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Tian, Guoqiang

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Studies on The Identification Problem of the Simultaneous Economic Models from the Viewpoint of Unique Determination of Parameters(I)*#

Tian Guoqiang

Huazhong(Central China) University of Science and Technology

Abstract

Proceeding from the viewpoint that the parameters to be estimated should be uniquely determined, we define the concepts of the distinction and identification of vectors such that under the basic assumptions of quite general nature, study the identification problem of the contemporaneous simultaneous models and obtain a number of results. Among them, multicollinearity problem is treated as an identification problem. More noticeable in this paper is the idea of removing the usual assumption that no linear identities connect the exogenous variables. The one-step identification method and two-step identification method are introduced. The usual major theorems about the identification for contemporaneous simultaneous models can be treated as special cases of our more general results. Also given in the paper are the concepts of almost identification and completely under identification. The proposal of the concepts of the distiction and identification of vectors is of great significance, in which the identification of economic models is abstractly included; further it has probability of linking up the relation between systems such as economic system, control system and biological system. In our following papers, we will study by using our viewpoint and methods the identifi-

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cation problems of the dynamic models(including the unstable ones), the nonlinear models, the error-shock models (also including the unstable ones) and obtain a number of theorems for the identification which are similar to contemporaneous models and are easy to verify.

§1. Introduction

The early work on the identification problem was done by Frisch[4], Haavelmo[6], Hurwicz[9], Koopmans and Reiersφl[10], and others. An extensive study on identification of the contemporaneous models and nonlinear models under various assumptions was seen in Fisher[3]. Hsiao[8] surveyed the past development and studied in particular the identification of dynamic models and error-shock models. Marvall[12] discussed in detail the error-shock models with an equation under various assumptions. It is generally true that the identification problem was studied on the basis of observational equivalence. Thus the scope of the study is usually restricted to dealing with only a part of the implication of the basic definition 2.1 in the next section.

§2. Definitions and Useful Theorems

We base our discussions on

Definition 2.1 If and only if all the unknown parameters of a model can be uniquely determined (or locally uniquely determined) from the observational and prior information, the model is said to be identified (or locally identified).

Definition 2.2 A model is underidentified if there are more than one parameter vector that lead to the same distribution for the endogenous variables y_t in the model.

Definition 2.3 The structure S which is only identified with all the independent prior information relevant to identification will be said to be just identified.

Definition 2.4 The structure S which can be identified with a part rather than all of the independent prior information relevant to identification will be called overidentifiable.

Without loss of generality, we assume that the prior information independent in the following. In addition, the following two definitions are given.

Definition 2.5 If the structure S is identified with probability one, it is called almost identifiable.

Definition 2.6 If the structure S is neither identifiable nor almost identifiable, it is called completely nonidentifiable.

Almost identifiability which is an extension of the concept of identifiability is essentially distinctive from complete underidentifiability. By almost identifiability, we mean that there is enough prior information required for the identification, but because of the chance errors of observation and (or) computation, it may result in underidentification whereas complete underidentifiability is a case in which the prior information is insufficient so that the structure is underidentifiable no matter how accurate are the observations ano computations and in whatever amount are the observations available. It will be shown in this paper that the distinction between almost identifiability and complete underidentifiability lies in whether the order conditions hold.

For the sake of discussion,we give below the concepts that vectors are distinguishable and identifiable according to certain characteristics they have and the theorems related to them. By such a mathematical abstract, the identification of economic models can be considered to be a special concrete case of the identification of vectors.

Definition2.7 For the ordered pair of vectors (α,β), $\alpha,\beta\in R^N$, if α has some characteristics,β also has, then α is said not to be distinguishable from β on the basis of these characteristics, otherwise, they are*.

Definition 2.8 For the vector $\alpha\in R^N$ having certain characteristics, if there are no other vectors in R^N having the same characteristics, then, α is uniquely determined, and we say that α is identifiable.

From the above definitions, we derive the following theorems.

Theorem 2.1 Given an $N\times K$ matrix $W(N>K)$, if A is a basis for the row kernel of W(i. e. A is a $G\times N$ matrix whose rows constitute a basis for the solution vectors of the equation $XW=0$), then the g-th row A_g of A can be distinguished from an N- component row vector α on the basis of $XW=0$ alone if and only if α is not a linear combination of the row vectors of A.

Theorem 2.2 If A is a basis for the row kernel of W, then A_g is identified under the normalization rule if and only if A_g is distinguished from the linear combination of the row vectors of A.

Theorem 2.1 means that for the elements of A with no constraints other than $Aw=0$, premultiplying Eq. $AW=0$ by any $G\times G$ nonsingular matrix F, we obtain

$$F(AW)=(FA)W=0.$$

Denote $A^*=FA$. The g-th row of A^* is denoted by A_j^*, where $A_j^*=F_gA$. since both A_g and A_g^* are the solutions of $XW=0$, A_g is not distinguished from A_g^*. But

* According to the definition of distinguishability, α and β may have ordered relation, such that while α is not distinguished from β, β may be distinguished from α. From example, $\alpha=(A,B)$, $\beta=(A,B,C)$, where A,B,C each represent a certain characteristic.

if some restrictions are put on A_g, then A_g is said not to be distinguishable from A_g^* only when A_g^* also satisfics the same restrictions, otherwise A_g is distinguished from A_g^*, thus ruling out many A^*'s that may result from the invertible linear transformation matrices F's if the A^*'s are required to satisfy the same restriction. For the sake of discussion, the concepts of admissible transformation and admissible matrix of vector are given.

Definition 2.9 Let a $G\times G$ matrix A with certain characteristics be a basis for the row kernel of W. If, for a certain $G\times G$ invertible matrix F, FA also has the same characteristics, then the linear transformation, denoted by the matrix F, is called admissible transformation and the corresponding matrix F is called admissible matrix.

When A_g has enough constraints such that it is uniquely determined, we know by Theorem 2.2 that the elements of the g-th row of the admissible matrix F are all zero except for the g-th, which is unity.

Thus we have

Theorem 2.3 A_g is identified if and only if the admissible matrix F is merely

$$F=\begin{array}{c} \left(\begin{array}{ccccc} & & \vdots & & \\ 0 & \cdots & 010 & \cdots & 0 \\ & & \vdots & & \\ & & \text{the } g\text{-th column} & & \end{array}\right) \end{array} \text{the } g\text{-th row}$$

and A is identified if and only if the admissible matrix F is merely

$$F=\begin{pmatrix} 1 & & 0 \\ & 1 \ddots & \\ 0 & & 1 \end{pmatrix}.$$

It is known that when an equation has a definite meaning (i. e. the parameters in the equation can be determined), multiplying by a nonzero constant both sides of the equation will not change the meaning of the equation. Hence, the normalization rule does not effect the identification problem, therefore the diagonal elements of the matrix F in Theorem 2.3 can be taken as nonzero constant.

Thus when A is a basis for the kernel of W, we only need to find conditions such that the admissible matrix F can only be one like the matrix in theorem 2.3. If we also know that the restrictions on A_g are

$$A_g\phi_g=d_g$$

where ϕ_g is an $N\times R_g$ known constant matrix; d_g is a $1\times R_g$ known row vector, then we have

Theorem 2.4 A necessary and sufficient condition for the identifiability of A_g under the constraints $A_gW=0$ and $A_g\phi_g=d_g$ is

$$\operatorname{rank}(A\phi_g)=\begin{cases}G & d_g\neq 0,\\ G-1 & d_g=0,\end{cases}$$

where $d_g\neq o$ means that there is at least one component which is not zero; $d_g=0$ means that all the components are zero. The former means that there is a normalization rule whereas the latter does not.

Proof: 1° $(d_g\neq 0)$ For $F_g=e_g{}'$ (e_g' is the g-th row of identity matrix), we have $A_g^{*}\phi_g=e_g'A\phi_g=A_g\phi_g=d_{g'}$ then it is known that e_g' is a solution of Eq. $E_g(A\phi_g)=d_g$. The solution of the equation is unique if and only if rank $(A_g)=G$ (including the solution of a contradiction equation).

2° $(d_g=0)$ For $F_g=ae_g'$, where a is a nonzero constant, ae_g' is a solution of Eq. $F_g(A\phi_g)=0$, the solution is proportional if and only if $\operatorname{rank}(A\phi_g)=G-1$.

Corollary 2.1 A necessary condition for the identifiability of the g-th row A_g of A is

$$R_g\geqslant\begin{cases}G & d_g\neq 0,\\ G-1 & d_g=0.\end{cases}$$

We also need two lemmas in linear algebra for the following discussions.

Lemma 2.1 For the matrix A satisfying $AW=0$(A and W are $G\times N$ and $N\times K$ matrixes respectively), we have

$$\operatorname{rank}(A)\leqslant N\text{-rank }(W),$$

when

$$\operatorname{rank}(A)=N\text{-rank }(W),$$

it is true that we can choose N-rank(W) linearly independent vectors from the row vectors of A as a basis for the row kernel of W. In particular, when $G=N$-rank(W), A constitutes a basis for the row kernel of W. Similarly, when $K=N$-rank(A), W constitutes a basis for the column kernel of A.

Lemma 2.2 For $AW=0$, if A has full row rank and $G=\operatorname{rank}(A)<N$-rank (W), then there must exist solution vectors $C_1,C_2,\ldots,C_{N-G-\operatorname{rank}(W)}$ of Eq $XW=0$ such that

$$\bar{A}=\begin{bmatrix}A\\ C_1\\ C_2\\ \vdots\\ C_{N-G-\operatorname{rank}(W)}\end{bmatrix}$$

is a basis for the row kernel of W.

We intend to discuss the identification problem in two ways: one is the onestep identification method, by which we mean that all the characteristics of A_g $(g=1,2,\cdots,G)$ are grouped together for the identification and the other is the two-step identification method, which is meant that, first by applying $XW=0$, we distinguish A_g from row vectors which are not linear combinations of the rows of $\bar{A}$, then by the characteristics of A alone, we distinguish A_g from row vectors which are linear combinations of the rows of $\bar{A}$.

We ignore the information contained in variance covariance matrix for the time being.

§3. Contemporaneous Simultaneous Equation Models

Much work has been done on the identification problem of contemporaneous simultaneous equation models[3][8], which is described under the assumption that no linear identities connect the exogenous variables. In this section, we deal with the identification problem of the more genefal models by removing the assumption that no linear identities connect exogenous variables. The usual major theorems for the identification are only a special case of our results[3][8]. This also leads to a new point that multicollinearity may be regarded as an identification problem.

Suppose the models have the following form

$$BY_t+\Gamma X_t=U_t \tag{3.1}$$

where Y_t, X_t and u_t are respectively $G\times 1$, $K\times 1$ and $G\times 1$ vectors of observed endogenous variables, observed exogenous variables and unobserved disturbance terms at time t; B and Γ are respectively $G\times G$ and $G\times K$ matrixes of coefficients to be estimated.

Assumption 3.1 B is nonsingular.

Assumption 3.2 $Eu_t=0$, $Eu_tXs'=0$*.

Letting $C_X=EX_tX_t'$, $CY_X=EY_tX_t'$, $CY=EY_tY_t'$,

we have $BC_{YX}+\Gamma C_X=0$;

$$\begin{aligned}\Sigma &= EU_tU_t'=E(BY_t+\Gamma X_t)(BY_t+\Gamma X_t)'\\ &=BC_YB'+BC_{YX}\Gamma'+\Gamma C_{XY}B'+\Gamma C_X\Gamma'.\end{aligned} \tag{3.2}$$

Simplifying the above, we have

$$BC_YB'=\Gamma C_X\Gamma'+\Sigma. \tag{3.3}$$

* When U_t are serially uncorrelated, the assumption can be relaxed in the way that X_t can be allowed to contain lagged endogenous variables because $\operatorname{plim}\frac{\sum_{t=0}^{T} x_tu_t'}{T}=0$.

It can be seen from (3.3) that Σ is also identified when B and Γ are identified. .

Let's first restrict ourselves to identifying the models with the two-step identification method. As the first step, define

$$A=(B,\Gamma), \tag{3.4}$$

$$W=\begin{bmatrix} C_{YX} \\ C_X \end{bmatrix} \tag{3.5}$$

where W contains all the sample observational information**(the observational information is reflected by the sample second order moments.) Then,(3.2) can be rewritten as

$$AW=0. \tag{3.6}$$

Suppose rank$(W)=K^{\circ}$. By Assumption 3.1, A has full row rank. We know rank$(W)=K^{\circ}\leqslant K$ by Lemma 2.1. when C_X is nonsingular, i. e. $K^{\circ}=K$, from Lemma 2.1, we know that A constitutes a basis for the row kernel of W. when C_X is singular, i. e. $K^{\circ}<K$, A is not a basis for the row kernel of W. By Lemma 2.2, there exist $k-k^{\circ}$ solution vectors $C_1, C_2, \cdots, C_{K-K^{\circ}}$ of Eq. $XW=0$ which is linearly independent of the rows of A such that

$$\bar{A}=\begin{Bmatrix} A \\ C_1 \\ C_2 \\ \vdots \\ C_{K-K^{\circ}} \end{Bmatrix} \tag{3.7}$$

constitutes a basis for the row kernel of W. Thus we have

$$\bar{A}W=0. \tag{3.8}$$

By Theorem 2.1, the observational information W can only distinguish A_g from the row vectors which are not linear combinations of the rows of $\bar{A}$, but can not distinguish A_g from the row vectors which are linear combinations of the rows of $\bar{A}$. Thus, if there are not any yestrictions for parameters of (3.1), any $(C+K-K^{\circ})\times(G+K-K^{\circ})$invertible matrix F is an admissible matrix. Then we have

$$F\bar{A}W=0. \tag{3.9}$$

So $A_g(g=1,2,\cdots,C)$ is completely not identifiable.

Now we come to the second step.

If we also know that A_g has the constraints

$$A_g\Phi_g=d_g \tag{3.10}$$

** Here W is not the same as $W=\begin{bmatrix}\Pi \\ I\end{bmatrix}$ in Literatures [3, 8], where Π is a parameter of the reduced form of (3.1). Note that C_X may be singulax.

where ϕ_g is a $(G+K)\times R_g$ matrix with known elements, d_g is a $1\times R_g$ vector with known elements, then we have

Theorem 3.1 (rank condition) Under the restriction (3.10) and the assumptions 3.1–3.2, A_g is identified if and only if

$$\text{rank}\quad(\bar{A}\phi_g)=\begin{cases}G+K-K^\circ & d_g\neq 0,\\ G+K-K^\circ-1, & d_g=0.\end{cases}$$

By Theorem 2.4, this can be proved.

Corollary 3.1 (order condition) A necessary condition for the identifiability of A_g is

$$R_g\geqslant\begin{cases}G+K-K^\circ & d_g\neq 0,\\ G+K-K^\circ-1, & d_g=0.\end{cases}$$

Corollary 3.2 When C_X is nonsingular, A_g is identified if and only if

$$\text{rank}\quad(A\phi_g)=\begin{cases}G, & d_g\neq 0,\\ G-1, & d_g=0;\end{cases}$$

a necessary condition for the identification of A_g is

$$R_g\geqslant\begin{cases}G, & d_g\neq 0,\\ G-1, & d_g=0.\end{cases}$$

Thus, we obtain the usual rank conditions and the order condition

Theoren 3.2 If rank $(\bar{A}\ \phi_g)=\begin{cases}G+K-K^\circ, & d_g\neq 0,\\ G+K-K^\circ-1, & d_g=0,\end{cases}$ (3)

then A_g is just identifiable if and only if

$$R_g=\begin{cases}G+K-K^\circ\ , & d_g\neq 0,\\ G+K-K^\circ-1, & d_g=0;\end{cases}$$

A_g is overidentifiable if and only if

$$R_g>\begin{cases}G+K-K^\circ, & d_g\neq 0,\\ G+K-K^\circ-1, & d_g=0.\end{cases}$$

When C_X is nonsingular (i.e.$K^\circ=K$) we obtain the usual results (3).

If the order conditions hold, but the rank conditions do not hold by computation, we consider that this arises owing to the observational error and/or computational error. Since the point set, on which the determinants of $G\times G$ submatrixes of the matrix $\bar{A}\ \phi_g$ are equal to zero, is of Lebesque measure zero, thus we have

Theorem 3.3 A_g is almost identifiable if and only if the order condition holds.

we shall focus on the identifiability of the model but not on just identifiability, over identifiability and almost identifiability. This is because only discussing the number of restrictions will result in corresponding conclusions.

Note that the two-step identification method can only conveniently be applicable to restrictions on the parameters of a particular equation. For restrictions on parameters of different equations (cross-equation restrictions), the one-step identification method is more appropriate.

$A_g W=0$ and $A_g\phi_g=d_g$ can be rewritten as

$$A_g(W \vdots \phi_g)=(0 \vdots d_g). \tag{3.11}$$

Theorem 3.4 The g-th row A_g of A is jdentified if and only if

$$\operatorname{rank}(W \vdots \phi_g)=\begin{cases} G+K, & d_g\neq 0, \\ G+K-1, & d_g=0. \end{cases}$$

Now consider the identifiability of A as a whole. We stack A in one row which is denoted by $\vec{A}$, i.e., $\vec{A}=(A_1, A_2, \cdots, A_G)'$. Then $AW=0$ can be rewritten as

$$(I_G\otimes W')\vec{A}=0, \tag{3.12}$$

where $(I_G\otimes W')$ is $GK\times G(G+K)$ matrix; $\otimes$ denotes the Kronecker product, i. e., $(I_G\otimes W')$ is

$$(I_G\otimes W')=\begin{bmatrix} W' & & & \\ & W' & & \\ & & \ddots & \\ & & & W' \end{bmatrix}, \tag{3.13}$$

We now assume that additional prior information including normalization rules is available in the form of R linear restrictions on $\vec{A}$, i. e.,

$$\phi\vec{A}=d, \tag{3.14}$$

where ϕ is an $R+G(G+K)$ matrix and d is an $R\times 1$ vector with known elements. Letting

$$Q=\begin{pmatrix} I\otimes W' \\ \phi \end{pmatrix}, \qquad \bar{d}=\begin{pmatrix} O \\ d \end{pmatrix}, \tag{3.15}$$

we have

$$O\vec{A}=\bar{d}. \tag{3.16}$$

Theorem 3.5 $\vec{A}$ is identified if and only if $\operatorname{rank}(Q)=G(G+K)$.

Corollary 3.3 A necessairy condition for the identifiability of $\vec{A}$ is $R\geqslant G(G+K-K^\circ)$, which becomes $R\geqslant G^2$ when G_X is nonsingular.

We obtain the usual result when C_X is nonsingular(8).

It is often that $\vec{A}$ is not identifiable, but a part of the elements of $\vec{A}$ is. In this case, the method of proof in Literature (8) can be used to obtain the results similar to those in (8).

When C_X is nonsingular, by (3.2), we have

$$C_{YX}C_X^{-1}=-B^{-1}\Gamma=\pi. \tag{3.17}$$

The posterior information suffices to determine the parameters of the reduced form π. In this case, π can be consistently estimated. However, when C_X is singular, the condition of proximity theorem(3) is not satisfied, and the consistent estimator of π can not be obtained on the basis of observational information alone*. Thus we come to the conclusion that when C_X is singular, π is not identified unless we have prior information about A or π

If (B, Γ) is identified, then of course is π by Eq. $\pi = -B^{-1}T$. When C_X is singular, we can not estimate π and A by using ordinary least-squares or Twostage least-squares. But when A is identifiable, it can be estimated by combining instrumental variable with the prior information of A. The estimator $\hat{\Pi}$ of π is determined by $-\hat{B}^{-1}\hat{\Gamma}$. $\hat{A}$ and $\hat{\Pi}$ thus obtained are both consistent. Now we are led to an important view-point: multicollinearity may be viewed as an iden tification problem rather than a relation between variables. As long as there is enough prior information, the consistent estimator of A can always be obtained no matter how serious the multicollinearity is. We may also consider the muiticollinearity problem in another way. Where ther exists multicollinearity, there must be some exogenous variables which can be linearly or nearly linearly expressed by other variables. We might regard these exogenous variables as endogenous variables. Thus we face a new model, and more prior restrictions on A are necessary for identification. To demonstrate these, we consider the model

$$P_t = a_0 + a_1 W_t + a_2 L_t + a_3 I_t + e_t \tag{3.18}$$

where P_t, W_t, I_t, L_t and e_t represent respectively production, wage, input, labour, and unobserved disturbace terms at time t.

Suppose that wage and labour are exactly linearly related: $W_t = KL_t$. Then whatever a^{**}_1 is, we have the same value P_t:

$$P_t = a_0 + a_1 W_t + a_2 L_t + a_3 I_t + e_t, \tag{3.19}$$

$$P_t = a_0 + (a_1 - a^*_1)W_t + (a_2 + Ka^*_1)L_t + a_3 I_t + e_t. \tag{3.20}$$

Thus an infinite number of parameter estimators correspond to the same P_t. In terms of identifiability, Eq. (3.18) is completely underidentified. Substituting $W_t = KL_t$ into Eq. (3.18), we have

$$P_t = a_0 + (a_1 K + a_2)L_t + a_3 I_t + e_t, \tag{3.21}$$

Suppose that the remainding variables have no multicollinearity. Regressing on Eq. (3.21). we can obtain the consistent estimators of a_0, $b \hat{=} (a_1 K + a_2)$ and a_3. Then for the parameters of (3.18), we have the restriction

$$Ka_1 + a_2 = b. \tag{3.22}$$

* Consistent estimators are unique only in the asymptotic sense.

** Since we know that there is a linear relation between W_t and L_t, we only need to take W_t or L_t as an instrumental variable in estimation.

By Theorem 3.1, the parameters of (3.18) can be uniquely determined under the constraint (3.22)*.

The problem may be viewed in still another way. Regard

$$P_t = a_0 + a_1 W + a_2 L_t + a_3 I_t + \epsilon_{1t},$$
$$W_t = KL_t + e_{2t} \qquad (e_{2t} \equiv 0) \tag{3.23}$$

as a simultaneous equation. Thus

$$B = \begin{pmatrix} 1 & -a_1 \\ & 1 \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} -a_0 & -a_2 & -a_3 \\ 0 & -K & 0 \end{pmatrix}. \tag{3.24}$$

Clearly $|B| = 1 \neq 0$. If there is no prior information, (3.18) can not be distinguished from the linear combination of the rows of (3.23). Hence, the first equation of(3.23) is completely underidentified, unless some prior information is piven(e. g.,$Ka_1 + a_2 = b$)

§ 4. Comments

It is seen from the above discussions that our viewpoint and methods have concised the statement of many of the principal results in identification theory and made it easy to understand what the identication is. In the following paper, we will remove the usual assumption that the roots of the polynomial equation $|B_0 + B_1 L + \cdots + B_P L^P| = 0$ lie outside the unit circle when dealing with the dynamic models and the dynamic error-shock models, so that we can study the unstable dynamic models and the unstable dynamic error-Shock models. We will prove that, even for the unstable economic models, the consistent estimators of their parameters still exist as long as the estimators for the parameters of the models can be uniquely determined. It is belived that the study of the unstable models is of great significance. This is because the economic systems are almost unstable (some people do think so[11]), and disturbance at every initial point will keep on dispersion with the shift of time, one relies on artificial restrictions to hold back such dispersion. Thus it seems necessary to study the identification of the unstable dynamic models.

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从参数唯一确定的观点论联立经济模型的识别问题(I)

田 国 强
(华中工学院数学系)

提 要

联立经济模型的识别问题是计量经济学的主要课题之一。现在已有的结果，大部分总结在文献〔3〕和〔8〕中。不过所用的方法大多是从观测上等价的角度得出来的。具体的说来，就是如何利用先验信息和观测信息将允许矩阵 F 和允许多项式矩阵 $F(L)$ 约束为单位矩阵。这样就大大地增添了识别的复杂性和局限性，特别是对于动态模型，只能研究部分的稳定动态模型的识别性。本文基于联立方程解的数学理论，从待估参数应是唯一确定的基本观点出发，定义了向量可区分和可识别的概念从而重新表述了经济计量模型的识别问题和方法并且获得了更为系统而简洁的数学描述。在非常一般的基本假定下，用统一的方法探讨了同期模型的识别，并且得到了若干新结果。其中包括对于同期模型识别的已有结果可以视为我们更一般结果的特例。更值得注意的是文中去掉了“外生变量不由任何线性恒等式连接”的通常假定，从而提出了可以把多重共线性问题视为识别问题。我们还给出了一步识别和二步识别两种识别方法及几乎能识别和完全不能识别的概念。向量可区分和可识别的概念的提出具有重要的意义。经济模型的识别可以抽象地被认为向量的识别。这些概念的提出，有可能沟通各种系统(如经济系统、控制系统、生物系统等)间的识别关系。随同另文，我们将利用我们的观点和方法来讨论动态模型、非线性模型和误差冲击模型的识别问题。特别值得一提的是：当讨论动态模型和动态误差冲击模型时，我们将去掉“要求滞后算子的多项式矩阵方程的根全部位于单位圆外”的通常假定，从而可以讨论非稳定的模型的识别，得到的结果也容易验证。