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Imperfect Evaluation in Project Screening

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Abstract

This paper studies a model in which an agent considers proposing a project of unknown quality to an evaluator, who decides whether or not to accept it. First, we show that there exist instances where an agent with a better track record of producing high-quality projects should be subjected to more stringent standards. Second, we show that an increase in the submission fee may lead to a decrease in the quality of projects that are implemented because of its effects on the evaluator's acceptance policy.

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1 Introduction

We study a game of two-sided incomplete information in which an agent considers proposing a project to an evaluator, who has the choice of whether or not to accept it. Each player learns a private informative signal about the quality of the project, while the available public information translates into a common prior. The agent's payoff upon submission is determined by the evaluator's decision. The evaluator's payoff is determined by the quality of the projects that he accepts. Making the proposal incurs a monetary or non-monetary submission fee on the agent.

The situations that are captured by the above setting are abundant and diverse. A possible example is that of a firm interested in undertaking a project, such as the development of an economic activity in an environmentally sensitive area, that requires filing a costly application with a regulatory agency. In this case, the firm is concerned with the agency's decision of whether to approve the proposal, while the regulatory agency is usually concerned with the social welfare implications of that activity.¹

A first finding in this setup relates a change in the common prior with the equilibrium strategy of the evaluator. We identify the condition under which the evaluator becomes less stringent in his acceptance policy when the prior about the agent is higher. This condition is not always satisfied and therefore, an agent with a higher prior, such as one with a better reputation of producing high-quality projects, may need to face *more* stringent requirements.

We then investigate the efficiency effects of a change in the submission fee. Leslie (2005) considers a model of one-sided incomplete information, in which the evaluator can perfectly assess the quality of a submitted project, and shows that submission fees and time delays at academic journals increase the quality of papers submitted for review by discouraging long-shot submissions. In our model, a higher submission fee also increases the quality of projects that the agent submits, but this induces the evaluator to lower his standards of acceptance and therefore to accept projects with less favorable private signals. We identify the condition under which, on net, a higher submission fee increases the expected quality of projects that are *implemented*. Since this condition is not always satisfied, it is possible for a higher submission fee to *lower* the expected quality of these projects. Thus, in a model with two-sided incomplete information in which the evaluator's response to an increase in the quality of projects submitted by the agent, higher submission fees can decrease the quality of projects that are implemented.²

The closest paper to ours is Taylor and Yildirim (2011), which studies a model in which an agent chooses the amount of effort to exert in generating a project, and investigates the moral hazard

¹Other examples are the introduction of a new product with potential undesirable effects, a merger, a patent application, or the submission of an article to an academic journal.

 $^{^{2}}$ Cotton (2012) shows that moderate submission fees and delays are optimal when authors are heterogenous with respect to willingness to pay submission fees and deal with delays.

effects of the potentially available public information. Ottaviani and Wickelgren (2009) analyze the trade-off between ex-ante and ex-post approval of an activity when the evaluator may reconsider his approval at the time when the quality of the project is revealed. Boleslavsky and Cotton (2011) study a model in which an evaluator has to select one of several competing proposals, and investigate the effect of the limited capacity on the incentives of the proposers to produce information.

2 The Model

There are two players, an agent (\mathcal{A}) and an evaluator (\mathcal{E}) . \mathcal{A} owns a project and considers proposing it to \mathcal{E} . The project is of either high (h) or low (l) quality. The common prior probability of h is π_0 . An accepted project yields \mathcal{A} a payoff 1, irrespective of its ex-post observed quality. Submitting the project incurs a fee on \mathcal{A} whose monetary equivalent is $c \in (0, 1)$. \mathcal{A} 's payoff from not submitting the project is 0. Upon receiving a project from \mathcal{A} , \mathcal{E} has the choice of whether to accept it or not. \mathcal{E} 's payoff from accepting a high-quality project is 1, while his loss from accepting a low-quality project is $L \in (0, 1)$. \mathcal{E} 's payoff from rejecting a project is normalized to $0.^3$

Prior to taking their decisions, \mathcal{A} and \mathcal{E} perform assessments of the project that result in a subjective evaluation of its quality.⁴ \mathcal{A} 's assessment yields a private signal $\mu \in [0, 1]$; \mathcal{E} 's assessment yields a private signal $\sigma \in [0, 1]$. For quality $q \in \{h, l\}$, $G^q(\mu)$ and $F^q(\sigma)$ denote the cumulative distribution functions of \mathcal{A} 's and \mathcal{E} 's signals, respectively, and $g^q(\mu) > 0$ and $f^q(\sigma) > 0$ are the corresponding density functions. We make the following assumption.

Assumption 1 (i) g^q and f^q are bounded and twice continuously differentiable for $q \in \{h, l\}$; (ii) $\frac{d}{d\sigma} \left[\frac{f^h(\sigma)}{f^l(\sigma)} \right] > 0, \ \frac{d}{d\mu} \left[\frac{g^h(\mu)}{g^l(\mu)} \right] > 0; \ (iii) \ \lim_{\sigma \to 0} \frac{g^h(\mu)}{g^l(\mu)} = 0 \ and \ \lim_{\sigma \to 1} \frac{g^h(\mu)}{g^l(\mu)} = \infty$

Part (*ii*) of the assumption is the usual monotone likelihood ratio. Part (*iii*) imposes that for extreme signals of \mathcal{E} , information becomes almost perfect.

3 The Equilibrium

Consider some arbitrary strategies of \mathcal{A} and \mathcal{E} , respectively, $\Lambda^{ag} : [0,1] \to \{s,n\}$ and $\Lambda^{ev} : [0,1] \to \{a,r\}$, with the obvious interpretation of the action labels. \mathcal{A} submits a project of quality signal μ if and only if

$$\Pr(\{a\}|\mu) \cdot 1 - c \ge 0 \iff \Pr(\{a\}|\mu) \ge c \tag{1}$$

 $^{^{3}}$ The analysis does not change in a meaningful way if the agent's payoff also depends on the quality of the project or if the evaluator is also concerned with the quality of projects that he rejects. Since the submission fee may often take a non-monetary form, we do not include it in the evaluator's payoff.

⁴In the case of an application for economic development in a environmentally sensitive area, both the firm and the regulatory agency can acquire private information through expert analysis about the likelihood that the activity is welfare improving, while, for instance, the firm's environmental record is public information.

where the event $\{a\} \equiv \{\sigma \in [0,1] : \Lambda^{ev}(\sigma) = a\}$. \mathcal{E} accepts a submitted project if and only if

$$\Pr(h|\{s\},\sigma) \cdot 1 + [1 - \Pr(h|\{s\},\sigma)](-L) \ge 0 \iff \Pr(h|\{s\},\sigma) \ge \frac{L}{1+L}$$

$$\tag{2}$$

where $\{s\} \equiv \{\mu \in [0,1] : \Lambda^{ag}(\mu) = s\}.$

The next lemma, whose proof is in the online appendix A1, states that in any Bayesian Nash equilibrium, the two players adopt cutoff strategies with respect to their informative signals.

Lemma 3.1 Any equilibrium of the game is characterized by two values $(\mu^*, \sigma^*) \in [0, 1] \times [0, 1]$, such that \mathcal{A} submits a project if and only if $\mu \geq \mu^*$ and \mathcal{E} accepts it if and only if $\sigma \geq \sigma^*$.

Therefore, a player's equilibrium strategy can be defined in terms of the corresponding cutoff. For the rest of the paper, σ_s and μ_s will denote generic cutoff *strategies* of the two players, $\overline{\sigma}(\mu_s)$ and $\overline{\mu}(\sigma_s)$ will denote best responses, while σ^* and μ^* will refer to equilibrium strategies.

The next lemma describes the two players' best responses. Its proof is in appendix A2.

Lemma 3.2 (i) There exist two thresholds $\sigma_1, \sigma_2 \in (0, 1)$, with $\sigma_1 < \sigma_2$, such that \mathcal{A} submits a project if and only if $\mu \geq \overline{\mu}(\sigma_s) \in [0, 1]$, where: (1) $\overline{\mu}(\sigma_s) = 0$, for $\sigma_s \in [0, \sigma_1]$; (2) $\overline{\mu}(\sigma_s)$ is defined implicitly by

$$\frac{\pi_0}{1-\pi_0} \frac{g^h(\overline{\mu})}{g^l(\overline{\mu})} = \frac{c - \left[1 - F^l(\sigma_s)\right]}{\left[1 - F^h(\sigma_s)\right] - c} \tag{3}$$

when $\sigma_s \in (\sigma_1, \sigma_2)$; and (3) $\overline{\mu}(\sigma_s) = 1$, for $\sigma_s \in [\sigma_2, 1]$.

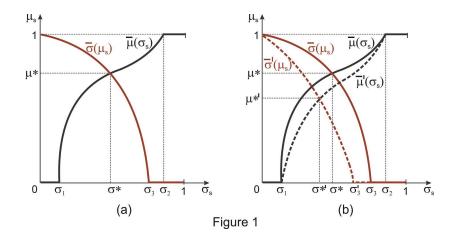
(ii) \mathcal{E} accepts a project if and only if $\sigma \geq \overline{\sigma}(\mu_s)$, where $\overline{\sigma}(\mu_s)$ is defined implicitly by

$$\frac{\pi_0}{1-\pi_0} \frac{f^h(\overline{\sigma})}{f^l(\overline{\sigma})} \frac{1-G^h(\mu_s)}{1-G^l(\mu_s)} = L \tag{4}$$

In (4), $\frac{1-G^{h}(\mu_{s})}{1-G^{l}(\mu_{s})}$ is the likelihood of h as inferred by \mathcal{E} from the fact that \mathcal{A} submitted the proposal. Therefore, \mathcal{E} accepts a proposal if and only if the likelihood of h, as inferred by \mathcal{E} from the prior $\left(\frac{\pi_{0}}{1-\pi_{0}}\right)$, his informative signal $\left(\frac{f^{h}(\sigma)}{f^{l}(\sigma)}\right)$, and the fact that \mathcal{A} submitted the proposal $\left(\frac{1-G^{h}(\mu_{s})}{1-G^{l}(\mu_{s})}\right)$, exceeds L. On the other hand, (3) states that \mathcal{A} submits a proposal if and only if the likelihood of h, as inferred by \mathcal{A} from the prior and from his informative signal, exceeds the corresponding ratio between the expected loss when the project is of low-quality, $\left(c - \left[1 - F^{l}(\sigma_{s})\right]\right)$, and the expected benefit when it is of high quality, $\left(\left[1 - F^{h}(\sigma_{s})\right] - c\right)$.⁵

The two best-response functions and the equilibrium are depicted in panel (a) of Figure 1.

 $^{{}^{5}}c - [1 - F^{l}(\sigma_{s})]$ is the expected loss from submitting a project of low quality as it is the difference between the submission cost c and the expected benefit $[1 - F^{l}(\sigma_{s})] \cdot 1$.



The next proposition states the existence and uniqueness of the Bayesian Nash equilibrium; its proof is in appendix A3.

Proposition 1 \mathcal{A} 's best response $\overline{\mu}(\sigma_s)$ is increasing. \mathcal{E} 's best response $\overline{\sigma}(\mu_s)$ is decreasing. There exists a unique equilibrium of the game, (μ^*, σ^*) , with $\mu^* \in [0, 1)$ and $\sigma^* \in (0, 1)$.

4 Results

The first comparative statics of interest are with respect to the two players' relative costs of their respective actions, L and c.

Proposition 2 (i) $\frac{d\mu^*}{dL} > 0$, $\frac{d\sigma^*}{dL} > 0$; (ii) $\frac{d\mu^*}{dc} > 0$, $\frac{d\sigma^*}{dc} < 0$.

When L increases, the right hand side of (4) increases, and thus the curve $\overline{\sigma}(\mu_s)$ shifts to the right. Since $\overline{\mu}(\sigma_s)$ remains unchanged, this leads to an increase in both μ^* and σ^* . Intuitively, if L increases, the incentive for \mathcal{E} to accept a project decreases, and this induces an increase in σ^* . Anticipating a more stringent acceptance policy by \mathcal{E} , \mathcal{A} exerts more project screening and increases μ^* . Second, when c increases, the curve $\overline{\sigma}(\mu_s)$ stays fixed. Taking the derivative of the right hand side of (3) with respect to c, and using the fact that $F^l(\sigma_s) > F^h(\sigma_s)$, ⁶ it follows that the curve $\overline{\mu}(\sigma_s)$ moves up. This leads to a decrease in σ^* and an increase in μ^* . Thus, when c increases, \mathcal{A} exerts more project screening and increases μ^* . Knowing this, \mathcal{E} increases his belief about the quality of projects that he receives, and lowers σ^* .

Next, we examine the effect of a change in π_0 on the equilibrium strategies. Note that when π_0 increases, the values of $\overline{\mu}$ and $\overline{\sigma}$ that satisfy equations (3) and (4), respectively, decrease. Thus, as depicted in panel (b) of Figure 1, both best-response curves shift down. μ^* therefore unequivocally

⁶The monotone likelihood property in assumption 1(i) implies stochastic dominance.

decreases: an agent who is exante more likely to produce a high-quality project is more confident in submitting marginal projects. The change in σ^* is ambiguous, as the direct effect on beliefs of the higher π_0 can be offset by the decrease in posterior beliefs generated by \mathcal{A} 's strategy. σ^* will decrease if \mathcal{E} 's strategy is more elastic with respect π_0 than with respect to \mathcal{A} 's strategy.

Proposition 3 We have: (i) $\frac{d\mu^*}{d\pi_0} < 0$; (ii) $\frac{d\sigma^*}{d\pi_0} < 0$ if and only if $\frac{d}{d\mu} \ln \frac{g^h(\mu^*)}{a^l(\mu^*)} > \frac{d}{d\mu} \ln \frac{1-G^h(\mu^*)}{1-G^l(\mu^*)}$.

Proof. Part (i) follows from the preceding argument. For (ii), writing (3) and (4) in equilibrium, dividing them, and taking logarithms, we obtain

$$\ln\left(\frac{g^{h}(\mu^{*})}{g^{l}(\mu^{*})}\right) - \ln\left(\frac{1 - G^{h}(\mu^{*})}{1 - G^{l}(\mu^{*})}\right) = \ln\left(\frac{1}{L}\frac{f^{h}(\sigma^{*})}{f^{l}(\sigma^{*})}\right) + \ln\left(\frac{c - \left[1 - F^{l}(\sigma^{*})\right]}{\left[1 - F^{h}(\sigma^{*})\right] - c}\right)$$
(5)

The term in the right hand side of (5) is increasing in σ^* .⁷ Since $\frac{d\mu^*}{d\pi_0} < 0$, it follows that $\frac{d\sigma^*}{d\pi_0} < 0$ if and only if the term in the left hand side of (5) is increasing in μ^* .

Thus, \mathcal{E} becomes more lenient if the elasticity of the likelihood of h as inferred from \mathcal{A} 's threshold signal μ^* is higher than the elasticity of the likelihood of h as inferred by \mathcal{E} from the fact that \mathcal{A} submitted the project. It can be shown by counterexample that the condition in proposition 3(ii)is not always satisfied. This suggests that there exist instances where an agent with a better track record of producing high-quality projects should be subjected to more stringent standards.

Next, we investigate the effect of an increase in c on the expected quality of projects that are implemented, which as is isomorphic with $\Pr(h|\mu \ge \mu^*, \sigma \ge \sigma^*)$.⁸

Proposition 4 $\frac{d}{dc} \Pr\left(h|\mu \ge \mu^*, \sigma \ge \sigma^*\right) > 0$ if and only if $\frac{d}{d\sigma} \ln \frac{f^h(\sigma^*)}{f^l(\sigma^*)} > \frac{d}{d\sigma} \ln \frac{1-F^h(\sigma^*)}{1-F^l(\sigma^*)}$.

Proof. By Bayes' Rule, we have

$$\Pr(h|\mu \ge \mu^*, \sigma \ge \sigma^*) = \frac{\Pr(\mu \ge \mu^*, \sigma \ge \sigma^*|h) \Pr(h)}{\Pr(\mu \ge \mu^*, \sigma \ge \sigma^*|h) \Pr(h) + \Pr(\mu \ge \mu^*, \sigma \ge \sigma^*|l) \Pr(l)}$$
$$= \frac{\pi_0}{\pi_0 + (1 - \pi_0) \frac{\Pr(\mu \ge \mu^*, \sigma \ge \sigma^*|l)}{\Pr(\mu \ge \mu^*, \sigma \ge \sigma^*|h)}}$$
$$= \frac{\pi_0}{\pi_0 + \frac{1 - \pi_0}{\frac{1 - G^h(\mu^*)}{1 - G^l(\mu^*)} \frac{1 - F^h(\sigma^*)}{1 - F^l(\sigma^*)}}}$$

where we used the conditional independence of the two players' signals. Therefore, $\Pr(h|\mu \ge \mu^*, \sigma \ge \sigma^*)$ increases if and only if $\ln \frac{1-G^h(\mu^*)}{1-G^l(\mu^*)} + \ln \frac{1-F^h(\sigma^*)}{1-F^l(\sigma^*)}$ increases. From (4), we have $\ln \frac{1-G^h(\mu^*)}{1-G^l(\mu^*)} =$

⁷The formal proof of the fact that $\frac{c-[1-F^l(\sigma^*)]}{[1-F^h(\sigma^*)]-c}$ is increasing in σ^* , for $\sigma^* \in (\sigma_1, \sigma_2)$, is presented in appendix A2.

⁸ The expected quality of the projects that are implemented is $l + (h - l) \Pr(h|\mu \ge \mu^*, \sigma \ge \sigma^*)$.

 $\ln L - \ln \frac{\pi_0}{1-\pi_0} - \ln \frac{f^h(\sigma^*)}{f^l(\sigma^*)}$, so $\Pr(h|\mu \ge \mu^*, \sigma \ge \sigma^*)$ increases if and only if $\ln \frac{1-F^h(\sigma^*)}{1-F^l(\sigma^*)} - \ln \frac{f^h(\sigma^*)}{f^l(\sigma^*)}$ increases. Since σ^* is decreasing in c, the proof of the proposition is complete.

The condition in proposition 4 is not always satisfied, and thus, while a higher submission fee always increases the expected quality of projects that are received for review (as elicited by the increase in μ^*), it may lead to an inferior equilibrium outcome by lowering the evaluator's standards of acceptance to an extent that more than offsets the increase in the quality of projects that are submitted.⁹ On the other hand, when the condition in proposition 4 is satisfied, the efficiency of the outcome can be improved by increasing c and shifting the project-screening onto the agent.

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⁹Barbos (2012) shows that this concern is even stronger in the presence of multiple heterogenous evaluators.

Appendix (NOT FOR PUBLICATION)

Appendix A1. Proof of Lemma 3.1

Consider some arbitrary strategy Λ^{ag} of \mathcal{A} . Then, for \mathcal{E} 's beliefs, by Bayes' Rule we have

$$\Pr(h|\{s\},\sigma) = \frac{j(\{s\},\sigma|h)\Pr(h)}{j(\{s\},\sigma|h)\Pr(h) + j(\{s\},\sigma|l)\Pr(l)}$$
(6)

where $j(\cdot|\cdot)$ denotes the conditional probability density function of the relevant continuous random variable. Since \mathcal{A} 's action and the signal σ are conditionally independent, it follows that

$$\Pr(h|\{s\},\sigma) = \frac{\Pr(\{s\}|h)f^{h}(\sigma)\pi_{0}}{\Pr(\{s\}|h)f^{h}(\sigma)\pi_{0} + \Pr(\{s\}|l)f^{l}(\sigma)(1-\pi_{0})}$$

$$= \frac{\Pr(\{s\}|h)\frac{\pi_{0}}{1-\pi_{0}}\frac{f^{h}(\sigma)}{f^{l}(\sigma)}}{\Pr(\{s\}|h)\frac{\pi_{0}}{1-\pi_{0}}\frac{f^{h}(\sigma)}{f^{l}(\sigma)} + \Pr(\{s\}|l)}$$
(7)

Since, the last term in (7) is increasing in $\frac{f^h(\sigma)}{f^l(\sigma)}$, the fact that $\frac{d}{d\sigma} \left[\frac{f^h(\sigma)}{f^l(\sigma)} \right] > 0$, as imposed by assumption 1, implies $\frac{d}{d\sigma} \Pr(h|\{s\}, \sigma) > 0$. Thus, given (2), for any Λ^{ag} , \mathcal{E} responds with a cutoff strategy by accepting a submitted project if and only if $\sigma \geq \overline{\sigma}(\Lambda^{ag})$, with $\overline{\sigma}(\Lambda^{ag}) \in [0, 1]$.

On the other hand, given some arbitrary strategy Λ^{ev} of \mathcal{E} , for \mathcal{A} 's belief we have

$$\Pr(\{a\}|\mu) = \Pr(\{a\}|\mu, h) \Pr(h|\mu) + \Pr(\{a\}|\mu, l) \Pr(l|\mu)$$

=
$$[\Pr(\{a\}|h) - \Pr(\{a\}|l)] \Pr(h|\mu) + \Pr(\{a\}|l)$$
(8)

where the second equality follows from the fact that μ is redundant for \mathcal{A} 's inference about \mathcal{E} 's action when conditioning on the quality of the project. Since in any equilibrium, \mathcal{E} uses a cutoff strategy, we have $\Pr(\{a\}|h) - \Pr(\{a\}|l) = \Pr(\sigma \geq \overline{\sigma}|h) - \Pr(\sigma \geq \overline{\sigma}|l) = F^l(\overline{\sigma}) - F^h(\overline{\sigma})$. The monotone likelihood ratio property implies first order stochastic dominance, and thus $F^l(\overline{\sigma}) - F^h(\overline{\sigma}) > 0$. On the other hand, by Bayes' Rule we have

$$\Pr(h|\mu) = \frac{g^h(\mu)\pi_0}{g^h(\mu)\pi_0 + g^l(\mu)(1-\pi_0)} = \frac{\frac{g^h(\mu)}{g^l(\mu)}\frac{\pi_0}{1-\pi_0}}{\frac{g^h(\mu)}{g^l(\mu)}\frac{\pi_0}{1-\pi_0} + 1}$$
(9)

which is increasing in $\frac{g^{h}(\mu)}{g^{l}(\mu)}$, and thus increasing in μ since $\frac{d}{d\mu} \left[\frac{g^{h}(\mu)}{g^{l}(\mu)} \right] > 0$ by assumption 1. Therefore, when \mathcal{E} employes a cutoff strategy, $\frac{d}{d\mu} \Pr(\{a\}|\mu) > 0$, and thus \mathcal{A} 's best response is a cutoff strategy.

Appendix A2. Proof of Lemma 3.2

(i) Given an arbitrary cutoff strategy σ_s of \mathcal{E} , we have $\Pr(\{a\}|q) = \Pr(\sigma \geq \sigma_s|q) = 1 - F^q(\sigma_s)$, for $q \in \{h, l\}$. Employing lemma 3.1 and (9) in $\Pr(\{a\}|\mu) = \Pr(\{a\}|h) \Pr(h|\mu) + \Pr(\{a\}|l) \Pr(l|\mu)$, it follows that

$$\Pr(\{a\}|\mu) = \frac{\frac{\pi_0}{1-\pi_0} \frac{g^h(\mu)}{g^l(\mu)}}{\frac{\pi_0}{1-\pi_0} \frac{g^h(\mu)}{g^l(\mu)} + 1} \left[1 - F^h(\sigma_s)\right] + \frac{1}{\frac{\pi_0}{1-\pi_0} \frac{g^h(\mu)}{g^l(\mu)} + 1} \left[1 - F^l(\sigma_s)\right]$$
(10)

From (1), we have then that given σ_s , \mathcal{A} submits a project if and only if

$$\frac{\frac{\pi_{0}}{1-\pi_{0}}\frac{g^{h}(\mu)}{g^{l}(\mu)}}{\frac{\pi_{0}}{1-\pi_{0}}\frac{g^{h}(\mu)}{g^{l}(\mu)}+1}\left[1-F^{h}(\sigma_{s})\right] + \frac{1}{\frac{\pi_{0}}{1-\pi_{0}}\frac{g^{h}(\mu)}{g^{l}(\mu)}+1}\left[1-F^{l}(\sigma_{s})\right] \geq c \iff \frac{\pi_{0}}{1-\pi_{0}}\frac{g^{h}(\mu)}{g^{l}(\mu)}\left\{\left[1-F^{h}(\sigma_{s})\right]-c\right\} \geq c-\left[1-F^{l}(\sigma_{s})\right] \iff \frac{\pi_{0}}{1-\pi_{0}}\frac{g^{h}(\mu)}{g^{l}(\mu)}\left\{\left[1-F^{h}(\sigma_{s})\right]-c\right\} \geq \frac{c-\left[1-F^{l}(\sigma_{s})\right]}{\left[1-F^{h}(\sigma_{s})\right]-c} \qquad (11)$$

Note now that if $\sigma_s < \sigma'_1 \equiv (F^l)^{-1} (1-c) \in (0,1)$, then (11) is satisfied for any μ . To see this, note that $\sigma_s < \sigma'_1$ implies that $1-c-F^l(\sigma_s) > 0$, which together with $F^h(\sigma_s) < F^l(\sigma_s)$ implies $1-c-F^h(\sigma_s) > 0$, and thus $[1-F^h(\sigma_s)] - c > 0 > c - [1-F^l(\sigma_s)]$. Second, if $\sigma_s > \sigma'_2 \equiv (F^h)^{-1} (1-c) \in (0,1)$, then (11) is satisfied for no μ . To see this, note that $\sigma_s > \sigma'_2$ implies $F^h(\sigma_s) - 1 + c > 0$, which then implies $F^l(\sigma_s) - 1 + c > 0$, and thus, $[1-F^h(\sigma_s)] - c < 0 < c - [1-F^l(\sigma_s)]$. Finally, note that $\sigma'_1 < \sigma'_2$. By taking the derivative of $\frac{c-[1-F^l(\sigma_s)]}{[1-F^h(\sigma_s)]-c}$ with respect to σ_s , since $[1-F^h(\sigma_s)] - c > 0$ and $c - [1-F^l(\sigma_s)] > 0$ when $\sigma_s \in [\sigma'_1, \sigma'_2]$, it follows immediately that this term is increasing in σ_s on $[\sigma'_1, \sigma'_2]$.

Let $\overline{S} \equiv \lim_{\mu \to 1} \frac{g^h(\mu)}{g^l(\mu)}$ and $\underline{S} \equiv \lim_{\mu \to 0} \frac{g^h(\mu)}{g^l(\mu)}$, where $\{\overline{S}, \underline{S}\} \subset \mathbb{R}_+ \cup \{0, +\infty\}$. If $\overline{S} = \infty$ and $\underline{S} = 0$, then let $\sigma_1 \equiv \sigma'_1$ and $\sigma_2 \equiv \sigma'_2$, and note that as σ_s increases from σ_1 to σ_2 , $\frac{c-[1-F^l(\sigma_s)]}{[1-F^h(\sigma_s)]-c}$ increases continuously from 0 to ∞ . Therefore, for any σ_s there exists $\overline{\mu} \in [0, 1]$ such that $\frac{g^h(\overline{\mu})}{g^l(\overline{\mu})} = \left(\frac{c-[1-F^l(\sigma_s)]}{[1-F^h(\sigma_s)]-c}\right) / \left(\frac{\pi_0}{1-\pi_0}\right)$. Assume now that $\overline{S} < \infty$ and $\underline{S} > 0$. Let σ_1 be defined implicitly by $\underline{S} = \left(\frac{c-[1-F^l(\sigma_1)]}{[1-F^h(\sigma_1)]-c}\right) / \left(\frac{\pi_0}{1-\pi_0}\right)$, and σ_2 be defined implicitly by $\overline{S} = \left(\frac{c-[1-F^l(\sigma_2)]}{[1-F^h(\sigma_2)]-c}\right) / \left(\frac{\pi_0}{1-\pi_0}\right)$. Note that since $\lim_{\sigma_s \to \sigma'_1^+} \frac{c-[1-F^l(\sigma_s)]}{[1-F^h(\sigma_s)]-c} = 0$ and $\lim_{\sigma_s \to \sigma'_2^-} \frac{c-[1-F^l(\sigma_s)]}{[1-F^h(\sigma_s)]-c} = \infty$, we have $\sigma_1 > \sigma'_1$ and $\sigma_2 < \sigma'_2$. Since $[\sigma'_1, \sigma'_2] \subset (0, 1)$ and $\sigma_1 < \sigma_2$, it follows that $[\sigma_1, \sigma_2] \subset (0, 1)$. Then, when σ_s increases from σ_1 to σ_2 , $\left(\frac{c-[1-F^l(\sigma_s)]}{[1-F^h(\sigma_s)]-c}\right) / \left(\frac{\pi_0}{1-\pi_0}\right)$ increases continuously from \underline{S} to \overline{S} . Therefore, for any σ_s there exists $\overline{\mu} \in [0, 1]$ such that $\frac{g^h(\overline{\mu})}{g^l(\overline{\mu})} = \left(\frac{c-[1-F^l(\sigma_s)]}{[1-F^h(\sigma_s)]-c}\right) / \left(\frac{\pi_0}{1-\pi_0}\right)$. Finally, if $\overline{S} < \infty$ and $\underline{S} = 0$, then let

 $\sigma_1 \equiv \sigma'_1 \text{ and } \sigma_2 \text{ be defined implicitly by } \overline{S} = \left(\frac{c - [1 - F^l(\sigma_2)]}{[1 - F^h(\sigma_2)] - c}\right) / \left(\frac{\pi_0}{1 - \pi_0}\right), \text{ while if } \overline{S} = \infty \text{ and } \underline{S} > 0,$ then let σ_1 be defined implicitly by $\underline{S} = \left(\frac{c - [1 - F^l(\sigma_1)]}{[1 - F^h(\sigma_1)] - c}\right) / \left(\frac{\pi_0}{1 - \pi_0}\right) \text{ and } \sigma_2 \equiv \sigma'_2.$

(ii) For the evaluator, from (2) and (7) it follows that, given an arbitrary cutoff strategy μ_s ,

$$\begin{split} \Pr(h|\{s\},\sigma) &\geq \frac{L}{1+L} \Longleftrightarrow \\ \frac{\Pr(\{s\}|h)\frac{\pi_0}{1-\pi_0}\frac{f^h(\sigma)}{f^l(\sigma)}}{\Pr(\{s\}|h)\frac{\pi_0}{1-\pi_0}\frac{f^h(\sigma)}{f^l(\sigma)} + \Pr(\{s\}|l)} &\geq \frac{L}{1+L} \Longleftrightarrow \\ \frac{\left[1-G^h(\mu_s)\right]\frac{\pi_0}{1-\pi_0}\frac{f^h(\sigma)}{f^l(\sigma)} + \Pr(\{s\}|l)}{\left[1-G^h(\mu_s)\right]\frac{\pi_0}{1-\pi_0}\frac{f^h(\sigma)}{f^l(\sigma)} + \left[1-G^l(\mu_s)\right]} &\geq \frac{L}{1+L} \Longleftrightarrow \\ \frac{\pi_0}{1-\pi_0}\frac{f^h(\sigma)}{f^l(\sigma)}\frac{1-G^h(\mu_s)}{1-G^l(\mu_s)} &\geq L. \blacksquare \end{split}$$

Appendix A3. Proof of Proposition 1

First, \mathcal{A} 's best-response function $\overline{\mu}(\sigma_s)$, as elicited from (3), is increasing because $\frac{d}{d\mu} \left[\frac{g^h(\mu)}{g^l(\mu)} \right] > 0$ and $\frac{c - [1 - F^l(\sigma_s)]}{[1 - F^h(\sigma_s)] - c}$ is increasing in σ_s . To show that $\overline{\sigma}(\mu_s)$ is decreasing, since $\frac{d}{d\sigma} \left[\frac{f^h(\sigma)}{f^l(\sigma)} \right] > 0$, it is enough to show that $\frac{1 - G^h(\mu_s)}{1 - G^l(\mu_s)}$ is increasing in μ_s . Taking the derivative, we have

$$\frac{\partial}{\partial \mu_s} \left[\frac{1 - G^h(\mu_s)}{1 - G^l(\mu_s)} \right] = \frac{-g^h(\mu_s) \left[1 - G^l(\mu_s) \right] + g^l(\mu_s) \left[1 - G^h(\mu_s) \right]}{\left[1 - G^l(\mu_s) \right]^2} > 0$$

To show this inequality, note that $\frac{d}{d\mu} \left[\frac{g^h(\mu)}{g^l(\mu)} \right] > 0$ implies $\frac{g^h(\mu_s)}{g^l(\mu_s)} \leq \frac{g^h(x)}{g^l(x)}$ for $x \in [\mu_s, 1]$, and thus that $g^h(\mu_s) g^l(x) \leq g^l(\mu_s) g^h(x)$. Integrating this inequality with respect to x between μ_s and 1, we obtain $g^h(\mu_s) \left[1 - G^l(\mu_s) \right] \leq g^l(\mu_s) \left[1 - G^h(\mu_s) \right]$ which immediately proves the result.

Thus $\overline{\sigma}(\mu_s)$, as defined implicitly by (4), is strictly decreasing. Let $\sigma_3 \equiv \overline{\sigma}(0)$ satisfying $\frac{\pi_0}{1-\pi_0} \frac{f^h(\sigma_3)}{f^I(\sigma_3)} = L$. Then, $\overline{\sigma}(\cdot)$ decreases on [0, 1] from σ_3 to 0. Define the inverse $\overline{\sigma}^{-1} : [0, \sigma_3] \to [0, 1]$, and note that it is decreasing and bijective on its domain. Assume first that $\sigma_3 > \sigma_1$. Then $\overline{\sigma}^{-1}(\sigma_s)$ and $\overline{\mu}(\sigma_s)$ must be equal at some value $\sigma^* \in (\sigma_1, \sigma_3)$. Let $\mu^* \equiv \overline{\mu}(\sigma^*)$, and note that $\sigma^* = \overline{\sigma}(\overline{\sigma}^{-1}(\mu^*)) = \overline{\sigma}(\overline{\mu}(\sigma^*)) = \overline{\sigma}(\mu^*)$. Thus, we conclude that (μ^*, σ^*) as defined is the unique equilibrium of the game. Moreover, since $\overline{\sigma}^{-1}(\cdot)$ is strictly decreasing on [0, 1], we must have $\mu^* \in [0, 1)$ and $\sigma^* \in (0, 1)$. Finally, if $\sigma_1 \geq \sigma_3$ then $\mu^* = 0$.