.

Let \(\lambda_t(h, z)\) be the distribution of individuals across state space \(H \times Z\) in period \(t\). We need to know how the distribution evolves over time. The transition of \(z\) is given by equation (2), and the transition of \(h\) is determined by the policy function \(h'(h, z)\). Therefore once the policy function is known, we can track the transition of individuals from one generation to another.

To describe the evolution of \(\lambda_t(h, z)\) over time formally, let \(H \times Z\) be a typical subset of \(H \times Z\). Define \(Q((h, z), H \times Z)\) as the probability that the next generation of the adults with current state \((h, z)\) transmits to the set \(H \times Z\). Formally

\[
Q((h, z), H \times Z) = \int_Z I\{h'(h, z) \in H\}dz
\]  

(12)

where \(I\) is the indicator function. Then the transition of distribution from
period \( t \) to period \( t+1 \) is

\[
\lambda_{t+1}(\mathcal{H} \times \mathcal{Z}) = \int_{\mathcal{H} \times \mathcal{Z}} Q((h, z), \mathcal{H} \times \mathcal{Z}) d\lambda_t(h, z)
\]  

(13)

### 2.4.2 Stationary Equilibrium

We have shown the existence of a steady state when the model is stripped of random shocks. Adding shocks back to the model, the economy converges to a stationary distribution \( \lambda^*(h, z) \) in which the economy operates around \( h = \bar{h} \) and \( z = 1 \). For any given tax rate \( \tau \), a stationary equilibrium is the set of policy functions \( h'(h, z), c(h, z), e(h, z), n(h, z), g(h, z) \), the public investment in education \( P \), and a distribution \( \lambda^*(h, z) \) such that

1. \( h'(h, z), c(h, z), e(h, z), n(h, z), g(h, z) \) solve the adult’s optimization problem.

2. Government budget is balanced. i.e., \( \int_{\mathcal{H} \times \mathcal{Z}} \tau(1 - e - n)hd\lambda^*(h, z) = P \).

3. For any subset \( \mathcal{H} \times \mathcal{Z} \), the distribution \( \lambda^*(h, z) \) is time invariant. Mathematically

\[
\lambda^*(\mathcal{H} \times \mathcal{Z}) = \int_{\mathcal{H} \times \mathcal{Z}} Q((h, z), \mathcal{H} \times \mathcal{Z}) d\lambda^*(h, z)
\]

In the qualitative analysis below, we start from the stationary equilibrium without public policy. After introducing the public policy into the economy, we solve for the evolution of distribution \( \lambda_t \) computationally until the economy reaches a new stationary equilibrium. Using the distributions during the transition period, we are able to compute the transitions of wage, consumption, time allocation, as well as earnings persistence and wage inequality which we formally define now.
2.4.3 Definition of Earnings Persistence and Wage Inequality

Given any distribution $\lambda(h, z)$, we define wage inequality as the coefficient of variation of wage rate. We define inter-generational earnings persistence in a standard way. Let the earning of adult $i$ be $y_i = (1-e-n)w$, and the earning of the next generation be $y_i'$, then earnings persistence is the coefficient $b_1$ in the following regression

$$\ln y_i' = b_0 + b_1 \ln y_i$$

3 Quantitative Results I: Effects of Public Policy

We have shown analytically that parental time investment increases with wage rate within our framework. In this section we quantitatively study how the public policy changes inter-generational earnings persistence and wage inequality. We also examine the transition paths of resource allocation and wage structure after the implementation of the public policy. To do so, we estimate the model parameters via the Simulated Method of Moments.

The quantitative analysis involves solving the general equilibrium model numerically. We provide details of the model solution strategy in the Appendix.

3.1 Estimation

The model has 7 parameters: $A, \beta, \alpha, \sigma, \gamma, \rho, \nu$. $A$ is a scaling parameter that shifts the levels of human capital stock and other variables. We normalize
Table 1: Model parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>degree of altruism toward children</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>relative importance of time investment</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>substitutability between time and goods investment</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>relative importance of leisure in preference</td>
</tr>
<tr>
<td>( \rho )</td>
<td>inter-generational persistence of learning ability</td>
</tr>
<tr>
<td>( \nu )</td>
<td>standard deviation of shocks to learning ability</td>
</tr>
</tbody>
</table>

The steady state human capital stock to one, which implies

\[
A = \frac{1 + \gamma + \beta}{\beta} \left[ 1 + \left( \frac{\alpha}{1-\alpha} \right)^{1/(1-\sigma)} \right]^{\sigma-1} \frac{\sigma-1}{(1 - \alpha)^{\frac{1}{\sigma}}}
\]

Therefore, once we estimate the remainder of the 6 parameters, \( A \) is pinned down.\(^{10}\) Table 1 recapitulates the definitions of the 6 parameters to be estimated.

3.1.1 Data moments

We estimate the parameters using the Simulated Method of Moments as formalized in Ingram and Lee (1991). Basically we choose moments that characterize a set of key data features, then search for the parameter values that minimize the distance between data moments and model moments from

\(^{10}\)It is easy to show that when \( A \) is defined like this, human capital stock is convergent. Based on the estimation results, \( A \) is around 11.5 which is not reported in the table of estimation results.
Table 2: Data moments

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Standard error</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average consumption</td>
<td>0.469*</td>
<td>0.003</td>
<td>CEX 2003</td>
</tr>
<tr>
<td>Average goods investment</td>
<td>0.025*</td>
<td>0.016</td>
<td>CEX 2003</td>
</tr>
<tr>
<td>Average time investment</td>
<td>0.086**</td>
<td>0.002</td>
<td>ATUS 2003</td>
</tr>
<tr>
<td>Wage elasticity of goods invest</td>
<td>1.067</td>
<td>0.371</td>
<td>CEX 2003</td>
</tr>
<tr>
<td>Wage elasticity of time invest</td>
<td>0.206</td>
<td>0.077</td>
<td>ATUS 2003</td>
</tr>
<tr>
<td>Inter-generational earnings correlation</td>
<td>0.449</td>
<td>0.095</td>
<td>Solon (1992)</td>
</tr>
<tr>
<td>Coefficient of variation of wage</td>
<td>0.663</td>
<td>0.021</td>
<td>ATUS 2003</td>
</tr>
</tbody>
</table>

* relative to wage/(1-saving rate)

** a fraction of total time available

simulated data. The distance is defined by $l^2$-norm, weighted by the inverse of the variance of data moments.\(^{11}\)

Seven moments are used. Six of them are calculated from the 2003 waves of the American Time Use Survey and the Consumer Expenditure Survey. The remaining moment, inter-generational earnings persistence, is taken from Solon (1992). Table 2 presents the data moments and their sources.

The first three moments are the mean levels of consumption, goods investment and time investment. These moments pin down the allocation of parents’ resources. In addition they are informative about the relative im-

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\(^{11}\)Ideally we should have used the optimal weighting matrix which is the inverse of the variance-covariance matrix of the data moments. However the off-diagonal components are not available from the data, due to the lack of panel data and the fact that moments are taken from different sources.
portance of time investment in human capital production and the degree of altruism towards the next generation.

The fourth and fifth moments are wage elasticities of parental investment, obtained by regressing the logarithm of goods and time investment on the logarithm of wage rate, instrumented by educational attainment.\(^\text{12}\)

The sixth moment is the inter-generational correlation in earnings. According to Solon (1992), the correlation is around 0.45. This moment is particularly informative on \(\rho\), the persistence of learning ability over generations.

The last moment is the coefficient of variation in wage rate. This moment measures the overall wage inequality in the economy. It is very responsive to the size of random shocks to learning ability. From ATUS 2003, the number is around 0.66. The literature on economic inequality uses the variance of logarithm of wage rate as a typical measure of inequality. Since wage rate in the model is much smaller in scale than in the data, we use the coefficient of variation so that the model moment is comparable to that in the data.

### 3.1.2 Estimation results

Table 3 reports the estimation results. Not surprisingly, \(\sigma\) is negative, which means the elasticity of substitution between goods and time investment is \(\eta = \frac{1}{1-\sigma} = \frac{1}{1-(-0.909)} \approx 0.524\). The estimate of inter-generational altruism, \(\beta\), is 0.246. If we consider one generation to be 20 years, then this is equivalent to

\(^{12}\)We use education attainment to instrument wage rate because we want to capture the human capital element in wage rate. In particular, this filters out shocks to wage rate which are not directly related to human capital.
Table 3: Estimation results

<table>
<thead>
<tr>
<th>Moment</th>
<th>Goods invest.</th>
<th>Time invest.</th>
<th>Consump.</th>
<th>( \frac{\partial g}{\partial h} \times \frac{h}{g} )</th>
<th>( \frac{\partial e}{\partial n} \times \frac{h}{e} )</th>
<th>Earnings persist.</th>
<th>Wage ineq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>0.026</td>
<td>0.086</td>
<td>0.469</td>
<td>0.744</td>
<td>0.188</td>
<td>0.449</td>
<td>0.663</td>
</tr>
<tr>
<td>Data</td>
<td>0.025</td>
<td>0.086</td>
<td>0.469</td>
<td>1.067</td>
<td>0.207</td>
<td>0.449</td>
<td>0.663</td>
</tr>
</tbody>
</table>

The table reports estimated parameter values and standard errors. Moments from simulated data are reported together with those from real data.

to an annual discount factor of 0.932. Regarding the two parameters for the unobservable stochastic learning ability process, we find that \( \rho = 0.315 \) and \( \nu = 0.522 \). Overall our model delivers the data facts very well, except that the wage elasticities of time and goods investment are a little lower in the model than in the data.

### 3.2 Inter-generational persistence and wage inequality

Earnings persistence and wage inequality have two sources: exogenous learning ability and parental investment, which correspond to “nature” and “nurture” effects in the literature respectively. In order to show the roles played by the latter, we decompose earnings persistence and wage inequality quantitatively. First we calculate them in the case in which every individual makes a steady state level time and goods investment. Next, we keep time investment fixed at the steady state level, but allow goods investment to be
endogenously chosen. Finally, we allow both types of investment to be endogenous. Different results from these cases, reported in Table 4, reveal the roles played by time and goods investment.

Table 4 shows that parental investment accounts for a relatively small portion of the wage inequality and earnings persistence. Fixing $e$ and $g$ at the steady state levels, wage inequality is 0.590, while it is 0.663 when both $e$ and $g$ are endogenous. Loosely speaking, parental investment, the so-called “nurture effect”, accounts for 11% of overall wage inequality. It accounts for 36.4% of earnings persistence. Within “nurture effect”, both time investment and goods investment are important contributors.\(^{13}\)

<table>
<thead>
<tr>
<th>Table 4: Wage inequality and earnings persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>$e = \tilde{e}$, $g = \tilde{g}$</td>
</tr>
<tr>
<td>$e = \tilde{e}$, $g = g(h,z)$</td>
</tr>
<tr>
<td>$e = e(h,z)$, $g = g(h,z)$</td>
</tr>
</tbody>
</table>

The table reports the decomposition of wage inequality and earnings persistence. The “with policy” case corresponds to an economy where a 5% income tax is imposed with the revenue invested in public schooling. $\tilde{e}$ and $\tilde{g}$ are time and goods investment in the steady state.

Columns (2) and (5) of the table report wage inequality and earnings

\(^{13}\)As pointed out by Cunha and Heckman (2007), “nurture effect” and “nature effect” on inter-generational persistence are nonlinear, and they interact with each other. Our production function of human capital is clearly non-linear.
persistence in the model with public schooling, assuming 5% tax rate. Here \( \tilde{c} \) is still the steady state time investment in the absence of the policy, and \( \tilde{g} \) is defined as public goods investment plus steady state private goods investment. As we allow goods investment to be endogenous and keep time investment fixed, both inequality and earnings persistence increase, but to a much less extent compared with the no policy case. Columns (3) and (6) show the policy effects. Allowing both time and goods investment to be endogenous, the public policy reduces inequality by 6.6% and persistence by 14.9%.

We further decompose the above policy effects by conducting two experiments. Results are reported in Table 5. First, we impose 5% tax, but assume no public investment in education. This leads to slightly higher inequality and persistence, because income tax strengthens the inequality in time investment. In addition, the “tax only” policy leads to slightly lower time and goods investment. Next, we consider the “public investment only” policy that keeps public investment from the baseline model unchanged but assumes away income taxation. The results reveal that much of the policy effects in the previous discussion are due to public investment, rather than to income taxation.

### 3.3 Transition paths

Starting from the original stationary equilibrium, the economy converges to a new equilibrium after the policy is implemented. Given \( \tau = 0.05 \), we plot the transition paths in Figure 2.

The top panels of Figure 2 plot the percentage changes in time alloca-
Figure 2: Transition paths ($\tau = 5\%$)

The figure shows percentage changes due to the implementation of the public policy. Tax rate is 5%. The top panels plot paths for time allocation, the middle panels for goods allocation and the lower panels for wage structure.
Table 5: Further decomposition of policy effects

<table>
<thead>
<tr>
<th></th>
<th>Time investment</th>
<th>Goods investment</th>
<th>Wage rate</th>
<th>Wage inequality</th>
<th>Earnings persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td></td>
</tr>
<tr>
<td>Tax + public inv.</td>
<td>6.1%</td>
<td>-84.6%</td>
<td>8.4%</td>
<td>-6.6%</td>
<td>-14.9%</td>
</tr>
<tr>
<td>Tax only</td>
<td>-0.8%</td>
<td>-4.2%</td>
<td>-1.6%</td>
<td>0.4%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Public inv. only</td>
<td>6.4%</td>
<td>-82.3%</td>
<td>9.1%</td>
<td>-6.3%</td>
<td>-14.0%</td>
</tr>
</tbody>
</table>

The table reports percentage changes due to different policies. The “Tax only” case imposes 5% income tax rate, but tax revenue is not invested in public schooling. The “Public investment only” case keeps public investment from the baseline model unchanged, but assumes away income taxation.

The policy shifts parents’ time from work to leisure and human capital investment. Consistent with the predictions in Proposition I, private goods investment decreases during the transition, but total goods investment and wage rate (human capital) increase, which reflects the reinforcing mechanism among time investment, goods investment and human capital accumulation.

Regarding wage structure, the policy increases average wage rate and decreases wage inequality and intergenerational persistence. Consumption is also decreased initially, then rises above the original level.
4 Quantitative Results II – the role of time investment

The reinforcing mechanism illustrated in Figure 1 exists in our model, but does not exist in the traditional-type model in which parental time investment is exogenous. Therefore in our model, the policy is much more effective in promoting human capital accumulation, and in reducing inter-generational earnings persistence and wage inequality. We quantitatively compare the policy effects between our model and a version of the traditional model in which time investment is exogenous.

4.1 The traditional model

We consider a traditional type model that is identical to our baseline model, except that the human capital production function has the following form

\[ h' = zA[B + (1 - \alpha)(g + P)^\sigma]\frac{1}{\sigma} \]

Here, we have replaced $\alpha e^\sigma$ in our baseline model with a constant $B$.

4.2 Comparing policy effects

To compare the policy effects, for each model, we compute the stationary equilibria in two regimes: with and without public policy, then compute the percentage changes of variables due to public policy. Differences in results between the two models reflect the roles played by time investment. Columns (1)-(4) of Table 6 present the results.
Table 6: Effects of public policy (percentage change)

<table>
<thead>
<tr>
<th></th>
<th>Baseline parameters</th>
<th>Alternative parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tau = 5% )</td>
<td>( \tau = 2% )</td>
</tr>
<tr>
<td>our RU (1)</td>
<td>(2) (3) (4) (5) (6)</td>
<td>(7) (8)</td>
</tr>
<tr>
<td>( \Delta ) work hours</td>
<td>-3.10 -0.22 -1.44 -0.37</td>
<td>-3.22 -0.18 -1.58 -0.43</td>
</tr>
<tr>
<td>( \Delta ) leisure</td>
<td>2.46 0.26 1.41 0.45</td>
<td>2.21 0.18 1.32 0.43</td>
</tr>
<tr>
<td>( \Delta ) time invest.</td>
<td>6.10 0.00 1.56 0.00</td>
<td>6.31 0.00 1.64 0.00</td>
</tr>
<tr>
<td>( \Delta ) parental goods invest.</td>
<td>-84.6 -86.5 -38.2 -39.3</td>
<td>-83.3 -85.4 -38.5 -39.4</td>
</tr>
<tr>
<td>( \Delta ) total goods invest.</td>
<td>18.1 14.8 0.7 -0.3</td>
<td>21.0 17.6 0.7 -0.1</td>
</tr>
<tr>
<td>( \Delta ) wage rate</td>
<td>8.43 3.88 1.16 0.01</td>
<td>9.29 4.66 1.15 0.02</td>
</tr>
<tr>
<td>( \Delta ) wage inequality</td>
<td>-6.60 -3.71 -0.94 -0.13</td>
<td>-7.66 -4.68 -1.12 -0.22</td>
</tr>
<tr>
<td>( \Delta ) persistence</td>
<td>-14.9 -11.0 -0.6 1.3</td>
<td>-14.1 -10.7 -0.6 1.1</td>
</tr>
</tbody>
</table>

The table reports policy effects in our model as opposed to the traditional model (RU) in which parental time investment is exogenous. Columns (1)-(4) show the comparison based on baseline parameter values (Table 3). Columns (5)-(8) are based on parameter values that minimize the distance between data moments and moments from the traditional model (Table 7).
Both models predict a decrease in work hours, which is due to the reduced after-tax wage rate, but the decrease is more significant in our model where individuals have the option to make more time investment. Both models predict increased leisure, and the increase is again much more significant in our model. Similar changes of time use patterns are found under different tax rates.

Private goods investment demonstrates huge drops in both models. The drop is larger in the traditional model. This is because the marginal return on goods investment is relatively lower due to the inability of individuals to increase time investment. Total goods investment is increased less in the traditional model. The policy increases average consumption in both regimes, but to a greater extent in our model.

Regarding wage structure, the public policy is significantly more effective in increasing wage rate, and in reducing wage inequality and earnings persistence in our model. With $\tau = 5\%$, average wage rate is increased by 8.43\%, as opposed to 3.88\% in the traditional model. Wage inequality and earnings persistence are decreased by 6.6\% and 14.9\% respectively in our model, but only 3.7\% and 11.0\% in the traditional model.

Columns (5)-(8) report the results from an alternative set of parameters given in Table 7. These are the parameters that minimize the distance between data moments and moments from the traditional model. Clearly, we can draw the same conclusions as under the baseline parameterizations.
Table 7: Calibration based on the traditional model

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.230</td>
<td>0.854</td>
<td>-0.697</td>
<td>0.883</td>
<td>0.315</td>
<td>0.522</td>
</tr>
</tbody>
</table>

The table reports parameter values that minimize the distance between data moments and simulated moments from the traditional model.

5 Further Discussions

5.1 Higher efficiency of time investment by more educated parents

Ramey and Ramey (2010) point out that more educated parents should be more efficient in time investment, hence wage/education gradient of parental time should be positive. We extend our baseline model to admit this potentially important channel.\(^\textit{14}\)

5.1.1 Human capital production function and optimal time investment

Let the human capital production function be

$$h_i' = z_i A \left[ \alpha(h_i^\delta e_i)^\sigma + (1 - \alpha) g_i^\sigma \right]^{\frac{1}{\sigma}} \quad (14)$$

The term $h_i^\delta$ in equation (14) allows for the channel in Ramey and Ramey (2010). Since goods investment is absent in their framework, our human capital production function includes

\(^{14}\)We are grateful to a referee for suggestions that led to the development of this subsection.
capital production function is essentially the same as in Ramey and Ramey (2010) if $\alpha=1$.

When $\delta = 0$, we are back in the baseline model. With non-zero $\delta$, it is straightforward to show the following.

\[
\frac{\partial g_t}{\partial h_t} = \frac{(1-\sigma\delta) \left( \frac{1-\alpha}{\alpha} \right)^{1/\sigma-1} h_t^{(1-\delta)/\sigma-1}}{\left( \frac{1+\gamma}{\beta} + 1 \right) \left( \frac{1-\alpha}{\alpha} \right)^{1/\sigma-1} h_t^{(1-\delta)/\sigma-1} + 1} \quad (15)
\]

\[
\frac{\partial e_t}{\partial h_t} = \frac{-\left( \frac{1-\sigma\delta}{1-\sigma} \right) \left( \frac{1-\alpha}{\alpha} \right)^{1/\sigma-1} h_t^{1-\delta/\sigma}}{\left( \frac{1+\gamma}{\beta} + 1 \right) \left( \frac{1-\alpha}{\alpha} \right)^{1/\sigma-1} h_t^{1-\delta/\sigma} + 1} \quad (16)
\]

Given these equations, there exist two cases in which wage/education gradients of goods and time investment are positive.

Case (i), $\delta < 1$ and $\sigma < 0$. In this case, for parents with more human capital, the efficiency of time investment does not outweigh their advantage on the labor market. Complementarity between two types of investment is still needed for the wage/education gradients to be positive.

Case (ii), $\delta > 1$ and $\sigma > 0$. In this case, parents with higher human capital are much more efficient in time investment. $\frac{\partial e_t}{\partial h_t}$ will be positive if time and goods investment are substitutes. In addition, from equation (15), if $\delta$ is large enough, goods investment will decrease with human capital. This is because for parents with more human capital, the advantage in parenting outweighs the advantage on the labor market, and they substitute goods investment for time investment.

Since more educated women are more likely to be in the labor force, it is likely that education has the nonneutral effect of raising the productivity of
labor market time more than that of time spent in parenting. Therefore we focus our discussion on case (i). When the same public policy is introduced into our model, we find that all the results in proposition I and II hold. The exception is that the sufficient condition for positive wage gradient of time investment becomes $\sigma < -\frac{\beta(1-\delta\sigma)}{(1+\gamma)(1-\delta)}$.  

5.1.2 Re-estimation of the model

To conduct a quantitative study based on the extended model, we re-estimate the model with $\delta$ as an additional parameter. The estimation yields $\delta = 0.045$. However, the standard errors of $\sigma$, $\rho$ and $\delta$ are large. The model fails to precisely identify time-goods substitutability ($\sigma$), persistence in learning ability ($\rho$) and higher productivity of more able parents ($\delta$), because the product of $\delta$ and $\sigma$ enters the exponential term in the production of human capital.  

We proceed by fixing $\delta$ at different values and estimating the rest of the parameters. Generally we find the distance between model and data moments increases slightly with $\delta$, supporting a low $\delta$. Table 8 reports the results with $\delta$ fixed at 0.045 and 0.3. When $\delta$ is higher, the estimated persistence in learning ability $\rho$ is much lower. Substitutability between

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15 Proof of the proposition with $\delta$ in human capital production function is available upon request.

16 We thank Dean Corbae for pointing this out.

17 The existing literature provides some indirect evidence in support of a positive $\delta$. For example, more educated parents are less likely to use non-parental child care (Leibowitz (1974a)). Subsidized non-parental care tends to have positive impact on outcomes for disadvantaged children whose parents on average have lower income and education attainment, but negative impact for children from more educated families. See Blau and Currie (2006) for a comprehensive survey.
goods and time is also lower, which enables the model to match the positive wage gradient of goods investment in the data.

Table 8: Estimation results with fixed $\delta$

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$\gamma$</th>
<th>$\rho$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.045$</td>
<td>0.246</td>
<td>-0.915</td>
<td>0.894</td>
<td>0.884</td>
<td>0.321</td>
<td>0.519</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.05)</td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(0.08)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$0.3$</td>
<td>0.246</td>
<td>-2.159</td>
<td>0.971</td>
<td>0.884</td>
<td>0.066</td>
<td>0.521</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.21)</td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(0.08)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

The table reports estimation results for the extended model for two cases, $\delta = 0.045$ and $\delta = 0.3$. Implied model moments are also reported.

5.1.3 Policy effects in the extended model

Table 9 reports the policy effects given different values of $\delta$. A higher $\delta$ means more of the inter-generational effect is driven by time investment. Compared with the baseline case, the public policy induces even more parental investment, hence is less effective in reducing the inter-generational effect. The table also shows that if time investment is fixed ($e=\bar{e}$), then policy effects increase with $\delta$.

Figure 3 presents the transition paths for the case of $\delta = 0.2$, along with the baseline case in which $\delta = 0$. As shown in Table 9, higher $\delta$ implies that public policy is less effective in reducing inequality and inter-generational
Table 9: Wage Inequality and earnings persistence from different $\delta$

<table>
<thead>
<tr>
<th></th>
<th>Wage inequality</th>
<th></th>
<th>Earnings persistence</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>without policy</td>
<td>with policy</td>
<td>policy effect</td>
<td>without policy</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>$\delta = 0.045$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e=e^*, g=\tilde{g}$</td>
<td>0.589</td>
<td>0.589</td>
<td>0.283</td>
<td>0.283</td>
</tr>
<tr>
<td>$e=e^*, g=g(h,z)$</td>
<td>0.631</td>
<td>0.605</td>
<td>-4.1%</td>
<td>0.398</td>
</tr>
<tr>
<td>$e=e(h,z), g=g(h,z)$</td>
<td>0.663</td>
<td>0.623</td>
<td>-6.1%</td>
<td>0.449</td>
</tr>
<tr>
<td>$\delta = 0.3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e=e^*, g=\tilde{g}$</td>
<td>0.585</td>
<td>0.585</td>
<td>0.246</td>
<td>0.246</td>
</tr>
<tr>
<td>$e=e^*, g=g(h,z)$</td>
<td>0.628</td>
<td>0.600</td>
<td>-4.4%</td>
<td>0.388</td>
</tr>
<tr>
<td>$e=e(h,z), g=g(h,z)$</td>
<td>0.663</td>
<td>0.665</td>
<td>0.4%</td>
<td>0.449</td>
</tr>
</tbody>
</table>

The table reports the decomposition of persistence and inequality for two cases, $\delta = 0.045$ and $\delta = 0.3$. Overall policy effects are weaker when $\delta$ is larger.
Figure 3: Comparing transition paths from different $\delta$

The figure shows percentage changes due to the implementation of the public policy, given different value of $\delta$. Tax rate is 5%.

persistence. However, the policy is more efficient in promoting human capital accumulation, because the reinforcing mechanism illustrated in Figure 1 is even stronger with a positive $\delta$.

5.2 The substitutability between goods and time investment

The substitutability between time and goods investment plays a key role in this paper. In reality, some activities, such as baby sitting, appear to be more easily substituted by purchased child care. Others, such as breastfeeding, are much more difficult to substitute. In our model, the parameter $\sigma$ measures
the overall substitutability for which there exists little direct evidence. The observed positive wage/education gradient of parental time can serve as an indirect evidence. In this subsection we provide a summary of relevant studies in the literature.\footnote{We are grateful to a referee for suggestions which led to the development of this subsection.}

The literature of home production typically finds a large elasticity of substitution between goods input and home production time. However, the production of children’s human capital should be different from usual home production because time with children has totally different patterns from other home production time. Due to the lack of direct evidence, some theoretical papers assume unit elasticity so that time and goods are combined in Cobb-Douglas form (Trostel (1993)).

A number of empirical observations imply that parental child care can be substituted to some extent. Leibowitz (1974a) shows that child care time is reduced when non-parental child care is present, and such an effect is most significant among less educated mothers. Government programs that provide universal day care typically induce more maternal participation in the labor market, and crowd out some child care time from mothers (Baker et al. (2008), Havnes and Mogstad (2009) and Gupta and Simonsen (2010)).

On the other hand, there exist a number of empirical observations that are consistent with a low substitutability.

First, maternal labor market participation only leads to a small-sized reduction in mother’s child care time, and leads to increased child care time of fathers (See Leibowitz (1974a), Bianchi (2000), Sandberg and Hofferth
(2001) and references therein). After entering the labor market, mothers tend to cut back leisure and home production time unrelated to child care, but try to preserve their time with children. This phenomenon is more pronounced among more educated women. Using Dutch data, Annemarie Nelen and Fouarge (2011) even find that maternal working hours are positively related with planned activities with children, such as going together to a museum or library.

Second, recent studies revealed the negative impact of maternal labor market participation on child outcome. Although earlier studies on this issue had reached different conclusions, more recent ones generally document negative effects. Ruhm (2004) shows that maternal employment has a negative impact on a child’s verbal ability, and reading and mathematics achievement.\textsuperscript{19} Bernal and Keane (2011) further document a significantly negative and sizeable effect of the increased single mother employment caused by the 1996 Welfare Reform Law along with earlier state policy changes adopted under federal waivers.

Third, the literature on the correlation between child outcomes and provision of universal child care programs by governments generally favors a low substitutability. Baker et al. (2008) use data from Quebec and find that children are worse off by a wide range of measures which are likely to be caused by less and lower quality maternal care. Based on Danish data, Gupta and Simonsen (2010) show that, compared to parental care, low-quality non-

\textsuperscript{19} Ruhm (2004) reviewed the existing literature and found that his conclusion is consistent with the most recent analysis, and suggests that the inconsistency with earlier ones is largely due to the failure to control for heterogeneity among mothers.
parental care (obtained from so-called family day care) significantly worsens child outcomes\textsuperscript{20}. Havnes and Mogstad (2009) show that the subsidized child care program crowds out informal care arrangements from friends, relatives and unlicensed care givers, leaving parental care virtually unchanged.

Fourth, there exists strong evidence that children with lower birth order (those who are born earlier) have better outcomes. These children on average receive much more parental time investment than their younger siblings (Price (2008)). On the other hand, they are likely to receive less goods investment, because parent’s income on average increases with age. The significantly better outcomes of children with lower birth order are consistent with our low substitutability assumption.

From the above discussion, it is not unreasonable to assume that the substitutability between time and goods is low on average, even though some parenting activities are clearly more substitutable. Leibowitz (2003) provides a discussion about why the substitutability appears to be low. First, it is difficult to monitor the quality of purchased child care. A hired nanny or babysitter may not spend her time investing in the child’s human capital, but enjoying her own time at the expense of the child. Secondly, the psychological and sociological literature emphasizes that parent-child bonding affects child’s development, which is further supported by neurobiologists (See the references in Leibowitz (2003). Also see Belsky (1988).) Thus, even for activities such as baby sitting, it is not completely safe to say they can

\textsuperscript{20}Gupta and Simonsen (2010) also show that enrollment in high-quality preschool does not lead to significantly different child outcomes, but longer hours in non-parental care lead to poorer child outcomes.
be substituted by purchased care.

5.3 Parental time and public spending on education – cross country comparison

In Proposition I, we make two predictions regarding parental time and public spending on education. We roughly check these predictions with cross country data.\footnote{We are grateful to a referee for suggestions which led to the development of this subsection.} We use the parental time reported in Guryan et al. (2008) and public spending on education reported by the World Bank. Figure 4 plots child care time by working women against public spending on education per capita. Consistent with proposition I, child care time increases with public investment. This relationship also holds for working men. On the other hand, we find a very weak correlation between education gradient of parenting time by working mothers and public spending – correlation coefficient is 0.18 with standard error equals 0.31. In summary we find strong support for the positive correlation between parental time and public investment, but little support for the wage gradient of parental time to decreasing with public investment.

5.4 Other forms of parental altruism

For transparency and tractability, this paper assumes a simple form of parental altruism – parents care only about the wage rate of their children. In this
The figure shows the correlation between public spending on education and child care hours across countries.
subsection we discuss other forms of altruism.\footnote{We are grateful to a referee for suggestions which led to the development of this subsection.}

5.4.1 Financial transfer

We extend our model to admit financial transfer from parents to children. Let $b_i$ be the transfer from the $i^{th}$ parents who maximize

$$\max_{c_i, n_i, e_i, g_i, b_i} \ln c_i + \gamma \ln n_i + \beta \ln h_i' + \phi \ln b_i$$

subject to

$$c_i + g_i + b_i = (1 - e_i - n_i)w_i$$

(17)

Let $g^*$ and $e^*$ be the optimal goods and time investment in the baseline model given $\{h_i, z_i\}$, the optimal goods and time allocation in this extended model is

$$g = \frac{1 + \gamma + \beta}{1 + \gamma + \beta + \phi}g^*$$

and

$$e = \frac{1 + \gamma + \beta}{1 + \gamma + \beta + \phi}e^*$$

Therefore compared with the baseline model, optimal time and goods investment are reduced by a fixed proportion. As a result, the analytical properties regarding time allocation and policy effects derived in the previous sections still hold. Since parents allocate relatively less time and goods to human capital production compared with the baseline case, the extended model must attribute a larger part of observed intergenerational persistence
to the “nature effect”, i.e., the persistence in learning ability. Hence quantitatively public policy would be less effective.

5.4.2 Infinite horizon

Another form of parental altruism is to assume that parents care about the value of their children, which include children’s own utility and the value of grandchildren. The problem essentially becomes an infinite horizon one with the following objective function for parents.

$$\max_{c,n,e,g} E \sum_{t=1}^{\infty} \beta^t u(c_t, n_t)$$

where $u(c_t, n_t) = \ln c_t + \gamma \ln n_t$ is the utility of the $t^{th}$ generation, and $E$ is the expectation operator, taken with respect to the learning ability of future generations.

In this setup, the time-goods investment ratio is the same as in the baseline model which is $\frac{c}{g} = \left( \frac{l-h}{\alpha} \right)^{\frac{1}{1-\alpha}} h^{\frac{1}{1-\alpha}}$. However, income tax would reduce goods investment to a greater extent\(^{23}\). This is because forward-looking parents care about the future generations’ consumption and leisure which are directly affected by tax. To see this point, we derive the first order conditions with respect to time and goods investment given income tax rate $\tau$.

\[
(1 - \tau)w_t \frac{\partial u(c_t, n_t)}{\partial c_t} = \frac{\partial h_{t+1}}{\partial c_t} \left[ (1 - \tau)(1 - n_{t+1} - e_{t+1}) \frac{\partial u(c_{t+1}, n_{t+1})}{\partial c_{t+1}} \right]
\]

\[
\frac{\partial u(c_t, n_t)}{\partial c_t} = \frac{\partial h_{t+1}}{\partial g_t} \left[ (1 - \tau)(1 - n_{t+1} - e_{t+1}) \frac{\partial u(c_{t+1}, n_{t+1})}{\partial c_{t+1}} \right]
\]

It is easy to show that the first order conditions are exactly the same as those in the baseline model, except for the extra term in brackets which is

\(^{23}\)We thank a referee for pointing this out.
the marginal value of human capital. In the baseline model, the marginal value is 1, because parents care about children’s human capital itself. Here, the marginal value is measured by marginal utility, and lowered by tax rate \( \tau \). Therefore, compared with the baseline case, parental goods investment is reduced to a greater extent. However, the public investment still stimulates parental time investment due to time-goods complementarity.

6 Conclusion

This paper is motivated by the data observation that parents with higher wage or education attainment spend more time teaching and taking care of their children. We show this is consistent with a model of inter-generational transmission of human capital. A key feature of the model is the low substitutability between goods and parental time investment in the production of children’s human capital. We also show that the positive wage/education gradient of parental investment is reduced by the policy that taxes income to finance public schooling.

We use the parameterized model to quantitatively study the impact of the public policy on resource allocation and wage structure. We first show that parental investment contributes to wage inequality and earnings persistence, but the contribution is decreased by the policy. Then we derive the transition paths of the economy from the old equilibrium without public policy to the new one. The policy leads to increased wage rate, but decreased wage inequality and earnings persistence. Consumption is decreased initially, but then increases to higher levels in the new equilibrium.
The public policy triggers a reinforcing mechanism among time investment, public investment and human capital accumulation. This reinforcing mechanism does not exist in the traditional model where time investment is exogenous. Therefore the policy is more effective in our model relative to a traditional one. This point is made clear as we quantitatively compare the results between our model and a version of the traditional model. Given the strong evidence that parents use time investment as a means of inter-generational human capital transmission, we recommend our model as a framework to analyze relevant public policies.

As an extension to our model we allow parents with more human capital to be more efficient in time investment. Compared with the baseline case, public policy becomes less effective in reducing inequality and intergenerational persistence, but more effective in promoting human capital accumulation.

We also discuss briefly the implications of alternative forms of parental altruism. It is worthwhile to study some other extensions. For example, the model can be enriched by embedding a life-cycle model in the OLG framework, which facilitates the study of rich dynamics among consumption, saving and human capital production. This would also allow us to examine how credit constraints impede human capital formation, and how the problem can be relieved by public policy. We leave detailed discussion of these extensions and their policy implications for future research.
7 Appendix

7.1 Data Appendix

In this paper we estimate the model parameters by matching seven key moments in the model to their data counterparts. Except for the moment of inter-generational correlation in earnings which is taken from Solon (1992), all the moments are obtained from the 2003 waves of the American Time Use Survey and the Consumer Expenditure Survey.

The American Time Use Survey is conducted by the U.S. Bureau of Labor Statistics (BLS) and the data are available from the BLS website. Using the time use taxonomy introduced in Aguiar and Hurst (2007), we consider three categories of parental time with children: basic child care, teaching, and playing. Teaching children includes activities such as reading to/with children, talking/listening to children, and helping children with their homework. Clearly these should be regarded as time investment. Much of the time categorized as basic child care and playing with children is beneficial or auxiliary to the development of human capital. For example, “basic child care” includes activities such as picking up children from school, attending children’s events; “playing with children” includes playing sports with children, doing arts and crafts with children, etc. Therefore we take the sum of the three categories as the proxy for parental time investment. We proxy leisure with the time spent on activities related to the following: lawn, garden and houseplants, animals and pets, socializing and relaxing, sports, exercise and recreation, telephone calls, household and personal mail, travel related to these events, eating, sleeping and personal care. This definition of leisure
is precisely “leisure measure two” in Aguiar and Hurst (2007).

Consumer Expenditure Survey data are publicly available from the NBER collection. Educational expenditure has three categories in the data. The first one is tuition for college (higher education). The second one is the tuition for Nursery, Elementary, and Secondary Education. This category includes tuition for elementary and high school, payment for private school bus, and other expenses for day care centers and nursery schools. The third category, other educational service, includes tuition for other schools, rental of books and equipment and other school-related expenses, and contributions to educational organizations. We take the sum of the three as the proxy for goods investment in children’s human capital. The survey provides no information on whether these expenditures are for children or not, therefore we delete the observations of families whose head of household is younger than 25 or not married or have no children. Consumption is measured by the sum of expenditures on nondurable non-educational goods and services.

We need to calculate educational expenditure normalized by wage rate. We also need to regress educational expenditure on wage rate to compute the response of goods investment with respect to parents’ wage rate. Since the data set provides only family level expenditures, the corresponding wage rate should also be at the family level. We take wage rate to be the sum of both spouses’ wage divided by the sum of work hours. i.e., \( w = \frac{w_{\text{husband}} + w_{\text{wife}}}{\text{hours}_{\text{husband}} + \text{hours}_{\text{wife}}} \).

Note that for both spouses, CEX provides information about annual wage, weeks worked in the year and hours worked in each week.

In both ATUS and CEX, we delete observations that have one of the following characteristics: (1) no complete information on age, education at-
tainment, number of children, age of children, and wage rate, labor income, marital status, child care time, leisure, work hours; (2) Respondents (in ATUS) or heads of household (in CEX) having zero wage; (3) not married; (4) having no children. In ATUS we also drop respondents who are younger than 21 or older than 65 in order to focus on parents in their working age who allocate time among work, leisure and child care.

7.2 Model solution strategy

In the regime with public investment in education, for a given $\tau$, we take the following steps to solve for the stationary equilibrium and the transition paths.

1. Draw random shocks to learning ability for $H$ individuals and $T$ periods, each period corresponding to one generation. Shocks are denoted by $\{\{\epsilon_{i,t}\}_{i=1}^{H}\}_{t=1}^{T}$.

2. Use $\ln z_{i,t} = \rho \ln z_{i,t-1} + \epsilon_{i,t}$ to obtain the paths of stochastic learning abilities for all the individuals. For the $i^{th}$ individual, the path is $\{z_{i,t}\}_{t=1}^{T}$.

3. Guess a sequence of public investment in education $\{P_t\}_{t=1}^{T}$

4. For each individual, given $\tau$, $\{P_t\}_{t=1}^{T}$, $\{z_{i,t}\}_{t=1}^{T}$, derive the optimal paths of human capital $\{h_{i,t}\}_{t=1}^{T}$, time investment $\{e_{i,t}\}_{t=1}^{T}$ and leisure $\{n_{i,t}\}_{t=1}^{T}$. Whenever analytical solutions are not available (i.e., whenever the nonnegativity constraint for goods investment binds), we resort to numerical solutions. Then we calculate tax obligation for the $i^{th}$ individual as $\Phi_{i,t} = (1 - e_{i,t} - n_{i,t}) \tau h_{i,t}$.
5. Integrate over all the individual to calculate the path of aggregate tax revenue \( \{ \Phi_t \}_{t=1}^T \).

6. Compare tax revenue with public investment in education. Specifically, if the L-infinity norm \( |x|_\infty = \max_t \{ \Phi_t \}_{t=1}^T - \{ P_t \}_{t=1}^T \) is greater than a pre-set tolerance level, find a new guess of \( \{ P_t \}_{t=1}^T \) and repeat steps (3)-(5). To find the new guess, whenever tax revenue is less than the expenditure, \( P_t \) is reduced, otherwise \( P_t \) is increased.

7. When \( |x|_\infty \) is less than the tolerance level, the corresponding \( \{ P_t \}_{t=1}^T \) satisfies the balanced budget condition. Then we solve for the corresponding \( \{ h_{i,t}, C_{i,t}, \epsilon_{i,t}, n_{i,t}, I_{i,t} \}_{t=0}^\infty \) for each of the individual.

8. Check the distribution of individuals in period \( T-1 \) and \( T \). If for any subset \( \mathcal{H} \times \mathcal{Z} \)

\[
\lambda_T(\mathcal{H} \times \mathcal{Z}) = \int_{\mathcal{H} \times \mathcal{Z}} Q((h, z), \mathcal{H} \times \mathcal{Z}) d\lambda_{T-1}(h, z)
\]

then the economy has converged to a new stationary equilibrium. Otherwise increase \( T \) and repeat steps (1)-(7).

### 7.3 Proof of the Propositions

**Proof of Proposition I** First we derive the following closed-form solutions for \( g, e, h' \). Let \( \hat{h} = (1 - \tau)h \) be the after-tax wage rate, define the constants \( M = \left( \frac{\alpha}{1-\alpha} \right)^{\frac{1}{\gamma}} \) and \( D = \left( \frac{\beta A}{1+\gamma+\beta} \right)^{\frac{\sigma}{1-\sigma}} (1-\alpha)^{\frac{1}{1-\sigma}} \), then

\[
g = \left( \frac{\beta}{1+\gamma+\beta} \right) \left( \frac{\hat{h} + P}{M h^{\frac{\sigma}{1-\sigma}} + 1} \right) - P \quad (18)
\]
\[ e = \left( \frac{\beta M}{1 + \gamma + \beta} \right) \left( \frac{\hat{h} + P}{M\hat{h} + \hat{h}^{\frac{1}{\sigma}} - \sigma} \right) \]

and

\[ h' = zD \left( M\hat{h}^{\frac{\sigma}{\sigma-1}} + 1 \right)^{\frac{1}{\sigma}} \left( \hat{h} + P \right) \]

When the non-negativity constraint of goods investment is not binding, \( g > 0 \). From equation (18), this implies

\[ P < \frac{\beta\hat{h}}{1 + \gamma + (1 + \gamma + \beta)M\hat{h}^{\frac{\sigma}{\sigma-1}}} \] (19)

From these solutions, it is straightforward to show that private investment \( g \) decreases with \( P \), time investment \( e \) increases with \( P \), and human capital production also increases with \( P \).

The first-order derivatives are

\[ \frac{\partial g}{\partial h} = \left( \frac{\beta M}{1 + \gamma + \beta} \right) \left( \frac{1}{1 + M\hat{h}^{\frac{\sigma}{\sigma-1}}} \right)^2 \left( 1 + \frac{M}{1 - \sigma} \hat{h}^{\frac{\sigma}{\sigma-1}} - \frac{\sigma MP}{\sigma - 1} \hat{h}^{\frac{1}{\sigma-1}} \right) \] (20)

and

\[ \frac{\partial e}{\partial h} = \left( \frac{\beta M}{1 + \gamma + \beta} \right) \left( \frac{1}{M\hat{h} + \hat{h}^{\frac{1}{\sigma}}} \right)^2 \left( -\frac{\sigma}{1 - \sigma} \hat{h}^{\frac{1}{\sigma-1}} - MP - \frac{1}{1 - \sigma} \hat{h}^{\frac{\sigma}{\sigma-1}} P \right) \] (21)

From equations (20) and (21), it is obvious that both \( \frac{\partial g}{\partial h} \) and \( \frac{\partial e}{\partial h} \) decrease with \( P \). Thus \( \frac{\partial g}{\partial h} \) and \( \frac{\partial e}{\partial h} \) decrease with \( P \).

To show \( \frac{\partial g}{\partial h} > 0 \), we just need to show that the term in the last parenthesis of equation (20) is positive. That is, \( \left( 1 - M\frac{\hat{h}^{\frac{\sigma}{\sigma-1}} - \frac{\sigma MP}{\sigma - 1} \hat{h}^{\frac{1}{\sigma-1}}}{\frac{1}{\sigma-1}} \right) > 0 \), which is equivalent to

\[ P < \frac{1}{\sigma} + \frac{\hat{h}^{\frac{\sigma}{\sigma-1}}}{\frac{\sigma}{\sigma-1}M} \] (22)
To show equation (22) is implied by equation (19), it is sufficient to show
\[
\hat{h} \left( -\frac{1}{\sigma} + \hat{h} \frac{\alpha}{\alpha - \sigma} - \frac{\alpha}{\alpha - \sigma} M \right) > \frac{\beta \hat{h}}{1 + \gamma + (1 + \gamma + \beta) M \hat{h} \frac{\sigma}{\alpha - 1}}
\]
which is equivalent to
\[
-\frac{1}{\sigma} + \frac{\hat{h} \frac{\alpha}{\alpha - \sigma}}{-\frac{\alpha}{\alpha - \sigma} M} > \frac{\beta \hat{h} \frac{\alpha}{\alpha - \sigma}}{(1 + \gamma) \hat{h} \frac{\alpha}{\alpha - \sigma} + (1 + \gamma + \beta) M}
\]
which is implied by
\[
-\frac{1}{\sigma} + \frac{\hat{h} \frac{\alpha}{\alpha - \sigma}}{-\frac{\alpha}{\alpha - \sigma} M} > \frac{\beta \hat{h} \frac{\alpha}{\alpha - \sigma}}{(1 + \gamma + \beta) M}
\]
Since \(-\frac{1}{\sigma} > 0\), it is sufficient to show \(\frac{\beta}{(1 + \gamma + \beta) M} - \frac{1}{\frac{\alpha}{\alpha - \sigma} M} < 0\), which is equivalent to \(0 < 1 + \gamma + \beta - \sigma (1 + \gamma)\), clearly true.

To show \(\frac{\partial e}{\partial \hat{h}} > 0\), we just need to show the last term in the parenthesis of equation (21) is positive, which is equivalent to
\[
P < \frac{\hat{h}}{-\frac{1}{\sigma} - \frac{1 - \sigma}{\hat{h} \frac{\sigma}{\alpha - 1}}} \quad (23)
\]
To show (23) is also implied by equation (19), it is sufficient to show
\[
-\frac{1}{\sigma} - \frac{1 - \sigma}{\hat{h} \frac{\sigma}{\alpha - 1} M \hat{h} \frac{\sigma}{\alpha - 1}} > \frac{\hat{h}}{\frac{1 + \gamma}{\beta} + \left( \frac{1 + \gamma + \beta}{\beta} \right) \hat{h} \frac{\sigma}{\alpha - 1}} \quad (24)
\]
Comparing the denominators in inequality (24) reveals that a small \(\sigma\) (large \(\sigma\) in absolute value) guarantees that the inequality holds. The sufficient conditions are \(-\frac{1}{\sigma} < \frac{1 + \gamma}{\beta}\) and \(-\frac{1 - \sigma}{\hat{h}} < \frac{1 + \gamma + \beta}{\beta}\), which are equivalent to
\[
\sigma < -\frac{\beta}{1 + \gamma}
\]
This condition is clearly satisfied.
The case of binding non-negativity constraint of goods. If the government makes excessive public investment, the nonnegativity constraint for $g$ becomes binding. It is easy to show that, when $g = 0$, the optimal time investment satisfies the following equation

$$\alpha \left(1 + \frac{\beta}{1 + \gamma}\right) e^\sigma - \frac{\alpha \beta}{1 + \gamma} e^{\sigma - 1} + (1 - \alpha) P^\sigma = 0$$

We cannot solve for $e$ explicitly from the above equation. But since we know $P \geq 0$, it is easy to show that

$$e < \frac{\beta}{1 + \gamma + \beta}$$  \hspace{1cm} (25)

To show that $e$ decreases with $P$, define $Q = (1 - \alpha) P^\sigma$. It is sufficient to show that $\frac{\partial Q}{\partial e} < 0$. Since $Q = \frac{\alpha \beta}{1 + \gamma} e^{\sigma - 1} - (\alpha + \frac{\alpha \beta}{1 + \gamma}) e^\sigma$, we have

$$\frac{\partial Q}{\partial e} = \frac{\alpha}{1 + \gamma} e^{\sigma - 2} [\beta (\sigma - 1) - (1 + \gamma + \beta) \sigma e]$$

Clearly, $\frac{\partial Q}{\partial e} < 0$ when $\sigma < 0$ and $e < \frac{\beta}{1 + \gamma + \beta}$ which is given in (25).

Proof of Proposition II  When $\sigma > 0$, from (21), it is straightforward to see that $\frac{\partial e}{\partial h}$ is negative and decreases with $P$. From equation (20), $\frac{\partial g}{\partial h}$ is positive and increases with $P$. Given that $\hat{h} = (1 - \tau) h$, clearly the properties in Proposition II are true.
References


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