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Abstract

McAfee and McMillan (1992) investigate the coordinated biding strategies in a strong cartel, in which the cartel members can exclude new entrants and can make transfer payments, and show that the strong cartel mechanism is incentive-compatible and efficient. In this paper however, we show the strong cartel mechanism is no longer ratifiable in the presence of participation cost, in which case the bidder with the highest value in the cartel would have incentive to veto for the collusive mechanism. This behavior could make the maximum benefit for the winning bidder.

1 Introduction

Asymmetric information is pervasive in economic activities, and often brings about collusive behaviors. In reality, the collusive problem often happens in an industry with heterogeneous firms and private information. In order to reduce the possibility of inefficiency due to collusion among bidders, people often use auction mechanism to enhance efficiency.

Myerson (1981) considers the optimal auction design problem of a seller who has an indivisible object to sell to one of several buyers. While there is imperfect information of the paying price between the seller and buyers, the seller's problem is to design an auction game that gives her the highest possible expected profit at Nash equilibrium. Thus, the seller must consider all probabilities of buyers' evaluation and try to choose an auction design which offers her the highest expected revenue. Alternatively speaking, bidders also want to choose an optimal bidding strategy to maximize their benefits. If asymmetric information exists, bidders might cooperate to gain the monopoly power to earn the most surplus in the auction.

Is optimal auction mechanism able to prevent collusive behavior? The answer may be no. Bidders could still find a way to profitably collude and share the generated collusive surplus by their private information. Cooper (1977) finds the anecdotal evidence from antiques and artwork auctions which indicates retaliatory strategies are in fact used to enforce collusion. When one of the ring members goes against his partners, or the ring falls out for one reason or another, then it works very well to the seller's advantage as vindicative competition leads to crazy prices.

Roberts (1983) studies the incentive mechanism design problem in the presence of collusive behavior. He finds if types of the firm are sufficiently similar, then the monopoly collusive mechanism cannot be achieved without side payments. If side payments are allowed, such collusion is possible with a dominant strategy and this mechanism is essentially equivalent to the second-price auction. Cramton and Palfrey (1995) attempt to find what cartel agreements are possible when firms have private information about production costs. They characterize a set of cartel agreements that can be supported while recognizing incentives and participation constraints. The incentive problem in large cartels is severe enough to prevent the cartel from achieving the monopoly outcome in either Cournot or Bertrand competition. However, if the cartel agreement requires less than unanimous ratification by members, the incentive problem can be overcome in large cartels. With common cost of uncertainty, perfect collusion is possible in large cartels, regardless of the ratification rule.

However, as McAfee and McMillan (1992) (hereafter MM) point out, the efficient colluding mechanism is generally not freely implemented. They discuss the bidding strategies in first-price sealed-bid auction in two cases: weak cartel, in which bidders cannot make transfer payments and strong cartel, in which the cartel members exclude new entrants and can make transfer payments. They show in the setting of weak cartel, the optimal outcome can be implemented without coordination. This means bidders can bid identical bid in the legit auction to achieve the optimal situation, because in the absence of side-payments, incentive compatibility requires the good should be awarded stochastically, with equal probability of being awarded to anyone whose value is larger than the minimum price: any attempt to arrange that the highest-value bidder wins generates incentives for the bidders to misstate their valuations. By submitting equal bids, the bidders in effect use the seller as their randomizing device. If bidders' values are lower than or equal to the reserve price, they have negative or zero revenue to win the auction. Thus any transfer scheme cannot be lump-sum and the bidder who fails to receive a transfer payment with the highest valuation has an incentive to overstate his valuation. But in strong cartel concept, the optimal cartel mechanism can be implemented by a prior auction, and reach the efficiency.

When people consider (interim) individual rationality constraint following the standard mechanism design approach, they compare bidders' interim payoffs from the collusive mechanism with those from an equilibrium of the seller's auction, i.e., the status quo. However, that the status quo here is a strategic interaction, and its outcome is affected by bidders' beliefs about others' values. This observation is important if bidders are making interim participation decisions for the collusive mechanism. Others can make inferences about a bidder's type from her choice between the collusive mechanism and the status quo, which in turn may affect the outcome of the status quo game if it is played. In Carmton and Palfrey (1995) paper, they refer to this possibility as the information leakage problem from participation decisions. To address this problem, they consider a two-stage process: Players simultaneously vote for or against the proposed mechanism. This vote occurs at the interim stage, i.e., when players have private information. If the mechanism is unanimous;y ratified, then it is implemented; otherwise the status quo game is played under revised beliefs where these satisfy consistency requirements in the spirit of rational expectations.

The fundamental structure of the auction with participation costs is one that an indivisible object is allocated to one of many potential buyers, and in order to participate in the auction, buyers must incur some costs, like entry fees or participation costs. After the cost is incurred, a bidder can submit a bid. Sometimes pre-bid costs are also required to get bidders involved in auctions, as Mills (1993) points out the bidding cost in a government procurement auction often runs into millions of dollars. If there is participation cost, the result for a collusive mechanism might change. According to this, we should consider the MM's model with participation costs to make the model more realistic than the model in the original setting. If a bidder's expected profit from attending an auction is less than the participation cost, he will not attend the auction. Even though the bidder's valuation is greater than the participation cost, he may not have the same bidding strategy as in an auction without participation costs.

Tan and Yilankaya (2007) investigate properties of second price auctions with equal participation costs when bidders' values are private information. They apply Carmton and Palfrey's (1995) method to show that the standard efficient collusive mechanisms characterized in the literature for the independent private values environment are not ratifiable when the status quo is a second-price auction and there are participation costs, however small they may be.

In MM paper, they interpret their efficiency model in first-price sealed-bid auction. Due to the cartel might not always be formed, there may incur information leakage problem. In this paper, we focus on the ratifiability of the cartel mechanism in first-price sealed-bid auction. We use the bidding function and equilibrium found in Cao and Tian (2009) paper to show the efficient cartel mechanism defined in "strong cartel" would not be ratifiable in the first-price sealed-bid auction with participation costs. This means there is a discontinuity between $c = 0$ and $c > 0$.

In section 2 we briefly describe the basic model setting. Section 3, we discuss the intuition about our investigation. We show equilibrium with different cutoff values in section 4. Section 5 is conclusion. All proofs are included in the appendix.

2 Model

In our setting, the seller and bidders are all risk-neutral, and rational. The bidder i's valuation for an indivisible object is v_i $(i = 1, 2, ..., n)$, and v_i is private information to the other bidders. We assume v_i is independently distributed [1] with a cumulative distribution function $F_i(\cdot)$, and it is continuously differentiable with a density function $f_i(\cdot) > 0$, within $[0, 1]$. Following the MM model with homogeneous bidders, we discuss the case that bidders' valuations $(v_1, v_2, \ldots, v_n)$ are drawn from the same distribution function; i.e., $F_1(\cdot) = F_2(\cdot) = \cdots = F_n(\cdot)$ We denote a participation cost, c, which bidders must pay before they bid by $c \in [0, 1]$. The seller values her object at $r \in [0, 1]$. Only the winner in the legit auction has to pay the reserve price but all bidders who participate in the legit auction have to pay the participation cost. This cost would make losers earn negative profit. The seller's behavior is passive in this model. She announces a reserve price and sells at the highest bid to the bidder in a first-price sealed-bid auction. We also assume that the seller knows neither she faces a cartel nor she is bound to the rules of the sealed-bid auction.

According to MM paper, we assume that the efficient cartel designs an incentive-compatible

[1] As mentioned in MM paper, the assumption of independent distribution rules out correlated valuations. This may make the model not compatible with some auctions where collusion occurred in real world. However, this assumption is not inconsistent while there are still some factors could also affect bidders' valuations. If there is a limited edition car on an auction with its actual value $V_i = P + s_i$, where s_i represents each bidder's personal valuation of the car and P is its future market price, the same for all bidders and which is independent of the s_i, then the independent private value assumption could still be applied to situations with common value elements. This provides the independence assumption in the model that all bidders with the same information about the common aspects would be satisfied.

mechanism in order to maximize the ex ante (before the valuations are known) sum of bidders' expected profits in the auction which means this cartel is self-enforcing.

The information setting is interim. Bidders only know their own private information before they bid in the prior auction in the cartel and they do not know others' valuations when they makes their own decisions. The cartel would choose the bidder with the highest bid in the prior auction to bid in the legit auction.

The timing of our model among the bidders is as follows. Considering possible cartel formation between date 1 and date 2,

- At date 1, a cartel mechanism exists and all bidders are in this mechanism at beginning.
- At date $1\frac{1}{5}$, nature draws a private valuation v_i for each bidder i.
- At date $1\frac{2}{5}$, each bidder bids in the prior auction.
- At date $1\frac{3}{5}$, bidders update their beliefs and vote for or against the collusive mechanism.
- At date $1\frac{4}{5}$, the game form specified in the collusive mechanism is played at date 2, if all bidders accepted the collusive mechanism at date $1\frac{3}{5}$. If at least one bidder rejects the collusive mechanism, no collusion occurs. Bidders bid in the legit auction at date 2 noncooperatively.

Note that the collusion is formed after the agents make participation decisions. We also assume when a bidder is indifference between staying in and vetoing for the cartel, he would choose to stay in the cartel.

An efficient cartel mechanism provides bidder i to win with probability $p_i(v_i, v_{-i})$ if and only if his value exceeds the minimum price r plus the cost c and all the other bidders' values v_{-i} . The latter occurs with probability, $F(v_i)^{n-1}$. The efficiency[2] is then characterized by

$$E_{-i}p_i(v_i, v_{-i}) = \begin{cases} F(v_i)^{n-1} & v_i \geq r + c \\ 0 & v_i < r + c. \end{cases}$$

The profit function is $\pi_i(\cdot) \geq 0$. Besides, the winner in the prior auction becomes the only bidder in the legit auction and would win the auction for sure, because there is no other bidder

[2]If a cartel member whose value is less than or equal to the minimum price r plus the participation cost c is constrained to earn zero profit, then efficiency implies noncooperative profit levels.

outside the cartel. So, he would pay the reserve price r plus the participation cost c, and pay the transfer payment[3], $Z(\cdot)$[4], to all members in the cartel.

The individual action set for any bidder can be characterized as $N_0 \cup [0, 1]$, where "N_0" denotes not submitting a bid. An equilibrium strategy of whether to participate is given by a profile of the bidders' cutoff points, which are a vector of the minimum valuations for each bidder i to cover the cost. Assume that a bidder with zero probability of winning would not bid more than his true value when he participates in the auction[5]. Given the strategies of all other bidders, a bidder's expected revenue from participating in the legit auction is a non-decreasing function of his own valuation[6]. Let $v^* = (v_1^*, \ldots, v_n^*)$ denote the profile of bidders' cutoff points. The bidding decision function $b_i(\cdot)$ of each bidder is characterized by

$$b_i(v_i, v^*) = \begin{cases} \lambda_i(v_i, v^*) & 1 \geq v_i \geq v_i^* \\ No & v_i < v_i^*. \end{cases}$$

where $\lambda_i(v_i, v^*)$ is a contingent bidding function when bidder i participates in the auction.

Once a bidder enters the prior auction, he can observe who have also entered the prior auction and can update his belief about others' valuation. If we observe that bidder i participates in the prior auction, it can be inferred that bidder i's value is bigger than or equal to v_i^*. Using Bayes' rule, we believe that bidder i's value is distributed on $[v_i^*, 1]$ with

$$Pr(\xi \leq v | v \geq v_i^*) = \frac{Pr(v_i^* \leq \xi < v)}{Pr(\xi \geq v_i^*)} = \frac{F_i(v) - F_i(v_i^*)}{1 - F_i(v_i^*)}.$$

The corresponding density function is given by $\frac{f_i(v)}{1-F_i(v_i^*)}$.

[3]As McAfee and McMillan (1992) show in the first-price sealed-bid auction, if the highest bid in this prior auction exceeds r, the winner then bids r in the legit auction and pays each of the losers an equal share of the difference between his bid in the prior auction and r.

[4]We use $Z(\cdot)$ in this paper which is equal to $T(\cdot) - r$ in MM paper.

[5]Without this assumption a bidder with value v_i, who in optimum has zero probability of winning, can sometimes bid more than his value. However, this bidding strategy can be eliminated by a trembling-hand argument. Once a bidder bids above his value, he may have positive probability to win the object which gives him a negative revenue. For a bidder, bidding below his value when he has zero probability of winning can also be supported in an optimal bidding strategy. However the allocation is the same as the optimal bidding strategy where he bids his value. For simplicity, we eliminate it.

[6]Lu and Sun (2007) show that for any auction mechanism with participation costs, the participating and nonparticipating types of any bidder are divided by a nondecreasing and equicontinuous shutdown curve. Thus in our framework, when participation cost is given, the participating and nonparticipating types of any bidder can be divided by a cutoff value and the threshold form is the only form of equilibria.

3 Intuition

A profit-maximizing cartel clearly prefers efficiency in the absence of incentive constraints, so that the pie to be divided among the members is as large as possible.

We can easily find the bidder with the highest value in the cartel would have incentive to veto for the mechanism in strong cartel with participation costs. The profit for bidder to bid alone in the legit auction is larger than he stays in the cartel. While bidders participating in the prior auction, they report their own bid to the cartel mechanism, and the ith bidder would win the object with probability $p_i(v_i, v_{-i})$. Then, bidder i's expected benefit is

$$\pi_i = E_{-i}[v_i p_i(v_i, v_{-i}) - Z_i(v_i, v_{-i}) - r - c].$$

where E_{-i} is the expectation over v_{-i}, Z_i is the expected transfer payment by bidder i, and c is participation cost. If the transfers are prohibited, Z_i equals 0. In strong cartel setting, it is able to make transfer payments among its members. Each losing bidder in the cartel could get

$$z_i = \frac{Z_i(v_i, v_{-i})}{n-1}.$$

If we play the game step by step, the bidder i, who has the highest valuation in the prior auction, would represent the cartel to bid in the auction. He will pay the participation cost c, and bid the value r to win the object. The seller will get r, and the winner i's benefit would be $\pi_i^s = v_i - Z_i(v_i, v_{-i}) - r - c \geq 0,$, where π_i^s is the profit he stays in the cartel. π_i^v is the profit he vetoes for the cartel.

There are two reasons to explain the winner have incentive to veto for the cartel.

(1) After the prior auction, he knows that no one has higher valuation than himself. If he is the only one who enters the auction, he can just pay $r + c$ and win this auction. At this time, his benefit is $\pi_i^v = v_i - r - c \geq \pi_i^s = v_i - Z_i(v_i, v_{-i}) - r - c \geq 0$.

(2) All the other bidders know who has the highest bid in the cartel after prior auction. If the winner in the prior auction bids his bid in the legit auction, they have no chance to win. After they update their belief, no one wants to join the auction because they would lose the auction for sure and have to pay the participation cost c, which makes them get negative profits.

According to the reasons mentioned above, the winner vetoing for the cartel could earn the maximum benefit, and others get zero benefit in this game. In this intuition, the cartel mechanism could not be sustainable.

4 Equilibrium

4.1 MM Model: The Benchmark Case

We set the participation cost, c, equal to zero to discuss the benchmark case of the strong cartel model in MM paper.

The strong cartel mechanism works as follows. While the bidders reporting their values to the cartel mechanism, the ith bidder is awarded the good with probability $p_i(w_i, w_{-i})$, w_i is the value reported from bidder i. Then, bidder i's expected profit, if he has value v_i and reports w_i, is

$$\pi_i = E_{-i}[v_i p_i(w_i, w_{-i}) - Z_i(w_i, w_{-i})] - r.$$

where E_{-i} is the expectation over w_{-i}. After dropping the bidder indices to simplify the notation, we can show:

$$\begin{aligned} \pi &= [v - Z(w)]F(w)^{n-1} \\ &+ [1 - F(w)^{n-1}] \int_w^{v_h} [\frac{Z(u)}{n-1}] \frac{(n-1)F(u)^{n-2} f(u)}{1 - F(w)^{n-1}} du \\ &= [v - Z(w)]F(w)^{n-1} \\ &+ \int_w^{v_h} [Z(u)]F(u)^{n-2} f(u) du \end{aligned}$$

where v_h is the highest value in the cartel. Finally, we get $\frac{\partial \pi}{\partial w}|_{w=v} = 0$, which means everyone in the cartel would report their value truthfully, which means the incentive compatible condition is satisfied.

They get the following result to specify the optimal direct mechanism:

Proposition 1 *Suppose $c = 0$, the strong cartel mechanism is incentive-compatible and efficient. Before the auction, the cartel members report their valuations to the mechanism. If no report exceeds r, the cartel does not bid in the auction. If at least one bid exceeds r, the bidder making the highest report v obtains the item and pays a total of*

$$Z(v) + r = F(v)^{-n} \int_r^v (u - r)(n-1)F(u)^{n-1} f(u) du + r.$$

and each losing bidder receives from the winner $[Z(v)/(n-1)]$

This cartel mechanism can be implemented by holding a prior first-price sealed-bid auction. Everyone can update the information about the highest valuation in the prior auction. This mechanism has the property that all losing bidders receive a transfer which means bidders whose value is less than the minimum price earn positive profits.

This cartel convinced under the Sherman antitrust Act of 1980, six cast-iron-pipe manufacturers, operated a knockout. In those cities not reserved for a particular firm, the price was fixed by a central committee (this is our r), Before a contract was let, the central committee accepted bids from the cartel mechanism members for the right to the contract. The lowest bidder then bid the prearranged price in the legitimate auction, and the others submitted higher bids. The surplus was periodically distributed to the cartel members in proposition to their production capacities (stigler, 1966)

Proposition 2 *An efficient cartel mechanism has the property that the winner transfers to each of the losers an amount equal to*

$$z_i = \frac{E[v_{(2)} - r | v_{(1)} \geq r]}{n}$$

where $v_{(j)}$ represents the jth order statistic and the expectation is taken over the distribution of the highest valuation. The winner's expected rent is this amount plus the rent he would have earned if the auction had been noncooperative.

If the bidding is noncooperative, the total expected profit earned by the winning bidder would be the expected difference between the highest valuation and the second-highest valuation, because the bidder would drive the price up to the second-highest valuation. With collusion, the total expected profit earned by the cartel is the difference between the expected highest valuation and the minimum price, because the price paid to the seller is the minimum price. Thus, the extra expected profit resulting from the cartel's activity, to be shared among the cartel members, is the difference between these two profit levels, or the expected second-highest valuation minus the minimum price. The equation above is essential that a first-price sealed-bid auction be used by the cartel, and has each of the cartel members getting an equal share of this sum (including the winner, whose total payment including his transfers to fellow cartel members is this amount less than what it would have been under noncooperative bidding). Because there is no cost entering the legit auction, if there is any bidder vetoing for the cartel after the prior auction, bidders can enter the legit auction for free and bid as much as possible to make the vetoer win the auction without any profit. This behavior makes the winner be indifference between staying in and vetoing for the mechanism.

4.2 Auction with Participation Costs

Following the MM model with homogeneous bidders, we discuss the case that bidders' valuations $(v_1, v_2, \ldots, v_n)$ are drawn from the same distribution function; i.e., $F_1(\cdot) = F_2(\cdot) = \cdots = F_n(\cdot)$. In order to simplify the calculation and notation, we assume the reserve price r equal to 0. For bidders using the same cutoff point v^*, the supports of their updated valuation distributions have the same lower bound when they participate in the prior auction. Thus when $v_i = v^*$, bidder i can only win the item when all others do not participate in the prior auction. In equilibrium we have

$$c = v^* F(v^*)^{n-1}.$$

Since $\rho(v) = vF(v)^{n-1} - c$ is an increasing function of v with $\rho(0) < 0$ and $\rho(1) > 0$, a unique symmetric equilibrium exists. When bidders use the same threshold and participate in the auction, the optimal bidding function is unique, which is symmetric and monotonic increasing, and is given by

$$\lambda(v_i, v^*) = v_i - \frac{\int_{v^*}^{v_i} (F(y) - F(v^*))^{n-1} dy}{(F(v_i) - F(v^*))^{n-1}}$$

In order to show the winning bidder has incentive to choose not to stay in the cartel, we have to define a credible veto set and prove if the bidder's value belongs in the veto set, he will veto for the cartel.

Definition 1 $A \subseteq [0, 1]$ *is a credible veto set, which means* $v_i \in A$ *if the individual rational (IR) condition,* $\pi_i^s > \pi_i^v$*, could not be satisfied, such that*

(1) $A \neq \emptyset$

(2) $\pi_i^v(v) > \pi_i^s(v) \Rightarrow v \in A$

(3) $\pi_i^v(v) < \pi_i^s(v) \Rightarrow v \notin A$

Definition 2 *Let* b^* *be an equilibrium of the first-price auction with participation cost* c *where others believe that* $i's$ *type is in* A*.*

For example, if $A = [v_N, 1]$, then b^* is an equilibrium of the auction where ratifiers' values are distributed on $[0, 1]$ according to $F(\cdot)$, and the vetoer's value is believed to be distributed on $[v_N, 1]$ according to $F_N(v) \equiv \frac{F(v) - F(v_N)}{1 - F(v_N)}$, which is derived from $F(\cdot)$ using Bayes' rule.

Definition 3 *The cartel mechanism is ratifiable against the seller's auction if for all* i *either*

(1) there does not exist a credible veto set for i *or*

(2) there exists a credible veto set A *and a corresponding equilibrium* b^* *in the post-veto auction such that* $\pi^v(v, b^*) = \pi^s(v)$ *for all* $v \in A$

To present above intuition formally, we will construct a credible veto set. We will then show that there is no credible veto set where all types belonging to it would be indifferent between vetoing and ratifying. Suppose when one of the bidders vetoes the cartel, others believe that his value exceeds a cutoff point $v_N < 1$. We will show that there is an equilibrium of the auction with these revised beliefs such that the vetoer's payoff in this equilibrium is larger than his cartel payoff if his value is larger than v_N. In other words, it is precisely the types in $[v_N, 1]$ who wants to veto the cartel if a veto causes other bidders to believe that the vetoer's type is in $[v_N, 1]$, so that $A = [v_N, 1]$ is a credible veto set for any bidder i.

Consider the first-price auction where the ratifiers' values are distributed on $[0, 1]$ according to $F(\cdot)$ and the vetoer $i's$ value is distributed on $[v_N, 1]$ according to $F_N(v)$, defined earlier. The equilibrium we consider in this auction , denoted by b^*, is given by

$$b_i^*(v) = \lambda(v) \quad for \;\; all \;\; v \in [v_N, 1]$$

$$b_j^*(v) = \begin{cases} N_0 & v \leq v_Y \\ \lambda(v, v_Y) & v > v_Y \;\; for \;\; all \;\; j \neq i. \end{cases}$$

In this equilibrium, the ratifiers use a common cutoff v_Y, which is determined by the indifference to participation condition. For any ratifier, the maximum of others' values is distributed on $[v_N, 1]$ according to $\hat{G}(y) \equiv F(y)^{n-2}F_N(y)$. Let $\tilde{v}_Y$ be the solution to

$$\int_{v_N}^{\tilde{v}_Y} (\tilde{v}_Y) d\hat{G}(y) = c.$$

where the left hand side denotes the payoff of a $\tilde{v}_Y$ type ratifier whenever $\tilde{v}_Y \leq 1$. We have $v_Y = min\{1, \tilde{v}_Y\}$ and $v_N < v_Y \leq 1$. Notice that v_Y is a strictly increasing function of v_N until it reaches 1 for some value of v_N and stays there for higher values of v_N. The payoff of any type-v vetoer i is

$$\pi^v(v, b^*) = max\{\int_0^{max(v, v_Y)} (v - \lambda(y)) dG(y) - c, 0\}$$

Comparing this payoff with the one from the cartel leads to the following lemma. All Proofs are in appendix.

Lemma 1 *There exists a (unique)* $v_N \in (c, 1)$ *such that*

$$\pi^v(v; b^*) \geq \pi^s(v) \;\; iff \;\; v \geq v_N$$

Lemma 1 shows that if, after a veto of the cartel mechanism by bidder i, others believe that his value exceeds v_N (and the equilibrium b^* is played), then i would benefit from such a veto iff his value exceeds v_N. Thus, $A = [v_N, 1]$ is a credible veto set for player i. Furthermore, there does not exist any credible veto set for which all types in this set are indifferent between vetoing and ratifying. This is stated in the following lemma.

Lemma 2 *There does not exist a credible veto set A (for any i) and a corresponding equilibrium b^* in the post-veto auction such that $\pi^v(v; b^*) = \pi^s(v)$ for all $v \in A$.*

Thus, proposition 3 then follows from definition 3 and lemmas 1 and 2.

Proposition 3 *In first-price seal-bid auction format, while participation cost is not equal to zero,*

- *The cartel mechanism is not ratifiable.*
- *There is a discontinuity between $c = 0$ and $c > 0$.*

Finally, we should notice when $c = 0$, the model reduces to the benchmark case of MM model, so the strong cartel could be supported. While c is larger than 0, whether how small it would be, the cartel mechanism could not be ratifiable. This means there is a discontinuity between $c = 0$ and $c > 0$.

4.3 Standard Auction

In this subsection, we simply discuss our result in last subsection with standard auction designed mechanism. In first subsection in this chapter, we already show the standard optimal direct mechanism defined in MM paper. Due to the information leakage problem, this optimal cartel mechanism might not be supported, either. We define some easy notations to give the intuition, but we do not give a formal proof in our paper. $T(w) = F(w)^{-n} \int_{r+c}^{v} (u - r - c)(n - 1)F(u)^{n-1} f(u)du + r + c$ is the total payment for the winning bidder staying in the cartel. $\pi_v = [v - r - c]F(w)^{n-1}$ is the expected profit he vetoing for the cartel, and π_s is the expected profit for the bidder staying in the cartel, which is defined by

$$\begin{aligned}
\pi_s &= [v - T(w)]F(w)^{n-1} \\
&+ [1 - F(w)^{n-1}] \int_w^v [\frac{T(u) - r - c}{n-1}] \frac{(n-1)F(u)^{n-2} f(u)}{1 - F(w)^{n-1}} du \\
&= [v - T(w)]F(w)^{n-1} \\
&+ \int_w^v [T(u) - r - c]F(u)^{n-2} f(u)du
\end{aligned}$$

Now we can calculate $\pi_v - \pi_s$:

$$\begin{aligned}\pi_v - \pi_s &= [v - r - c - v + T(w)]F(w)^{n-1} - \int_w^v [T(u) - r - c]F(u)^{n-2} f(u)du \\ &\geq 0\end{aligned}$$

From the calculation above, we know the profit for a vetoer is at least equal to or higher than he staying in the cartel with standard auction format. While the participation cost is not zero, the strong cartel mechanism is not ratifiable whether the auction format is first-price, second-price or standard auction design.

Proposition 4 *In standard auction format, while participation cost is not equal to zero,*

- *The cartel mechanism is not ratifiable.*

5 Conclusion

The fundamental structure of the auction with participation costs is one through which an indivisible object is allocated to one of many potential buyers, and in order to participate in the auction, buyers must incur some costs. In some cases, like government procurement auction, the bidding costs often run into millions of dollars. This cost would affect bidders' bidding strategies. After the cost is incurred, a bidder can submit a bid. As McAfee and McMillan point out that an cartel agreement is worthless without some way of enforcing it. Since contracts to fix prices cannot usually be written and illegal, any collusive agreement must be designed to be self enforcing. If individual rational condition and incentive constrain could not be supported, the efficient colluding mechanism is generally not freely implemented. When bidders' actions are strategic interaction, and their outcome is affected by bidders' beliefs about others' values, the standard mechanism design approach may have information leakage problem from participation decisions. This observation is important if bidders are making interim participation decisions for the collusive mechanism. Besides, in real would, pre-bid costs are also required to get bidders involved in auctions.

In MM paper, they interpret their efficiency model in first-price sealed-bid auction can be supported. In our model Once a bidder participates in the auction, the bidding strategy depends on the valuation distributions. For the investigation above, we find the strong cartel mechanism is no longer sustainable in the presence of participation cost, in which case the bidder with higher value would bid his true value and veto for the collusive mechanism. This behavior could let him earn the maximum benefit, and all the others get zero profit in this auction, which means the individual rational condition cannot be hold. As such, the strong cartel mechanism might not be supported in MM model with participation costs in first-price sealed-bid auction.

How does the cartel mechanism investigated in the literatures be supported in other auction mechanism? This is still a open question here. Except this, when the incentive-compatible condition could not be hold, the model investigate here might become very complicated to find out an equilibrium that is dominant strategy for each bidder. Bidders might cheat and bid higher value to win the prior auction. Then, bidders would update their belief to prepare the legit auction. This is also an interesting topic for future research. How do people change their behavior if moral hazard exists in this economy? Besides, we could loose the assumption of the cartel members could update their information about the winner's valuation. Then, we can study how the victims' strategies destabilize the cartel.

Appendix: Proofs

Proof of Lemma 1:

We have

$$\pi^v(v) = \begin{cases} 0 & v \leq \frac{c}{G(v_Y)} \\ vG(v_Y) - c & \frac{c}{G(v_Y)} \leq v \leq v_Y \\ vG(v) - \int_{v_Y}^{v} \lambda(y)dG(y) - c & v > v_Y \end{cases}$$

and

$$\pi^s(v) = \begin{cases} z & v \leq c \\ z + \int_c^v G(y)dy & v > c \end{cases}$$

We want to show:

$$\pi^v(v, b^*) \geq \pi^s(v) \Rightarrow v \geq v_N$$

We will first find a v_N for which $\pi^v(v_N, b^*) = \pi^s(v_N)$, and then check the inqualities.

Step 1 :To show

$$\pi^v(v_N, b^*) = \pi^s(v_N)$$

$$\because c < v_N \leq v_Y$$

We can write:

$$\pi(v_N, b^*) = v_N G(v_Y) - c$$

and

$$\pi^s(v_N) = z + \int_c^{v_N} G(y)dy$$

i.e.

$$vG(v_Y) - c - z - \int_c^v G(y)dy = 0$$

let

$$\phi(v) = vG(V_Y(v)) - \int_c^v G(y)dy - c - z$$

and

$$\begin{aligned} \phi'(v) &= G(v_Y(v)) + vG'(v_Y(v))v_Y'(v) - G(v) \\ &= [G(V_Y(v)) - G(v)] + vG'(v_Y(v))v_Y' \end{aligned}$$

$$\because G(\cdot) \text{ is a cdf increasing function, we get } v_Y(v) \geq v$$

$$\therefore G(v_Y(v)) - G(v) > 0$$

$$\Rightarrow v_Y' > 0$$

$$\therefore \phi^{'}(v) > 0$$

$$\phi(c) = cG(v_Y(c)) - c - z = c[G(v_Y(c)) - 1] - z < 0$$

$$\begin{aligned}
\phi(1) &= G(v_Y(1)) - \int_c^1 G(y)dy - c - z \\
&= 1 - c - \int_c^1 G(y)dy - z
\end{aligned}$$

To prove $\int_c^1 [1 - G(y)]dy > z$, where $z(y) = \int_c^1 [y - c - \frac{1-F(y)}{f(y)}]G(y)dF(y)$. We know z is an increasing function of y. When $y = 1$, z reaches its maximum at

$$z = \int_c^1 [1 - c]dy = (1 - c)^2$$

$\because T(y) = \int_c^1 [1 - G(y)]dy$ is a decreasing function of y

$\therefore T(y)$ reaches its minimum at $y = 1$.

In equilibrium condition, $v^* G(v^*) = c$.

$$\begin{aligned}
T(1) &= \int_c^1 [1 - G(1)]dy \qquad where \quad 1G(1) = c \\
&= \int_c^1 (1 - c)dy \\
&= (1 - c)^2
\end{aligned}$$

$$\therefore T(y) \geq z(y)$$

$\because G(\cdot)$ is strictly increasing, it cannot reach equality. $\therefore \phi(1) > 0$

for $v < 1$, since $v_Y(v) \geq v$. Since $\phi(v)$ is continuous, $\phi(c) < 0$, and $\phi(1) > 0$, a unique solution to $\phi(v) = 0$ exists, and is our candidate for v_N.

Step 2: We want to show

$$\pi^v(v, b^*) > \pi^s(v),\ for\, v > v_N$$

Fix c, and hence v_N and v_Y are fixed, we have $c \leq \frac{c}{G(v_Y)} \leq v_N \leq v_Y$. The payoff difference $\pi^v(v; b^*) - \pi^s(v)$ is continuous and given by:

$$\pi^v(v, b^*) - \pi^s(v) = \begin{cases} -z & v < c & I \\ -z - \int_c^v G(y)dy & c \leq v < \frac{c}{G(v_Y)} & II \\ vG(v_Y) - c - z - \int_c^v G(y)dy & \frac{c}{G(v_Y)} \leq v \leq v_Y & III \\ vG(v) - \int_{v_Y}^v \lambda(y)dG(y) - c - z - \int_c^v G(y)dy & v > v_Y & IV \end{cases}$$

I,II:

$$\pi^v(v,b^*) - \pi^s(v) < 0,\ when\, v < \frac{c}{G(v_Y)}$$

III:

let $\varphi(v) = vG(v_Y) - c - \int_c^v G(y)dy - \pi$

$$\because \varphi(v_N) = \phi(v_N) = 0$$

$$\varphi'(v) = G(v_Y) - G(v) > 0,\ when\, v < v_Y$$

$$\varphi'(v_Y) = 0$$

$$\therefore \varphi(v) \begin{cases} < 0 & v < v_N \\ = 0 & v = v_N \\ > 0 & v > v_N \end{cases}$$

IV:

$$\begin{aligned}
\frac{d}{dv} &= [\pi^v(v,b^*) - \pi^s(v)] \\
&= \frac{d}{dv}[vG(v) - \int_{v_Y}^{v} \lambda(y)dG(y) - c - \int_c^v G(y)dy - \pi] \\
&= G(v) + vG'(v) - \lambda(v)G'(v) - G(v) \\
&= [v - \lambda(v)]G'(v) > 0, \\
\because &\ \ [v - \lambda(v)] > 0 \\
\therefore &\ \ increasing\ \ when\ \ v > v_Y
\end{aligned}$$

Q.E.D.

Proof of Lemma 2:

This proof is by contradiction. suppose there exists a credible veto set A and a corresponding equilibrium in the post-veto auction b^* such that

$$\pi^v(v,b^*) = \pi^s(v) \quad for\ \ all\ \ v \in A \qquad (1)$$

Note that it must be the case that $\pi^v(v;b^*) > 0$ for all $v \in A$, because $\pi^s(v) > 0$ for all $v \in [0,1]$. Let $a^* = inf A$. Since any vetoer type's payoff would be zero if she were not participating in the auction, the vetoer's cutoff in b^* must be $a^* \geq c$, i.e., all vetoer types participate the auction with probability one. Moreover, the continuity of payoff functions imply that $v = a^*$ satisfies (1) as well. Let $a_1 < a_2 < \cdots < a_J$ be the cutoffs of the ratifiers where $a_j \in [0,1]$ is used by n_j bidders and $\sum_{j=1}^J n_j = n - 1$.

Suppose that $a^* < 1$. It must be that $a_1 < a^*$, since otherwise the ratifiers using a_1 are sure to lose in the auction, and thus have a negative payoff. For vetoer types $v \in (a^*, a_1)$ we have

$$\pi^v(v; b^*) - \pi^s(v) = v \prod_{j=1}^{J} F(a_j)^{n_j} - c - \int_c^v G(y)dy - z,$$

which is strictly increasing in v, since $\prod_{j=1}^{J} F(a_j)^{n_j} \geq F(a_1)^{n-1} > G(v)$. So, types in (a^*, a_1) have strict incentive to veto, and therefore must also belong to the veto set A, which contradicts (1). Now suppose that $a^* = 1$, i.e., $A = 1$. Given that the vetoer is participating with probability one, none of the ratifiers will participate, and thus

$$\pi^v(1; b^*) = 1 - c > \pi^s,$$

a contradiction.

Q.E.D.

Proof of Standard Auction:

Calculate $\pi_v - \pi_s$:

$$\begin{aligned}
\pi_v - \pi_s &= [v - r - c - v + T(w)]F(w)^{n-1} \\
&- \int_w^v [T(u) - r - c]F(u)^{n-2} f(u)du \\
&= [T(w) - r - c]F(w)^{n-1} \\
&- \int_w^v [T(u) - r - c]F(u)^{n-2} f(u)du \\
&= F(w)^{-n} \int_{r+c}^v (u - r - c)(n-1)F(u)^{n-1} f(u)duF(w)^{n-1} \\
&- \int_w^v [F(u)^{-n} \int_{r+c}^v (u - r - c)(n-1)F(u)^{n-1} f(u)du]F(u)^{n-2} f(u)du \\
&= F(w)^{-1} \int_{r+c}^v (u - r - c)(n-1)F(u)^{n-1} f(u)du \\
&- \int_{r+c}^v (u - r - c)(n-1)F(u)^{n-1} f(u)du \int_w^v F(u)^{-2} f(u)du
\end{aligned}$$

$\because \int_w^v F(u)^{-2} f(u)du = \int_w^v F(u)^{-2} dF(u)$, Let $F(u) = x$, we can get:

$$\begin{aligned}
\int_w^v F(u)^{-2} dF(u) &= \int_{F(w)}^{F(v)} x^{-2} dx \\
&= -x^{-1}|_{F(w)}^{F(v)} \\
&= -\frac{1}{F(v)} + \frac{1}{F(w)}
\end{aligned}$$

Let $\int_{r+c}^{v}(u-r-c)(n-1)F(u)^{n-1}f(u)du = G(u)$, we can get:

$$\begin{aligned} F(w)^{-1}G(u) - G(u)(-\frac{1}{F(v)} + \frac{1}{F(w)}) &= \frac{1}{F(v)}G(u) \\ &\geq 0 \end{aligned}$$

Q.E.D.

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