The Effect of Pension on the Optimized Life Expectancy and Lifetime Utility Level

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September 2012

The Effect of Pension System on the Optimized Life Expectancy and Lifetime Utility Level

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Abstract

In this paper, we analyze the effect of a pension system on the life expectancy and the lifetime utility level using a cross country data and an optimal dynamic problem of individuals who live in continuous and finite time. From the data, we find that 1) Happiness can be almost explained by income per capita. 2) Depending on income per capita, the pension system can make life span longer or shorter and can raise or reduce the level of happiness. Our model yields some results which are consistent with the results from the data: i) Life expectancy is not always proportional to lifetime utility. ii) The pension system can make life expectancy longer or shorter. This paper suggests that it is not always true that the pension system improves the lifetime utility level.

JEL Classification Codes: C61, H55, I31

Keywords: pension system, optimized life expectancy, lifetime utility level, health investments

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This paper is revised from MPRA Paper No. 41375, posted 17, September 2012. I especially added the empirical analysis in Section 2.

1 Introduction

According to an anecdote in Europe, as soon as a pension system was introduced, the number of people who jog in the park for their health increased. Believe it or not, under the pension system, it looks like a good deal, if we live long enough. This paper analyzes the effect of a pension system on the life expectancy and the lifetime utility level. There are many literatures on the effect of rising longevity on some economic variables as saving rate, growth rate, labor market, education, etc. For example, Bloom et al. (2007) and Dushi et al. (2010), Lee et al. (2000) etc. examine the effects of improvements in health or life expectancy on social security system and saving rate. Weil (2007), Acemoglu and Johnson (2007), Zhang and Zhang (2004), Zhang et al. (2001), etc. analyze the effects of improvements in health or life expectancy on economic growth. Zhang et al. (2003) shows that rising longevity encourages both savings and earlier retirement. Gorski et al. (2007) studies the effects of a pension reform on the educational level of the economy. Pecchenino and Utendorf (1999), de la Croix and Licandro (1999), Cipriani (2000), Boucekkine, et al. (2002, 2003), Pecchenino and Pollard (1997, 2002) etc. analyze the effect of longer life span on economic growth through the level of schooling and human capital accumulation. Recently, Pestieau and Ponthiere (2012) surveys the various contributions to the impact of changes in longevity on various public policies. However, there is little research on the opposite direction, that is, the economic variables, except the level of income, affect life expectancy or health. This paper is different from previous researches in that a pension system is the cause, and the life expectancy and the lifetime utility level are its effects.

A vast amount of empirical and theoretical researches about the economic welfare of a pension system has been accumulated. The main results of some previous studies on pension system and economic welfare can be summarized as follows: under a fully funded system, the economic welfare is not affected, however, under a pay-as-you-go pension system, depending on the economic situations and generations, the economic welfare might be both improved or worsened. The public pension system as a risk-hedging device can increase welfare by providing a certainty in the imperfect market. (Shiller, 1999, Krueger and Kubler, 2002, Sánchez-Marcos and Sánchez-Martin, 2006, Bohn, 2009, etc.) The compulsory pension system which is one of the forced saving policies can lead to high saving rates. Meanwhile, the public pension system crowds out the private savings. It can have a negative effect on the capital accumulation and can retard growth. (Cutler and Gruber, 1996, Feldstein and Liebman, 2002, Zhang and Zhang, 2004, etc.) The overall welfare impact depends on the balance between the insurance effect and the crowding-out effect. In this paper, we show that the pension system has both positive and negative effect on welfare using a cross country data. More specifically, the pension system has a positive effect on welfare when income per capita is low, but the pension system has a negative effect on welfare when income per capita is high.

We compare the utility level under the restriction of the pension system as a compulsory saving with the utility level without the restriction. Many previous researches analyze economic welfare using overlapping generation models. We use the optimal dynamic problem of individuals who live in continuous time, but not discrete time which is used in the overlapping generation models. (e.g. Chakraborty, 2004, Sánchez-Marcos and Sánchez-Martin, 2006, Ponthiere, 2009, etc.) This is one of the differences of our model from the previous models. To tell more specifics, in many previous overlapping generation models, the maximum life span is given (e.g., two-period or three-period) and the survival probability is introduced and the life expectancy is calculated by the average of the longevity of the people who live to the maximum life span and the people who die before the maximum life span depending on the the survival probability. Actually, in two-period model, only two kinds of ages (i.e, one-period-old and two-period-old) exist and nobody survives more than the given period even thougn the life expectancy has variations. However, our model is handled under continuous time and the agents decide the terminal point of the continuous model for oneself to maximize his/her life time utility. Like lifetime uncertainty models, e.g., Pecchenino and Pollard (1997), Chakraborty (2004), Momota, et al (2005), etc., we assume that it is possible to extend life span by the effort of an individual through health investments.[1] For example, eating good food, taking some nutritional supplements, getting in shape by going to the gym, investing in the development of medical technology, etc., longevity will arise due to the given examples on health investments. In reality, it is well known that coronary heart disease (CHD) mortality is highly influenced by the major risk factors, e.g., serum cholesterol, systolic blood pressure, diabetes, smoking habits, high alcohol consumption, lack of exercise and stress, etc. Lifestyle changes through individual's efforts (e.g., healthier diet, physical exercise, cessation of smoking and drinking) and medications have been shown to be effective in reducing coronary disease. If we can eliminate the risk factors, the life expectacny will undoubtedly grow.

An individual distributes his/her budget to his/her basic needs and to his/her health invest-

[1]Lifetime uncertainty models assume that the health investments can increase the surviving probability. However, we do not consider uncertainty in our model.

ments to maximize his/her lifetime utility. We consider that the individual's longevity is based from the result of the individual's utility maximization problem. This means that the individual's longevity is an endogenous variable and not exdogenous variable. We investigate how the optimized life span and the lifetime utility level can be changed by a pension system.

In Section 2, using a cross country data, we show the following: 1) Income per capita can almost explain happiness. The positive correlationship between life expectancy and happiness is a spurious relationship in which two variables have no direct causal connection. In reality, income per capita which is an unseen variable has caused both. And the life span does not have much influence on happiness. 2) Depending on income per capita, the pension system can make life span longer or shorter. 3) Depending on income per capita, the pension system can raise or reduce the level of happiness. In Section 3, our model yields two important results: i) Life expectancy is not always proportional to lifetime utility level. ii) Pension system can make the life span longer or shorter. The life span depends on the type of pension system. From the combination of the results i) and ii), it is possible that A) the pension system makes the life span longer and increases the utility level. B) the pension system makes the life span longer, however decreases the utility level. C) the pension system makes the life span shorter and decreases the utility level. Case 1 is preferable. But Case 2 and 3 are not preferable cases, but could possibly happen. Our results A), B) and C) from the optimal problem are consistent with 2) and 3) that we will see in the data in Section 2.

This paper is organized as follows: In Section 2, we confirm the relationship among income per capita, the presence or absence of a pension system, happiness and life expectancy using a cross country data. Section 3 presents the model and drives the benchmark outcomes. In section 4, we introduce the pension system to the benchmark. Section 5 solves the models numerically and analyzes the results and concludes. And finally, we include an Appendix.

2 Empirical Facts

2.1 Data

We take up income per capita, life expectancy and the presence or absence of a pension system as the determinants of happiness. The three variables are thought to be the variable that look at an economic side, a biological side and a social systematic side to decide the level of happiness, respectively. We can clarify how a pension system influences happiness using a data. The data used by this paper can be easily acquired on the internet. The happiness index and the life expectancy are available from World Database of Happiness and GDP per capita is also found in Penn World table. The presence or absence of a pension system is obtained from Table 1 of Bloom et al. (2007). World Database of Happiness releases the averages of the happiness index and the life expectancy from 2000 to 2009 by country, respectively. The happiness index run from 0 to 10. GDP per capita uses the variable "rgdpch" in Penn World Table 7.1. According to the Penn World Table 7.1, the definition of variable "rgdpch" is that PPP converted GDP per capita (chain series), at 2005 constant prices. We calculate the average of GDP per capita from 2000 to 2009 to meet the happiness index and the life expectancy in World Database of Happiness. We used the logarithm for GDP per capita, in the following analysis. The pension data which are dummy variables for the presence or absence of a pension system show the situation in 2002. The value is one when they have a pension system and the value is zero when they do not have a pension system. The pension data use "Universal Coverage". According to Bloom et al. (2007), the definition is that the dummy variable of universal coverage indicates whether the system covers all workers or not. Table 1 shows the detailed data source. World Database of Happiness, Penn World Table 7.1 and the pension data in 2002 of Bloom et al. (2007) listed 149, 190 and 61 countries, respectively. We focus on the 61 countries which are included in all the three data sets. Table A1 in Appendix contains the basic information of the 61 countries.

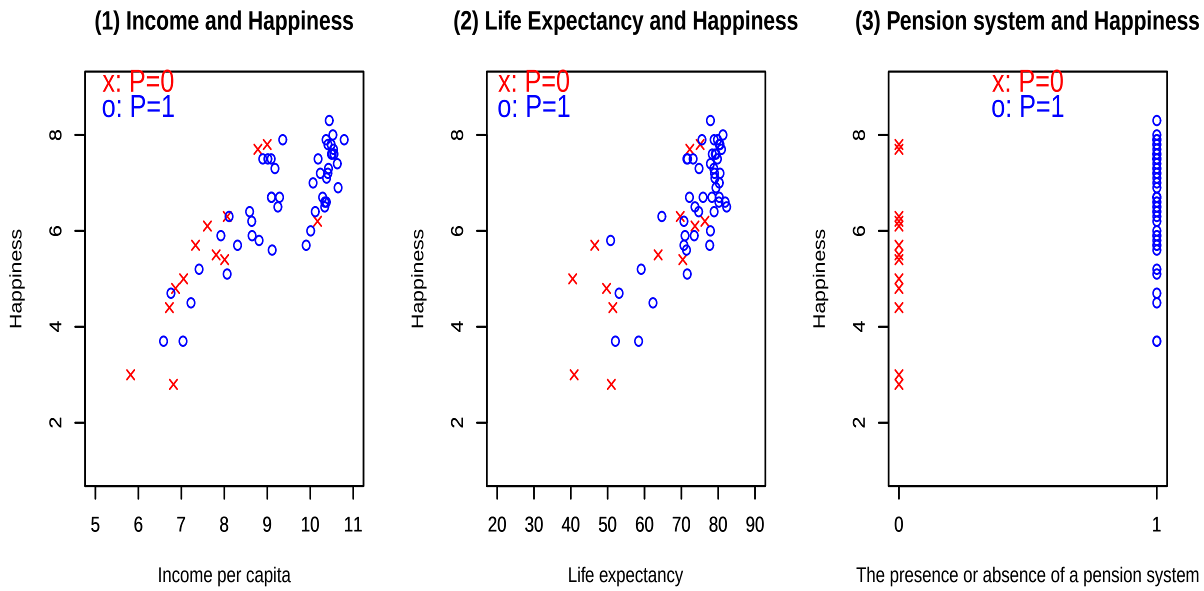

Figure 1: Income, life expectancy, pension system and happiness

Figure (1) plots the relationship among GDP per capita, life expectancy, the presence or

Table 1: Data sources

Indicators	Database	URL
Satisfaction with life (Happiness)	World Database of Happiness	http://worlddatabaseofhappiness.eur.nl/
Life expectancy	World Database of Happiness	http://worlddatabaseofhappiness.eur.nl/
Life expectancy at birth, total (years)	World Bank	http://data.worldbank.org/indicator/SP.DYN.LE00.IN
Happy life years*	World Database of Happiness	http://worlddatabaseofhappiness.eur.nl/
Per capita GDP	Penn World Table	https://pwt.sas.upenn.edu/

*) Happy life years are calculated by the product of Satisfaction with life and Life expectancy.

absence of a pension system and happiness. In Figure (1), each vertical axis shows the happiness. o's and +'s represent the countries that have the pension system and do not, respectively. It appears that there are positive relationships between income per capita and happiness, between life expectancy and happiness, between pension and happiness, respectively. The coefficients of correlation are $r(\text{income per capita}, \text{happiness}) = 0.833$, $r(\text{life expectancy}, \text{happiness}) = 0.788$, $r(\text{pension}, \text{happiness}) = 0.390$.

2.2 Regression Analysis

2.2.1 Happiness

We investigate the relationships between happiness and income per capita, between happiness and life expectancy, between happiness and pension system. The happiness is treated as the dependent variable. We use linear regression models as Eq. (1) to (4). Eq. (1) has a single regressor, income per capipa. Eq. (2) has two regressors, income per capita and life expectancy. And, Eq. (3) and Eq. (4) have two regressors, income per capita and the presence or absence of a pension system. We estimate the variables by the maximum likelihood esimation (MLE). The maximum likelihood estimation is a method to maximize the likelihood function in estimating the variables. H_i, y_i, L_i and P_i represent the happiness level, income per capita, life expectancy and the presence or absence of a pension system in a country i, respectively. Let us assume that the errors ϵ_i is identically distributed, independent random variables with $\epsilon_i \sim N(0, \sigma^2)$. Table 2 is the estimation results of Eq. (1) to (4), respectively.

$$H_i = \beta_0 + \beta_1 y_i + \epsilon_i \tag{1}$$

$$H_i = \beta_0 + \beta_1 y_i + \beta_2 L_i + \epsilon_i \tag{2}$$

$$H_i = \beta_0 + \beta_1 y_i + \beta_2 P_i + \epsilon_i \tag{3}$$

$$H_i = (1 - P_i)(\beta_0 + \beta_1 y_i) + P_i(\gamma_0 + \gamma_1 y_i) + \epsilon_i \tag{4}$$

Table 2: Estimation results

Parameters	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)
β_0	-0.756	-0.926	-0.904	-2.535
t value	-1.229	-1.515	-1.422	-1.870
β_1	0.789	0.587	0.824	1.036
t value	11.747	4.204	10.476	5.941
β_2	-	0.028	-0.220	-
t value	-	1.632	-0.852	-
γ_0	-	-	-	-0.632
t value	-	-	-	-0.768
γ_1	-	-	-	0.772
t value	-	-	-	8.922
σ	0.706	0.691	0.702	0.692
t value	11.047	11.045	11.045	11.045
log likelihood value	-65.32	-64.02	-64.96	-64.05
turning point $\frac{\gamma_0 - \beta_0}{\beta_1 - \gamma_1}$	-	-	-	7.206

The upper and lower show the estimated values and t values, respectively.

The estimated all β_1's for income per capita (y) in Eq. (1), (2) and (3) are positive and significant, but the estimated β_2's in Eq. (2) and Eq. (3) for life expectancy (L) and the presence or absence of a pension system (P) are not significant, respectively. We know that happiness can be almost explained by income per capita and that the life expectancy and the pension system contribute little to happiness. And we also know that from Eq. (4), the intercept ($\beta_0 < \gamma_0$) and the slope ($\beta_1 > \gamma_1$) of the line relating per capita income and happiness are statistically and significantly different by the presence or absence of a pension system. From the results, we can say that when income per capita is low, happiness is higher with the pension system than without it, otherwise, when income per capita is high, happiness is higher without the pension system than with it. The turning point is about income per capita $1,347 = \exp(7.206)$ which is small enough. When income level is low, the pension system make people happy. On the other hand, when income level is high, the pension system which is a kind of the forced saving policies prevents individuals from maximizing his/her utility. In other words, it suggests that to plan his/her own future and to optimize his/her utility by himself/herself can be happier without the pension.

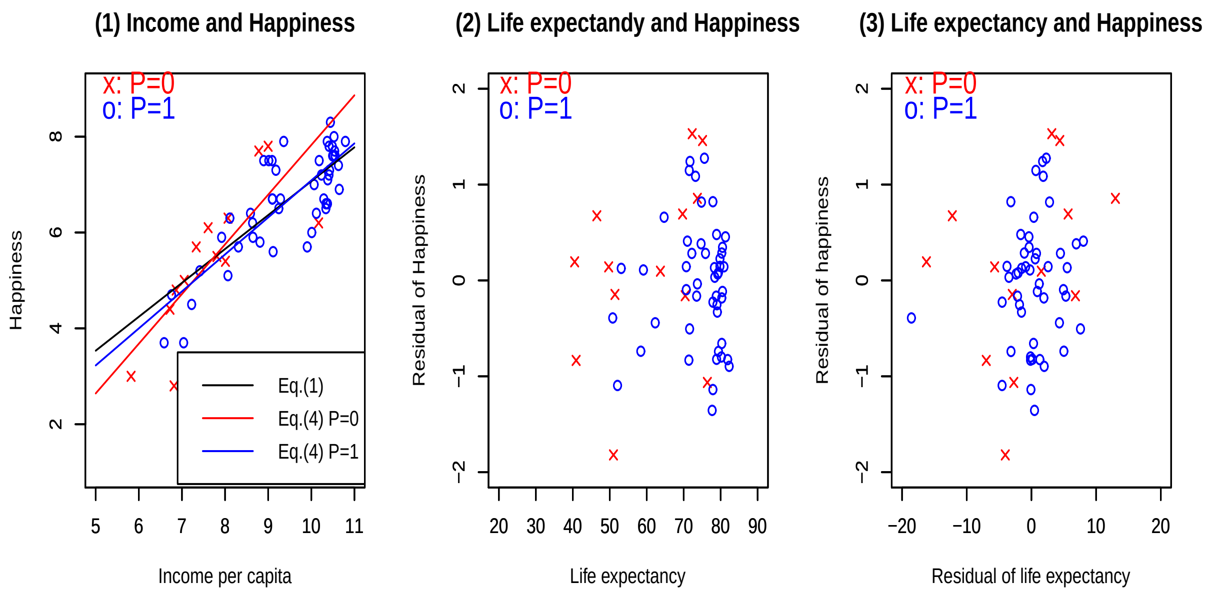

Figure 2: Income, life expectancy and happiness

We visualize the regression results from Eq. (1) to Eq. (4) in Figure 2. Figure 2 (1) shows regression lines based on Eq. (1) and Eq. (4). In comparison with the red line ($P = 0$) and the blue line ($P = 1$), the slope of the red line is steeper than that of the blue line. We can find that when income per capita is low, the blue line is upper than the red line, that is, happiness is higher with the pension than without it, but when income per capita is high, the opposite occures. The red line is upper than the blue line, that is, happiness is higher without the pension than with it. We draw the black line that best fits the data point in Figure 2 (1) from Eq. (1), and then, for each country, we measure the gap between the country's actual level of happiness and the level predicted by the fitted line. Figure 2 (2) shows the relationship between the residual part of happiness and life expectancy. Because happiness is explained by income per capita, there is not high corellatonship between the residuals and life expectancy. The correlation coefficient is (r=0.096). Figure 2 (2) means that β_2 of Eq. (2) is not significant. Figure 2 (3) shows the relationship between the residuals of life expectancy of the independent variable income per capita and the residuals of happiness of the independent variable income per capita. When we remove the influence of income on life expectancy and happiness, the relationship between both variables is not so strong (r=0.205). From the result of Figure 2 (2) and (3), we know that the positive correlationship between life expectancy and happiness in Figure 1 (2) is a spurious relationship. Even if we can see the relationship that the longer life expectancy looks happier, we know that two variables have no direct causal connection and the income per capita actually works behind the two variables, the life expectancy and happiness. The increasing life span without increasing income does not necessarily increase happiness. It unfortunately suggests that the survival itself is not always making utility high.

2.2.2 Life Expectancy and Happy Life Year

First, we investigate the relationships between the life expectancy and income per capita, between the life expectancy and the pension system. The life expectancy is treated as the dependent variable. We do regression analysis using Eq. (5) to Eq. (7). Eq. (5) has a single regressor, income per capita. Eq. (6) and Eq. (7) have two regressors, income per capita and the presence or absence of a pension system. We estimate the variables by the maximum likelihood esimation like as what we did in Section 2.2.1.

Second, we also investigate the relationships between the happy life year and income per capita, between the happy life year and the pension system. The happy life year is treated as the dependent variable. The happy life years are calculated by the product of the level of happiness and life expectancy. We do regression analysis using Eq. (8) to Eq. (10). HL_i represents the happy life year.

$$L_i = \beta_0 + \beta_1 y_i + \epsilon_i \tag{5}$$

$$L_i = \beta_0 + \beta_1 y_i + \beta_2 P_i + \epsilon_i \tag{6}$$

$$L_i = (1 - P_i)(\beta_0 + \beta_1 y_i) + P_i(\gamma_0 + \gamma_1 y_i) + \epsilon_i \tag{7}$$

$$HL_i = \beta_0 + \beta_1 y_i + \epsilon_i \tag{8}$$

$$HL_i = \beta_0 + \beta_1 y_i + +\beta_2 P_i + \epsilon_i \tag{9}$$

$$HL_i = (1 - P_i)(\beta_0 + \beta_1 y_i) + P_i(\gamma_0 + \gamma_1 y_i) + \epsilon_i \tag{10}$$

Table 3: Estimation results

Parameters	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (8)	Eq. (9)	Eq. (10)
β_0	6.070	7.568	-17.126	-39.142	-39.587	-55.242
t value	1.351	1.638	-1.834	-7.683	-7.480	-4.921
β_1	7.183	6.825	10.034	9.458	9.565	11.599
t value	14.652	11.947	8.356	17.016	14.611	8.034
β_2	-	2.223	-	-	-0.663	-
t value	-	1.183	-	-	-0.308	-
γ_0	-	-	17.240	-	-	-35.525
t value	-	-	3.043	-	-	-5.216
γ_1	-	-	6.036	-	-	9.064
t value	-	-	10.131	-	-	12.653
σ	5.156	5.097	4.762	5.845	5.841	5.726
t value	11.045	11.045	11.045	11.045	11.045	11.045
log likelihood value	-186.60	-185.91	-181.76	-194.26	-194.21	-193.00
turning point $\frac{\gamma_0-\beta_0}{\beta_1-\gamma_1}$	-	-	8.596	-	-	7.779

The upper and lower show the estimated values and t values, respectively.

Table 3 shows the estimation results by Eq. (5) to Eq. (10). The results in Table 3 are very similar to the results obtained with happiness as an independent variable in Table 1. The estimated all β_1's for income per capita (y) are positive and significant, but the estimated β_2's for the presence or absence of a pension system (P) are not significant. From Eq. (6) and Eq. (9), we know that because both the life expectancy and the happy life year are explained by income per capita, the pension system itself has no explanatory power to both the life expectancy and the happy life year. It means that the pension system itself cannot extend the life span and the happy life year. From Eq. (7) and Eq. (10), we know that depending on the presence or absence of a pension system, the intercept ($\beta_0 < \gamma_0$) and the slope ($\beta_1 > \gamma_1$) of the regression line are statistically and significantly different. From the results, we can say that when income per capita is low, the life expectancy and the happy life year are longer with the pension system than without it, otherwise, when income per capita is high, the life expectancy and the happy life year are longer without the pension system than with it. The turning points of Eq. (7) and Eq. (10)

are about income per capita $5,410 = \exp(8.596)$ and $2,390 = \exp(7.779)$, respectively. Anyhow, the values are low. In countries with low income levels, both the life expectancy and the happy life year are increased by the pension system, on the other hand, in countries with high income levels, both the life expectancy and the happy life year are decreased by the pension system. It suggests that in countries with a high level of income, the individual's utility optimizations are being hampered by the savings forced by the pension system.

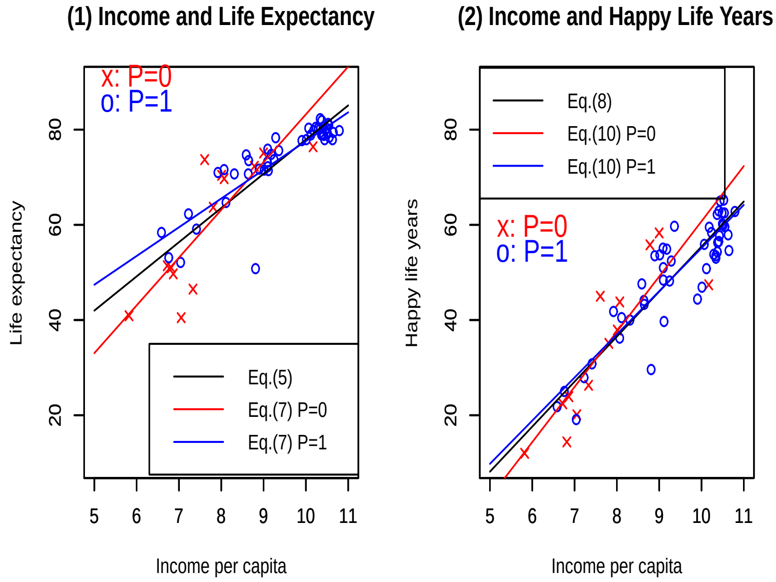

Figure 3: Income, life expectancy and happiness

Figure 3 shows regresion lines. Figure 3 (1) shows the relationship between income per capita and the life expectancy. Figure 3 (2) shows the relationship between income per capita and the happy life year. When we compare the red lines ($P = 0$) and the bule lines ($P = 1$), Figure 3 shows that the regression lines without the pension system are steeper than those with it.

2.2.3 Nonparametric estimation

We loosen the assumption of linear relationship between variables which we used in Section 2.2.1 and Section 2.2.2. We estimate the relationship by a nonparametric estimation which estimates the regression function without assuming any specific form. Eq. (11) to Eq. (14) are the unknown regression equations. The dependent variables in Eq. (11) to Eq. (14) are happiness, the life expectancy and the happy life years and happiness, respectively. Each equation from Eq. (11) to Eq. (13) have one regressor, income per capita and Eq. (14) has two regressors, income per capita and the pension system.

$$H_i = f(y_i) + \epsilon_i \tag{11}$$

$$L_i = f(y_i) + \epsilon_i \tag{12}$$

$$HL_i = f(y_i) + \epsilon_i \tag{13}$$

$$H_i = f(y_i, P_i) + \epsilon_i \tag{14}$$

Figure 4 (1) to (4) are the regression results of Eq. (11) to Eq. (14), respectively. The estimation results in Figure 4 (1) to (3) are not much different to the results obtained by assuming a linear relationship like as what we did before. The red lines ($P = 0$) without the pension system are steeper than the bule lines ($P = 1$) with the pension system. Because Eq. (14) has two regressors, we depict the regression result by the contour line in Figure 4 (4). The contour line shows the same level of happiness in the different combination of two variables which are income per capital and pension system. When income per capita is low, it can be seen that the contour lines are vertically-tilted. It means that regardless of whether the pension system is involved, the level of happiness is almost the same. However, as income per capita increases, the contour lines are no longer vertically-tilted. The contour lines begin to turn clockwise. For example, point A where income per capita which is 8.5 without the pension system and point B where income per capita which is 10.0 with the pension system have the same level of happiness, that is, the same level of happiness is seven. In other words, if they have the same income, it is happier when there is no pension system than when there is.

2.2.4 Check of Diminishing Marginal Increase of Life Expectancy

In countries with low income per capita, the increase of their life expectancy has a tendency to be big, in contrast, in countries with high income per capita, the increase of their life expectancy has a tendency to be small, that is, as income is increased, the marginal increase of the life expectancy is apt to decrease. We check whether the diminishing phenomenon is the cause that the regression lines in countries with low income per capita have steep slopes and the regression lines in countries with high income per capita have glacis slopes, because it is possible that countries with high income per capita tend to establish a pension system, in contrast, the countries with low income per capita tend not to establish a pension system.

We do regression analysis using Eq. (15) and Eq. (16).[2] We regress the change of life

[2] Because World Database of Happiness releases the average from 2000 to 2009 for life expectancy, there is no

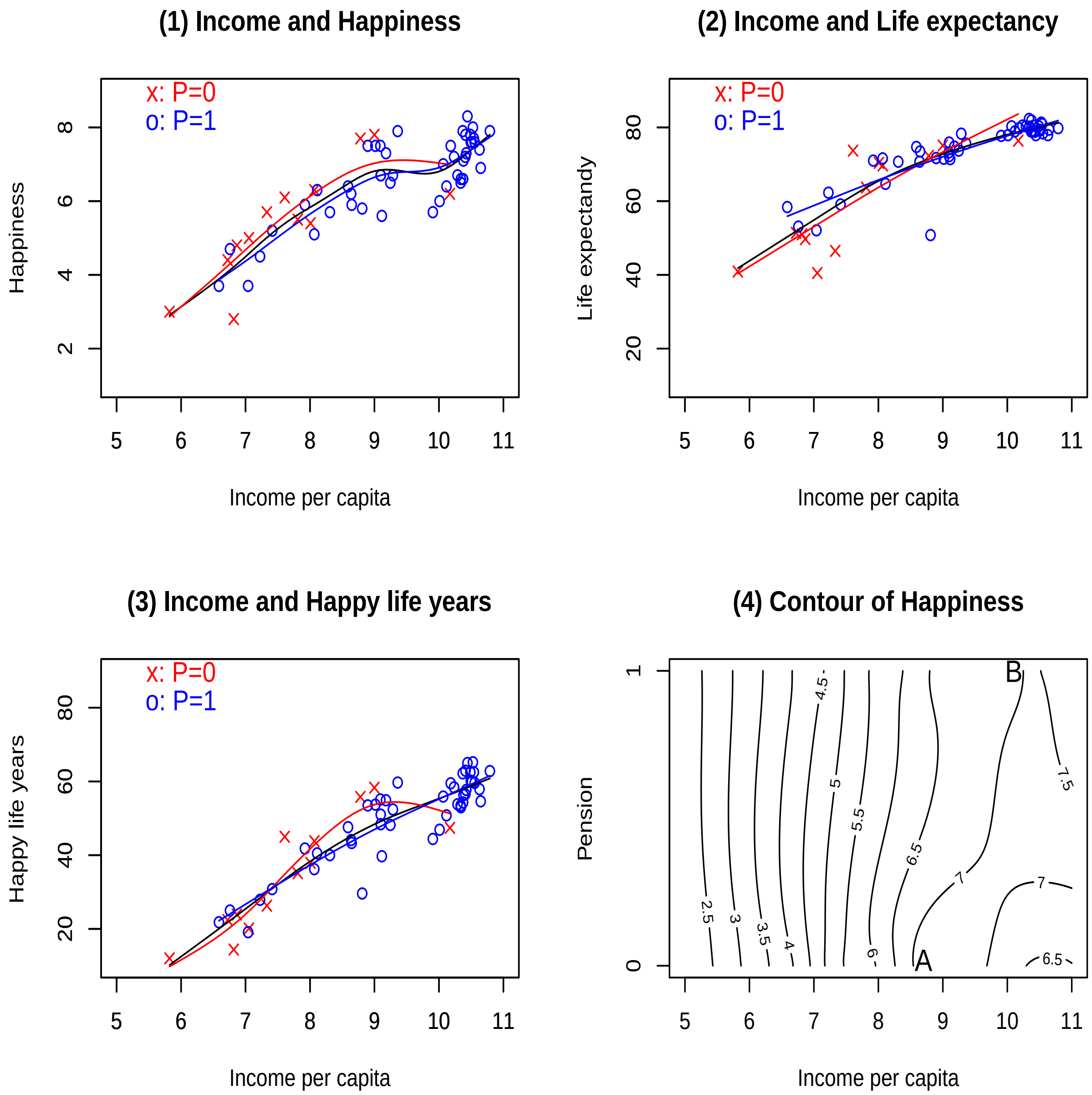

Figure 4: The results of nonparametric estimation

expectancy to the change of income $\left(\frac{\Delta L}{\Delta y}\right)$ on initial income (y_0) with Eq. (15). And, we also regress the elasticity of the life expectancy for the income changes $\left(\frac{\Delta L}{\Delta y}\frac{y_0}{L_0}\right)$ on initial income (y_0) with Eq. (16). y_0 and L_0 show the income per capita and life expectancy in 2000 as initial values, respectively. ΔL and Δy show the difference of values in the income per capita between 2000 to 2009 and the difference of values in the life expectancy between 2000 to 2009, respectively. The regression results from Eq. (15) and Eq. (16) show that the estimations for initial incomes are not significant and the coefficients of determination (R^2) are almost zero. Because we have used the

way to calculate the difference of life expectancy, $\Delta L = L_{2009} - L_{2000}$. Therefore, we use the annual data for the life expectancy and income per capita in World Bank and Penn World Table 7.1, respectively. The number of observations in common is 145.

logarithm for GDP per capita in this paper, it can be said that the diminishing marginal increase of the life expectancy are eliminated from the beginning. It is considered that the logarithm of income per capita and the life expectancy have a relationship close to linear.

$$\begin{aligned}\left(\frac{\Delta L}{\Delta y}\right)_i =&15.455 - 0.874 \times y_{0i} + \epsilon_i \\ t \text{ value } (0.324)\ &(-0.157)\ R^2 = 0.000\end{aligned} \tag{15}$$

$$\begin{aligned}\left(\frac{\Delta L}{\Delta y}\frac{y_0}{L_0}\right)_i =&2,030 - 0.120 \times y_{0i} + \epsilon_i \\ t \text{ value } (0.328)\ &(-0.167)\ R^2 = 0.000\end{aligned} \tag{16}$$

3 The benchmark Model

In this section, we build a utility maximization model to analyze the relationship between the life expectancy and happiness which we have seen in the previous section. The level of happiness and the lifetime utility will be used in the same meaning.

3.1 Setting

We consider an individual's utility maximization problem under the finite period. He can live up to T years old and die at the age of T. There is no uncertainty in the model and individuals have perfect foresight. An individual maximizes his lifetime utility which is affected by consumption. The instantaneous utility function is specified in log form as follows:

$$u(c) = \ln c \tag{17}$$

where c is a consumption. We think that it is possible to extend the life span by the efforts of the individual. We assume that there is a linear relationship between health investment and the life span as follows

$$T = a + bz,\ (a > 0,\ b > 0) \tag{18}$$

where T and z are life span and health investment, respectively. And a and b are positive constants. We assume that the health investments do not affect the utility directly.[3] We also assume that the interest earning is the only source of income of the individual. And to simplify,

[3]We can divide consumption c into two categories. These are the general consumption c^G and the consumption for health improvement c^H. The effect of the latter c^H on the utility of individual is unclear whether positive or negative. For example, there might be a person who drinks wheat grass for his health maintenance even though it

a small country is assumed, then the interest rate is constant at all period. Let us denote the individual's asset as x, then his budget constraint is written as:

$$\dot{x} = rx - c - z \tag{19}$$

where r is an interest rate.

An individual's utility maximization problem can be written as follows:

$$\begin{aligned} \max_{c(t),T} \int_0^T e^{-\rho t} \ln c(t)\ dt, \quad (0 < \rho < 1) \\ \text{s.t} \quad \dot{x}(t) = rx(t) - c(t) - z \end{aligned} \tag{20}$$

where ρ is a discount rate. We assume the ρ is constant, that is, our model is a exponential discounting model, not a hyperbolic discounting model which is treated in behavioral economics. We assume $r \geq \rho$.[4] For simplification, we assume that z has a constant value from initial period until T period and that z is decided at the initial period. In an unrealistic assumption, we assume that as an individual is born, he decides how much he invests for his health and how long he lives under a social environment.

3.2 Solving the Model

The maximization problem is solved in two stages. At the first stage, we do not consider the Eq. (18), that is, T is a given value, not a control variable. At the second stage, we consider the Eq. (18). First, maximize over c and x for any given T and z, and then the objective function maximized with respect to c and x could be described as a function of T and z. Second, maximize over T and z taking into account c obtained in the first stage.

3.2.1 The First Stage

We use the Hamiltonian method to solve the maximization problem. The Hamiltonian is written as follows:

$$H = \ln c + \lambda(rx - c - z) \tag{21}$$

is unpalatable, while there might be a person who drinks it with the thinking that it is delicious. There might be a person who commutes to the gym for his health maintenance though it is painful, while there might be a person who goes happily to the gym. Nutritional supplements are benificial for health but are not delicious. Therefore, we can assume that the consumption for health improvement c^H is neutral to an individual's utility. This means $\frac{\partial u(c^H)}{\partial c^H}$=0 and $u(c^H) = u(c^G, c^H)$.

[4]If $r = \rho$, there is no transitional path, because the jump from the initial state upto the terminal state occurs. If $r < \rho$, there is an overshooting, the economy turns back to the terminal state and has a negative growth rate. We do not consider the negative growth in our analysis.

By differentiating Eq. (21) with respect to c and x, we can get Eq. (22) and Eq. (23).

$$\frac{\partial H}{\partial c} = \frac{1}{c} - \lambda = 0 \quad \Rightarrow \quad c = \lambda^{-1}, \tag{22}$$

$$\frac{\partial H}{\partial x} = \rho\lambda - \dot{\lambda} = \lambda r \quad \Rightarrow \quad \frac{\dot{\lambda}}{\lambda} = \rho - r. \tag{23}$$

We integrate Eq. (23) to time t, then we get

$$\ln \lambda = (\rho - r)t + k \tag{24}$$

where k is a constant of integration. Taking exponential both sides of Eq. (24), then we can get

$$\lambda = C_1 e^{(\rho - r)t} \tag{25}$$

where $C_1 = e^k$. Substituting Eqs. (22) and (25) into Eq. (19), we obtain the following

$$\dot{x} - rx + z = -C_1^{-1} e^{-\rho t} e^{rt}. \tag{26}$$

This differential equation is solved as follow

$$x = \frac{1}{C_1}\Big(\frac{e^{-\rho t} - 1}{\rho}\Big)e^{rt} + C_2 e^{rt} + \frac{z}{r} \tag{27}$$

where C_2 is a constant. See Appendix A.1 for the detailed calculation. C_1 and C_2 can be obtained from substituting the initial condition and transversality condition. Let us $x(0) = x_0$, then we get C_2 as follows

$$C_2 = x_0 - \frac{z}{r}. \tag{28}$$

To maximize his utility, when dying, he uses up all his assets and leave nothing. In other words, $x(T) = 0$. We get C_1 as follows

$$C_1 = \frac{1}{\rho}\frac{1 - e^{-\rho T}}{x_0 - (1 - e^{-rT})\frac{z}{r}}. \tag{29}$$

Substituting Eqs. (28) and (29) into Eq. (27), we obtain the following

$$x(t) = \frac{x_0 - (1 - e^{-rT})\frac{z}{r}}{1 - e^{-\rho T}}(e^{-\rho t} - 1)e^{rt} + (x_0 - \frac{z}{r})e^{rt} + \frac{z}{r}. \tag{30}$$

Substituting Eq. (25) into Eq. (22), we can get

$$c(t) = \rho\frac{x_0 - (1 - e^{-rT})\frac{z}{r}}{1 - e^{-\rho T}}e^{(r-\rho)t}. \tag{31}$$

Eqs. (30) and (31) are the optimal paths of x and c, respectively by regarding the variable T and z as fixed.

3.2.2 The Second Stage

In the second stage, to maximize his lifetime utility, the individual chooses his optimal T with considering Eq. (18). We can rewrite the utility maximization problem as follows:

$$\begin{aligned} \max_T \int_0^T e^{-\rho t} \ln\Big(\rho \frac{x_0 - (1-e^{-rT})\frac{z}{r}}{1-e^{-\rho T}} e^{(r-\rho)t}\Big)\, dt \\ \text{s.t} \quad T = a + bz \end{aligned} \tag{32}$$

We solve the integral in Eq. (32), then we can induce Eq. (33)

$$\begin{aligned} &\int_0^T e^{-\rho t} \ln\Big(\rho \frac{x_0 - (1-e^{-rT})\frac{z}{r}}{1-e^{-\rho T}} e^{(r-\rho)t}\Big)\, dt \\ =& -\ln\Big(\rho \frac{x_0 - (1-e^{-rT})\frac{z}{r}}{1-e^{-\rho T}}\Big)\Big(\frac{e^{-\rho T}-1}{\rho}\Big) - (r-\rho)\Big(\frac{(\rho T+1)e^{-\rho T}-1}{\rho^2}\Big) \end{aligned} \tag{33}$$

See Appendix A.2 for the detailed calculation. Substituting Eq. (18) into Eq. (33), Eq. (32) can be rewritten as Eq. (34) which has no integral and has only one control variable T. Eq. (34) is just a static maximization problem, not a dynamic one.

$$\max_T \ \ln\Big(\rho \frac{x_0 - (1-e^{-rT})\frac{T-a}{rb}}{1-e^{-\rho T}}\Big)\Big(\frac{1-e^{-\rho T}}{\rho}\Big) + (r-\rho)\Big(\frac{1-(\rho T+1)e^{-\rho T}}{\rho^2}\Big) \tag{34}$$

We take the derivative of Eq. (34) with respect to T and set the first derivative to zero.

$$e^{-\rho T} \ln\Big(\rho \frac{x_0 - (1-e^{-rT})\frac{T-a}{br}}{1-e^{-\rho T}}\Big) - \frac{1-e^{-\rho T}}{\rho b} \frac{e^{-rT}(T-a) + (1-e^{-rT})\frac{1}{r}}{x_0 - (1-e^{-rT})\frac{T-a}{br}} - e^{-\rho T} + (r-\rho)Te^{-\rho T} = 0 \tag{35}$$

Eq. (35) is an implicit function as $f(x_0, T|a, b, r, \rho) = 0$ which is highly non linear and difficult to solve analytically.

4 Pension System

We introduce a pension system into the benchmark model. There is no uncertainty in the model. The purpose of this paper is to analyze the effect of the pension system on the maximized utility by comparing the maximized utility under the constraint by compulsory savings such as a pension system and the maximized utility without the constraint. The presence of uncertainty is not the essence in our model.[5]

He pays a pension p from 0 to s period, gets a pension q after s period to death. The government decides p, q and s which are constants as given to individuals. This pension system

[5] Even if uncertainty is introduced in the model, the same conclusion could be reached, because the true substance of this problem is the existence or lack of the constraint which prevents the utility from maximizing. We leave a model whcih introduces an uncertainty for further study.

plays as a compulsory saving for individuals. For simplification, we do not consider the balanced budget of the government for the pension system. It can be a fully funded system or a pay-as-you-go pension system, because we do not need to consider where the financial resources of pension come from, under the situation where there is no need for the balanced budget.

We shall call the period from 0 to s period as young period and after s period as old period. His budget constraint Eq. (19) is changed to Eq. (36).

$$\dot{x} = \begin{cases} rx - c - z - p, & \text{if} \quad 0 \le t \le s \\ rx - c - z + q, & \text{if} \quad s < t \le T. \end{cases} \tag{36}$$

The way to solve the model with this pension system is similar to that of the benchmark model even though we have to divide it into young period and old period. Eq. (27) is changed as follow

$$x = \begin{cases} \frac{1}{C_1^Y}\left(\frac{e^{-\rho t}-1}{\rho}\right)e^{rt} + C_2^Y e^{rt} + \frac{z+p}{r}, & \text{if} \quad 0 \le t \le s \\ \frac{1}{C_1^O}\left(\frac{e^{-\rho t}-1}{\rho}\right)e^{rt} + C_2^O e^{rt} + \frac{z-q}{r}, & \text{if} \quad s < t \le T. \end{cases} \tag{37}$$

where, C_1^Y, C_2^Y, C_1^O and C_2^O are constants of integration which are as follows:

$$C_1^Y = \frac{1}{\rho}\frac{1 - e^{-\rho s}}{x_0 - (1 - e^{-rs})\frac{z+p}{r} - x(s)e^{-rs}} \tag{38}$$

$$C_2^Y = x_0 - \frac{z+p}{r} \tag{39}$$

$$C_1^O = \frac{1}{\rho}\frac{(e^{-\rho T} - e^{-\rho s})e^{rs}}{\frac{z-q}{r}(1 - e^{r(s-T)}) - x(s)} \tag{40}$$

$$C_2^O = \frac{\frac{z-q}{r}(1 - e^{r(s-T)}) - x(s)}{(e^{-\rho T} - e^{-\rho s})e^{rs}}(1 - e^{-\rho T}) - \frac{z-q}{r}e^{-rT} \tag{41}$$

where, $x(s)$ is interpreted as both the terminal value of young period and the initial value of old period at the same time. By the same way as the previous, Eq. (31) is changed as follows

$$c(t) = \begin{cases} \frac{1}{C_1^Y}e^{(r-\rho)t}, & \text{if} \quad 0 \le t \le s \\ \frac{1}{C_1^O}e^{(r-\rho)t}, & \text{if} \quad s < t \le T. \end{cases} \tag{42}$$

Substituting Eq. (42) into the utility function, we obtain the following

$$\int_0^s e^{-\rho t}\ln\Big(\frac{1}{C_1^Y}e^{(r-\rho)t}\Big)\,dt + \int_s^T e^{-\rho t}\ln\Big(\frac{1}{C_1^O}e^{(r-\rho)t}\Big)\,dt \tag{43}$$

We integrate Eq. (43) to time t, then we get

$$\ln\Big(\frac{1}{C_1^Y}\Big)\frac{1 - e^{-\rho s}}{\rho} + \ln\Big(\frac{1}{C_1^O}\Big)\frac{e^{-\rho s} - e^{-\rho T}}{\rho} - (r-\rho)\Big(\frac{(\rho T + 1)e^{-\rho T} - 1}{\rho^2}\Big). \tag{44}$$

There are z's in C_1^Y, C_2^Y, C_1^O and C_2^O. We substitute $z = \frac{T-a}{b}$ into C_1^Y, C_2^Y, C_1^O and C_2^O, then, the original dynamic optimization problem with the pension system is nothing less than the static optimization problem with respect to T and $x(s)$ as seen in Eq. (45). In other words, all he has to do is to decide his own life expectancy and the initial asset at the old period.

$$\begin{aligned}\max_{T,x(s)} U\big(T,x(s)\big) = &\ln\Big(\frac{1}{C_1^Y\big(T,x(s)\big)}\Big)\frac{1-e^{-\rho s}}{\rho} + \ln\Big(\frac{1}{C_1^O\big(T,x(s)\big)}\Big)\frac{e^{-\rho s}-e^{-\rho T}}{\rho} \\ &-(r-\rho)\Big(\frac{(\rho T+1)e^{-\rho T}-1}{\rho^2}\Big)\end{aligned} \tag{45}$$

5 Results and Conclusion

Taking the derivative of Eq. (45) with respect to T and $x(s)$, and setting each first derivatives to zero, and solving the system of equations, we could obtain the optimal T^* and $x(s)^*$. Since the profit function of Eq. (45) is highly nonlinear, however, it is very difficult to get an exact analytical solution for this problem. The alternative option is to provide the solutions numerically. The suitable parameter values are used for the calculation, though they are arbitrary. The parameter values that we use to calculate are the following: $a = 20$, $b = 10$, $x_0 = 100$, $\rho = 0.01$, $r = 0.02$. In order to investigate the effects of only the pension system, not including the effect of income, we put initial income as a constant. And we control the parameters for pension system i.e., p, q and s which are the amount of payment for pension, the amount of pension gratuity and the period of payment for pension, respectively. To show the effects of p, q and s on the life expectancy and the lifetime utility, p and q are controlled from 0.0 to 2.0, respectively, and s is controlled from 0.0 to 20.

Each panel in Figure 5 shows the results as the contour lines. Figure 5 (1) and 5 (2) show the results of the life expectancy and the lifetime utility level, respectively, when s is fixed at 10.0 while p and q are changed. In Figure 5 (1) and 5 (2), the values on the left-upper side are high and the values on the right-lower side are low. Under fixed s, when p is small and q is big, the life expectancy is longer and the lifetime utility level is higher. Figure 5 (3) and 5 (4) show the results of the life expectancy and the lifetime utility level, respectively, when q is fixed at 1.0 while p and s are changed. In Figure 5 (3) and 5 (4), the values on the left-lower side are high and the values on the right-upper side are low. Under fixed q, when p is small and s is short, the life expectancy is longer and the lifetime utility level is higher. Figure 5 (5) and 5 (6) show

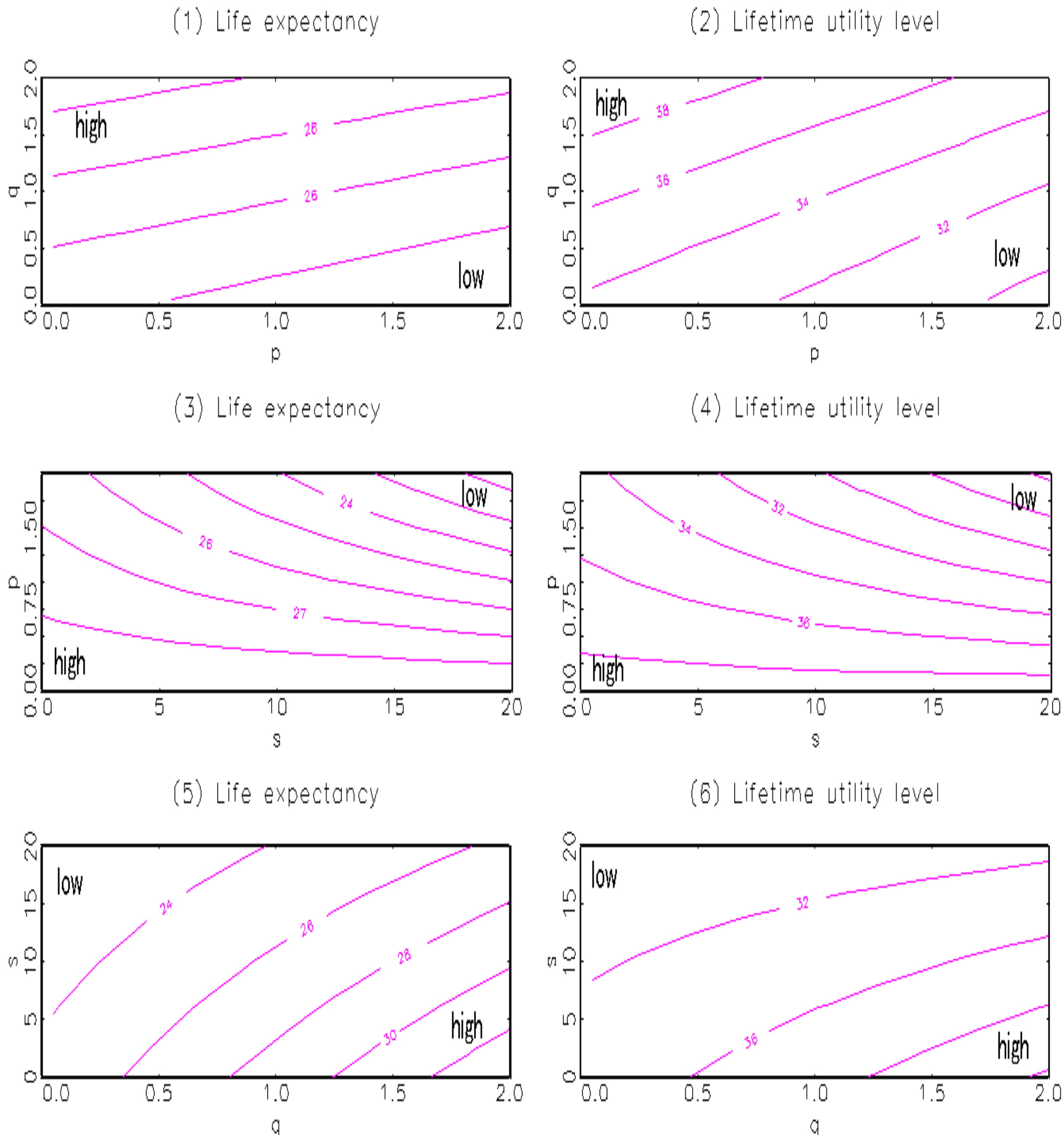

Figure 5: Life expectancy and lifetime utility level

the results of the life expectancy and the lifetime utility level, respectively, when p is fixed at 1.0 while q and s are changed. In Figure 5 (5) and 5 (6), the values on the right-lower side are high and the values on the left-upper side are low. Under fixed p, when q is big and s is short, the life expectancy is longer and the lifetime utility level is higher.

To summarize these results, when p is small, when q is big, and when s is short, that is, when an individual pays a small amount of money for a short period of time and gets a big amount of money from his pension, the life expectancy is longer and the lifetime utility level is higher. The increase in lifetime utility is hardly astonishing because the lifetime budget constraint of an individual is shifted to the right when an individual receives more benefits from the pension system and has not to pay for it. These results accord with intuition.

Next, we show the relationship among the life expectancy, the lifetime utility and the pension system through the combination of p, q and s and compare them with the results from the cross section data in Section 2.

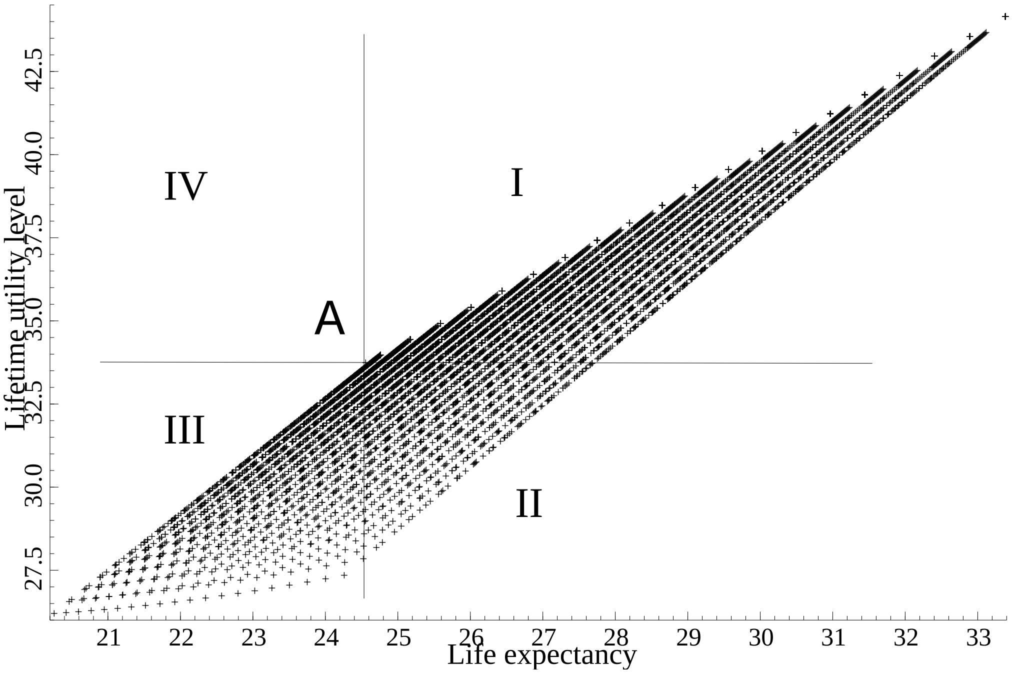

Figure 6: Comparison of the results with and without the pension system

Figure 6 plots the relationship between the life expectancy and the lifetime utility level. By changing of the paremeters for pension system which are the amount of payment for pension, the amount of pension gratuity and the period of payment for pension, respectively, we get the combinations of life expectation and life utility. The horizontal line and the vertical line show the life expectancy and the lifetime utility level, respectively.[6] The +'s in Figure 6 are the corresponding values of the life expectancy in Figure 5 (1), 5 (3) and 5 (5), and the lifetime utility level in Figure 5 (2), 5 (4) and 5 (6). And point A (24.556, 33.742) shows the pair of the life expectancy and the lifetime utility level obtained from the benchmark model. All of these +'s

[6]The figures of the life expectancy tell nothing about the relative length of life expectancy. As the concept of the ordinal utility, the differences in the figures of the life expectancy are treated as meaningless. The figures do not mean the number of years.

except point A show the pairs when the pension system exists in some way or another. We draw a vertical and horizontal line from point A and divide the plain into 4 areas. In area I, the life expectancy is longer and the lifetime utility level is higher compared to point A. In area II, the life expectancy is longer but the lifetime utility level is lower compared to point A. In area III, the life expectancy is shorter and the lifetime utility level is lower compared to point A. There is no pair in area IV.

The life expectancy is not always proportional to the lifetime utility level. Comparing any '+' in area II and point A, even though the life expectancy is longer, the lifetime utility level is lower. And the pension system can make life expectancy longer or shorter and can make lifetime utility level higher or lower. If we get big amounts of pension in the future, the life expectancy can be extended and the lifetime utility can go up. It is the most preferable, however, in today's reality, the pension system cannot avoid the problem of financial resources.

The pension system could also lead to some kind of infirmity as follows: 1) Even though the life expectancy is extended, the lifetime utility level goes down. By that, an individual is forced to pay the pension during his young period, the pension system leads to less personal consumption in his young period. Even though he tries to prolong his life for a long time to get his money back which he paid mandatorily, his lifetime utility level can go down compared to a case of no pension system. As rising longevity incited by the pension system, the years they gain in life expectancy may not be healthy ones, so the increase in life expectancy requires more savings for health-care spending in his/her old age and less consumption through his/her wholelife. It is also confirmed from the data in Section 2 that the increase in life expectancy without an increase in income does not affect too much utility. For example, this is the case of the right shift from point A on the horizontal line. 2) The life expectancy is decreased, moreover, the lifetime utility level goes down. This is the worst scenario. An individual can choose a short life to refuse to pay the pension until such period s and to increase his consumption in his young period.[7]

The public pension system which is a compulsory savings crowds out the private savings and can prevent the utility maximization. It is not always true that the pension system improves the lifetime utility level as shown in area II and III in Figure 6. As we have seen in Section 2 with the cross section data in the developed countries, the level of happiness and life excpectancy in the case of no pension system are higher then those in the case with pension system. Not only the

[7]There was an accident reported in South Korea last 2005, where in a person who was against the compulsory pension system and was in arrears with his pension took away his life.

government has to exert effort to avoid infirmities as stated above, but the government also has to reconsider about the raison d'etre (the reason for existence) of the compulsory pension system.

Appendix

A.1 Derivation of Eq. (27)

Let us put $B = -C_1^{-1}$. Multiplying both sides of Eq. (26) by e^{-rt} and integrating to time t, we get the following

$$\begin{aligned}(\dot{x} - rx + z)e^{-rt} &= Be^{-\rho t} \\ xe^{-rt} - \frac{z}{r}e^{-rt} + D_1 &= -B\frac{e^{-\rho t}}{\rho} + D_2\end{aligned} \tag{A1}$$

where D_1 and D_2 are constants of integration. Eq. (A1) can be arranged as follows

$$\begin{aligned}xe^{-rt} - \frac{z}{r}e^{-rt} &= -B\frac{e^{-\rho t}}{\rho} + B\frac{1}{\rho} + C_2 \\ \Big(x - \frac{z}{r}\Big)e^{-rt} &= -B\Big(\frac{e^{-\rho t} - 1}{\rho}\Big) + C_2\end{aligned} \tag{A2}$$

where $C_2 = D_2 - D_1 - B\frac{1}{\rho}$. Multiplying both sides of Eq. (A2) by e^{-rt} and substituting $B = -C_1^{-1}$ into Eq. (A2), Eq. (A2) can be arranged as Eq. (27).

A.2 Derivation of Eq. (33)

Let us put $A = \ln\Big(\rho\frac{x_0 - (1-e^{-rT})\frac{z}{r}}{1-e^{-\rho T}}\Big)$.

$$\begin{aligned}&\int_0^T e^{-\rho t}\ln\Big(\rho\frac{x_0 - (1-e^{-rT})\frac{z}{r}}{1-e^{-\rho T}}e^{(r-\rho)t}\Big)\,dt &&= \int_0^T \big[Ae^{-\rho t} + (r-\rho)te^{-\rho t}\big]\,dt \\ =&A\int_0^T e^{-\rho t}\,dt + (r-\rho)\int_0^T te^{-\rho t}\,dt &&= A\Big[-\frac{e^{-\rho t}}{\rho}\Big]_0^T - (r-\rho)\Big[\frac{(\rho t+1)e^{-\rho t}}{\rho^2}\Big]_0^T \\ =&-A\Big(\frac{e^{-\rho T}-1}{\rho}\Big) - (r-\rho)\Big(\frac{(\rho T+1)e^{-\rho T}-1}{\rho^2}\Big)\end{aligned} \tag{A3}$$

Substituting $A = \ln\Big(\rho\frac{x_0 - (1-e^{-rT})\frac{z}{r}}{1-e^{-\rho T}}\Big)$ into Eq. (A3), Eq. (A3) can be arranged as Eq. (33).

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Table 4: Data Set

Country	Pension	Happiness	Life expectancy	Happy life years	GDP per capita
Argentina	1	7.3	74.8	54.9	9687
Australia	1	7.7	80.9	62.5	37863
Austria	1	7.6	79.4	60.2	36075
Belgium	1	7.3	78.8	57.7	33656
Bolivia	1	6.3	64.7	40.5	3331
Brazil	1	7.5	71.7	53.5	7297
Burkina Faso	0	4.4	51.4	22.4	831
Canada	1	7.8	80.3	62.5	35836
Chile	1	6.7	78.3	52.4	10783
Colombia	0	7.7	72.3	55.8	6514
Denmark	1	8.3	77.9	65.0	34283
Dominican Republic	1	7.5	71.5	53.7	8215
Ecuador	1	6.4	74.7	47.6	5374
Egypt	1	5.7	70.7	40.0	4059
Finland	1	7.9	78.9	62.2	31840
France	1	6.6	80.2	53.0	30952
Germany	1	7.1	79.1	56.3	32214
Ghana	1	5.2	59.1	30.8	1659
Greece	1	6.4	78.9	50.8	24764
Hong Kong	1	6.6	81.9	54.3	32048
India	0	5.5	63.7	35.1	2466
Indonesia	0	6.3	69.7	43.8	3192
Ireland	1	7.6	78.4	59.6	38301
Israel	1	7.0	80.3	55.9	23533
Italy	1	6.7	80.3	53.8	29384
Jamaica	1	6.7	72.2	48.4	8930
Japan	1	6.5	82.3	53.5	30858
Kenya	1	3.7	52.1	19.1	1141
Korea, South	1	6.0	77.9	46.9	22254
Madagascar	1	3.7	58.4	21.8	725
Malaysia	1	6.5	73.7	48.2	10366
Mali	1	4.7	53.1	25.0	862
Mexico	1	7.9	75.6	59.7	11611
Morocco	0	5.4	70.4	37.9	3006
Netherlands	1	7.6	79.2	59.9	36687
New Zealand	1	7.5	79.8	59.5	26436
Nigeria	0	5.7	46.5	26.3	1527
Norway	1	7.9	79.8	62.8	48534
Panama	0	7.8	75.1	58.3	8095
Peru	1	6.2	70.7	44.1	5642
Philippines	1	5.9	71.0	41.8	2753
Portugal	1	5.7	77.7	44.4	20032
Senegal	1	4.5	62.3	27.9	1374
Singapore	1	6.9	79.4	54.6	42103
South Africa	1	5.8	50.8	29.6	6697
Spain	1	7.2	80.5	58.4	27852
Sri Lanka	1	5.1	71.6	36.2	3186
Sweden	1	7.8	80.5	62.9	33235
Switzerland	1	8.0	81.3	65.2	37295
Taiwan	0	6.2	76.4	47.4	26107
Tanzania	0	2.8	51.0	14.4	913
Tunisia	1	5.9	73.5	43.3	5692
Turkey	1	5.6	71.4	39.7	9084
Uganda	0	4.8	49.7	23.9	959
United Kingdom	1	7.2	79.0	56.5	33166
United States	1	7.4	77.9	57.9	41327
Uruguay	1	6.7	75.9	51.0	8927
Venezuela	1	7.5	73.2	55.1	8876
Viet Nam	0	6.1	73.7	45.0	2014
Zambia	0	5.0	40.5	20.1	1156
Zimbabwe	0	3.0	40.9	12.0	337
Average	0.79	6.4	71.2	46.6	16622
Min	0	2.8	40.5	12.0	337
Max	1	8.3	82.3	65.2	48534

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