# Patents versus RD subsidies in a Schumpeterian growth model with endogenous market structure

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# Patents versus R&D Subsidies in a Schumpeterian Growth Model with Endogenous Market Structure

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**Abstract**

In this note, we explore the different implications of patent breadth and R&D subsidies on economic growth and endogenous market structure in a Schumpeterian growth model. We find that these two policy instruments have the same positive effect on economic growth when the model exhibits counterfactual scale effects under an exogenous number of firms. However, when the model becomes scale-invariant under an endogenous number of firms, R&D subsidies increase economic growth but decrease the number of firms, whereas patent breadth expands the number of firms but reduces economic growth. Therefore, R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating economic growth.

*JEL classification*: O30, O40
*Keywords*: economic growth, endogenous market structure, patents, R&D subsidies

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# 1 Introduction

What are the different implications of patent breadth and R&D subsidies on economic growth and market structure? To explore this question, we consider a second-generation R&D-based growth model, pioneered by Peretto (1998), Young (1998), Howitt (1999) and Segerstrom (2000). To our knowledge, this is the first study that analyzes patent policy in a second-generation R&D-based growth model that is free of scale effects.[1] The model features two dimensions of technological progress. In the vertical dimension, firms improve the quality of existing products. In the horizontal dimension, firms invent new products. In this scale-invariant Schumpeterian growth model with endogenous market structure (EMS) measured by the number of firms in equilibrium, we find some interesting differences between patent breadth and R&D subsidies. At the first glance, these two policy instruments should have similar effects on innovation and economic growth. On the one hand, patent breadth improves the incentives of innovation by increasing the private return to R&D investment. On the other hand, R&D subsidies improve the incentives of innovation by reducing the private cost of R&D investment. For example, an interesting study by Li (2001) shows that both of these policy instruments contribute to increasing innovation and economic growth in a quality-ladder growth model that features scale effects. However, in a scale-invariant Schumpeterian growth model with EMS, we find that patent breadth and R&D subsidies have drastically different implications on economic growth and market structure. Specifically, when the number of firms is exogenous, we find that the model exhibits counterfactual scale effects and that both patent breadth and R&D subsidies have positive effects on economic growth as in previous studies. Interestingly, when we endogenize the number of firms to make the Schumpeterian model scale-invariant, we find that patent breadth expands the number of firms but decreases economic growth, whereas R&D subsidies increase economic growth but reduce the number of firms.

Intuitively, R&D subsidies decrease the cost of R&D investment and improve the incentives of R&D. As for patent breadth, although it increases the profit margin of monopolistic firms, which improves the incentives of R&D, it also leads to a decrease in the scale of production, which has a negative effect on the incentives of R&D. It turns out that the negative effect dominates the positive effect, so that a larger patent breadth decreases economic growth. These contrasting effects of patent breadth and R&D subsidies suggest that R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating economic growth. The negative effect of patent protection on R&D is consistent with empirical evidence in Qian (2007) and Lerner (2009), who find that strengthening patent protection may have a negative effect on innovation.[2] As for the positive effect of R&D subsidies on R&D, it is also consistent with empirical evidence; see Hall and Van Reenen (2000) for a survey of empirical studies.

This study relates to the literature on R&D-driven economic growth; see Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) for seminal studies. Subsequent studies in this literature often apply variants of the R&D-based

[1] See for example Laincz and Peretto (2006) for a discussion on how scale effects in R&D-based growth models are inconsistent with empirical evidence.

[2] See also the case-study evidence in Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008).

growth model to analyze the effects of policy instruments, such as patent breadth and R&D subsidies, on economic growth and innovation; see for example, Segerstrom (2000), Li (2001), Goh and Olivier (2002), Iwaisako and Futagami (2012), Chu (2011) and Chu and Furukawa (2011). However, these studies do not analyze the effects of patent policy on EMS;[3] therefore, the present study contributes to the literature with a novel analysis of patent breadth in a Schumpeterian growth model with EMS. Furthermore, we contrast the different effects of patent breadth and R&D subsidies and find that under a scale-invariant Schumpeterian growth model with EMS, the effects of patent breadth and R&D subsidies are drastically different suggesting the importance of taking into consideration EMS when performing policy analysis in R&D-based growth models. O'Donoghue and Zweimuller (2004), Horii and Iwaisako (2007), Furukawa (2007, 2010), Chu (2009), Chu *et al.* (2012) and Chu and Pan (2012) also find that increasing the strength of other patent policy levers, such as blocking patents and patentability requirement, can have a negative effect on economic growth; therefore, the present study complements these studies by proposing a novel mechanism (i.e., EMS) through which patent breadth also has a negative effect on R&D and economic growth.

The rest of this note is organized as follows. Section 2 presents the Schumpeterian growth model with EMS. Section 3 analyzes the effects of patent breadth and R&D subsidies.

# 2 A Schumpeterian growth model with EMS

In summary, the growth-theoretic framework is based on the Schumpeterian model with in-house R&D and EMS in Peretto (1996, 1998, 1999). We incorporate into the model patent breadth and R&D subsidies to analyze their different implications on economic growth and market structure.

## 2.1 Households

There is a representative household, who has the following lifetime utility function

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt, \tag{1}$$

where $C_t$ denotes consumption of final goods (numeraire) at time $t$. The parameter $\rho > 0$ determines the rate of subjective discounting. The household maximizes (1) subject to the following asset-accumulation equation

$$\dot{A}_t = r_t A_t + w_t L - \tau_t - C_t. \tag{2}$$

$A_t$ is the real value of assets owned by each household, and $r_t$ is the real interest rate. The household has a labor endowment of $L$ units and supplies them inelastically to earn a real

[3] See Peretto (1996, 1999) for seminal studies in R&D-based growth models with EMS and Etro (2012) for an excellent textbook treatment of this topic.

wage rate $w_t$. The household also faces a lump-sum tax $\tau_t$ from the government. From standard dynamic optimization, the familiar Euler equation is

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{3}$$

## 2.2 Final goods

Final goods $Y_t$ are produced by competitive firms that aggregate intermediate goods $X_t(i)$ for $i \in [0, N_t]$ using a standard CES aggregator given by

$$Y_t = \left( \int_0^{N_t} [X_t(i)]^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}, \tag{4}$$

where $\varepsilon > 1$ is the elasticity of substitution. From profit maximization, the conditional demand function for $X_t(i)$ is

$$X_t(i) = [p_t(i)]^{-\varepsilon} Y_t, \tag{5}$$

where $p_t(i)$ is the price of $X_t(i)$ and the price of $Y_t$ is normalized to unity.

## 2.3 Intermediate goods and in-house R&D

There is a continuum of industries producing differentiated intermediate goods. Each industry is dominated by a monopolistic firm with the following production function

$$X_t(i) = Z_t^{\theta}(i) L_{x,t}(i), \tag{6}$$

where $\theta \in (0, 1)$ is the elasticity of output with respect to technology. $L_{x,t}(i)$ is the number of workers employed in industry $i \in [0, N_t]$ for production. The law of motion for technology $Z_t(i)$ is

$$\dot{Z}_t(i) = Z_t^{\alpha} [Z_t(i)]^{1-\alpha} L_{z,t}(i), \tag{7}$$

where $\alpha \in (0, 1)$ determines the degree of technology spillovers,[4] and $Z_t \equiv \frac{1}{N_t} \int_0^{N_t} Z_t(i) di$ is the average level of technology in the economy. $L_{z,t}(i)$ is the number of workers employed in industry $i$ for R&D.

The value of the monopolistic firm in industry $i$ is

$$V_t(i) = \int_t^{\infty} \exp\left( - \int_t^u r_v dv \right) \pi_u(i) du. \tag{8}$$

The profit flow $\pi_t(i)$ at time $t$ is

$$\pi_t(i) = p_t(i) X_t(i) - w_t L_{x,t}(i) - (1 - s) w_t L_{z,t}(i), \tag{9}$$

[4] This specification nests the specification in Peretto (1998) as a special case with $\alpha = 1$.

where the parameter $s \in [0, 1)$ is the rate of R&D subsidies. The monopolistic firm maximizes (8) subject to (5) and (7). The current-value Hamiltonian for this optimization problem is

$$H_t(i) = \pi_t(i) + \lambda_t(i)\dot{Z}_t(i). \tag{10}$$

We solve this optimization problem in Appendix A and find that the unconstrained profit-maximizing markup ratio is $\varepsilon/(\varepsilon - 1)$. To analyze the effects of patent breadth, we impose an upper bound $\mu > 1$ on the markup ratio.[5] Therefore, the equilibrium price becomes

$$p_t(i) = \min\left\{\mu, \frac{\varepsilon}{\varepsilon - 1}\right\} \frac{w_t}{Z_t^\theta(i)}. \tag{11}$$

For the rest of this study, we assume that $\mu \leq \varepsilon/(\varepsilon - 1)$. In this case, a larger patent breadth $\mu$ leads to a higher markup, and this implication is consistent with Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

## 2.4 Entrants

A firm that is active at time $t$ must have been born at some earlier date. Following Peretto (1996, 1998, 1999), we consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0, N_t]$, by assuming that any new entry at time $t$ has access to the level of aggregate technology $Z_t$.[6] A new firm incurs $F$ units of labor as a setup cost to set up its operation and introduce a new variety of products to the market. We refer to this process as entry. Suppose entry is positive (i.e., $\dot{N}_t > 0$). Then, the no-arbitrage condition is[7]

$$V_t = w_t F. \tag{12}$$

The familiar Bellman equation implies that the return to entry is

$$r_t = \frac{\pi_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{13}$$

## 2.5 Government

The government chooses an exogenous rate $s$ of R&D subsidies. To balance the fiscal budget, the government levies a lump-sum tax $\tau_t$ on households. The balanced budget condition is

$$\tau_t = s w_t \int_0^{N_t} L_{z,t}(i)di. \tag{14}$$

[5] Intuitively, the presence of monopolistic profits attracts potential imitators. However, stronger patent protection increases the production cost of imitative products and allows monopolistic firms to charge a higher markup without losing market share to imitators; see also Li (2001), Goh and Olivier (2002), Chu (2011), Chu and Furukawa (2011) and Iwaisako and Futagami (2012) for a similar formulation.

[6] Peretto (1996, 1999) justifies this assumption by arguing that knowledge is embodied in workers. Therefore, when a new firm recruits workers from existing firms, it also gains access to their knowledge.

[7] We follow the standard approach in this class of models to treat entry and exit symmetrically (i.e., the scrap value of exiting an industry is also $w_t F$); therefore, $V_t = w_t F$ always holds. If $V_t > w_t F$ ($V_t < w_t F$), then there would be an infinite number of entries (exits).

## 2.6 Decentralized equilibrium

The equilibrium is a time path of allocations $\{A_t, C_t, Y_t, X_t(i), L_{x,t}(i), L_{z,t}(i)\}$ and prices $\{r_t, w_t, p_t(i), V_t(i)\}$. Also, at each instant of time, the following holds:

- Households maximize utility taking $\{r_t, w_t\}$ as given;
- Competitive final goods firms maximize profits taking $\{p_t(i)\}$ as given;
- Incumbents in the intermediate goods sector choose $\{p_t(i), L_{x,t}(i), L_{z,t}(i)\}$ to maximize $\{V_t(i)\}$ taking $\{r_t\}$ as given;
- Entrants make entry decisions taking $\{V_t\}$ as given;
- The value of all existing monopolistic firms adds up to the value of households' assets such that $A_t = N_t V_t$;
- The market-clearing condition of labor holds such that $L = N_t(L_{x,t} + L_{z,t}) + \dot{N}_t F$;
- Finally, the market-clearing condition of final goods holds such that $C_t = Y_t$.

## 2.7 Balanced growth path

In this subsection, we discuss the dynamic properties of the model. In Appendix B, we provide the derivations of the dynamic system represented by two differential equations $\dot{N}_t = f(N_t, L_{x,t})$ and $\dot{L}_{x,t} = h(N_t, L_{x,t})$. $N_t$ is a state variable, and $L_{x,t}$ is a jump variable. We find that given an initial value of $N_t$, the economy jumps to a saddle path and gradually converges to a unique and stable steady state; see Appendix B for the phase diagram.

# 3 Patent breadth versus R&D subsidies

We first present the aggregate equations. Imposing symmetry across $i \in [0, N_t]$ yields the labor-market clearing condition given by

$$L = N_t(L_{x,t} + L_{z,t}) + L_{n,t}, \tag{15}$$

where $L_{n,t} = \dot{N}_t F$ and $\dot{N}_t = 0$ in the steady state. Substituting (6) into (4) yields the aggregate production function of $Y_t$ given by

$$Y_t = (N_t)^{\varepsilon/(\varepsilon-1)} Z_t^{\theta} L_{x,t}. \tag{16}$$

The resource constraint implies $C_t = Y_t$. The rest of the analysis focuses on the balanced growth path, and we drop the time subscript for convenience.

At the steady state, $N$ and $L_x$ are stationary. Therefore, the steady-state equilibrium growth rate is

$$g \equiv \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \theta \frac{\dot{Z}}{Z} = \theta L_z. \tag{17}$$

We will show that the equilibrium growth rate is free of scale effects (i.e., $g$ is independent of $L$) when the number of firms is endogenous. Solving the dynamic optimization in (10),[8] we obtain the following equilibrium condition from the profit maximization of firms

$$L_z = \frac{1}{\alpha}\left[\frac{\mu\theta}{1-s}\left(\frac{\varepsilon-1}{\varepsilon}\right)L_x - \rho\right], \tag{18}$$

which describes a positive relationship between $L_x$ and $L_z$.

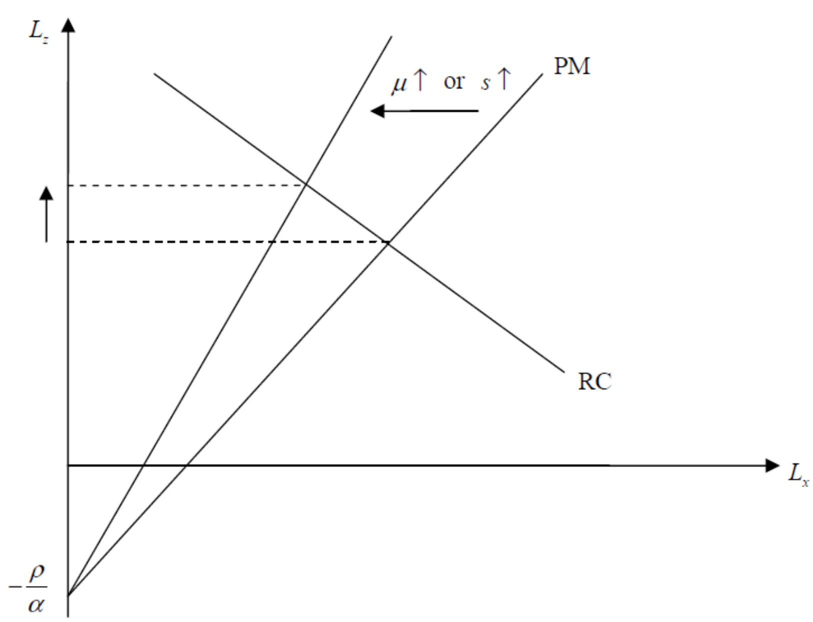
### 3.1 Scale effects under an exogenous number of firms

Combining (18) and the steady-state version of the resource constraint in (15) yields the steady-state equilibrium allocation of R&D labor as a function of $N$.

$$L_z(\underset{+}{\mu}, \underset{+}{s}) = \frac{(L/N)\theta(\varepsilon-1)/\varepsilon - \rho(1-s)/\mu}{\alpha(1-s)/\mu + \theta(\varepsilon-1)/\varepsilon}, \tag{19}$$

Suppose the number of firms $N$ is exogenous. Then, (19) shows that the model exhibits scale effects (i.e., $g = \theta L_z$ is increasing in $L$). In this case, $g = \theta L_z$ is increasing in patent breadth $\mu$ and R&D subsidies $s$; see also Li (2001) who finds a similar symmetric effect of patent breadth and R&D subsidies on economic growth. In Figure 1, we plot the solution of the profit-maximization (PM) condition in (18) and the resource constraint (RC) in (15) in the $(L_x, L_z)$ space to demonstrate the comparative statics with respect to $\mu$ and $s$.

Figure 1: Comparative statics of $\mu$ or $s$ under an exogenous number of firms




**Proposition 1** *Under an exogenous number of firms, the equilibrium growth rate is increasing in patent breadth and R&D subsidies.*

[8] Derivations are relegated to Appendix A.

## 3.2 Scale invariant under an endogenous number of firms

In this subsection, we consider an endogenous number of firms. In this case, market structure measured by the number of firms responds to the aggregate market size. Specifically, a larger supply of labor $L$ increases the aggregate market size and induces firms to enter the market raising the number of firms in equilibrium. As a result, the market size of each firm measured by the number of workers per firm (i.e., $L/N$) is independent of $L$ in the steady state. Because of this scale invariant property of the model, the equilibrium growth rate is free of scale effects.

When the number of firms is endogenous, we solve the model using the entry condition $V_t = w_t F$, which implies, from (13), $\pi_t/\rho = w_t F$ on the balanced growth path. Substituting (6) and (11) into (9) yields

$$\pi_t = \rho w_t F \Leftrightarrow L_z = \frac{(\mu - 1)L_x - \rho F}{1 - s}, \tag{20}$$

which describes another positive relationship between $L_x$ and $L_z$. We use (18), (20) and then (15) to solve for the steady-state equilibrium values of $\{L_z, L_x, N\}$.

$$L_z(\underset{-}{\mu}, \underset{+}{s}) = \left[\alpha - \frac{\mu\theta}{\mu - 1}\left(\frac{\varepsilon - 1}{\varepsilon}\right)\right]^{-1}\left[\frac{\mu\theta}{\mu - 1}\left(\frac{\varepsilon - 1}{\varepsilon}\right)\frac{F}{1 - s} - 1\right]\rho, \tag{21}$$

$$L_x(\underset{-}{\mu}, \underset{+}{s}) = \frac{(1 - s)L_z(\mu, s) + \rho F}{(\mu - 1)}, \tag{22}$$

$$N(\underset{+}{\mu}, \underset{-}{s}) = \frac{L}{L_x(\mu, s) + L_z(\mu, s)}, \tag{23}$$

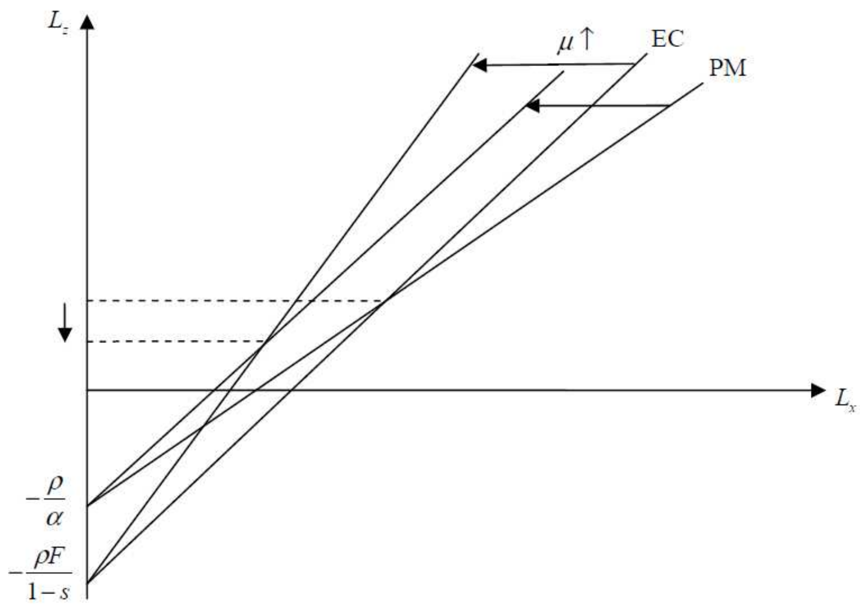
where $N$ is proportional to $L$ in the steady state. To ensure $L_z > 0$, we impose the following parameter condition

$$\alpha > \frac{\mu\theta}{\mu - 1}\left(\frac{\varepsilon - 1}{\varepsilon}\right) > \frac{1 - s}{F}. \tag{P1}$$

In Appendix C, we show that this parameter condition coincides with the stability condition in the capital market.

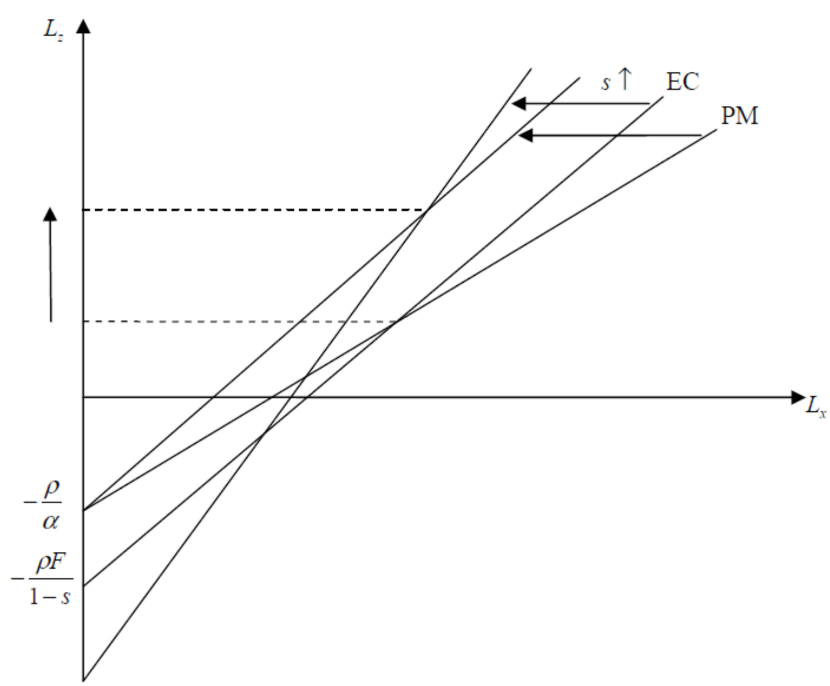
Recall the equilibrium growth rate is $g = \theta L_z$; therefore, (21) shows that $g$ is independent of $L$. In this model, $g$ is increasing in the discount rate $\rho$ due to endogenous entries. As for the effects of patent breadth, $L_z$ is decreasing in $\mu$, and $N$ is increasing in $\mu$. Intuitively, a larger $\mu$ increases the markup and the profit margin of each intermediate goods firm; however, it also decreases the scale of production $L_x$, which affects the incentives of R&D. It turns out that the negative effect dominates the positive effect such that R&D $L_z$ decreases. Given that both $L_x$ and $L_z$ decrease, the equilibrium number of firms $N$ must increase in order to satisfy the resource constraint on labor. In Figure 2, we plot the PM condition in (18) and the entry condition (EC) in (20) in the $(L_x, L_z)$ space to demonstrate the comparative statics with respect to $\mu$.

Figure 2: Comparative statics of $\mu$ under an endogenous number of firms




As for the effects of R&D subsidies, $L_z$ is increasing in $s$, and $N$ is decreasing in $s$. Intuitively, an increase in $s$ decreases the cost of R&D and leads to a larger $L_x$ increasing the scale of production. Both of these effects contribute to increasing R&D. Given that both $L_x$ and $L_z$ increase, the equilibrium number of firms $N$ must decrease in order to satisfy the resource constraint on labor. In Figure 3, we demonstrate the comparative statics with respect to $s$.

Figure 3: Comparative statics of $s$ under an endogenous number of firms



**Proposition 2** *Under an endogenous number of firms, a larger patent breadth increases the number of firms but decreases the equilibrium growth rate, whereas a higher rate of R&D subsidies increases the equilibrium growth rate but decreases the number of firms.*

Previous studies, such as Li (2001), Goh and Olivier (2002), Chu (2011), Chu and Furukawa (2011) and Iwaisako and Futagami (2012), also analyze the effects of patent breadth on R&D and economic growth. These studies generally find that a larger patent breath increases R&D because they consider an R&D-based growth model that features scale effects and exogenous market structure, which give rise to the different result from the present study. Therefore, the present study complements them by providing a novel analysis of patent breadth on economic growth under endogenous market structure and by showing the contrasting effects of patent breadth and R&D subsidies. Peretto (1998) and Segerstrom (2000) also analyze the effects of R&D subsidies in a scale-invariant Schumpeterian growth model. Segerstrom (2000) finds that R&D subsidies can have either positive or negative effects on economic growth, and this interesting result is driven by the tradeoff between quality improvement and variety expansion on economic growth. In contrast, economic growth is solely based on quality improvement in the present study and in Peretto (1998), who also finds a positive effect of R&D subsidies on economic growth. See for example Peretto and Connolly (2007) who show that quality improvement is the only plausible engine of economic growth in the long run.

# 4 Conclusion

In this note, we have analyzed the different implications of two important policy instruments, patent breadth and R&D subsidies, on economic growth and market structure in a scale-invariant Schumpeterian growth model with EMS. When we fix the number of firms in the model, we find that patent breadth and R&D subsidies serve to increase economic growth as in previous studies. However, when we consider a more realistic framework with an endogenous number of firms, we find that these two commonly discussed policy instruments have surprisingly opposing effects on economic growth and market structure. Specifically, patent breadth decreases economic growth but expands the number of firms, whereas R&D subsidies reduce the number of firms but increase economic growth. These contrasting effects of patent breadth and R&D subsidies suggest that R&D subsidy is perhaps a more suitable policy instrument than patent breadth for the purpose of stimulating economic growth. This finding is consistent with evidence from empirical studies and case studies discussed in the introduction.

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## Appendix A

**Derivations of $L_x$.** Substituting (5), (6) and (9) into (10) yields

$$H_t(i) = (Y_t)^{1/\varepsilon} [Z_t(i)]^{\theta(\varepsilon-1)/\varepsilon} [L_{x,t}(i)]^{(\varepsilon-1)/\varepsilon} - w_t L_{x,t}(i) - (1-s) w_t L_{z,t}(i) + \lambda_t(i) Z_t^{\alpha} [Z_t(i)]^{1-\alpha} L_{z,t}(i). \tag{A1}$$

The first-order conditions include

$$\frac{\partial H_t(i)}{\partial L_{x,t}(i)} = \frac{\varepsilon-1}{\varepsilon} (Y_t)^{1/\varepsilon} [Z_t(i)]^{\theta(\varepsilon-1)/\varepsilon} [L_{x,t}(i)]^{-1/\varepsilon} - w_t = 0, \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial L_{z,t}(i)} = -(1-s) w_t + \lambda_t(i) Z_t^{\alpha} [Z_t(i)]^{1-\alpha} = 0, \tag{A3}$$

$$\frac{\partial H_t(i)}{\partial Z_t(i)} = \theta \left( \frac{\varepsilon-1}{\varepsilon} \right) (Y_t)^{1/\varepsilon} [Z_t(i)]^{\theta(\varepsilon-1)/\varepsilon-1} [L_{x,t}(i)]^{(\varepsilon-1)/\varepsilon} + (1-\alpha) \lambda_t(i) \left[ \frac{Z_t}{Z_t(i)} \right]^{\alpha} L_{z,t}(i) = r_t \lambda_t(i) - \dot{\lambda}_t(i). \tag{A4}$$

Using (5) and (6), we can derive from (A2) the unconstrained profit-maximizing price in (11). Substituting (A3), (5) and the constrained markup price in (11) into (A4), we obtain

$$\frac{\mu\theta}{1-s} \left( \frac{\varepsilon-1}{\varepsilon} \right) \left[ \frac{Z_t}{Z_t(i)} \right]^{\alpha} L_{x,t}(i) + (1-\alpha) \left[ \frac{Z_t}{Z_t(i)} \right]^{\alpha} L_{z,t}(i) = r_t - \frac{\dot{\lambda}_t(i)}{\lambda_t(i)}. \tag{A5}$$

Applying $r = \rho + g$ and imposing symmetry and balanced growth on (A5) yield (18). ■

## Appendix B

**Dynamics of the model.** In this appendix, we provide the derivations of the dynamic system. First, we use the resource constraint on labor to derive a differential equation in $N_t$. Second, we use the firm's profit-maximization conditions to derive a differential equation in $L_{x,t}$. Third, we use the entry condition to derive a static equilibrium condition for $L_{x,t}$ and $L_{z,t}$. Finally, putting these conditions together, we have an autonomous dynamic system in $\dot{N}_t = f(N_t, L_{x,t})$ and $\dot{L}_{x,t} = h(N_t, L_{x,t})$. We find that its dynamics is characterized by saddle-path stability.

Substituting $\dot{N}_t = L_{n,t}/F$ into (15) yields our first differential equation given by

$$\dot{N}_t = \frac{1}{F}\left[L - N_t(L_{x,t} + L_{z,t})\right]. \tag{B1}$$

To derive our second differential equation, noting $Z_t(i) = Z_t$, we use (A3) to derive

$$\frac{\dot{\lambda}_t}{\lambda_t} = \frac{\dot{w}_t}{w_t} - \frac{\dot{Z}_t}{Z_t}. \tag{B2}$$

Substituting (B2) and the Euler equation (3) into (A5) yields

$$\frac{\dot{C}}{C} - \frac{\dot{w}_t}{w_t} = \frac{\mu\theta}{1-s}\left(\frac{\varepsilon - 1}{\varepsilon}\right) L_{x,t} - \alpha L_{z,t} - \rho, \tag{B3}$$

where we have also used $\dot{Z}_t/Z_t = L_{z,t}$. Using (5), (6), (11) and (16), we derive

$$w_t = \frac{1}{\mu}\left(N_t\right)^{1/(\varepsilon-1)} Z_t^{\theta}. \tag{B4}$$

Taking the log of (B4) and (16) and then differentiating them with respect to $t$ yield

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{w}_t}{w_t} = \frac{\dot{N}}{N} + \frac{\dot{L}_{x,t}}{L_{x,t}}, \tag{B5}$$

where we have also used $C_t = Y_t$. Substituting (B1) and (B3) into (B5) yields

$$\frac{\dot{L}_{x,t}}{L_{x,t}} = \frac{\mu\theta}{1-s}\left(\frac{\varepsilon - 1}{\varepsilon}\right) L_{x,t} - \alpha L_{z,t} - \rho + \frac{1}{F}\left(L_{x,t} + L_{z,t} - \frac{L}{N_t}\right). \tag{B6}$$

To derive a static equilibrium condition for $L_{z,t}$, we use the entry condition (12) to derive

$$\frac{\dot{w}_t}{w_t} = \frac{\dot{V}_t}{V_t} = r_t - \frac{\pi_t}{V_t}, \tag{B7}$$

where the second equality uses (13). Substituting (3) into (B7) yields

$$\frac{\pi_t}{V_t} = \rho + \frac{\dot{C}_t}{C_t} - \frac{\dot{w}_t}{w_t} = \frac{\mu\theta}{1-s}\left(\frac{\varepsilon - 1}{\varepsilon}\right) L_{x,t} - \alpha L_{z,t}, \tag{B8}$$

where the second equality uses (B3). Using (6), (9), (11) and (12), we obtain

$$\frac{\pi_t}{V_t} = \frac{(\mu - 1)\, L_{x,t} - (1-s) L_{z,t}}{F}. \tag{B9}$$

Combining (B8) and (B9) yields

$$L_{z,t} = \left(\alpha - \frac{1-s}{F}\right)^{-1} \left[\frac{\mu\theta}{1-s}\left(\frac{\varepsilon - 1}{\varepsilon}\right) - \frac{(\mu-1)}{F}\right] L_{x,t}. \tag{B10}$$

The parameter restriction (P1) ensures that the coefficient on $L_{x,t}$ in (B10) is positive; therefore, (B10) describes a positive relationship between $L_{x,t}$ and $L_{z,t}$.

Equations (B1), (B6) and (B10) together form an autonomous dynamic system. We first use (B1) to derive the $\dot{N}_t = 0$ locus given by

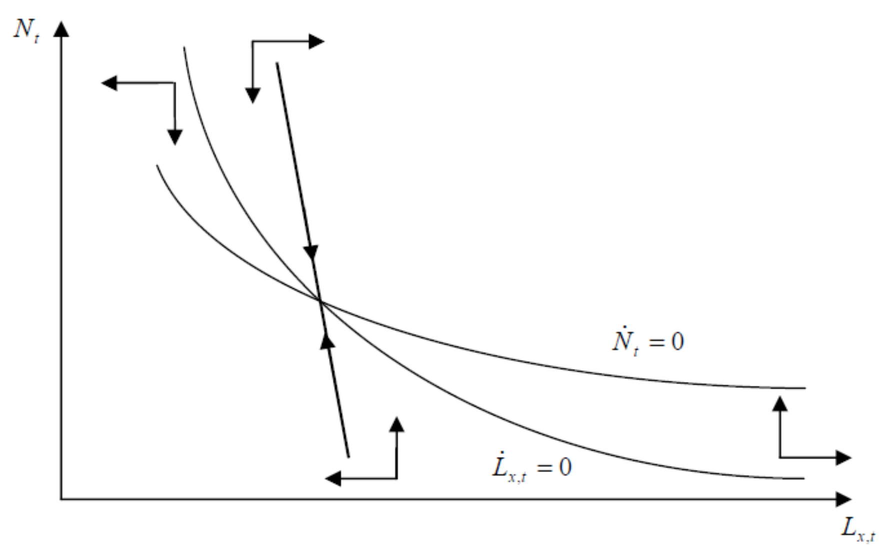
$$\dot{N}_t = 0 \Leftrightarrow N_t = \frac{L}{L_{x,t} + L_{z,t}}, \tag{B11}$$

where $L_{z,t}$ is given by (B10). Then, we use (B6) to derive the $\dot{L}_{x,t} = 0$ locus given by

$$\dot{L}_{x,t} = 0 \Leftrightarrow N_t = \left\{L_{x,t} + L_{z,t} + \left[\frac{\mu\theta}{1-s}\left(\frac{\varepsilon-1}{\varepsilon}\right) L_{x,t} - \alpha L_{z,t} - \rho\right] F\right\}^{-1} L, \tag{B12}$$

where $L_{z,t}$ is also given by (B10). Both (B11) and (B12) describe a negative relationship between $N_t$ and $L_{x,t}$.[9] If we plot the two curves in the $(L_{x,t}, N_t)$ space, then the $\dot{N}_t = 0$ locus is below (above) the $\dot{L}_{x,t} = 0$ locus when $L_{x,t}$ is small (large). Their intersection point is the steady-state equilibrium allocation of $L_x$ in (22). Figure 4 plots the phase diagram of this dynamic system and shows that given an initial value of $N_t$, $L_{x,t}$ jumps to a saddle path that gradually converges to a unique and stable steady state. ■

Figure 4: Phase diagram




[9] Using (B10) and (P1), one can show that $\frac{\mu\theta}{1-s}\left(\frac{\varepsilon-1}{\varepsilon}\right) L_{x,t} - \alpha L_{z,t}$ is increasing in $L_{x,t}$.

## Appendix C

**Stability of the capital market.** To confirm the stability of the equilibrium condition in the capital market, we show that for a given $N$, the rate of return to R&D crosses the rate of return to entry from above in the $(L_z, r)$ space; see Peretto (1998) for a more detailed discussion. From (A5), the rate of return to R&D is

$$r^{R\&D} = \frac{\mu\theta}{1-s}\left(\frac{\varepsilon-1}{\varepsilon}\right)L_x + (1-\alpha)L_z + \dot{\lambda}_t/\lambda_t, \tag{C1}$$

where $\dot{\lambda}_t/\lambda_t = (\theta-1)L_z$. Substituting $L = N(L_x + L_z)$ into (C1) yields

$$r^{R\&D} = \frac{\mu\theta}{1-s}\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{L}{N} - L_z\right) + (\theta-\alpha)L_z. \tag{C2}$$

Substituting (12) into (13) and imposing balanced growth yield

$$r^{entry} = \frac{\pi}{wF} + \theta L_z = \frac{(\mu-1)L_x - (1-s)L_z}{F} + \theta L_z. \tag{C3}$$

Substituting $L = N(L_x + L_z)$ into (C3) yields

$$r^{entry} = \frac{1}{F}\left[(\mu-1)\frac{L}{N} - (\mu-s)L_z\right] + \theta L_z. \tag{C4}$$

The parameter condition in (P1) implies that $r^{R\&D}$ crosses $r^{entry}$ from above in the $(L_z, r)$ space; see Figure 5 for an illustration. Therefore, whenever $L_z$ is to the right (left) of the intersection, $r^{R\&D} < (>)\ r^{entry}$ and labors are reallocated from R&D to entry (from entry to R&D) confirming the stability of $r^{R\&D} = r^{entry}$. ■

Figure 5: Equilibrium of the capital market