Costs of Low Productivity: Intensive and Extensive Margins

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Abstract

This paper discusses welfare costs of a decrease in productivity and argues that there are two important channels which cause a reduction in welfare: a decrease in output per firm (*intensive margin*) and a decrease in number of operating firms (*extensive margin*). Traditional Dixit-Stiglitz monopolistic competition framework with constant elasticity of substitution utility and common productivity across firms fail to capture the extensive margin. To address this problem, this paper introduces "continuum-quadratic" utility (i.e. linear demand system) while keeping the other assumptions unchanged and finds that lowering productivity affects not only the intensive but extensive margin as well.

1. Introduction

A straightforward result of an average productivity decrease in a market is the reduction of the average output per firm (keeping everything else fixed). However, productivity decrease may also indeed a decrease in number of operating firms. Throughout the paper, the first and the latter effects of a productivity decrease are called intensive and extensive margins, respectively. The reasoning behind the latter effect is as follows: Productivity decrease indeeds a reduction in total production, hence in total real income and consumption. From the demand side, a decrease in real income yields both a decrease in the number of varieties demanded in the consumption basket and less demand for each variety. Hence, a demand of smaller basket (i.e. less varieties) yields a decrease in number of operating firms since some firms exit the market due to lack of demand. Moreover, less demand for each variety caueses a lower production of each firm. As a result, these two effects together yields a lower welfare due to a decrease in productivity.

In a traditional Dixit-Stiglitz monpolostic competition framework with C.E.S. (constant elasticity of substitution) utility, sunk entry cost and common productivity (i.e. 1 over marginal cost) across firms fail to capture the extensive margin (i.e. number of operating firms in an industry does not affected form the productivity level).[1] Hence, this result yields the following conclusion for these type of models: A decrease in productivity level affects the welfare only with one channel: a decrease in output per firm. To address this problem, this paper introduces "continuum-quadratic" utility (i.e. linear demand system) while keeping the other assumptions unchanged and finds that lowering productivity affects not only the intensive but extensive margin as well. Therefore, a decrease in productivity affects welfare through two channels : a decrease in output per firm and a decrease in number of operating firms.

The present paper modifies the standard Dixit-Stiglitz framework by changing the demand structure. Among others Neary (2003) and Neary (2009) use a similar utility in an oligopolistic competition environment. Among others, Ottaviano, Tabuchi, Thisse (2002) also uses a similar utility with an an outside sector and an additional term which captures the competition effect in a monopolistically competitive market.

[1] See Dixit and Stiglitz (1977) for details. See also Neary (2004) for a detailed discussion of Dixit-Stiglitz monpolostic competition framework. Among others, one of the most well-known application of Dixit-Stiglitz framework is the seminal paper by Krugman (1980).

Paper organizes as follows: Section 2 presents the model. Section 3 discusses costs of low productivity and section 4 concludes.

2. Model

This section presents a general equilibrium model with a monopolistic competition under continuum-quadratic utility assumption.

2.1. Preferences and Demand

There is a measure of L identical consumers, where each supplies one unit of labor inelastically. Preferences are defined over a continuum of differentiated variety indexed by $z \in \Omega$. All consumers share the same non-homothetic utility function and a representative consumer maximizes,

$$\max_{q(z)} \alpha \int_{z\in\Omega} q(z) - 1/2\ \gamma \int_{z\in\Omega} (q(z))^2\, dz \qquad (1)$$

s.t.

$$\int_{z\in\Omega} p(z)q(z)dz = w, \qquad (2)$$

where $q(z)$ and $p(z)$ denote the quantity and price of a variety z, respectively. $z \in \Omega^* \subset \Omega$, where Ω and Ω^* represents the set of all potential varieties and the subset of demanded varieties (which is going to be determined endogenously), respectively. w denotes the nominal wage income of a consumer. $\alpha, \gamma > 0$ are the demand parameters where γ indexes the degree of product differentiation.[2] Hence, the demand function of a representative consumer is given by

$$q(z) = \begin{cases} \dfrac{\alpha - \lambda p(z)}{\gamma}, & p(z) < p^{max} \\ 0, & p(z) \geq p^{max}, \end{cases} \qquad (3)$$

where λ is the Lagrange multiplier of the budget constraint and p^{max} denotes the maximum price level which is affordable by a representative consumer. Hence $p^{max} \equiv \alpha/\lambda$ is the threshold price which makes the demand zero.[3] Clearly, market demand is given by $q^m(z) = L\, q(z)$.

2.2. Supply

Each variety z is produced by a different firm and therefore each firm is a monopolist for that variety. All firms share a common productivity $\boldsymbol{\phi}$. Firms pay a sunk entry cost f_e before entering the market. After paying the sunk entry cost and entering the market, firms are producing according to constant returns to scale production function: $q = \phi l$, where q denotes the output of a firm and l denotes the amount of labor employed by a firm where labor is supplied in competitive market. Since the

[2] $\gamma = 0$ implies perfect substitution between varieties.

[3] Since the marginal utilities are bounded for all varieties, demand may be equal to zero even with a finite price.

productivity level of all firms' are identical, firms' production and pricing strategies are symmetric.[4] Hence, given the demand function, a representative firm with productivity level ϕ solves

$$\max_{p} \quad \pi = pqL - qL\frac{w}{\phi} \tag{4}$$

s.t.

$$q = \frac{\alpha - \lambda p}{\gamma}, \tag{5}$$

where π denotes the profit of a represantative firm**.** F.O.C. with respect to price, p yields

$$p = \frac{1}{2}\left(\frac{\alpha}{\lambda} + \frac{w}{\phi}\right). \tag{6}$$

After substituting λ in to p, we get[5]

$$p = \frac{w}{\phi}\left(\frac{\alpha - \gamma q}{\alpha - 2\gamma q}\right), \tag{7}$$

where $\alpha - 2\gamma q$ is positive under $f_e, \phi, \alpha, \gamma, L \epsilon (0, \infty)$ assumption and this yields a positive values for price, p.[6] Hence, given the optimal pricing rule, we can rewrite the profit, $\pi = pqL - qL\frac{w}{\phi}$, as a function of output q

$$\pi = \frac{w\gamma L}{\phi}\frac{q^2}{\alpha - 2\gamma q}. \tag{8}$$

Since $\alpha - 2\gamma q$ is positive, profits are also positive for operating firms.

2.3. Equilibrium Conditions

In the equilibrium

- Free Entry (i.e. zero profit)
- Labor Market Clearing
- Spending = Income

conditions have to be satisfied.

2.3. 1. Free Entry Condition

Free entry condition implies that each firm's profit should be equal to the entry cost:

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[4] Using this symmetry, we can ignore the z notation in the firms' problem.

[5] Λ can be obtained from equation (5).

[6] See Appendix A for details.

$$\frac{w\,\gamma L}{\phi}\,\frac{q^2}{\alpha-2\gamma q}=f_e w. \tag{9}$$

Hence this condition implies the following quadratic equation for q,

$$q^2+\frac{2f_e\phi}{L}\,q-\frac{f_e\phi\alpha}{\gamma L}=0. \tag{10}$$

Lemma 1. Given $f_e,\phi,\alpha,\gamma,L\,\epsilon\,(0,\infty)$, equation (10) has *a uniqe positive real root* which is

$$q=\sqrt{\frac{a^2\gamma+a\alpha}{\gamma}}-a, \tag{11}$$

where $a=\frac{f_e\phi}{L}$.

Proof. See Appendix A.

Moreover, price p can be obtained by substituting q into equation (7).

$$p=\frac{w}{\phi}\left(\frac{\alpha-\gamma\left(\sqrt{\frac{a^2\gamma+a\alpha}{\gamma}}-a\right)}{\alpha-2\gamma\left(\sqrt{\frac{a^2\gamma+a\alpha}{\gamma}}-a\right)}\right). \tag{12}$$

2.3.2. Labor Market Clearing Condition

Labor Market Clearing condition implies that sum of employed labor should be equal to the amount of labor supplied by consumers:

$$n\left(\frac{qL}{\phi}+f_e\right)=L, \tag{13}$$

where n is the measure of operating firms. Given q, n can be obtained from labor market clearing condition:

$$n=\frac{\phi}{\sqrt{\frac{a^2\gamma+a\alpha}{\gamma}}}. \tag{14}$$

2.3.3. Spending=Income Condition

Finally, given the equilibrium results for n, p, q, it can be shown that the last equilibrium condition (spending=income) is satisfied automatically by Walras law: $npqL = Lw$.[7]

2.4. Welfare Analysis

Among others, real per capita output, nq, can be used as a measure of welfare, W. Due to the spending=income condition, nq is also equivalant to the real per capita income (i.e. real wages). In order to decompose the total welfare into 2 parts (intensive and extensive margins) "nq" is used as a measure of welfare in the rest of the paper:

$$W = \underbrace{n}_{extensive\ margin} \times \underbrace{q}_{intensive\ margin}. \tag{15}$$

The extensive margin captures the measure of varieties which are consumed (i.e. measure of operating firms), and the intensive margin captures the quantity of consumption of each variety. Hence, welfare is given by

$$W = \phi - \frac{a\phi}{\sqrt{\frac{a^2\gamma + a\alpha}{\gamma}}}. \tag{16}$$

In the equilibrium equation (16) is positive since n and q are positive.[8]

3. Costs of Low Productivity: Intensive and Extensive Margins

A decrease in common productivity clearly reduces the output per firm. However, the negative effect of a decrease in productivity on total economy is not bounded with the intensive margin loss, since the number of operating firms in the market is decreasing as well (extensive margin). In the model, besides the intensive margin effect, some firms are shutting down due to lack of demand which indeeds a decrease in number of operating firms (varieties). Moreover, labor is reallocating from the firms which are already shut down to the ones who are still operating. Altough the operating firms have more workers, they produce lower output per firm compared to the inital case due to lower productivity. Formally,

Proposition 1. Measure of operating firms and output per firm are both strictly monotone function of productivity.

Proof. See Appendix B for details.

However, as a well-known result, with the standard C.E.S. utility (keeping everything else same) a decrease in productivity decreases the total output *solely* with one channel, a decrease in output per firm. In other words, a decrease in productivity does not affect the number of operating firms.[9]

4. Conclusion

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[7]Since labor market is perfectly competitive, nominal wages w can be normalized to 1.

[8] See Appendix B for details.
[9] See Appendix C for details.

This paper discusses welfare costs of a decrease in productivity and argues that there are two important channels which cause a reduction in welfare: a decrease in output per firm (*intensive margin*) and a decrease in number of operating firms (*extensive margin*). Intensive margin is a straightforward result of a decrease in productivity as stated in standard Dixit-Stiglitz monopolistic competition framework with CES utility. However, these models fail to capture the latter effect which is less obvious than the intensive margin effect. The modified model (linear demand system) captures both intensive and extensive margins of welfare costs due to a decrease in productivity.

In a nutshell, this paper highlights the importance of a second channel and argues that this channel should taken into account while calculating the loss of welfare due to a decrease in productivity.

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Appendix A

In order to show that $\alpha - 2\gamma q > 0$, let's first prove the Lemma 1. Given $f_e, \phi, \alpha, \gamma, L \,\epsilon\, (0, \infty)$, $q^2 + \frac{2f_e\phi}{L}\, q - \frac{f_e\phi\alpha}{\gamma L} = 0$ implies that

- one of the roots is negative since $-\frac{2f_e\phi}{L} < 0$ and the other root is positive since $4a\alpha > 0$ (i.e. the latter guarantees $\left(\left(\frac{2f_e\phi}{L}\right)^2 + \frac{4f_e\phi\alpha}{\gamma L}\right)^{0.5} - \frac{2f_e\phi}{L} > 0$).
- Moreover, it is enough to show that $\alpha - 2\gamma\left(\sqrt{\frac{a^2\gamma + a\alpha}{\gamma}} - a\right) > 0$ (using the positive root) for proving $\alpha - 2\gamma q > 0$. Simple algebra implies $(\alpha + 2T)^2 > 4T^2 + 4\alpha T$, where $T = \frac{f_e\phi\gamma}{L}$. This condition is always satisfied since $\alpha^2 > 0$. $\alpha - 2\gamma q > 0$ implies that $\alpha - \gamma q > 0$, which implies $p > 0$.

Appendix B

- In order to show that $W > 0$, it is enough to show that $\sqrt{\frac{a^2\gamma + a\alpha}{\gamma}} > a$ and this condition is satisfied whenever $a\alpha > 0$.
- $\frac{\partial n}{\partial \phi} = \frac{1}{2}\frac{k_2}{(k_1\phi + k_2)(k_1\phi(k_1\phi + k_2))^{0.5}} > 0$
- $\frac{\partial q}{\partial \phi} = \frac{1}{2}\frac{k_1\left(-2(k_1\phi(k_1\phi + k_2))^{0.5} + 2k_1\phi + k_2\right)}{(k_1\phi(k_1\phi + k_2))^{0.5}} > 0$, where $k_1 = \frac{f_e}{L} > 0$ and $k_2 = \frac{\alpha}{\gamma} > 0$. $k_2^2 > 0$ guarentees that $-2\left(k_1\phi(k_1\phi + k_2)\right)^{0.5} + 2k_1\phi + k_2 > 0$.

Appendix C

A represantative consumer maximes her utility under C.E.S. assumption: $\max_{q(z)}(\int_{z\epsilon\Omega}(q(z)^{\frac{\sigma-1}{\sigma}}dz)^{\frac{\sigma}{\sigma-1}}$ s.t $\int_{z\epsilon\Omega} p(z)q(z)dz = w$, where $\sigma > 1$ represents the elasticity of substitution between varities. Hence, the demand function is $q(z) = \left(\frac{p(z)}{P}\right)^{-\sigma}\frac{w}{P}$, where $P = \left(\int_{z\,\epsilon\Omega} p(z)^{1-\sigma}\,dz\right)^{\frac{1}{1-\sigma}}$. Given the demand function, a represantative firm maximizes the profits: $\max_p \pi = pqL - qLw/\phi$ s.t. $q = \left(\frac{p}{P}\right)^{-\sigma}\frac{w}{P}$.[10] F.O.C. with respect to p yields $p = \frac{\sigma}{\sigma-1}\frac{w}{\phi}$. Hence profit of a firm is $\pi = w\left(\frac{q\,L}{(\sigma-1)\phi}\right)$. Moreover, free entry condition implies that $q = \frac{f_e\phi(\sigma-1)}{L}$. *Hence, this result shows that* $\frac{\partial q}{\partial \phi} > 0$. Using the Labor Market Clearing Condition measure of operating firm is obtained: $n = \frac{L}{f_e\sigma}$. Hence, $\frac{\partial n}{\partial \phi} = 0$.

[10] Due to the symmetry of firms z notation is ignored.