Using Monte Carlo simulation to calculate match importance: the case of English Premier League

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USING MONTE CARLO SIMULATION TO CALCULATE MATCH IMPORTANCE: THE CASE OF ENGLISH PREMIER LEAGUE

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This paper presents a new method of calculating match importance (a common variable in sports attendance demand studies) using Monte Carlo simulation. Using betting odds and actual results of 12 seasons of English Premier League, it is shown that the presented method is based on realistic predictions of match results and season outcomes. The Monte Carlo method provides results closest to Jennett's approach; however, it does not require ex-post information and can be used for any type of season outcome.

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1 INTRODUCTION

In national team sports leagues, clubs typically compete to win the championship, get promoted to a higher league, qualify for other competitions, or to avoid relegation. Because the leagues usually use a round-robin tournament system (sometimes combined with play-offs), not all matches have the same impact on the final outcome of the season – there are highly important last-round matches where a single result can decide which team becomes the new league champion, on the other hand, there are unimportant matches between clubs without a realistic chance of being either promoted or relegated. It has been well established in the sports attendance demand literature that important matches attract more spectators; however, there is little agreement on how to actually calculate the importance of a specific match. The problem with using a suboptimal method is that impact of match importance on attendance is underestimated (or the null hypothesis cannot be rejected at all).

This paper presents a new method of calculating match importance using Monte Carlo simulation. As an example, data about 12 seasons of soccer matches (2000/01-2011/12) in the English Premier League are used. First, probabilities of all individual match results until the end of season are estimated based on past performances of all teams. These match result probabilities are then used to simulate the rest of the season and estimate probabilities of various season outcomes (final team ranks). The importance of a specific match for a specific team is then the association between the match result and the final season outcome (e.g. being relegated or not). Using both actual results and betting odds as a benchmark, it is shown that the presented method is based on realistic predictions of both individual match results and season outcomes. The Monte Carlo method is also compared with other common methods for calculating match importance.

2 LITERATURE

There are two distinct components of match importance: first, how likely a team is to achieve a certain outcome, such as championship, promotion, or relegation (this is usually called seasonal uncertainty); second, how much a specific match can influence the probability of this outcome. The literature offers various approaches addressing one or both of these components differing in complexity and utilized information.

Probably the simplest possible method is to use a dummy variable that equals one for all matches in the last X rounds of the season (assuming that late matches tend to be more important). This method can be found, for example, in Paul (2003), who used a dummy variable for all NHL matches played in March and April.

The second method is to use the concept of mathematical certainty/impossibility – for example, when a team leads a competition by seven points, there are two rounds remaining and a win is worth three points, the team is mathematically certain to win the competition (there is not even a theoretical possibility of another outcome). For example, Baimbridge et al. (1996) defined dummy variables for the certain championship and certain relegation in the English Premier League, García and Rodríguez (2002) used a similar approach for Spanish soccer, and Feddersen et al. (2012) for multiple European soccer leagues.

The third approach is to use a more complex rule based on the current team positions, numbers of points, and the number of remaining matches; Baimbridge et al. (1996) and Simmons and Forrest (2006) used dummies for English soccer teams both being in the promotion zone or in the relegation zone; Goddard and Asimakopoulos (2004) asked if a team could be promoted or relegated if all other competing teams got one point in each of their remaining matches; Benz et al. (2009) employed a dummy variable equal to one if a German Bundesliga soccer team was no more than two points behind the current leader and there were at most six rounds until the end of the season.

The fourth main approach, which treats match importance as an interval (rather than binary) variable, was introduced by Jennett (1984) for Scottish soccer and later used by others (Borland and Lye 1992; Dobson and Goddard 1992). Jennett's approach applied to the uncertainty of winning the championship works in this way: first, take the number of points that were eventually necessary to win the championship (of course, this ex-post information is not actually available before the end of the season, but it could be argued that it is possible to estimate it) – let's say it is 65. If it is still theoretically possible for a team to reach 65 points, set match importance for this team to 1/(number of matches necessary to reach 65 points), otherwise set match importance to zero. At the beginning of the season, all teams are able to reach 65 points, but they would need at least 22 matches (assuming 3 points for a win), so match importance equals 1/22. As the season progresses, importance for a specific team either increases towards 1 or drops to zero (when it is no longer possible to win). The match in which the eventual winner reaches 65 points must have the importance equal to one (this can happen in the last round or sooner).

While this short overview of methods of calculating match importance is by no means exhaustive, other methods are usually quite similar or just combine elements of the approaches described above. More thorough discussion can be found in García and Rodríguez (2009).

All the methods described above, while easy to implement, have apparent limitations. Using a dummy variable for the last X rounds of the season is very crude (there are many last-round matches that do not decide anything) and does not distinguish between the importance for the home and away team. Using mathematical certainty/impossibility is too conservative; a team is expected not to win the championship much sooner than it is mathematically eliminated. When using the current team positions, numbers of points, and the number of remaining rounds, the chosen rule is necessarily arbitrary – as argued, for example, in Cairns (1987) and Peel and Thomas (1992) – and unlikely to work well in all possible cases. Jennett's method is the only one that correctly treats match importance as a variable with more than just two possible values, but cannot be used for predictions (uses ex-post information) and cannot be easily adapted for other outcomes other than the championship.[1] Also, none of the methods above takes into account strengths of the remaining opponents or final table ranking criteria. In the rest of this paper, it is shown that the proposed Monte Carlo method of calculating match importance can solve all these problems.

[1] For example, if Jennett's method is used for European qualification, the key qualification matches for the eventual league winner will happen when its qualification is not really in doubt anymore. This problem would be compounded when using Jennett's method for relegation; however, Jennett (1984) proposed to modify the fraction denominator to the number of matches remaining until the end of the season, which partially solves the problem for this specific criterion.

3 DATA

To show how the Monte Carlo method works and to verify that it gives realistic results, data about English Premier League are used; however, the method can be easily adapted to any other similar competition. The dataset consists of days/times and final results of all 4,560 matches played in 12 seasons (2000/01-2011/12).

In each season of Premier League, there are 20 teams playing two matches (one home and one away) against each other, so each team plays 38 matches per season. Winning a match is worth three points, drawing a match one point, and losing a match zero points. English Premier League does not use the head-on matches criterion to rank teams with the same number of points, so the final table ranking criteria are total points, total goal difference, and total goals scored (in this order). The first team wins a championship title and the last three teams are relegated to a lower competition – therefore, winning championship and not being relegated are two primary goals that enter into match importance calculations. Other possible goals could be qualifying for a European competition (usually first five teams) or just placing as well as possible.

To verify predictions of individual match results, the latest available betting odds of a big British bookmaker William Hill are used – these odds have been obtained for all but 16 matches played since December 26th, 2005; altogether for 2,477 matches. The odds have been converted to implied probabilities of a home win, draw, and away win.[2] All data including betting odds have been exported from a sports database Trefík.[3]

[2] For example, the odds for the Liverpool – Chelsea match played on May 8th, 2012, were 2.10 (home win) – 3.30 (draw) – 3.50 (away win). This traditional form of betting odds indicates what multiple of the original sum the bettor gets if the result actually happens. To convert it to probabilities, the numbers are first inverted: 0.476 – 0.303 – 0.286. The new numbers add up to more than one – in this case 1.065 – to keep the betting agency profitable, so it is necessary to divide them by their sum to get the final probabilities 0.447 – 0.284 – 0.269. This common method is described, for example, in Benz et al. (2009).

[3] The database is available at www.trefik.com. The data were exported on June 7th, 2012. The match result data were also selectively cross-checked against the website Soccerway (www.soccerway.com).

4 METHOD

As stated above, the importance of a specific match for a specific team can be decomposed into two components – first, the probability that the team reaches a particular season outcome, such as championship or not being relegated; second, how this probability depends on the match result. More formally, it is necessary to calculate the probabilities of various outcomes conditional on the specific match result. The match importance can then be expressed as a measure of association between the match result and the outcome.

Multiple relevant outcomes imply multiple types of match importance; a particular match can have a small influence on the probability that a team wins the competition, a large influence on the final rank, and a zero influence on the probability that the team is relegated. These various types of match importance are likely to be valued differently by potential match spectators.

The proposed Monte Carlo method of consists of three steps leading to the desired result: first, calculate the result probabilities of all the remaining matches of the season; second, use these probabilities to estimate the probabilities of final team ranks; third, calculate the association between the match result and various season outcomes .

4.1 STEP 1: ESTIMATING PROBABILITIES OF MATCH RESULTS

There are two basic approaches to the first step of finding result probabilities of a specific match – use betting odds or use the past results of both teams. While betting odds could reflect all available information not necessarily included in the past results, such as injuries and suspensions of key players (see Peel and Thomas 1992), there are many papers showing systematic biases – for example, Cain et al. (2000) analyzed betting odds quoted by William Hill for UK soccer matches and found that bets on favorites (as opposed to longshots) had a higher expected value. However, the key disadvantage of betting odds is their unavailability for matches further in the future. Therefore, the only feasible choice is to use the past results.

To calculate probabilities of results of a specific match, this paper uses a method commonly employed in the sports betting literature and described, for example, in Dixon and Coles (1997). For a given match, this method works in the following way: first, calculate the home team's average score in the last 19 home matches (one rolling season) and the away team's average score in the last 19 away matches;[4] second, assume that the number of goals scored by the home team and the

[4] If there are fewer than 19 matches available, the averages are calculated from only these matches. It is recommended to have at least one extra season of match results available; however, there will always be situations where there is a team with zero match history (e.g. just promoted). Therefore, it is necessary to

number of goals scored by the away team are two independent[5] Poisson-distributed variables with the following expected values:

$$\lambda_{\text{home team}} = \frac{\text{average goals scored by home team} + \text{average goals conceded by away team}}{2}$$

$$\lambda_{\text{away team}} = \frac{\text{average goals conceded by home team} + \text{average goals scored by away team}}{2}$$

Based on the expected values, it is possible to construct corresponding probability distributions of goals scored by each team, compute joint probabilities of all possible match scores, and subsequently also the probabilities of a home win, draw, and away win.

The length of the period used to calculate average scores is a compromise between capturing the current strengths of both teams and using a sample size that is not too small. Possible modifications would be to change the length of the period or to give more weight to recent results. The key assumption is that team strengths do not change much over time, since results of past matches are utilized to calculate probabilities of all match results until the end of the season.

4.2 Step 2: Estimating probabilities of final ranks

In the second step, probabilities of all match results until the end of the season are combined with the actual results so far[6] to find out the probabilities of final team ranks conditional on a particular match result. In most cases, the final rank probabilities cannot be found analytically – the number of all possible scenarios is simply too high. However, it is possible to use the Monte Carlo method to estimate these probabilities with any desired level of precision.

initialize histories of all teams with at least one home match and one away match result. This paper uses one 1.522:1.105 home win and one 1.105:1.522 away loss, where the score is based on the average score of all matches in the dataset. These artificial matches drop from the history after one full season of results is available.

[5] In the Premier League dataset, the independence assumption seems close to the reality; the correlation between goals scored by home and away teams is close to zero (-0.050 for the whole dataset and +0.048 for the dataset restricted to matches where no team scored more than one goal). Some possible modifications, such as using bivariate Poisson distribution or various ad-hoc adjustments, are discussed in Dixon and Coles (2007).

[6] In this paper, matches played on the same day as the match whose importance is being calculated are considered to take place in the future (so their results are not known). This lag can be modified depending on spectators' behavior – for example, if people decide to attend a match several days in advance, even the match results several days in the past might be considered unknown.

The method is implemented as follows: first, start with a league table based on all matches played so far; second, use the match result probabilities derived in the previous step to randomly choose results (exact scores) of all the remaining matches (including the match whose importance is being calculated); third, calculate the final league table (using any ranking criteria applicable); fourth, note down the result of the match whose importance is being calculated and final ranks of both teams. Repeat these steps as many times as necessary to reach a desired precision. For each team, categorize the match results into a win, a draw, and a loss,[7] and construct a contingency table where one variable is the match result and the other variable is the final team rank (also repeat this step for the other team). From the contingency table, it is possible to calculate relative frequencies of various match results, final ranks, and their combinations.

These relative frequencies are estimates of true probabilities that could (theoretically) be found out analytically by going through all possible scenarios. The standard error of a probability estimate equals $\sqrt{[p(1-p)/n]}$, where p is the actual probability and n is the number of simulation runs, so by increasing the number of simulation runs the estimate precision increases as well.[8]

4.3 STEP 3: CALCULATING RESULT-OUTCOME ASSOCIATION

In the third step, the contingency table can finally be used to calculate the importance of a particular match from the point of view of a specific team using a specific desired outcome. As defined above, the importance can be expressed as a measure of association between the match result and the final outcome.

If the analyzed outcome is anything else than the final rank, the contingency table must first be prepared by aggregating appropriate columns – for example, if the analyzed outcome is relegation, final ranks 1 to 17 are aggregated into the first column (not being relegated) and final ranks 18 to 20 are aggregated into the second column (being relegated). Consequently, the new contingency table has 3 rows (win/draw/loss) and 2 columns (no relegation/relegation). Similarly, to analyze match impact on winning the championship, final rank 1 becomes the first column and final ranks 2 to 20 are aggregated into the second column. To get a positive association value, rows and columns should be ordered so that the best result/best outcome combination is in the top left corner.

To make the results intuitively understandable and comparable to other methods, the chosen measure of association should be on the scale from zero (no importance) to one (maximum possible importance). There are two obvious extreme cases – first, the outcome probability is already zero or

[7] This step transforms the result into an ordinary variable and simplifies the following computation. Another alternative would be to use a score difference.

[8] The maximum practical value of n is limited by available computing power and time. Using an optimized MATLAB/Octave implementation, 1,000,000 simulation runs should be expected to take one to two hours per season running a single thread on a desktop Intel i5/i7 Sandy/Ivy Bridge CPU.

one and therefore does not depend on the match result at all; second, the outcome hangs in the balance (e.g. the probability of being relegated is 1/2) and is solely determined by the match result. The chosen measure of association should be equal to zero in the first case and one in the second case. The measure should also take into account that both variables (result and outcome) are ordinal. Kendall-Stuart tau-c is a measure of association between two ordinal variables that fulfills all these conditions and is therefore used throughout this paper.

		Relegation	
		No	Yes
Match result	Win	50	0
	Draw	30	0
	Loss	20	0
tau-c = 0			

		Relegation	
		No	Yes
Match result	Win	50	0
	Draw	30	0
	Loss	0	20
tau-c = 0.64			

		Relegation	
		No	Yes
Match result	Win	50	0
	Draw	0	30
	Loss	0	20
tau-c = 1			

TABLE 1: EXAMPLES OF TAU-C VALUES BASED ON 100 SIMULATION RUNS

Table 1 shows tau-c values for three simple contingency tables for a relegation outcome, each based on 100 simulation runs.[9] In the left panel, the outcome (no relegation) is already determined before the match and the match importance correspondingly equals the lowest possible value of zero. In the middle panel, the outcome is completely determined by the match result, however, the before-the-match probability of relegation is just 0.2. Therefore, the match importance indicated by the tau-c value (0.64) is high, but not the maximum possible. Finally, the right panel represents the ideal case of maximum match importance; the outcome is completely determined by the match result, the before-the-match probability of relegation is exactly 0.5, and tau-c reaches its maximum possible value of one.

Match importance values obtained from the Monte Carlo simulation are just estimates of their true values, since they are based on estimated probabilities of various result-outcome combinations. Computing standard errors of tau-c estimates is not simple; however, the worst-case match importance standard error for a 3x2 contingency table can be roughly approximated by $1/\sqrt{n}$, where n is the number of simulation runs.[10]

[9] For simplicity, it is assumed that relative frequencies in these contingency tables exactly correspond to true probabilities.

[10] This estimate is based on a simple Monte Carlo simulation for a 3x2 contingency table with all outcome-result probabilities equal to 1/6. The simulated standard error was about 10% higher than $1/\sqrt{n}$ for n between 1,000 and 10,000,000 and decreased slowly when deviating from the original contingency table probabilities.

5 VERIFICATION

The rest of this paper is based on 10,000,000 simulation runs for each of 4,560 matches of 12 seasons of English Premier League. The worst-case standard error of various outcome probability estimates is $1.58 * 10^{-4}$ and the worst-case standard error of match importance estimates can be roughly approximated by $3.16 * 10^{-4}$, so it is clear that any potential biases in estimated numbers would be due to imperfect assumptions and not to the Monte Carlo method itself. In this chapter, match and season predictions are verified against betting odds and actual results.

5.1 VERIFYING MATCH PREDICTIONS

To show that the Monte Carlo method is based on plausible individual match predictions, this part uses data about 2,477 English Premier League matches with available betting odds to compare both probabilities derived from the Poisson-distribution algorithm described above and probabilities implied by betting odds against actual match results. For the algorithm, histories of both teams just before the match are used to make the test comparable for both methods. The null hypothesis in all tests is that the predicted/implied probabilities are equal to the true probabilities of home win, draw, or away win.

In the first test, predicted/implied probabilities of a particular result are added together and divided by the total number of matches to obtain a point estimate of the actual relative frequency of each result. Given the null hypothesis, each match prediction is an alternative distribution with variance p(1-p) (where p is the probability of a particular result); these variances can be used to calculate standard errors and 95% confidence intervals of all point estimates.

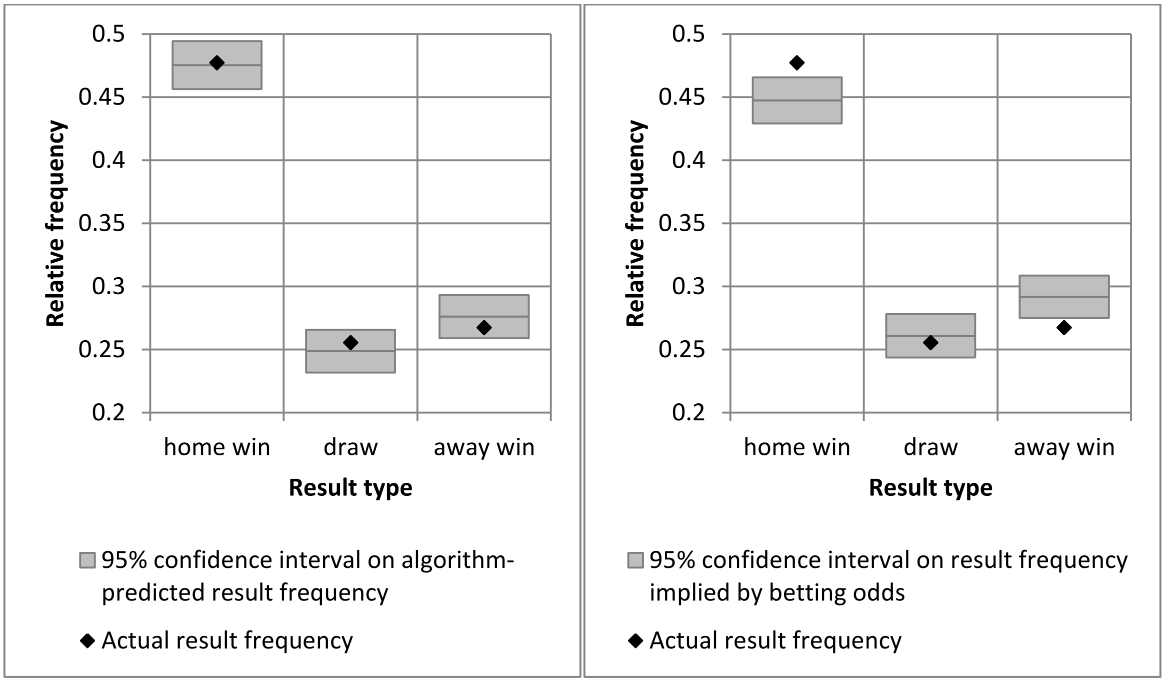

FIGURE 1: PREDICTED/IMPLIED VS. ACTUAL FREQUENCIES OF MATCH RESULTS

Figure 1 summarizes first test outcomes for both algorithm-predicted result frequencies (left) and result frequencies implied by betting odds (right). For algorithm predictions, all predicted relative frequencies are within one percentage point of the actual relative frequencies and the null hypothesis cannot be rejected at $\alpha = 0.05$ for any kind of result. For betting odds, implied relative frequencies of home win and away win differ from the actual relative frequencies by two to three percentage points and the null hypothesis is rejected in both cases. While the algorithmic method does not exhibit a detectable overall bias, betting odds slightly underestimate probabilities of home win and slightly overestimate probabilities of away win.

To see whether there are any biases associated with a particular type of matches, the second test orders all matches from the lowest to the highest predicted/implied probability of a specific result (so there are six orderings – two prediction methods times three possible results) and splits each ordering into five equally-sized bins. For matches in each bin, the implied/predicted relative frequency is computed along with its 99% confidence interval. The implied/predicted frequencies are then compared with actual frequencies for each bin. The probability that at least one implied/predicted frequency lies outside the corresponding 99% confidence interval given the null hypothesis almost exactly equals $\alpha = 0.05$.

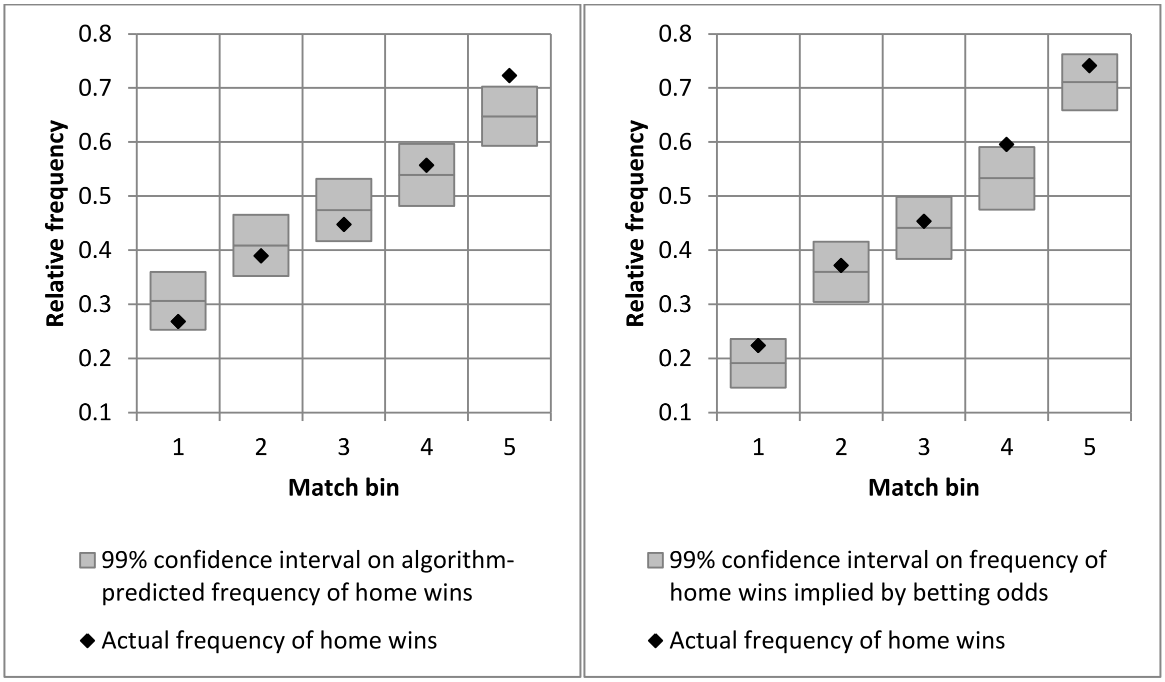

FIGURE 2: BINNED PREDICTED/IMPLIED VS. ACTUAL FREQUENCIES OF HOME WIN

Figure 2 shows the test results for algorithm-predicted (left) and betting-odds-implied (right) home win frequencies. In both cases, the null hypothesis is just rejected at $\alpha = 0.05$; the Poisson-distribution algorithm on average underestimates home win probability if the home team is already a heavy favorite (bin 5), while the betting odds underestimate probability of home win across all bins, but critically in bin 4, where the home team is a moderate favorite. The pattern is similar for away wins – the algorithm seems to slightly underestimate favorites and overestimate outsiders, betting odds underestimate away team in general, and the null hypothesis is rejected for both methods. For draw probabilities, the null hypothesis cannot be rejected in either case.

The tests above show that while exhibiting a slight favorite-outsider bias, the simple Poisson-distribution algorithm used in the Monte Carlo method provides predictions that are qualitatively comparable to betting odds and close enough to the actual match results.

5.2 VERIFYING SEASON PREDICTIONS

In this part, estimated probabilities of various season outcomes are tested against outcomes that actually happened. This is also an indirect test that the match prediction algorithm works for matches further in the future.

Data about 11 seasons and 4,180 matches are used (the first season is left out to make sure that all matches are preceded by at least one full season of historical results). For each match, there are two sets (one for each team) of estimated probabilities of 20 different final ranks, so altogether there are 4,180 * 2 * 20 = 167,200 individual final rank probabilities with the outcome actually happening in 167,200/20 = 8360 cases. Again, the null hypothesis is that these estimated probabilities are equal to the actual ones.

First, it is interesting to look at probabilities estimated to be very close to zero or one. There are 38,758 final rank predictions with estimated probabilities $p < 10^{-5}$; the expected number of outcomes actually happening obtained by aggregating all probabilities is 0.021. In reality, none of the outcomes happened. Symmetrically, there are 62 predictions with estimated probabilities $p > 1 - 10^{-5}$ and the expected number of outcomes $62 - 3 * 10^{-7}$. In reality, the outcome happened in all 62 cases. Therefore, it can be concluded that the Monte Carlo method correctly predicts practical certainty or practical impossibility of outcomes.

After eliminating the extreme probability predictions from the data, there are 128,380 predictions with estimated probabilities between 10^{-5} and $1 - 10^{-5}$. Unfortunately, this whole dataset does not lend itself easily to traditional statistical tests, since the individual events are clearly not independent; a team placing much better or much worse than could be expected leads to many predictions related to this team being wrong simultaneously. To alleviate this problem, only every 21st prediction selected by systematic sampling (with extreme-probability predictions subsequently thrown out) is used further, thus reducing the original 128,380 predictions to a sample of 6,111.

Similarly to testing individual match results, these 6,111 predictions are ordered from the lowest to the highest estimated probability and split into five equal bins to analyze any potential biases associated with a specific range of estimated probabilities. For predictions in each bin, the predicted relative frequency of outcome is computed along with its 99% confidence interval and compared with the actual frequency. Given the null hypothesis, the probability that at least one predicted frequency lies outside the corresponding confidence interval is again $\alpha = 0.05$.

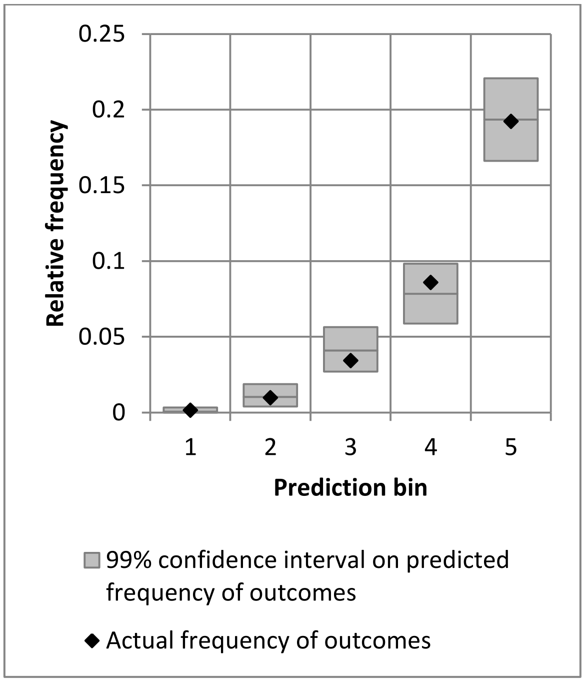

FIGURE 3: BINNED PREDICTED VS. ACTUAL FREQUENCIES OF OUTCOME

As shown in Figure 3, all actual relative frequencies are comfortably inside their respective confidence intervals, so the null hypothesis cannot be rejected. Since the difference between predicted and actual frequencies is at most one percentage point in each bin, any undetected bias should be small.

To summarize, the Monte Carlo method of calculating match importance relies on slightly biased individual match predictions (qualitatively comparable to predictions based on betting odds), but the estimated season outcome probabilities (including probabilities extremely close to zero or one) do not exhibit a discernible bias.

6 Comparison with other methods

As shown in the previous chapter, the Monte Carlo method provides match importance values that are derived from realistic predictions of both match results and season outcomes. Unlike other methods, it also takes into account factors such as strengths of the remaining opponents and final table ranking criteria. This chapter looks more closely on the computed match importance values using championship and relegation criteria and compares them against numbers obtained by using other common methods.

Data about 11 seasons and 4,180 matches are used. For each match, there are four associated match importance values – one championship and one relegation importance value for each team. Altogether, there are 8,360 championship importance values and 8,360 relegation importance values. Their basic descriptive statistics are provided in Table 2.

	Average	Standard deviation	Skewness	Min	Percentiles			Max
					90^{th}	99^{th}	99.9^{th}	
Championship	0.0165	0.0453	4.7893	0.0000	0.0692	0.1856	0.4747	0.8256
Relegation	0.0530	0.0744	2.4665	0.0000	0.1502	0.3007	0.6361	0.8747

Table 2: Descriptive statistics of championship and relegation importance, N = 8,360

Both championship and relegation importance distributions are extremely skewed towards low values; there are simply not that many important matches. On average, relegation importance is higher, since there are more relegation than championship spots and typically more teams in contention for avoiding relegation than for winning championship. Maximum observed values are close to one in both cases, indicating matches deciding the outcome hanging in the balance.

To simplify the following analysis, all match importance values are classified into the five following groups: zero importance (importance ≤ 10^{-6}; this value indicates that the outcome probability was extremely close to zero or one), very low importance (10^{-6} < importance ≤ 0.01), low importance (0.01 < importance ≤ 0.1), medium importance (0.1 < importance ≤ 0.2), and high importance (importance > 0.2). Figure 4 shows relative frequencies of grouped relegation importance values in different competition rounds.

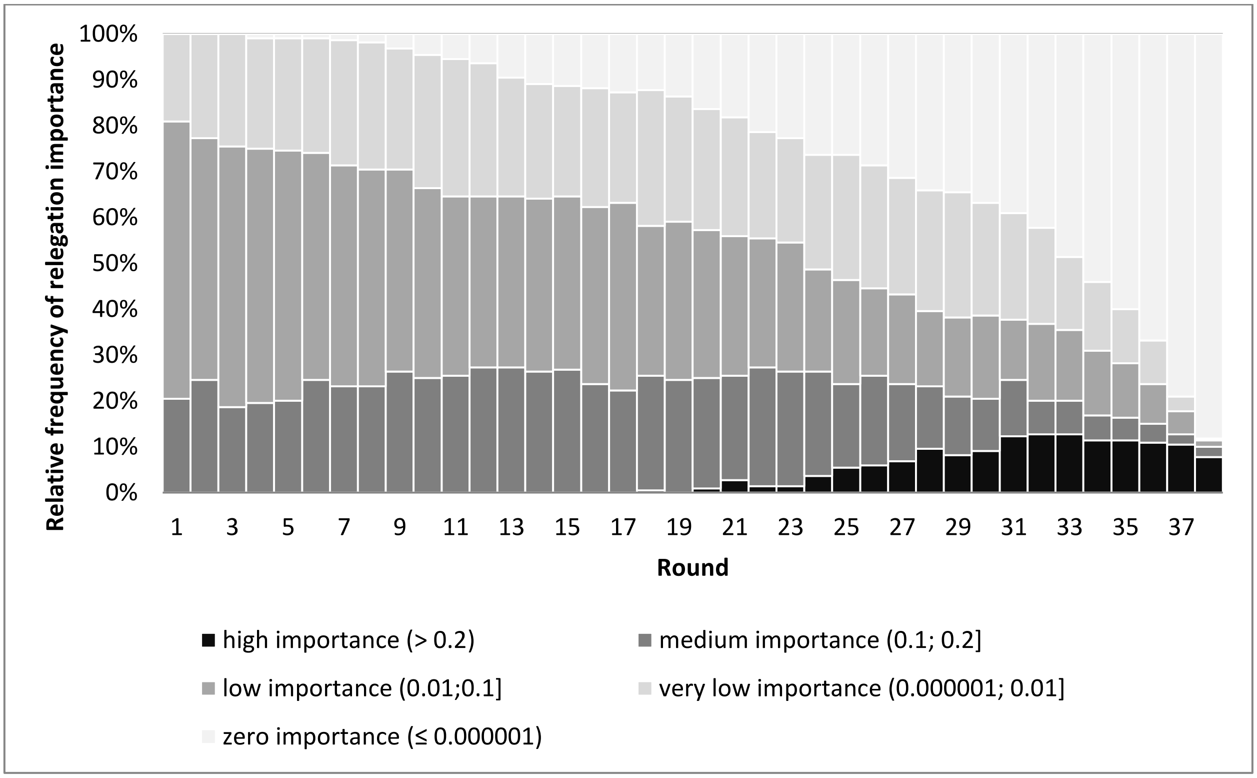

FIGURE 4: RELATIVE FREQUENCIES OF RELEGATION IMPORTANCE VALUES IN DIFFERENT COMPETITION ROUNDS

Figure 4, which shows relative frequencies of grouped relegation importance values in different competition rounds, immediately demonstrates problems of the simplest match importance method of assuming that all matches in the last X rounds are important; first, there is no obvious cutoff round where important matches start to appear; second, most matches even in the last several rounds are simply not that important for any team.[11] Therefore, just using a dummy variable for all matches in the last X rounds severely underestimates impact of match importance on attendance.

Another match importance method (mathematical certainty) assumes that a match is important for a given team if a specific outcome is not yet mathematically certain or impossible. A modified approach (used, for example, in Goddard and Asimakopoulos 2004) assumes that in the worst-case scenario, a team will get only X points in each remaining match with all the other teams getting full three points (and placing better if having the same number of points), while in the best-case scenario, a team will get full three points in each remaining match with all the other teams getting X points (and placing worse if having the same number of points). A team is sure to be relegated if it finishes no better than 18th in the best-case scenario; similarly, it cannot be relegated if it finishes no worse than 17th in the worst-case scenario. For X = 0, this method is exactly the same as mathematical certainty, while higher values of X eliminate low-importance matches more aggressively.

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[11] Even when including the European qualification criterion, only about one half of all matches in the last four rounds have at least medium importance on at least one criterion for at least one team.

		Number of matches	Percentage of matches classified as important (modified mathematical certainty)					
			X=0	X=0.5	X=1	X=1.5	X=2	X=2.5
Relegation importance (Monte Carlo)	zero	1,942	47.6	39.2	30.6	19.8	9.0	1.1
	very low	1,919	100	98.6	96.4	88.4	73.4	39.8
	low	2,612	100	99.9	99.7	98.6	95.7	82.9
	medium	1,568	100	99.8	99.8	99.7	99.2	94.8
	high	319	100	99.4	99.4	98.7	97.5	82.4
	Sum	8,360						

TABLE 3: CLASSIFICATION OF RELEGATION IMPORTANCE VALUES BY MONTE CARLO VS. MODIFIED CERTAINTY

Table 3 shows how both the Monte Carlo method and the modified mathematical certainty method with different values of X classify 8,360 relegation importance values. It is evident that the traditional mathematical certainty method (X = 0) is not nearly aggressive enough; 48% of zero-importance and full 100% of very-low-importance matches are still classified as important.

The best setting for the English Premier League dataset seems to be X = 2,[12] which is higher than commonly used in the literature; for example, Goddard and Asimakopoulos (2004) used X = 1. For X = 2, just several percent of medium- and high-importance matches are misclassified as not important, while only 9% of zero importance matches are misclassified as important. This could be considered a good result given the computational simplicity of this method; however, importance is still treated as a binary variable, so a lot of information is necessarily lost.

Similarly to the Monte Carlo method, Jennett's method treats match importance as a variable with many possible values between zero and one. To better compare both methods, match importance values generated by Jennett's method are also classified into the five following groups matching the distribution of the Monte Carlo values as closely as possible: zero importance (importance = 0), very low importance (0 < importance ≤ 1/25), low importance (1/25 < importance ≤ 1/18), medium importance (1/18 < importance ≤ 1/5), and high importance (importance > 1/5).

		Championship importance (Jennett)					
		zero	very low	low	medium	high	Sum
Championship importance (Monte Carlo)	zero	3,363	92	29	0	12	3,496
	very low	828	1,748	689	63	3	3,331
	low	78	386	295	195	10	964
	medium	25	78	147	243	13	506
	high	6	0	3	25	29	63
	Sum	4,300	2,304	1,163	526	67	8,360

TABLE 4: CLASSIFICATION OF CHAMPIONSHIP IMPORTANCE VALUES BY MONTE CARLO VS. JENNETT

[12] This is also true for championship importance values.

Table 4 shows that both methods classify the championship importance values into the corresponding categories quite similarly – 78% of values classified as highly important by Jennett's method have at least low importance assigned by the Monte Carlo method, 90% of values classified as highly important by the Monte Carlo method are considered to have at least low importance by Jennett's method, and only 3.7% of value pairs are more than one category apart. The Spearman rank correlation between uncategorized values of championship importance is 0.781.

Manual inspection of matches where the two methods disagree reveals that there are three reasons Jennett's method occasionally fails: first, it uses ex-post information unknowable before the match (this leads to mistakenly assigning zero importance to ex-ante important matches); second, it does not take into account who the opponent is (and therefore not assigning higher importance to matches against teams competing for the same outcome); third, it ignores any table ranking criteria besides total points.

Based on the data, Jennett's method is obviously the best alternative to the more computationally complex Monte Carlo method for championship importance if the season is already over and an occasional misclassification is not a big problem. In sports attendance demand studies, these two conditions are usually met. However, Jennett's approach does not work as well for other criteria, such as relegation or qualifying for European competitions,[13] and cannot be used for prediction.

[13] The Spearman correlation is 0.586 for relegation and 0.536 for qualifying for Europe.

7 Conclusion

As shown throughout the paper, the Monte Carlo method of calculating match importance relies on slightly biased individual match predictions, but it is able to estimate probabilities of various season outcomes without a detectable bias. The method can used to derive match importance values expressed as continuous variables for any team in any match given any desired outcome (such as championship, promotion, or avoiding relegation) and does not need any information unknowable before the match. When applied in sports attendance demand research, the method is expected to produce more precise and nuanced estimates of relationship between match importance and attendance.

The Monte Carlo method can be also used as a benchmark for other, less complex approaches. Using the Premier League dataset, Jennett's method of estimating championship importance is found to produce results that are quite close to the Monte Carlo simulation. However, Jennett's method requires ex-post information and does not work as well for other criteria besides championship. If ex-post information is not available and a simple dummy variable for match importance for each team is considered sufficient, the modified mathematical certainty approach provides a reasonable approximation when using the best-case/worst-case scenarios of obtaining one more/less point than the other teams in each remaining match.

There are three potential avenues of improvement of the presented method at the cost of added complexity and computation time: first, adjust the individual match prediction algorithm by modifying the history length/weights used to compute the expected number of goals or by dropping the independence assumption; second, introduce additional uncertainty to matches further in the future to account for events such as injuries, transfers, and staff changes that have long-term impact on performance; third, consider the possibility that match importance impacts team effort and thus the expected match result.[14]

[14] This topic was recently addressed by Feddersen et al. (2012).

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