Transaction Costs can Encourage Coasean Bargaining

Robson, Alex

Griffith University

August 2012

Transaction Costs can Encourage Coasean Bargaining

by Alex Robson*

Abstract

When there are three parties, it is well known that the Coase Theorem may not hold even when there are no transaction costs, due to the emptiness of the core of the corresponding cooperative game [Aivazian and Callen (1981)]. We show that the standard Coasean bargaining game involving three parties is strategically equivalent to an asymmetric three player majority game. Hence, when there are three parties, the Coase Theorem fails if and only if the core of the corresponding three player majority game is empty. We use this equivalence result to derive *all* instances in which the Coase Theorem will and will not hold with three parties, and show that the Coase Theorem will actually hold most (over 80 per cent) of the time. We also demonstrate, in contrast to Aivazian and Callen (2003), that it is always possible to find a set of transaction costs which, when introduced into a frictionless bargaining situation, will cause an empty core to become non-empty. In other words, with suitably designed transaction costs, it is possible for the Coase Theorem to hold in cases where, in the absence of those transaction costs, it would fail to hold. When there are three parties, rather than hindering agreements, transaction costs can encourage Coasean bargaining.

Keywords: Coase Theorem, externalities, transaction costs, cooperative games

JEL Classification Codes: C71, C78, D23, D62, K0

* Department of Accounting, Finance and Economics, Griffith Business School, Nathan Campus, Nathan, Qld, 4111. email: a.robson@griffith.edu.au

1. Introduction

There are two versions of the Coase (1960) theorem that are commonly referred to in the literature.[1] The *efficiency version* states that if transaction costs are sufficiently low, then the initial legal regime or assignment of property rights will not hinder the parties from reaching an agreement in which all possible gains from trade have been exhausted. The *invariance version* states that if transaction costs are sufficiently low, then bargaining will always lead to the *same* level of the activity which generates the externality, irrespective of the initial legal regime or assignment of property rights.[2]

In an important and insightful paper, Aivazian and Callen (1981) showed that when there are three parties, both versions of the Coase Theorem can fail to hold even when transaction costs are low. This can happen because of the *empty core problem*, where an agreement between all three parties is unstable. It is possible that agreements between two parties which exclude the third are so profitable that the opportunity cost of those two parties entering an agreement with the third party may exceed the benefits. Hence any agreement between all three parties will be susceptible to coalitions of two players breaking away, and both versions of the Coase Theorem can fail to hold. [3] More recently, Aivazian and Callen (2003) extend these arguments, and argue that "if the core is empty in the absence of coalition formation costs, then it is necessarily empty with such costs."

How likely is the kind of instability identified by Aivazian and Callen, and how robust is their conclusion that the introduction of transaction costs will not improve matters? This paper makes a number of contributions to the literature. First, we develop a method for deriving *all* possible examples in which the Coase Theorem fails or holds when there are three parties. Second, we use this method to demonstrate that instability is relatively unlikely, and so the Coase Theorem will hold most of the time. Finally, we demonstrate that, in contrast to Aivazian and Callen (2003), the

[1] See Parisi (2008) for a recent summary of the literature on the Coase Theorem.

[2] Cooter and Ulen (2012), Robson and Skaperdas (2008) and Robson (2012) distinguish between these two versions.

[3] Bernholz (1997) argues that the empty core problem can be solved by the appropriate use of penalty clauses which increase the costs of *breaking* agreements. These kinds of costs are important, but are not the focus of the present analysis. Instead, we focus on the costs of *making* agreements.

introduction of transaction costs can cause an empty core to become non-empty. In other words, when transaction costs are introduced, the Coase Theorem can hold in cases where, in the absence of those transaction costs, it would fail to hold. Rather than hindering agreements, the existence of transaction costs can encourage Coasean bargaining.

2. The Basic Setup

2.1. The Aivazian-Callen (1981) Example

To illustrate the main issues, consider the following example, which is well known. There are two factories, 1 and 2, and a group of residents, R. Both factories emit pollution and this reduces the wellbeing of the residents. In the absence of production by the firms, the residents are assumed to enjoy utility of 40. If both firms produce, the residents' utility falls to 24. Therefore, production by both factories imposes a negative external cost of 16 (=40-24) on the residents. Suppose factory 1 imposes a negative external cost of 9 on the residents, whilst factory 2 imposes a negative external cost of 4. The marginal external cost to the residents is therefore not constant. Finally, assume that if the factories produce alone, they can earn profits of 3 and 8 respectively. If they merge and produce together, then they can jointly earn profits of 15. This reflects an assumption of economies of scale in production.

Note that in this example there is actually more than one class of externality at work.[4] The first two externalities are negative, and are caused by the factories reducing the residents' wellbeing. The second externality is a positive externality – both factories producing together increases their joint profits. We shall see, however, that the mere existence of this positive externality is not sufficient to make the Coase Theorem fail – it is the size of the positive externality that matters.

The efficient outcome in this example is for both factories not to produce. To better see how the example works, we transform the example into a cooperative game with transferable utility. Let the characteristic function of this game be v. Suppose first that the factories can produce as much as they wish (a rule of no liability). Denote the value of the grand coalition by v_N. Then $v_N = 40$, the total utility available to the

[4] Mueller (2003), page 30, also makes this point.

parties when the factories and the residents agree that the factories should shut down production. Notation for other possible coalitions is defined in a similar fashion. Under the no liability rule, the characteristic function of this cooperative game is:

$$v_1 = 3,\ v_2 = 8,\ v_R = 24,\ v_{12} = 15,\ v_{1R} = 31, v_{2R} = 36,\ v_N = 40 \qquad (1)$$

Aivazian and Callen show that the core of this game is empty, and so there is no efficient agreement between the parties which is stable against threats by groups of two players to reach an agreement on their own.

Now suppose that the factories must first obtain the residents' permission to produce. Again, let the characteristic function of this game be v, and denote the value of the grand coalition by v_N. Then again we have $v_N = 40$. The characteristic function in this situation is:

$$v_1 = 0,\ v_2 = 0,\ v_R = 40,\ v_{12} = 0, v_{1R} = 40,\ v_{2R} = 40,\ v_N = 40$$

There is no payment that the factories could make to persuade the residents to let them produce. The core is non-empty and has a very simple structure: it is simply the point $x_1 = 0, x_2 = 0, x_R = 40$. Note, however, that the outcome is not the same as that under the no liability rule. Hence both versions of the Coase Theorem fail to hold in this example.

2.2. Modifying the Example

Only a slight modification of the previous example is needed to show how the previous result changes. Consider the same example, but now suppose that if the factories merge, they can realise joint profits of 12 rather than 15. This seems like a trivial modification - after all, our previous example featured economies of scale, this example simply makes those synergies slightly smaller. However, modifying the example in this way reduces the opportunity cost of firms 1 and 2 entering into agreement with the residents. This reduction in opportunity cost means that the firms require less compensation for entering such an agreement, which in turn means that the residents' net benefit will be higher than in the previous case.

This slight change now renders efficient agreement stable. The characteristic function is now:

$$v_1 = 3,\ v_2 = 8,\ v_R = 24,\ v_{12} = 12,\ v_{1R} = 31,\ v_{2R} = 36, v_N = 40 \qquad (2)$$

It is straightforward to show the core of this game is non-empty. For example, any set of agreements which satisfies:

$$x_1 = 4 - \alpha,\ x_2 = 8 + \alpha\ ,\ x_R = 28$$

where $0 < \alpha < 1$, is feasible and will be stable against deviations by any deviation by a single party, as well as any deviation by a subcoalition of two parties. Hence the core is non-empty, and both versions of the Coase Theorem will hold.

3. Deriving All Three Player Examples

How ubiquitous is the example derived by Aivazian and Callen? To understand when the Coase Theorem will hold and when it will not when there are three parties, we consider the following asymmetric *three player majority game*.[5] In this game there are again three players, $1, 2$ and R. When all players all act together, they can obtain a payoff of \$1. If any two of them acting together can obtain a payoff smaller than 1, with different two player coalitions possibly obtaining different amounts. The characteristic function of this cooperative game is:

$$v_S = \begin{cases} 0 & \text{if } |S| = 1 \\ \theta_{12} & \text{if } S = \{1,2\} \\ \theta_{1R} & \text{if } S = \{1,R\} \\ \theta_{2R} & \text{if } S = \{2,R\} \\ 1 & \text{if } |S| = 3 \end{cases} \tag{3}$$

where $\{\theta_{12}, \theta_{1R}, \theta_{2R}\} \in (0,1)^3$. Suppose that the payments x_1, x_2, x_R are in the core of this game. Then these payments must be non-negative, and we must have $x_1 + x_2 + x_R = 1$ and $x_1 + x_2 \geq \theta_{12}, x_1 + x_R \geq \theta_{1R}, x_2 + x_R \geq \theta_{2R}$. But if this second set of inequalities holds, they must also hold if we sum them together, so we must have:

$$2(x_1 + x_2 + x_R) \geq \theta_{12} + \theta_{1R} + \theta_{2R}$$

But since $x_1 + x_2 + x_R = 1$, this implies the core is non-empty if and only if $\theta_{12} + \theta_{1R} + \theta_{2R} \leq 2$.

---

[5] This game is a more general case the symmetric majority game, which is studied by (for example) Osborne and Rubinstein (1995), page 259.

To understand the economic structure of the Aivazian-Callen example and how it relates to this asymmetric majority game, consider the *0-1 normalisation* of a cooperative game, which works as follows.[6] Suppose that we have cooperative game with a characteristic function v_S, and which has $v_i \neq 0$ for some i, and $v_N \neq 1$. We seek to transform the payoffs of the game such that it is strategically equivalent to the original game, but which has $v_i = 0$ for all i, and $v_N = 1$. This can be done by adding (not necessarily positive) numbers z_i to the individual payoff of each individual to give them $v_i = 0$. In other words, set $z_i = -v_i$. This number must be added to every coalition of which i is a member. In particular, the new value of the grand coalition must be:

$$K = v_N + \sum_i z_i \tag{4}$$

Note that in the context of Coasean bargaining, $K = v_N + \sum_i z_i$ has the interpretation of the total available gains from trade, relative to autarky. Now, divide the new value of every coalition by K. The resulting characteristic function is strategically equivalent to the original game, since each individual's payoff has been scaled up by z_i and divided by a constant K. Moreover, we have $v_i = 0$ for all i, and $v_N = 1$. Thus we have shown:

Lemma: (a) *Every three player game in which the Coase Theorem fails to hold is strategically equivalent to an asymmetric three player majority game with an empty core.*

(b) *Every three player game in which the Coase Theorem holds is strategically equivalent to an asymmetric three player weighted majority game with a non-empty core.*

Recall that a cooperative game is *cohesive* if $v_N \geq \sum_{k=1}^{K} v_{S_k}$ for all partitions $\{S_1, \ldots, S_K\}$

---

[6] See, for example, Ordeshook (1986), page 323.

of N. To study the Coase Theorem, we want to focus on situations where it is efficient for the grand coalition to form. Hence we restrict our attention to cohesive games. But the asymmetric majority game is cohesive; hence the Lemma provides us with an algorithm for constructing all three player cohesive cooperative games in which the Coase Theorem will and will not hold.

Proposition 1: *Let v_1, v_2 and v_R be any three positive numbers, and let $v_N > v_1 + v_2 + v_R$. Then there exists numbers $\bar{v}_{12}$, $\bar{v}_{1R}$ and $\bar{v}_{2R}$ with $v_n > \bar{v}_{ij} > v_i + v_j$ for $i, j = 1, 2, R$, $i \neq j$ so that:*

(a) If $v_{12} + v_{1R} + v_{2R} > \bar{v}_{12} + \bar{v}_{1R} + \bar{v}_{2R}$, then the Coase Theorem will not hold; and

(b) If $v_{12} + v_{1R} + v_{2R} \leq \bar{v}_{12} + \bar{v}_{1R} + \bar{v}_{2R}$, the Coase Theorem will hold.

Moreover, the resulting cooperative game is cohesive.

Proof: To prove the result, we use the 0-1 normalisation and the previous Lemma. Let v_1, v_2 and v_R be any three positive numbers, let $v_N > v_{12} + v_{1R} + v_{2R}$, and let $K = v_N - [v_1 + v_2 + v_R]$ be the efficiency gains from full cooperation. Let $\theta_{12}, \theta_{1R}, \theta_{2R}$ be any three numbers on the interval (0,1) with the property that $\theta_{12} + \theta_{1R} + \theta_{2R} = 2$.

Define $\bar{v}_{12}, \bar{v}_{1R}$ and $\bar{v}_{2R}$ implicitly by:

$$\frac{\bar{v}_{12} - v_1 - v_2}{K} = \theta_{12}$$

$$\frac{\bar{v}_{1R} - v_1 - v_R}{K} = \theta_{1R}$$

and

$$\frac{\bar{v}_{2R} - v_2 - v_R}{K} = \theta_{2R}$$

Then, by construction, any game with payoffs such that $v_{12} + v_{1R} + v_{2R} > \bar{v}_{12} + \bar{v}_{1R} + \bar{v}_{2R}$ will be equivalent to an asymmetric majority game with an empty core, and the Coase Theorem will fail to hold. Similarly, any game with payoffs such that $v_{12} + v_{1R} + v_{2R} \leq \bar{v}_{12} + \bar{v}_{1R} + \bar{v}_{2R}$ will be equivalent to an asymmetric majority game with a non-empty core, and the Coase Theorem will hold.

Note, too, that as long as the resulting game is derived from the asymmetric majority game, it will be cohesive, even if $\theta_{12}+\theta_{1R}+\theta_{2R}>2$. For example, as long as v_{12} is constructed according to the above procedure, we will have:

$$\begin{aligned}
&v_{12}+v_R = K\theta_{12}+v_1+v_2+v_R \\
&=\left\{v_N-[v_1+v_2+v_R]\right\}\theta_{12}+v_1+v_2+v_R \\
&=\theta_{12}v_N+\left(1-\theta_{12}\right)\left[v_1+v_2+v_R\right] \\
&<\theta_{12}v_N+\left(1-\theta_{12}\right)v_N=v_N
\end{aligned}$$

■

The equivalence between the Aivazian-Callen example and the asymmetric majority game is not just a theoretical curiosum. It allows us to construct examples and counterexamples. More importantly, it allows us to investigate how frequently we should expect to see the Coase Theorem failing because of the problems of instability associated with the empty core, as well as the effects of transaction costs. These two issues are addressed in the remainder of the paper.

4. How Likely is it that the Coase Theorem Will Fail to Hold?

As discussed above, the equivalence between the three player bargaining game and the asymmetric majority game allows us to make precise statements about how likely it is that the core will be empty and the Coase Theorem will fail to hold. Since the two classes of games are equivalent, the failure of the Coase Theorem will hold with the same frequency that the core of the corresponding asymmetric majority game is non-empty, which requires $\left(\theta_{12},\theta_{1R},\theta_{2R}\right)\in(0,1)^3$ and $\theta_{12}+\theta_{1R}+\theta_{2R}\le 2$. We therefore can show:

Proposition 2: *Suppose that all possible payoffs are equally likely. Then the Coase Theorem holds most of the time. More precisely, it will hold with probability 5/6.*

Proof: We need to find the probability that $\theta_{12}+\theta_{1R}+\theta_{2R}\le 2$, given that the points $\left(\theta_{12},\theta_{1R},\theta_{2R}\right)$ are uniformly distributed on the open unit cube $(0,1)^3$. This probability is given by:

$$
\begin{aligned}
&\Pr(\theta_{12}+\theta_{1R}+\theta_{2R}<2)=1-\Pr(\theta_{12}+\theta_{1R}+\theta_{2R}\geq 2)\\
&=1-\int_0^1\int_{1-\theta_{2R}}^1\int_{2-\theta_{1R}-\theta_{2R}}^1 d\theta_{12}d\theta_{1R}d\theta_{2R}\\
&=1-\int_0^1\int_{1-\theta_{2R}}^1 1-\left(2-\theta_{1R}-\theta_{2R}\right)d\theta_{1R}d\theta_{2R}\\
&=1-\int_0^1\int_{1-\theta_{2R}}^1 \left(\theta_{1R}+\theta_{2R}-1\right)d\theta_{1R}d\theta_{2R}\\
&=1-\int_0^1\left[\frac{\theta_{1R}^{\ 2}}{2}+\theta_{2R}\theta_{1R}-\theta_{1R}\right]_{1-\theta_{2R}}^1 d\theta_{2R}\\
&=1-\left\{\int_0^1\left[\frac{1}{2}+\theta_{2R}-1\right]-\left[\frac{\left(1-\theta_{2R}\right)^2}{2}+\theta_{2R}(1-\theta_{2R})-(1-\theta_{2R})\right]d\theta_{2R}\right\}\\
&=1-\left\{\int_0^1\left[\theta_{2R}-\frac{1}{2}-\frac{\left(1-\theta_{2R}\right)^2}{2}-\left(\theta_{2R}-1\right)(1-\theta_{2R})\right]d\theta_{2R}\right\}\\
&=1-\int_0^1\left[\theta_{2R}-\frac{1}{2}+\frac{1-2\theta_{2R}+\theta_{2R}^{\ 2}}{2}\right]d\theta_{2R}\\
&=1-\int_0^1\left[\frac{\theta_{2R}^{\ 2}}{2}\right]d\theta_{2R}=1-\left[\frac{\theta_{2R}^{\ 3}}{6}\right]_0^1=\frac{5}{6}
\end{aligned}
$$

■

The interpretation of this result is straightforward: whilst it is always possible to construct combinations of payoffs such as those obtained by Aivazian and Callen (1981) which result in an empty core, with three players these combinations are relatively unlikely to occur. In three player games, the failure of the Coase Theorem due to coalitional instability and the emptiness of the core is the exception rather than the rule.

5. Transaction Costs

One of the most important insights of Coase's (1960) analysis is that if transaction costs are sufficiently large, then the legal rule or structure of initial property rights matters for efficiency. Hence, there will exist efficiency enhancing and efficiency maximizing rules.

Avazian and Callen (2003) argue, using a special functional form for coalition formation costs, that *"if the core is empty in the absence of coalition formation costs,* *then it is necessarily empty with such costs."* Specifically, they assume that transaction costs take the following functional form:

$$C_S = \begin{cases} |S|^k & \text{if } |S| > 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where $k > 1$.

These transaction costs are very specific. Is the result true in general? Proposition 1 can be used to show that it is not.

Proposition 3: *For every three player bargaining game without transaction costs which has an empty core, there exists a transformed bargaining game with a set of positive, non-decreasing transaction costs which possesses a non-empty core.*

Proof: Let the original payoffs be $v_1, v_2, v_R, v_{12}, v_{1R}, v_{2R}$ and v_N, and suppose that the core is empty. Choose transaction costs in the following way. For the grand coalition, we want C_N to be sufficiently small so that there are still positive gains from trade. Thus:

$v_N - (v_1 + v_2 + v_R) - C_N = K - C_N > 0$.

On the other hand, we do not want C_N to be so small that they are exceeded by the transaction costs of two-player coalitions. Hence we will also require:

$C_N > K\left[3\max\{\theta_{12}, \theta_{1R}, \theta_{1R}\} - 2\right]$, which implies that $\frac{C_N}{3} > K\left[\max\{\theta_{12}, \theta_{1R}, \theta_{1R}\} - \frac{2}{3}\right]$

Now if $\frac{v_{12} - (v_1 + v_2)}{K - C_N} \leq \frac{2}{3}$, set $C_{12} = 0$. If not, set C_{12} so that $\frac{v_{12} - C_{12} - (v_1 + v_2)}{K - C_N} = \frac{2}{3}$.

Repeat these steps for v_{1R} and v_{2R}. Note that if $C_{12} > 0$ we have:

$$v_{12} - C_{12} - (v_1 + v_2) = \frac{2}{3}(K - C_N) \tag{6}$$

so that:

$$C_{12} = \theta_{12}K - \frac{2}{3}(K - C_N) = \frac{2}{3}C_N + \left(\theta_{12} - \frac{2}{3}\right)K < \frac{2}{3}C_N + \frac{C_N}{3} = C_N$$

where the last inequality follows from the choice of that C_N, which satisfied $C_N > K\left[3\max\{\theta_{12},\theta_{1R},\theta_{1R}\}-2\right]$. Note that this game is strategically equivalent to a cohesive symmetric majority game with $\theta = 2/3$, the core of which is non-empty. ■

Hence, as long as the transaction costs associated with reaching an agreement between two parties are sufficiently high (relative to the transaction costs associated with agreement making between three parties), it is possible that the introduction of transaction costs will reduce the opportunity costs of the grand coalition reaching an efficient agreement, and a game which has an empty core will become a game with a non-empty core.

It is straightforward to apply this result to the Aivazian and Callen (1981) example with an empty core. In their example, $K = 5$, and $\theta_{12} = \theta_{1R} = \theta_{2R} = 0.8$. Hence, the proof of Proposition 3 suggests that we will need $C_N > K\left[3\max\{\theta_{12},\theta_{1R},\theta_{1R}\}-2\right] = 5\times(2.4-2) = 2$. So, choose $C_N = 3$. Also, the proof of Proposition 3 suggests that we should set:

$$C_{12} = \theta_{12}K - \frac{2}{3}(K - C_N) = \frac{2}{3}C_N + \left(\theta_{12} - \frac{2}{3}\right)K = 2 + \left(0.8 - \frac{2}{3}\right)5 = 2\tfrac{2}{3}$$

with $C_{12} = C_{1R} = C_{2R}$.

Then the characteristic function for the new game with transaction costs is:

$$v_1 = 3,\ v_2 = 8,\ v_R = 24,\ v_{12} = 12\tfrac{1}{3},\ v_{1R} = 28\tfrac{1}{3}, v_{2R} = 33\tfrac{1}{3}, v_N = 37$$

which is cohesive and has a non-empty core. Hence, the introduction of suitably chosen transaction costs into the Aivazian and Callen (1981) makes a non-empty core empty. Transaction costs can encourage Coasean bargaining.

6. Conclusion

This paper has demonstrated the equivalence between the three party bargaining game studied by Aivazian and Callen (1981) and an asymmetric majority game to derive a number of results. We showed that when there are no transaction costs, the instability identified by Aivazian and Callen will occur relatively infrequently. Indeed, if all payoff combinations are equally likely, then Coasean bargaining is five times more likely than not. In the those cases where instability could arise due to the emptiness of the core, it is possible to find a set of transaction costs which completely eliminates this as a source of instability and a barrier to Coasean bargaining.

References

Aivazian,V. and Callen, J.(1981) "The Coase Theorem and The Empty Core", *Journal of Law and Economics*, 175-181.

Aivazian, V. and Callen, J. (2003) "The Core, Transaction Costs, and the Coase Theorem," *Constitutional Political Economy,* 14: 287-299.

Benoit, J-P. and Kornhauser, L. (2002) "Game Theoretic Analysis of Legal Rules and Institutions, Chapter 60 in Aumann, R. and Hart, S. (eds) *Handbook of Game Theory with Economic Applications*, Volume 3, London: North Holland.

Bernholz, P. (1997) "Property Rights, Contracts, Cyclical Social Preferences, and the Coase Theorem: A Synthesis," *European Journal of Political Economy,* 13: 419-442.

Coase, R. (1960) "The Problem of Social Cost," *Journal of Law and Economics*, 3: 1-44.

Cooter, R. and Ulen, T. (2012) *Law and Economics*, 6th edition, New York: Prentice Hall.

Mueller, D. (2003) *Public Choice III,* Cambridge: Cambridge University Press.

Ordeshook, P. (1986) *Game Theory and Political Theory*, Cambridge: Cambridge University Press.

Osborne, M. and Rubinstein, A. (1994) *A Course in Game Theory,* Cambridge, MA: MIT Press.

Parisi, F. (2008) "Coase Theorem" in Durlauf, S. and Blume, L. (eds) *The New Palgrave Dictionary of Economics,* London: Palgrave Macmillan.

Robson, A. and Skaperdas, S. (2008) "Costly Enforcement of Property Rights and the Coase Theorem," *Economic Theory,* 36: 109-128.

Robson, A. (2012) *Law and Markets,* London: Palgrave Macmillan.