Equilibria in Second Price Auctions with Information Acquisition

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Abstract

This paper studies equilibria in second price auctions with information acquisition in an independent private value setting. We focus on the existence and uniqueness of equilibrium in the information acquisition stage for both homogenous and heterogenous bidders. It is shown that, when the relative probability gain of information acquisition is increasing, there always exists an equilibrium and further it is symmetric and unique when bidders are homogenous. Moreover, we show that different type of bidders must choose different information levels, and further the advantaged groups with lower marginal information cost have stronger incentive to acquire information. An illustrative example with two bidders and Gaussian specification is presented to provide intuition and implications on equilibrium behavior of bidders.

Key words: second price auctions, information acquisition, heterogenous bidders, the existence and uniqueness of equilibrium.

JEL Classification: C70; D44; D82.

1 Introduction

1.1 Overview

The literature on equilibria involving mechanism design studies the behavior of agents in a game form designed by a principal to achieve a certain goal given that agents may hold private information and play strategically. A typical assumption made in this literature is that information

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held by agents is exogenous.[1] However, the private information of agents is acquired costly rather than endowed in many real world situations. For instance, in an auction where buyers cannot detect the quality of the objective they are going to bid, the buyers may incur some cost to assess the objective to avoid the risk of bidding a low-quality objective at a high-quality price. Even they are endowed with partial initial information about the objective, agents may have incentive to acquire more accurate information to improve their welfare. This strategy of privately accessing more information prior to participation is called information acquisition (**IA** for short). Basically, an agent may be interested in two types of information: the information about the objective such as the estimated valuation of the objective in an auction, and the private information of opponents such as their valuations about the objective. Consequently, the IA of a bidder can be divided into two types: one is to acquire information about himself (or the objective); the other one is to acquire information about other agents.

Most papers have contributed to the first type of IA issue, such as Milgrom and Weber (1981), Matthews (1984), Persico (2000), Bergemann and Pesendorfer (2001), Ganuza (2004), Rezende (2005), Compte and Jehiel (2006), Larson (2006), Eso and Szentes (2007), and Shi (2007). There are also some studies on the second type of IA. Fang and Morris (2006) studied the revenue equivalence and incentives for bidders to acquire costly information about opponents' valuations. Tian and Xiao (2007) extended the settings of Fang and Morris by endogenizing information acquisition, and considered the real multidimensional auction problem, where the type and strategy spaces of the bidder are both multidimensional.

However, most of these studies on the first type of IA are based on symmetric equilibrium in economic environments with homogenous bidder, in which both prior beliefs on valuations and information cost are the same among bidders. These assumptions are clearly unrealistic. For instance, in art auctions, each bidder has essentially a different belief on the art. Even in auctions such as contract bidding where potential contractors have the same initial information about the project, they probably have different opportunity costs for any further necessary information about the project. Thus, both prior beliefs on the true valuations and information costs can be different among bidders. When this is the case, under what conditions the existence and uniqueness of equilibrium can be guaranteed? This is the issue we focus on in this paper. Also, the existing literature involves binary decision of acquiring information—to decide whether to acquire more information or not.[2] However, in many applications, bidders not only have to decide whether they need more information, but also to choose what level of information they

[1] For instance, in Hurwicz (1960) and Hurwicz and Reiter (1990), the initial dispersion of information is represented as a factorization of an exogenous parameter space.

[2] See Bergemann, Shi and Valimaki (2007).

should acquire, considering other bidders' strategic reactions. Therefore, the existence and uniqueness of equilibrium in auctions allowing bidders to choose information in a *continuous* level should be reconsidered.

1.2 Objectives of the Paper

This paper contributes to the auction literature with information acquisition and extend the existing literature to a more general setting. We focus on individuals' behavior in second price auctions with IA in an independent private value environment. While most studies on the information acquisition assume homogeneity of bidders, we allow for heterogenous bidder whose valuations are drawn from general distributions and heterogeneous information acquisition cost (**IA cost** for short).

Specifically, we consider an second price auction where an indivisible object is to be sold to one of several potential bidders. Each bidder's valuation for the object is unknown ex-ante to other bidders as well as himself. But before participation, bidders can privately acquire information about their valuations by costly doing experiments in which private signals will return. The bidders update their estimates of the valuations according to the signals. They improve the informativeness of the received signals by choosing high levels of information, but with an increasing cost. The beliefs on the true valuations can be updated but the true valuations can never be completely revealed.

The timing of the auction is scheduled as a two-stage game: (1) *the information acquisition stage*(the IA stage) – bidders decide how much information to acquire after observing the object and update their valuations about the object based on the acquired information, and (2)*the bidding stage* – each buyer submits a report about his private information to the seller and the bidder with the highest bid is rewarded with the object and pay at the second highest bid. The auction in the second stage is the standard second price auction format which is invoked by Vickrey (1961). In fact, after the IA stage, the bidding stage is much alike conventional exogenous information auctions. While the equilibrium issue of these conventional auctions has been well studied (cf. Maskin and Riley (2000b), Blume and Heidhues (2004)), studies on individuals' behavior in the IA stage have been largely neglected.

We study the existence and uniqueness of equilibrium in the information acquisition stage in a general setting of second price auctions and economic environments. We maintain the basic information structure in Shi (2007) but offer a complete perspective of equilibrium in the IA stage rather than the optimal reserve price. We provide sufficient conditions which are general enough to cover many applications that literature has considered.

To provide intuition and understand our results easily, we start with our analysis by considering an economy with two bidders and Gaussian specification. The true valuations of both bidders are normally distributed and are ex-ante unobservable to both bidders. However, each bidder can acquire a costly experiment returning a noisy signal, which is the sum of the true valuation and a normally distributed error. To increase the informativeness of their signals, the bidders can reduce the variance of the error but with an increasing cost. The equilibrium is guaranteed if the mean valuations are the same. For two homogeneous bidders, the equilibrium must be symmetric and unique. For two heterogeneous bidders, the equilibrium must be intuitive: the advantaged bidder with lower marginal cost (or the bidder with less prior precision) has incentive to require more information, given other factors are the same.

We then extend our analysis to more general settings. We allow general prior distributions of bidders' valuations and different IA cost functions. Following the intuition of the two-bidder case, we provide the notion of the expected marginal value of information, taking into account of other bidders' strategies. We show that an equilibrium is guaranteed for convex IA cost functions and a broad range of posterior distributions. Besides, there is a unique equilibrium that is symmetric if bidders are homogeneous. And the symmetric equilibrium is unique if the posterior distribution is rotation ordered. Furthermore, if bidders can be divided into groups so that bidders are homogeneous in the same group and heterogenous across groups, then the equilibrium must be type-symmetric which is *intuitive*: bidders in the same group choose the same information level; bidders in different groups choose different information levels; and the incentive to acquire information will be weakened for a higher marginal IA cost in equilibrium.

1.3 Related literature

There is voluminous theoretical literature about equilibrium of auctions with private value. However, most of the discussions rely on a critical assumption that the private values of bidders are informed to them at the beginning of the auction. The auctions with heterogenous bidders are also involved in classical papers, such as Graham and Marshall (1987) and Maskin and Riley (2000b). However, models with costly information acquisition as an *endogenous* element of the bidding process receive less attention.

One of the earliest papers explicitly combining information acquisition with bidding strategy is Milgrom (1981). In this seminal paper, a bidding model is developed which has the market-like features that bidders act as price takers and that prices convey information. Our model shares his model with a similar information structure that bidders may acquire information at a cost before bidding, but his model devotes to extending the theory of rational expectations market equilibrium and ignores the heterogeneity in bidders that we incorporate.

Another model is provided by Schweizer and Ungern-Sternberg (1983), in which they presented a common-value auction where each bidder draws an estimate of the valuation from an interval centered on the true value. Bidders can costly narrow the length of the interval to approach the true value. However, the analysis mainly resorts to simulations for the case of two bidders. Lee (1984), in turn, developed a two-agent model with incomplete information in the sense that bidders are not sure whether their opponents are informed. In this model, the potential valuation of the objective being auctioned can take two values, which can be fully revealed. Lee characterized a symmetric equilibrium and showed that each bidder acquires information with a positive probability in the information acquisition stage and uses a randomized strategy whose distribution depends on the value he discovers in the bidding stage. In Lee (1985), the model is extended to the case with larger number of bidders.

Matthews (1984) studied information acquisition and discussed a symmetric equilibrium in a first price auction, in which each bidder shares a common, but unknown value. Hausch and Li (1993) developed a common-value model much like Lee's. The objective has two potential valuations which can be reflected by a signal with a cost. The accuracy of the signal is positively related to the amount of cost spent on it. The authors characterized a symmetric equilibrium and the analysis is extended to the private-value case in Hausch and Li (1993a). Nicola Persico (2000) studied the incentive to acquire information and investigated the value of information by introducing the notion of risk-sensitivity. He established that the value of information is higher in decision problems in which bidders are more risk-sensitive. In his model, he obtained an explicit expression for the marginal value of information. In contrast, our paper specified the properties of the marginal value of information without giving any explicit expression and in turn, the general analysis of equilibrium is established by these properties.

The main differences between our model and the existing literature are: First, costly information acquisition is incorporated as an endogenous element of bidding process. Second, heterogeneities—a continuum of feasible values, asymmetric IA cost and beliefs on true valuations—is allowed. Third, the information structure is general—the beliefs on true valuations can be updated by acquiring information, but the true valuations can never be fully revealed. Finally, instead of focusing on prevailing issues such as optimal reserve price and revenue equivalence, this paper devotes to a modest but important issue—the existence and uniqueness of equilibrium. In sum, this paper incorporates heterogeneity and endogenous element into conventional models to analyze the equilibrium issue in the second price auction.

The remainder of the paper is organized as follows. Section 2 introduces a general setup in which the basic information structure is similar to Shi (2007). Section 3 studies equilibria in a simply case with two bidders and Gaussian specification. Section 4 discusses the existence and uniqueness of equilibrium in economies with many bidders and general distributions. We conclude in section 5. All proofs are relegated to the appendix, unless otherwise noted.

2 The Setup

In a second price auction, a single object is sold to n bidders, who are indexed by $i \in \{1, 2, \ldots, n\}$. The buyers' true valuations about the objective $\{\omega_i, i = 1, 2, \ldots, n\}$, unknown ex-ante, are independently drawn from a family of distributions $\{F_i, i = 1, 2, \ldots, n\}$ with supports $[\underline{\omega}_i, \overline{\omega}_i]$ correspondingly. Each F_i has a strict positive and differentiable density f_i, with the mean μ_i.

A bidder with valuation ω_i gets utility u_i if he wins the object and pays t_i:

$$u_i = \omega_i - t_i.$$

The auction is a two-stage game: the first stage is the information acquisition stage (the IA stage) and the second stage is the bidding stage. In the IA stage, bidder i can acquire a costly signal s_i about ω_i, with $s_i \in [\underline{s}_i, \overline{s}_i] \subset \Re$. Let α_i denote the level of information acquired by bidder i, and s_i is the signal received by bidder i conditional on the level of information he chooses. Signals received by different bidders are private and independent among bidders. In fact, bidder i acquires information by choosing a differentiable joint distribution of (s_i, ω_i) from a family of joint distributions $G_i(s_i, \omega_i|\alpha_i) : [\underline{s}_i, \overline{s}_i] \times [\underline{\omega}_i, \overline{\omega}_i] \to [0, 1]$, given the information choice α_i. The information choice corresponds to a statistical experiment, which returns a private signal s_i. The signal with higher α_i is more informative because it implies a more precise posterior belief on the true valuation. The joint distribution $G_i(s_i, \omega_i|\alpha_i)$ is referred to as the information structure.

Notice that the beliefs on the true valuations can be updated but cannot be identified completely. The cost of performing an experiment α_i is $C_i(\alpha_i)$, which is increasing and convex in α_i. There may be asymmetries in the information structures and cost functions. Bidders may resort to different experiments to acquire information. Even the information choices are the same, bidders may also have different (opportunity) costs to implement it. The feasible information level of bidder i is $[\underline{\alpha}, \overline{\alpha}]$, where $\underline{\alpha}$ is the endowed information level that the bidder can receive at no cost, and $\overline{\alpha}$ is the maximum information level that the bidder is able to afford. As usual, we use $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ to denote the information choices of all bidders, and $\alpha_{-i} = (\alpha_1, \ldots .\alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n)$ to denote the choices of all bidders except bidder i.

Let $G_i(\omega_i|s_i, \alpha_i)$ denote the distribution of valuation ω_i conditional on s_i and α_i, and

$G_i(s_i|\omega_i, \alpha_i)$ denote the distribution of signal s_i conditional on ω_i and α_i. With a little abuse of notations, $G_i(\omega_i|\alpha_i)$ and $G_i(s_i|\alpha_i)$ are used to denote the marginal distribution of ω_i and s_i conditional on information choice α_i, respectively. They are defined in the usual way, that is, $G_i(\omega_i|\alpha_i) = \mathbb{E}_{s_i}[G_i(s_i, \omega_i|\alpha_i)]$ and $G_i(s_i|\alpha_i) = \mathbb{E}_{\omega_i}[G_i(s_i, \omega_i|\alpha_i)]$. Again, to simplify notations, we use $g_{\alpha_i}(\omega_i|s_i)$, $g_{\alpha_i}(s_i|\omega_i)$, $g_{\alpha_i}(\omega_i)$ and $g_{\alpha_i}(s_i)$ to denote the corresponding densities.

After observing the signal s_i from the experiment corresponding to α_i, bidder i updates his prior belief on ω_i according to Bayes rule:

$$g_{\alpha_i}(\omega_i|s_i) = \frac{g_{\alpha_i}(s_i|\omega_i)f_i(\omega_i)}{\int_{\underline{\omega}_i}^{\overline{\omega}_i} g_{\alpha_i}(s_i|\omega_i)f_i(\omega_i)d\omega_i}.$$

Let $v_i(s_i, \alpha_i)$ denote bidder i's updated estimate of ω_i after performing experiment α_i and observing s_i:

$$v_i(s_i, \alpha_i) \equiv \mathbb{E}[\omega_i|s_i, \alpha_i] = \int_{\underline{\omega}_i}^{\overline{\omega}_i} \omega_i g_{\alpha_i}(\omega_i|s_i)d\omega_i.$$

We use v_i to denote the updated estimate $v_i(s_i, \alpha_i)$, and use $\boldsymbol{v}$ to denote the n-vector $(v_1, v_2, \ldots, v_n)$. Sometimes, $\boldsymbol{v}$ is also written as (v_i, v_{-i}), where $v_{-i} = (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$. Assume $v_i(s_i, \alpha_i)$ is increasing in s_i. That is, a higher signal induces a higher posterior estimate given the information choice. Let $H_{\alpha_i}(v_i)$ denote the distribution of v_i with corresponding density $h_{\alpha_i}(v_i)$, given the information choice. Then, $H_{\alpha_i}(v_i)$ is the posterior distribution of the true value[3]:

$$H_{\alpha_i}(v_i) \equiv Pr\{\mathbb{E}[\omega_i|s_i, \alpha_i] \leq x\} = \int_{\underline{s}_i}^{v_i^{-1}(x,\alpha_i)} g_{\alpha_i}(s_i)ds_i.$$

Since $v_i(s_i, \alpha_i)$ is increasing in s_i, the upper limit of the integral is well defined. That is, $H_{\alpha_i}(x)$ is the probability that the bidder i's posterior estimate v_i is below x, given his information choice α_i. Note that the support $[\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i}]$ corresponding to the updated distribution $H_{\alpha_i}(v_i)$ may change with the information choice. However, we suppose throughout the paper that $\frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i} = \frac{\partial \overline{\omega}_{\alpha_i}}{\partial \alpha_i} = 0$ for all i [4].

After the IA stage, the bidding stage begins, which is a second price auction. In fact, the behavior of bidders in this stage is the same as those in conventional second price auctions, considering the information acquired in the IA stage as given. Note that even if the prior beliefs are ex ante symmetric, after the IA stage, the posterior beliefs may be asymmetric. However, such asymmetries among bidders do not affect bidding behavior in the second price auction—it is still a weakly dominant strategy for each bidder to bid his (updated) valuation. For this reason,

[3] $H_{\alpha_i}(v_i)$ is in fact a distribution function of s_i, because given α_i, v_i is a function of s_i. Thus, $H_{\alpha_i}(v_i)$ can also be derived by the distribution of s_i and Bayes rule.

[4] For prior distributions with support $(-\infty, +\infty)$, such as Gaussian distribution satisfy this assumption.

we only need to focus on bidders' interactions in the IA stage. The equilibrium we analyze in the paper is specified as below:

Definition 1 (Equilibrium in Auctions with Information Acquisition) The information choice profile $\boldsymbol{\alpha}^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*)$ in the information acquisition stage consists an equilibrium if and only if, for all $i = 1, 2, \ldots, n$ and α_i,

$$\mathbb{E}_{\boldsymbol{v},\alpha_{-i}^*}\{u_i[v_i(s_i, \alpha_i^*)]\} - C_i(\alpha_i^*) \geq \mathbb{E}_{\boldsymbol{v},\alpha_{-i}^*}\{u_i[v_i(s_i, \alpha_i)]\} - C_i(\alpha_i).$$

That is, $\alpha_i^* \in \text{argmax}\{\mathbb{E}_{\boldsymbol{v},\alpha_{-i}^*}\{u_i[v_i(s_i, \alpha_i)]\} - C_i(\alpha_i)\}$.[5]

The definition says that in equilibrium, each bidder maximizes his net expected revenue given other bidders' information choices. The equilibrium involves strategic interactions in both interrelated stages. The choice in the IA stage will update the belief on the true valuations and in turn improves the gross expected revenue in the second stage[6]; the expected gain from the second stage also influences the information choice in the IA stage directly. Bidders then face a trade-off: more gains in the bidding stage resulted from high information choice but high information choice also implies high cost in the IA stage.

Remark 1 The second price auction format simplifies our analysis to a great extent. In the second price auction, the asymmetry in beliefs does not affect the symmetric equilibrium bidding strategies. Consequently, the symmetric equilibrium allows us to concentrate the equilibria in the IA stage, although a complete equilibrium in such an auction is consisted of the equilibrium in the IA stage as well as the bidding stage. However, it is difficult to just focus on the IA stage in other auction formats, since any equilibrium strategy profile in the bidding stage may be asymmetric and probably depends on the information level acquired in the IA stage. For example, in the first price auction, even though bidders are homogenous ex ante, the updated beliefs may be asymmetric after IA stage. And such an asymmetry may lead to many complications: equilibrium strategies depends on information levels, and different equilibria may imply different expected payments for each bidder.

Thus, given bidder i's true valuation, v_i, his expected payoff is his payoff minus his payment if he wins:

$$\mathbb{E}_{v_{-i},\alpha_{-i}^*}\{u_i(v_i)\} = v_i Q_i(v_i) - \int_{\underline{\omega}_{\alpha_i}}^{v_i} x Q_i(x) dx = \int_{\underline{\omega}_{\alpha_i}}^{v_i} Q_i(x) dx$$

[5] α_{-i}^* means the expectation conditions on the opponents' strategies.

[6] Shi (2007) indicated that a bidder achieve higher expected payoff for high information level under conditions compatible with our assumptions in this paper

where $Q_i(x) = \Pi_{l \neq i} H_{\alpha_l}(x)$ is the expected probability of bidder i to win the object when his valuation is x.

Then, since the v_i is unknown ex ante, then the expected payoff is:

$$\begin{aligned}
\mathbb{E}_{\boldsymbol{v},\alpha^*_{-i}}\{u_i(v_i)\} &= \mathbb{E}_{v_i}\{\int_{\underline{\omega}_{\alpha_i}}^{v_i} Q_i(x)dx\} \\
&= \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \int_{\underline{\omega}_{\alpha_i}}^{v_i} Q_i(x)dxh_{\alpha_i}(v_i)dv_i \\
&= \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} (1 - H_{\alpha_i}(v_i))Q_i(v_i)dv_i.
\end{aligned}$$

Therefore, the information choice profile $\boldsymbol{\alpha}^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*)$ in the IA stage consists an equilibrium if and only if

$$\alpha_i^* \in \text{argmax} \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} (1 - H_{\alpha_i}(v_i))Q_i(v_i)dv_i - C_i(\alpha_i)\}.$$

Remark 2 The common knowledge should be emphasized: each bidder's true valuation is drawn from a known distribution; acquired information (signals) is private but the way (Bayesian rule) of using information to update belief is common knowledge; the IA cost for each bidder is also made public. Thus, while the second stage is an incomplete information game, the first stage is considered as a complete information game.

3 Two Bidders and Gaussian Specification

We start with a simple economy with two bidders in the second price auction. We focus on a special information structure in this section: Gaussian distribution and linear cost function. We first investigate the buyers' IA decision problem and establish the existence of equilibrium in this setting. We then show that there is no asymmetric equilibrium and the symmetric equilibrium is unique when bidders are homogeneous. When bidders are heterogeneous, the equilibrium must be *intuitive equilibrium*: the disadvantaged bidder with high marginal information cost resorts to less information if the only difference between bidders is the information cost, and the bidder with low precision chooses more information if the only difference between bidders is the prior beliefs.

3.1 The Existence of Equilibrium

It is assumed that bidder i's true valuation ω_i is drawn from a normal distribution with mean μ_i and precision β_i:

$$\omega_i \sim N(\mu_i, \frac{1}{\beta_i}).$$

That is, the true valuation ω_i is normally distributed with an expectation μ_i and a standard variance $\frac{1}{\beta_i}$. A lower β indicates that the prior distribution is more spread out, yielding more potential gains from information acquisition.

By doing a costly experiment corresponding to α_i, the bidder i can observe a signal s_i:

$$s_i = \omega_i + \varepsilon_i$$

where the additive error ε_i is independent of ω_i and $\varepsilon_i \sim N(0, \frac{1}{\alpha_i})$. The higher α_i, the more precise the signal is. Thus, α_i is interpreted as the informativeness or precision of bidder i's signal. Each bidder is endowed with an initial signal precision, $\underline{\alpha}$, which is positive. To increase the precision, bidder i can require a higher information level, α_i. Thus, $(\alpha_i - \underline{\alpha})$ is the part that bidder i invests in IA. The IA cost is linear in the incremental precision. That is,

$$C_i(\alpha_i) = c_i(\alpha_i - \underline{\alpha}),$$

where c_i is the constant marginal cost of one additional unit of precision. Assume the information choice set of the bidders is $[\underline{\alpha}, \overline{\alpha}]$.

Note that no symmetry is imposed so far. The bidders may have different prior beliefs, and choose different experiments to acquire information. The IA cost may also be different.

The bidders update their beliefs on their valuations after observing signals. Here we acquire the posterior valuation distribution conditional on the signal s_i by the standard normal updating technique:

$$\omega_i | s_i, \alpha_i \sim N(\frac{\alpha_i s_i + \beta_i \mu_i}{\alpha_i + \beta_i}, \frac{1}{\alpha_i + \beta_i}).$$

It immediately follows that $v_i(s_i, \alpha_i) \equiv \mathbb{E}(\omega_i | s_i, \alpha_i) = \frac{\alpha_i s_i + \beta_i \mu_i}{\alpha_i + \beta_i}$.

Thus, the distribution of the posterior estimate v_i, $H_{\alpha_i}(v_i)$ is normal: $v_i \sim N(\mu_i, \sigma_i^2(\alpha_i))$, where the standard variance $\sigma_i(\alpha_i) = \sqrt{\frac{\alpha_i}{\beta_i(\alpha_i + \beta_i)}}$.

Then,

$$H_{\alpha_i}(v_i) = \int_{-\infty}^{v_i} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\{-\frac{(x - \mu_i)^2}{2\sigma_i^2}\} dx \tag{1}$$

Remark 3 The variance of the posterior estimate v_i is increasing in the information level α_i. This indicates that the distribution is more spread out for a more precise signal. Although the variance of the posterior valuation should be smaller after the IA stage (because of the received private signals), the ex ante variance (prior to the private signals) need not to be smaller, comparing to the prior distribution. The relationship between the variance and the informativeness when different information levels are chosen is demonstrated by two graphs in Shi (2007). It is shown that the higher the information level, the more the expected revenue is.

We first investigate bidders' IA decision problem. Each bidder chooses α_i to maximize his expected payoff

$$\pi_i(\alpha_i, \alpha_j) \equiv \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} (1 - H_{\alpha_i}(v_i))H_{\alpha_j}(v_i)dv_i - C_i(\alpha_i). \tag{2}$$

In the specification of this section, the IA decision problem of bidder i is

$$\max_{\alpha_i} \int_{-\infty}^{+\infty} (1 - H_{\alpha_i}(v_i))H_{\alpha_j}(v_i)dv_i - c_i(\alpha_i - \underline{\alpha}_i). \tag{3}$$

With some algebra, we have

$$\frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = \frac{1}{2\sqrt{2\pi}(\alpha_i + \beta_i)^2} \frac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}} \exp\{-\frac{(\mu_1 - \mu_2)^2}{2(\sigma_1^2 + \sigma_2^2)}\} - c_i, \tag{4}$$

Lemma 1 *Suppose* $(\mu_1 - \mu_2)^2 \leq \frac{1}{\beta_1} + \frac{1}{\beta_2}$. *Then* $\pi_i(\alpha_1, \alpha_2)$ *is strictly concave in* α_i, $i = 1, 2$.

This lemma indicates that when the difference of mean valuations is not too larger, bidder i's expected payoff is concave in his own information level. A large difference in mean valuations may induce an"irregular" payoff function. The intuition is straightforward: for example, given the valuation of bidder 2, v_2, largely exceeds μ_1 then the bidder 1 has a great incentive to acquire more information, since more information implies more expected benefit ex ante. In this case, the marginal information level is increasing in his information choice. If $|\mu_1 - \mu_2|$ is large, which indicates that the valuation of bidder 2 has a great probability of largely exceeding bidder 1's valuation, in the specification of Gaussian. This implies that the marginal information value may be increasing on some interval.

Proposition 1 *When* $(\mu_1 - \mu_2)^2 \leq \frac{1}{\beta_1} + \frac{1}{\beta_2}$, *there is an equilibrium.*

Proof. It is well-known that a Nash equilibrium exists if for (i) each player's strategy space is a nonempty, convex, and compact subset of an Euclidean space; (ii) $\pi_i(\alpha_1, \alpha_2)$ is continuous in (α_1, α_2), and concave in α_i (cf. Mas-colell, Whinston and Green, 1995). Requirement (i) is satisfied since the strategy space of bidder i is $[\underline{\alpha}, \overline{\alpha}]$, and (ii) is also satisfied by Lemma 1 and the fact that $\pi_i(\alpha_1, \alpha_2)$ is continuous in (α_1, α_2). Thus, there is an equilibrium. ■

3.2 Homogeneous Bidders

We now focus on equilibria in a homogeneous environment: $\mu_i = \mu$, $\beta_i = \beta$ and $\underline{\alpha}_i = \underline{\alpha}$ for $i = 1, 2$. An equilibrium (α_1^*, α_2^*) is called symmetric equilibrium if $\alpha_1^* = \alpha_2^*$, i.e., the bidders choose the same information level; otherwise, the equilibrium is said to be asymmetric.

The following proposition shows that no asymmetric equilibrium exists in a homogeneous setting.

Proposition 2 *Suppose bidders are homogeneous. If an equilibrium exists, it must be symmetric and unique.*

Proof. The Kuhn-Tucker first order condition for bidder i's problem is

$$\begin{cases} \dfrac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = \dfrac{1}{2\sqrt{2\pi}(\alpha_i + \beta)^2} \dfrac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}} - c = -\lambda_i + \gamma_i \\ \lambda_i(\alpha_i - \underline{\alpha}) = 0 \\ \gamma_i(\alpha_i - \overline{\alpha}) = 0 \\ \lambda_i, \gamma_i \geq 0, \end{cases} \tag{5}$$

where λ_i and γ_i are the Lagrange multipliers for the restrictions $\alpha_i \geq \underline{\alpha}$ and $\alpha_i \leq \overline{\alpha}$, respectively

Suppose, by way of contradiction, that there is an asymmetric equilibrium (α_1^*, α_2^*). Without loss of generality, assume $\alpha_1^* < \alpha_2^*$. This implies that $\alpha_1^* < \overline{\alpha}$ and $\alpha_2^* > \underline{\alpha}$. Then according to (5), $\gamma_1 = 0$ and $\lambda_2 = 0$. Thus, $\frac{\partial \pi_1(\alpha_1^*, \alpha_2^*)}{\partial \alpha_1} = -\lambda_1 \leq 0$ and $\frac{\partial \pi_2(\alpha_1^*, \alpha_2^*)}{\partial \alpha_2} = \gamma_2 \geq 0$.

Then we have

$$c = C_1'(\alpha_1^*) \geq \frac{1}{2\sqrt{2\pi}(\alpha_1^* + \beta)^2} \frac{1}{\sqrt{\sigma_1^{*2} + \sigma_2^{*2}}} > \frac{1}{2\sqrt{2\pi}(\alpha_2^* + \beta)^2} \frac{1}{\sqrt{\sigma_1^{*2} + \sigma_2^{*2}}} \geq C_2'(\alpha_2^*) = c,$$

a contradiction. Therefore, the symmetry of equilibrium is established.

Now we show that the equilibrium is unique. To do so, we first show the interior equilibrium is unique. According to equation (5), $\frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = 0$ should be satisfied for the interior equilibrium. Since the equilibrium is symmetric, this equation reduces to $\frac{\partial \pi_i(\alpha, \alpha)}{\partial \alpha_i} = 0$. That is, $\frac{1}{2\sqrt{2\pi}(\alpha+\beta)^2} \frac{1}{\sqrt{\sigma^2+\sigma^2}} - c = 0$. The uniqueness of interior equilibrium is a result of strict monotonicity of function $\frac{1}{2\sqrt{2\pi}(\alpha+\beta)^2} \frac{1}{\sqrt{\sigma^2+\sigma^2}}$ in α.

We then show the uniqueness of corner equilibrium. Note that, since the equilibrium must be symmetric, there are only two possible corner equilibria: $\alpha_1^* = \alpha_2^* = \underline{\alpha}$ and $\alpha_1^* = \alpha_2^* = \overline{\alpha}$. The two equilibria can not both exist. Suppose not. Then equation (5) implies $\frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = -\lambda_i \leq 0$ at $\alpha_1^* = \alpha_2^* = \underline{\alpha}$; and $\frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = \gamma_i \geq 0$ at equilibrium $\alpha_1^* = \alpha_2^* = \overline{\alpha}$.

Then,

$$c \geq \frac{1}{2\sqrt{2\pi}(\underline{\alpha}+\beta)^2}\frac{1}{\sqrt{\underline{\sigma}^2+\underline{\sigma}^2}} > \frac{1}{2\sqrt{2\pi}(\overline{\alpha}+\beta)^2}\frac{1}{\sqrt{\overline{\sigma}^2+\overline{\sigma}^2}} \geq c$$

by noting that $\underline{\alpha} < \overline{\alpha}$ and $\sigma_i(\alpha_i) = \sqrt{\frac{\alpha_i}{\beta_i(\alpha_i+\beta_i)}}$ is increasing in α_i, a contradiction. Thus, the corner equilibrium must be unique.

Finally we show the equilibrium is unique. That is, it is either an interior or corner equilibrium, but not both. Suppose not. Without loss of generality, assume there are an interior equilibrium $(\alpha_1^*, \alpha_2^*) = (\alpha, \alpha)$ and a corner equilibrium $(\hat{\alpha}_1, \hat{\alpha}_2) = (\underline{\alpha}, \underline{\alpha})$. From equation (5), we have $\frac{\partial \pi_i(\alpha,\alpha)}{\partial \alpha_i} = 0$ at interior equilibrium (α, α) and $\frac{\partial \pi_i(\underline{\alpha},\underline{\alpha})}{\partial \alpha_i} = -\lambda_i \leq 0$ at the corner equilibrium $(\underline{\alpha}, \underline{\alpha})$. Then we have

$$c = \frac{1}{2\sqrt{2\pi}(\alpha+\beta)^2}\frac{1}{\sqrt{\sigma^2+\sigma^2}} < \frac{1}{2\sqrt{2\pi}(\underline{\alpha}+\beta)^2}\frac{1}{\sqrt{\underline{\sigma}^2+\underline{\sigma}^2}} = c$$

by noting that $\underline{\alpha} < \alpha$ and $\sigma_i(\alpha_i) = \sqrt{\frac{\alpha_i}{\beta_i(\alpha_i+\beta_i)}}$ is increasing in α_i, a contradiction. Therefore, the equilibrium must be unique. ■

This proposition helps to simplify any attempt to analyze equilibria: in a homogeneous setting, only symmetric equilibrium deserve to be considered, because the ex ante symmetry of bidders automatically implies the ex post symmetry of information choices. What's more, it is shown that there is only one equilibrium: it is either an interior or corner equilibrium.

3.3 Heterogeneous Bidders

Now we consider equilibria for economic environments where either the marginal IA cost or the prior beliefs are different but the mean valuations are the same, i.e., either $c_1 \neq c_2$ or $\beta_1 \neq \beta_2$, and $\mu_1 = \mu_2$.

It is intuitive that if the only difference between the bidders is the variances of the prior distribution, then the one with less prior precision has incentive to acquire higher level of information, which is indicated by a higher α. Indeed, a smaller β indicates a larger variance, yielding more potential gains even if the bidder resorts to the same level of information. Thus, in equilibrium, he will acquire relatively more information than the one with high prior precision. That is, $\beta_1 > \beta_2$ implies $\alpha_1^* < \alpha_2^*$ in an equilibrium, and vice versa. Similarly, given other factors the same between bidders, the one with lower marginal cost has more incentive to acquire information, i.e., $c_1 > c_2$ implies $\alpha_1^* < \alpha_2^*$, and vice versa. An equilibrium with either such property is called an *intuitive equilibrium*. The following proposition indicates that the only equilibrium in the heterogeneous environment is the intuitive equilibrium.

Proposition 3 *For economic environments considered in this subsection, if* (α_1^*, α_2^*) *is an equilibrium, we have:*

(i) If $\beta_1 = \beta_2$*, the equilibrium must be intuitive, i.e.,* $\alpha_1^* \geq \alpha_2^*$ *whenever* $c_1 < c_2$*, vise verse;*

(ii) If $c_1 = c_2$*, the equilibrium must be intuitive, i.e.,* $\alpha_1^* \geq \alpha_2^*$ *whenever* $\beta_1 < \beta_2$*, vise verse.*

Proof. To prove (i), suppose $c_1 < c_2$ but $\alpha_1^* < \alpha_2^*$. We then have $\alpha_1^* < \overline{\alpha}$ and $\alpha_2^* > \underline{\alpha}$. With the same logic of the proof of Proposition 2, equation (5) implies $\frac{\partial \pi_1(\alpha_1^*, \alpha_2^*)}{\partial \alpha_1} = -\lambda_1 \leq 0$ and $\frac{\partial \pi_2(\alpha_1^*, \alpha_2^*)}{\partial \alpha_2} = \gamma_2 \geq 0$. Consequently,

$$c_2 \leq \frac{1}{2\sqrt{2\pi}(\alpha_2^* + \beta)^2} \frac{1}{\sqrt{\sigma_1^{*2} + \sigma_2^{*2}}} < \frac{1}{2\sqrt{2\pi}(\alpha_1^* + \beta)^2} \frac{1}{\sqrt{\sigma_1^{*2} + \sigma_2^{*2}}} \leq c_1,$$

a contradiction. Thus we have $\alpha_1^* \geq \alpha_2^*$ whenever $c_1 < c_2$. Similarly, if $c_1 > c_2$, we must have $\alpha_1^* \leq \alpha_2^*$.

To prove (ii), suppose $\beta_1 < \beta_2$ but $\alpha_1^* < \alpha_2^*$. With the same logic, equation (5) implies a contradiction to $c_1 = c_2$:

$$c_2 \leq \frac{1}{2\sqrt{2\pi}(\alpha_2^* + \beta_2)^2} \frac{1}{\sqrt{\sigma_1^{*2} + \sigma_2^{*2}}} < \frac{1}{2\sqrt{2\pi}(\alpha_1^* + \beta_1)^2} \frac{1}{\sqrt{\sigma_1^{*2} + \sigma_2^{*2}}} \leq c_1.$$

Similarly, we have $\alpha_1^* \leq \alpha_2^*$ if $\beta_1 > \beta_2$. ■

If there is no restriction on the means of the true valuations in this heterogenous setting, that is, μ_1 and μ_2 can either be the same or different, then there may be multiple equilibria. However, if we impose the condition $\mu_1 = \mu_2$, then not only the existence is guaranteed by Proposition 1, but also the uniqueness of interior equilibrium is established.

Proposition 4 *If there exists an interior equilibrium, it is unique.*

Proof. From equation (5), an interior equilibrium (α_1^*, α_2^*) must satisfy $\frac{\partial \pi_i(\alpha_1^*, \alpha_2^*)}{\partial \alpha_i} = 0$ for each bidder i. That is,

$$\frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = \frac{1}{2\sqrt{2\pi}(\alpha_i + \beta_i)^2} \frac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}} - c_i = 0, \tag{6}$$

for $i = 1, 2$.

Comparing the above equation for $i = 1, 2$, we obtain

$$\sqrt{c_1}(\alpha_1 + \beta_1) = \sqrt{c_2}(\alpha_2 + \beta_2), \tag{7}$$

which defines $\alpha_2(\alpha_1)$ as an increasing function of α_1 along the path defined by (7).

Without loss of generality, suppose $(\hat{\alpha}_1, \hat{\alpha_2})$ is another interior equilibrium with $\hat{\alpha}_1 < \alpha_1^*$. Then equation (7) implies $\hat{\alpha}_2 < \alpha_2^*$. Then it is easy to see that the first order condition (6) is satisfied for $(\hat{\alpha}_1, \hat{\alpha_2})$:

$$\frac{1}{2\sqrt{2\pi}(\hat{\alpha}_i+\beta_i)^2}\frac{1}{\sqrt{\hat{\sigma}_1^2+\hat{\sigma}_2^2}} > \frac{1}{2\sqrt{2\pi}(\alpha_i^*+\beta_i)^2}\frac{1}{\sqrt{\sigma_1^{*2}+\sigma_2^{*2}}} = c_i.$$

Thus, $(\hat{\alpha}_1, \hat{\alpha_2})$ is not an interior equilibrium. ■

This result also reveals that, as long as the mean valuations are the same for the two bidders, other asymmetries, such as marginal IA cost, have no influence on the uniqueness of interior equilibrium.

The following proposition gives a comparative static analysis.

Proposition 5 *Suppose $c_1 = c_2 = c$. At the interior equilibrium, a larger marginal cost corresponds to a less incentive to acquire information, that is, $\frac{\partial \alpha_i^*}{\partial c} < 0$ for $i = 1, 2$.*

Proof. For interior equilibrium, the Kuhn-Tucker condition can be simplified as

$$8\pi c_1^2(\alpha_1+\beta_1)^4\left(\frac{\alpha_1}{\beta_1(\alpha_1+\beta_1)}+\frac{\alpha_2}{\beta_2(\alpha_2+\beta_2)}\right) = \exp\{-\frac{(\mu_1-\mu_2)^2}{\sigma_1^2+\sigma_2^2}\},$$

with $\sqrt{c_1}(\alpha_1+\beta_1) = \sqrt{c_2}(\alpha_2+\beta_2)$

For $c_1 = c_2 = c$ and $\mu_1 = \mu_2$, the equation can be reduced to

$$2\sqrt{2\pi}(\alpha_1+\beta_1)^{\frac{2}{3}}\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}\right)^{\frac{1}{2}} = \frac{1}{c}$$

with $\alpha_1 + \beta_1 = \alpha_2 + \beta_2$.

The first derivative with respect to c is

$$2c\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}\right)\frac{\partial\alpha_1}{\partial c} + c(\alpha_1+\beta_1)\left(\frac{1}{\beta_1}\frac{\partial\alpha_1}{\partial c}+\frac{1}{\beta_2}\frac{\partial\alpha_2}{\partial c}\right) = -2(\alpha_1+\beta_1)\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}\right),$$

with $\dfrac{\partial\alpha_1}{\partial c} = \dfrac{\partial\alpha_2}{\partial c}$.

Then $\dfrac{\partial\alpha_i}{\partial c} = -\dfrac{2(\alpha_1+\beta_1)(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2})}{2c(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2})+c(\alpha_1+\beta_1)(\frac{1}{\beta_1}+\frac{1}{\beta_2})} < 0.$ ■

This proposition indicates that with the same marginal IA cost mean valuation, the incentive of each bidder to acquire information is weakened by a higher marginal cost.

The above propositions provide a fairly clear description of the case of heterogeneous bidders: (1) Only intuitive equilibrium can exist in the heterogeneous environment. (2) With the same mean valuation, there is only one interior intuitive equilibrium. (3) A higher the marginal IA cost implies a less incentive to acquire information.

4 Many Bidders and General Distribution Functions

In this section, we consider a more general setting to analyze the existence and uniqueness of equilibrium in the second price auction with many bidders and general distribution functions. Most of the insights from the previous section can be carried through under certain assumptions. For simplicity of exposition, we assume the prior beliefs are the same.

4.1 The Existence of Equilibrium

To start with, we define a term that is used in the following discussions.

Definition 2 (Expected Marginal Value of Information, EMVI) Given information choices of other bidders, the expected marginal value of information of bidder i, EMVI for short, is defined as $\mathcal{V}_i(\alpha_i, \alpha_{-i}) = -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} \Pi_{l \neq i} H_{\alpha_l}(v_i) dv_i\}$.

To understand the meaning of the term, consider the following example.

Example 1 (EMVI for the two-bidder's case) Suppose there are only two bidders, as discussed in Section 3, the problem of bidder 1 is

$$\max_{\alpha_1}\{\int_{-\infty}^{+\infty}(1 - H_{\alpha_1}(v_1))H_{\alpha_2}(v_1)dv_1 - C_1(\alpha_1)\}.$$

The first term is the expected revenue without considering the IA cost, and it is obviously that this term is influenced by the level of information acquired by the bidder. Indeed, the expected revenue will be changed by $-\int_{-\infty}^{+\infty} \frac{\partial H_{\alpha_1}(v_1)}{\partial \alpha_1} H_{\alpha_2}(v_1)dv_1$ if the bidder chooses to rise his information level by differential amount. Thus, the first partial derivative of the revenue with respect to α_1 is considered as the expected marginal value of information. The EMVI defined above is a general version of the two-bidder case.

Bidder i's expected marginal information value in Definition 2 is a function of information choices of other bidders as well as of himself. The EMVI can be altered by every change in bidders' strategies. Thus, all interactions are captured in this expected marginal value of information.

Let $\Pi_i(\alpha_i, \alpha_{-i})$ denote the expected payoff of bidder i:

$$\Pi_i(\alpha_i, \alpha_{-i}) \equiv \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} (1 - H_{\alpha_i}(v_i))\Pi_{l \neq i} H_{\alpha_l}(v_i)dv_i - C_i(\alpha_i) \quad (8)$$

The decision problem of bidder i is:

$$\max_{\alpha_i} \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} (1 - H_{\alpha_i}(v_i))\Pi_{l \neq i} H_{\alpha_l}(v_i) dv_i - C_i(\alpha_i).$$

Then we need the following assumption to characterize the equilibrium in the IA stage.

Assumption 1 (Monotonicity) For each i, $\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i} \frac{1}{H_{\alpha_i}(v)}$ is strictly increasing in α_i for all v.

Assumption 1 means the relative probability gain from information acquisition, $\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i} \frac{1}{H_{\alpha_i}(v)}$, is higher when information acquisition is higher. If the family of distributions can be ranked by the first order condition, this assumption is satisfied. We use the the truth-or-noise technology (cf. Lewis and Sappington, 1994; Shi, 2007) with uniform prior distribution as an illustrative example.

Example 2 The buyers' true valuations $\{\omega_i\}$ are independently drawn from a distribution $F_i(v)$. Buyer i can acquire a costly signal s_i about ω_i. With probability $\alpha_i \in [\underline{\alpha}, 1]$, the signal s_i perfectly matches the true valuation ω_i, and with probability $1 - \alpha_i$, s_i is a noise independently drawn from F. The truth-or-noise technology with uniform prior distribution on $[\underline{\omega}_i, \overline{\omega}_i]$ and a noise with the mean $\underline{\omega}_i$ satisfies Assumptions 1 and 2.

The proof is relegated in the Appendix.

Before presenting the main results, we need the following lemmas.

Lemma 2 *Under Assumption 1, $H_{\alpha_i}(v)$ is strictly convex in α_i.*

Lemma 3 *Suppose $H_{\alpha_i}(v)$ is convex in α_i. Then, the expected payoff function $\Pi_i(\alpha_i, \alpha_{-i})$ is concave in α_i.*

Lemma 4 *Suppose $H_{\alpha_i}(v)$ is strictly convex in α_i. Then $\frac{\partial \mathcal{V}_i(\alpha_i, \alpha_{-i})}{\partial \alpha_i} < 0$ for all i.*

We then have the following result on the existence of equilibrium.

Proposition 6 (Existence of Equilibrium) *If $H_{\alpha_i}(v)$ is convex in α_i, there is an equilibrium.*

This proposition says that when the regularities of information acquisition and posterior distributions are satisfied, the equilibrium is guaranteed. Assumption 3 is not required in this proposition. Note that heterogeneities in the the prior beliefs and IA cost are allowed in the result.

4.2 Homogeneous Bidders

We first consider bidders have the same distribution and IA cost functions, i.e., $F_i(\cdot) = F(\cdot)$, $G_i(\cdot) = G(\cdot)$, and $C_i(\alpha) = C(\alpha)$ for all i. The homogeneous prior beliefs have an important implication: for any two bidders, i and j, their marginal value of information functions have a special property: $\mathcal{V}_i(\alpha_i, \alpha_j, \alpha_{-i,j}) = \mathcal{V}_j(\alpha_j, \alpha_i, \alpha_{-i,j})$. For example, $\mathcal{V}_1(\alpha_1, \alpha_2) = \mathcal{V}_2(\alpha_2, \alpha_1)$ for two bidders' case. This property is intuitive but seminal because it excludes any asymmetric equilibrium in this environment.

Proposition 7 *Suppose Assumptions 1 is satisfied and bidders are homogeneous. Then the equilibrium is symmetric and unique.*

This proposition clarifies the essence in the complex situation: the ex ante symmetric environment implies the ex post symmetry if each bidder has the property that other bidders' information choices have greater effect on his own EMVI. Also, in many situations, it is possible that there are multiple symmetric equilibria. However, this proposition indicates that the symmetric equilibrium is unique in this setting if posterior distributions are rotation ordered.

4.3 Heterogenous Bidders

When the bidders are heterogenous, i.e., they may be different either in prior beliefs or the IA cost, or both, the analysis of the equilibrium becomes much more complex. However, if the bidders are ex ante symmetric in a group and the only heterogeneity across groups is the IA cost, then the analysis becomes much simpler.

Consider there are K types of bidders, who are ex ante symmetric within each type. The kth type is consisted of n_k bidders with the same distribution functions $F_k(\cdot)$ and $G_k(\cdot)$, and the same IA cost function $C_k(\alpha)$. We focus the situation where distribution functions $F_k(\cdot)$ and $G_k(\cdot)$ are the same for all bidders, but $C'_k(\alpha) \neq C'_s(\alpha)$ for $k \neq s$ and all α. The total number of bidders is $n = \sum_1^K n_k$. For convenience, we denote the information choice of the bidder i in type k by α_{ik}.

Definition 3 *An equilibrium is called the type-symmetric equilibrium, if the information choices of bidders in the same type are identical, i.e., $\alpha^*_{ik} = \alpha^*_k$ for each type k and every member i of this type.*

There may be some non-type-symmetric equilibria, in which even bidders in the same type may choose different information levels. Of course, there also other special equilibria, where not only the bidders in the same type choose the same information level, but the bidders in different types make the same choice. In an extreme case, all bidders may choose the same information level despise their differences in the IA cost. However, the following proposition shows that there are cases where only type-symmetric equilibria exist.

Proposition 8 *In the economic environment under consideration, if an equilibrium exists, it must be type-symmetric.*

This proposition predicts that it is impossible for bidders of the same type to choose different level of information in equilibrium. Every ex ante symmetric bidder has the same behavior in the IA stage. This result is compatible with the case when bidders are homogeneous. If there is only one type of bidders, then this proposition indicates that all of them will choose the same information level in equilibrium.

The following proposition verifies the intuitive relationship of choices among bidders in different types.

Proposition 9 *For bidders in any different types k and s in the economic environment under consideration, the type-symmetric equilibria have the following properties:*

(i) Different types of bidders must choose either different information levels, i.e., $\alpha_k^* \neq \alpha_s^*$, or the same information level $\underline{\alpha}$ or $\overline{\alpha}$;

(ii) $\alpha_k^* \leq \alpha_s^*$ whenever $C_k'(\alpha) > C_s'(\alpha)$ for all α.

Therefore, in the second price auction with heterogeneous bidders, if Assumptions 1-3 are satisfied, then any equilibrium must be an type-symmetric, in which bidders of different types choose different information levels. Besides, lower information cost implies high information level in equilibrium.

5 Conclusion

This paper investigated the existence and uniqueness of equilibrium in the second price auction with information acquisition, which have been usually neglected in the literature. The existence of equilibrium is valid even for heterogeneous prior beliefs and IA cost. For cases with bidders being homogeneous in a group but heterogeneous across groups, the existence and uniqueness of type-(a)symmetric equilibria are examined. This distinguishes our model from the existing literature on information acquisition where homogeneity of bidders is required.

We first analyzed economic environments with two bidders and Gaussian specification. Then we extended our results to more general economic environments with heterogenous prior beliefs and IA cost functions. For a broad range of posterior distributions, we establish the existence of equilibrium. It is also shown that the only equilibrium is symmetric if bidders with the rotation ordered posterior distribution are homogeneous. The equilibrium must be *type-symmetric* if bidders can be divided into types: bidders in the same group choose the same information level while bidders in different groups choose different information levels. Besides, the advantaged groups with lower marginal information cost have stronger incentive to acquire information.

6 Appendixes

Proof of Lemma 1: The derivative of the expected payoff in respect of α_i is

$$\frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = -\int_{-\infty}^{+\infty} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} H_{\alpha_j}(v_i) dv_i - c_i. \tag{9}$$

Since

$$H_{\alpha_i}(v_i) = \int_{-\infty}^{v_i} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\{-\frac{(x-\mu_i)^2}{2\sigma_i^2}\} dx$$

and

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} = -\frac{(x-\mu_i)}{2\sqrt{2\pi}} \exp(-\frac{(x-\mu_i)^2}{2\sigma_i^2}) \sqrt{\frac{\beta_i^3}{\alpha_i^3(\alpha_i+\beta_i)}},$$

then,

$$\frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} = \frac{1}{2\sqrt{2\pi}(\alpha_i+\beta_i)^2} \frac{1}{\sqrt{\sigma_1^2+\sigma_2^2}} \exp\{-\frac{(\mu_1-\mu_2)^2}{2(\sigma_1^2+\sigma_2^2)}\} - c_i, \tag{10}$$

where $\sigma_i(\alpha_i) = \sqrt{\frac{\alpha_i}{\beta_i(\alpha_i+\beta_i)}}$, which is increasing in α_i.

$$\frac{\partial^2 \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i^2} = -\frac{1}{2\sqrt{2\pi}(\alpha_i+\beta_i)^4} \frac{1}{\sqrt{\sigma_1^2+\sigma_2^2}} \exp\{-\frac{(\mu_1-\mu_2)^2}{2(\sigma_1^2+\sigma_2^2)}\}[2(\alpha_i+\beta_i)+\frac{\sigma_i}{\sigma_1^2+\sigma_2^2}(1-\frac{(\mu_1-\mu_2)^2}{\sigma_1^2+\sigma_2^2})].$$

When $(\mu_1-\mu_2)^2 \le \frac{1}{\beta_1}+\frac{1}{\beta_2}$, we have $\frac{(\mu_1-\mu_2)^2}{\sigma_1^2+\sigma_2^2} < 1$, then $\pi_i(\alpha_1, \alpha_2)$ is strictly concave in α_i. ■

Proof of Example 2 For the truth-or-noise technology with uniform prior distribution on $[\underline{\omega}_i, \overline{\omega}_i]$ and a noise with the same mean $\underline{\omega}_i$ ($\underline{\omega}_i = \underline{\omega}$ for all i), we have $f(x) = \frac{1}{\overline{\omega}_i - \underline{\omega}_i}$ on $[\underline{\omega}_i, \overline{\omega}_i]$ and $f(x) = 0$ otherwise. And $F(x) = \frac{x-\underline{\omega}_i}{\overline{\omega}_i - \underline{\omega}_i}$ on $[\underline{\omega}_i, \overline{\omega}_i]$, $F(x) = 0$ when $x < \underline{\omega}_i$ and $F(x) = 1$ when $x > \overline{\omega}_i$.

We suppose $\alpha_i \in [\underline{\alpha}_i, 1]$, $\underline{\alpha}_i > 0$. According to Shi (2007), after observes a realization s_i with precision α_i, bidder i will revise his posterior estimate as follows:

$$v_i(s_i, \alpha_i) = \mathbb{E}(\omega_i | s_i, \alpha_i) = \alpha_i s_i + (1-\alpha_i)\underline{\omega}_i$$

The distribution and density of the posterior estimate are:

$$H_{\alpha_i}(v_i) = F(\frac{v_i - (1-\alpha_i)\underline{\omega}_i}{\alpha_i})$$

and

$$h_{\alpha_i}(v_i) = \frac{1}{\alpha_i} f(\frac{v_i - (1-\alpha_i)\underline{\omega}_i}{\alpha_i}).$$

Then we have

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} = -\frac{v_i - \underline{\omega}_i}{\alpha_i^2} f(\frac{v_i - (1-\alpha_i)\underline{\omega}_i}{\alpha_i}) \tag{11}$$

and thus

$$\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i}\frac{1}{H_{\alpha_i}(v)} = -f(\frac{v_i - (1-\alpha_i)\underline{\omega}_i}{\alpha_i})\frac{1}{F(\frac{v_i-(1-\alpha_i)\underline{\omega}_i}{\alpha_i})}\frac{v_i - \underline{\omega}_i}{\alpha_i^2} = -\frac{1}{\alpha_i}, \tag{12}$$

which implies $\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i}\frac{1}{H_{\alpha_i}(v)}$ is strictly increasing in α_i. Therefore, Assumption 1 is satisfied. ■

Proof of Lemma 2: Since $\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i}\frac{1}{H_{\alpha_i}(v)}$ is strictly increasing in α_i for all v, then

$$\frac{\partial}{\partial \alpha_i}\{\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i}\frac{1}{H_{\alpha_i}(v)}\} = \frac{1}{H_{\alpha_i}^2(v)}[\frac{\partial^2 H_{\alpha_i}}{\partial \alpha_i^2}H_{\alpha_i} - (\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i})^2] > 0$$

for almost everywhere on $[\underline{\alpha}, \overline{\alpha}]$. Then $\frac{\partial^2 H_{\alpha_i}}{\partial \alpha_i^2} > 0$ for almost everywhere on $[\underline{\alpha}, \overline{\alpha}]$, and thus $H_{\alpha_i}(v)$ is strictly convex in α_i. ■

Proof of Lemma 3: Note that, given other bidders choose α_{-i}, bidder i chooses α_i to maximize his payoff. The support is independent from the information choice, i.e., $\frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i} = 0$, and $\frac{\partial \overline{\omega}_{\alpha_i}}{\partial \alpha_i} = 0$. As the discussion is Section 4, the payoff of bidder i is

$$\Pi_i(\alpha_i) = \int_{\underline{\omega}_i}^{\overline{\omega}_i}(1 - H_{\alpha_i}(v_i))\Pi_{l\neq i}H_{\alpha_l}(v_i)dv_i - C_i(\alpha_i).$$

Then the second partial derivative then is

$$\frac{\partial^2 \Pi_i(\alpha_i)}{\partial \alpha_i^2} = -\int_{\underline{\omega}_i}^{\overline{\omega}_i}\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2}\Pi_{l\neq i}H_{\alpha_l}(v_i)dv_i\} - C_i''(\alpha_i).$$

Since $C_i''(\alpha_i) \geq 0$ and $\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2} \geq 0$ by the concavity of $H_{\alpha_i}(v_i)$, we have $\frac{\partial^2 \Pi_i(\alpha_i)}{\partial \alpha_i^2} \leq 0$, which means $\Pi_i(\alpha_i)$ is concave in α_i. ■

Proof of Lemma 4: Recall the definition of EMVI for bidder i is

$$\mathcal{V}_i(\alpha_i, \alpha_{-i}) = -\int_{\underline{\omega}_i}^{\overline{\omega}_i}\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}\Pi_{l\neq i}H_{\alpha_l}(v_i)dv_i.$$

We then have

$$\frac{\partial \mathcal{V}_i(\alpha_i, \alpha_{-i})}{\partial \alpha_i} = -\int_{\underline{\omega}_i}^{\overline{\omega}_i}\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2}\Pi_{l\neq i}H_{\alpha_l}(v_i)(v_i)dv_i < 0$$

by the strict convexity of $H_{\alpha_i}(v_i)$. ■

Proof of Proposition 6: The proof is similar to the proof of Proposition 1. Since the strategy space is a non-empty convex compact set and $\pi_i(\alpha_i, \alpha_j)$ is continuous in (α_i, α_{-i}) and concave in α_i by Lemma 3, then a Nash equilibrium exists in this setting. ■

Proof of Proposition 7: The Kuhn-Tucker condition for bidder i's problem is

$$\begin{cases} \dfrac{\partial \Pi_i(\alpha_i, \alpha_{-i})}{\partial \alpha_i} = \mathcal{V}_i(\alpha_i, \alpha_{-i}) - C_i'(\alpha_i) = -\lambda_i + \gamma_i \\ \lambda_i(\alpha_i - \underline{\alpha}) = 0 \\ \gamma_i(\alpha_i - \overline{\alpha}) = 0 \\ \lambda_i, \gamma_i \geq 0. \end{cases} \tag{13}$$

We first show the equilibrium must be symmetric. Suppose not. Without loss generality, assume $\alpha_1^* < \alpha_2^*$. This implies $\alpha_1^* < \overline{\alpha}$ and $\alpha_2^* > \underline{\alpha}$. Then by (13), we have $\gamma_1 = 0$ and $\lambda_2 = 0$. Thus, $\frac{\partial \pi_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2})}{\partial \alpha_1} = -\lambda_1 \leq 0$ and $\frac{\partial \pi_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2})}{\partial \alpha_2} = \gamma_2 \geq 0$. Hence we have:

$$\begin{aligned} C'(\alpha_1^*) &\geq \mathcal{V}_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) \\ C'(\alpha_2^*) &\leq \mathcal{V}_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*). \end{aligned}$$

Note that

$$\begin{aligned} &\mathcal{V}_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) - \mathcal{V}_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) \\ &\quad = -\int_{\underline{\omega}_i}^{\overline{\omega}_i} \left[\frac{\partial H_{\alpha_1^*}(v)}{\partial \alpha_1^*} H_{\alpha_2^*}(v) - \frac{\partial H_{\alpha_2^*}(v)}{\partial \alpha_2^*} H_{\alpha_1^*}(v) \right] \Pi_{l=3}^n H_{\alpha_l^*}(v) dv \\ &\quad = -\int_{\underline{\omega}_i}^{\overline{\omega}_i} \left[\frac{\frac{\partial H_{\alpha_1^*}(v)}{\partial \alpha_1^*}}{H_{\alpha_1^*}(v)} - \frac{\frac{\partial H_{\alpha_2^*}(v)}{\partial \alpha_2^*}}{H_{\alpha_2^*}(v)} \right] \Pi_{l=1}^n H_{\alpha_l^*}(v) dv. \end{aligned}$$

Also, note that posterior estimated distribution functions are the same for all homogeneous bidders and $\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i} \frac{1}{H_{\alpha_i}(v)}$ is strictly increasing in α_i for all v. Then, when $\alpha_1^* < \alpha_2^*$, we have

$$\frac{\frac{\partial H_{\alpha_1^*}(v)}{\partial \alpha_1^*}}{H_{\alpha_1^*}(v)} < \frac{\frac{\partial H_{\alpha_2^*}(v)}{\partial \alpha_2^*}}{H_{\alpha_2^*}(v)},$$

and thus, $\mathcal{V}_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) > \mathcal{V}_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*)$. Therefore, we have

$$C'(\alpha_2^*) \leq \mathcal{V}_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) < \mathcal{V}_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) \leq C'(\alpha_1^*),$$

which implies $\alpha_1^* > \alpha_2^*$, a contradiction. Hence the equilibrium must be symmetric.

We now show the equilibrium is unique. To do so, we first show the interior equilibrium is unique. For the interior equilibrium, the Kuhn-Tucker condition (13) reduces to

$$\mathcal{V}_i(\alpha_i, \alpha_{-i}) - C'(\alpha_i) = 0.$$

Then for any symmetric interior equilibrium, we have $\mathcal{V}_i(\boldsymbol{\alpha}) - C'(\alpha) = 0$, where $\boldsymbol{\alpha} = (\alpha, \ldots, \alpha)$.

Let $\mathcal{G}_i(\alpha) = \mathcal{V}_i(\boldsymbol{\alpha}) - C'(\alpha)$. Then the first derivative is

$$\mathcal{G}_i'(\alpha) = \sum_{j=1}^{n} \frac{\partial \mathcal{V}_i(\boldsymbol{\alpha})}{\partial \alpha_j} - C''(\alpha).$$

Recall that $C''(\alpha) \geq 0$, and by Lemma 4, $\frac{\partial \mathcal{V}_i(\alpha_i)}{\partial \alpha_i} < 0$. Also, when $\boldsymbol{\alpha} = (\alpha, \ldots, \alpha)$ and bidders are homogeneous, we have $\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} \equiv \frac{\partial H_{\alpha_j}(v_i)}{\partial \alpha_j}$ for all v_i, and thus

$$\begin{aligned}
\frac{\partial \mathcal{V}_i(\boldsymbol{\alpha})}{\partial \alpha_j} &= -\int_{\underline{\omega}_i}^{\overline{\omega}_i} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} \frac{\partial H_{\alpha_j}(v_i)}{\partial \alpha_j} \Pi_{l \neq i,j} H_{\alpha_l}(v_i) dv_i \\
&= -\int_{\underline{\omega}_i}^{\overline{\omega}_i} \left(\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} \right)^2 \Pi_{l \neq i,j} H_{\alpha_l}(v_i) dv_i \\
&\leq 0 \quad \forall j \neq i.
\end{aligned}$$

Thus $\mathcal{G}_i(\alpha)' < 0$. Therefore, the strict monotonicity implies the uniqueness of symmetric equilibrium.

We then show the uniqueness of corner equilibrium. Since it must be symmetric, there are only two possible corner equilibria: $(\underline{\alpha}, \ldots, \underline{\alpha})$ and $(\overline{\alpha}, \ldots, \overline{\alpha})$.

By the Kuhn-Tucker condition (13), we have $\mathcal{V}_i(\underline{\alpha}, \ldots, \underline{\alpha}) \leq C'(\underline{\alpha})$ for the equilibrium $(\underline{\alpha}, \ldots, \underline{\alpha})$; and $\mathcal{V}_i(\overline{\alpha}, \ldots, \overline{\alpha}) \leq C'(\overline{\alpha})$ for the equilibrium $(\overline{\alpha}, \ldots, \overline{\alpha})$.

Since $\mathcal{V}_i(\cdot)$ is strictly decreasing in α_i by Lemma 4, we have $\mathcal{V}_i(\underline{\alpha}, \ldots, \underline{\alpha}) > \mathcal{V}_i(\overline{\alpha}, \ldots, \overline{\alpha})$ and thus the two possible equilibria cannot exist simultaneously.

We last show that the interior equilibrium and the corner equilibrium cannot exist simultaneously. Suppose not. Without loss generality, there are an interior equilibrium $(\alpha, \ldots, \alpha)$ and a corner equilibrium $(\underline{\alpha}, \ldots, \underline{\alpha})$. Then, the above discussion implies $\mathcal{V}(\alpha, \ldots, \alpha) = C'(\alpha)$ and

$\mathcal{V}(\underline{\alpha}, \ldots, \underline{\alpha}) \leq C'(\underline{\alpha})$. This indicates

$$C'(\alpha) = \mathcal{V}(\alpha, \ldots, \alpha) < \mathcal{V}(\underline{\alpha}, \ldots, \underline{\alpha}) \leq C'(\underline{\alpha})$$

and consequently, we have $\alpha < \underline{\alpha}$, a contradiction.

Therefore, the equilibrium must be symmetric and unique. ■

Proof of Proposition 8: By Proposition 7, symmetric bidders of the same type must choose the same information level. This indicates that equilibrium must be type-symmetric. ■

Proof of Proposition 9: To show the first part of the proposition, one only has to recall that, for any two homogeneous bidders, their updated distributions have a special property: if $\alpha_i = \alpha_j$, then $\mathcal{V}_i(\alpha_i, \alpha_{-i}) = \mathcal{V}_j(\alpha_j, \alpha_{-j})$. Suppose that bidder ik and js are in different groups, with IA cost function $C_k(\alpha)$ and $C_s(\alpha)$ respectively. If they choose the same information level other than $\underline{\alpha}$ and $\overline{\alpha}$, then by equation (13) we must have $\mathcal{V}_{ik}(\alpha_i, \alpha_{-i}) = C'_k(\alpha_k)$ and $\mathcal{V}_{js}(\alpha_j, \alpha_{-j}) = C'_s(\alpha_s)$.

Then,

$$C'_k(\alpha^*_k) = \mathcal{V}_{ik}(\alpha^*_i, \alpha^*_{-i}) = \mathcal{V}_{js}(\alpha^*_j, \alpha^*_{-j}) = C'_s(\alpha^*_j)$$

by noting that, by assumption, distribution functions $F_k(\cdot)$ and $G_k(\cdot)$ are the same for all bidders. However, since $C'_k(\alpha) \neq C'_s(\alpha)$, we have $\alpha^*_k \neq \alpha^*_s$, a contradiction. Thus, when bidders' cost functions are different, bidders must choose different information levels in equilibrium.

We now show the second part. We need to show that $\alpha^*_k \leq \alpha^*_s$ provided $C'_k(\alpha) > C'_s(\alpha)$.

Suppose not. From the proof of Proposition 7, we know that $C'_k(\alpha^*_k) = \mathcal{V}_k(\alpha^*_k, \alpha^*_{-k}) \leq \mathcal{V}_s(\alpha^*_s, \alpha^*_{-s}) = C'_s(\alpha^*_s)$. But this can not be true because $\alpha^*_k > \alpha^*_s$ and $C'_k(\alpha) > C'_s(\alpha)$. Thus $\alpha^*_k \leq \alpha^*_s$. ■

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