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Abstract

In this study, we test the hypothesis that psychological barriers exist in 5 European Equity Market indices [ATX, CAC, DAX, FTSE, SMI]. We employ both a traditional methodology that assumes a uniform distribution of M-Values and a modified approach that accounts for the fact that the digits of stock prices may be distributed in accordance with Benford's law. In addition, we test the validity of the various assumptions employed in these tests using a Monte Carlo Simulation and Kuiper's Modified Kolmogorov-Smirnov Goodness of Fit Test. We find evidence for barriers in 1 index [SMI] at the 1000 level under the assumption of uniformity but no significant evidence of barriers at the 100 level or at the 1000 level in the remaining indices. We also find evidence that substantiates the criticism of the use of the uniformity assumption for tests at the 1000 level in favour of a distribution consistent with Benford's Law. However, we do not reach a different conclusion on the presence of psychological barriers when tests are performed without the implicit use of that uniformity assumption. In addition, we find possible evidence of price clustering around round numbers at the 1000 level in 2 indices [CAC, DAX] even after adjusting for the expected concentration within the region due to Benford-specific effects.

Keywords: Benford's Law; psychological barriers in stock prices; significance of round numbers in stock prices;
JEL Classification Codes: C12, C15, G02, G14, G15

1. Introduction

Since the creation of the first stock exchange and the subsequent trading of stocks became widespread, the movement of stock prices has been of interest to a vast group of people. The region around round numbers, in particular, has long captivated the interest of market commentators with conventional wisdom asserting that round numbers act as regions of natural support and resistance, or function as "psychological barriers" as it has come to be known.

Such a bold assertion can hardly remain without academic interest and a raft of papers have emerged in attempts to assess the validity of such claims. As with most tests on controversial issues, the empirical evidence is mixed. One reason for this phenomenon is the lack of agreement on the appropriate methodology to use in testing for the presence of barriers. The early papers on the topic of psychological barriers employed tests based on the assumption that the digits of stock prices should be uniformly distributed. This assumption is fairly intuitive and can be explained by appealing to examples that illustrate the idea that a stock price is an arbitrarily scaled random figure that by itself should have no attributes that would determine its relative frequency of occurrence. For example, there should be no reason for a stock price with ending digits of 32 to occur more frequently than, for example, a stock price with ending digits of 53. Over a long period of time, what we should expect to see is, on average, a stock index closing with ending digits of 32 as frequently as it does with ending digits of 53. This same line of reasoning is then applied to regions around round numbers to assert that in the absence of barriers round numbers should occur with similar frequency to numbers in other regions and the digits of stock prices should hence be uniformly distributed. Tests were then conducted around this assumption of uniformity to test for evidence of non-uniformity that could be attributed to deviations in the region of round numbers and this was then use to support the hypothesis that psychological barriers exist at round numbers. Based on such a methodology, a number of papers in the 1990s found evidence of barriers around round numbers in a number of major equity markets around the world.

However, there exists another group of academics that criticises the uniformity assumption based on a much less intuitive mathematical concept known as Benford's Law. Benford's Law asserts that the distribution of digits in natural phenomenon tends to follow a specific distribution due to a number of reasons and the predictions from this concept imply that the distribution of digits in stock prices should not be uniformly distributed. Modified tests have been proposed to account for this effect and the conclusion from most studies that take this into account has been that barriers do not exist in round numbers.

In this study, we attempt to contribute to the debate on the significance of round numbers by considering both approaches over a recent time period and a number of significant stock indices within the European equity markets. Through the series of tests that we employ, we critically examine the significance of round numbers as potential regions of resistance in 5 European equity indices based on different assumptions for the distribution of M-values. Beyond presenting the result and implications from tests based on different assumptions, we also examine the empirical validity of the underlying assumptions of these tests by modelling and testing their fit to the available data to provide a more holistic approach to the assessment of the results obtained.

2. Background

2.1. The Significance of Round Numbers

When we refer to "the significance of round numbers", we are simply referring to the regions around the 00 region of a stock index, hence at the 100, 200, 300, 400,.., 1000, 2000, etc level and whether stock prices tend to move in a different manner when it enters the proximity of these regions. The importance of round numbers as a natural region of support and resistance has its roots in Technical Analysis. *Technical Analysis of Stock Trends* by Edwards, Maggee and Basetti presents the following analysis on the significance of round numbers:

"There are certain other levels that may, at times, evidently produce considerable Resistance or Support without any reference to a previous "vested interest." We have in mind the "round" Figures 20, 30, 50, 75, 100, etc. In setting a goal for taking profits when we buy a stock, it is natural for us to think in terms of such round prices… In fact, any time an issue gets out into new all-time high ground, where there is nothing in its chart history to indicate otherwise, it is a fairly safe bet that Resistance will appear at the round figures."

A region of support is a level or area on the chart under the market where buying interest is sufficiently strong to overcome selling pressure and a region of resistance is a level or area on the chart over the market where selling interest is sufficiently strong to overcome buying pressure [Murphy, 1999]. If psychological barriers around round numbers exist, then what we would expect is evidence to show that price levels around round numbers tend to provide support in a downtrend and resistance in an uptrend.

2.2. Proposed Explanations for the Significance of Round Numbers

There are a range of plausible reasons that have been suggested for the potential existence of psychological barriers around round numbers. Explanations form technical analysts often rest on the assertion that people use round numbers as natural points to take profits or cut losses. This common anchor point is then used as a part of the investment decision making process of many individuals which then manifests itself as overhanging supply or demand around those regions that cause the phenomenon of support and resistance around these points to be observed.

Beyond simple thought experiments, other explanations have also been offered that often appeal to research from behavioural finance or business theory. Some concepts often quoted as possible reasons include the importance of odd pricing [Schindler and Kirby, 1997] coordination within a limited price set [Harris, 1991] and bounded rationality [Sonnemans, 2006]. However, while all these reasons and thought experiments provide explanations of why psychological barriers may exists, none of them provide a reason for why psychological barriers must necessarily be present, and the presence or absence of psychological barriers in European equity markets remains fundamentally an empirical and not a theoretical question.

2.3. Objectives and Significance of Study

There are 3 main objectives of this research paper. Firstly, we aim to present empirical evidence from tests on the significance of round numbers in the stock indices of 5 major European markets using the traditional approach to the testing of barriers based on the methodology from Donaldson and Kim (1993). This study provides an updated investigation over a recent time period as well as results for some previously untested European markets.

Next, we move to investigate the key criticism of the traditional approach of testing for barriers, that of the uniformity assumption. We evaluate the robustness of our earlier results by comparison with a Monte Carlo simulation with results drawn from cyclical permutations of returns as presented in De Ceuster et al (1997). By construction, this simulation would exhibit no psychological barriers and this approach is hence often regarded by critics of the uniformity assumption as a more appropriate approach to test for the presence of barriers that would not lead to the erroneous conclusion of the presence of barriers due only to an expected distribution of digits in a manner consistent with Benford's Law. We present the empirical results from this test to help provide a more complete picture of the significance of round numbers in these European markets.

Finally, we extend the approach adopted by present studies in this area through a direct investigation of the empirical validity of criticisms based on Benford's Law. We test, by means of Kuiper's modified Kolmogorov-Smirnov Goodness-of-Fit Test the fit of a distribution of M-values based on Benford's law to the distribution of M-values found in the 5 European stock indices. This enables us to examine the appropriateness of criticisms of the uniformity assumption based on Benford's law and thus provide a more comprehensive picture of the results obtained from the various tests.

2.4. M-Values

In empirical studies on psychological barriers, the analysis is often restricted to 2 digits to isolate effects at the 1000 or 100 level. These 2 digit values are known as M-Values, with M^{1000} denoting the 2 digits for tests of barriers at the 1000 level and M^{100} denoting the 2 digits for tests of barriers at the 100 level. To illustrate the calculation of M-values with an example, if a stock closes at 2430.5, then M^{1000}=43 and M^{100}=30. If psychological barriers exist at the 100 level, what we would expect to see is the index closing less frequently at the 100, 200, 300, .., 1100, 2100, 3200, etc. level and hence the M^{100} value of 00 and the M^{100} values in that immediate region occurring less frequently.

Mathematically, M-values can be expressed with the following equations:

$$M_t^{100} = [P_t] \bmod 100.$$

And

$$M_t^{1000} = [100 \times 10^{(\log_{10} P_t) \, mod 1}] \bmod 100.$$

Where $[P_t]$ is the integer part of P_t and mod 100 denotes reduction modulo 100.

2.5. Benford's Law

The law of anomalous numbers (now more commonly known as Benford's Law) states that for commonly observed empirical data, regularities should occur in the First Significant Digits (FSDs) of the data. Benford (1938) proposes that for the FSDs {1,…, 9}, the frequency observed of the each digit $D_1 \in \{1, \ldots, 9\}$ should be approximately $\log_{10}(1+\frac{1}{D_1})$. For example, the frequency at which we should observe a FSD of 1 = $\log_{10}(1+\frac{1}{1})$ = 0.301. Stated in a probabilistic manner, Benford's Law dictates that:

$$P(D_1 = k) = \log_{10}[1 + \left(\frac{1}{k}\right)] \; ; \; k = 1, \ldots, 9$$

This result has been established via combinatorial arguments [Cohen, 1976] as well as other statistical derivations [Hill, 1995]. Arguments for the use of Benford's Law have been put forth in areas such as fraud detection [Nigrini, 1996] as well as in tests of auction prices [Giles, 2007] and other areas of statistical analysis [Judge and Schechter, 2009].

2.5.1. Implications for Tests on the Significance of Round Numbers

The conventional approach to tests on the significance of round numbers in stock prices relies on the critical assumption of uniformity in the distribution of M-values. De Ceuster et al (1997) proposed the first criticism of this assumption based on Benford's Law and showed using a test based on cyclical permutations of returns that there was no evidence for psychological barriers in the DJIA as had been previously suggested by Donaldson and Kim (1993). Intuitively, if Benford's Law holds, M-values cannot be uniformly distributed as we would expect to see 1s occurring with a different frequency than 5s, for example, and tests based on a uniform distribution of M-values are likely to recover significant differences due to the distribution of frequencies according to a manner consistent with Benford's Law which would be wrongly attributed to the presence of psychological barriers.

2.5.2. Application of Benford's Law to M-Values

The general form of Benford's Law gives a realistic model for the distribution of the digits of stock indices. Although Benford's Law is stated for FSDs, the joint distribution for second and higher significant digits is invariant to scale [Pinkham, 1961] and can be stated in the following manner:

$$P(D_1 = d_1, \ldots, D_k = d_k) = \log_{10}[1 + \sum_{i=1}^{k} (d_i \times 10^{k-i})^{-1}],$$

For $d_i \in \{1,2, \ldots, 9\}$ and $d_j \in \{0,1,2, \ldots, 9\}$, j>1.

De Ceuster et al (1997) derived the limit distribution of M-values, which we state here and have applied in parts of this paper:

$$\lim_{t \to \infty} P(M_t^{1000} = k) = \sum_{i=1}^{9} \log_{10}\left(\frac{i \times 10^2 + k + 1}{i \times 10^2 + k}\right)$$

And

$$\lim_{t\to\infty} P(M_t^{100} = k) = \lim_{n\to\infty} \sum_{i_1=1}^{9} \sum_{i_2=0}^{9} \cdots \sum_{i_{n-1}=0}^{9} \log_{10}\left(\frac{\sum_{r=1}^{n-1} i_r \times 10^{n-r+1} + k + 1}{\sum_{r=1}^{n-1} i_r \times 10^{n-r+1} + k}\right) = \frac{1}{100}$$

Hence, based on results derived from Benford's Law, we see that the expected frequencies are non-uniform for M^{1000} but uniform for M^{100} values, consistent with the intuition that the n-th significant digit in an arbitrarily scaled random number is closer to being uniformly distributed when n tends to be large.

3. Methodology

3.1. Tests on the Distribution of M-Values

The first test evaluates evidence for the significance of certain regions of numbers by testing the distribution of M-Values using a Chi-Squared-Goodness-of-Fit test. In the absence of barriers, the traditional assumption is that we can expect M-Values to approximately follow a uniform distribution over a long time period. The X^2 test is hence conducted to evaluate the fit of the M-values to a uniform distribution.

The M-values are aggregated into 10 disjunct categories centred on the 00 round number region, i.e. 96-05, 06-15, … , 76-85, 86-95 and the frequency of occurrence of M-values for each category is recorded. The expected number of M-values in each category if it followed a uniform distribution is calculated as follows:

$$E_i = \frac{N}{10}$$

Where E_i is the expected number of observations in category I where I = 1, 2, …, 10 and N represents the total number of observed M-Values.

A X^2 test with the following hypotheses is then conducted:

H_0: The M-Values follow a uniform distribution

H_1: The M-Values do not follow a uniform distribution

The X^2 statistic is calculated as follows:

$$X^2 = \sum_{i=1}^{I} \frac{(O_i - E_i)^2}{E_i}$$

The number of degrees of freedom for each test is 9. The results of the tests are reported in **Table 2**.

3.2. Tests on the Behaviour of Prices Around Round Numbers

While the Chi-Squared-Goodness-of-Fit test reveals some information on the distribution of M-values and has the potential to provide evidence for the possible presence of barriers, proof of a non-uniform distribution of M-values is, at best, a necessary but not sufficient result for the proof of the presence of barriers around round numbers. One primary limitation of the uniformity test is its inability to isolate which regions differ as well as the lack of information on directionality which makes it impossible to conclude if the M-values exhibit evidence of price clustering or price barriers and whether that occurs around the round number region. In order to obtain information on directionality, we conduct tests using 2 regression models, the Barrier Proximity Test from Donaldson and Kim (1993) and the Barrier Hump Test based on a model of price movements as proposed by Bertola and Caballero (1992).

3.2.1. Barrier Proximity Test

The barrier proximity test evaluates the presence of barriers through the use of a simple ordinary least squares regression. A vector F(M) is created with a length of 100 which registers the relative frequency of M-Values occurring in each region. If the M-Values are uniformly distributed, then we would expect each M-value to have a relative frequency of 1. We adopt the specification used in Koedijk and Stork (1997) as it yields results that are more interpretable with non-overlapping categories than the specification employed in Donaldson and Kim (1993).

The regression is then run with the following specification:

$$F(M) = C + \beta_1 D_1 + \beta_2 D_2 + \beta_3 D_3 + \varepsilon$$

Where C represents a constant term and ε the error term and D_i represents dummy variables, where the dummy variables are defined as follows:

D_1 = 1 for M = {98, …, 02} and D_1 = 0 for M = {03, ..., 97}

D_2= 1 for M = {92, …, 97}, {03, …, 08} and D_2 = 0 for M = {98, …, 02}, {09, …, 91}

D_3 = 1 for M = {85, ..., 91}, {09, …, 15} and D_3 = 0 for M = {92, ..., 08}, {16, …, 84}

Where {a,..b} represents the set of values inclusive of and between a and b. For example, {98, …, 02} represents the set of values {98, 99, 00, 01, 02}.

The results of the Barrier Proximity Test are presented in **Tables 3 and 4**. If there are no barriers, we would expect a significant constant value at 1 and all the β values to be close to zero and not statistically significant. A significant and negative β value would represent evidence for price barriers in that category (under the assumption that prices are uniformly distributed) while a positive β value would suggest evidence of price clustering within that region.

3.2.2. Barrier Hump Test

The barrier hump test is based on the work of Bertolla and Caballero (1992), who look at trading in the foreign exchange market and consider the case of a perforate barrier zone in which a support level or resistance can be crossed but only with some difficulty. Once such a barrier is crossed, a realignment occurs during which time prices move by an unusually large amount and a new barrier zone is established that contains that new price level. The authors prove that given the sudden change in prices associated with the crossing of the barrier, the absence of unexploited excess profit opportunities implies that the amount of time the price spends close to barriers must be less than the amount of time it spends away from the edges of the zone. Thus, the ergodic distribution of price realizations within a perforate barrier zone should be hump-shaped, with less frequent price realizations close to the edges of the zone and more frequent realizations in the centre of the zone. This is what the barrier hump test assumes in its model of price movements and the presence of barriers is tested by running a regression against a quadratic specification as defined by the following equation that has been adapted from Donaldson and Kim (1993):

$$F(M) = \alpha + \gamma M + \delta M^2 + \varepsilon$$

In the absence of barriers, we would expect δ to be zero and in the presence of barriers we would expect δ to be negative. The results of the barrier hump test are reported in **Tables 5 and 6**.

3.3. Cyclical Permutation of Returns

We use a Monte Carlo simulation to generate a price series that, by construction, would contain no price barriers. The Monte Carlo set-up consists of simulating returns $\widetilde{R}_t$, t = 2, …, T, then computing the stock recursively from the following equations:

$$\tilde{P}_1 = P_1$$

$$\tilde{P}_t = P_{t-1} \exp(R_t), t = 2, \ \ldots, \mathrm{T}$$

Let $R = (R_2, \ldots, R_T)$ be the vector of actual returns over the period. Returns are calculated using the following equation:

$$R_t = \ln(\frac{P_t}{P_{t\text{-}1}}), \ \mathrm{t} = 2, \ \ldots, \mathrm{T}$$

Returns are then simulated using cyclical permutations of R. A cyclical permutation of R is any (R_t, R_{t+1}, … , R_T, R_2, R_3, …, R_{t-1}) and we conduct simulations for all possible cyclical permutations of R. The advantage of using a simulation based on cyclical permutations is that it allows us to simulate price paths with the same starting and ending value. Next, it allows us to consider calendar effects and preserves high volatility clusters and other anomalies in the observed stock index and hence provides the 'closest' approximation to the actual data generating process [De Ceuster et al, 1997]. The simulated data series is employed in tests in section 3.4.1 and 3.5.

3.4. Criticisms of the Uniformity Assumption

In this section, we perform 2 types of tests, we use a Chi-Squared Goodness of Fit Test based on the results from the Monte Carlo Simulation to evaluate the validity of the uniformity assumption and we use Kuiper's modified Kolmogorov-Smirnov test to test the fit of Benford's Law as an alternative hypothesis for the distribution of M-values.

3.4.1. Uniformity Tests

For each price obtained from the simulation we calculate the corresponding M^{1000} and M^{100} value and each run thus gives an empirical frequency distribution of M-values. In this section we test the assumption of uniformity of the simulated runs using the Chi-Square-Goodness-of-Fit-Test as specified in section 3.1. This allows us to test the uniformity assumption on a simulated price series that by construction does not contain price barriers at round numbers. If the M-values from the simulated price series consistently show deviations from a uniform distribution as well, we can conclude that the uniformity assumption for the observed price series is not valid and conclusions of the presence of price barriers based on evidence of deviations from a uniform distribution are hence invalid as well. **Tables 8 and 9** report the results of these uniformity tests.

3.4.2. Kuiper's modified Kolmogorov-Smirnov Goodness of Fit Test [KST]

In this section, we test whether the observed data exhibits characteristics consistent with the predictions from Benford's Law. Stated formally, we investigate whether the empirical data has characteristics consistent with the limit distributions for the M-values as stated in section 2.4.2. The Chi-Squared-Goodness-of-Fit test is not employed for this investigation as it has been shown to exhibit low statistical power in tests for Benford's Law when used with small samples. The KST and other common non-parametric tests such as the Cramér-von Mises test have also been deemed to be unsuitable for this investigation due to the "circular" nature of M-values [Giles (2007)]. What we mean when we refer to the "circular" nature of M-values is the fact that 99, for example, is very close to and not very far from 00 as would be the case for a unidirectional linear data set. We employ the KST because it recognizes the ordinality and circularity of the data and does not depend on the choice of origin. One additional feature of this test that is particularly useful is the fact that the null distribution of the test statistic is invariant to the hypothesised distribution, for all N.

The KST is conducted with the following hypotheses:

H_0: The M-values are distributed in a manner consistent with the limit values derived from Benford's Law

H_1: The M-values are not distributed in a manner consistent with the limit values derived from Benford's Law

The statistic for this test, the V_N statistic, is calculated using the following equation:

$$V_N = \text{Max}_x\,[F_e(x) - F_b(x)] + \text{Max}_x[F_b(x) - F_e(x)]$$

Where $F_e(x)$ is the empirical CDF and $F_b(x)$ is the CDF based on a distribution that follows Benford's Law.

The critical values for the null distribution of the transformed statistic are then calculated as follows:

$$V_N^* = V_N(N^{0.5} + 0.155 + 0.24N^{-0.5})$$

Studies that employ the use of the KST normally use the critical values tabulated by Stephens (1970) in evaluating the results of the test. However, the results of Stephens (1970) have been shown to be too conservative once Benford-specific values are derived and hence we employ the values presented in Morrow (2010) instead. These critical values have been derived from an application of the central limit theorem to a multivariate Bernoulli variable that corresponds to a random variable that exactly satisfies Benford's Law and thus yields critical values that have been shown to be more appropriate for tests of Benford's Law than those presented in Stephens (1970).The results of the KST are presented in **Table 9**.

3.5. Testing the Presence of Psychological Barriers without the Uniformity Assumption

We follow the methodology proposed by De Ceuster et al (1997) for the testing of barriers in a way that does not rely on an assumption of a uniform distribution of M-values. This is done by comparing the observed frequency of M-values around regions of potential psychological barriers with the corresponding frequency in the simulated stock indices from the Monte Carlo simulation.

Let ω be a set of M-values representing the region around a round number. We consider the following regions for a set-up similar to that used in section 3.2.1:

ω={00}, {98, …, 02}, {96, …, 05}, {90, … , 09}

For a given choice of ω, the stock index P_t is considered to be in the region of a barrier if $M_t \in \omega$, i.e. if $I_\omega(M_t)$=1, where I_ω is the indicator function of ω. Let τ denote the relative amount of time spent by an index in the neighbourhood of a psychological barrier as indicated by its relative frequency, then:

$$\tau = \sum_{t=1}^{T} \frac{I_\omega(M_t)}{T}$$

Tables 9 and 10 report the value of τ for each ω and each type of M-value. A small τ lends support to the psychological barrier hypothesis for each ω selected, and we formally test the hypothesis by comparing the percentage of τ calculated for the simulated stock indices that are smaller than or equal to the corresponding τ statistic calculated from the observed prices of each stock index. These percentages that are reported in **Tables 10 and 11** are essentially left tail percentages and can be interpreted in an analogous way to p-values from normal statistical tests. This one-sided test is similar to the type of test employed in section 3.2.1 but does not use uniformity as a benchmark.

4. Results and Discussion

4.1. Data

In this study, we examine a 10 year period from January 2001 to December 2011 for 5 European Stock Indices [FTSE, CAC, DAX, ATX, SMI] and obtained data on the daily closing prices of each index. The presence of psychological barriers is more likely to manifest itself in indices that are closely watched by participants and that are actively traded on a daily basis, hence these indices were chosen based on their importance in the European markets. In addition, as we are investigating the presence of barriers at the 100 and 1000 level, only indices with a sufficiently large range were considered in our shortlist and a summary of the data for these 5 indices can be found in **Table 1**.

4.1.1. Summary of Markets Investigated

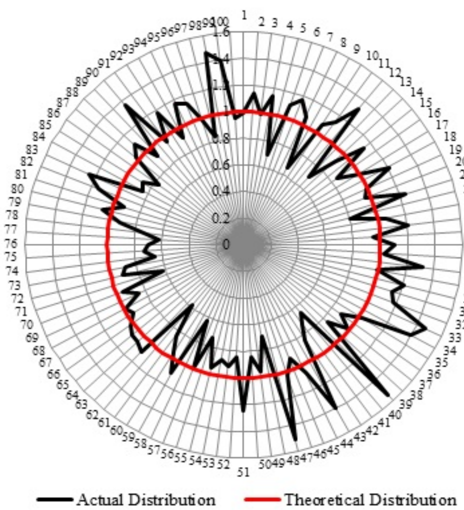
Table 1: Key Statistics of Markets Investigated

Symbol	Market	Low	High
ATX	Austria	1003.72	4981.87
CAC	France	2403.04	6168.15
DAX	Germany	2202.96	8105.69
FTSE	United Kingdom	3287.00	6732.40
SMI	Switzerland	3675.40	9531.50

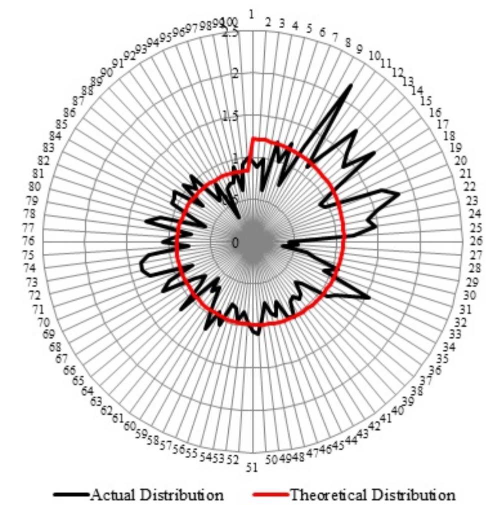
4.1.2. Frequency Distribution of M-Values

The following figures illustrate the differences observed between the actual and theoretical distributions (according to Benford's Law) of M100 and M1000 values for the 5 markets:

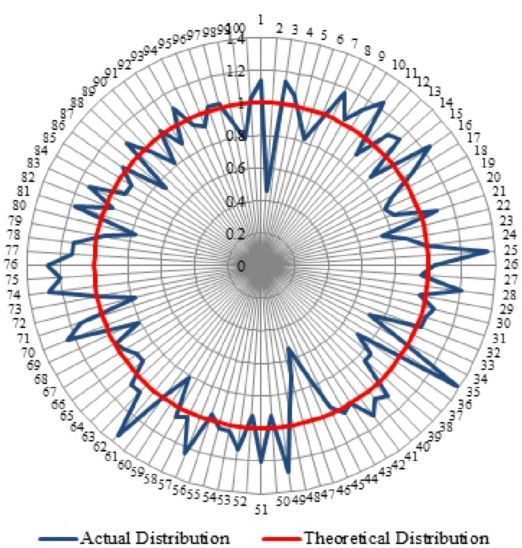
ATX M100



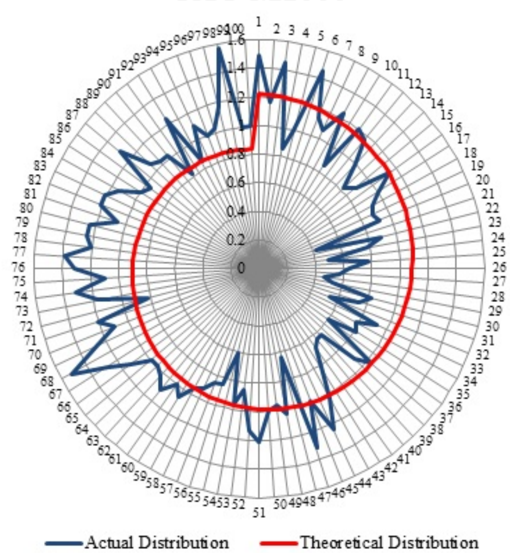
ATX M1000



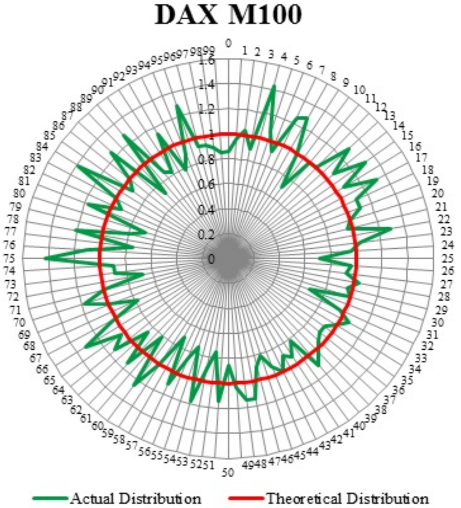
CAC M100



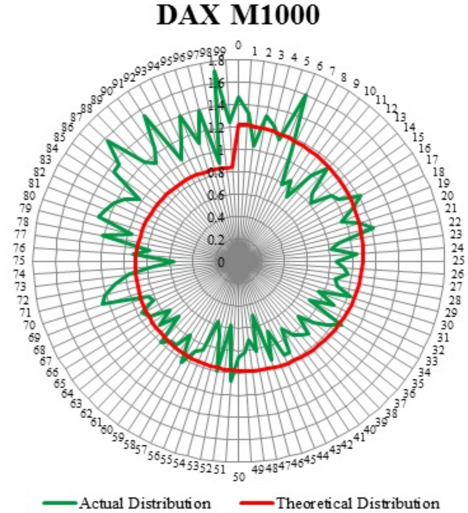
CAC M1000



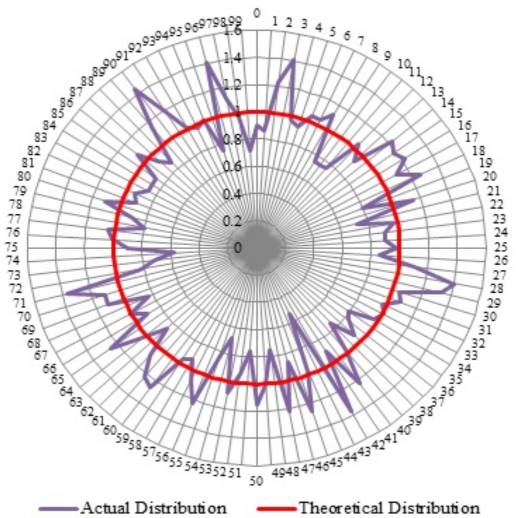
DAX M100



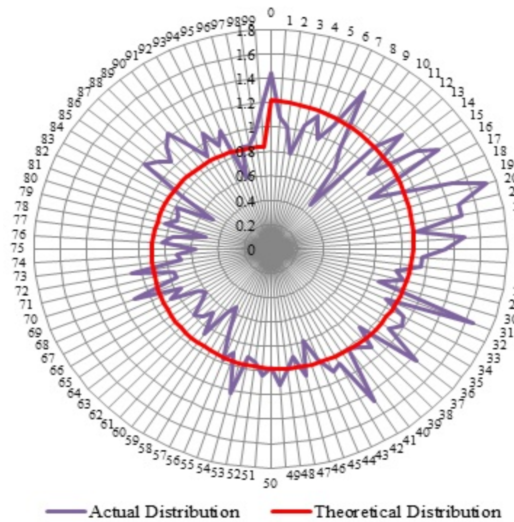
DAX M1000



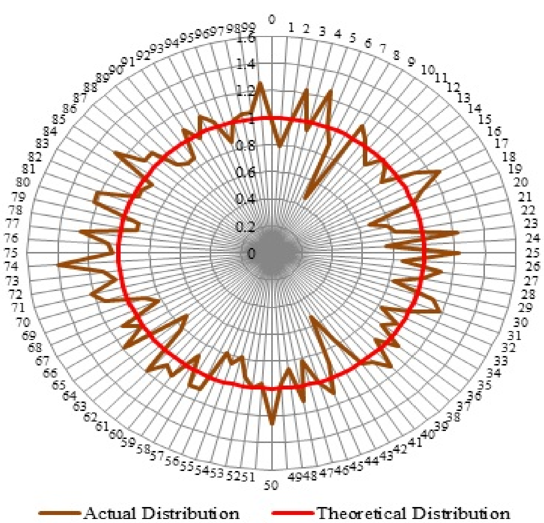
FTSE M100



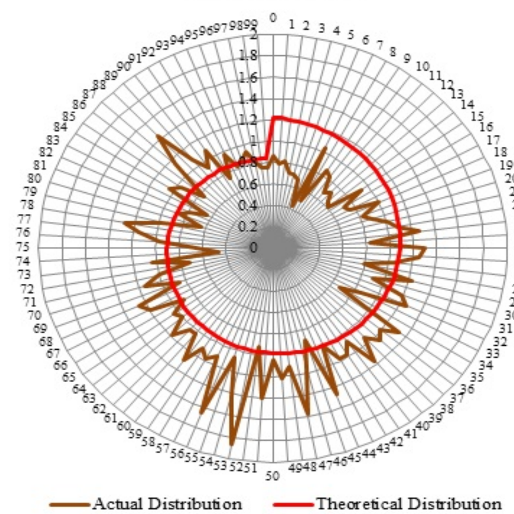
FTSE M1000



SMI M100



SMI M1000



4.2. Test on the Distribution of M-Values

Table 2: X^2-Statistics for M^{100} and M^{1000} values

Symbol	X^2 -Statistic M^{1000}	X^2 -Statistic M^{1000}
FTSE	8.774658	90.56731***
CAC	7.625222	80.98224***
DAX	4.807487	78.58289***
ATX	22.89358***	106.3138***
SMI	6.668103	76.48132***

*** Represents results that are significant at the 99% confidence level

Table 2 reports the results from the Chi-Squared-Goodness-of-Fit-Test. From this test, we see strong evidence to reject the hypothesis that M-values follow a uniform in the M^{1000} values of all the indices and in the M^{100} values of the ATX index. There is not sufficient evidence to reject the hypothesis that the M-values follow a uniform distribution in the M^{100} values of the FTSE, CAC, DAX and SMI index.

4.3. Tests on the Behaviour of Prices Around Round Numbers
4.3.1. Barrier Proximity Test

Table 3: Regression of M^{100} Frequencies on M^{100} Value Dummies

M^{1000}	ATX	CAC	DAX	FTSE	SMI
Variables	Freq100[a]	Freq100	Freq100	Freq100	Freq100
D_1^b	0.1118	-0.0961	-0.0712	-0.0548	0.0036
	(0.0989)	(0.0791)	(0.0824)	(0.0889)	(0.0763)
D_2^c	0.0488	-0.0138	0.0756	0.0742	-0.0173
	(0.0668)	(0.0534)	(0.0556)	(0.0601)	(0.0515)
D_3^a	0.0484	0.0070	0.0251	0.0224	-0.0015
	(0.0626)	(0.0500)	(0.0521)	(0.0563)	(0.0483)
Constant	0.9818***	1.0055***	0.9910***	0.9907***	1.002***
	(0.0257)	(0.0205)	(0.0214)	(0.0231)	(0.0198)

Robust standard errors in parentheses: *** $p<0.01$, ** $p<0.05$, * $p<0.1$
a – Freq100 is the relative frequency of M100 values in the index.
b – D_1 is the dummy for M100 values between 98 and 2.
c – D_2 is the dummy for M100 values between 92-97 and 3-8.
d – D_3 is the dummy for M100 values between 85-91 and 9-15.

Table 4: Regression of M^{1000} frequencies on M^{1000} Value Dummies

M^{1000}	ATX	CAC	DAX	FTSE	SMI
Variables	Freq1000[a]	Freq1000	Freq1000	Freq1000	Freq1000
D_1^b	-0.0403	0.2665**	0.4387***	0.1506	-0.2760**
	(0.1454)	(0.1081)	(0.0931)	(0.1229)	(0.1090)
D_2^c	-0.1431	0.1664**	0.2663***	0.0162	-0.3012***
	(0.0982)	(0.0730)	(0.0628)	(0.0830)	(0.0736)
D_3^a	0.2291**	0.0797	0.2146	0.0770	-0.1677**
	(0.0921)	(0.0684)	(0.0589)	(0.0778)	(0.0690)
Constant	0.9871***	0.9555***	0.9161***	0.9797***	1.0734***
	(0.0378)	(0.0281)	(0.0242)	(0.0319)	(0.02833)

Robust standard errors in parentheses: *** $p<0.01$, ** $p<0.05$, * $p<0.1$
a – Freq100 is the relative frequency of M100 values in the index.
b – D_1 is the dummy for M100 values between 98 and 2.
c – D_2 is the dummy for M100 values between 92-97 and 3-8.
d – D_3 is the dummy for M100 values between 85-91 and 9-15.

From the Barrier Proximity Test, we see evidence for the presence of barriers at the 1000 level in the SMI and possible evidence of price clustering around round numbers at the 1000 level in the CAC and DAX indices.

4.3.2. Barrier Hump Test

Table 5: Regression of M^{100} Frequencies on M^{100} and squared M^{100} values

M^{100}	ATX	CAC	DAX	FTSE	SMI
Variables	Freq100[a]	Freq100	Freq100	Freq100	Freq100
M_b	-0.00360	-0.00245	-0.00124	0.00311	0.00469**
	(0.00331)	(0.00226)	(0.00216)	(0.00280)	(0.00188)

Table 5: Regression of M^{100} Frequencies on M^{100} and squared M^{100} values - continued

M_c^2	2.53e-05	2.63e-05	6.44e-06	-2.79e-05	-5.57e-05***
	(3.14e-05)	(2.23e-05)	(2.08e-05)	(2.62e-05)	(1.76e-05)
Constant	1.105***	1.037***	1.043***	0.940***	0.949***
	(0.0758)	(0.0462)	(0.0498)	(0.0594)	(0.0407)

Robust standard errors in parentheses: *** p<0.01, ** p<0.05, * p<0.1
a – Freq100 is the relative frequency of M100 values in the index.
b – *M* is the variable containing M100 values.
c – M^2 is the variable that contains the squared M100 values.

Table 6: Regression of M^{1000} Frequencies on M^{1000} and squared M^{1000} values

M^{1000}	ATX	CAC	DAX	FTSE	SMI
Variables	Freq1000[a]	Freq1000	Freq1000	Freq1000	Freq1000
M_b	0.00905	-0.00204	0.00133	0.00509	0.0177***
	(0.00649)	(0.00504)	(0.00362)	(0.00513)	(0.00467)
M_c^2	-7.74e-05	2.77e-05	-1.91e-05	-3.11e-05	-0.000161***
	(6.16e-05)	(4.69e-05)	(3.15e-05)	(7.43e-05)	(4.60e-05)
Constant	0.839***	1.002***	1.006***	0.868***	0.574***
	(0.0822)	(0.0636)	(0.0901)	(0.0796)	(0.109)

Robust standard errors in parentheses: *** p<0.01, ** p<0.05, * p<0.1
a – Freq1000 is the relative frequency of M1000 values in the index.
b – *M* is the variable containing M1000 values.
c – M^2 is the variable that contains the squared M1000 values.

From the Barrier Hump Test, we find evidence that confirms the results from the Barrier Proximity Test of barriers around round numbers at the 1000 level for the SMI. There is also statistically significant evidence of barriers at the 100 level for the SMI. However, the δ value for that regression is fairly small and unlikely to be indicative of any operationally significant barriers around that region. There is no statistically significant evidence of possible barriers in the remaining indices at both the 100 and 1000 levels.

4.4. Testing the Uniformity Assumption
4.4.1. Monte Carlo Simulation [Uniformity Tests]

Table 7: Rejection of uniformity test

Rejections of Uniformity of M-Values in Simulations (%)						
M-Values	**Level of Significance**	**ATX**	**CAC**	**DAX**	**FTSE**	**SMI**
M^{100}	99%	36%	2%	1%	1%	1%
M^{1000}	99%	99%	88%	69%	73%	88%

Table 8: Percentage of simulation statistics greater than observed statistic

Simulation Statistic Greater than the Observed Statistic (%)						
M-Values	**Level of Significance**	**ATX**	**CAC**	**DAX**	**FTSE**	**SMI**
M^{100}	99%	32%	61%	83%	44%	72%
M^{1000}	99%	90%	63%	8%	7%	32%

For the M-values obtained from the Monte Carlo simulation, we see very few rejections of uniformity at the 100 level for 4 out of 5 of the indices (CAC, DAX, FTSE, SMI) consistent with the expected limit distribution of the M-values according to Benford's Law [which approaches a uniform distribution at the lmit]. The ATX index shows a surprising result, with a significant proportion of the simulations returning M-Values with non-uniform distributions at the 100 level. This could be due to the smaller range of the ATX index and could explain the non-uniform distribution of M-values at the 100 level in the observed prices as well.

At the 1000 level, we see a significant proportion of M-values from the simulation exhibiting characteristics of a non-uniform distribution for all 5 indices. This is consistent with the expected results if the M-values are indeed distributed according to Benford's Law, and the results from this simulation provide indirect evidence to support the criticism of the uniformity assumption that traditional tests have been predicated upon.

4.4.2. Kuiper's Modified Kolmogorov-Smirnov Test

From Morrow (2010), the critical values for this test are 1.191 at the 90% confidence level, 1.321 at the 95% confidence level and 1.579 at the 99% confidence level.

Table 9: Kolmogorov-Smirnov Test results

Symbol	V_N^* Statistic M^{1000}	V_N^* Statistic M^{100}
FTSE	0.50213	0.18416
CAC	1.03219	0.19798
DAX	0.95697	0.22499
ATX	0.54861	0.42490
SMI	0.91475	0.21206

From this test, we see that there is not sufficient evidence at the 90% confidence level to reject the hypothesis that the distribution of M-values conforms to that derived from Benford's Law in all of the indices at both the M^{1000} and M^{100} values and hence we conclude that the criticism of the uniformity assumption applied in earlier tests based on Benford's law is valid.

4.5. Testing the Presence of Psychological Barriers Without the Uniformity Assumption

Table 10: τ statistic for each category (ω) for M^{100} values in each index

	M^{100}			
ω	**{0}**	**{98,…,02}**	**{96,…,05}**	**{90,…,09}**
τ(ATX)	0.99	5.47	10.72	21.03
τ(ATX Simulation) [%]	46.24	79.52	77.54	74.53
τ(CAC)	1.14	4.55	9.27	19.50
τ(CAC Simulation) [%]	75.24	11.37	8.10	27.32
τ(DAX)	0.86	4.60	9.84	20.07
τ(DAX Simulation) [%]	39.18	13.55	34.80	51.62
τ(FTSE)	0.90	4.68	10.51	20.84
τ(FTSE Simulation) [%]	27.57	23.15	82.04	86.11
τ(SMI)	0.97	5.03	10.49	19.90
τ(SMI Simulation) [%]	41.77	49.64	79.06	45.51

Table 11: τ statistic for each category (ω) for M^{1000} values in each index

	M^{1000}			
ω	**{0}**	**{98,…,02}**	**{96,…,05}**	**{90,…,09}**
τ(ATX)	0.95	4.73	9.10	18.53
τ(ATX Simulation) [%]	50.39	52.11	48.99	47.78
τ(CAC)	1.49	6.11	12.15	22.49
τ(CAC Simulation) [%]	93.85	78.85	79.75	65.68
τ(DAX)	1.46	6.77	12.76	24.60
τ(DAX Simulation) [%]	90.05	99.32	98.72	98.04
τ(FTSE)	1.44	5.65	10.37	20.52
τ(FTSE Simulation) [%]	97.05	84.95	73.04	71.89
τ(SMI)	0.86	3.99	7.87	16.13
τ(SMI Simulation) [%]	23.99	7.47	3.77	2.80

From the test, we see that all of the indices besides the SMI do not have evidence of psychological barriers when results are compared with the simulated return series at both the 100 and 1000 levels. Only the SMI index shows significant evidence of barriers at the 1000 level. In addition, the results also suggest evidence of price clustering around round numbers in the CAC and DAX indices at the 1000 level even after the expected concentration within the region due to Benford-specific effects are accounted for.

5. Conclusion

In this study, we test for the significance of round numbers in 5 European equity markets under different assumptions for the distribution of digits. We find evidence for barriers in 1 index [SMI] at the 1000 level under the assumption of uniformity but no significant evidence of barriers at the 100 level and at the 1000 level in the remaining indices. This result is consistent with the findings of Dorfleitner and Klein (2009) and in line with the notion that barriers have disappeared in many of these indices after knowledge of their location became widespread. Instead, there seems to be evidence of price clustering around the regions of round numbers in 2 of these indices [CAC, DAX] at the 1000 level which could be due to prolonged periods of uncertainty when prices enter the region that manifest itself as a trading range within the region instead of a barrier around the region.

Next, we tested for the appropriateness of the uniformity assumption by comparison with a simulated price series that had no price barriers (by construction). We find evidence that the uniformity assumption is appropriate in most indices [ATX, CAC, DAX, FTSE] at the 100 level, as predicted by the limit distribution for these digits derived from Benford's Law. For the 1000 level, we find results consistent with the predictions from Benford's Law as well that substantiates the argument that the application of the uniformity assumption for tests at the 1000 level is inappropriate in all 5 indices.

Next, having established that the uniformity assumption is inappropriate for tests at the 1000 level, we investigate whether Benford's Law provides a good alternative model to explain this phenomenon by explicitly testing the empirical fit using Kuiper's Modified Kolmogorov-Smirnov Goodness of Fit Test. For the 5 indices, we find not sufficient evidence to reject the hypothesis that the observed M-values are distributed according to Benford's Law.

Finally, we re-examine the evidence of the significance of round numbers using a test that does not assume a uniform distribution of M values. We compare the observed frequency distribution of M values with that derived from a Monte Carlo simulation based on cyclical permutations of returns and find evidence of price barriers in 1 index [SMI] at the 1000 level and possible evidence of price clustering in 2 indices [CAD, DAX] at the 1000 level. This is consistent with the findings from the tests based on the uniformity assumption and hence while there may be evidence against the use of the uniformity assumption in tests at the 1000 level, the conclusions drawn from a test that implicitly incorporates the predicted outcomes from Benford's Law has not materially changed.

References

[1] Benford, F. (1938). The law of anomalous numbers. *Proceedings of the American Philosophical Society,* 78, 551-572.

[2] Bertola, G., & Caballero, R. (1992). Target zones and realignments. *American Economics Review,* 82, 520-536.

[3] Cohen, D. (1976). An explanation of the first digit phenomenon. *Journal of Combinatorial Theory*, 20, 367-370.

[4] De Ceuster, M. K. J., Dhaene, J., & Schatteman, T. (1998). On the hypothesis of psychological barriers in stock markets and benford's law. *Journal of Empirical Finance*, 5, 263-279.

[5] Donaldson, R. G., & Kim, H. Y. (1993). Price barriers in the dow jones industrial average. *Journal of Financial and Quantitative Analysis*, 28(3), 313-330.

[6] Dorfleitner, G., & Klein, C. (2009). Psychological barriers in european stock markets: where are they?. *Global Finance Journal*, 268-285.

[7] Edwards, R. D., Magee, J., & Bassetti, W. H. C. (2007). Technical analysis of stock trends. 9th ed., 245.

[8] Giles, D. E. (2007). Benford's law and naturally occurring prices in certain ebay auctions. *Applied Economics Letters*, 14, 157-161.

[9] Harris, L. (1991). Stock price clustering and discreteness. *Review of Financial Studies* 4, 389-415.

[10] Hill, T. P. (1995). A statistical derivation of the significant-digit law. *Statistical Science*, 10, 354-363.

[11] Hill, T. P. (1995). Base-invariance implies benford's law. *Proceedings of the American Mathematical Society*, 123, 887-895.

[12] Judge, G., & Schechter, L. (2009). Detecting problems in survey data using benford's law. *Journal of Human Resources*, 44, 1-24.

[13] Koedijk, K. G., & Stork, P. A. (1994). Should we care? Psychological barriers in stock markets. *Economics Letters*, 427-432.

[14] Ley, E., & Varian, H.R. (1994). Are there psychological barriers in the Dow-Jones index? *Applied Financial Economics*, 4, 217-224.

[15] Murphy, J. J. (1999), Technical analysis of the financial markets: a comprehensive guide to trading methods and applications. 2nd ed, 55.

[16] Morrow, J. (2010). Benford's law, families of distributions and a test basis.

[17] Nigrini, M. (1996). A taxpayer compliance application of benford's law. *Journal of the American Taxation Association*, 18, 72-91.

[18] Pinkham, R. S. (1961). On the distribution of the first significant digits. *Annals of Mathematical Statistics*, 32, 1223-1230.

[19] Schindler, R. M., & Kirby, P. N. (1997). Patterns of rightmost digits used in advertised prices: implications for nine-ending effects. *The Journal of Consumer Research*, 24:2, 192-201.

[20] Sonnemans, J. (2006). Price clustering and natural resistance points in the dutch stock market: a natural experiment. *European Economic Review*, 1937-1950.

[21] Stephens, M. A. (1970). Use of the kolmogorov-smirnov, cramer-von-mises and related statistics without extensive tables. *Journal of the Royal Statistical Society. Series B (Methodological),* 32, 115-122.