Endogenous Information Acquisition on Opponents' Valuations in Multidimensional First Price Auctions

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Abstract

This paper investigates bidder's covert behavior of endogenous information acquisition on her opponents' valuations in first price auction model with independent private values. Such an information acquisition setting leads to bidimensional type space and bidimensional strategy space. We consider two different specifications of the environments: the *ex ante* information acquisition setting and the *interim* information acquisition setting. In equilibria the expected payoffs of the bidder under these specifications could exceed the counterpart payoffs of the corresponding standard sealed-bid auctions without information acquisition as long as the cost is small, but the auctioneer has lower payoffs in these models than those of the standard ones. Moreover, the incurred information cost becomes the deadweight loss, resulting in inefficient outcomes.

Key words: First-price sealed-bid auctions, endogenous information acquisition, opponents' valuation, profitability.

JEL Classification: C70; D44; D82.

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1 Introduction

The information acquisition problem in auctions occurs when a bidder does not have a clear idea about her exact willingness to pay for the underlying auctioned object, which can happen in both common value auctions and private value auctions. Much work has been done on this problem. This paper considers another type of information acquisition problem in which the bidder knows her valuation of the auctioned object and also tries to learn the other bidders' valuations. Thus each bidder has a bidimensional type that consists of her own valuation (the valuation type) and the signal about her opponent's valuation (information type).

In realistic economic activities, like auctions or procurement, and other similar situations, bidders can have information about their own estimations of the auctioned object or project, and also information about their opponents' evaluations of the underlying object besides their common prior. The latter information may come from bidders' strategic actions or from some other parties, which helps update bidders' beliefs on opponents. In other words, this is an environment where bidders have multidimensional information, or multidimensional types in game theory terminology. For example, in China, many middle-size and small-size state-owned enterprises have been privatized by way of procurement in the past several years. There are usually two or three potential buyers. The interactions between these buyers are rather involved and they may know part of their opponents' inside information. Another example is the NBA draft. The recruiting competition can be compared to an auction over the top-ranked candidates by assuming that the preferences of candidates only rely on monetary benefit. It has been observed during the period that clubs take measures to hide their own beliefs about candidates' values, and employ "detectives" to investigate the confidential information on other clubs' prices for recruiting their desired candidate(s). We can see that a club could have multidimensional information, its own evaluation and possibly other clubs' evaluations of candidates in this competitive environment.

This paper tries to capture such a multidimensional information structure by way of information acquisition[1] on opponents. For independent private values, the bidder's own valuation does not tell her any information about other bidders' valuations. This means the bidder cannot update her belief based on her own valuation signal. A natural question, then, is whether the bidder has incentives to acquire information about her opponents' valuations. Though it is somewhat difficult to find an example in the context of independent private value such that it is fully consistent with this kind of information acquisition behavior, our paper makes the first

[1]For simplicity, we always use "information acquisition" to refer to the bidder's acquiring information about her opponents' private valuations. Sometimes information acquisition is abbreviated as IA. This is different from the meaning elsewhere in the literature. Sometimes information acquisition, information acquiring, information collection, and collecting information are used interchangeably.

attempt to model bidders' bidimensional types and bidimensional strategies in a simple setting.

The auction format we mainly focus on is the first price sealed-bid auction (abbreviated as FPA), since this sealed-bid auction has been widely used in practice, such as in auctioning mineral rights in government-owned land, the sales of artwork and real estate as well as procurement. Examples include the U.S. offshore oil and gas lease auctions, the UK Treasury securities auctions which are the multiunit equivalent of the first-price auction (every winner pays her own bid).[2] As Klemperer (2003, pp.2) has claimed, sealed-bid designs frequently (but not always) both attract a larger number of serious bidders and are better at deterrence of collusion than ascending designs, see also Klemperer (1998, 2000, 2002). Maskin and Riley (1984) show that first price sealed-bid auction is the most profitable one amongst the standard auctions when bidders are risk-averse. When considering bidders have outside options, Kirchkamp et al. (2008) show that bidders in first-price auctions have more overbidding with outside options than without, and first-price auctions yield more revenue than second-price auctions do. As for second-price sealed-bid auctions (SPA) or Vickrey Auctions, it might be trivial to consider such a type of information acquisition on opponents' valuations because opponents' signals do not play any role in the formulation of strategies in SPA. All these reasons lead us to consider first price auctions.

In our paper, the bidder can covertly incur a cost to achieve a noisy signal conveying information about her opponents' valuations. We assume neither the bidder's information acquisition choice nor the signal received by the bidder can be observed by other bidders. For simplicity, the accuracy of the signal and the cost of the signal are exogenously determined. In our private value auction model, we analyze only the two-bidder case, where each bidder's valuation of the auctioned object is independently drawn from an identical distribution that has only two possible values.

We consider two different specifications of the environments: the *ex ante* information acquisition setting where bidders invest in opponents' valuations before observing their own valuations and the *interim* information acquisition setting where bidders invest in opponents' valuations after observing their own valuations. This is because both the interim and ex-ante information acquisitions are equally interesting in daily economic activities. In some case one can obtain information about one's opponents prior to or when evaluating her own strength exactly in the competition. For example, a firm needs to know about the incumbents of the market when it considers entering that industry. In some case one may intentionally get to know his or her competitors, namely, an incumbent would like to expect other firms' reactions when it introduces a new round of competition since the rivals' strategies are of particular importance. In other

[2] See the survey paper by Porter (1995) and footnote 14 in Klemperer (2003, chapter 1). Besides, the U.S. Treasury auctions used to be run with the multiunit version of first price auctions too.

cases, one may know partial information on opponents as a byproduct with one's own value and then decide whether to invest in more.

Most studies of information acquisition in auctions focus on how to improve the estimate of true valuation, since the bidder does not know her valuation when the auction begins, i.e., Matthews (1984) and Larson (2006) in the context of common values, Persico (2000) with affiliated values, Shi and Valimaki (2007) with interdependent values, Rezende (2005), Compte and Jehiel (2007), Shi (2007) in private value settings. All these papers share the same characteristic that bidders do not know their exact valuations at the beginning of the auction. Tan (1992) and Arozamena and Cantillon (2004) investigate the incentives for investment before auctions, and Hernando-Veciana (2006) considers the incentives of a bidder to acquire information in an auction when her information acquisition decision is observed by the other bidders before they bid.[3]

Our models have some similarities with that of Persico (2000), in which the bidder's strategy is bidimensional (consisting of the cost to induce more accurate signal and the bidding rule). In our paper, the bidder's strategy is also two-dimensional, namely, the bidder has freedom to choose the probability to incur a fixed cost for receipt of a signal with certain accuracy and specifies her bidding rules accordingly. However, the basic structures of the two papers are quite different. Our paper also shares some common characteristics with Fang and Morris (2006). First, both papers study the independent private value auction model where the bidder knows exactly what her valuation is at the beginning of the auction. In addition, the costly information acquisition behavior refers to collecting information about opponents' valuations.

However, these two models have the following distinctions. First, in Fang and Morris' model, the behavior of acquiring information is exogenously given. Our paper aims to solve this problem by endogenizing the choice of information acquisition, since in many realistic economic activities the choice of acquiring information is part of bidders' strategies. Secondly, there is no social welfare loss in Fang and Morris' binary-valuation model since no information cost is taken into consideration. As such, the outcome is efficient from the view of social planner, which may be regarded as "first best" outcome. However, contrary to Fang and Morris' benchmark case, in our model, costly information acquisition that can be regarded as an information rent leads to deadweight loss in total social surplus, and consequently, it results in inefficient outcome, which may be regarded as "second best" outcome[4]. In other words, generally information acquisition

[3] There is also a strand of papers studying information acquisition problem where the auctioneer has controlled the information resources, such as Bergemann and Pesendorfer (2002) and Ganuza (2004), Eso and Szentes (2007). Milgrom and Weber (1982) also investigate the seller's information disclosure policy in affiliated setting.

[4] The equilibrium outcome in our model may be even worse than second best due to the dissipative information cost, since it is not clear what exactly the second best outcome is.

involves efficiency loss and the first best outcome cannot be achieved. Thirdly, our strategy space is two-dimensional, while in Fang and Morris' model it is of single dimension.

The contribution of this paper is as follows. First, our model takes an initial step in endogenizing information acquisition on opponent's valuations in independent private value models. Second, in our endogenous model, both the type space and strategy space of bidders are bidimensional,[5] which differs from the current literature. Third, we find that costly information acquisition leads to deadweight loss in social surplus, and therefore, the equilibrium outcome is inefficient. This tells that the efficient outcome is generally not achievable with costly information acquisition, and the corresponding policy implication would be that the information acquisition behavior should be prevented to maximize social welfare.

The remainder of this paper is organized as follows. Section 2 describes the basic setting of the model. Section 3 discusses bidders' behavior when bidders can make ex ante information acquisition choices. Section 4 considers a more relaxed environment in which bidders make their information acquisition choices at an interim stage. Section 5 discusses the comparative statics and implications: efficiency, surplus, and revenue across FPA and SPA formats. Section 6 discusses some variants of the model. Section 7 concludes. All the proofs are relegated to the appendix.

2 Preliminaries

There are two risk neutral bidders indexed by $i \in \mathcal{N} = \{1, 2\}$ who compete for an underlying object. The object is worth zero to the seller. Every bidder $i \in \mathcal{N}$ has a valuation $v_i \in \mathcal{V} = \{V_l, V_h\}$ for the object, where $0 < V_l < V_h$. The valuations v_i are private and independently drawn from identical distribution. The prior distribution of bidder i's valuation v_i is given by $Pr\{v_i = V_l\} = p_l$ $(0 < p_l < 1)$, and $Pr\{v_i = V_h\} = p_h = 1 - p_l$ for $i \in \mathcal{N}$. Then $v = (v_1, v_2) \in \mathcal{V} \times \mathcal{V}$ is a profile of possible valuations. The auction format is the first price sealed-bid auction. The reservation price of the seller is normalized to zero.

Each bidder i can incur an exogenous cost $c (c \geqq 0)$ to get a noisy signal s_i which induces information about her opponent's private valuation. If the bidder does not want to collect information about her opponent's valuation, she does not incur any cost and cannot receive any signal. Let $a_i \in \{1, 0\}$ denote bidder i's choice of acquiring information, where $a_i = 1$ means

[5] In our independent private value auction model, the type of the bidder is determined by the bidder's valuation type and the signal received, and the strategy of the bidder includes the information acquisition choice and bidding rule. Therefore, our model relaxes the restrictive approach of encoding multidimensional types into a single-dimensional variable. From the point of view in general mechanism design (cf. Hurwicz (1979), Mount and Reiter (1974), and Tian (1994, 2004, 2006)), the information smuggling of changing multi-dimensional types (say, by the inverse Peano curve) into a single-dimensional variable is essentially equivalent to one-dimensional type problem.

acquiring information and $a_i = 0$ means not.

To clearly describe the information structure, we assume neither the bidder's information acquisition choice nor the signal received by the bidder can be observed by her opponent.

The signal s_i received by bidder i is drawn from the support $\mathcal{S} = \{L, H\}$ if she acquires information. For convenience, we denote s_0 as receiving no signal, and let $\bar{\mathcal{S}} = \{L, H, s_0\}$. Given bidder j's valuation and bidder i's IA choice, the probability of the transmitted signal s_i taking on each possible value is characterized by

$$\begin{aligned} \Pr\{s_i = L|v_j = V_l, a_i = 1\} = \Pr\{s_i = H|v_j = V_h, a_i = 1\} = q, \\ \Pr\{s_i = s_0|v_j = V_l, a_i = 0\} = \Pr\{s_i = s_0|v_j = V_h, a_i = 0\} = 1; \end{aligned} \tag{1}$$

for $i \neq j,\ i, j \in \mathcal{N}$; note that $\Pr\{s_i = L|v_j = V_l, a_i = 1\} + \Pr\{s_i = H|v_j = V_l, a_i = 1\} = 1$ and $\Pr\{s_i = H|v_j = V_h, a_i = 1\} + \Pr\{s_i = L|v_j = V_h, a_i = 1\} = 1$. Since all the signals are characterized by one parameter q, we call q the accuracy of the signal. Without loss of generality, we assume $0.5 < q \leqq 1$. Bidder j updates her belief about i's information type s_i according to the signal distribution specified by equation (1).

The auction game we will analyze is a Bayesian game of incomplete information. The type space of each bidder is $\mathcal{T} \equiv \{V_l, V_h\} \times \{L, H, s_0\}$. One representative type of bidder i is $t_i = (v_i, s_i) \in \mathcal{T}$. We call v_i the *valuation type*, and s_i *the information type*. Given bidder i's signal s_i, bidder i updates her belief on her opponent's valuation type according to the Bayes' rule as follows. For $s_i \in \{L, H\}$, $i \neq j,\ i, j \in \mathcal{N}$,

$$\begin{aligned} \Pr\{v_j = V_l|s_i = L\} &= \frac{p_l q}{p_l q + p_h(1-q)}, \\ \Pr\{v_j = V_l|s_i = H\} &= \frac{p_l(1-q)}{p_l(1-q) + p_h q}, \\ \Pr\{v_j = V_l|s_i = s_0\} &= p_l; \end{aligned} \tag{2}$$

and the conditional probability is also a probability distribution, i.e., $\Pr\{v_j = V_h|s_i\} = 1 - \Pr\{v_j = V_l|s_i\}$ for $s_i \in \bar{\mathcal{S}}$.

The primitives of the model are a tuple of five independent parameters as follows,

$$\mathscr{E} = \left\{ < V_l, V_h, p_l, q, c >:\ V_h > V_l > 0, p_l \in (0,1), q \in (0.5, 1],\ c \geqq 0 \right\}.$$

An element $e \in \mathscr{E}$ is called an auction environment.

3 Ex Ante Endogenous Information Acquisition

3.1 The Model

In this section we consider an auction economy in which bidders can acquire information on their opponents' valuations *ex ante*. That is, the bidder can decide whether to collect information before knowing her own valuation.

The procedures of this auction game are as follows:

- Stage 1. Bidders simultaneously decide whether to incur a cost c to collect information on her opponent's valuation, and this decision is not observed by her opponent.
- Stage 2. Nature draws a valuation for each bidder and tells the bidder only what her own valuation is. Every bidder receives a signal s_i revealing her opponent's valuation if she incurs a cost c in Stage 1. Otherwise, she receives no signal. Bidders' signal is unobservable for her opponent.
- Stage 3. Bidders submit their bids simultaneously based on their own valuations, and possibly their updated belief from the signal according to (2).
- Stage 4. The bidder whose bid is the highest receives the object and pays what she bids.[6]

In this *ex ante* IA auction game, bidder has bidimensional type (valuation type and information type) and bidimensional strategy (acquiring information choice and bidding rule). To analyze this auction economy, we first define a set of notations.

For interested primitive values, bidders may randomize IA choice and bid submitted. Let $\mathcal{F}^a(\{0,1\})$ denote the collection of probability distributions defined on the set $\{0,1\}$, and let $\mathcal{F}^b([0,V_h])$ denote the collection of probability distributions defined on the interval $[0,V_h]$. We define the bidder's (mixed) strategy as follows. The (mixed) information acquisition choice of bidder i is one element of $\mathcal{F}^a(\{1,0\})$, which can be simply characterized by one number $\pi_{iE} = \Pr\{a_i = 1\}$ where $1 > \pi_{iE} > 0$ (note that $1 - \pi_{iE} = \Pr\{a_i = 0\}$). A bidding rule[7] is a mapping $\mathcal{B} \ni \mathbb{b} : \mathcal{T} \longrightarrow \mathcal{F}^b([0,V_h])$. The strategy space of bidder i is then characterized by

[6] When there is a tie, we choose the fair tie-breaking rule for $q \in (0.5, 1)$ and Vickrey tie-breaking rule for $q = 1$. The fair tie-breaking rule says in the tie each bidder wins the object with equal probability and pays what she bids conditional on winning. The Vickrey tie-breaking rule is as follows: in the tie both bidders are asked to report a number, either 0 or ϵ^* where ϵ^* is a sufficiently small positive number. If only one bidder reports ϵ^*, then this bidder gets the object and pays her bid; else the bidders get the object with equal probability and the bidder having the object pays her bid plus her reported number. Actually it does not matter which tie-breaking rule is used for $q < 1$, we just choose the fair tie-breaking rule to explicitly and conveniently specify the strategic form of the auction economy. For $q = 1$, however, the fair tie-breaking rule does not promise an equilibrium in our auction game and the Vickrey rule will reinstall the equilibrium.

[7] In principle, the bid support is a subset of the real line. Without loss of generality, we focus on the bounded support $[0, V_h]$.

$\mathcal{A}_i^E = \mathcal{F}^a(\{1,0\}) \times \mathcal{B}$, and a typical element of $\mathcal{A}_i^E$ would be $A_i^E = [\pi_{iE}, (\mathbb{b}_i(t))_{t \in \mathcal{T}}]$. Note that $\mathbb{b}_i(t)$ represents a random variable (or equivalently a distribution) with potential support $[0, V_h]$, not a single bid. Then $\mathcal{A}^E = \mathcal{A}_1^E \times \mathcal{A}_2^E$ denotes the strategy space of the auction game.

Given a strategy profile $(A_1^E, A_2^E) \in \mathcal{A}^E$, there would be an *ex ante* joint distribution of the random vector $(a_1, v_1, b_1; a_2, v_2, b_2)$, $\Gamma(a_1, v_1, b_1; a_2, v_2, b_2 | A_1^E, A_2^E)$ induced by (A_1^E, A_2^E), here b_i refers to the bid that bidder i submits. The *ex post* payoff $g^E \equiv (g_1^E, g_2^E) : \{1,0\}^2 \times \mathcal{V}^2 \times [0, V_h]^2 \longrightarrow \Re^2$ simply takes the form as follows according to the first price auction rule

$$g_i^E(a_1, v_1, b_1; a_2, v_2, b_2) = (v_i - b_i) \cdot \left[\mathbb{1}\{b_i > b_j\} + \frac{1}{2}\mathbb{1}\{b_i = b_j\}\right] - a_i c, \ i \neq j, i = 1, 2, \quad (3)$$

where $\mathbb{1}\{\cdot\}$ is the indicator function. Hence, the *ex ante* expected payoff $G^E \equiv (G_1^E, G_2^E)$: $\mathcal{A}^E \longrightarrow \Re^2$ is given by

$$G_i^E(A_1^E, A_2^E) = \mathbb{E}_{\Gamma(\cdot | A_1^E, A_2^E)}\left[g_i^E(a_1, v_1, b_1; a_2, v_2, b_2)\right], \ i = 1, 2, \quad (4)$$

where the expectation of g_i^E is taken with respect to the joint distribution of random vector $(a_1, v_1, b_1; a_2, v_2, b_2)$ induced by (A_1^E, A_2^E).

A tuple $< \mathcal{A}^E, G^E >$ with primitive $e \in \mathscr{E}$ is then called an *ex ante* IA auction game.

3.2 Equilibrium

We first have the following lemma that tells type-V_l bidder bids her valuation V_l for sure in any equilibrium.

Lemma 1 *In any equilibrium of the ex ante IA auction game* $< \mathcal{A}^E, G^E >$*, type-*(V_l, s) *bidders bid* V_l *in pure strategy for* $s \in \{H, L, s_0\}$*.*

With this lemma in hand, we establish that the equilibrium for this FPA auction game $< \mathcal{A}^E, G^E >$ is unique and symmetric. The characterization of the equilibrium is summarized in the following proposition.

Proposition 1 *The ex ante IA auction game* $< \mathcal{A}^E, G^E >$ *with primitive* $e \in \mathscr{E}$ *has a unique equilibrium. More specifically:*

(i). When the informative cost is small, i.e. $c < p_l p_h^2 (V_h - V_l)(2q - 1)$*, then the unique equilibrium of the FPA is symmetric and can be described as follows. For* $i = 1, 2$

1. *With probability* π_E*, the bidder collects information about her opponent's private valuation; with probability* $(1 - \pi_E)$ *the bidder does not collect information.*

2. $b_i(V_l, s) = V_l$ *for* $s \in \{s_0, L, H\}$.

3. *When* $q = 1$, *type-*(V_h, L) *bidder submits* $b_i(V_h, L) = V_l$ *for sure; when* $q < 1$, *type-*(V_h, L) *bidder mixes over* $(V_l, \delta_1]$ *according to the cumulative distribution function (c.d.f)* $J^l(\cdot)$ *specified by*

$$J^l(b) = \frac{p_l q}{\pi_E p_h (1-q)^2} \cdot \frac{b - V_l}{V_h - b}, \quad b \in (V_l, \delta_1]. \tag{5}$$

4. *Type-*(V_h, s_0) *bidder mixes over* $[\delta_1, \delta_2]$ *according to the c.d.f.* $J^{s_0}(\cdot)$ *specified by*

$$J^{s_0}(b) = \frac{p_l + \pi_E(1-q)p_h}{p_h(1-\pi_E)} \cdot \frac{b - \delta_1}{V_h - b}, \quad b \in [\delta_1, \delta_2]. \tag{6}$$

5. *Type-*(V_h, H) *bidder mixes over* $[\delta_2, \delta_3]$ *according to the c.d.f.* $J^h(\cdot)$ *specified by*

$$J^h(b) = \frac{p_l(1-q) + p_h q(1-\pi_E q)}{\pi_E p_h q^2} \cdot \frac{b - \delta_2}{V_h - b}, \quad b \in [\delta_2, \delta_3]. \tag{7}$$

Here

$$\delta_1 = \frac{p_l q V_l + \pi_E p_h (1-q)^2 V_h}{p_l q + \pi_E p_h (1-q)^2}, \tag{8}$$

$$\delta_2 = \frac{p_h(1-\pi_E)V_h + [p_l + \pi_E(1-q)p_h]\delta_1}{p_l + p_h(1-\pi_E q)}, \tag{9}$$

$$\delta_3 = \frac{\pi_E p_h q^2 V_h + [p_l(1-q) + p_h q(1-\pi_E q)]\delta_2}{p_l(1-q) + p_h q}, \tag{10}$$

and π_E *is determined by the following condition:*

$$\begin{aligned} & p_h\big\{p_l q(V_h - V_l) + [p_l(1-q) + p_h q(1-\pi_E q)](V_h - \delta_2)\big\} - c \\ =& p_h\big[p_l + \pi_E(1-q)p_h\big](V_h - \delta_1). \end{aligned} \tag{11}$$

(ii). When $c \geqq p_l p_h^2 (V_h - V_l)(2q-1)$, *the unique equilibrium of the FPA is symmetric and can be described as follows. For* $i = 1, 2$

1. $\pi_{iE} = 0$. *No bidder acquires information.*

2. $b_i(V_l, s_0) = V_l$.

3. *Type-*(V_h, s_0) *bidder mixes over* $(V_l, \gamma]$ *according to the c.d.f.* $\bar{J}(\cdot)$ *specified by*

$$\bar{J}(b) = \frac{p_l}{p_h} \cdot \frac{b - V_l}{V_h - b}, \quad b \in (V_l, \gamma], \tag{12}$$

where

$$\gamma = p_l V_l + p_h V_h. \tag{13}$$

Remark 1 Provided that the informative cost is negligible, i.e., $c = 0$, the information acquisition probability π_E reduces to 1, which is the same equilibrium outcome as that of the exogenous information acquisition (Proposition 1 in Fang and Morris, 2006).

Remark 2 When $q = 1$, the switch of tie-breaking rule does not change the variables of interest. For instance, the bidder's expected surplus and seller's revenue are continuous at $q = 1$. The reason is that the equilibria are exactly the same under fair tie-breaking rule and Vickrey tie-breaking rule for $q < 1$.

Remark 3 For equilibrium strategies in part (i), if $\pi_E = 0$, type-(V_h, L) and type-(V_h, H) bidders disappear, and the formula $J^{s_0}(\cdot)$ remains valid; if $\pi_E = 1$, type-(V_h, s_0) bidder disappears, and the formulas $J^l(\cdot)$ and $J^h(\cdot)$ remain valid.

The type of equilibrium depends on the value of the informative cost. There is information acquisition only if the cost is small, i.e., $c < p_l p_h^2(V_h - V_l)(2q-1)$. In equilibrium, the bidder with valuation V_l always bids her valuation V_l. If bidders acquire information, the type-(V_h, L), type-(V_h, s_0) and type-(V_h, H) bidders play mixed strategy in equilibrium, and type-(V_h, L)'s support is lower than type-(V_h, s_0)'s support which is also lower than type-(V_h, H)'s. This is because type-(V_h, L) bidder perceives her opponent more likely to be type-V_l bidder, type-(V_h, H) bidder regards her opponent more likely as type-V_h bidder, and type-(V_h, s_0) bidder still takes her ex ante belief. Then their best responses would result in such a support ranking. When the informative cost is large, namely, $c \geqq p_l p_h^2(V_h - V_l)(2q-1)$, no bidder acquires information and it degenerates to the standard FPA equilibrium with primitive $e \in \mathscr{E}$.

The proof of the proposition is a little bit involved, but the basic idea is simple. It is always true that if the bidder plays mixed strategy in submitting her bid, she should get exactly the same expected surplus from any bid drawn from the corresponding support. This fact (indifference condition implied by mixed strategy) can help pin down the c.d.f of the bid as well as the upper or lower bound of the support. The same idea applies that the bidder should be indifferent between acquiring information and not acquiring information if she plays mixed strategy in IA.

3.3 Demand for Information

We call π_E the *demand for ex ante information*, since for any informative cost c, the equilibrium is symmetric and unique.

Substituting δ_1, δ_2 and δ_3 into (11) and rearranging terms yields the formula of the reverse demand for information:

$$c = p_l p_h (V_h - V_l) \cdot \frac{p_l p_h q(2q-1)}{p_l q + p_h(1-q)^2 \pi_E} \cdot \frac{(1-\pi_E)}{(1-p_h q \pi_E)}. \tag{14}$$

Immediately, we have the following intuitive result as a corollary of Proposition 1.

Corollary 1 *(Demand for Ex Ante Information) The probability of ex ante IA, π_E, has the following properties:*

(i). For all given values $q \in (0.5, 1]$, π_E is continuous in c for $c \geqq 0$, and strictly decreasing in c for $c \in [0, p_l p_h^2 (V_h - V_l)(2q-1)]$.

(ii). For all values $c \geqq 0$, π_E is (weakly) increasing in accuracy q.

When the cost of the signal is small, i.e., $c \leqq p_l p_h^2 (V_h - V_l)(2q-1)$, the probability of acquiring information, π_E, satisfies the usual law of demand. This is intuitive because a more costly signal provides less incentive for the bidder when the accuracy keeps fixed.

On the other hand, a more informative signal will help the bidder evaluate her opponent more effectively, and hence is more attractive to the bidder, provided the cost does not change.

Quantitatively, Figure 1 presents the simulated results for the impacts of cost c and accuracy q on IA probability π_E, respectively. It verifies the comparative results for π_E. The primitive values are $V_h = 2$, $V_l = 1$ and $p_l = p_h = 0.5$, which are chosen for computing convenience. When considering the impact of the informative cost c, we choose two fixed values of accuracy, $q = 0.8$ which is a representative of generic accuracy and $q = 1$ which represents the perfect accuracy. When considering the impact of the accuracy q, we fix the cost to be $c = 0$ (special costless case) and $c = 0.02$ (a representative of generic cost). These numeric values will be used continually in the following sections for consistency. The kink points in the figure correspond to the critical values of c and q that satisfy the condition $c = p_l p_h^2 (V_h - V_l)(2q-1)$. In the left sub-figure, the kink points are at critical value $c = 0.075$ for $q = 0.8$ and $c = 0.125$ for $q = 1$. In the right sub-figure, the kink point is at the critical value $q = 0.58$ for $c = 0.02$.

In equilibrium, we see the type-V_l bidder ignores the signal received. Thus, one can expect that if we relax the endogenous information acquisition assumption, i.e., permitting collecting information at *interim* stage, the type-V_h's demand for information can even be intensified. This is investigated in the next section. We will discuss the welfare analysis and comparative statics of ex ante IA equilibrium in section 5.

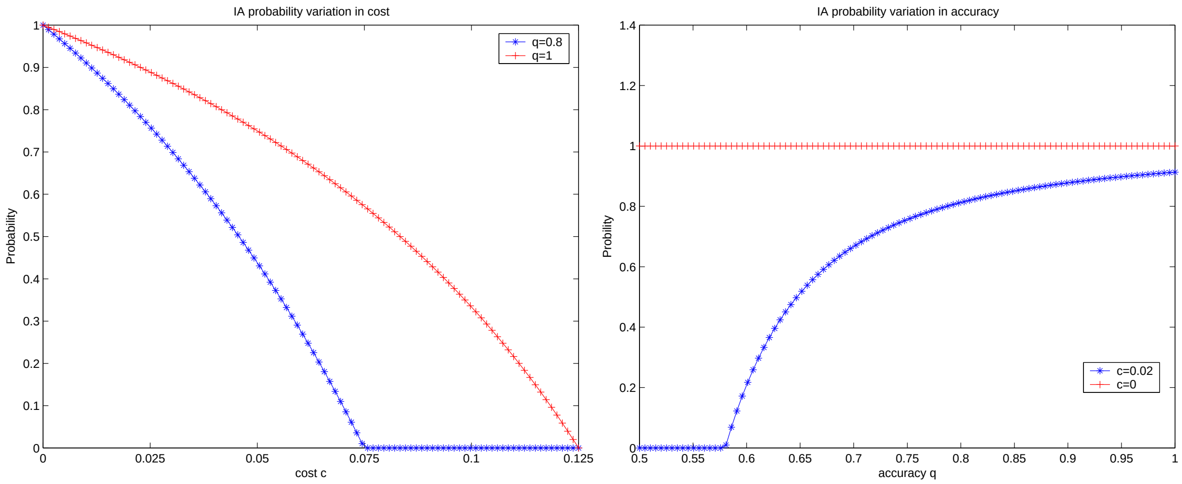

Figure 1: Cost and Accuracy's Impacts on IA Probability π_E
Primitive: $V_h = 2, V_l = 1, p_l = p_h = 0.5$.

4 Interim Information Acquisition

In the previous section we consider the auction economy in which the bidder can freely choose to collect information about her opponent's valuation *ex ante*, but it is still very restrictive in the sense that the type-V_l bidder in the interim stage would definitely ignore the signal received. In other words, if the realized valuation is V_l, the bidder cannot benefit from the signal she receives because she cannot do better than bid her valuation V_l. In this section we consider the sealed-bid auction economy in which information acquisition choice is made after bidders' valuations are realized, which will give the bidders more freedom to formalize their strategy in the auction game.

4.1 The Model

The procedures of this auction game are as follows:

- Stage 1. Nature draws a valuation for each bidder and tells the bidder only what her own valuation is.
- Stage 2. Every bidder decides whether to incur a cost c to receive a signal s_i based on her valuation, and this decision as well as the signal received cannot be observed by her opponent.
- Stage 3. Bidders submit their bids simultaneously based on their own valuations, and possibly the updated belief from the signal received according to (2).
- Stage 4. The bidder whose bid is the highest receives the object and pays what she bids.

In this interim IA auction game, bidder i's strategy is a bidimensional mapping $(\mathbb{c}_i, \mathbb{b}_i)$, such that $\mathcal{C} \ni \mathbb{c}_i : \{V_l, V_h\} \longrightarrow \mathcal{F}^a(\{0,1\})$ represents the IA choice rule and $\mathcal{B} \ni \mathbb{b}_i : \mathcal{T} \longrightarrow \mathcal{F}^b([0, V_h])$ represents the bidding rule, where $\mathcal{A}_i^I = \mathcal{C} \times \mathcal{B}$ is bidder i's strategy space. Denote $((\mathbb{c}_1, \mathbb{b}_1), (\mathbb{c}_2, \mathbb{b}_2)) = (A_1^I, A_2^I) \in \mathcal{A}_1^I \times \mathcal{A}_2^I \equiv \mathcal{A}^I$ as one strategy profile.[8]

Similar as the ex ante IA setting, given a strategy profile $(A_1^I, A_2^I) \in \mathcal{A}^I$, there would be an *ex ante* joint distribution of the random vector $(a_1, v_1, b_1; a_2, v_2, b_2)$, $\Gamma(a_1, v_1, b_1; a_2, v_2, b_2 | A_1^I, A_2^I)$ induced by (A_1^I, A_2^I), here (a_i, v_i, b_i) refers to the realization profile of bidder i's information acquisition choice, valuation and bid sumbitted. The *ex post* payoff $g^I \equiv (g_1^I, g_2^I) : \{1,0\}^2 \times \mathcal{V}^2 \times [0, V_h]^2 \longrightarrow \mathfrak{R}^2$ takes the same form as g^E in (3), i.e.,

$$g_i^I(a_1, v_1, b_1; a_2, v_2, b_2) = (v_i - b_i) \cdot \left[\mathbb{1}\{b_i > b_j\} + \frac{1}{2}\mathbb{1}\{b_i = b_j\} \right] - a_i c, \ i \neq j, i = 1, 2. \tag{15}$$

Hence, the *ex ante* expected payoff $G^I \equiv (G_1^I, G_2^I) : \mathcal{A}^I \longrightarrow \mathfrak{R}^2$ is given by

$$G_i^I(A_1^I, A_2^I) = \mathbb{E}_{\Gamma(\cdot | A_1^I, A_2^I)} \left[g_i^I(a_1, v_1, b_1; a_2, v_2, b_2) \right], \ i = 1, 2, \tag{16}$$

A tuple $< \mathcal{A}^I, G^I >$ with primitive $e \in \mathscr{E}$ is then called an *interim* IA auction game.

4.2 Equilibrium

The argument of Lemma 1 in the appendix does not require the detailed structure of information acquisition, and therefore, the result that type-V_l bidder bids V_l in pure strategy in any equilibrium remains valid in our interim IA setting. Then only the type-V_h bidder has incentives to receive costly informative signal. Similarly, the equilibrium of the interim IA auction game $< \mathcal{A}^I, G^I >$ is unique, symmetric and can be summarized as follows.

Proposition 2 *The interim IA auction game $< \mathcal{A}^I, G^I >$ with primitive $e \in \mathscr{E}$ has a unique equilibrium. More specifically:*

(i). When the informative cost is small, i.e. $0 \leqq c < p_l p_h (V_h - V_l)(2q - 1)$, then the unique equilibrium of the FPA is symmetric and can be described as follows. For $i = 1, 2$

1. *$\pi_{iI}^l = 0$. The type-V_l bidder does not incur any cost to receive any signal.*
2. *$\pi_{iI}^h = \pi_I$. The type-V_h bidder, with probability π_I, chooses to incur a cost c to collect information on her opponent's private valuation; with probability $1 - \pi_I$ she does not incur any cost.*

[8] $\mathbb{c}_i$ is effectively referred to two distributions on binary support $\{1, 0\}$. Hence, $\mathbb{c}_i$ can be represented as $((\pi_{iI}^l, 1 - \pi_{iI}^l), (\pi_{iI}^h, 1 - \pi_{iI}^h))$ where $\pi_{iI}^l = \Pr\{a_i = 1 | v_i = V_l\}$ denotes the IA probability of the type-V_l bidder i and $\pi_{iI}^h = \Pr\{a_i = 1 | v_i = V_h\}$ denotes that of the type-V_h bidder i.

3. $b_i(V_l, s_0) = V_l$.

4. *When* $q = 1$, *type-*(V_h, L) *bidder submits* $b_i(V_h, L) = V_l$ *for sure; when* $q < 1$, *type-*(V_h, L) *bidder mixes over* $(V_l, \beta_1]$ *according to the c.d.f.* $Q^l(\cdot)$ *specified by*

$$Q^l(b) = \frac{p_l q}{\pi_I p_h (1-q)^2} \cdot \frac{b - V_l}{V_h - b}, \quad b \in (V_l, \beta_1]. \tag{17}$$

5. *Type-*(V_h, s_0) *bidder mixes over* $[\beta_1, \beta_2]$ *according to the c.d.f.* $Q^{s_0}(\cdot)$ *specified by*

$$Q^{s_0}(b) = \frac{p_l + \pi_I(1-q)p_h}{p_h(1-\pi_I)} \cdot \frac{b - \beta_1}{V_h - b}, \quad b \in [\beta_1, \beta_2]. \tag{18}$$

6. *Type-*(V_h, H) *bidder mixes over* $[\beta_2, \beta_3]$ *according to the c.d.f.* $Q^h(\cdot)$ *specified by*

$$Q^h(b) = \frac{p_l(1-q) + p_h q(1-\pi_I q)}{\pi_I p_h q^2} \cdot \frac{b - \beta_2}{V_h - b}, \quad b \in [\beta_2, \beta_3]. \tag{19}$$

Here

$$\beta_1 = \frac{p_l q V_l + \pi_I p_h (1-q)^2 V_h}{p_l q + \pi_I p_h (1-q)^2}, \tag{20}$$

$$\beta_2 = \frac{p_h(1-\pi_I)V_h + [p_l + \pi_I(1-q)p_h]\beta_1}{p_l + p_h(1-\pi_I q)}, \tag{21}$$

$$\beta_3 = \frac{\pi_I p_h q^2 V_h + [p_l(1-q) + p_h q(1-\pi_I q)]\beta_2}{p_l(1-q) + p_h q}, \tag{22}$$

and π_I *is determined by the following condition*

$$\begin{aligned} &p_l q(V_h - V_l) + [p_l(1-q) + p_h q(1-\pi_I q)](V_h - \beta_2) - c \\ =&[p_l + \pi_I(1-q)p_h](V_h - \beta_1). \end{aligned} \tag{23}$$

(ii). When $c \geqq p_l p_h (V_h - V_l)(2q-1)$, *the unique equilibrium of the FPA is symmetric and can be described as follows. For* $i = 1, 2$

1. $\pi_{iI}^l = \pi_{iI}^h = 0$. *No bidder acquires information.*

2. $b_i(V_l, s_0) = V_l$.

3. *Type-*(V_h, s_0) *bidder mixes over* $(V_l, \gamma]$ *according to the c.d.f.* $\bar{J}(\cdot)$, *where* $\bar{J}(\cdot)$ *and* γ *are given by (12) and (13), respectively.*

Remark 4 As in the unique equilibrium of ex ante information acquisition, when the informative cost is negligible, i.e., $c = 0$, the information acquisition probability π_I reduces to 1.

Remark 5 For equilibrium strategies in part (i), if $\pi_E = 0$, type-(V_h, L) and type-(V_h, H) bidders disappear, and the formula $Q^{s_0}(\cdot)$ remains valid; if $\pi_E = 1$, type-(V_h, s_0) bidder disappears, and the formulas $Q^l(\cdot)$ and $Q^h(\cdot)$ remain valid.

In the unique equilibrium of interim information acquisition specified by Proposition 2, type-V_l bidder has no incentive to acquire information and bids her valuation V_l, type-V_h bidder plays a mixed strategy in information acquisition choice. The type-(V_h, L), type-(V_h, s_0) and type-(V_h, H) bidders randomly draw their bids in equilibrium, and type-(V_h, L)'s support is lower than type-(V_h, s_0)'s, while the latter is also lower than type-(V_h, H)'s. This is similar to the bidding rule of ex ante information acquisition, so is the reason behind.

4.3 Demand for Interim Information

We call π_I the *demand for interim information*, since for any informative cost c, the equilibrium is symmetric and unique.

The strategies in interim IA equilibrium share the same formulas as those in ex ante IA equilibrium, except for the indifference condition that determines the probability π_I. Obviously in interim IA setting, the type V_l bidder always gets zero surplus and has no incentive to acquire information, and the equation to pin down π_I only concerns the type-V_h bidder. Therefore, the multiplier p_h on both sides of (11) is eliminated, which yields (23). Again rearranging this equation yields the reverse demand for information in interim equilibrium, namely,

$$c = p_l p_h (V_h - V_l) \cdot \frac{p_l q(2q-1)}{p_l q + p_h(1-q)^2 \pi_I} \cdot \frac{(1-\pi_I)}{(1-p_h q \pi_I)}. \tag{24}$$

Parallel to the expression for π_E, we have the property that π_I decreases as the cost goes up, the same as that in ex ante setting. And moreover, the interim demand for information, π_I, is higher than the ex ante demand π_E when facing the same cost in a proper range. These results are summarized in the following corollary.

Corollary 2 *(Demand for Interim Information) The probability of interim IA, π_I, has the following properties:*

(i). For all given values $q \in (0.5, 1]$, π_I is continuous in informative cost c for $c \geqq 0$, and strictly decreasing for $c \in [0, p_l p_h (V_h - V_l)(2q-1)]$.

(ii). For all values $c \geqq 0$, π_I is (weakly) increasing in accuracy q.

(iii). $\pi_I > \pi_E$ for $c \in (0, p_l p_h (V_h - V_l)(2q-1)), q \in (0.5, 1]$.

We have known qualitatively how bidders submit their bids in equilibrium. Now we give a numeric example to illustrate their bidding rules and expected payoffs in equilibrium.

Example 1 *Let $V_h = 2$, $V_l = 1$, $q = 0.8$, $p_l = p_h = 0.5$, $c = 0.02$. Then Table 1 gives the probabilities of acquiring information at ex ante and interim stage, the endpoints of bidding supports and the corresponding expected payoffs (the formula of the expected payoff will be given in the next section). Figure 2 shows the mixing bid supports for this numeric example. Note that in standard first price auction without IA, the payoff of the bidder is 0.25, which is equal to $p_l p_h(V_h - V_l)$.*

Ex Ante		Interim	
π_E	0.7170	π_I	0.8572
δ_1	1.0346	β_1	1.0411
δ_2	1.2261	β_2	1.1453
δ_3	1.5812	β_3	1.6142
S^E	0.2760	S^I	0.2804
No IA			
γ	1.5		
S_0	0.25		

Table 1: Example of IA and Bidding

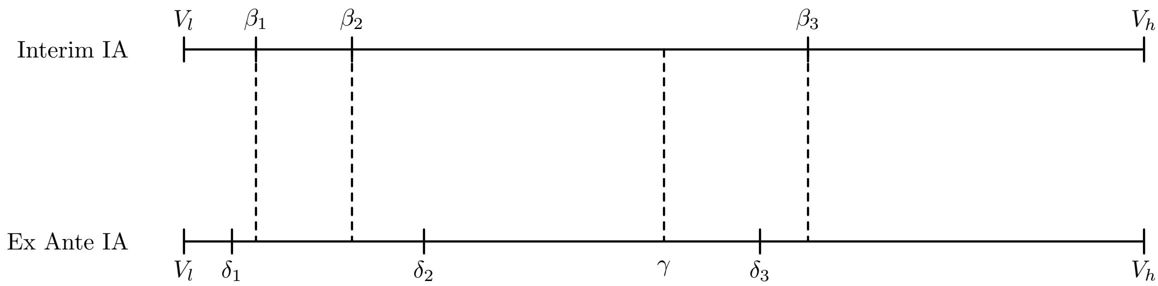

Figure 2: Bid Support

5 Discussion: Welfare Analysis and Comparative Statics

5.1 Ex Ante Information Acquisition

One central question that motivates our project is whether bidders benefit from acquiring information concerning opponent's valuations. The answer is affirmative. Information acquisition indeed improves the bidder's surplus, albeit it cannot improve the allocative efficiency (the efficient allocations have been obtained in standard FPA with the same primitive e). However, since the information is costly and the expected social value of the object is fixed, the seller necessarily raises less revenue. We will discuss how cost c and accuracy q impact bidder's surplus and seller's revenue as well as total social surplus in the following subsections.

5.1.1 Bidder's Surplus

Bidder's expected payoff in ex ante IA equilibrium has been given in the indifference condition (11), either left or right hand side specifies the net surplus. By substituting δ_1 into the right hand side of (11), we have a simple formula for bidder's expected payoff

$$S^E = \frac{p_l q + p_h \pi_E q(1-q)}{p_l q + p_h \pi_E (1-q)^2} \cdot p_l p_h (V_h - V_l), \tag{25}$$

which is strictly increasing in the probability of acquiring information, π_E when the signal is noisy but informative, i.e., $q \in (0.5, 1)$. The standard FPA with the same primitive e will give the bidder surplus $S_0 = p_l p_h (V_h - V_l)$, which can be obtained by setting $\pi_E = 0$ in (25). Note that the demand for information π_E is decreasing in informative cost c by Corollary 1, then we have the results summarized in the following proposition.

Proposition 3 *Bidder's net payoff in the ex ante IA auction game $< \mathcal{A}^E, G^E >$ with primitive e, S^E, has the following properties:*

(i). S^E exceeds the payoff in the standard FPA with primitive e, i.e., $S^E \geqq S_0$ and the inequality is strict for $q \in (0.5, 1)$ and $c < p_l p_h^2 (V_h - V_l)(2q-1)$.

(ii). In the presence of noisy but informative signal, namely, $q \in (0.5, 1)$, S^E is strictly decreasing in informative cost c for $c \leqq p_l p_h^2 (V_h - V_l)(2q-1)$.

The proposition says the competition between bidders is not so fierce as that of the standard FPA auctions. This is because by knowing more about the opponents' valuations, type-(V_h, L) and type-(V_h, s_0) bidders bid more conservatively, see the numerical Example 1. For instance, the type-(V_h, L) bidder submits her bid very closely to V_l. The type-(V_h, H) bidder seems to bid more aggressively, i.e. in Example 1, the upper bound of her bid support $\delta_3 = 1.5812$ is higher than $\gamma = 1.5$, the upper bound of type-V_h bidder's support in standard FPA. However, on average, bidders with valuation V_h bid less than they do in the standard FPA.

The informative cost c plays an important role in determining the bidder's expected surplus. To consider the interested comparative statics, suppose $q \in (0.5, 1)$. As the cost increases, bidder's incentive to acquire information reduces and π_E decreases. Therefore, on average the type-V_h bidder submits bid in a less pointed manner, which leads to less expected payoff.

The accuracy q has a disparate impact on bidder's payoff. An increment in accuracy has two effects. On one hand, a more accurate signal improves the bidder's updated belief and the bidder will bid more effectively if receiving the signal, which can increase the expected payoff. On the other hand, a more informative signal reduces the bidder's private information, and thus, increases the competitiveness and leads bidder's potential surplus to decrease. Theoretically, it is not clear which effect dominates the other at all possible values of q. However, for extreme cases, we can see how q affects bidder's surplus.

A very noisy signal provides bidders with no incentive to acquire information. It is easy to see that the upper bound of the cost, beyond which no bidder will choose to acquire information, $p_l p_h^2 (V_h - V_l)(2q-1)$ will be smaller than any given positive c if q stays sufficiently close to $\frac{1}{2}$. Therefore, $\pi_E = 0$ for q in some small neighborhood of $\frac{1}{2}$. As the informativeness of the signal improves, π_E will be positive, hence the expected surplus of bidder goes above S_0 in this corresponding range of q. On the other hand when the signal is completely accurate, i.e., $q = 1$, bidder's expected surplus falls back to S_0 by (25), which seems to be a little bit counterintuitive. Note that when $q = 1$, a type-(V_h, s) bidder cannot confront with a type-(V_h, L) opponent due to complete informativeness of the signal for all $s \in \{H, L, s_0\}$. This reduces the probability of winning against a relatively "weak" opponent, and therefore, leads to less expected surplus. When q is somewhat informative but noisy, we do not know theoretically how bidder's surplus changes as q increases. However, it seems that quantitatively the payoff is likely to increase first and then decrease as q goes up.

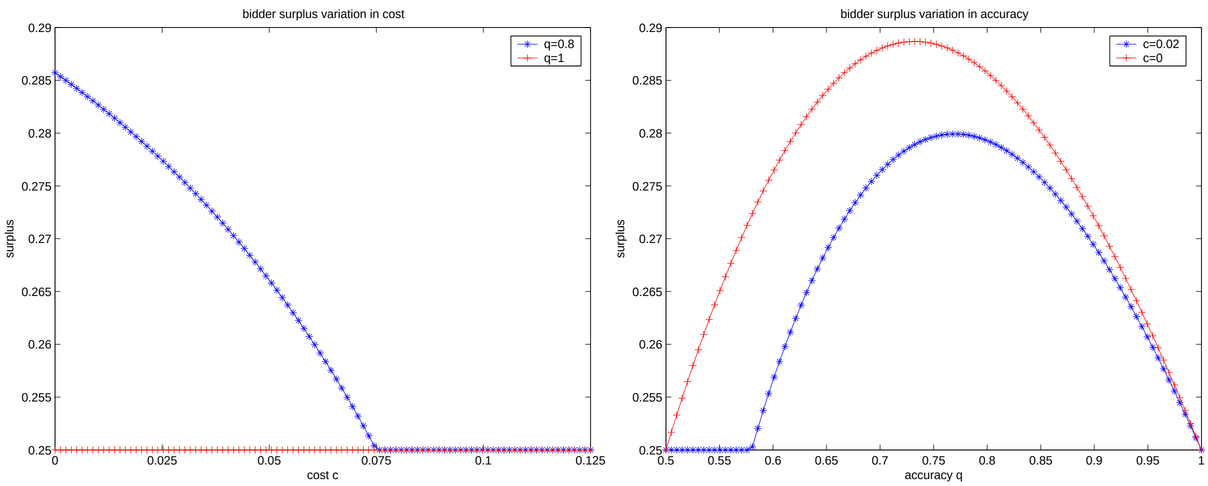

Figure 3: Cost and Accuracy's Impacts on Bidder's Payoff
Primitive: $V_h = 2,\ V_l = 1,\ p_l = p_h = \frac{1}{2}$.

Numerically, the comparative results can be seen in Figure 3, which confirms the comparative results for S^E. Note that $S^E = S_0$ for $q = 1$ regardless of the value of π_E, and $\pi_E = 1$ for $c = 0$

regardless of the value of q.

5.1.2 Total Surplus

In the unique equilibrium of ex ante information acquisition, the object is always optimally allocated, so the total ex ante expected social welfare is equal to the expected value of $\max\{v_1, v_2\}$ minus total expected informative cost. Since $\mathbb{E}[\max\{v_1, v_2\}] = p_l^2 V_l + (1-p_l^2)V_h$ and each bidder incurs expected informative cost $\pi_E c$, the total expected surplus then would be

$$TS^E = p_l^2 V_l + (1-p_l^2)V_h - 2\pi_E c. \tag{26}$$

Note that if there is no information acquisition, the expected total social surplus would be $TS_0 = p_l^2 V_l + (1-p_l^2)V_h$ by setting $\pi_E = 0$ in (26). We can see the expected information cost $2\pi_E c$, which can be regarded as an information rent, becomes the deadweight loss. Hence, the ex ante IA equilibrium outcome is inefficient[9] if bidders incur cost to acquire information, which differs from the result in Fang and Morris' binary value model that the exogenous IA equilibrium outcome is always efficient due to zero-cost setting, which can be regarded as "first best" outcome. Then the equilibrium outcome in our ex ante IA model may be regarded as "second best" outcome, compared to the ideal case in Fang and Morris' model.

The impacts of cost and accuracy on total surplus is then summarized in the following proposition:

Proposition 4 *The expected total social surplus in the ex ante IA auction game $< \mathcal{A}^E, G^E >$ with primitive e, TS^E, has the following properties:*

(i). TS^E is less than the social surplus in the standard FPA with the primitive e, i.e., $TS^E \leqq TS_0$ and the inequality is strict for $q \in (0.5, 1)$ and $0 < c < p_l p_h^2 (V_h - V_l)(2q-1)$.

(ii). Suppose the accuracy $q \in (0.5, 1]$ is given, then TS^E is decreasing in cost c for $c \in [0, c_{TS}^E(q))$ and increasing in cost c for $c \in [c_{TS}^E(q), p_l p_h^2(V_h - V_l)(2q-1)]$, where $c_{TS}^E(q) \in (0, p_l p_h^2 (V_h - V_l)(2q-1))$ is a constant dependent on q.

(iii). Suppose the cost $c \in (0, p_l p_h^2(V_h - V_l))$ is given, then TS^E is weakly decreasing in accuracy $q \in (0.5, 1]$.

We know that π_E is decreasing when cost c goes up. It turns out that their product, the expected cost $\pi_E c$ first increases when cost is moderate and then decreases as cost becomes

[9] The efficiency here refers to whether the social surplus is at its maximum level, i.e., $\mathbb{E}[\max\{v_1, v_2\}]$, not to allocative efficiency.

large. Therefore, the total social surplus has a “U” shape response to cost. Provided the cost of information acquisition is fixed, a more informative signal will increase the IA probability π_E, hence the resulted social surplus decreases due to more expected cost. In other words, more accurate signal actually hurts social benefit since the incurred information cost is deadweight loss.

Figure 4 shows how social surplus reacts to the variation in cost c and to the variation in accuracy q, respectively, using the same primitive values as in previous numeric simulations. We can see no IA is the best outcome for the whole auction economy, and more accurate signal makes the total welfare worse-off.

The above analysis implies that in realistic auction environment with information acquisition, the efficient outcome is not generally achievable due to dissipative information cost. One policy implication would be that information acquisition should be prevented to maximize the social surplus. This can be done by means of raising the information cost or reducing the accuracy of the signal or both. Or if possible, the central planner can directly control the IA choices to ease social welfare loss.

However, when the cost is not a loss and is actually transferred to some party who provides information, then information acquisition has no impact on social surplus. We will discuss it in more detail in section 6.1.

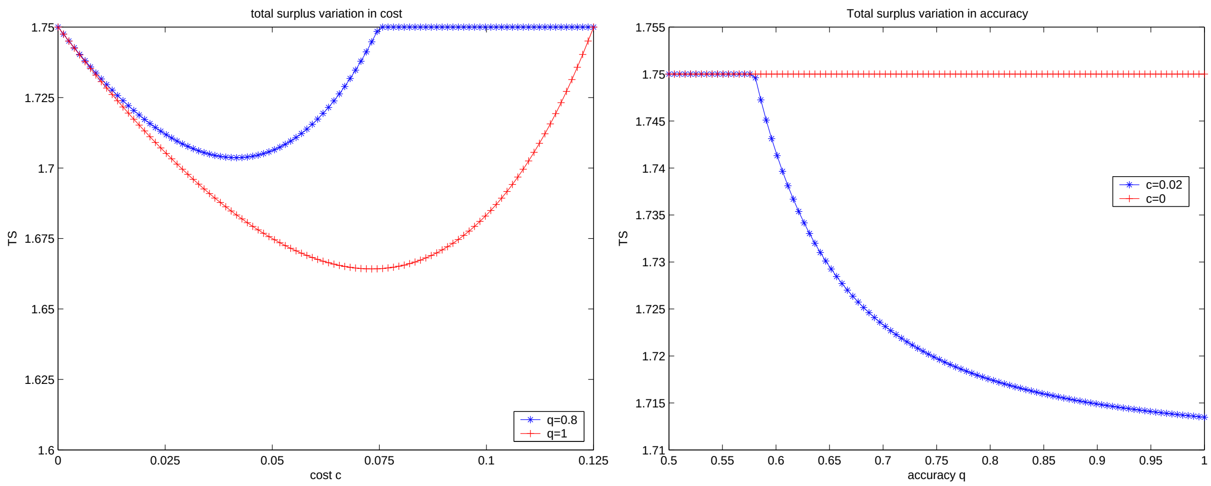

Figure 4: Cost and Accuracy's Impacts on Total Surplus
Primitive: $V_h = 2,\ V_l = 1,\ p_l = p_h = \frac{1}{2}$.

5.1.3 Seller's Revenue

The total social surplus contains two parts, the revenue of the seller and the surplus of the bidder. Therefore, seller's expected revenue χ^E equates to the expected total surplus minus two bidders' expected payoffs, namely,

$$\chi^E = [p_l^2 V_l + (1 - p_l^2) V_h] - 2\pi_E c - 2S^E. \tag{27}$$

Immediately by setting $\pi_E = 0$ in (27), we know the revenue would be $\chi_0 = [p_l^2 V_l + (1 - p_l^2) V_h] - 2S_0 = (1 - p_h^2) V_l + p_h^2 V_h$ in standard FPA. The equation (27) also tells us that the seller effectively bears the informative cost that is incurred by bidders. Note that $S^E(q, \pi_E) \geqq S_0$, therefore, seller always gets revenue less than χ_0 in ex ante IA auction. Approximately, bidder's surplus and seller's revenue change in opposite directions. To summarize, we have the following comparative statics for seller's revenue.

Proposition 5 *Seller's revenue in the ex ante IA auction game* $< \mathcal{A}^E, G^E >$ *with primitive* e, χ^E, *has the following properties:*

(i). χ^E *is less than* χ_0, *seller's revenue in standard FPA with primitive* e, *and the inequality is strict for* $c < p_l p_h^2 (V_h - V_l)(2q - 1)$ *and* $q \in (0.5, 1)$.

(ii). Suppose the signal is noisy but informative, namely, $q \in (0.5, 1)$. *Then there exists some constant* $c^E(q) \in (0, p_l p_h^2 (V_h - V_l)(2q - 1))$ *dependent on* q, *such that* χ^E *is decreasing in informative cost* c *for* $c \in [0, c^E(q))$ *and increasing in cost* c *for* $c \in [c^E(q), p_l p_h^2 (V_h - V_l)(2q - 1)]$.

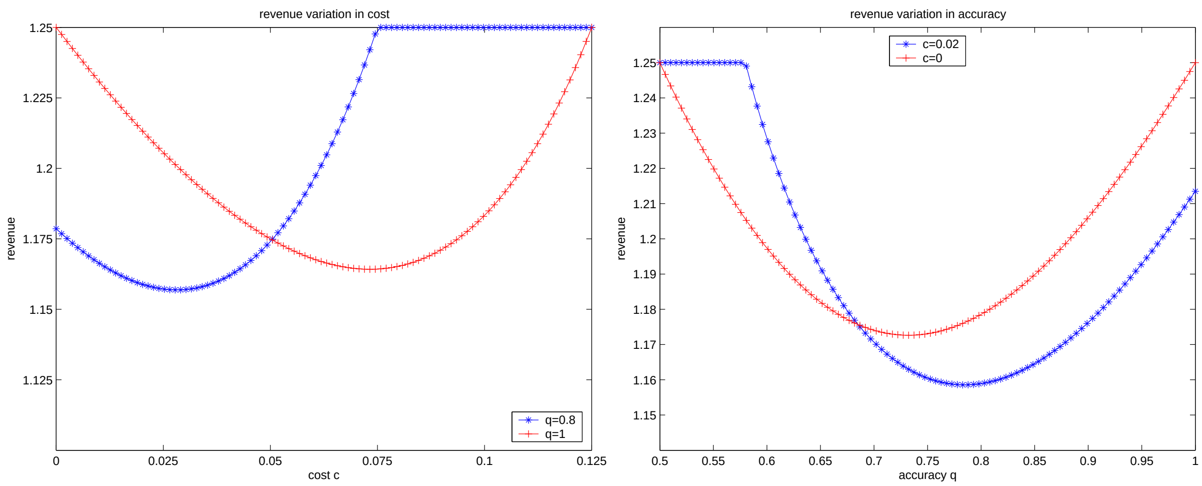

Figure 5: Cost and Accuracy's Impacts on Seller's Revenue
Primitive: $V_h = 2,\ V_l = 1,\ p_l = p_h = \frac{1}{2}$.

Figure 5 shows the behaviors of revenue in response to the cost c and the accuracy q, both of which demonstrate a "U" shape response, which verifies the properties of χ^E stated in the Proposition 5.

Obviously, seller's revenue achieves its maximum χ_0 only when bidders do not acquire information ($\pi_E = 0$, or $c \geqq p_l p_h^2 (V_h - V_l)(2q-1)$) or the signal is costless and completely informative ($q = 1$ and $c = 0$), according to the formula (27). Due to efficient allocation, bidders' benefits would necessarily conflict with seller's interest, which implies the seller would definitely prefer no information acquisition. Then one policy implication would be that to raise more revenue, the seller should prevent bidders from acquiring information, or induce either very noisy signal or completely accurate signal whenever possible.

5.2 Surplus and Revenue in Interim IA Equilibrium

5.2.1 Bidder's Surplus

Similar to ex ante IA equilibrium, bidder's expected payoff in interim IA equilibrium has been given in either side of the indifference condition (23). By substituting β_1 into the right hand side of (23), we have a simple formula for bidder's expected payoff

$$S^I = \frac{p_l q + p_h \pi_I q(1-q)}{p_l q + p_h \pi_I (1-q)^2} \cdot p_l p_h (V_h - V_l). \tag{28}$$

Hence we have the comparative results summarized in the following proposition.

Proposition 6 *In the presence of noisy signal, i.e. $q \in (0.5, 1)$, and for $c \leqq p_l p_h (V_h - V_l)(2q-1)$, we have*

(i). Bidder's net payoff S^I is decreasing in informative cost c, and hence exceeds its counterpart payoff, S_0 from standard FPA without IA;

(ii). $S^I \geqq S^E$, and the inequality is strict for $c \in (0, p_l p_h (V_h - V_l)(2q-1))$ and $q \in (0.5, 1)$, provided the auctions $< \mathcal{A}^E, G^E >$ and $< \mathcal{A}^I, G^I >$ have the same primitive $e \in \mathscr{E}$.

This proposition is parallel to Proposition 3, and it says the impact of cost on bidder's payoff is similar to that of the ex ante IA case. When the signal is more costly, the type-V_h bidder would be less likely to acquire information, and bidder's expected payoff goes down. Also the accuracy of the signal has similar influence on bidder's payoff as in ex ante case.

The information acquisition choice in interim stage becomes valuation-type-specific and turns to be more efficient in terms of the utilization of the information. Hence bidders get higher ex ante expected payoffs in auction game $< \mathcal{A}^I, G^I >$ than in $< \mathcal{A}^E, G^E >$.

5.2.2 Total Surplus

In equilibrium of interim information acquisition, the object is optimally allocated, hence the total expected social welfare equates to the total expected surplus minus total expected informative cost

$$TS^I = p_l^2 V_l + (1 - p_l^2)V_h - 2p_h \pi_I c. \tag{29}$$

Again similar to the ex ante IA case, the information cost $2p_h\pi_I c$ turns to the deadweight loss, and the interim IA equilibrium outcome is not efficient, either.

Immediately, we have the following comparative results:

Proposition 7 *The expected total social surplus in the interim IA auction game* $< \mathcal{A}^I, G^I >$ *with primitive e,* TS^I*, has the following properties:*

(i). TS^I *is less than the social surplus in the standard FPA with the primitive e, i.e.,* $TS^I \leqq TS_0$ *and the inequality is strict for* $q \in (0.5, 1)$ *and* $0 < c < p_l p_h^2 (V_h - V_l)(2q - 1)$.

(ii). Suppose the accuracy $q \in (0.5, 1]$ *is given, then* TS^I *is decreasing in cost c for* $c \in [0, c_{TS}^I(q))$ *and increasing in cost c for* $c \in [c_{TS}^I(q), p_l p_h^2 (V_h - V_l)(2q-1)]$*, where* $c_{TS}^I(q) \in (0, p_l p_h^2 (V_h - V_l)(2q - 1))$ *is a constant dependent on q.*

(iii). Suppose the cost $c \in (0, p_l p_h^2 (V_h - V_l))$ *is given, then* TS^I *is weakly decreasing in accuracy* $q \in (0.5, 1]$.

This proposition is parallel to Proposition 4, and the comparative results are similar in two different IA settings. Unlike the bidder's surplus, we cannot unambiguously rank the total surpluses TS^E and TS^I. This is because the expected cost $2\pi_E c$ in ex ante IA setting can be larger or smaller than the expected cost $2p_h\pi_I c$ in interim IA setting.

5.2.3 Seller's Revenue

Since total surplus includes seller's revenue and bidders' surpluses, then the revenue has the following expression,

$$\chi^I = p_l^2 V_l + (1 - p_l^2)V_h - 2p_h \pi_I c - 2S^I. \tag{30}$$

Then we have the following results.

Proposition 8 *Seller's revenue in interim IA auction game* $< \mathcal{A}^I, G^I >$ *with primitive e,* χ^I*, has the following properties:*

(i). χ^I *is less than* χ_0*, seller's revenue in standard FPA with primitive e, and the inequality is strict for* $c < p_l p_h (V_h - V_l)(2q - 1)$ *and* $q \in (0.5, 1)$.

(ii). *Suppose the signal is noisy but informative, namely,* $q \in (0.5, 1)$. *Then there exists some constant* $c^I(q) \in (0, p_l p_h(V_h - V_l)(2q-1))$ *dependent on* q, *such that* χ^I *is decreasing in informative cost* c *for* $c \in [0, c^I(q))$ *and increasing in cost* c *for* $c \in [c^I(q), p_l p_h(V_h - V_l)(2q-1)]$.

Quantitatively, bidder's expected surplus and seller's revenue have similar responses to the cost and the accuracy of the signal as those in ex ante equilibrium, and the only difference between ex ante and interim environments is the magnitude. We cannot generally tell whether seller gets more revenue in the ex ante IA auction $< \mathcal{A}^E, G^E >$ or in the interim IA auction $< \mathcal{A}^I, G^I >$ despite the unambiguous ranking for bidders' surpluses in two different IA settings because it is not clear in which setting the expected cost is higher.

5.3 Comparison Across Auction Formats

It is obvious that the object is always efficiently allocated in our first price IA auction model since bidders have binary values. As we have discussed at the very beginning, information acquisition plays no role in formulating bidders' strategies in second price auction (SPA) since each bidder has weakly dominant strategy to bid truthfully, but it remains worth making comparisons across FPA and SPA formats because the revenue equivalence still holds between these auctions without any type of information acquisition.[10]

Consider SPA in our basic setting with primitive $e \in \mathscr{E}$. The seller gets V_h if and only if both bidders have values V_h, which occurs with probability p_h^2, otherwise the seller receives V_l. Therefore in SPA, seller's expected surplus, denoted by χ^{SPA}, would be

$$\chi^{\text{SPA}} = (1 - p_h^2)V_l + p_h^2 V_h. \tag{31}$$

On the other hand, each bidder obtains positive surplus, i.e. $(V_h - V_l)$ if and only if her own value is V_h and her opponent has value V_l, which occurs with probability $p_l p_h$. Then bidders' expected surplus from SPA, denoted by S^{SPA}, is

$$S^{\text{SPA}} = p_l p_h (V_h - V_l). \tag{32}$$

This yields the following results:

Proposition 9 *Suppose* $0 < c < p_l p_h^2(V_h - V_l)(2q-1)$, $q \in (0.5, 1)$. *Provided (FPA) IA games* $< \mathcal{A}^E, G^E >$, $< \mathcal{A}^I, G^I >$, *and SPA are permitted in the basic setting* $e \in \mathscr{E}$, *then we have*

[10]This can be seen in the Appendix of Fang and Morris (2006).

1. *Bidder's expected surplus from FPA is strictly higher than that from SPA, regardless of the stage at which the IA choice is made, i.e.,* $S^E > S^{SPA}$ *and* $S^I > S^{SPA}$.

2. *Seller gets strictly less expected revenue from FPA than that from SPA, no matter at which stage bidder can acquire information, i.e.,* $\chi^E < \chi^{SPA}$ *and* $\chi^I < \chi^{SPA}$.

The first item could follow from Proposition 3 and Proposition 6, since the revenue equivalence still holds with discrete values across standard FPA and SPA without information acquisition, i.e., $S^{SPA} = S_0$, and IA plays no role in formulating strategies in SPA.

The revenue ranking is unambiguously clear across FPA and SPA for our binary value model.[11] However, this ranking does not hold generally for endogenous IA. For example, when valuation has continuous support, then it is possible that seller gets more revenue in FPA than in SPA.

It is obvious that bidders have strong incentives to acquire information about their opponents' valuations intentionally in FPA if it is permitted, but not in SPA. The other fact is that the auctioneer is worse-off in FPA, if such an information acquisition occurs, which differs from the result that revenue increases associated with traditional information acquisition. The natural policy implication would be that the seller may need to deter bidders' potential information acquisition to maximize her revenue if bidders' private values are independent.

6 Some Variants

6.1 Third Party Charges the Informative Cost

In our basic setting, we do not model where the informative signal comes from, i.e., who provides the information. Therefore, in our welfare analysis, the informative cost becomes the deadweight loss of total social surplus. Also, we leave it open for pricing the signal. It seems that the two issues can be resolved by introducing the third party who actually collects information and sells it to bidders.

Suppose there exists such a third party at ex ante stage(the case is similar at interim stage). The party has the access to collect noisy information concerning bidders' valuations without incurring any cost, and aims to maximize its total revenue from providing information. Therefore,

[11] Note that here the revenue equivalence fails. It's true that allocations do not pin down the revenue in discrete type settings, which might contribute to the failure of revenue equivalence in standard auctions. However, Fang and Morris (2006) already show that in standard auctions without information acquisition, the revenue equivalence holds in discrete setting. It follows that the realized asymmetric information structure from IA contributes to the failure of the RET; besides, the informative cost c also plays a role here.

the third party has objective function

$$\tau^E(c) = 2\pi_E c. \tag{33}$$

By Proposition 1, the possible informative cost charged by this party is the interval $[0, p_l p_h^2 (V_h - V_l)(2q-1)]$. If the accuracy of the transmitted signal q cannot be controlled by this party, then the optimal cost for this party would be $c_{TS}^E(q)$, which is the constant given in Proposition 4 item (ii). The interesting thing is that if the accuracy can be chosen by this party, then definitely the party will choose optimal accuracy $q^* = 1$ to maximize the expected cost $2\pi_E c$ because π_E is increasing in q by Corollary 1. However, bidders cannot benefit from this completely accurate signal since they can just get expected surplus $S^E = S_0$. This optimal choice may favor the seller to some extent, for bidders' having less surplus means that seller may get more revenue (increment in expected cost will reduce revenue at the same time).

One key point is that the expected total surplus now is always a constant $TS = TS_0$ since the expected cost – the deadweight loss – now turns to be the benefit of the third party. Therefore, the IA auction game $< \mathcal{A}^E, G^E >$ becomes one mechanism that redistributes the total surplus amongst bidders, seller and the information provider. Moreover, part of seller's revenue is redistributed to bidders and the information provider compared to the standard FPA with the same primitive, which can be easily seen from the formula (27).

6.2 Observable Information Acquisition

This paper considers covert information acquisition on opponent's valuation, and it is crucial to maintain the assumption that the choice of acquiring information is unobservable to the opponent. It will result in two problems without this assumption. We consider the interim IA setting. First, the signal structure is not inherently consistent. This is a model with binary valuations, hence only the type-V_h bidder has incentive to acquire information. If this action is public information, then it is a perfect signal that tells bidder j $v_i = V_h$ when i acquires information. Also bidder j could be a type-V_h bidder who already acquires information and receives a signal s_j, then s_j is meaningless for the bidder, thus why should the bidder incur a cost to acquire information? Second, there could be no equilibrium in some subcases. Suppose bidder i has valuation V_h and acquires information and bidder j does not, and this is common knowledge. In this case, no equilibrium bidding strategy exists with fair tie breaking rule.[12] Hence observable information acquisition is undesirable for our modeling.

[12]This case is the same as the Example 4.4 on pp.53 of Krishna's book, which is another version of the example in Maskin and Riley (2000). The equilibrium could be restored but we need Vickrey tie breaking rule.

6.3 More Realizations

Besides the binary values, more points of valuation support could be added in. As Fang and Morris (2006) have observed: it does not admit any equilibrium for exogenous information acquisition with generic parameters even in binary valuation support model. Thus for more points, the equilibrium analysis would also be problematic. As for continuous valuation support, the formation of bidding rule with multiple arguments turns to be a rather involved problem, which is the essential question with multidimension. This is an open question for future research.

6.4 Beyond Private Values

Admittedly, we specify an independent private value setting, which seems to be restrictive in some sense. This endogenous IA specification can be extended to other non-private value environment. For example, it turns out that in endogenous IA first-price auction with interdependent discrete valuations, bidder can also get higher payoff than that of standard FPA without IA, which is the environment we haven't considered here but are working on.

7 Conclusion

This paper mainly investigates bidders' covert information acquisition on opponents' valuations in first price sealed-bid auctions with independent private values, which leads to bidimensional strategy space and bidimensional type space. We present two different specifications: ex ante information acquisition and interim information acquisition. We find that: (i) bidder's expected payoff exceeds that of the counterpart standard FPA when informative cost is small and the signal is informative and noisy; (ii) the auctioned object is always efficiently allocated due to the settings of binary values; (iii) in all cases in which bidder acquires information with positive probability, the auctioneer's expected revenue is lower than that of the standard auction, which contradicts the results in the current information acquisition literature; (iv) the incurred informative cost becomes the deadweight loss in total social surplus and the equilibrium outcome is inefficient. The natural implication of our analysis would be that seller needs to prevent bidders from acquiring information to raise more revenue. Also from the social planner's point of view, information acquisition should be prevented to maximize social welfare. We also analyze how the accuracy and the informative cost impact on bidders' expected surplus, seller's revenue and total surplus. Some possible variations of the model are discussed at last.

Our paper takes the first step to explore the properties of endogenous information acquisition on opponents' valuations in the context of independent private value auctions. We do find bidders have incentives to take this IA action. To see the robustness of the results, continuous support necessitates exploration. It seems that the difficulty rests with the multidimensional bidding rules. Moreover, the probability of acquiring information is probably a function of bidder's valuation. These are open questions for future exploration.

8 Appdendix

Proof of Lemma 1: For $t_i \in \{V_l\} \times \{L, H, s_0\}$, let type-$t_i$ bidder i's bidding support be either $[b_1^{t_i}, b_2^{t_i}]$ with $b_1^{t_i} \leqq b_2^{t_i} \leqq V_l$ or $(b_1^{t_i}, b_2^{t_i}]$ with $b_1^{t_i} < b_2^{t_i} \leqq V_l$(the upper bound of the support interval may be open, but this is not important for the argument). Let $t \in \arg\min_{t_1 \in \{V_l\} \times \{L,H,s_0\}} b_1^{t_1}$ and $t' \in \arg\min_{t_2 \in \{V_l\} \times \{L,H,s_0\}} b_1^{t_2}$.

We first show that type-t bidder 1 and type-t' bidder 2 should bid in pure strategies. It is obvious that $b_1^t = b_1^{t'} = b_1$ since b_1^t and $b_1^{t'}$ are the possible smallest bids for bidders 1 and 2, respectively. There are four possible cases that can be ruled out in equilibrium.

Case 1: type-t bidder 1 has support $(b_1, b_2^t]$ and type-t' bidder 2 has support $(b_1, b_2^{t'}]$ where $b_1 < b_2^t$ and $b_1 < b_2^{t'}$. We can see that the bid $b = \frac{b_1 + \min\{b_2^t,\, b_2^{t'}\}}{2}$ will yield both type-t bidder 1 and type-t' bidder 2 positive surplus. However, bids close to b_1 will win with probability almost zero, hence both bidder 1 and type-t' bidder 2's surpluses will approach zero by submitting a bid close to b_1. This is a contradiction with the indifference condition given by mixed strategy, so this pattern of the supports cannot be the case.

Case 2: type-t bidder 1 has support $(b_1, b_2^t]$ and type-t' bidder 2 has support $[b_1, b_2^{t'}]$ where $b_1 < b_2^t$ and $b_1 < b_2^{t'}$, or type-t bidder 1 has support $[b_1, b_2^t]$ and type-t' bidder 2 has support $(b_1, b_2^{t'}]$ where $b_1 < b_2^t$ and $b_1 < b_2^{t'}$. Since the two subcases are symmetric, consider the former one. It is easy to see a bid b_1 yields bidder 2 exactly zero surplus due to zero winning probability. However, a bid $b = \frac{b_1 + \min\{b_2^t,\, b_2^{t'}\}}{2}$ yields bidder 2 strictly positive expected surplus, a contradiction with the indifference condition. This pattern cannot be true either.

Case 3: type-t bidder 1 has support $[b_1, b_2^t]$ and type-t' bidder 2 has support $[b_1, b_2^{t'}]$ where either $b_1 < b_2^t$ and $b_1 < b_2^{t'}$, or $b_1 = b_2^t$ and $b_1 < b_2^{t'}$, or $b_1 = b_2^{t'}$ and $b_1 < b_2^t$. Consider the subcase where $b_1 < b_2^t$. A bid $\frac{b_1 + b_2^t}{2}$ will yield type-t bidder 1 positive surplus. Hence by indifference condition, a bid b_1 will give type-t bidder 1 positive surplus, which requires her opponent—bidder 2 to submit a bid b_1 with positive probability so that type-t bidder 1 can win against bidder 2 with positive probability by submitting b_1. However, then, if type-t bidder 1 submits a bid slightly higher than b_1, this bid will yield a jump in winning probability against bidder 2, hence a jump in expected surplus of type-t bidder 1. Again a contradiction with the requirement of mixed strategy. The subcase where $b_1 < b_2^{t'}$ follows the same argument.

Case 4: type-t bidder 1 has support $(b_1, b_2^t]$ with $b_1 < b_2^t$, type-t' bidder 2 bids b_1 with probability 1; or type-t' bidder 2 has support $(b_1, b_2^{t'}]$ with $b_1 < b_2^{t'}$, type-t bidder 1 bids b_1 with probability 1. By symmetry, it suffices to consider the former subcase. It is obvious that type-t' bidder 2's strategy is not optimal. The bid b_1 wins against none of bidder 1's bids, hence yields bidder 2 zero surplus. By submitting a bid $\frac{b_1+b_2^t}{2}$, type-t' bidder 2 gets strictly positive expected surplus, which is a profitable deviation.

Since all those cases are excluded from equilibrium, we conclude that it must be true in equilibrium that $b_1 = b_2^t = b_2^{t'}$. In other words, type-t bidder 1 and type-t' bidder 2 bid in pure strategies.

We then show that $b_1 = V_l$. Suppose $b_1 < V_l$, then either type-t bidder 1 or type-t' bidder 2 can deviate by submitting a slightly higher bid $b_1 + \epsilon$ with probability 1, which will be a profitable deviation by choosing ϵ arbitrarily close to zero. A contradiction.

Note that b_1 is the possible lowest bid for all types $t_i \in \{V_l\} \times \{L, H, s_0\}$, $i = 1, 2$ and those bidders will never bid more than their valuation V_l. The proof is complete.□

Proof of Proposition 1: We take several steps to complete this argument. First we verify that the provided strategies in the proposition actually constitute an equilibrium. Then we show that there is no other symmetric equilibrium. Last we argue that there is no asymmetric equilibrium either.

One point we would like to clarify before the formal proof is that in part (i) for $q \in (0, 1)$, when q approaches 1 from below, the limit of type-(V_h, L)'s bid support $(V_l, \delta_1]$ would be an empty set due to $\lim_{p\to 1} \delta_1 = V_l$. Hence type-$(V_h, L)$ bidder's best bidding strategy doesn't exist given all other types' bidding strategies. That's why the tie-breaking rule needs to switch at $q = 1$. Under Vickrey tie-breaking rule, type-(V_h, L)'s bid support could be closed interval $[V_l, \delta_1]$(the limit of this interval is singleton $\{V_l\}$ as $p \to 1$), and any other type bidder's strategy does not change. The following argument considers the setting for $q \in (0, 1)$, and the same argument can go through for $q = 1$ (possibly with some slight modification) by replacing type-(V_h, L)'s bid support $(V_l, \delta_1]$ with closed interval $[V_l, \delta_1]$.

The argument follows several intermediate claims:

Claim 1. *Given that the bidder's opponent follows the provided strategy in the proposition, the bidder's best response is the same strategy given in the proposition.*

Note that by Lemma 1, it suffices to check the strategies of the bidders with valuation V_h. We just need to verify that the bidder gets exactly the same expected payoff if she submits any bid drawn from the support of provided mixed strategy, and that the bidder cannot obtain more expected payoff by deviation. In other words, the provided strategy is an equilibrium. Since it is straightforward, the verification procedure is omitted.

Claim 2. *For $c < p_l p_h^2(V_h - V_l)(2q-1)$, there exists no type of symmetric equilibrium other than the one in part (i) of the proposition.*

The argument takes 4 steps.

Step 1: We show that in any symmetric equilibrium, both bidders would choose to acquire information with positive probability. In other words, we only need to show the case where no bidder chooses to acquire information is not an equilibrium. Suppose instead, this case can be an equilibrium. Then it reduces to the standard FPA equilibrium strategy given in part (ii) of the proposition; that is,

- None of the bidders incurs any cost to receive any signal.
- $b_i(V_l) = V_l$.
- Type-V_h bidder mixes over $(V_l, \gamma]$ according to cumulative distribution function $\bar{J}(\cdot)$ specified in (12).

If two bidders follow this strategy, then their surplus under this strategy profile is $p_l p_h(V_h - V_l)$. Now suppose bidder 1 follows the above strategy. Bidder 2 acquires information and adopts the strategy as follows:

- She acquires information in the first stage.
- $b_i(V_l) = V_l$, $b(V_h, L) = V_l + \epsilon$ and $b(V_h, L) = \gamma$.

Her payoff will be given by

$$\begin{aligned}
&p_h\{[p_l q + p_h(1-q)\bar{J}(V_l+\epsilon)](V_h - V_l - \epsilon) + [p_l(1-q) + p_h q](V_h - \gamma)\} - c \\
>&p_h\{p_l q(V_h - V_l - \epsilon) + [p_l(1-q) + p_h q]\cdot(V_h - \gamma)\} - c \qquad (34)\\
=&p_l p_h(V_h - V_l) + p_l p_h^2(2q-1)(V_h - V_l) - c - p_l p_h q\epsilon.
\end{aligned}$$

Since $c < p_l p_h^2(V_h - V_l)(2q-1)$, this deviation is profitable by choosing ϵ arbitrarily close to zero. Hence, it cannot be an equilibrium.

Step 2: We show that in any symmetric equilibria type-(V_h, s_0), type-(V_h, L), and type-(V_h, H) bidders should bid in mixed strategies. We show it by way of contradiction.

Suppose the statement is not true, i.e., a type-(V_h, s) $s \in \{s_0, L, H\}$ bidder i bids a fixed amount $b^* < V_h$ with probability 1. Then the same type-(V_h, s) bidder $j (j \neq i)$ can deviate to make more surplus by bidding $b^* + \epsilon$, where ϵ is arbitrarily close to zero. This deviation will lead to a positive jump in type-(V_h, s) bidder j's winning probability, and hence is profitable.

Step 3: We prove that in any symmetric mixed strategy equilibria, the supports of $J^l(\cdot)$ and $J^{s_0}(\cdot)$ and the supports of $J^{s_0}(\cdot)$ and $J^h(\cdot)$ are contiguous and non-overlapping. Note that the contiguity of supports follows from the same ϵ-deviation argument.

Now suppose that the supports of $J^l(\cdot)$ and $J^{s_0}(\cdot)$ overlap over an interval $[b_1, b_2]$ where $b_1 < b_2$. Mixed strategy requires that the expected surplus be constant for both type-(V_h, L) and type-(V_h, s_0) bidders from any bid $b \in [b_1, b_2]$. Hence we have that the following equations hold for some constants $C(V_h, s_0)$ and $C(V_h, l)$:

$$(V_h - b)\big[p_l + p_h\pi_E(1-q)J^l(b) + p_h(1-\pi_E)J^{s_0}(b)\big] = C(V_h, s_0), \tag{35}$$

$$(V_h - b)\Big[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\cdot\pi_E(1-q)J^l(b)}{p_l q + p_h(1-q)} + \frac{p_h(1-q)(1-\pi_E)J^{s_0}(b)}{p_l q + p_h(1-q)}\Big] = C(V_h, l). \tag{36}$$

Multiplying equation (35) by $(1-q)$ and equation (36) by $-[p_l q + p_h(1-q)]$, and summing up, we have

$$(V_h - b)p_l(1-2q) = (1-q)C(V_h, s_0) - [p_l q + p_h(1-q)]C(V_h, l). \tag{37}$$

Since the right hand side of equation (37) is a constant, this equation cannot hold for any value of bid $b \in [b_1, b_2]$ for $q > \frac{1}{2}$. Therefore, the supports for $J^{s_0}(\cdot)$ and $J^l(\cdot)$ are non-overlapping in the symmetric equilibrium.

By the same method, if the supports of $J^{s_0}(\cdot)$ and $J^h(\cdot)$ overlap over an interval $[b_1', b_2']$ where $b_1' < b_2'$, then for some constants $\hat{C}(V_h, s_0)$ and $\hat{C}(V_h, H)$, we have

$$(V_h - b)\big[p_l + p_h\pi_E(1-q) + p_h(1-\pi_E)J^{s_0}(b) + p_h\pi_E q J^h(b)\big] = \hat{C}(V_h, s_0), \tag{38}$$

$$\begin{aligned}(V_h - b)\Big[&\frac{p_l(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q\cdot\pi_E(1-q)}{p_l(1-q) + p_h q} \\ &+ \frac{p_h q(1-\pi_E)J^{s_0}(b)}{p_l(1-q) + p_h q} + \frac{p_h q \pi_E J^h(b)}{p_l(1-q) + p_h q}\Big] = \hat{C}(V_h, h).\end{aligned} \tag{39}$$

Multiplying equation (38) by q and equation (39) by $-[p_l(1-q) + p_h q]$, and summing up, we have

$$(V_h - b)p_l(2q-1) = q\hat{C}(V_h, s_0) - [p_l(1-q) + p_h q]\hat{C}(V_h, h). \tag{40}$$

Again the left hand side of equation (40) is a constant. This equation cannot hold for any value of bid $b \in [b_1', b_2']$ for $q > \frac{1}{2}$. Therefore, the supports for $J^{s_0}(\cdot)$ and $J^h(\cdot)$ are non-overlapping in the symmetric equilibrium.

Step 4: Now we show that the support of the type-(V_h, L) bidder should be lower than that of the type-(V_h, s_0) bidder, and the support of the type-(V_h, s_0) bidder should be lower than that of the type-(V_h, H) bidder. Since we have three possible type bidders who use mixed strategies, there exist six possible rank permutations for the three corresponding supports. We should rule out all the other five cases[13]. Now let $[V_l, \tilde{b}_1]$ be the support of the type-(V_h, s_0) bidder's mixed strategy with cumulative distribution function (cdf) $\tilde{J}^{s_0}(\cdot)$. Let $[\tilde{b}_1, \tilde{b}_2]$ be the support of the type-(V_h, L) bidder's mixed strategy with cdf $\tilde{J}^l(\cdot)$, and let $[\tilde{b}_2, \tilde{b}_3]$ be the support of the type-(V_h, H) bidder's mixed strategy with cdf $\tilde{J}^h(\cdot)$. Since type-(V_h, L) bidders randomize on the interval $[\tilde{b}_1, \tilde{b}_2]$, their expected surpluses should be constants for any bids drawn from the interval; that is

$$\begin{aligned}(V_h - b)&\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)(1-\tilde{\pi}_E)}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\tilde{\pi}_E(1-q)}{p_l q + p_h(1-q)}\tilde{J}^l(b)\right]\\ &= \frac{p_l q + p_h(1-q)(1-\tilde{\pi}_E)}{p_l q + p_h(1-q)}(V_h - \tilde{b}_1).\end{aligned}$$

From this equation we can solve for $\tilde{J}^l(b)$:

$$\tilde{J}^l(b) = \frac{p_l q + p_h(1-q)(1-\tilde{\pi}_E)}{p_h(1-q)^2\tilde{\pi}_E}\frac{b - \tilde{b}_1}{V_h - b}. \tag{41}$$

Assume bidder 1 follows the provided strategy. That is, the type-(V_h, L) bidder 1 randomizes over $[\tilde{b}_1, \tilde{b}_2]$ according to cdf $\tilde{J}^l(\cdot)$, the type-(V_h, s_0) bidder 1 randomizes over $[V_l, \tilde{b}_1]$ according to cdf $\tilde{J}^{s_0}(\cdot)$, and the type-(V_h, H) bidder 1 randomizes over $[\tilde{b}_2, \tilde{b}_3]$ according to cdf $\tilde{J}^h(\cdot)$. By submitting bid $b \in [\tilde{b}_1, \tilde{b}_2]$, the expected surplus for type-(V_h, s_0) bidder 2 is given by

$$\begin{aligned}&(V_h - b)\left[p_l + p_h(1-\tilde{\pi}_E) + p_h\tilde{\pi}_E(1-q)\tilde{J}^l(b)\right]\\ =\ & \frac{(2q-1)p_l}{1-q}\cdot b + \left[(p_l + p_h(1-\tilde{\pi}_E))V_h - \frac{p_l q + p_h(1-q)(1-\tilde{\pi}_E)\tilde{b}_1}{1-q}\right],\end{aligned}$$

which is strictly increasing in b for $b \in [\tilde{b}_1, \tilde{b}_2]$. Hence the type-$(V_h, s_0)$ bidder has incentive to deviate while her opponent follows the prescribed strategy. This is a contradiction that means the supposed case cannot be an equilibrium.

The other four cases can be ruled out using the same method.

Claim 3: *There exists no asymmetric equilibrium in which bidder 1 and bidder 2 acquire information with disparate probabilities.*

[13] Denote $S(V_h, s)$ as the mixed strategy support of type-(V_h, s) bidder where $s \in \{H, L, s_0\}$ and $\ll$ is interpreted as "lower than." The five cases are $S(V_h, s_0) \ll S(V_h, l) \ll S(V_h, h)$, $S(V_h, s_0) \ll S(V_h, h) \ll S(V_h, l)$, $S(V_h, l) \ll S(V_h, h) \ll S(V_h, s_0)$, $S(V_h, h) \ll S(V_h, l) \ll S(V_h, s_0)$, $S(V_h, h) \ll S(V_h, s_0) \ll S(V_h, l)$.

Suppose there exists such an equilibrium. Denote $\hat{\pi}_{1E}$ and $\hat{\pi}_{2E}$ as bidder 1 and bidder 2's information acquisition probability respectively. Without loss of generality, let $\hat{\pi}_{1E} > \hat{\pi}_{2E} > 0$. The following several steps show that the distribution functions and the parameters satisfy some necessary conditions imposed by the mixed strategy. Remember that similar arguments can be used to show that: in an asymmetric equilibrium, the support of type-(V_h, L) bidder i must be lower than, non-overlapping, and contiguous with that of type-(V_h, s_0) bidder i; the support of type-(V_h, s_0) bidder i must be lower than, non-overlapping, and contiguous with that of type-(V_h, H) bidder i, for $i = 1, 2$ (refer to Steps 3 and 4 in Claim 2). Then bidder 1 and bidder 2's strategies should be as follows:

- Bidder 1 acquires information with probability $\hat{\pi}_{1E}$. The type-(V_h, L) bidder 1 randomizes over the interval $(V_l, \hat{b}_{11}]$ according to some cdf $\hat{J}_1^l(\cdot)$, the type-(V_h, s_0) bidder 1 randomizes over the interval $[\hat{b}_{11}, \hat{b}_{12}]$ according to some cdf $\hat{J}_1^{s_0}(\cdot)$, and the type-(V_h, H) bidder 1 randomizes over the interval $[\hat{b}_{12}, \hat{b}_{13}]$ according to some cdf $\hat{J}_1^h(\cdot)$, where $V_l < \hat{b}_{11} < \hat{b}_{12} < \hat{b}_{13} < V_h$.

- Bidder 2 acquires information with probability $\hat{\pi}_{2E}$. The type-(V_h, L) bidder 2 randomizes over the interval $(V_l, \hat{b}_{21}]$ according to some cdf $\hat{J}_2^l(\cdot)$, the type-(V_h, s_0) bidder 2 randomizes over the interval $[\hat{b}_{21}, \hat{b}_{22}]$ according to some cdf $\hat{J}_2^{s_0}(\cdot)$, and the type-(V_h, H) bidder 2 randomizes over the interval $[\hat{b}_{22}, \hat{b}_{23}]$ according to some cdf $\hat{J}_2^h(\cdot)$, where $V_l < \hat{b}_{21} < \hat{b}_{22} < \hat{b}_{23} < V_h$.

Note that the upper bound of the support for type-(V_h, H) bidder 1 and type-(V_h, H) bidder 2 should be the same; that is, $\hat{b}_{13} = \hat{b}_{23} = \hat{b}_3$. Then we derive the necessary conditions that pin down the cumulative distribution functions and the critical parameters.

Step 1: We first show that $\hat{b}_{11} > \hat{b}_{21}$ and $\hat{b}_{12} < \hat{b}_{22}$. Suppose instead $\hat{b}_{11} \leqq \hat{b}_{12}$. Then both type-$(V_h, L)$ bidder 1 and type-(V_h, L) bidder 2 should be indifferent between those bids that belong to the interval $(V_l, \hat{b}_{11}]$. By drawing a bid from $(V_l, \hat{b}_{11}]$, type-(V_h, L) bidder 1's expected surplus is given by

$$
\begin{aligned}
&(V_h - b)\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{2E}(1-q)}{p_l q + p_h(1-q)}\hat{J}_2^l(b)\right] \\
=&(V_h - V_l)\frac{p_l q}{p_l q + p_h(1-q)} && b = V_l^+ \\
=&(V_h - \hat{b}_{11})\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{2E}(1-q)}{p_l q + p_h(1-q)}\hat{J}_2^l(\hat{b}_{11})\right] && b = \hat{b}_{11},
\end{aligned}
$$

where $b = V_l^+$ means the bid b approaches V_l from above and the surplus is given at $b = V_l^+$ by taking the limit as $b \to V_l$, and the meaning of this notation is preserved in the following argument. The type-(V_h, L) bidder 2's expected surplus is given by

$$
\begin{aligned}
&(V_h - b)\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{1E}(1-q)}{p_l q + p_h(1-q)}\hat{J}_1^l(b)\right] \\
=&(V_h - V_l)\frac{p_l q}{p_l q + p_h(1-q)} && b = V_l^+ \\
=&(V_h - \hat{b}_{11})\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{1E}(1-q)}{p_l q + p_h(1-q)}\right] && b = \hat{b}_{11}.
\end{aligned}
$$

From these two equations we get $\hat{\pi}_{1E} = \hat{\pi}_{2E}\hat{J}_2^l(\hat{b}_{11})$, contradicting to the assumption $\hat{\pi}_{1E} > \hat{\pi}_{2E}$.

Now suppose $\hat{b}_{12} \geqq \hat{b}_{22}$. Mixed bidding strategy requires that both type-(V_h, H) bidder 1 and type-(V_h, H) bidder 2 should be indifferent between those bids that belong to the interval $[\hat{b}_{12}, \hat{b}_3]$. By drawing a bid from $[\hat{b}_{12}, \hat{b}_3]$, type-$(V_h, H)$ bidder 1's expected surplus is given by

$$
\begin{aligned}
&(V_h - b)\left[\frac{p_l(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q\hat{\pi}_{2E}(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q(1-\hat{\pi}_{2E})}{p_l(1-q) + p_h q} + \frac{p_h q\hat{\pi}_{2E} q\hat{J}_2^h(b)}{p_l(1-q) + p_h q}\right] \\
=&(V_h - \hat{b}_{12})\left[\frac{p_l(1-q) + p_h q(1 - q\hat{\pi}_{2E})}{p_l(1-q) + p_h q} + \frac{p_h q\hat{\pi}_{2E} q\hat{J}_2^h(\hat{b}_{12})}{p_l(1-q) + p_h q}\right] && b = \hat{b}_{12} \\
=&V_h - \hat{b}_3 && b = \hat{b}_3,
\end{aligned}
$$

and the type-(V_h, H) bidder 2's expected surplus is given by

$$
\begin{aligned}
&(V_h - b)\left[\frac{p_l(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q\hat{\pi}_{1E}(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q(1-\hat{\pi}_{1E})}{p_l(1-q) + p_h q} + \frac{p_h q\hat{\pi}_{1E} q\hat{J}_1^h(b)}{p_l(1-q) + p_h q}\right] \\
=&(V_h - \hat{b}_{12})\left[\frac{p_l(1-q) + p_h q(1 - q\hat{\pi}_{1E})}{p_l(1-q) + p_h q}\right] && b = \hat{b}_{12} \\
=&V_h - \hat{b}_3 && b = \hat{b}_3.
\end{aligned}
$$

From these two equations we obtain $\hat{\pi}_{2E} - \hat{\pi}_{1E} = \hat{\pi}_{1E}\hat{J}_2^h(\hat{b}_{12}) > 0$, which contradicts the fact that $\hat{\pi}_{2E} < \hat{\pi}_{1E}$.

Now we have the rank for the extreme points of the supports as $V_l < \hat{b}_{21} < \hat{b}_{11} < \hat{b}_{12} < \hat{b}_{22} < \hat{b}_3 < V_h$, demonstrated in Figure 6.

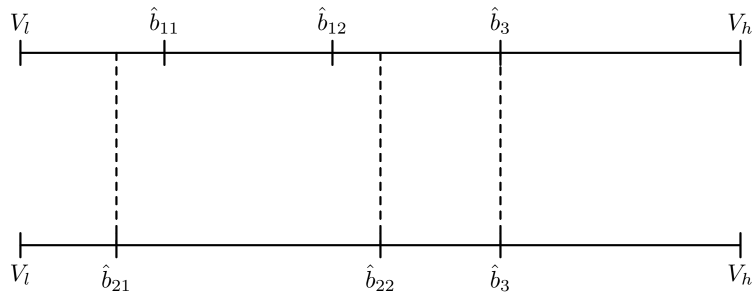

Figure 6: Possible Bid Support

Step 2: Now we consider type-(V_h, L) bidder 1's surplus. Since she randomizes over the interval $(V_l, \hat{b}_{11}]$, her expected surplus is given by

$$\begin{aligned}
&(V_h - b)\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{2E}(1-q)}{p_l q + p_h(1-q)}\hat{J}_2^l(b) + \frac{p_h(1-q)(1-\hat{\pi}_{2E})}{p_l q + p_h(1-q)}\hat{J}_2^{s_0}(b)\right] \\
=&(V_h - V_l)\frac{p_l q}{p_l q + p_h(1-q)} && b = V_l^+ \\
=&(V_h - \hat{b}_{21})\left[\frac{p_l q + p_h(1-q)\hat{\pi}_{2E}(1-q)}{p_l q + p_h(1-q)}\right] && b = \hat{b}_{21} \\
=&(V_h - \hat{b}_{11})\left[\frac{p_l q + p_h(1-q)\hat{\pi}_{2E}(1-q) + p_h(1-q)(1-\hat{\pi}_{2E})\hat{J}_2^{s_0}(\hat{b}_{11})}{p_l q + p_h(1-q)}\right] && b = \hat{b}_{11},
\end{aligned}$$

which should be a constant for all bids in this interval. By this indifference condition we get

$$\hat{b}_{21} = \frac{p_l q V_l + p_h(1-q)^2\hat{\pi}_{2E}V_h}{p_l q + p_h(1-q)^2\hat{\pi}_{2E}}; \tag{42}$$

$$\hat{J}_2^l(b) = \frac{p_l q}{p_h(1-q)^2\hat{\pi}_{2E}} \cdot \frac{b - V_l}{V_h - b}, \quad b \in (V_l, \hat{b}_{21}]; \tag{43}$$

$$\hat{J}_2^{s_0}(b) = \frac{p_l q + p_h(1-q)^2\hat{\pi}_{2E}}{p_h(1-q)(1-\hat{\pi}_{2E})} \cdot \frac{b - \hat{b}_{21}}{V_h - b}, \quad b \in [\hat{b}_{21}, \hat{b}_{11}]. \tag{44}$$

We turn to the type-(V_h, L) bidder 2. Since she randomizes over the interval $(V_l, \hat{b}_{12}]$, her expected surplus is given by

$$\begin{aligned}
&(V_h - b)\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\hat{\pi}_{1E}(1-q)}{p_l q + p_h(1-q)}\hat{J}_1^l(b)\right] \\
=&(V_h - V_l)\frac{p_l q}{p_l q + p_h(1-q)} && b = V_l^+ \\
=&(V_h - \hat{b}_{21})\left[\frac{p_l q + p_h(1-q)\hat{\pi}_{1E}(1-q)\hat{J}_1^l(\hat{b}_{21})}{p_l q + p_h(1-q)}\right] && b = \hat{b}_{21},
\end{aligned}$$

which should be a constant for all bids in this interval $[V_l, \hat{b}_{12}]$. Since type-(V_h, L) bidder 1 and type-(V_h, L) bidder 2 have the same payoff according to the above indifference conditions, we have relation

$$\hat{\pi}_{2E} = \hat{\pi}_{1E}\hat{J}_1^l(\hat{b}_{21}). \tag{45}$$

Now we turn to the type-(V_h, s_0) bidder 2. Since this type bidder 2 randomizes over the interval $[\hat{b}_{21}, \hat{b}_{22}]$, her expected surplus is given by

$$\begin{aligned}
&(V_h - b)\left[p_l + p_h\hat{\pi}_{1E}(1-q)\hat{J}_1^l(b) + p_h(1-\hat{\pi}_{1E})\hat{J}_1^{s_0}(b) + p_h\hat{\pi}_{1E}q\hat{J}_1^h(b)\right] \\
=&(V_h - \hat{b}_{21})\left[p_l + p_h\hat{\pi}_{1E}(1-q)\hat{J}_1^l(\hat{b}_{21})\right] && b = \hat{b}_{21} \\
=&(V_h - \hat{b}_{11})\left[p_l + p_h\hat{\pi}_{1E}(1-q)\right] && b = \hat{b}_{11},
\end{aligned}$$

which should be a constant for all bids in $[\hat{b}_{21}, \hat{b}_{22}]$. Then we have

$$\hat{b}_{11} = \frac{[p_l + p_h(1-q)\hat{\pi}_{2E}]\hat{b}_{21} + p_h(1-q)(\hat{\pi}_{1E} - \hat{\pi}_{2E})V_h}{p_l + p_h(1-q)\hat{\pi}_{1E}}. \tag{46}$$

Step 3: Now we give the conditions that determine the key parameters $\hat{\pi}_{1E}$ and $\hat{\pi}_{2E}$. Note that when bidder 1 or bidder 2 uses a mixed strategy to acquire information, the surplus should be the same between acquiring information and not acquiring information. Suppose bidder 1 acquires information. With probability $p_h[p_l q + p_h(1-q)]$, bidder 1's type is (V_h, L) and the corresponding surplus is $\frac{p_l q(V_h - V_l)}{p_l q + p_h(1-q)}$. With probability $p_h[p_l(1-q) + p_h q]$ bidder 1's type is (V_h, H) and the corresponding surplus is $(V_h - \hat{b}_3)$. The type-V_l bidder has zero surplus. Then the total surplus is $p_h\big[(V_h - V_l)p_l q + (p_l(1-q) + p_h q)(V_h - \hat{b}_3)\big] - c$. Now suppose bidder 1 does not acquire information. With probability p_h her type is (V_h, s_0) and the corresponding surplus is $(V_h - \hat{b}_{11})\big[p_l + p_h\hat{\pi}_{2E}(1-q) + p_h(1-\hat{\pi}_{2E})\hat{J}_2^{s_0}(\hat{b}_{11})\big]$. With probability p_l her valuation is V_l and the corresponding surplus is zero. Then bidder 1's total non-IA surplus is $p_h(V_h - \hat{b}_{11})\big[p_l + p_h\hat{\pi}_{2E}(1-q) + p_h(1-\hat{\pi}_{2E})\hat{J}_2^{s_0}(\hat{b}_{11})\big]$. In equilibrium, it should be true that

$$\begin{aligned} &p_h\big[(V_h - V_l)p_l q + (p_l(1-q) + p_h q)(V_h - \hat{b}_3)\big] - c \\ &\qquad = p_h(V_h - \hat{b}_{11})\big[p_l + p_h\hat{\pi}_{2E}(1-q) + p_h(1-\hat{\pi}_{2E})\hat{J}_2^{s_0}(\hat{b}_{11})\big]. \end{aligned} \tag{47}$$

By the same analysis, we have a similar condition for bidder 2:

$$\begin{aligned} &p_h\big[(V_h - V_l)p_l q + (p_l(1-q) + p_h q)(V_h - \hat{b}_3)\big] - c \\ &\qquad = p_h(V_h - \hat{b}_{21})\big[p_l + p_h\hat{\pi}_{1E}(1-q)\hat{J}_1^l(\hat{b}_{21})\big]. \end{aligned} \tag{48}$$

Note that the left hand sides of the two bidders' indifference conditions are the same and $\hat{\pi}_{2E} = \hat{\pi}_{1E}\hat{J}_1^l(\hat{b}_{21})$. Then we obtain one necessary condition for $\hat{\pi}_{1E}$ and $\hat{\pi}_{2E}$:

$$\hat{\pi}_{1E}(1-q) = \hat{\pi}_{2E}(1-q) + (1-\hat{\pi}_{2E})\hat{J}_2^{s_0}(\hat{b}_{11}). \tag{49}$$

Substituting $\hat{b}_{11}$ and $\hat{J}_2^{s_0}(\cdot)$ (defined in Step 2) into the equation yields

$$(\hat{\pi}_{1E} - \hat{\pi}_{2E})(1-q) = \frac{(\hat{\pi}_{1E} - \hat{\pi}_{2E})[p_l q + p_h(1-q)^2\hat{\pi}_{2E}]}{p_l + p_h(1-q)\hat{\pi}_{2E}}, \tag{50}$$

which holds only when $q = \frac{1}{2}$. This contradicts the fact $q > \frac{1}{2}$. Then there exists no such equilibrium that bidder 1 and bidder 2 acquire information with different probabilities.

Claim 4: *It cannot be an equilibrium that one bidder acquires information and the other* *bidder does not acquire information.*

By way of contradiction, suppose we can construct such an equilibrium. Without loss of generality, let bidder 1 choose not to acquire information. Then she will bid V_l if her valuation is V_l, and mix over the interval $(V_l, \tilde{\gamma}]$ for some constant $\tilde{\gamma}$ according to some cdf $\tilde{H}(\cdot)$. Assume bidder 2 acquires information with probability $\tilde{\pi}$. Then in equilibrium the type-V_l bidder 2 will bid V_l, the type-(V_h, s_0) bidder 2 will mix over the same interval $(V_l, \tilde{\gamma}]$ according to some cdf $\tilde{H}_2^{s_0}$, the type-(V_h, L) bidder 2 will mix over the same interval $(V_l, \tilde{\gamma}]$ according to some cdf $\tilde{H}_2^l$, and the type-(V_h, H) bidder 2 will mix over the same interval $(V_l, \tilde{\gamma}]$ according to some cdf $\tilde{H}_2^h$. Note that by Lemma 1, the bidder with valuation V_l bids V_l in any equilibrium, and by ϵ-deviation argument, the type-V_h bidder should randomize to submit bids over some interval. Since bidder 2 knows that the type-V_h bidder 1 never bids more than $\tilde{\gamma}$, the type-V_h bidder 2 also randomizes over the same interval.

By the requirement of mixed strategy, the expected surplus for the type-(V_h, L) bidder 2 and the type-(V_h, H) bidder 2 must be constant. Hence for some constants $\tilde{C}_l$ and $\tilde{C}_h$, the following equations should hold, which is impossible unless $q = \frac{1}{2}$:

$$\begin{aligned}
&\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)}{p_l q + p_h(1-q)} \cdot \tilde{H}_1(b)\right](V_h - b) = \tilde{C}_l, \\
&\left[\frac{p_l(1-q)}{p_l(1-q) + p_h q} + \frac{p_h q}{p_l(1-q) + p_h q} \cdot \tilde{H}_1(b)\right](V_h - b) = \tilde{C}_h,
\end{aligned}$$

for all $b \in (V_l, \tilde{\gamma}]$. Thus we have a contradiction.

Claim 5: *There is no asymmetric equilibrium in which two bidders acquire information with the same probability and submit bids according to different strategies.*

Proof. First, similar arguments can be used to show that in an asymmetric equilibrium, the support of a type-(V_h, L) bidder must be lower than that of a type-(V_h, s_0) bidder, and the support of a type-(V_h, s_0) bidder must be lower than that of a type-(V_h, H) bidder (see Step 4 in Claim 2).

Let $\tilde{\pi}$ be the probability that bidders 1 and 2 acquire information about their opponents' valuations. Now suppose that the type-(V_h, L) bidder 1 plays a mixed strategy over the support of $(V_l, \tilde{b}_1^l]$ with c.d.f $\tilde{T}_1^l(\cdot)$, and the type-(V_h, L) bidder 2 plays a mixed strategy over the support of $(V_l, \tilde{b}_2^l]$ with c.d.f $\tilde{T}_2^l(\cdot)$. Without loss of generality, assume that $\tilde{b}_1^l > \tilde{b}_2^l$. Note that the type-$(V_h, L)$ bidder 1 is indifferent among all bids in $(V_l, \tilde{b}_2^l]$. Then $\tilde{T}_2^l(\cdot)$ should satisfy

$$\begin{aligned}
&(V_h - b)\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\tilde{\pi}(1-q)\tilde{T}_2^l(b)}{p_l q + p_h(1-q)}\right] \\
=&(V_h - V_l)\frac{p_l q}{p_l q + p_h(1-q)},
\end{aligned} \tag{51}$$

from which we obtain

$$\tilde{T}_2^l(b) = \frac{p_l q}{p_h(1-q)^2\tilde{\pi}} \cdot \frac{b - V_l}{V_h - b}, \tag{52}$$

and $\tilde{T}_2^l(\tilde{b}_2^l) = 1$. Since bidder 2 is also indifferent among all bids in $(V_l, \tilde{b}_2^l]$, $\tilde{T}_1^l(\cdot)$ should also satisfy

$$\begin{aligned}&(V_h - b)\left[\frac{p_l q}{p_l q + p_h(1-q)} + \frac{p_h(1-q)\tilde{\pi}(1-q)\tilde{T}_1^l(b)}{p_l q + p_h(1-q)}\right]\\ =&(V_h - V_l)\frac{p_l q}{p_l q + p_h(1-q)}.\end{aligned} \tag{53}$$

Then we have

$$\tilde{T}_1^l(b) = \frac{p_l q}{p_h(1-q)^2\tilde{\pi}} \cdot \frac{b - V_l}{V_h - b}, \tag{54}$$

which is exactly the same as $\tilde{T}_2^l(b)$. Then $\tilde{T}_2^l(\tilde{b}_2^l) = 1$, which contradicts the fact that $\tilde{T}_2^l(\tilde{b}_2^l) < 1$ for bidder 1 mixes over the interval $(V_l, \tilde{b}_1^l]$. Hence it should be the case that type-(V_h, L) bidders 1 and 2 both mix in the same interval $(V_l, \tilde{b}^l]$.

Now suppose that the type-(V_h, s_0) bidder 1 mixes over the interval $[\tilde{b}^l, \tilde{b}_1^{s_0}]$ according to some c.d.f. $\tilde{T}_1^{s_0}(\cdot)$, and the type-(V_h, s_0) bidder 2 mixes over the interval $[\tilde{b}^l, \tilde{b}_2^{s_0}]$ according to some c.d.f. $\tilde{T}_2^{s_0}(\cdot)$. Without loss of generality, assume $\tilde{b}_1^{s_0} > \tilde{b}_2^{s_0}$. Then by applying the same analysis above, we will obtain $\tilde{b}_1^{s_0} = \tilde{b}_2^{s_0}$ and $\tilde{T}_1^{s_0}(\cdot) = \tilde{T}_2^{s_0}(\cdot)$, which in turn implies that the type-(V_h, H) bidders 1 and 2 will use exactly the same mixed strategies. This completes the claim.

Claim 6: *π_E is uniquely determined by (11) when $0 \leqq c < p_l p_h^2(V_h - V_l)(2q - 1)$.*

Proof. The probability of IA, or the demand for information, π_E, is pinned down by bidder's indifference condition between IA and no IA. Consider the bidder who acquires information by incurring cost c. This bidder obtains positive surplus only when her valuation type is V_h which occurs with probability p_h. Conditional on her action of IA, $a = 1$, the type-V_h bidder receives a signal L that comes with probability $p_l q + p_h(1-q)$, or a signal H with probability $p_l(1-q) + p_h q$. In equilibrium, the type-(V_h, L) bidder submits a bid randomly drawn from $(V_l, \delta_1]$. Since any bid in this interval yields type-(V_h, L) bidder the same expected payoff, suppose she submits a bid with value V_l. This bid wins only against an opponent whose value is V_l, which occurs with probability $\frac{p_l(1-q)}{p_l(1-q)+p_h q}$ conditional on signal L. Therefore, the type-(V_h, L) bidder's expected payoff is $S_{(V_h,L)} = \frac{p_l(1-q)}{p_l(1-q)+p_h q}(V_h - V_l)$. Now turn to the type-$(V_h, H)$ bidder. Suppose her bid is δ_2 (again this bidder submits bid randomly and which bid she submits wouldn't matter). The bid δ_2 can win against a type-V_l opponent, a type-(V_h, L) opponent and a type-(V_h, s_0) opponent, which occur with probabilities $\frac{p_l(1-q)}{p_l(1-q)+p_h q}$, $\frac{p_h q \pi_E(1-q)}{p_l(1-q)+p_h q}$ and $\frac{p_h q(1-\pi_E)}{p_l(1-q)+p_h q}$, respectively, conditional on her own type (V_h, H). Therefore, type-(V_h, H) bidder has expected payoff $S_{(V_h,H)} = (V_h - \delta_2)[\frac{p_l(1-q)}{p_l(1-q)+p_hq} + \frac{p_hq\pi_E(1-q)}{p_l(1-q)+p_hq} + \frac{p_hq(1-\pi_E)}{p_l(1-q)+p_hq}]$. The total ex ante expected payoff then would be $p_h[(p_lq + p_h(1-q))S_{(V_h,L)} + (p_l(1-q) + p_hq)S_{(V_h,H)}] - c$, which yields the left hand side of (11) by simplifying terms. Using the same analysis, the bidder's expected payoff without IA can be obtained, which is demonstrated on the right hand side of (11). Since bidder plays mixed strategy in information acquisition choice, different IA actions must give her exactly the same expected payoff, hence (11) holds.

Substituting δ_1, δ_2 and δ_3 into (11) and rearranging terms yields the reverse demand for information

$$c = p_lp_h(V_h - V_l) \cdot \frac{p_lp_hq(2q-1)}{p_lq + p_h(1-q)^2\pi_E} \cdot \frac{(1-\pi_E)}{(1-p_hq\pi_E)}. \tag{55}$$

The right hand side (RHS) is strictly decreasing in π_E, and RHS$= p_lp_h^2(V_h - V_l)(2q-1)$ when $\pi_E = 0$ and RHS$= 0$ when $\pi_E = 1$. Hence, π_E is uniquely determined for $0 \leqq c < p_lp_h^2(V_h - V_l)(2q-1)$ as claimed.

Claim 7: *For $c \geqq p_lp_h^2(V_h - V_l)(2q-1)$, no bidder acquires information in equilibrium.*

Proof. If both bidders acquire information, then both bidders choose the same IA probability that is implicitly given by (11) according to Claims 3 and 5 and the same reasoning in the argument of Claim 2, Steps 2 to 4. However, the resulting probability π_E given by (11) or equivalently by (14) is either zero or does not exist for $c \geqq p_lp_h^2(V_h - V_l)(2q-1)$. Since Claim 3 implies either both bidders acquire information with positive probability or neither acquires information in equilibrium, the result follows.

Claim 8: *For $c \geqq p_lp_h^2(V_h - V_l)(2q-1)$, the equilibrium is symmetric, unique and given in part (ii) of the proposition.*

Proof. When $c \geqq p_lp_h^2(V_h - V_l)(2q-1)$, the *ex ante* IA auction game $< \mathcal{A}^E, G^E >$ is equivalent to a standard FPA game by Claim 7. By Lemma 1, type-(V_l, s_0) bidder bids V_l in pure strategy. The same reasoning as in the argument of Step 2, Claim 2 shows that type-(V_h, s_0) bidder plays mixed strategy. Obviously, the supports of the type-(V_h, s_0) bidders 1 and 2 are the same, namely, $(V_l, \gamma]$ for some $V_l < \gamma < V_h$, since they just consider winning against each other (they win against type-(V_l, s_0) opponent for sure). The indifference condition that requires any bid drawn on the support yield the same expected surplus will pin down the parameter γ and c.d.f.$\bar{J}(\cdot)$, as given in (13) and (12), respectively. This implies the equilibrium is unique and symmetric.

The series of Claims 1 to 8 together with Lemma 1 imply that the equilibrium of the game $< \mathcal{A}^E, G^E >$ is unique and symmetric, and fully characterized in the proposition. The proof of the proposition now is complete. □

Proof of Corollary 1: Part (i). For $c \in [0, p_l p_h^2(V_h - V_l)(2q-1))$, we know π_E is continuous and decreasing in c, and π_E goes to zero as c approaches $p_l p_h^2(V_h - V_l)(2q-1)$ from equation (14). Also from part (ii) of Proposition 1, $\pi_E = 0$ for $c \in [p_l p_h^2(V_h - V_l)(2q-1), +\infty)$. The result then follows.

Part (ii). For $c = 0$, $\pi_E = 1$ for all $q \in (0.5, 1]$ since π_E is determined by (14). For $0 < c < p_l p_h^2(V_h - V_l)$, there are two cases to consider. If $0.5 < q \leqq \frac{1}{2}[1 + \frac{c}{p_l p_h^2(V_h - V_l)}]$, then $\pi_E = 0$ by part (ii) of Proposition 1. If $\frac{1}{2}[1 + \frac{c}{p_l p_h^2(V_h - V_l)}] < q \leqq 1$, then $\pi_E > 0$ and is determined by (14). Let $B_1 = \frac{q(2q-1)}{p_l q + p_h \pi_E (1-q)^2}$, and $B_2 = \frac{1-\pi_E}{1-p_h q \pi_E}$. It is easy to see that both B_1 and B_2 are increasing in q and decreasing in π_E. Therefore, the right hand side of (14) is increasing in q and decreasing in π_E. Since the left hand side of (14) does not depend on q, we can conclude that π_E is increasing in accuracy q for $\frac{1}{2}[1 + \frac{c}{p_l p_h^2(V_h - V_l)}] < q \leqq 1$. For $c \geqq p_l p_h^2(V_h - V_l)$, we simply have $\pi_E = 0$ by part (ii) of Proposition 1. Hence the conclusion follows. □

Proof of Proposition 2. All procedures of the proof are exactly parallel to those of Proposition 1. The only difference is that the bidder with valuation V_l will always choose not to acquire information since she cannot benefit from it. Note that type-V_l bidder completely ignores the signal if she acquires information in ex ante IA equilibrium. This indifference also implies the equation that determines π_I now turns to be the indifference condition for type-V_h bidder. We do not intend to replicate the argument, hence omit the proof. □

Proof of Corollary 2: Part (i). For $c < p_l p_h (V_h - V_l)(2q-1)$, π_I is given by equation (24); therefore, it is obvious that π_I is strictly decreasing in c and $\lim_{c \nearrow p_l p_h (V_h - V_l)(2q-1)} \pi_I = 0$. For $c \geqq p_l p_h (V_h - V_l)(2q-1)$, $\pi_I = 0$ by part (ii) of Proposition 2. Hence, π_I is continuous in c.

Part (ii). It's self-evident that $\pi_I = 1$ when $c = 0$ from (24). When $p_l p_h (V_h - V_l) > c > 0$, there are two different ranges of q to consider. If $0.5 < q \leqq \frac{1}{2}[1 + \frac{c}{p_l p_h (V_h - V_l)}]$, $\pi_I = 0$ by part (ii) of Proposition 2. If $\frac{1}{2}[1 + \frac{c}{p_l p_h (V_h - V_l)}] < q \leqq 1$, $\pi_I > 0$ and π_I is characterized by (24). Let $\hat{B}_1 = \frac{p_l q(2q-1)}{p_l q + p_h (1-q)^2 \pi_I}$ and $\hat{B}_2 = \frac{(1-\pi_I)}{(1-p_h q \pi_I)}$. It is easy to figure out that both $\hat{B}_1$ and $\hat{B}_2$ are strictly increasing in q and strictly decreasing in π_I. Therefore, the right hand side of (24) is strictly increasing in q and strictly decreasing in π_I, which implies π_I increases as q goes up for $\frac{1}{2}[1 + \frac{c}{p_l p_h (V_h - V_l)}] < q \leqq 1$. When $c \geqq p_l p_h (V_h - V_l)$, obviously $\pi_I = 0$ by part (ii) of Proposition 2.

Part (iii). We define a function of π as

$$\phi(\pi) = p_l^2 p_h^2 (V_h - V_l) q(2q-1) \cdot \frac{1-\pi}{[p_l q + p_h (1-q)^2 \pi](1 - p_h q \pi)}, \tag{56}$$

which is decreasing in π. Actually this function is the right hand side of (14). Then we have

$$\phi(\pi_E) = \frac{c}{p_h} > c = \phi(\pi_I); \tag{57}$$

hence $0 < \pi_E < \pi_I$ for $0 < c < p_l p_h^2(V_h - V_l)(2q-1)$. When $p_l p_h^2(V_h - V_l)(2q-1) \leqq c < p_l p_h(V_h - V_l)(2q-1)$, it is easy to see $\pi_I > 0 = \pi_E$. This completes the proof. $\square$

Proof of Proposition 3. Part (i) follows directly from (25), noting that $\pi_E \geqq 0$ and $q \in (0.5, 1]$. Part (ii) follows from the result that π_E is decreasing in c by Corollary 1 and the fact that S^E is increasing in π_E for $q \in (0.5, 1)$.

Proof of Proposition 4. Part (i) is self evident, since $\pi_E > 0$ for the given ranges of c and q by Proposition 1.

Part (ii). First for $c < p_l p_h^2(V_h - V_l)(2q-1)$, π_E is characterized by equation (14), and differentiating both sides with respect to c and rearranging terms yields

$$\frac{\partial \pi_E}{\partial c} = -\frac{[p_l q + p_h(1-q)^2\pi_E]^2[1 - p_h q \pi_E]^2}{p_l^2 p_h^2 (V_h - V_l) q (2q-1) \left[p_h(1-q)^2(1-\pi_E)(1-p_h q \pi_E) + (1-p_h q)(p_l q + p_h(1-q)^2\pi_E)\right]}. \tag{58}$$

Therefore, simple algebra shows that

$$\begin{aligned} \frac{\partial TS^E}{\partial c} &= -2(\pi_E + c \cdot \frac{\partial \pi_E}{\partial c}) \\ &= -2 \cdot \frac{p_h[p_h(1-q)^3 - p_l(2q-1)]\pi_E^2 + 2p_l q \pi_E - p_l q}{\left[p_h(1-q)^2(1-\pi_E)(1-p_h q \pi_E) + (1-p_h q)(p_l q + p_h(1-q)^2\pi_E)\right]} \end{aligned} \tag{59}$$

where the second line follows by substituting c and $\frac{\partial \pi_E}{\partial c}$ using (14) and (58), respectively.

Let the numerator be $N(\pi_E) = p_h[p_h(1-q)^3 - p_l(2q-1)]\pi_E^2 + 2p_l q \pi_E - p_l q$. $N(\cdot)$ has the properties that $N(0) = -p_l q < 0$, $N(1) = p_h^2(1-q)^3 + p_l[q - p_h(2q-1)] > 0$, and $N(\cdot)$ is strictly increasing in π_E for $\pi_E \in [0,1]$ regardless of the sign of $[p_h(1-q)^3 - p_l(2q-1)]$. The increasing property can be seen as follows. If $p_h(1-q)^3 - p_l(2q-1) = 0$, then $N(\cdot)$ is obviously increasing in π_E. If $p_h(1-q)^3 - p_l(2q-1) < 0$, then $N(\cdot)$ is a quadratic function of π_E and is increasing in π_E for $\pi_E < -\frac{2p_l q}{2p_h[p_h(1-q)^3 - p_l(2q-1)]}$; note that $-\frac{2p_l q}{2p_h[p_h(1-q)^3 - p_l(2q-1)]} > \frac{2p_l q}{2p_l p_h(2q-1)} > 1$, hence $N(\cdot)$ is increasing for $\pi_E \in [0,1]$. If $p_h(1-q)^3 - p_l(2q-1) > 0$, then the increasing range of $N(\cdot)$ would be $\pi_E > -\frac{2p_l q}{2p_h[p_h(1-q)^3 - p_l(2q-1)]}$, which includes the interval $[0,1]$. Then, $N(\cdot)$ is strictly increasing for $\pi_E \in [0,1]$.

Therefore, there exists a unique critical value $\hat{\pi}_E \in (0,1)$ such that $\frac{\partial TS^E}{\partial c} < 0$ for $\pi_E \in [0, \hat{\pi}_E)$ and $\frac{\partial TS^E}{\partial c} > 0$ for $\pi_E \in [\hat{\pi}_E, 1]$. By Corollary 1, π_E is strictly decreasing in c and is uniquely determined by equation (14), then the result follows by setting $c^E_{TS}(q) = \phi(\hat{\pi}_E)$, where $\phi(\cdot)$ is defined in (56).

Part (iii). By Corollary 1 item (ii), π_E is weakly increasing in q, then it follows that TS^E is weakly decreasing in accuracy. □

Proof of Proposition 5. Part (i). Since $\pi_E c \geqq 0$, and $S^E \geqq S_0$ by Proposition 3 part (i), the result follows.

Part (ii). Simple algebra shows that

$$\begin{aligned}\frac{d\chi^E}{dc} &= -2\left[\pi_E + \left(c + \frac{\partial S^E}{\partial \pi_E}\right)\cdot \frac{\partial \pi_E}{\partial c}\right]\\ &= -2\left\{\pi_E + \left[c + p_l^2 p_h^2 (V_h - V_l)\frac{q(1-q)(2q-1)}{[p_l q + p_h(1-q)^2\pi_E]^2}\right]\cdot \frac{\partial \pi_E}{\partial c}\right\}.\end{aligned}$$

Using (14) and (58) to substitute c and $\frac{\partial \pi_E}{\partial c}$ yields

$$\frac{d\chi^E}{dc} = 2\frac{(1-p_h q)\left[p_h(2q-1)\pi_E^2 - 2q\pi_E + 1\right]}{p_h(1-q)^2(1-\pi_E)(1-p_h q\pi_E) + (1-p_h q)(p_l q + p_h(1-q)^2\pi_E)}. \tag{60}$$

Therefore, the sign of $\frac{d\chi^E}{dc}$ is the same as the term $\xi(\pi_E) = [p_h(2q-1)\pi_E^2 - 2q\pi_E + 1]$. It is easy to see $\xi(\pi_E) > 0$ for $\pi_E \in [0, \pi_E^*)$ and $\xi(\pi_E) < 0$ for $\pi_E \in (\pi_E^*, 1]$ where $\pi_E^* = \frac{q-\sqrt{q^2-2p_h q + p_h}}{p_h(2q-1)} \in (0,1)$. Define the critical value $c^E(q) = \phi(\pi_E^*)$, where $\phi(\cdot)$ is given by (56). Then $c^E(q) \in (0, p_l p_h^2(V_h - V_l)(2q-1))$ as $\pi_E \in (0,1)$.

Since π_E is a strictly decreasing function of c when $c \in [0, p_l p_h^2(V_h - V_l)(2q-1)]$, we have $\xi(\pi_E) > 0$ for $c \in (c^E(q), p_l p_h^2(V_h - V_l)(2q-1)]$ and $\xi(\pi_E) < 0$ for $c \in [0, c^E(q))$. Then the result of Part (ii) follows. □

Proof of Proposition 6. Part (i) follows from the fact that π_I is increasing in π_I and π_I is decreasing in c by Corollary 2.

Part (ii). Note that $\pi_I \geqq \pi_E$ implies $S^I \geqq S^E$, and $\pi_I > \pi_E$ and $q \in (0,5,1)$ imply $S^I > S^E$. For $q \in (0.5, 1)$ and $0 < c < p_l p_h(V_h - V_l)(2q-1)$, $\pi_I > \pi_E$ by Corollary 2. Then the strict inequality follows. We also know that $\pi_I = \pi_E = 1$ for $c = 0$ and $\pi_I = \pi_E = 0$ for $c = p_l p_h(V_h - V_l)(2q-1)$ according to Propositions 1 and 2, then $\pi_I \geqq \pi_E$ holds. Part (ii) follows. □

Proof of Proposition 7. The argument is parallel to that of Proposition 4, hence is omitted. □

Proof of Proposition 8. The argument is parallel to that of Proposition 5, hence is omitted.

□

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