Evaluating the forecasting performance of linear and nonlinear monetary policy rules for South Africa

Kasai, Ndahiriwe and Naraidoo, Ruthira

Department of Economics, Kigali Independent University, Rwanda, Department of Economics, University of Pretoria, South Africa

June 2011
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Ndahiriwe Kasaï
Department of Economics
Kigali Independent University, Rwanda
e-mail: dr.kasai.ndahiriwe@ulk.ac.rw or kasaind@yahoo.fr

and

Ruthira Naraadoo
Department of Economics
University of Pretoria, South Africa
e-mail: ruthira.naraadoo@up.ac.za

Abstract

This paper compares forecast performance of linear and nonlinear monetary policy rules using South African data. Recursive forecasts values are computed for 1- to 12-steps ahead for the out-of-sample period 2006:01 to 2010:12. For the nonlinear models we use bootstrap method for multi-step ahead forecasts as opposed to point forecasts approach used for linear models. The aim of the paper is to evaluate the performance of three competing models in an out-of-sample forecasting exercise. Overall ranking reveals the superiority of the nonlinear model that distinguishes between downward and upward movements in the business cycles in closely matching the historical record. As such, forecasting performance tests reveal that the South African Reserve bank pays particular attention to business cycles movements when setting its policy rate.

Key words: Monetary policy rules, forecast evaluation.

JEL classification: C22; C52; C53; E17; E43; E52; E58
1. Introduction

Empirical evidence in the field of monetary policy continues to prove that the behaviour of central bankers is not symmetric either around a certain level of policy instrument, the inflation target or potential output. More recent examples include Hayat and Mishra (2010) and Martin and Milas (2010a) among others. In these cases, besides the failure to reject the null hypothesis of linearity, nonlinear models are found to outperform their rival linear models in terms of goodness-of-fit. It is well known that one of the prime benefits of robust economic models is the predictive accuracy they have. In the field of monetary policy, for instance, a robust monetary policy reaction function can help monetary authorities to predict more accurately the future values of the policy instrument.

Given the recent in-sample outperformance of nonlinear monetary policy reaction functions, one can expect the latter to predict the behaviour of central banks better than a simple linear policy rule. However, early in the 1990s, De Gooijer and Kumar (1992) concluded that there was no clear evidence in favour of non-linear over linear models in terms of forecast performance. More than a decade later, Clements et al. (2004) suspect that the situation has not changed very much, as we had not gone very far in the area of non-linear forecast models. The literature review by Clements et al. (2004) suggests that the forecasting performance of nonlinear models is on average not particularly good relative to rival linear models. As far as monetary policy rules are concerned, Qin and Enders (2008) find more challenging results as they report that the univariate models forecast better than the Taylor rules, linear and nonlinear. More recently, Naraidoo and Paya (2010) compare linear and nonlinear parametric models and, non-parametric and semi-parametric models in forecasting the South African Reserve Bank’s repurchase

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1 As far as the South African reserve bank (SARB) is concerned, Reid and Du Plessis (2010) advocate for greater transparency that could be achieved if it were to publish a forecast of the expected path of its policy instrument. Furthermore, as propounded by the same authors, forecasts of the policy instrument would shed some lights on the forward-looking nature of monetary policy and thereby enhance the predictability of the SARB’s policy stance.
rate. They find that a semi-parametric model that relaxes the functional form of the monetary policy rule outperforms other models especially in long horizon forecasting.

This paper contributes to the scarce literature that uses Taylor rules to forecast the nominal interest rate out-of-sample. Some notable exceptions are Qin and Enders (2008), Moura and Carvalho (2010) and Naraidoo and Paya (2010). In this study about South Africa, we construct the forecasts from linear and nonlinear Taylor type rule models under a backward looking expectations formation for the target variables and examine their forecasting gains over the period 2006:M01 to 2010:M12. The main aim of the paper is to evaluate predictive accuracy of three competing models based on a number of forecasting tests; namely the mean squared prediction error (MSPE), the median squared prediction error (MedSPE) the modified Diebold and Mariano, the encompassing and Cark and West tests.

The rest of the paper is organized as follows. In the next section, we discuss the linear and nonlinear Taylor rule versions to be evaluated for predictive ability. Section 3 discusses the data and forecasting methodology. Section 4 passes into review in-sample properties of the four alternative models by comparing their goodness of fit in terms of the Akaike information criterion (AIC). Section 5 reports an in-depth forecasting evaluation of different models with the aim to determine the best model in predictive ability. Section 6 concludes.

2. Alternative models

In this paper we make comparative forecasting evaluation among the models discussed in Kasai and Naraidoo (2012 and 2013). However, for forecasting purposes we consider backward looking versions rather than the forward looking ones. Although Kasai (2011) suggests that forward looking version of the Taylor rule describes better the behaviour of the SARB than the backward-looking, an out-of-sample forecasting exercise cannot
use future values of variables in the pure forecasting sense. Therefore, models Kasai and Naraidoo (2012 and 2013) are rewritten in their backward looking versions.

**Model 1:**

\[
i_t = \rho_i(L)i_{t-1} + (1 - \rho_i)\left\{\rho_0 + \rho_x(\pi_{t-1} - \pi^x) + \rho_y y_{t-1} + \rho_j f_{t-1}\right\}
\]

(1)

where, \(i_t\) is the actual nominal interest rate, \(\rho_i(L) = \rho_{i1} + \rho_{i2}L + \ldots + \rho_{iL}L^{L-1}\) (we have used \(\rho_i \equiv \rho_i(1)\) as a measure of interest rate persistence), \(\pi_{t-1}\) is the inflation rate at time \((t-1)\), \(y_{t-1}\) is the output gap at time \((t-1)\), \(f_{t-1}\) is a measure of financial conditions index\(^2\) at time \((t-1)\) used to augment the original rule, and \(\varepsilon_t\) is an error term. Parameter \(\rho_j\) is the weight on variable \(j\).

Equation (1) is characterized by three modifications made on the original simple Taylor rule, namely interest rate smoothing, the backward-looking version and the inclusion of financial condition index. The theoretical justification for including the financial conditions measure might either be that it enters the aggregate demand curve, similar to Castro (2008) or Goodhart and Hoffman (2002) or still the policymaker might have preferences for this index being close to equilibrium as in Naraidoo and Raputsoane (2010). For instance, Walsh (2009) points out that when financial factors cause distortions, these distortions will in general introduce corresponding terms in a loss function for monetary policy (see for example the theoretical model of Martin and Milas, 2010b). An alternative theoretical justification for the inclusion of the financial index in the policy rule is that the index determines movements in the differential between policy rates and 3-month interbank rates, the latter being the benchmark for private sector interest rates (see for example Martin and Milas, 2009).

\(^{2}\) See Kasai and Naraidoo (2012) for the construction of the index.
Other alternatives consist to allow for nonlinearities in interest rate setting behaviour of the monetary authorities (see Kasai and Naraidoo, 2012 for more discussion). The first nonlinear version is axed on the widespread belief that central bankers’ interventions through changes in a short-term interest rate are influenced by the state of the business cycle (see for instance, Bec et al., 2002). This being the case, the following nonlinear policy rule is considered.\textsuperscript{3}

**Model 2:**

\[ i_t = \rho_j i_{t-1} + (1 - \rho_j) \left\{ \rho_0 + \theta_j^j(y_{t-1}; \gamma^j; \tau)M_{lt} + (1- \theta_j^j(y_{t-1}; \gamma^j; \tau)M_{2t} \right\} + \epsilon_t \]  

(2)

where \( M_{jt} = \rho_{jt} \pi_{t-1} + \rho_{jt} y_{t-1} + \rho_{jt} f_{t-1} \) for j=1,2 and the function \( \theta_j^j(y_{t-1}; \gamma^j; \tau) \) is the weight defined below in (3). \( M_{lt} \) is a linear Taylor rule that represents the behaviour of policymakers during business cycle recessions and \( M_{2t} \) is a linear Taylor rule that represents the behaviour of policymakers during business cycle expansions. The weight \( \theta_j^j(y_{t-1}; \gamma^j; \tau) \) is modelled using the following logistic function (see e.g. van Dijk et al., 2002):

\[ \theta_j^j(y_{t-1}; \gamma^j; \tau) = 1 - \frac{1}{1 + e^{-\gamma^j(y_{t-1}; \theta_j^j y_{t-1})}} \]  

(3)

In (3) the smoothness parameter \( \gamma^j > 0 \) determines the smoothness of the transition regimes. We follow Granger and Teräsvirta (1993) and Teräsvirta (1994) in making \( \gamma^j \) dimension-free by dividing it by the standard deviation of \( y_{t-1} \).

In addition to the above non-linear version of the Taylor rule, Kasai and Naraidoo (2013) have reported that opportunistic approach to monetary policy also deserves its

\textsuperscript{3} Kasai and Naraidoo (2012) reported that the nonlinear Taylor rule improves its performance with the advent of the financial crisis, providing the best description of in-sample SARB interest rate setting behavior.
particular attention in the context of the South African economy. On this regard, we choose a quadratic logistic function that was reported in Kasai and Naraïdoo (2013) to outperform all other models. As such, equation (2) is revised to accommodate the two features of opportunistic approach to monetary policy. The model is specified as follows:

**Model 3:**

\[ i_t = \rho_i(L)i_{t-1} + (1 - \rho_i) \left\{ i^* + \theta_i \rho_{ZD} (\pi_{t-1} - \pi'_{t-1}) + (1 - \theta_i) \rho_{OZD} (\pi_{t-1} - \pi'_{t-1}) \right\} + \varepsilon_t \]  

(4)

where \( \pi'_{t} \) is the intermediate inflation target defined as \( \pi'_{t} = \mu \left( \frac{1}{n} \sum_{j=1}^{n} \pi_{t-j} \right) + (1 - \mu) \pi^* \) and \( \theta = \text{pr}\left\{ -\delta \leq E_t(\pi_{t-1} - \pi'_{t-1}) \leq \delta \right\} \) is the probability that inflation is within the zone of discretion.

\[ \theta = \text{pr}\left\{ -\delta \leq (\pi_{t-1} - \pi'_{t-1}) \leq \delta \right\} = 1 - \frac{1}{1 + e^{-\left( \frac{1}{\gamma} \left( \pi_{t-1} - \pi'_{t-1} \right) \right)}} \]  

(5)

Similarly, we follow Granger and Teräsvirta (1993) and Teräsvirta (1994) in making the smoothness parameter \( \gamma > 0 \) dimension-free by dividing it by the standard deviation of \( (\pi_{t-1} - \pi'_{t-1}) \). In equation (5) it is assumed that the policy maker responds to \( (\pi_{t-1} - \pi'_{t-1}) \).

The response is assumed to depend on whether the inflation is within the target zone or not.

Within sample we would expect the fit of such alternative models to be barely distinguishable, given the high correlations between the interest rate and its lags. However, the key distinguishing feature amongst linear and nonlinear models lies in their forecast implications, namely that the equilibrium to which the reaction function returns depends on the size of the shocks/inflation and business cycle states. A linear
Taylor type rule model will forecast the interest rate to stay roughly where it is if non-
stationary; or, if stationary, to revert to some deterministic equilibrium. Thus the
forecast implications of linear as opposed to nonlinear models are quite different. This
is kept in mind when forecasting out-of-sample in the section 5 below.

3. Data discussion

In this paper we review and evaluate the predictive accuracy of a variety of models of
monetary policy rules using monthly data for South Africa for the period spanning from
2000:M01 to 2010:M12. We split the sample into in-sample and out-of-sample
experiments. In-sample observations span from 2000:01 to 2005:12 and out-of-sample
observations covers the period spanning from 2006:01 to 2010:12.

The repurchase rate (repo rate) measures the nominal interest rate. Inflation is measured
as the annualized rate of change of consumer price index as the later is the SARB’s
preferred core measure of inflation. The output gap is measured as a difference between
the logarithm of industrial production and its Hodrick-Prescott (1997) trend. The
financial condition index is constructed as an average of (i) the real effective exchange
rate (REER) detrended by a HP filter and where the rand appreciation increases the
index, (ii) the real house price index (RH,) detrended by a HP filter and where the
house price index is an average price of all houses compiled by the ABSA bank, deflated
by the consumer price index (iii) the real stock price (RS,) which is measured by the
Johannesburg Stock Exchange All Share index, deflated by the consumer price index
(iv) the credit spread (CS,) which is the spread between the yield on the 10-year
government bond and the yield on A rated corporate bonds, and (v) the future spread

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4 On the expanding window versus fixed-length rolling window issue we note that according to Stock and
Watson (2005), “recursive forecasts are more accurate than the rolling forecasts” for the representative
macroeconomic dataset they study. On the other hand, however, Giacomini and White (2006) find that a
“rolling window procedure can result in substantial forecast accuracy gains relative to an expanding
window for important economic time series.”

5 For instance Burger (2008) argued that the spread between the mortgage rate and the 10-year
government bond rate is very much like an intermediate monetary target since its change leads to an
which is the change of spread between the 3-month interest rate futures contracts \(F_t\) in the previous quarter and the current short-term interest rate.

To tackle the end-point problem in calculating the HP trend (see Mise et al., 2005a&b), the sample has to be expended on both starting and ending points. With regards to the starting points, this study considers actual data for twelve months prior to 2000. With regard to the ending points, an autoregressive (AR\(n\)) estimation has been applied to the series under decomposition and the AR model is in turn used to forecast additional observations that have to be added to each of the series before applying the HP filter. The method is applied to the industrial production and the components of the financial index (with \(n\) set at 4 to eliminate serial correlation). The obtained smoothed representation \(t\) (trend) of a given time series is considered to be its potential level. The cyclical component \(s\) represents the fluctuations around the long-run pattern. A negative value of the cyclical component indicates that the short-term level of the series is below its potential level, while a positive value indicates that the short-term level is above the potential one.

The constructed financial index is expressed in standardized form, relative to the mean value of year 2000 and where the vertical scale measures deviations in terms of standard deviations; therefore, a value of 1 represents a 1-standard deviation difference from the mean. Additionally, all data are seasonally adjusted. The index is also in the spirit of the UK financial conditions index provided by the Bank of England’s Financial Stability Report (Bank of England, 2007).

4. In-sample evaluation

This section reviews the in-sample properties of backward looking models that are going to be tested for out-of-sample properties in the next section. Tables 1 and 2

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opposite change in output or price level.
report estimates of the Taylor rule Models 1, 2 and 3 for the in-sample period which runs from 2000:M1 to 2005:M12.

For linear Model 1, the last three rows report Hamilton’s (2001) λ-test, and the λ$_4$ and g-tests proposed by Dahl and González-Rivera (2003). Under the null hypothesis of linearity, these are Lagrange Multiplier test statistics following the χ$^2$ distribution. These tests are powerful in detecting non-linear regime-switching behavior like the one considered by Model 2 and 3. All three tests reject linearity. Model 3 in Table 2 exhibits the lowers AIC and shows that the inflation outside the zone of discretion, output gap and financial index effects are statistically significant but not the inflation within the zone of discretion. The results are in line with the opportunistic approach theory.

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*We run the tests using Gauss codes obtained from Hamilton’s web page at: [http://weber.ucsd.edu/~jhamilto/software.htm#other](http://weber.ucsd.edu/~jhamilto/software.htm#other). To account for the small sample, we report bootstrapped p-values of the three tests based on 1000 re-samples.*
### Table 1: In-sample estimates for Models 1 and 2

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Model 1 (Linear)</th>
<th>Model 2 (Nonlinear)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$</td>
<td>0.882***</td>
<td>6.876***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.478***</td>
<td>0.859***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>1.077***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.023**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>0.882***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>$\rho_{1\pi}$</td>
<td></td>
<td>0.697***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.03)</td>
</tr>
<tr>
<td>$\rho_{1y}$</td>
<td>0.286</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td></td>
</tr>
<tr>
<td>$\rho_{1f}$</td>
<td></td>
<td>0.059***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\rho_{2\pi}$</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>$\rho_{2y}$</td>
<td>0.116</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>$\rho_{2f}$</td>
<td></td>
<td>-0.024**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\tau$</td>
<td></td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00)</td>
</tr>
<tr>
<td>$\gamma^y$</td>
<td></td>
<td>5.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notes:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Where Model 1 is $i_t = \rho_0 i_{t-1} + (1 - \rho_0) \left( \rho_0 + \rho_\pi \pi_{t-1} + \rho_y y_{t-1} + \rho_f f_{t-1} \right) + \varepsilon_t$ and Model 2 is $i_t = \rho_0 i_{t-1} + (1 - \rho_0) \left( \rho_0 + \theta(y_{t-1}; \gamma^y; \tau) M_{\theta} + (1 - \theta(y_{t-1}; \gamma^y; \tau) M_{\theta}) \right) + \varepsilon_t$ with $M_{\theta} = \rho_\pi \pi_{t-1} + \rho_y y_{t-1} + \rho_f f_{t-1}$ for $j = 1, 2$ and $y_t$ is the transition variable.</td>
<td></td>
</tr>
<tr>
<td>(ii) Numbers in parentheses are standard errors. <em>(<strong>)</strong></em> indicate that the parameter is significant at a 10(5)[1] % level respectively. AIC is the Akaike Information Criterion. J-stat is the $p$-value of a chi-square test of the model's over-identifying restrictions (Hansen, 1982). The set of instruments includes a constant, 1-6, 9, 12 lagged values of repo rate, the inflation, the output gap, the 10-year government bond, money (M3) growth, and the financial index.</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: The Opportunistic Approach (Model 3)

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_i$</td>
<td>0.832*** (0.01)</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>0.396 (0.30)</td>
</tr>
<tr>
<td>$\rho_{ZD}$</td>
<td>0.523*** (0.03)</td>
</tr>
<tr>
<td>$\rho_{OZD}$</td>
<td>1.147*** (0.04)</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>0.008*** (0.00)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.530*** (0.03)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>2.05</td>
</tr>
</tbody>
</table>

S.E.  | 0.394 |
AIC   | **1.052** |
$\bar{R}^2$ | 0.972 |
$H_0: \rho_{ZD} = \rho_{OZD}$ (p value) | 0.000 |
J-statistic (p value) | 0.249 |

**Notes**: Numbers in parentheses are standard errors. 
*(**)[**][***] indicate that the parameter is significant at a 10(5)% level respectively. S.E is the regression standard error. AIC is Akaike Information criterion. J-statistic is the $p$-value of a chi-square test of the model's over-identifying restrictions (Hansen, 1982). The set of instruments includes a constant, 1-6, 9, 12 lagged values of repo rate, the inflation, the output gap, the 10-year government bond, money (M3) growth, and the financial index.

5. Out-of-sample evaluation

5.1. Forecasting methodology

In this paper, in-sample observations spans from 2000:01 to 2005:12 and out-of-sample observations covers the period spanning from 2006:01 to 2010:12. The number of in-
sample and out-of-sample observations is denoted by $R$ and $P$, respectively, so that the total number of observations is $T = R + P$. As we perform recursive out-of-sample forecasts, the in-sample observations increase from $R$ to $T - h$. In the recursive exercise, the parameters of the model are re-estimated by employing data up to time $t-1$ so as to generate forecast for the following $h$ horizons. The number of forecasts corresponding to horizon $h$ is equal to $P - h + 1$. The forecasting nonlinear monetary policy rule can be described by the following model

\[ i_t = F(X_{t-1}; \theta) + \varepsilon_t \]  

(6)

Where $\varepsilon_t \sim iid(0, \sigma^2)$ and $X_t$ is a $(k \times 1)$ vector of the exogenous variables and lagged repo rate as defined in Section 2. The optimal one-step-ahead forecast equals

\[ \hat{i}_{t+1/t} = E[i_{t+1} / X_t] = F(X_t; \theta) \]  

(7)

which is equivalent to the optimal one-step-ahead for the alternative linear model. An easy way of obtaining a 2-step-ahead forecast is to draw it from the 1-step-ahead forecast and have

\[ \hat{i}_{t+2/t}^{(n)} = F(X_{t+1}; \theta). \]  

(8)

However, this approach has been a subject of strong criticisms to the extent of being named ‘naïve’ by Brown and Mariano (1989) or ‘skeleton’ forecast by Tong (1990). These fair criticisms are based on the fact that equation (6) considers $E(\varepsilon_{t+1}/X_t) = 0$ and are supported by simulation evidence by Lin and Granger (1994) reporting substantial losses of efficiency.

As opposed to the so called ‘naïve’ or ‘skeleton’ approach numerical techniques are required in forecasting nonlinear models like the ones in section 2. Detailed discussions on the techniques are provided by Granger and Teräsvirta (1993), Franses and van Dijk (2000) and Fan and Yao (2003). In this paper, the residuals ($\hat{\varepsilon}_t$) of the estimated model
is obtained through bootstrapping. With this method, the density of \( \hat{\epsilon}_i \) is composed of \( N \) independent error vectors \( \{\epsilon_{i+1}^{(i)}, \ldots, \epsilon_{i+N}^{(i)}\} \) giving a better approximation of the 2-step-ahead forecast as follows:

\[
l_{t+2|t}^B = \left( \frac{1}{N} \right) \sum_{i=1}^{N} F \left( X_{t+1|t} + \epsilon_{i+1}^{(i)}; \theta \right)
\]

(9)

To obtain h-step-ahead, one generates \( \epsilon_{i+1}^{(i)}, \ldots, \epsilon_{i+h}^{(i)} \), \( i = 1, \ldots, N \) and sequentially computes \( N \) forecasts for \( l_{t+1|t}, \ldots, l_{t+h|t} \) with \( h \geq 2 \) and where a single point forecast for a particular point in time is obtained by simply averaging its corresponding \( N \) forecasts (see Teräsvirta, 2006).

Forecasting performance is evaluated using, among other tests, the Mean Squared Prediction Error (MSPE) and Median Squared Prediction Error (MedSPE) criteria. For robustness purpose, we also test the null hypothesis of equal forecasting accuracy using modified Diebold-Mariano statistics \( DM – t \), see Harvey et al., 1997). The \( DM – t \) for any two models denoted by 1 and 2 is computed as follows:

\[
DM – t = (P – h + 1)^{1/2} \frac{\bar{d}}{\hat{\delta}_{dd}^{1/2}}.
\]

where \( \hat{d}_{t+h} = \hat{\epsilon}_{1,t+h}^2 - \hat{\epsilon}_{2,t+h}^2 \); \( \hat{\epsilon}_{i,t+h} \) being h-step ahead prediction error for model \( i \);

\[
\bar{d} = (P – h + 1)^{-1} \sum_{t=R}^{T-h} \hat{d}_{t+h} = MSPE_1 - MSPE_2;
\]

\[
\hat{\Gamma}_{dd}(j) = (P – h + 1)^{-1} \sum_{t=R}^{T-h} d_{t+h} \hat{d}_{t+h-j} \text{ for: } j \geq 0 \text{ and } \hat{\Gamma}_{dd}(j) = \hat{\Gamma}_{dd}(-j);
\]

\[
\hat{\delta}_{dd} = \sum_{j=-\hat{j}}^{\hat{j}} K(j/M) \Gamma_{dd}(j)
\]

denotes the long-run variance of \( d_{t+h} \) estimated using a kernel-based estimator with function \( K(.,) \), bandwidth parameter \( M \) and maximum number of lags \( \hat{j} \). We follow Harvey et al. (1997) in correcting for small-sample bias and so the corrected test statistic is obtained by multiplying the above \( DM – t \) by
\[ \zeta = \sqrt{\frac{P-2h+h(h-1)/(P-h+1)}{(P-h+1)}} \]

The hypotheses to be tested are

\[ H_0 : \hat{\epsilon}_{1,t+h} - \hat{\epsilon}_{2,t+h} = 0 \]

and

\[ H_1 : \hat{\epsilon}_{1,t+h} - \hat{\epsilon}_{2,t+h} \neq 0 \]

The rejection of the null is based on Student’s \( t \) distribution with \((n - 1)\) degrees of freedom rather than the standard normal distribution (see Harvey et al., 1997). It is worth to mention that nonlinear Taylor rule equations nest the linear equations and therefore their population errors are identical under the null hypothesis making the variance \( d_{t+h} \) equal to zero (see McCracken, 2007). Indeed, it has been argued that asymptotic distribution theory for the Diebold and Mariano (1995) test does not hold for nested models (see McCracken, 2000; Clark and McCracken, 2001 and Teräsvirta, 2005). However, Giacomini and White (2006) showed that when in-sample size remains finite, the asymptotic distribution of the Diebold and Mariano statistic (DM statistic) is still standard normal when forecasts are compared from nested models. Bhardwaj and Swanson (2006) also argue that the \( DM-t \) statistic can still be used as an important diagnostic in predictive accuracy as the non-standard limit distribution is reasonably approximated by a standard normal in many contexts.

As far as the issue of nestedness is concerned, we apply the Clark and McCracken (2001) encompassing test \( ENC-t \) and Clark and West (2007). Both tests are designed to test the null hypothesis of equal forecasting accuracy for nested models. The \( ENC-t \) statistic is given by

\[ ENC-t = (P-1)^{1/2} \frac{\bar{\tilde{e}}}{\left( P^{-1} \sum_{t=h}^{T-1} (G_{t+h} - \bar{e}) \right)^{1/2}}, \]
where \( c_{t+h} = \hat{\epsilon}_{1,t+h}^2 - \hat{\epsilon}_{2,t+h}^2 \) and \( \bar{c} = P^{-1} \sum_{t=h+1}^{T-h} c_{t+h} \). The ENC \(-t\) has the same null hypothesis as the \( DM - t\) test, but the alternative is \( H_1: \hat{\epsilon}_{1,t+h} - \hat{\epsilon}_{2,t+h} > 0 \) which is more restrictive than the \( DM - t\) that considers \( H_1 = \hat{\epsilon}_{1,t+h} - \hat{\epsilon}_{2,t+h} \neq 0 \). For \( h=1\), the limiting distribution is \( N(0,1) \). By contrast, Clark and McCracken (2001) show that for multistep-ahead \((h>1)\) forecasts, the limiting distribution is non-standard. However, as noted by Bhardwaj and Swanson (2006), tabulated critical values are quite close to the \( N(0,1) \) values when Newey and West (1987)-type estimator is used for \( h>1 \). As such, standard normal distribution can be used as a rough guide for multistep-ahead forecasts comparison (see Clark and McCracken, 2001 for further details).

An alternative test for equal forecast errors is the Clark and West test (CW-test) statistics given by

\[
\hat{f}_{t+h} = (\hat{i}_{t+h} - \hat{i}_{t+h})^2 - [(i_{t+h} - \hat{i}_{1,t+h})^2 - (\hat{i}_{1,t+h} - \hat{i}_{2,t+h})^2].
\]

Where the period \( t \) forecast of the repo rate \( i_{t+h} \) from the two models are denoted \( \hat{i}_{u,t+h} \) and \( \hat{i}_{2,t+h} \) with corresponding period \( t+h \) forecast errors \( i_{t+h} - \hat{i}_{1,t+h} \) and \( i_{t+h} - \hat{i}_{2,t+h} \).

The test for equal MSPE is performed by regressing \( \hat{f}_{i+h} \) on a constant and using the resulting \( t \)-statistic for a zero coefficient (see Clark and West, 2007). As above, the null hypothesis is equal MSPE while the alternative is model 2 has a smaller MSPE than model 1. In line with Clark and West (2007), the null is rejected if the \( t \)-statistic is greater than +1.282 (for a one sided 0.10 test) or +1.645 (for a one sided 0.05 test).

### 5.2. Testing predictive ability

One of the prime usages of robust economic models is to predict the future pattern of economic series. Therefore, most economic models, linear or non-linear can be judged in terms of their forecasting performance. As such, this paper uses a variety of functional forms discussed in section 2 and section 4 with the aim of obtaining the best
model in predictive ability. The forecast evaluation based on the mean squared prediction error (MSPE) and the median squared prediction error (MedSPE) have been reported. These two forecast error statistics are scale dependent. According to the criteria, smaller errors show better predictive ability and therefore the closer to zero the better the predictive ability of the model. The ranks of the 3 competing models’ forecasts are shown in Tables 3 and 4. The comparison of forecast performance is made vertically for each horizon in terms of forecasting test. As shown in Tables 3 and 4, nonlinear Model 2 yields the smallest MSPE and MedSPE for the short and long horizons and so ranked the first in terms of these criteria. Comparing the remaining two models, one can observe that linear Model 1 is ranked the second best for the very short horizon. However, multi-step ahead \((h > 3)\) forecast evaluation reveals empirical evidence in favour of the nonlinear model 3 in terms of MSPE. It is known that significant in-sample evidence of predictability does not guarantee significant out-of-sample predictability. This might be due to a number of factors such as the power of tests (see Inoue and Kilian, 2004). In terms of MedSPE, the linear Model 1 is ranked second. Average ranking respectively based on MSPE and MedSPE is reported in the last columns of Table 3 and 4 showing the superiority of nonlinear model 2.
Table 3: Mean squared prediction error rank (recursive estimates)

<table>
<thead>
<tr>
<th>Model</th>
<th>h=1</th>
<th>h=2</th>
<th>h=3</th>
<th>h=4</th>
<th>h=5</th>
<th>h=6</th>
<th>h=7</th>
<th>h=8</th>
<th>h=9</th>
<th>h=10</th>
<th>h=11</th>
<th>h=12</th>
<th>Average rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (b)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2.75</td>
</tr>
<tr>
<td>2 (b)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3 (b)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Notes: The Table reports the out-of-sample forecasting ranks of Model 1, 2 and 3 across the recursive windows for forecasting horizons \(h=1,\ldots,12\), using the Mean Squared Prediction Error (MSPE). The last column reports the average forecasting rank. Model 1 is the linear estimation, Model 2 is nonlinear with output as transition variable and Model 3 is a nonlinear estimation that accommodates the opportunistic approach to disinflation.

Table 4: Median squared prediction error rank (recursive estimates)

<table>
<thead>
<tr>
<th>Model</th>
<th>h=1</th>
<th>h=2</th>
<th>h=3</th>
<th>h=4</th>
<th>h=5</th>
<th>h=6</th>
<th>h=7</th>
<th>h=8</th>
<th>h=9</th>
<th>h=10</th>
<th>h=11</th>
<th>h=12</th>
<th>Average rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (b)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2.16</td>
</tr>
<tr>
<td>2 (b)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3 (b)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2.83</td>
</tr>
</tbody>
</table>

Note: The Table reports the out-of-sample forecasting ranks of Model 1, 2 and 3 across the recursive windows for forecasting horizons \(h=1,\ldots,12\), using the Median Squared Prediction Error (MedSPE). The last column reports the average forecasting rank. See Table 3 for the forecasting model definitions.
The modified Diebold-Mariano (DM-h) test results are reported in Table 5. These examine the statistical significance of MSPE reductions with uniform weight placed on forecast losses. The Table provides pair wise out-of-sample forecast comparisons based on recursive estimates. Table 5 shows that the modified Diebold and Mariano (1995) test points to the superiority of Model 2 over the linear model for the short and medium term horizons ($2 \leq h \leq 8$), but such dominance disappears as the forecast horizon lengthens ($h \geq 9$). On the other hand, the nonlinear Model 3 is never significantly better than the linear one.

Turning to the tests designed to test the null hypothesis of equal forecasting accuracy for nested models, the judgment based on $ENC_t$ and $CW_t$, respectively reported in Tables 6 and 7, is not much different from the one based on MSPE above. In fact, the results in Tables 6 and 7 reveal strong empirical evidence in favour of nonlinear models. Relative to the linear Model 1, nonlinear Model 2 is reported to yield the best predictive accuracy for all horizons in terms of both the encompassing ($ENC_t$) and Clark and West ($CW_t$) tests. Comparing predictive accuracy for linear model 1 and nonlinear Model 3 it is also clear that for multi-step ahead ($h > 3$), the nonlinear Model 3 can be judged best ranked for these longer horizons. However, the linear Model 1 can predict the near future ($h \leq 3$) better than the nonlinear Model 3.

All in all, Model 2 is best in closely matching the historical record for all the horizons. Overall ranking also shows that the nonlinear Model 3 is second best in medium and long horizons. As such, the findings would alleviate the concern by Clements et al. (2004) who reported lack of predictive ability for most of nonlinear models relative to their benchmark linear ones.
Table 5: Forecast Accuracy Evaluation \( (DM - t) \)

<table>
<thead>
<tr>
<th></th>
<th>Step1</th>
<th>Step2</th>
<th>Step3</th>
<th>Step4</th>
<th>Step5</th>
<th>Step6</th>
<th>Step7</th>
<th>Step8</th>
<th>Step9</th>
<th>Step10</th>
<th>Step11</th>
<th>Step12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1 vs Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>0.02</td>
<td>1.46*</td>
<td>1.96**</td>
<td>2.06**</td>
<td>2.07**</td>
<td>1.78**</td>
<td>1.50*</td>
<td>1.31*</td>
<td>1.20</td>
<td>1.14</td>
<td>1.09</td>
<td>1.13</td>
</tr>
<tr>
<td>Model 3</td>
<td>-1.28</td>
<td>-1.08</td>
<td>-0.71</td>
<td>-0.34</td>
<td>-0.23</td>
<td>-0.18</td>
<td>-0.11</td>
<td>-0.08</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.03</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Model 2 vs Model 3

| Model 3 | -0.75 | -2.08 | -2.19 | -1.82 | -1.75 | -1.60 | -1.42 | -1.26 | -1.13 | -1.07  | -1.01  | -1.05  |

**Note:** Table 12 shows forecast comparisons based on modified Diebold-Mariano statistics \( (DM - t) \) for horizons extending from 1 to 12. The entries in the table show the test statistics for the null hypothesis that Model i’s forecast performance as measured by MSPE is not superior to that of Model j at the 5% and 10% significance level respectively denoted by two and one asterisks (indicating the rejection of the null hypothesis). For definitions of Models, see footnote for Table 3.
Table 6: Forecast Accuracy Evaluation ($ENC - t$)

<table>
<thead>
<tr>
<th>Model</th>
<th>Step1</th>
<th>Step2</th>
<th>Step3</th>
<th>Step4</th>
<th>Step5</th>
<th>Step6</th>
<th>Step7</th>
<th>Step8</th>
<th>Step9</th>
<th>Step10</th>
<th>Step11</th>
<th>Step12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1 vs Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td>1.38*</td>
<td>2.07**</td>
<td>2.43**</td>
<td>2.61**</td>
<td>2.91**</td>
<td>3.22**</td>
<td>3.54**</td>
<td>3.84**</td>
<td>4.20**</td>
<td>4.77**</td>
<td>5.14**</td>
<td>5.70**</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.73</td>
<td>0.47</td>
<td>0.66</td>
<td>1.31*</td>
<td>1.67**</td>
<td>1.92**</td>
<td>2.25**</td>
<td>2.42**</td>
<td>2.57**</td>
<td>2.88**</td>
<td>3.08**</td>
<td>3.45**</td>
</tr>
<tr>
<td>Model 2 vs Model 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 3</td>
<td>1.78**</td>
<td>1.23</td>
<td>0.98</td>
<td>1.20</td>
<td>1.16</td>
<td>1.09</td>
<td>1.02</td>
<td>0.93</td>
<td>0.77</td>
<td>0.70</td>
<td>0.65</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Note: Table 13 shows forecast comparisons based on Clark and McCracken (2001) encompassing test statistics ($ENC - t$) for horizons extending from 1 to 12. The entries in the table show the test statistics for the null hypothesis that Model i’s forecast performance as measured by MSPE is not superior to that of Model j at the 5% and 10% significance level respectively denoted by two and one asterisks (indicating the rejection of the null hypothesis). For definitions of Models, see footnote for Table 3.
Table 7: Forecast Accuracy Evaluation \((CW - t)\)

<table>
<thead>
<tr>
<th></th>
<th>Step1</th>
<th>Step2</th>
<th>Step3</th>
<th>Step4</th>
<th>Step5</th>
<th>Step6</th>
<th>Step7</th>
<th>Step8</th>
<th>Step9</th>
<th>Step10</th>
<th>Step11</th>
<th>Step12</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model 1 vs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Model 2</strong></td>
<td>1.732*</td>
<td>3.922**</td>
<td>4.693**</td>
<td>4.568**</td>
<td>4.796**</td>
<td>4.999**</td>
<td>5.017**</td>
<td>4.916**</td>
<td>4.705**</td>
<td>4.536**</td>
<td>4.309**</td>
<td>4.204**</td>
</tr>
<tr>
<td><strong>Model 3</strong></td>
<td>0.721</td>
<td>0.470</td>
<td>0.657</td>
<td>1.303*</td>
<td>1.635*</td>
<td>1.868**</td>
<td>2.173**</td>
<td>2.312**</td>
<td>2.432*</td>
<td>2.697**</td>
<td>2.855**</td>
<td>3.163**</td>
</tr>
</tbody>
</table>

**Model 2 vs**

| **Model 3**     | 1.761** | 1.221  | 0.969  | 1.192  | 1.140  | 1.064  | 0.980  | 0.889  | 0.731  | 0.652  | 0.598  | 0.503  |

**Note:** Table 14 shows forecast comparisons based on modified Clark and West statistics \((CW - t)\) for horizons extending from 1 to 12. The entries in the table show the test statistics for the null hypothesis that Model i’s forecast performance as measured by MSPE is not superior to that of Model j at the 5% and 10% significance level respectively denoted by two and one asterisks (indicating the rejection of the null hypothesis). For definitions of Models, see footnote for Table 3.
6. Conclusion

In this paper, three functional forms of a Taylor type policy rule have been used for forecasting exercise with the aim of obtaining the best model in predictive ability. Out-of-sample properties are assessed using point forecast for the linear model while forecast obtained by means of bootstrapping method is used for nonlinear models. The evaluation is based on several forecasting accuracy tests; namely the mean squared prediction error (MSPE), the median squared prediction error (MedSPE) the modified Diebold and Mariano, the encompassing and Cark and West tests.

Comparison of the forecasts from nonlinear functional forms with the benchmark linear model, show the advantage of considering nonlinearities in monetary policy reaction functions for most of the cases. Indeed, based on several forecasting accuracy tests, overall ranking reveals the superiority of the nonlinear model that distinguishes between downward and upward movements in the business cycles in closely matching the historical record. As such, forecasting performance tests reveal that the SARB pays particular attention to business cycles movements when setting its policy rate.
References


