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Implementing Lindahl allocations by a withholding mechanism

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This paper investigates the problem of designing mechanisms whose Nash allocations coincide with Lindahl allocations for public goods economies when initial endowments are private information and unreported endowments are consumed (withheld) but are not destroyed. It will be noted that the mechanism presented here is individually feasible, balanced, and continuous. Besides, we allow preferences of agents to be nontotal-nontransitive and discontinuous.

1. Introduction

Since Hurwicz (1972) formalized a general model to deal with the incentives problem which is a basic aspect that a social system in general and economic system in particular needs to consider, there have been many mechanisms which solve free-rider (incentives) problems in the sense that they result in Pareto efficient allocations for public goods economies when individual self-interested behavior is characterized by Nash-equilibrium strategies. Groves and Ledyard (1977) were the first to propose a mechanism that yields Pareto-efficient allocations through Nash equilibria. Since then there have been many mechanisms which implement the Lindahl correspondence at Nash equilibrium points such as those in Hurwicz (1979), Walker (1981), Hurwicz et al. (1984), Tian (1989, 1990, 1991), Li et al. (1990), Tian and Li (1991) among others.

Among these mechanisms, Hurwicz et al. (1984) are the first to consider the case where the initial endowments are private information. This situation would certainly increase the size of the message space[1] but would reduce the information requirements on the designer. They consider both cases of destroying and withholding unreported endowments. Their mechanisms allow each agent to reveal information about his own endowment in a way

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[1]As Hurwicz et al. (1984) demonstrated, any mechanism which provides feasible outcomes necessarily has message spaces which depend on agents' endowments.

that he can understate but not overstate his own endowment so as to guarantee the feasibility even at disequilibrium points. The intuition here is straightforward: if a mechanism allows agents to overstate their endowments, then it allows for infeasible outcomes – it will sometimes attempt to allocate more than it is possible, given the true aggregate endowment. Their mechanisms, however, are discontinuous; small variations in an agent's strategy choice may lead to large jumps in the resulting allocations, and they require each agent to announce not only his own endowment but also others' endowments, and thus the sizes of the message spaces are very large. Tian (1989, 1991) improved the results of Hurwicz et al. (1984) by giving continuous mechanisms with much lower dimensions of message spaces. Further these mechanisms have the advantage that each agent is required to announce only his own endowment but not others' endowments. But these mechanisms only consider the case where the unreported endowments are cancelled (destroyed) rather than consumed so that the mechanisms are merely weakly balanced but not balanced. Also, these mechanisms, like the mechanisms of Hurwicz et al. (1984), enforce agents to state correctly their endowments in equilibrium. In other words, agents do not choose to understate their endowments and would wish to overstate their endowments (which is not permitted). Consequently, there is not much left of an incentive compatibility problem.

Also, when a mechanism is merely weakly balanced, the allocation at a disequilibrium point may be less than the total endowments. Should this happen, some resources are not completely used even though preferences of agents are strictly monotone. Thus, although a weakly balanced mechanism guarantees the feasibility of allocations, there still remains an incentive problem unless some additional enforcement is carried out. In other words, one has to make sure that goods which do not belong to any agent by the rule of mechanism are destroyed or individuals will be better off if the unused resources are consumed by them. Even in the case where the unreported endowments are surely destroyed, there is another problem, namely, the mechanism results in Pareto inefficient allocations since the balancedness of allocations is a necessary condition for Pareto efficiency. On the other hand, if a mechanism is balanced, even if disequilibrium points are used to compute the allocations, one can guarantee that resources are not wasted or destroyed.[2]

A similar situation prevailed with regard to the Nash-implementation of

[2]Note that these disequilibrium allocations do not necessarily result in Pareto inefficient allocations since the set of Lindahl allocations, which coincides with the set of Nash allocations of the mechanism that implements the Lindahl correspondence, is, in general, much smaller than the set of Pareto efficient allocations. Thus a disequilibrium allocation can still be a Pareto efficient allocation.

the Walrasian correspondence until Hong (1990) designed a mechanism which implements the Walrasian correspondence in the case of withholding. In addition, her mechanism is individually feasible, balanced, and continuous.

This paper modifies the mechanisms of Tian (1989, 1991) to the case of withholding by giving a simple mechanism whose Nash allocations coincide with the Lindahl allocations for public goods economies. As will be noted, this mechanism is well-behaved in the sense that it is individually feasible, balanced (not merely weakly balanced), and continuous. Also, we allow preferences of agents to be nontotal-nontransitive. This situation, as Tian (1991) pointed out, is potentially very important since in many cases – in particular, in the case where economic entities are composed of more than one individual; it is natural that the preferences for such agents would be nontransitive or nontotal due to the problem of aggregating the individuals' preferences. This is particularly true for public goods (projects) since choices of public goods are likely to be determined by communities. Because of well known problems in aggregating preferences of individuals, it may be necessary (or desirable) to represent the preferences of groups (communities) as nontransitive or nontotal.

It should be noted that the mechanism presented in this paper has the advantage that agents are not required to report their true endowments even at equilibrium. This is a very interesting and important property. Since we do not need to assume that agents have to show their announced endowments to the designer or are enforced to report the true endowments, there is a big difference between this situation and one in which endowments are assumed to be known to the designer and the incentive compatibility problem is well taken. Another advantage is that each agent is required to announce only his own endowment but not others' endowments and thus it uses a message space of much lower dimension than those of Hurwicz et al. (1984).

The plan of this paper is as follows. Section 2 sets forth a public goods model and presents a mechanism which has the desirable properties mentioned above. Section 3 shows that this mechanism fully implements the Lindahl correspondence. Finally, the concluding remarks are offered in section 4.

2. Public goods model and mechanism

2.1. Economic environments

In an economy with public goods, there are n agents (groups, players, or voters) who consume one private good and K public goods, x being private (as a numeraire) and y public. The single private good x can be thought of as a Hicksian composite commodity or money, and public goods y can be thought of as K public projects. Denote by $N=\{1,2,\dots,n\}$ the set of agents. Each agent's characteristic is denoted by $e_i=(\mathring{w}_i, P_i)$, where $\mathring{w}_i$ is the initial endowment of the private good and P_i is the strict (irreflexive) preference relation defined on $\mathbb{R}_+^{1+K}$ which may be nontotal or nontransitive.[3] We assume that there are no initial endowments of public goods, but that the public goods can be produced from the private good under constant returns to scale. That is, the production function f_k is given by $y^k=f^k(x)=(1/q^k)x$ for each $k=1,\dots,K$. Thus each unit of public good y_k requires q^k units of private good. Hence the feasibility constraint becomes

$$\sum_{i=1}^{n} x_i + qy \leqq \sum_{i=1}^{n} \mathring{w}_i, \tag{1}$$

where $q=(q^1,\dots,q^K)\in\mathbb{R}_{++}^K$.

An economy is the full vector $e=(e_1,\dots,e_n)$ and the set of all such economies is denoted by E. The following assumptions are made on E:

Assumption 1. $n\geqq 3$.[4]

Assumption 2. $\mathring{w}_i>0$ for all $i\in N$.

Assumption 3. P_i is convex[5] and strict monotonically increasing in the private good.

Assumption 4. For all $i\in N$, $(x_i, y)\,P_i\,(x_i', y')$ for all $x_i\in\mathbb{R}_{++}$, $x_i'\in\partial\mathbb{R}_+$, and y, $y'\in\mathbb{R}_+^K$, where $\partial\mathbb{R}_+^m$ is the boundary of $\mathbb{R}_+^m$.

Remark 1. Assumption 4 was called 'indispensability of money' by Mas-Colell (1980). This assumption cannot be dispensed. Tian (1988) showed that the (constrained) Lindahl correspondence violates Maskin's (1977) monotonicity condition only under Assumptions 1–3 and thus cannot be Nash-implemented by an individually feasible and balanced mechanism.

[3] In this paper we only consider the case of strict preferences but the same theorems can be obtained for weak preferences. From the results of Kim and Richter (1986) and Tian (1992) we know that the weak preference approach and the strict preference approach are equally valid: definitions and theorems in one approach correspond to definitions and theorems in the other approach.

[4] This is a necessary condition for the balanced and continuous implementation. Kwan and Nakamura (1987) proved that there are no balanced and continuous mechanisms which implement the Lindahl correspondence for two-agent economies.

[5] P_i is convex if for bundles a, b, c with $0<\lambda\leqq 1$ and $c=\lambda a+(1-\lambda)b$, the relation $a\,P_i\,b$ implies $c\,P_i\,b$.

2.2. Lindahl allocations

An allocation $(x, y)=(x_1,\dots,x_n, y)$ is *feasible* for an economy e if $(x, y)\in\mathbb{R}_+^{n+K}$ and (1) holds.

An allocation (x^*, y^*) is a *Lindahl allocation* for an economy e if it is feasible and there are personalized price vectors $q_i^*\in\mathbb{R}_+^K$, one for each i, such that;

(1) $x_i^*+q_i^* y^*\leqq \mathring{w}_i$ for all $i\in N$;
(2) for all $i\in N$, there does not exist (x_i, y) such that $(x_i, y)P_i(x_i^*, y^*)$ and $x_i+q_i^* y\leqq \mathring{w}_i$;
(3) $\sum_{i=1}^n q_i^*=q$.

Denote by $L(e)$ the set of all such allocations.

An allocation (x, y) is *Pareto-optimal* with respect to the strict preference profile $P=(P_1,\dots,P_n)$ if it is feasible and there does not exist another feasible allocation (x', y') such that $(x_i', y')P_i(x_i, y)$ for all $i\in N$.

An allocation (x, y) is *individually rational* with respect to P if $\neg$ $(w_i, 0)P_i(x_i, y)$ for all $i\in N$. Here '$\neg$' stands for 'it is not the case that'.

2.3. Mechanism

In the following we will present an individually feasible, balanced, and continuous mechanism which fully Nash-implements the Lindahl correspondence when the withheld endowments are consumed but not destroyed.

Let M_i denote the ith message domain. Its elements are written as m_i and called messages. Let $M=\prod_{i=1}^n M_i$ denote the message space. The message spaces of agents are defined as follows.

For each $i\in N$, his/her message domain is of the form

$$M_i=(0,1]\times(0,\mathring{w}_i]\times\mathbb{R}^K\times\mathbb{R}^K. \tag{2}$$

A generic element of M_i is $(\delta_i, w_i, \phi_i, y_i)$ whose components have the following interpretations. The component δ_i denotes the degree of desirability for the private good. In particular, when $\delta_i=1$, agent i wishes that public goods would not be produced. The designer will use the smallest δ_i of all agents to determine the level of public goods [see eq. (4) below]. The component w_i denotes a profession of agent i's endowment, the inequality $0<w_i\leqq\mathring{w}_i$ means that the agent cannot overstate his own endowment; on the other hand, the endowment can be understated, but the claimed endowment w_i must be positive. The component ϕ_i denotes the price vector of public goods proposed by agent i for use in other agents' budget constraints, and the component y_i denotes the proposed level of public goods that agent i is

willing to contribute (a negative y_i means the agent wants to receive a subsidy from the society).

Define the personalized price of each public good k for the ith consumer by

$$q_i^k(m) = b_i^k + \sum_{j=1}^{n} a_{ij}^k \phi_j^k, \tag{3}$$

where $\sum_{i=1}^n b_i^k = q^k$, $\sum_{i=1}^n a_{ij}^k = 0$, $a_{ii}^k = 0$, and $\sum_{j=1}^n |a_{ij}^k| > 0$ for $i \in N$ and $k = 1, 2, \ldots, K$. In addition, the coefficients a_{ij}^k are chosen so that the rank of the matrices $[a_{ij}^k]$ is equal to $n-1$ for all k. Observe that, by construction, $\sum_{i=1}^n q_i(m) = q$ for all $m \in M$ and each agent's personalized prices are independent of his own messages [i.e., $q_i(m^*) = q_i(m^*/m_i, i)$ for any $m_i \in M_i$]. Here $q_i(m) = (q_i^1(m), \ldots, q_i^K(m))$. Note that even though $q_i(m)$ is only a function of the price-component, $(\phi_1, \ldots, \phi_n)$, of the message m, we can write it as a function of m without loss of generality.

Define a correspondence $B: M \to 2^{\mathbb{R}_+^K}$ by

$$B(m) = \{ y \in \mathbb{R}_+^K : (1 - \delta(m)) w_i - q_i(m) y \geqq 0 \; \forall i \in N \}, \tag{4}$$

which is clearly a continuous correspondence with non-empty compact convex values. Here $\delta(m) = \min\{\delta_1, \ldots, \delta_n\}$.

Define the outcome function for public goods $Y: M \to B$ by

$$Y(m) = \left\{ y: \min_{y \in B(m)} \|y - \tilde{y}\| \right\}, \tag{5}$$

which is the closest point to $\tilde{y}$. Here $\tilde{y} = \sum_{i=1}^n y_i$. Then $Y(m)$ is single-valued and continuous on M.[6]

For each individual i, define the taxing function $T_i: M \to \mathbb{R}$ by

$$T_i(m) = q_i(m) Y(m). \tag{6}$$

Then

$$\sum_{i=1}^{n} T_i(m) = q(m) Y(m). \tag{7}$$

The outcome function $X(m): M \to \mathbb{R}_+$ is given by

[6]This is because $Y(m)$ is an upper semi-continuous correspondence by Berge's Maximum Theorem [see Debreu (1959, p. 19)] and single-valued [see Mas-Colell (1985, p. 28)].

$$X_i(m)=w_i-q_i(m)Y(m). \tag{8}$$

Note that $X_i(m)>0$ by the definition of the constrained correspondence B and the total (final) consumption of agent i for the private good is the sum of $X_i(m)$ and $(\mathring{w}_i-w_i)$. That is, it is the sum of the amount of private good allocated by the mechanism and the unreported amount of his/her own endowment.

Thus the outcome function is continuous on M, $(X(m), Y(m))\in\mathbb{R}_+^{n+K}$, and

$$\sum_{i=1}^{n}[X_i(m)+\mathring{w}_i-w_i]+qY(m)=\sum_{i=1}^{n}\mathring{w}_i, \tag{9}$$

for all $n\in M$.

From (9), we have

$$\sum_{i=1}^{n}X_i(m)+qY(m)=\sum_{i=1}^{n}w_i, \tag{10}$$

which means the aggregate consumption of the private good allocated by the mechanism is equal to the aggregate of endowments reported by agents for all $m\in M$.

Denote $h: M\to\mathbb{R}_+^{n+K}$ the outcome function, or more explicitly, $h_i(m)=(X_i(m), Y(m))$. Then the mechanism consists of $\langle M,h\rangle$ defined on E. By the constructions of the mechanism, the mechanism $\langle M,h\rangle$ is *individually feasible* (i.e., $(X_i(m)+\mathring{w}_i-w_i, Y(m))\in\mathbb{R}_+^{1+K}$ for all $i\in N$ and all $m\in M$), *balanced* [i.e., (9) holds for all $m\in M$], and continuous.

A message $m^*=(m_1^*,\dots,m_n^*)\in M$ is said to be a *Nash equilibrium* of the mechanism $\langle M,h\rangle$ in the presence of withholding for an economy e if for any $i\in N$ and for all $m_i\in M_i$,

$$\neg[h_i(m^*/m_i,i)+(\mathring{w}_i-w_i,0)]\,P_i\,[h_i(m^*)+(\mathring{w}_i-w_i^*,0)], \tag{11}$$

where $(m^*/m_i,i)=(m_1^*,\dots,m_{i-1}^*,m_i,m_{i+1}^*,\dots,m_n^*)$. $[h(m^*)+(\mathring{w}-w^*,0)]$ is then called a *Nash (equilibrium) allocation* of the mechanism for the economy e. Denote by $V_{M,h}(e)$ the set of all such Nash equilibria and by $N_{M,h}(e)$ the set of all such Nash (equilibrium) allocations. The mechanism $\langle M,h\rangle$ is said to fully *Nash-implement* the Lindahl correspondence L on E, if, for all $e\in E$, $N_{M,h}(e)=L(e)$.

3. Implementation results

The remainder of this paper is devoted to the proof of equivalence between

Nash allocations and Lindahl allocations. Theorem 1 below proves that every Nash allocation is a Lindahl allocation. Theorem 2 below proves that every Lindahl allocation is a Nash allocation.

Theorem 1. Under Assumptions 1–4, if the withholding mechanism defined above has a Nash equilibrium m^*, then the Nash allocation $(X(m^*)+\mathring{w}-w^*, Y(m^*))$ is a Lindahl allocation with $(q_1(m^*),\ldots,q_n(m^*))$ as the Lindahl price vector, i.e., $N_{M,h}(e)\subseteq L(e)$.

Proof. Let m^* be a Nash equilibrium. Now we prove that $(X(m^*)+\mathring{w}-w^*, Y(m^*))$ is a Lindahl allocation with $(q_1(m^*),\ldots,q_n(m^*))$ as the Lindahl price vector. Since the mechanism is individually feasible and balanced, and $\sum_{i=1}^{n} q_i(m^*)=q$ as well as $[X_i(m^*)+\mathring{w}_i-w_i^*]+q_i(m^*)Y(m^*)=\mathring{w}_i$ for all $i\in N$, we only need to show that each individual is maximizing his/her preferences. Suppose, by way of contradiction, that there is some $(x_i, y)\in\mathbb{R}_+^{1+K}$ such that $(x_i, y)\,P_i\,(X_i(m^*)+\mathring{w}_i-w_i^*, Y(m^*))$ and $x_i+q_i(m^*)y\leqq \mathring{w}_i$. Because of monotonicity of preferences, it will be enough to confine ourselves to the case of $x_i+q_i(m^*)y=\mathring{w}_i$. Let

$$x_{\lambda i}=\lambda x_i+(1-\lambda)\,[X_i(m^*)+\mathring{w}_i-w_i^*],$$

$$y_\lambda=\lambda y+(1-\lambda)Y(m^*).$$

Then by convexity of preferences we have $(x_{\lambda i}, y_\lambda)\,P_i\,(X_i(m^*)+\mathring{w}_i-w_i^*, Y(m^*))$ for any $0<\lambda<1$. Also $(x_{\lambda i}, y_\lambda)\in\mathbb{R}_+^{1+K}$ and $x_{\lambda i}+q_i(m^*)y_\lambda=\mathring{w}_i$. Now suppose that player i chooses δ_i so that $\delta_i<\delta(m^*)$, $y_i=y_\lambda-\sum_{j\neq i}^{n} y_j^*$, and keeps w_i^* and ϕ_i^* unchanged. Then $\delta(m^*/m_i, i)=\delta_i<\delta(m^*)$ and thus $(1-\delta(m^*/m_i,i))w_j^*-q_j(m^*)$. $Y(m^*)>(1-\delta(m^*))w_j^*-q_j(m^*)Y(m^*)\geqq 0$ for all $j\in N$ by the construction of the mechanism. Thus, we have $(1-\delta(m^*/m_i,i))w_j^*-q_j(m^*/m_i,i)\,y_\lambda>0$ for all $j\in N$ as λ is sufficiently small. Hence $y_\lambda\in B(m^*/m_i,i)$ and therefore $Y(m^*/m_i,i)=y_\lambda$ and $X_i(m^*/m_i,i)=w_i^*-q_i(m^*)Y(m^*/m_i,i)=w_i^*-q_i(m^*)y_\lambda$. Then $X_i(m^*/m_i,i)+\mathring{w}_i-w_i^*=x_{i\lambda}$. From $(x_{i\lambda}, y_\lambda)\,P_i\,(X_i(m^*)+\mathring{w}_i-w_i^*, Y(m^*))$, we have

$$(X_i(m^*/m_i,i)+\mathring{w}_i-w_i^*, Y(m^*/m_i,i))\,P_i\,(X_i(m^*)+\mathring{w}_i-w_i^*, Y(m^*)).$$

This contradicts the hypothesis that $(X(m^*)+\mathring{w}_i-w_i^*, Y(m^*))\in N_{M,h}(e)$. □

Theorem 2. Under Assumptions 1–4, if (x^*, y^*) is a Lindahl allocation with the Lindahl price vector $q^*=(q_1^*,\ldots,q_n^*)$, then there is a Nash equilibrium m^* for the withholding mechanism defined above such that $X_i(m^*)+\mathring{w}_i-w_i^*=x_i^*$, and $q_i(m^*)=q_i^*$, for all $i\in N$, $Y(m^*)=y^*$, i.e., $L(e)\subseteq N_{M,h}(e)$.

Proof. We first note that $x^*\in\mathbb{R}_{++}^n$ by Assumption 4. We need to show that there is a message m^* such that (x^*, y^*) is a Nash allocation. Let $w_i^* = \mathring{w}_i$, let δ_i be sufficiently small so that $(1-\delta(m^*))\mathring{w}_i - q_i^* y^* > 0$, and let $(y_1^*, \ldots, y_n^*, \phi_1^*, \ldots, \phi_n^*)$ be a solution of the following linear equations system:

$$y^* = \sum_{i=1}^{n} y_i,$$
$$q_i^{*k} = b_i^k + \sum_{j=1}^{n} a_{ij}^k \phi_{kj} \phi_{kj}, \tag{12}$$

for $k = 1, \ldots, K$. Then, it can be easily verified that $Y(m^*) = y^*$, and $q_i(m^*) = q_i^*$, $X_i(m^*) = x_i^*$, for all $i \in N$. Notice that $q_i(m^*/m_i, i) = q_i(m^*)$ for all $m_i \in M_i$, $(X(m^*/m_i, i) + \mathring{w}_i - w_i, Y(m^*/m_i, i)) \in \mathbb{R}_+^{1+K}$ and $[X_i(m^*/m_i, i) + \mathring{w}_i - w_i] + q_i(m^*)$. $Y(m^*/m_i, i) = \mathring{w}_i$ for all $i \in N$ and $m_i \in M_i$. Therefore, we know that

$$\neg (X_i(m^*/m_i, i) + \mathring{w}_i - w_i, Y(m^*/m_i, i)) \, P_i \, (X_i(m^*), Y(m^*)),$$

for otherwise it contradicts the fact that $(X_i(m^*), Y(m^*))$ is a Lindahl allocation. $\square$

Remark 2. From Theorem 1, we can see that even at Nash equilibria agents are not necessarily reporting their true endowments since we may have $w_i^* \neq \mathring{w}_i$. Indeed, we can modify the proof of Theorem 2 so that the underreported endowment is a Nash-equilibrium strategy as long as the reported endowments are close enough to the true endowments. Thus every Lindahl allocation can be supported by a Nash equilibrium with the false announcement about endowments.

Since Lindahl allocations are Pareto optimal and individually rational, the mechanism yields Pareto-optimal and individually rational allocations.

Summarizing the above discussions, we conclude that for one private and K public goods economies E satisfying Assumptions 1–4, there exists an individually feasible, balanced, and continuous mechanism which fully Nash-implements the Lindahl correspondence in the presence of withholding.

4. Concluding remarks

In this paper, we have presented a simple mechanism which fully implements the Lindahl correspondence when endowments are private information and unreported endowments are consumed but not destroyed. This mechanism has the advantage that agents are not required to report their true endowments even at equilibrium and thus the incentive compatibility problem for endowments is well taken. In addition, this mechanism requires that each agent announce only his own endowment but not others' endowments and thus it uses a message space of much lower dimension than those of Hurwicz et al (1984). Besides, this mechanism is well-behaved in the sense that it is individually feasible, balanced, and continuous. Furthermore, we allow preferences of agents to be nontotal-nontransitive and discontinuous. Though this paper only considers Nash-implementation of the Lindahl correspondence for public economies with one private good and constant returns to scale, the mechanisms such as those presented in Li et al. (1990), Tian (1989), Tian and Li (1991) can be similarly modified to implement the Lindahl correspondence for public goods economies with any number of goods and decreasing returns to scale.

Finally it may be remarked that since system (12) has a large multiplicity of solutions, the mechanism has a (high dimension) continuum of Nash equilibria, even in economies with a unique Lindahl equilibrium. This multiple equilibrium problem, however, can be solved by slightly modifying the above mechanism and strengthening Assumption 4. Indeed, if we let the messages which determine the level of public goods and personalized prices be the same, i.e., let $\phi_i = y_i$ (so that the dimension of the message space of the modified mechanism is reduced by nK dimensions), and if we replace Assumption 4 by Assumption 4′: for all $i \in N$, $(x_i, y) P_i (x_i', y')$ for all $(x_i, y) \in \mathbb{R}^{1+K}_{++}$, $(x_i', y') \in \partial \mathbb{R}^{1+K}_{+}$, then the solution of system (12) is unique and we can prove that the modified mechanism Nash implements Lindahl allocations by using proofs similar to those given in Tian (1990, 1991). Of course, in this case, the modified mechanism only implements interior Lindahl allocations.[7]

[7]Since the multiple equilibrium problem is a general negative aspect of Nash implementation, other solution concepts such as those of subgame perfect equilibrium and undominated Nash equilibrium have been used in the literature [e.g., Moore and Repullo (1988), Abreu and Sen (1990), Palfrey and Srivastava (1991)]. We may also use these equilibrium concepts which are refinements of Nash equilibrium to solve the multiple equilibrium problem of the mechanism given in this paper.

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