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ABSTRACT

This paper develops an analytically tractable dynamic general-equilibrium model with a banking system to examine the macroeconomic implications of capital adequacy requirements. In contrast to the hypothesis of a credit crunch, we find that increasing the strength of bank capital requirements does not necessarily reduce the equilibrium quantity of loans, provided that banks have the option to respond to the capital requirements by accumulating more equity instead of cutting back on lending. Accordingly, we show that there is an inverted-U-shaped relationship between CAR and capital accumulation (and consumption). Furthermore, the optimal capital adequacy ratio for social-welfare maximization is lower than that for capital-accumulation maximization. In accordance with general empirical findings, the capital- accumulation maximizing capital adequacy ratio is procyclical with respect to economic conditions. We also find that monetary policy affects the real macroeconomic activities via the so-called bank lending channel, but the effectiveness of monetary policy is weakened by bank capital requirements.

***Keywords*:** Banking capital regulation; bank lending channel; the loan-deposit rate
***JEL Classification*:** O4, J5

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1. Introduction

Capital adequacy requirements (CAR) have become one of the most important banking regulations ever since more than 100 countries world-wide adopted the 1988 Basel Accords. Originally, the goal of the Basel Committee was to strengthen the stability of the international banking system by encouraging banking institutions to boost their capital positions (Basel Committee on Banking Supervision, 1999). Nowadays, CAR are the most common regulation in the banking industry in both developed and developing countries,[1] and as a result bank capital has become an important factor in bank asset-liability management and its importance has continued to increase.[2] For example, the credit crunch in the U.S. was at least viewed partly as a consequence of banks' scrambling to meet the 1992 deadline for CAR under the 1988 Basel agreement (Bernanke and Lown, 1991).

Even though more than sixteen years have passed since the regulation began to be implemented, its macro-oriented implications are not completely addressed. Despite the relatively large body of empirical literature that has focused on the related issues, there have been no definitive conclusions with regard to them.[3] In the literature, most studies dealing with CAR focus on a bank-level analysis of portfolio risk (Koehn and Santomero, 1980, Kim and Santomero, 1988, and Keeley and Furlong, 1990), the probability of failure (Dewatripont and Tirole, 1994), and the role of information asymmetries (Repullo and Suarez, 2000). Their analyses largely emphasize the informational microfoundations that are confined to a *partial-equilibrium* setting without taking into consideration the interactions between markets.[4] Few studies attempt to analyze the monetary policy implications and macroeconomic performance of CAR. The main reason for this neglect is that traditional monetary theory has mostly, if not entirely, ignored the role played by bank equity. The bank-related analyses that account for how monetary policy affects the real economy usually emphasize the so-called bank lending channel (the channel of interaction between bank lending and the real sector) and restrict their focus on the role of reserves (or reserve requirements) in determining the volume of bank loans, while treating bank capital as predetermined. However, as criticized by Friedman (1991), "[t]raditionally, most economists have regarded the fact that banks hold capital as at best a macroeconomic irrelevance and at worst a pedagogical inconvenience." This traditional simplification stands in stark contrast to the importance attached to capital adequacy in the regulation of banks, and thereby gives rise to the difficulty in addressing important CAR-related issues.

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[1] Most of the more than 100 adherents to the 1988 Basel Accords have already expressed intentions to adopt the new version, called Basel II.

[2] Santos claims that "[b]anking is undoubtedly one of the most regulated industries in the world, and the rules on bank capital are one of the most prominent aspects of such regulation."

[3] See Bernanke and Lown (1991), Hall (1993), Berger and Udell (1994), Hancock and Wilcox (1995), and Peek and Rosengren (1995a, 1995b) among others.

[4] Santos (2001) and VanHoose (2006) provide surveys of the literature on the CAR through the 1990s.

In this paper, we develop an analytically tractable *dynamic general-equilibrium* framework to systematically examine the *macroeconomic* implications of CAR. One of the earliest attempts to examine the macroeconomic implications of CAR was Blum and Hellwig (1995). Under specific parameter configurations, they argue that CAR may potentially amplify demand-side shocks. By following this line of research, Cecchetti and Li (2005) further suggest that CAR can also reinforce the effects of supply-side shocks. Seater (2001) points out that coordinated bank regulation and monetary policy influence both the mean and variance of aggregate output. Although insightful, the results of these studies are based on *ad hoc* IS-LM-style macro models that fail to fully account for endogenous responses of the banking system to regulation. Moreover, their work lacks a micro-foundation, and hence they are unable to perform a welfare analysis. By contrast, Van den Heuvel (2008) constructs a quantitative dynamic model and calibrates the model to U.S. banking data in order to perform a numerical analysis. He finds that the welfare cost of increasing CAR by 10% is equivalent to a permanent loss of consumption of at least 0.1-0.2%. By analogy, Aliaga-Díaz (2005) performs numerical simulations, and his results suggest that banks attempt to anticipate aggregate shocks by accumulating a buffer of capital over the regulatory minimum.

The analytical framework in this paper not only complements the earlier research cited above, but also allows us to provide the intuition on the macroeconomic effects of CAR. Financial intermediation requires real resource costs and provides firms with productive financial services, which give rise to an endogenous loan-deposit spread that affects the real-sector consumption and capital accumulation via the bank lending channel. The analytically tractable model allows us to examine the relationship between the CAR and capital accumulation (and consumption) and to highlight the important connections between the strength of the lending channel and the level of CAR in both *positive* and *normative* perspectives. The normative analysis is particularly important, since it is a new theoretical attempt to come up with the optimal level of CAR in terms of welfare maximization and capital-accumulation maximization.[5] Our theoretical analysis provides new insights into the assessment of the effects of CAR that have never been considered in a formal theoretical setting before.

There are three main results. The first is that increasing the strength of CAR does not necessarily reduce the equilibrium volume of loans, provided that banks have the option to respond to capital requirements by accumulating more equity as opposed to cutting back on lending. This result does *not* support *the hypothesis of a credit crunch.*[6] Given such a result, we show that there is an inverted-U-shaped relationship between CAR and capital accumulation (and consumption), which also stands in sharp contrast to the traditional prediction (see, for example, Santomero and Watson, 1977). The initial level of CAR is crucial in terms of governing the relationship between bank capital regulation and capital accumulation (or consumption). If the strength of CAR is relatively high, then

[5] Aliaga-Díaz (2005) includes this task in his agenda for future research.

[6] Not all empirical studies support the credit crunch hypothesis, either. See VanHoose (2006) for a survey of the empirical evidence in relation to the CAR.

increasing the level of CAR will cause the capital regulation to become too tight, thereby intensifying the distortion of CAR on a bank's asset and liability allocations. Under such a situation, CAR give rise to a harmful effect on capital accumulation (and consumption). By contrast, if the initial capital requirement ratio is relatively low, then appropriately increasing the strength of CAR gives rise to a favorable effect on capital accumulation (and consumption).

Secondly, our welfare analysis indicates that because bank capital requirements damage consumption more than capital accumulation, the optimal capital adequacy ratio for social-welfare maximization is lower than that for the maximization of capital accumulation. Furthermore, the capital-accumulation maximizing CAR is procyclical with respect to economic conditions. This procyclicality provides not only a convincing explanation, but also a solid economic foundation to the empirical finding of Borio (2003) and Bliss and Kaufman (2003) who point out that CAR in general have a procyclical tendency. Thirdly, our study confirms the existence of the bank lending channel. A contractionary monetary policy (implemented by increasing either the federal fund target rate or the reserve requirement ratio) decreases the equilibrium quantity of loans, and as a result capital accumulation and consumption fall in response. However, because banks can change the equity-debt financing mix by accumulating equity rather than by cutting back on loans, stricter CAR make the bank lending channel less powerful. In other words, CAR impinge upon the effectiveness of monetary policy.

Our analytical framework comprises two novel characteristics which distinguish this paper from the previous literature. First, by going beyond the existing macroeconomics literature, the model presented here incorporates a detailed balance sheet of banks in a setting with endogenous bank equity. We also allow banks to make portfolio allocations among assets (including loans, reserves, and federal funds) and to manage assets and liabilities by accumulating equities. In particular, it is costly for banks to manage the components of their balance sheets. Such a modeling allows us not only to analyze the effects of CAR on the loan market, but also to examine the role that bank lending plays in the monetary transmission in a macro-economy in which banks are increasingly able to issue non-reservable liabilities. Second, as emphasized by Schumpeter (1939, ch. III.D), "credit creation" on the part of the financial sector can play an important role in the process of accumulating a firm's productive capital. More explicitly, Robinson (1969, ch. 4) indicates that banks' loan services have the potential to affect firms' investment decisions by making cheaper external financing sources available. As for firms' investment behavior, Mayer (1990) and Schmidt (2001) provide empirical evidence in support of the contention that bank loans are the most important source of external funds for non-financial businesses in most countries. To capture a feature of reality, we allow the banking sector to play an active role in the process of accumulating productive capital by affecting firms' investment decisions via the provision of cheaper external financing sources and the provision of loan services in terms of market promotion and project evaluation. This set-up allows us to address how monetary policy and financial regulation affect the *real* macroeconomic activities via the bank lending channel and how CAR and the bank lending channel interact with each other. Apparently, this paper's focus is not on the information problem caused by CAR; instead, the emphasis is on a careful treatment of the bank's balance-sheet components and the resulting implications on bank lending and monetary policy. Therefore, the analysis is performed without taking into consideration moral-hazard problems.

2. The Model

There are four types of decision makers: households, firms, banks, and the government (solely represented by the monetary authority). Time is continuous (the time index is suppressed throughout the paper). There is a continuum of each type of agent (households, firms, and banks) with unit mass. Each household derives utility from consuming a final good and makes a portfolio choice among various types of assets. Firms operate in a competitive market producing the single final good with a Cobb-Douglas technology that uses physical capital and the banks' investment projects as inputs. Banks take deposits from households, make loans to firms, and manage assets and liabilities on the balance sheet. The monetary authorities balance the government's budget and impose two kinds of regulations on banks: reserve requirements and loan-based capital requirements.

2.1 Households

The economy is populated by a unit measure of identical, infinitely-lived households. The representative household, in facing its budget constraint, maximizes the discounted sum of future instantaneous utilities by optimally selecting a consumption bundle *C* and an asset portfolio allocation among (real) bank deposits *D*, and the stock of equities issued by banks S_B and by firms S_F, taking all prices and rates of return on equities as given. Given the initial conditions of assets, the optimization problem can be expressed by:

$$\max \int_0^\infty \ln C \cdot e^{-\rho t} dt,$$

$$\text{s.t. } \dot{D} + p_B \dot{S}_B + p_F \dot{S}_F = (r_D - \pi)D + \bar{i}_B p_B S_B + \bar{i}_F p_F S_F - C + TR,$$

$$D(0) = D^0,\ S_B(0) = S_B^0,\ S_F(0) = S_F^0, \tag{1}$$

where $\rho(>0)$ is a constant rate of time preference, r_D is the nominal deposit rate, π is the inflation rate, p_B (p_F) is the relative price of bank (firm) equity in terms of the consumption good, $\bar{i}_B$ ($\bar{i}_F$) is the rate of return (dividend) on bank (firm) equity, and *TR* is a lump-sum transfer provided by the government. For simplicity, labor is abstracted from the analysis.

Solving the household's problem (1) yields the following first-order conditions:

$$\frac{1}{C} = \lambda, \tag{2a}$$

$$\rho - \frac{\dot{\lambda}}{\lambda} = r_D - \pi, \tag{2b}$$

$$\rho - \frac{\dot{\lambda}}{\lambda} = \bar{i}_F + \frac{\dot{p}_F}{p_F}, \tag{2c}$$

$$\rho - \frac{\dot{\lambda}}{\lambda} = \bar{i}_B + \frac{\dot{p}_B}{p_B}, \tag{2d}$$

and the transversality conditions $\lim_{t\to\infty} \lambda D e^{-\rho t} = \lim_{t\to\infty} \lambda S_B e^{-\rho t} = \lim_{t\to\infty} \lambda S_F e^{-\rho t} = 0$, where λ is the shadow price associated with the budget constraint. Equation (2a) is the optimal condition in relation to consumption. Equations (2b)-(2d) give the optimal accumulation for assets – bank deposits, and the stock of equities issued by firms and banks, respectively.

2.2 Firms

In the economy each firm has an identical production technology. The representative firm invests physical capital *K* and purchases investment projects provided by banks *x* in order to produce its products. As stressed by Vinala and Berges (1988) and King and Levine (1993a, 1993b), financial institutions can provide these research, evaluative, and monitoring services more effectively and less expensively than individual investors; they also are better at mobilizing and providing appropriate financing to entrepreneurs than individuals. The evaluation and sorting of entrepreneurs lowers the cost of investing in terms of productivity enhancement. The positive incidence of financial intermediation on productive efficiency and capital productivity have been supported by the existing empirical works. To incorporate the feature of reality into the model, the production function is given by:

$$f(K,x) = AK^{\varepsilon}x, \text{ with } x = \theta \cdot g(\chi, L) = \theta \cdot \chi^{\sigma} L^{v}, \ 0 < \varepsilon, v < 1, \ 0 < \varepsilon + v < 1, \text{ and } \sigma > 1. \tag{3}$$

Here, A is a constant productivity coefficient. Equation (3) indicates that *effective* bank loan services *x* increase with the parameter of banking efficacy θ and the volume of bank loans *L*, but decrease with the fraction of bad loans $(1-\chi)$. The individual firm's capital can be converted from bank loans in the form of "fractional substitutes" and the conversion coefficient θ is regarded as a measure of banking efficacy. A higher θ captures the commercial banks' provision of loan services in terms of market promotion and project evaluation. It also captures "banks' monitoring services," namely, banks offer monitoring services to firms to ensure that their investment is sound. However, bank loans *L* cannot be completely converted to effective loan services which may deteriorate owing to bad loans (this will become clearer in the next subsection). Therefore, we point out that *x* is decreasing by the fraction $(1-\chi)$ (i.e., increasing in χ). This specification clearly indicates that loans are a risky investment for commercial banks. It also captures the economic conditions: the bad loan rate $(1-\chi)$ is higher during a period of economic recession and is lower during a period of economic expansion. Since χ is less than one, we should further introduce the restriction that $\sigma > 1$.

The firm's real "flow" profit is defined as $f(K,x) - (\delta - \pi)\chi L$ (where δ is the loan rate), which is either paid out in the form of dividends Ω_F to stockholders or retained as earnings RE_F to finance further investment *I*. By following Turnovsky (1990), we assume that firms and banks offer a dividend yield to stockholders on their equity according to the *fixed* dividend payout rule, i.e., $\bar{i}_F = \Omega_F/(p_F S_F)$.[7] A particular emphasis is that in addition to internal funds (retained earnings), there are two sources of external funds for firms: they can borrow from banks or borrow directly from households by issuing equity to households. Accordingly, the financing constraint facing firms is given by:

$$I = RE_F + p_F \dot{S}_F + \chi \dot{L}, \tag{4}$$

where $I = \dot{K} + dK$ (where d is the depreciation rate of capital), which also describes the evolution of capital.

While the optimization problem for firms is dynamic, we can simplify the problem as a static one by assuming that there is no adjustment cost for investment. Accordingly, given the initial values of capital and loans K^0 and L^0, the firm's optimization problem can be simply to choose capital and loans to maximize the following static objective function:[8]

$$\max \ \Pi_F = f(K,x) - (r_D - \pi)K - (\delta - r_D)\chi L - dK, \tag{5}$$

recalling that the production function is $f(K,x) = AK^{\varepsilon}(\theta\chi^{\sigma}L^{v})$, as reported in (3). The above equation indicates that firms employ capital at the rate of $(r_D - \pi)$ and employ loans at the rate of $(\delta - r_D)$. The unit cost of capital $(r_D - \pi)$ can be viewed as the firm's (or the shareholders') opportunity cost when the firm uses its funds to invest (accumulate capital), rather than earning the interest revenues. By analogy, $(\delta - r_D)$ reflects the opportunity cost when the firm chooses to purchase investment projects by making a loan from banks, rather than to make a direct investment by issuing equity to households.

Given (5), we have the first-order conditions for the firm's optimization as follows:

$$f_K - d = r_D - \pi, \tag{6a}$$

$$f_L = \chi(\delta - r_D), \tag{6b}$$

where $f_K = \varepsilon AK^{\varepsilon-1}(\theta\chi^{\sigma}L^{v})$ and $f_L = f_x x_L = vAK^{\varepsilon}(\theta\chi^{\sigma}L^{v-1})$. Equation (6a) indicates that the net marginal benefit from increasing capital ($f_K - d$) is equal to its opportunity cost, i.e., the real deposit rate ($r_D - \pi$). Equation (6b) indicates that the marginal benefit from increasing loans f_L is equal to the difference in the opportunity cost between obtaining loans from banks and issuing equity to

[7] The firm keeps the dividend rate $\bar{i}_F$ constant by adjusting the amount of the dividend. To focus on the macroeconomic effects of CAR, the analysis adopts the simply fixed dividend payout rule and ignores other relatively complicated rules. Nevertheless, this simplification has no significant loss of generality if we apply the dividend irrelevance theory, which purports that a firm's dividend policy has no effect on either its value or its cost of capital (see Modigliani and Miller, 1958).

[8] This simplification was pointed out to us by an anonymous referee, to whom we are grateful.

households $\chi(\delta - r_D)$. Of particular note, while the firm's optimization is simplified as a static one, the first-order conditions in (6a) and (6b) are identical to those derived by a complete intertemporal optimization.[9]

2.3 Banks

There is a continuum of banks of mass one, which take deposits *D* from households and extend loans to firms. As mentioned above, in our model loans are viewed as a risky investment for banks, i.e., there is a fraction $(1-\chi)$ of outstanding loans *L* that go bad during times of recession. By following Van den Heuvel (2008), we assume that these bad loans are written off by banks (i.e., banks absorb the losses resulting from the bad loans). A legal requirement imposes a lower bound on the fraction of deposits that banks hold, i.e., $R \geq \phi D$, where *R* is required reserves and ϕ is the corresponding reserve requirement rate. Aside from reserve requirements *R* and effective loans χL, the bank also holds federal funds *F* as its assets. These assets are financed by debt instruments – deposits *D* – and also by equity capital, $V_B (= p_B S_B)$, as shown by the following balance-sheet constraint:

$$R + F + \chi L = V_B + D. \tag{7}$$

Departing from the common macro-models in which there are only two forms of assets – namely, loans and reserves – the balance-sheet constraint (7) allows a bank to make a portfolio choice among its various assets or to engage in asset-liability management. As for firms, the gross profit of banks is either paid out in the form of dividends Ω_B to stockholders or retained as earnings RE_B. Moreover, the fixed dividend payout rule indicates that banks will offer a dividend yield to stockholders on their equity so that $\bar{i}_B = \Omega_B /(p_B S_B)$ is fixed.

For simplicity, we assume that the bank does not invest capital. Given this, the optimization problem can also be simplified as a static one. Such a static optimization problem is similar to that of Kopecky and VanHoose (2004a, p.637). Specifically, the bank optimally chooses those balance sheet quantities *R*, *L*, *D*, *F*, and S_B so as to maximize its profits Π_B, i.e.:

$$max \;\; \Pi_B = (\bar{r} - r_D)F + (\delta - r_D)\chi L - r_D R - \left[\frac{\alpha_L}{2}L^2 + \frac{\alpha_D}{2}D^2 + \frac{\alpha_S}{2}(p_B S_B)^2\right] \tag{8}$$

subject to

$$R \geq \phi D, \tag{9}$$

$$p_B S_B \geq \psi L, \tag{10}$$

$$R + F + \chi L = D + V_B, \tag{7}$$

where $\bar{r}$ is the rate of return on federal funds, i.e., the overnight interest rate (it can be simply thought

[9] See the earlier version of our paper for the more complete analysis which is available from the authors upon request or from the web site: http://www.econ.sinica.edu.tw/index.php?foreLang=tw.

of as the open market interest rate that is one of the monetary instruments of the central bank). In particular, in departing from the relevant literature, it should be noted that the banks' provision of an intermediation service is costly. By following Elyasiani et al. (1995) and Kopecky and VanHoose (2004a, 2004b, 2006), we assume the quadratic costs for managing the components on a bank's balance sheet.[10] The parameters α_L, α_D, and α_S have non-negative values that govern the magnitude of the resource costs for the individual bank's balance-sheet items.[11] Note that $\alpha_L \cdot L^2/2$ includes the costs of evaluating, managing, and funding the entrepreneurial activity that may enhance the firms' productivity. By facilitating the transfer of the most liquid funds among depository institutions, the federal funds market plays a major role in the execution of monetary policy.[12]

Loan-based capital requirements (10) require banks to hold an amount of equity equal to a fraction $\psi (\leq 1)$ of bank loans. To focus our point, we assume that capital requirements bind all banks in the macro-model. Universally binding capital requirements are a standard assumption in related studies. We emphasize that in our model the existence of bad loans will distort the efficiency of a market economy and consequently, this creates room for CAR to restore the efficiency of the market mechanism.[13] Given that η_1, η_2, and η_3 are the Lagrange multipliers associated with the constraints (9), (7), and (10), respectively, the optimal conditions necessary for this optimization problem allow us to derive the following results:

$$\eta_1 = \bar{r}, \tag{11a}$$

$$\eta_2 = r_D - \bar{r}, \tag{11b}$$

$$\eta_3 = \alpha_S p_B S_B + (r_D - \bar{r}), \tag{11c}$$

$$r_D = (1-\phi)\bar{r} - \alpha_D D, \tag{11d}$$

$$\delta = \bar{r} + \frac{1}{\chi}\{\alpha_L L + \psi \cdot [\alpha_S p_B S_B + (r_D - \bar{r})]\}. \tag{11e}$$

Equation (11d) is essentially the bank's demand for deposits and (11e) is its supply of loans. It is clear from (11a)-(11e) that monetary policy (including the required reserve ratio ϕ, the open market

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[10] For the costs of managing the components on a bank's balance sheet, $\dot{L}$ and $\dot{D}$ may be more appropriate than L and D to capture the adjustment costs. However, the cost of issuing new equities is closely related to the value of outstanding equities. To make our analysis tractable, we adopt the specification of (15). We are grateful to an anonymous referee for bringing this point to our attention.

[11] Ideally, bad loans $(1-\chi)L$ should increase banks' managing costs. It is more plausible to specify the cost of managing loans as $\alpha_L\{L^2 + [z(1-\chi)L]^2\}$, where z is a constant parameter. However, this complicated specification does not alter our main results.

[12] In order to maintain the effectiveness of monetary policy, the costs stemming from federal funds transactions are in general relatively low (see the report of the Federal Reserve Bank of New York, August, 2007). Thus, in line with Klein (1971) and Freixas and Rochet (1997), we ignore the transactions costs of federal funds for simplicity. This simplification allows us to decrease the mathematical complication and make our economic intuition clearer, without altering our main results.

[13] See Section 6 for the relevant discussions.

interest rate $\bar{r}$, and the capital required ratio ψ) and economic conditions (captured by the rate of bad loans χ) are important factors that affect the bank's asset-liability management and thus the bank lending effect. In addition, since (11d) indicates that $r_D - \bar{r} = -\phi\bar{r} - \alpha_D D < 0$, it is easy to learn that $\eta_2 = r_D - \bar{r} < 0$ and $\eta_3 = \alpha_S p_B S_B + (r_D - \bar{r}) > 0$.

2.4 The monetary authority (government)

The Federal Reserve's balance sheet is given by:

$$\overline{G} = R + F. \tag{12}$$

This balance sheet (12) contains reserve requirements R and federal funds (or reserves of depositary institutions) F as liability items, and gold certificates $\overline{G}$ as an asset item, which is fixed for simplicity. Reserves of depositary institutions are useful for making transfers between banks and for lending in the overnight market.

Since currency in circulation is abstracted from the model, the monetary base (high-power money) H is equal to the required reserves of banks R, i.e., $H = R = \phi D$. The law of motion governing the monetary base is given by:

$$\frac{\dot{H}}{H} = \mu - \pi, \tag{13}$$

where μ is the growth rate of high-power money. Accordingly, the monetary authorities target the level of the open market interest rate $\bar{r}$ by adjusting the amount of federal funds F. Thus, $\bar{r}$ serves as a tool of monetary policy in terms of governing the monetary base. Given (12), this implies that $\dot{F} = -\dot{R} = -\dot{H}$. Finally, the government finances its expenditures on lump-sum transfers by levying seigniorage taxes and the gains stemming from open market operations (net of the real interest payments on federal funds). Thus, the flow government budget constraint is:

$$\mu R + \dot{F} - (\bar{r} - \pi)F = TR. \tag{14}$$

3. Macroeconomic Equilibrium

This model economy defines an equilibrium as a sequence of prices $\{\delta, r_D, p_F, p_B, \pi\}_{t=0}^{\infty}$, real allocations $\{C, K\}_{t=0}^{\infty}$, stocks of financial assets $\{D, L, F, S_F, S_B\}_{t=0}^{\infty}$, and policy variables $\{\psi, \phi, \bar{r}, \mu, TR\}_{t=0}^{\infty}$, that satisfy:

(i) the optimizations of households (i.e., (2a)-(2d)0, firms ((6a) and (6b)), and banks ((11a)-(11e));

(ii) the budget constraints of households (1) and the government (14);

(iii) the financing constraints of firms (4) and the balance-sheet constraints of banks (7) and the Fed (12);

(iv) the evolution of capital and the monetary base (13);

(v) the requirements imposed on banks' reserves (9) and capital adequacy (10);

(vi) the market clearing conditions for the good market, the deposit market, the loan market, and the stock market (the following equations (15)-(18)).

By using the conditions reported in (ii)-(vi), we obtain the economy-wide resource constraint, i.e., the good-market clearing condition:

$$\dot{K} = f(K, x) - C - dK - \left[\frac{\alpha_L}{2}L^2 + \frac{\alpha_D}{2}D^2 + \frac{\alpha_S}{2}(p_B S_B)^2\right]. \tag{15}$$

Moreover, the households' supply of deposits (2b) with (2a) and the banks' demand for deposits (11d) allow us to have the equilibrium condition for the deposit market:

$$\frac{\dot{C}}{C} = r_D - \pi - \rho = (1-\phi)\bar{r} - \alpha_D D - \pi - \rho. \tag{16}$$

Combining the firms' demand for loans (6b) and the banks' supply of loans (11e) leads to the equilibrium condition of the loan market:

$$\frac{f_L}{\chi} + r_D = \delta = \bar{r} + \frac{1}{\chi}\{\alpha_L L + \psi \cdot [\alpha_S p_B S_B + (r_D - \bar{r})]\}. \tag{17}$$

Finally, in equilibrium, the real rate of return on deposits, bank equities, and firm equities must be equal. This leads to the following *no-arbitrage condition*:[14]

$$r_D - \pi = \bar{i}_F + \frac{\dot{p}_F}{p_F} = \bar{i}_B + \frac{\dot{p}_B}{p_B}, \tag{18}$$

We are now ready to state and to solve the system of macroeconomic dynamics. First, combining the balance-sheet constraints of banks (7) and the Fed (12) leads to the following relationship:

$$D = (\chi - \psi)L + \overline{G}. \tag{19}$$

Given (19) and the bank capital requirement ($p_B S_B = \psi L$), the equilibrium condition for the good market (15) can be rewritten as:

$$\dot{K} = f(K, x) - C - dK - \left\{\frac{\alpha_L}{2}L^2 + \frac{\alpha_D}{2}[(\chi - \psi)L + \overline{G}]^2 + \frac{\alpha_S}{2}(\psi L)^2\right\}. \tag{20}$$

Equation (20) differs from the common condition of the good market equilibrium without the banking sector in two significant respects. On the one hand, financial services (bank loans) play a crucial role in increasing the effectiveness of firms' investment and thus the real production efficacy. On the other hand, the asset-liability management of banks entails real resource costs.

Second, by substituting (6a) into the equilibrium condition of the market for deposits (16), we have the standard Keynes-Ramsey rule:

$$\frac{\dot{C}}{C} = f_K - d - \rho, \tag{21}$$

[14] Given these market-clearing conditions, the stock market equilibrium can also be obtained.

where $f_K = \varepsilon A K^{\varepsilon-1}(\theta\chi^{\sigma}L^{\nu})$ if the production function is characterized by (1). Third, substituting (11d) with (19) and $p_B S_B = \psi L$ into the equilibrium condition of the loan market (17) and applying the implicit theorem yields the instantaneous relationship involving loans as follows:

$$L = L(\underset{+}{K};\underset{?}{\psi},\underset{-}{\bar{r}},\underset{-}{\phi},\underset{+}{\chi},\underset{+}{\theta}). \tag{22}$$

See Appendix 1 for the derivation of (22).

By substituting (22) into the equilibrium conditions of the good and deposit markets, (20) and (21) thus construct the 2×2 dynamic system in terms of *C* and *K*. Once *C*, *K*, and *L* are solved from (20), (21), and (22), the equilibrium deposits *D* are determined by (19) and the price variables δ, r_D, and π in turn are solved from (11d), (11e) and (6a).

3.1 Comparative statics

Before analyzing more complex channels of interaction between bank lending and the real sector, we shall first briefly discuss the comparative statics of bank loans (22) that are central in terms of governing the bank lending channel. Figure 1 provides graphical illustrations for the results of (22). Given (10), (11d), and (19), it is easy to deduce from (17) that the loan demand curve, denoted by *LD*, is downward sloping and that the loan supply curve, denoted by *LS*, is upward sloping, as shown in Figure 1.[15]

With this understanding, Figure 1(a) indicates that more severe CAR (an increase in ψ) have an ambiguous effect on the equilibrium quantity of loans. Since a higher binding capital requirement, by forcing banks to tie together loans *L* and equity V_B, gives rise to an artificial loan management cost, this leads banks to reduce their lending (loan supply) and, accordingly, shifts the *LS* curve leftwards. However, more severe CAR also decrease the bank's available funds and, as a result, banks are inclined to increase the deposit rate r_D in order to attract more deposits from households. From the firm's standpoint, this reduces the difference in terms of the cost to banks of extending loans and issuing equity to households $(\delta - r_D)$, as indicated by (6b). Thus, the demand for loans increases, causing the *LD* curve to shift to the right, as shown in Figure 1(a). Since the effect of the demand side generates an opposite effect on the equilibrium quantity of loans, imposing more severe CAR has a mixed effect on loans, while it definitely increases the loan rate. This result does not support the hypothesis of a credit crunch.

An increase in the federal funds target increases the cost to the bank of using the federal funds.

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[15] The slopes of the loan demand and supply curves are, respectively:

$$\left.\frac{\partial\delta}{\partial L}\right|_{LD} = \frac{1}{\chi}f_{LL} - \alpha_D(\chi-\psi) < 0 \quad\text{and}\quad \left.\frac{\partial\delta}{dL}\right|_{LS} = \frac{1}{\chi}\{\alpha_L + \psi[\alpha_S\psi - \alpha_D(\chi-\psi)]\} > 0 .$$

In order to ensure a positive slope of the loan supply curve, we assume that the operational costs stemming from adjusting equity are substantially high, i.e., $\alpha_S > \alpha_D(\chi-\psi)/\psi$.

This results in a contraction of the bank's available funds and in turn reduces the bank's supply of loans. Therefore, Figure 1(b) indicates that the *LS* curve shifts to the left. On the other hand, since the bank faces a shortage of available funds, it raises the deposit rate in order to attract more deposits. Due to the reduction in the difference between the cost to banks of extending loans and issuing equity to households $(\delta - r_D)$, firms are willing to demand more loans from the banking sector, thereby shifting the *LD* curve to the right. Because the latter effect on the demand side is dominated by the former effect on the supply side, eventually the equilibrium quantity of loans decreases and the loan rate increases. Similar logic can be applied to explain the effect of the reserve requirement, as shown in Figure 1(c), which indicates that increasing the required rate in relation to bank reserves decreases the equilibrium quantity of loans and increases the loan rate.

Figure 1(d) shows that an improvement in banking efficacy θ (i.e., through the provision of high-quality loan services in terms of market promotion and project evaluation) induces firms to demand more loan services. Therefore, the *LD* curve shifts to the right and, consequently, both the quantity and the price in the loan market increase. By contrast, when the economy enters a recession (as captured by an increase in the bad loan rate $(1-\chi)$), banks, on the one hand, cut their lending in response, and on the other hand, firms reduce their demand for loans since bad loans cause the *effectiveness* of loan services to deteriorate. Thus, the quantity of bank loans falls, as in Figure 1(e).

3.2 Capital regulation

At the steady-state equilibrium, the dynamic system (presented by (20), (21), and (22)) is characterized by $\dot{C} = \dot{K} = 0$ and the stationary values are $\tilde{C}$ and $\tilde{K}$, respectively. Figure 2 depicts the existence and uniqueness of the competitive dynamic equilibrium. Since the dynamical system has one root with a negative real part and another with a positive real part, it is saddle-path stable. Accordingly, we establish the following proposition:

Proposition 1. ***(The Effects of Capital Regulation)*** *In the steady-state equilibrium,*

(i) there is an inverted-U-shaped relationship between capital accumulation and bank capital adequacy with the peak of capital at $\psi = \psi_K^*$*, as shown in Figure 3(a).*

(ii) Similarly, there is also an inverted-U-shaped relationship between consumption and bank capital adequacy with the peak of consumption at $\psi = \psi_C^*$*, as shown in Figure 3(b).*

Proof: See Appendix 2.

Proposition 1(i) provides us with an important policy implication. There is a tradition and common prediction, such as in Santomero and Watson (1977), whereby tighter capital regulation leads banks to reduce the credit they offer and consequently gives rise to a reduction in productive investment (and hence capital accumulation). This prediction is apparently based on the assumption that banks' equity is exogenously given. In sharp contrast to this prediction, we find that more severe CAR do not necessarily decrease capital accumulation and consumption in the real sector, and that the changes in these crucially depend on the sign of L_ψ. In the presence of a stock market, there may be a positive relationship between the CAR and capital accumulation (or consumption), provided that banks can increase their outstanding assets V_B, without a reduction in lending. By issuing equity, banks can, on the one hand, meet the capital regulation and, on the other hand, support the increase in the firms' loan demand, as shown in Figure 1(a). In other words, banks may respond to the change in the CAR by accumulating more equity rather than cutting back on loans. As such, stricter CAR increase, rather than decrease, the equilibrium quantity of loans, i.e., $L_\psi > 0$. In this case, the CAR favor capital accumulation and consumption.

Of importance, Proposition 1 further indicates that the initial level of capital adequacy is crucial in terms of governing the relationship between the CAR and capital accumulation (or consumption). If $\psi > \psi_K^*$ ($\psi > \psi_C^*$) at the status quo, then increasing ψ causes the capital regulation to become too tight, thereby intensifying the distortion in the bank's asset and liability allocations. Under such a situation, the CAR give rise to a harmful effect on capital (consumption). By contrast, if the initial capital requirement ratio is relatively low $\psi < \psi_K^*$ ($\psi < \psi_C^*$), then appropriately increasing the strength of the CAR gives rise to a favorable effect on capital accumulation (consumption).

4. Welfare Analysis

Proposition 1(a) and Figure 3(a) implicitly point out that ψ_K^* is the optimal capital adequacy ratio under the capital-accumulation maximization calculus. In this section we examine the optimal capital adequacy from the welfare point of view. Two questions are of particular interest here. One is whether or not the welfare-maximizing capital requirement is the same as the capital-maximizing one. The other is whether the optimal capital requirement ratio is affected by the bad loan rate $(1-\chi)$ and the parameter of operational cost stemming from managing equity α_S.

Given (1), the optimization problem of the social planner can be expressed by:

$$\max_{\psi} \quad \tilde{U} = \int_0^\infty \ln \tilde{C} \cdot e^{-\rho t} dt = \frac{1}{\rho} \ln \tilde{C}. \tag{23}$$

Thus, the welfare-maximizing capital requirement rate must satisfy the following first-order condition:

$$\frac{\partial \tilde{U}}{\partial \psi} = \frac{1}{\rho \tilde{C}} \frac{\partial \tilde{C}}{\partial \psi} = \frac{-1}{\rho \tilde{C} \Delta} \{[f_{KL}\rho - f_{KK}(\chi - \psi)\phi \bar{r}] L_\psi C + \Delta \alpha_S \psi L^2\}, \tag{24}$$

which indicates that $\text{sgn}(\partial \tilde{U} / \partial \psi) = \text{sgn}(\partial \tilde{C} / \partial \psi)$. It follows from Proposition 1 and Figure 3(b) that the socially optimal bank adequacy rate is ψ_C^*. Accordingly, we have the following proposition:

Proposition 2. ***(The Optimal Bank Capital Adequacy)*** *In the steady-state equilibrium,*

(i) *the optimal capital adequacy ratio of social welfare maximization is lower than that of capital accumulation maximization, i.e.,* $\psi_C^* < \psi_K^*$.

(ii) *The capital-maximizing CAR rate is decreasing in the bad loan rate ($1-\chi$) and in the banks' operational costs of managing equities α_S. The welfare-maximizing CAR rate has a negative relationship with α_S, while it has an inconclusive relationship with ($1-\chi$).*

Proof: See Appendix 3.

We glean the intuition for the result $\psi_C^* < \psi_K^*$ by examining the economy-wide resource constraint (20) with $\dot{K}=0$ and the Keynes-Ramsey rule (21) with $\dot{C}=0$. Given (23'), the equilibrium capital stock $\tilde{K}$ is recursively determined by (21) with $\dot{C}=0$. Once the equilibrium capital stock is determined, equilibrium consumption $\tilde{C}$ is solved from (20). With this understanding, (20) indicates that implementing the CAR gives rise to real resource costs to the bank that are caused by its management of assets and liabilities. Relative to capital, these costs generate an additional adverse effect in terms of reducing aggregate consumption. Therefore, the CAR give rise to greater damage to consumption than to capital. As a result, it follows from $\text{sgn}(\partial\tilde{U}/\partial\psi) = \text{sgn}(\partial\tilde{C}/\partial\psi)$ that the optimal rate of the welfare-maximizing CAR must be smaller than that of the capital-maximizing CAR.

Proposition 2(ii) offers two interesting corollaries to Proposition 2(i). In the capital-maximizing calculation, as indicated in Proposition 1, stricter CAR can increase, rather than decrease, equilibrium capital and consumption, provided that banks are able to respond to the change in the CAR by accumulating more equities as opposed to cutting back on loans. As such, the favorable effect turns out to be less powerful if issuing equity is more costly. Under such a situation, the optimal level of ψ_K^* is lower. This potentially points out that the optimal level of the CAR varies with the economy with different operational costs of issuing equity. The optimal CAR are lower in some developing countries where issuing equity is relatively costly for banks.

The CAR also decrease considerably during economic downturns, which in turn raise the bad loan risk for banks. By focusing on the case where $L_\psi < 0$, Figure 4(a) indicates that during an economic downturn (i.e., in the face of a higher bad loan rate $(1-\chi)$), the adverse effect of the CAR on the loan supply is reinforced, leading the *LS* curve to further shift leftwards.[16] This is because a rise in the bad loan rate intensifies the negatively distortionary effect caused by the regulation of the CAR and this leads banks to cut their lending in recessions. On the other hand, the firm's demand for loans also decreases in recessions since bad loans give rise to a deterioration in the quality of bank loan services. Thus, as shown in Figure 4(a), the favorable effect of the CAR on the demand for loans is

[16] To be more specific, a change in the bad loan rate influences the initial location of the equilibrium point and the slope of the loan supply. This gives rise to an induced effect that can be opposite to the direct effect of increasing the bad loan rate. However, to make our analysis meaningful, we rule out the possibility whereby a higher rate of bad loans alleviates the negatively distortionary effect of the CAR, leading banks to extend more loans to firms.

lessened – the *LD* curve shifts to the left. Given that bad loans have a negative effect on both the supply of and demand for loans, increasing the CAR results in a decline in the equilibrium quantity of loans. By applying similar logic, it is easy to provide the intuition to explain the case where $L_\psi > 0$. As shown in Figure 4(b), bad loans decrease not only the bank's loan supply, but also the firm's loan demand, leading *LS* to further shift leftwards and *LD* to shift to the left. The favorable effect of the CAR on bank loans is thereby dampened during a downturn. To summarize these two cases, we have $\partial L_\psi / \partial \chi > 0$.

As noted previously, the equilibrium bank loan is central in terms of affecting real activities (consumption and capital accumulation) via the bank lending channel. Given that the bad loan ratio reinforces the negative effect of the CAR on the equilibrium loan, we easily recognize that during recessions more severe CAR are more likely to harm consumption and capital accumulation. As a result, the optimal level of the CAR is decreasing in the bad loan rate. In other words, the optimal CAR are state-varying and procyclical with respect to economic conditions. This procyclical feature of the CAR provides not only a convincing explanation, but also a solid economic foundation for the empirical findings obtained by Borio (2003) and Bliss and Kaufman (2003), who point out that the CAR in general has a procyclical tendency.[17]

In the welfare-maximizing calculation, the effects of the bad loan rate $(1-\chi)$ and the parameter of operational costs α_S on the optimal ψ_C^* are more complicated. Like the effect of α_S on the capital-maximizing CAR, in the face of a larger α_S, increasing the strength of the CAR is more likely to lead banks to cut their lending. Thus, the welfare-maximizing CAR ψ_C^* is decreasing in α_S. However, the bad loan rate has an ambiguous effect on the optimal rate of the welfare-maximizing CAR. The reason is that a higher bad loan rate decreases the scale of banks' financial activity and in turn the equilibrium quantity of loans, as shown in Figure 1(e). By referring to the second term on the right-hand side of (24), the decrease in the volume of loans lowers the marginal cost of the CAR and, accordingly, gives rise to a favorable effect on the regulation of CAR. Therefore, the bad loan rate $(1-\chi)$ has an ambiguous effect on the optimal rate of the welfare-maximizing CAR.

5. Monetary Policy

We now turn to the bank lending effect of monetary (financial) policy. We in particular emphasize the importance of the connections between the strength of the lending channel and the regulatory level of the CAR. By so doing, we arrive at the following proposition:

Proposition 3. ***(The Effects of Monetary Policy versus the Bank Lending Channel)*** *In the steady-state equilibrium,*

[17] Some studies suggest that the impact of the procyclical feature of the CAR could be reduced through various deposit insurance or other mechanisms. See VanHoose (2006) for a comprehensive survey.

(i) capital (consumption) is decreasing in the federal funds target rate, $\partial \tilde{K} / \partial \bar{r} < 0$ ($\partial \tilde{C} / \partial \bar{r} < 0$), *and the reserve requirement ratio,* $\partial \tilde{K} / \partial \phi < 0$ ($\partial \tilde{C} / \partial \phi < 0$).

(ii) capital requirements, when binding, lower the effectiveness of monetary policy, (i.e., $\partial \left| \partial \tilde{K} / \partial \bar{r} \right| / \partial \psi < 0$ *and* $\partial \left| \partial \tilde{K} / \partial \phi \right| / \partial \psi < 0$).

Proof: See Appendix 4.

As is evident from (23'), a contractionary monetary policy (implemented by increasing either the interest rate on federal funds or the reserve requirement ratio) decreases the equilibrium quantity of loans (referring to Figure 1(b) and Figure 1(c)). Since the Cobb-Douglas production function, as reported in (3), implies that capital and banks' loan services are technical complements for each other, the decrease in L deteriorates the marginal productivity of physical capital. As a result, capital accumulation declines and aggregate consumption in turn falls. By contrast, an enhancement of banking efficacy θ increases the scale of banks' financial activity and in turn the equilibrium quantity of loans, as shown in Figure 1(d). Therefore, it accelerates capital accumulation and expands consumption.

It follows from Proposition 3(i) that there is one important caveat in that the CAR, when binding, lower the effectiveness of monetary policy via the bank lending channel. The economic intuition behind the result is straightforward. As shown in Figures 1(b) and 1(c), a contractionary monetary policy decreases the quantity of loans. Given a smaller quantity of loans, increasing the strength of the CAR ψ induces banks to increase their outstanding equities. Doing so allows banks to meet the capital requirement without a cut in loan supply. Since a larger stock of bank capital provides a buffer to the monetary shock, the negative effect of a contractionary monetary policy on capital accumulation is weakened.

6. Discussions and Extensions

In the benchmark model we have examined the optimal CAR rate. We here turn to the discussion of why the government introduces this CAR regulation into the economy. To this end, we need to further examine the equilibrium in an economy without the CAR requirement.

The previous sections have indicated that the levels of capital and welfare are increasing in the amount of loans in the economy. Thus, it is easy to explain why the introduction of CAR could be welfare improving by examining why the amount of loans in the economy with CAR can be larger than that in the economy without CAR. In the economy with CAR, the equilibrium quantity of loans can be solved from (21) with $\dot{C} = 0$ and (22) and satisfies the following equation:

$$\Sigma \cdot L^{\frac{\nu+\varepsilon-1}{1-\varepsilon}} = \left[\alpha_L + \psi^2 \alpha_S + (\chi - \psi)^2 \alpha_D \right] L + (\chi - \psi)(\phi \bar{r} + \alpha_D \bar{G}) , \tag{25}$$

where $\Sigma = \nu \cdot [\varepsilon / (\eta + \rho)]^{\frac{\varepsilon}{1-\varepsilon}} (A\theta)^{\frac{1}{1-\varepsilon}} \chi^{\frac{\sigma}{1-\varepsilon}}$. On the other hand, in the economy where there is no CAR, we can determine the equilibrium quantity of loans by the following equation:

$$\Sigma \cdot L^{\frac{\nu+\varepsilon-1}{1-\varepsilon}} = \left(\alpha_L + \frac{\alpha_D \alpha_S}{\alpha_D + \alpha_S} \chi^2 \right) L + \frac{\alpha_S}{\alpha_D + \alpha_S} \chi \cdot (\phi \bar{r} + \alpha_D \bar{G}) . \tag{26}$$

By using (25) and (26), we can compare the levels of bank loans in two distinct scenarios -- the economy with and without CAR. For ease of exposition, we further define $F = \left[\alpha_L + \psi^2 \alpha_S + (\chi - \psi)^2 \alpha_D \right] L + (\chi - \psi)(\phi \bar{r} + \alpha_D \bar{G})$ in the case with CAR and $N = (\alpha_L + \frac{\alpha_D \alpha_S}{\alpha_D + \alpha_S} \chi^2) L + \frac{\alpha_S}{\alpha_D + \alpha_S} \chi \cdot (\phi \bar{r} + \alpha_D \bar{G})$ in the case without CAR. Thus, Figure 5 indicates that, other things being equal, the amount of loans is determined by $\Sigma \cdot L^{\frac{\nu+\varepsilon-1}{1-\varepsilon}} = F$ in the presence of CAR and by $\Sigma \cdot L^{\frac{\nu+\varepsilon-1}{1-\varepsilon}} = N$ in the absence of CAR. As shown in Figure 5, the amount of loans in the economy with CAR, L^{CAR}, can be either larger or smaller than that in the economy without CAR, L^{NCAR}.

By using Figure 5, it is easy to further derive that, given the distortion caused by bad loans $(1-\chi)$, together with the costs for managing the components on a bank's balance sheet, $L^{CAR} > L^{NCAR}$ can be a case if the following necessary (but not sufficient) condition: $\psi > \chi \frac{\alpha_D}{\alpha_D + \alpha_S}$ is valid. Obviously, given a specific strength of CAR, a higher bad loan rate χ is more likely to meet this necessary condition; the amount of bank loans in the economy with CAR is more likely to be larger than that in the economy without CAR. This implies that introducing the regulation of the CAR can improve the welfare of a competitive economy in the situation where the bad loan rate is sufficiently large (a sufficiently low χ). Intuitively, the existence of bad loans gives rise to a distortionary effect on the efficiency of the market mechanism and, as a result, creates room for CAR to remedy the distortion caused by bad loans and restore this market efficiency. That is why the equilibrium amount of bank loans in the economy with CAR can be larger than that in the economy without CAR and the introduction of CAR can be welfare improving. This result also sheds light on the importance of analyzing the optimal CAR rate, just as in our study.

For the sake of simplicity, banks are not allowed to hold equities issued by firms in our benchmark model. Nevertheless, it may be more realistic for banks to hold the firms' equities in order to have better portfolio allocations among assets. Thus, it is natural to ask: whether the main result of this study is robust once we relax this restriction.

If banks hold equities issued by firms, denoted by S_F^B, the bank's balance-sheet constraint (7) should be modified as:

$$R + F + \chi L + p_F S_F^B = D + V_B , \tag{27}$$

and the costs for managing the components on a bank's balance sheet should change to:

$[\frac{\alpha_L}{2}L^2+\frac{\alpha_D}{2}D^2+\frac{\alpha_S}{2}(p_B S_B)^2+\frac{\alpha_{S_F}}{2}(p_F S_F^B)^2]$, where $\alpha_{S_F}(>0)$ captures the costs of managing the bank's holdings of equities issued by firms. Notice that the one-to-one relationship between L and $V_B(=p_B S_B)$ is now no longer valid, once banks hold equities issued by firms. That is, the CAR restriction now turns out to become:

$$p_B S_B \geq \psi(L+p_F S_F^B)\,. \tag{28}$$

Given these modifications, by following a similar procedure we can easily re-derive the effect of CAR on capital accumulation as follows:

$$\frac{\partial \tilde{K}}{\partial \psi}=-\frac{1}{\Delta}f_{KL}L_\psi C \lesseqgtr 0\,,\ \text{if}\ \ L_\psi=\frac{1}{M\vartheta^2}\left\{2\omega\left[\alpha_{S_F}+(1-\chi)(1-\psi)\alpha_D\right]L-\left[2\omega-(1-\chi)\alpha_D\vartheta\right](f_K-d-\bar{i}_F)\right\}\lesseqgtr 0\,,$$

where $M=f_{LL}+\frac{1}{\vartheta}\left[(1-\psi)(\chi-\psi)\alpha_D+\psi^2\alpha_{S_B}\right]f_{LK}-\alpha_L-\frac{1}{\vartheta}\left[(\chi-\psi)^2\alpha_D\alpha_{S_F}+\psi^2\alpha_{S_B}\alpha_{S_F}+(1-\chi)^2\psi^2\alpha_D\alpha_{S_B}\right]$,

$\vartheta=\alpha_{S_F}+\psi^2\alpha_{S_B}+(1-\psi)^2\alpha_D$, and $\omega=\psi\alpha_{S_B}\left[\alpha_{S_F}+(1-\chi)\alpha_D\right]-(\chi-\psi)\alpha_D\alpha_{S_F}$. It is evident from the above equation that the capital effect of CAR is similar to that in Proposition 1. The ambiguity confirms that tighter capital regulation does not necessarily lead banks to reduce the loans they offer and gives rise to a monotonically negative effect on the capital stock and consumption. Under some reasonable conditions, there still exists an inverted-U-shaped relationship between capital accumulation and bank capital adequacy, which allows us to determine the optimal CAR rate. In other words, our main result is still valid.

7. Concluding Remarks

Although CAR have become the most common regulation in the banking industry, several basic and important macroeconomic questions remain unanswered. On the one hand, the large body of empirical literature on related subjects does not reach definitive conclusions regarding them. On the other hand, there are still very few theoretical models offering explanations from a general equilibrium perspective. To fill this gap, this paper has built an analytically tractable macro model and has used it to address the macroeconomic implications of a capital adequacy requirement system which links bank lending to bank equity. Essentially, both positive and normative analyses have been conducted in the paper.

We have found that increasing the strength of the CAR does not necessarily reduce the equilibrium quantity of loans, provided that banks can change their equity-debt financing mix by accumulating more equity as opposed to cutting back on lending. This does not support the hypothesis of a credit crunch. Moreover, once the CAR increase the quantity of loans, our macro-model suggests that more severe CAR may also give rise to a favorable effect, rather than an adverse one, on capital accumulation and consumption. Of particular importance, there is an inverted-U-shaped relationship between the CAR and capital accumulation (and consumption). For the case where there is a relatively low status quo ratio for the CAR, appropriately increasing the strength of the CAR favors capital accumulation and consumption. Of particular note, as a benchmark, for banks the possibility of holding the equality issued by firms is abstracted for simplicity. However, in an extension we have shown that the above-mentioned results are robust when the banks are allowed to hold equities issued by firms.

We also find that the optimal capital adequacy ratio of social welfare maximization is lower than that of capital accumulation maximization. The capital-maximizing CAR rate is lower when the bad loan rate and the banks' operational costs of managing equities increase. The welfare-maximizing CAR rate also has a negative relationship with the banks' operational costs of managing equities, but it has an inconclusive relationship with the bad loan rate. Finally, there exists the so-called bank lending channel – a contractionary monetary policy decreases the equilibrium quantity of loans and, as a result, capital accumulation and consumption fall in response. However, since banks can change their equity-debt financing mix by accumulating equity rather than by cutting back on loans, stricter CAR lower the effectiveness of monetary policy.

There are several directions over which further theoretical research would be interesting. First, in this model we do not deal with the risk attitude of banks toward the uncertainty involving bad loans. Keeley and Furlong (1990) use the mean-variance approach to analyze the effects of bank capital regulation on the assets and bankruptcy risk of the insured. It is interesting to explore how the introduction of uncertainty into the model naturally leads to an examination of the effects of the bank's risk aversion on the macro consequences of the CAR. Secondly, currency in circulation is abstracted from our model. Although this simplification does not alter our results, an important direction for future research may be to further examine the monetary policy under various monetary regimes, such as inflation targeting and Taylor's rule, when the model incorporates currency in circulation. Finally and more interestingly, in future research we will perform a numerical exercise to compute the welfare costs of moderate rates of CAR and the weakening effects of CAR on monetary policy.

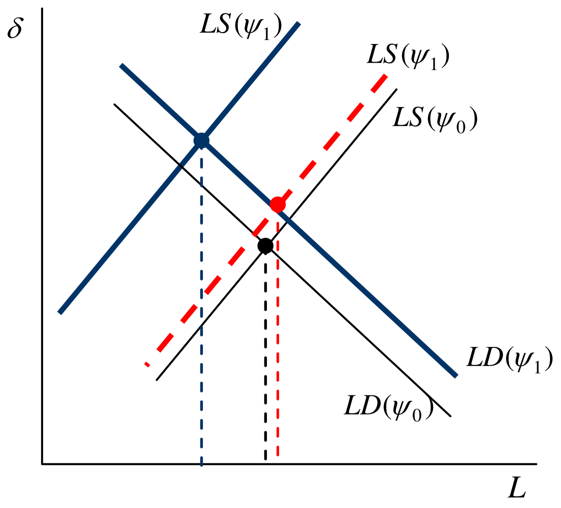


Figure 1(a): The loan effect of increasing ψ

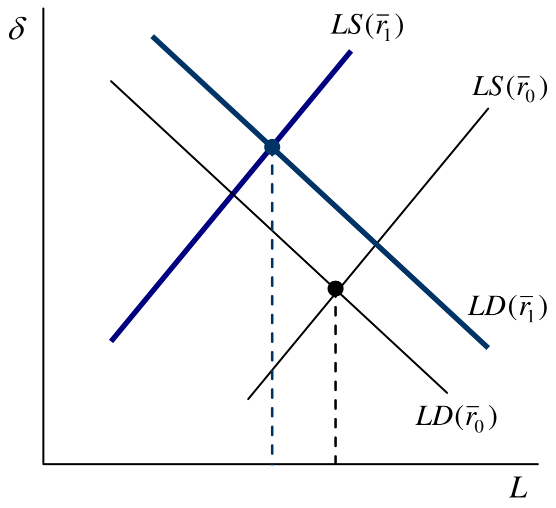


Figure 1(b): The loan effect of increasing $\bar{r}$

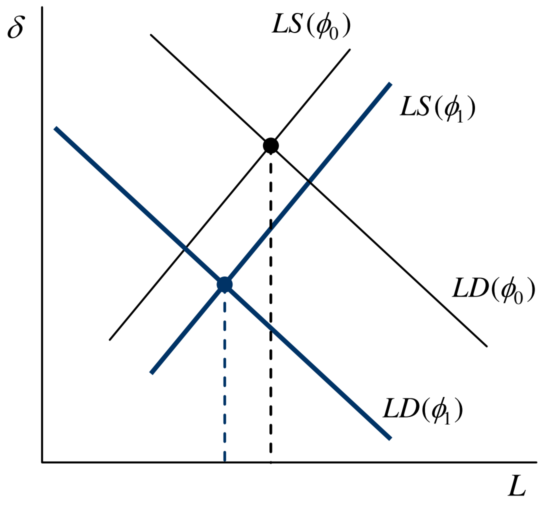


Figure 1(c): The loan effect of increasing ϕ

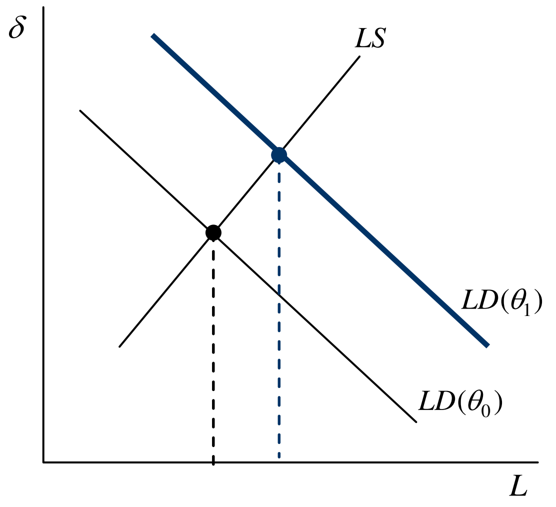


Figure 1(d): The loan effect of increasing θ

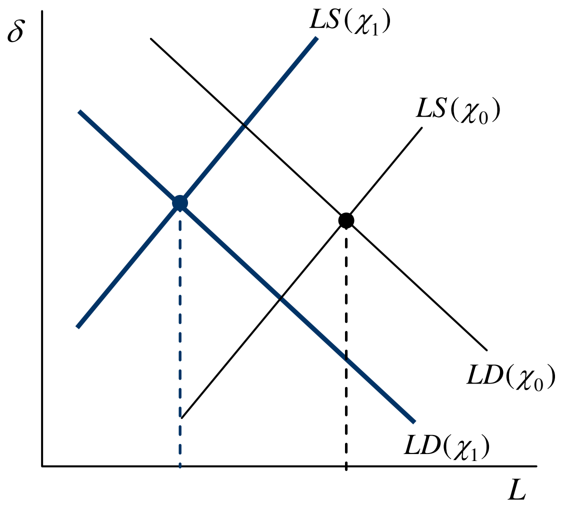


Figure 1(e): The loan effect of decreasing χ

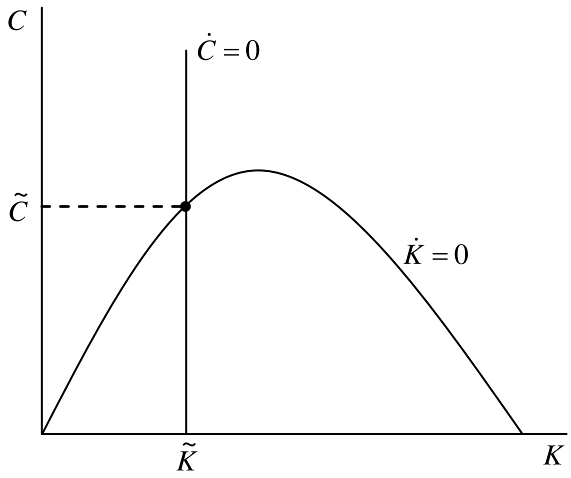


Figure 2 Macroeconomic equilibrium

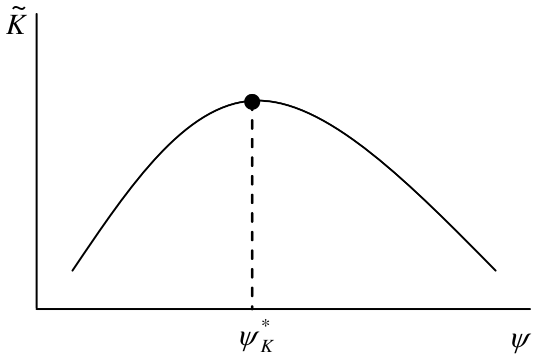


Figure 3(a) Capital-maximizing CAR

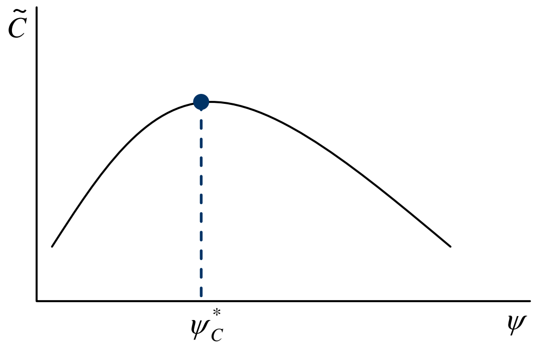


Figure 3(b) Consumption-maximizing CAR

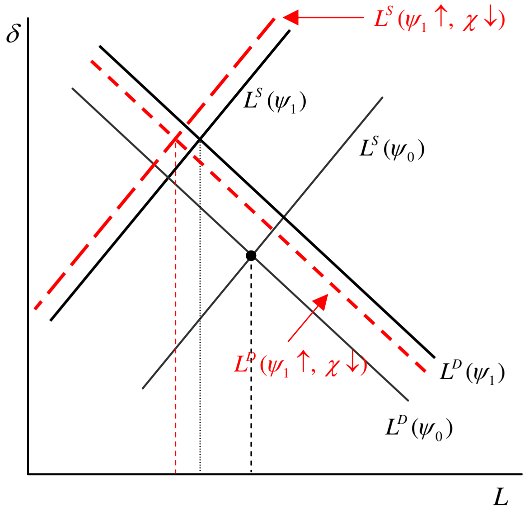


Figure 4(a) The case where $L_\psi < 0$

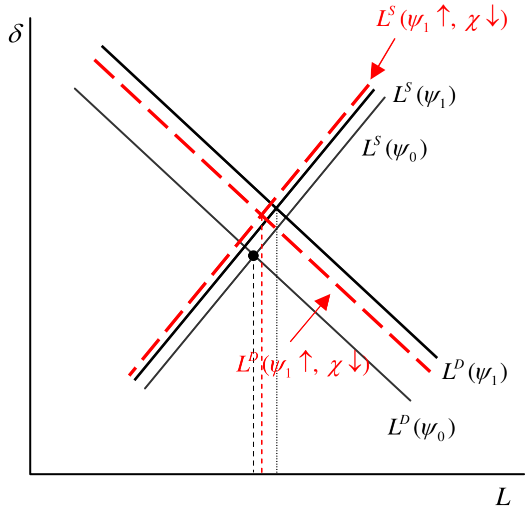


Figure 4(b) The case where $L_\psi > 0$

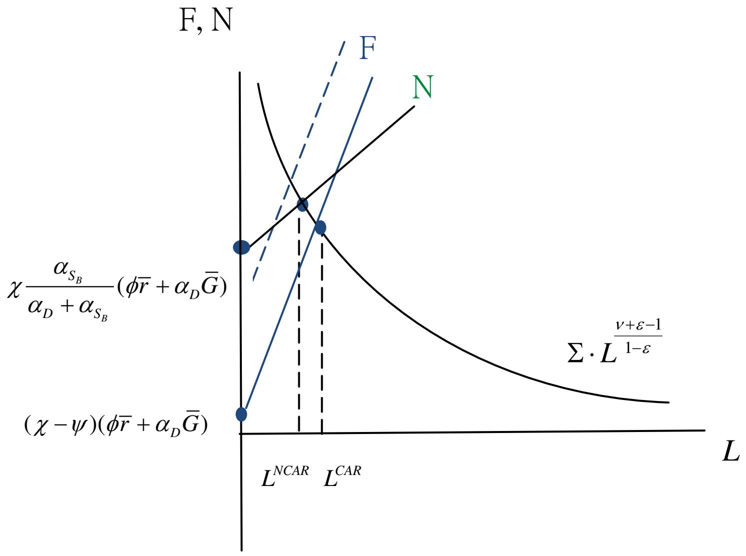


Figure 5 Loans in the economy with and without CAR

Appendix

Appendix 1 *(The derivation of (22))*

From (11d), (17), (19), and bank capital requirements $p_B S_B = \psi L$, it is easy to obtain:

$L_K = -f_{KL}/\Lambda > 0$, $L_\psi = \{2\psi\alpha_S L - [2(\chi-\psi)\alpha_D L + \phi\bar{r} + \alpha_D \bar{G}]\}/\Lambda \lesseqgtr 0$, $L_{\bar{r}} = (\chi-\psi)\phi/\Lambda < 0$,

$L_\phi = (\chi-\psi)\bar{r}/\Lambda < 0$, $L_\chi = -\{\psi\alpha_D L + (\delta-\bar{r}) + (f_{L\chi}\chi - f_L)/\chi - \chi\alpha_D L]\}/\Lambda > 0$, $L_\theta = -f_{L\theta}/\Lambda > 0$,

$\Lambda = f_{LL} - [\alpha_L + \psi^2\alpha_S + (\chi-\psi)^2\alpha_D] < 0$, $f_{KL} = \varepsilon v A K^{\varepsilon-1}(\theta\chi^\sigma L^{v-1}) > 0$, $f_{L\chi} = \sigma v A K^\varepsilon(\theta\chi^{\sigma-1}L^{v-1}) > 0$,

$f_{L\theta} = vAK^\varepsilon(\chi^\sigma L^{v-1}) > 0$, and $f_{LL} = v(v-1)AK^\varepsilon(\theta\chi^\sigma L^{v-2}) < 0$. ■

Appendix 2 *(The proof of Proposition 1)*

Suppose that the $\tilde{K}$ and $\tilde{C}$ are continuous and differentiable with respect to ψ. By substituting (22) into (20) and (21), we have the effects of increasing the strength of CAR on the steady-state capital stock and consumption as follows:

$$\frac{\partial\tilde{K}}{\partial\psi} = -\frac{1}{\Delta}f_{KL}L_\psi C \lesseqgtr 0, \quad if \ L_\psi \lesseqgtr 0 \Leftrightarrow \psi \gtreqless \psi_K^* \equiv \frac{\chi\alpha_D}{\alpha_S+\alpha_D} + \frac{\phi\bar{r}+\alpha_D\bar{G}}{2(\alpha_S+\alpha_D)L^0}, \tag{A1}$$

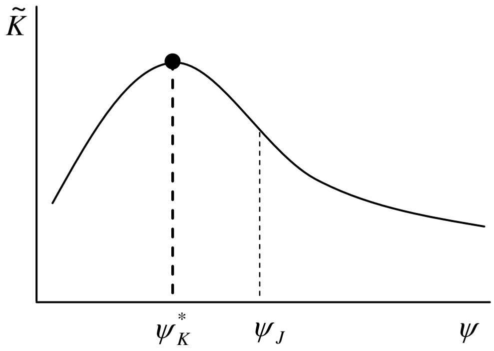
$$\frac{\partial\tilde{C}}{\partial\psi} = -\frac{1}{\Delta}\{[f_{KL}\rho - f_{KK}(\chi-\psi)\phi\bar{r}]L_\psi C + \Delta\alpha_S\psi L^2\} \lesseqgtr 0, \ if \ \psi \gtreqless \psi_C^*, \tag{A2}$$

where $\Delta = (f_{KK} + f_{KL}L_K)C < 0$. Even though we cannot explicitly solve the closed form of ψ_C^*, it is easy to obtain it by applying the implicit theorem.

According to (A1), we further derive:

$$\frac{\partial^2\tilde{K}}{\partial\psi^2} = -\frac{2f_{LK}}{(f_{KK}+f_{KL}L_K)\Lambda}\left\{(\alpha_S+\alpha_D)L + \left[2+\frac{(f_{LK})^2}{(f_{KK}+f_{KL}L_K)\Lambda}\right]\left[\psi\alpha_S - \alpha_D(\chi-\psi)\right]L_\psi\right\}. \tag{A3}$$

In deriving (A3) we ignore the third-order derivations of the production function. Thus, (A3) indicates that the equilibrium capital stock has a concave relationship with ψ ($\partial^2\tilde{K}/\partial\psi^2 < 0$) in the case where $L_\psi > 0$. However, in the case where $L_\psi < 0$, there exists an inflection point, say, *J*, such that concavity holds until point *J* is reached and thereafter it turns out to have convexity (because of $(\alpha_S+\alpha_D)L + \left\{2 + (f_{LK})^2/\left[(f_{KK}+f_{KL}L_K)\Lambda\right]\right\}\left[\psi\alpha_S - \alpha_D(\chi-\psi)\right]L_\psi < 0$), as shown in the figure below. This is confirmed from (A3) in that if $\psi \to \infty$, then the volume of loans is reduced to nil ($L = 0$) and as a result, (A3) becomes $\partial^2\tilde{K}/\partial\psi^2 > 0$.



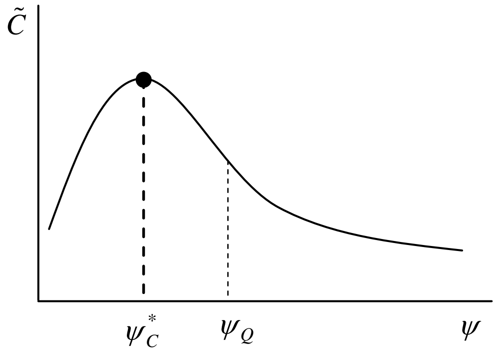
Without loss of generality, in this analysis we rule out the possibility that $\psi > \psi_J$, and hence the $\tilde{K}$ function simply plots an inverse U-shaped curve, as shown in Figure 3(a).

We turn to the second-order derivation of consumption with respect to ψ. From (A2), we derive:

$$\frac{\partial^2 \tilde{C}}{\partial \psi^2} = -\alpha_S L^2 - \frac{2[f_{KL}\rho - f_{KK}(\chi-\psi)\phi\bar{r}]}{(f_{KK}+f_{KL}L_K)\Lambda}\left\{(\alpha_S+\alpha_D)L + \left[2+\frac{(f_{LK})^2}{(f_{KK}+f_{KL}L_K)\Lambda}\right][\psi\alpha_S - \alpha_D(\chi-\psi)]L_\psi\right\}$$

$$-\left[\frac{f_{KK}\phi\bar{r}}{(f_{KK}+f_{KL}L_K)} + 2\alpha_S\psi L\right]L_\psi . \qquad \text{(A4)}$$

The formula of (A4) is very similar to that of (A3), and therefore it is easy to plot the relationship between $\tilde{C}$ and ψ by applying the same logic illustrated above. The following figure plots the relationship between $\tilde{C}$ and ψ:



Again, without loss of generality, we ignore the possibility of $\psi > \psi_Q$, and hence the $\tilde{C}$ function exhibits an inverse U-shaped relationship with ψ, as shown in Figure 3(b). ■

Appendix 3 ***(The proof of Proposition 2)***

Equation (24) indicates that $\text{sgn}(\partial\tilde{U}/\partial\psi) = \text{sgn}(\partial\tilde{C}/\partial\psi)$, and therefore we derive the welfare-maximizing CAR by letting $\partial\tilde{C}/\partial\psi = 0$. Given that:

$$\frac{\partial \tilde{K}}{\partial \psi} = -\frac{1}{\Delta} f_{KL} L_\psi C \quad \text{and} \qquad \text{(A5)}$$

$$\frac{\partial \tilde{C}}{\partial \psi} = -\frac{1}{\Delta}\{[f_{KL}\rho - f_{KK}(\chi-\psi)\phi\bar{r}]L_{\psi}C + \Delta\alpha_S\psi L^2\}, \tag{A6}$$

if $L_{\psi} = 0$, we then have $\partial\tilde{K}/\partial\psi = 0$ and $\partial\tilde{C}/\partial\psi = -\alpha_S\psi L^2 < 0$. This implies that the welfare-maximizing capital requirement ψ_C^* is lower than the capital-maximizing capital requirement ψ_K^*.

By focusing on the case of the welfare-maximizing CAR, we can easily derive:

$$\frac{\partial \psi_K^*}{\partial \chi} = \frac{\alpha_D}{\alpha_S + \alpha_D} > 0,$$

$$\frac{\partial \psi_K^*}{\partial \alpha_S} = \frac{-1}{(\alpha_S+\alpha_D)^2}\left[\chi\alpha_D + \frac{\phi\bar{r} + \alpha_D\bar{G}}{2L^0}\right] < 0.$$

In the welfare-maximizing calculation, we should further impose the restriction that $L_{\psi} > 0$ in order to focus our analysis on the interior solution (referring to (A6)) and assume that the second-order condition $\partial^2\tilde{C}/\partial\psi^2 < 0$ is satisfied. Thus, we have:

$$\frac{\partial \psi_C^*}{\partial \alpha_S} = \frac{1}{(\partial^2\tilde{C}/\partial\psi^2)}\left\{\frac{f_{KL}\rho - f_{KK}(\chi-\psi)\phi\bar{r}}{f_{KK}+f_{KL}L_K}L_{\psi\alpha_S} - \frac{f_{KL}\rho - f_{KK}(\chi-\psi)\phi\bar{r}}{(f_{KK}+f_{KL}L_K)^2}f_{KL}L_{K\alpha_S}L_{\psi} + 2\psi L^2(\frac{1}{2}-\zeta)\right\} < 0, \tag{A7}$$

$$\frac{\partial \psi_C^*}{\partial \chi} = \frac{1}{(\partial^2\tilde{C}/\partial\psi^2)}\left\{-\frac{f_{KK}\phi\bar{r}}{f_{KK}+f_{KL}L_K}L_{\psi} + \frac{f_{KL}\rho - f_{KK}(\chi-\psi)\phi\bar{r}}{f_{KK}+f_{KL}L_K}L_{\psi\chi} - \frac{f_{KL}\rho - f_{KK}(\chi-\psi)\phi\bar{r}}{(f_{KK}+f_{KL}L_K)^2}f_{KL}L_{K\chi}L_{\psi} + 2\alpha_S\psi LL_{\chi}\right\} \gtrless 0, \tag{A8}$$

where $L_{\psi\alpha_S} = \{2\psi L[f_{LL} - \alpha_L - \chi(\chi-\psi)\alpha_D] + \psi^2\Lambda L_{\psi}\}/\Lambda^2 < 0$, $L_{K\alpha_S} = -f_{LK}\psi^2/\Lambda^2 < 0$, $\zeta = -L_{\alpha_S}\frac{\alpha_S}{L} > 0$, and $L_{\alpha_S} = \psi^2 L/\Lambda < 0$. In (A7), it is convincing to impose the condition $\zeta < 1/2$, implying that the marginal cost resulting from increasing the strength of CAR increases if issuing equity becomes more costly. ■

Appendix 4 ***(The proofs of Proposition 3)***

For (20)-(22), we derive the effects of monetary policy (increasing in $\bar{r}$ and ϕ) and banking efficacy on $\tilde{K}$ and $\tilde{C}$ as follows:

$$\frac{\partial \tilde{K}}{\partial \bar{r}} = -\frac{1}{\Delta}f_{KL}L_{\bar{r}}C < 0,$$

$$\frac{\partial \tilde{K}}{\partial \phi} = -\frac{1}{\Delta}f_{KL}L_{\varphi}C < 0,$$

$$\frac{\partial \tilde{C}}{\partial \bar{r}} = -\frac{1}{\Delta}\{f_{KL}[\rho + (\chi - \psi)\phi\bar{r}L_K] - (f_{KK} + f_{KL}L_K)(\chi - \psi)\phi\bar{r}\}L_{\bar{r}}C < 0\,,$$

$$\frac{\partial \tilde{C}}{\partial \phi} = -\frac{1}{\Delta}\{f_{KL}[\rho + (\chi - \psi)\phi\bar{r}L_K] - (f_{KK} + f_{KL}L_K)(\chi - \psi)\phi\bar{r}\}L_{\phi}C < 0\,,$$

It is easy to further derive:

$$\frac{\partial\left|\partial \tilde{K} / \partial \bar{r}\right|}{\partial \psi} = -\frac{f_{KL}C}{\Delta\Lambda}\left\{\varphi - 2[\psi\alpha_S - (\chi - \psi)\alpha_D]L_{\bar{r}} \cdot (1 + \frac{f_{KL}{}^2C}{\Delta\Lambda})\right\} < 0 \quad \text{and}$$

$$\frac{\partial\left|\partial \tilde{K} / \partial \phi\right|}{\partial \psi} = -\frac{f_{KL}C}{\Delta\Lambda}\left\{\bar{r} - (1 + \frac{f_{KL}{}^2C}{\Delta\Lambda})2[\psi\alpha_S - (\chi - \psi)\alpha_D]L_{\varphi}\right\} < 0\,. \quad \blacksquare$$

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