Portfolio risk evaluation: An approach based on dynamic conditional correlations models and wavelet multiresolution analysis

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Abstract

We analyzed the volatility dynamics of three developed markets (U.K., U.S. and Japan), during the period 2003-2011, by comparing the performance of several multivariate volatility models, namely Constant Conditional Correlation (CCC), Dynamic Conditional Correlation (DCC) and *consistent* DCC (*c*DCC) models. To evaluate the performance of models we used four statistical loss functions on the daily Value-at-Risk (VaR) estimates of a diversified portfolio in three stock indices: FTSE 100, S&P 500 and Nikkei 225. We based on one-day ahead conditional variance forecasts. To assess the performance of the abovementioned models and to measure risks over different time-scales, we proposed a wavelet-based approach which decomposes a given time series on different time horizons. Wavelet multiresolution analysis and multivariate conditional volatility models are combined for volatility forecasting to measure the comovement between stock market returns and to estimate daily VaR in the time-frequency space. Empirical results shows that the asymmetric *c*DCC model of Aielli (2008) is the most preferable according to statistical loss functions under raw data. The results also suggest that wavelet-based models increase predictive performance of financial forecasting in low scales according to number of violations and failure probabilities for VaR models.

Key words: Dynamic conditional correlations, Value-at-Risk, wavelet decomposition, Stock prices

1. Introduction

Measuring market risk is the most interest of financial managers and traders. The most widely used measure of risk managment is the Value-at-Risk (VaR), which was introduced by Jorion (1996). Forecasting VaR is based on the volatility models. The well-known volatility model is the generalized autoregressive conditional heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986). The success of this model has subsequently led to a family of univariate and multivariate GARCH models which can capture different behavior in financial returns. The development of this family of models has led to the development of conditional VaR forecasts.

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The literature on multivariate GARCH models is large and expanding. Engle and Kroner (1995) defined a general class of multivariate GARCH (MGARCH) models. The popular ones are *diagonal* VECH model of Bollerslev et al. (1988), the BEKK model of Engle and Kroner (1995). While, popular, these models have limitations.[1] In particular, *diagonal* VECH lacks correlation between the variance terms, BEKK can have poorly behaved likelihood function (making estimation difficult, especially for models with more than two variables), and VECH has a large number of free parameters (which makes it impractical for models with more than two variables). To deal with the curse of the dimensionality Engle (2002) proposed the dynamic conditional correlations (DCC) model which generalizes the specification of Bollerslev (1990) by assuming a time variation of correlation matrix. Tse and Tsui (2002) defined a multivariate GARCH model which includes time-varying correlations and yet satisfies the positive-definite condition. Ling and McAleer (2003), McAleer et al. (2009) and Caporin and McAleer (2009, 2010) proposed another family of multivariate GARCH models which assume constant conditional correlations, and do not suffer from the problem of dimensionality, by comparison with the VECH and BEKK models. The convenient of these models is that modeling conditional variances allows large shocks to one variable to affect the variances of the other variables.

Another important topic in the financial econometrics is the asymmetric behavior of conditional variances. The basic idea is that negative shocks have a different impact on the conditional variance evolution than do positive shocks of similar magnitude. Cappiello et al. (2006), Aielli (2008) and Palandri (2009) extended the DCC model of Engle (2002) to an asymmetric DCC model which is a generalization of the DCC model (authors develop a model capable of allowing for conditional asymmetries not only in volatilities but also in correlations). Authors used several asymmetric versions of the DCC model. In our article, we based on version which allows asymmetry in conditional variance. This phenomenon was raised by Nelson (1991) in introducing the Exponential GARCH (EGARCH) model, and was also considered by Glosten et al. (1993) (GJR-GARCH models), Rabemananjara and Zakoian (1993) and Zakoian (1994) (Threshold GARCH models) for the univariate case. More recently, in the area of finance, several empirical studies based on symmetric and asymmetric multivariate GARCH models have been employed. For instance, Cappiello et al. (2006) used the asymmetric generalized-DCC (AG-DCC) specification to investigate asymmetries in conditional variances and correlation dynamics of three groups of countries (Europe, Australia and North America). They provide evidence that equity returns show strong evidence of asymmetries in conditional volatility, while little is found for bond returns. Chiang et al. (2007) applied a DCC model to nine Asian stocks and confirm a contagian effect during the Asian crisis. Ho et al. (2009) applied various multivariate GARCH models to investigate the evidence of asymmetry and time-varying conditional correlations between five sectors of Industrial Production of the United States. They provide also, strong evidence of asymmetric conditional volatility in all sectors and some support of

[1]See Bauwens et al. (2006) for more details.

time-varying correlations in various sectoral pairs. Büttner and Hayo (2011) used a bivariate DCC model to extract dynamic conditional correlations between European stock markets. Chang et al. (2011) employed several multivariate GARCH models (CCC, DCC, VARMA-GARCH and VARMA-AGARCH) to model conditional volatility in the returns of rubber spot and futures in major rubber futures and rubber spot Asian markets, in order to investigate volatility transmissions across these markets. Their results provide presence of volatility spillovers and asymmetric effects of positive and negative return shocks on conditional volatility. Kenourgios et al. (2011) applied the AG-DCC model and a multivariate regime-switching Gaussian copula model to capture non-linear correlation dynamics in four emerging equity markets (Brazil, Russia, India and China) and two developed markets (U.S. and U.K.). Arouri et al. (2011) employed a VAR-GARCH approach to investigate the return linkage and volatility transmission between oil and stock markets in Gulf Cooperation Council countries. Lahrech and Sylwester (2011) used DCC multivariate GARCH models to examine the dynamic linkage between U.S. and Latin American stock markets. Their results show an increase in the degree of co-movement between Latin American equity markets and U.S. equity ones.

In this study, we employ three multivariate GARCH models, such as the CCC model of Bollerslev (1990), the DCC model of Engle (2002) and the *c*DCC model of Aielli (2008). These models impose a useful structure on the many possible model parameters. However, parameters of the model can easily be estimated and the model can be evaluated and used in straightforward way. Our empirical methodology follows a two-step approach. The first step applies these dynamic conditional correlation models to model conditional volatility in the returns of three developed stock markets (U.K., U.S. and Japan stock markets), in order to examine the evidence of time-varying conditional variances and correlations between stock markets. Moreover, in order to show the asymmetric effects of positive and negative return shocks on conditional volatility the EGARCH and GJR-GARCH models of Nelson (1991) and Glosten et al. (1993), respectively, were employed for modeling univariate conditional volatility. In the second step, we re-examine the dynamic conditional correlation analysis among the three major developed stock markets through a novel approach, wavelet analysis. This technique is a very promising tool as it is possible to capture the time and frequency varying features of co-movement within an unified framework which represents a refinement to previous approaches. This wavelet-based analysis takes account the distinction between the short and long-term investor. From a portfolio diversification view, there exist a kind of investors whose are more interested in the co-movement of stock returns at higher frequencies (lower scales), that is, short-term fluctuations, and also, there exist a kind of investors whose focuse on the relationship at lower frequencies (higher scales), that is, long-term fluctuations. The study of the co-movement of stock market returns, i.e. dynamics of variances and correlations, across scales is crucial for risk assessment of a portfolio. In terms of risk management, a higher co-movement (higher covariances) among assets of a given portfolio implies lower gains. According to investors or traders, evaluating the co-movement of assets is a great importance to best assess the risk of a portfolio. Several applications of wavelet studying the co-movement of stock indices have been recently applied by Sharkasi et al. (2005), Rua and Nunes (2009), Rua (2010) and Masih et al. (2010).

In this paper, we investigate also the selection of the multivariate GARCH models used in the two approaches (with and without wavelet analysis) to identify which model has the best out-of-sample forecasting performance. The assessment of the forecast performance of these models is based on out-of-sample one-day ahead conditional matrix forecasts. However, to measure model performance we used four statistical loss functions.[2]

The empirical evidence showed that the conditional variances and correlations of U.K., U.S. and Japan stock market returns were dynamic and the three markets were highly correlated. We showed also, that the *c*DCC model of Aielli (2008) is preferable than the CCC and DCC models based on one-day ahead out-of-sample forecasts. With regard to wavelet-based multivariate conditional volatility approach, our findings suggested a multi-scale behavior of the three markets under study, which decomposes the total spillover into three sub-spillovers and decomposes the market risk measured by VaR into wavelet VaR (WVaR) estimates. In addition, this new approach help traders and investors to reduce risk management on their investing time horizons.

The purpose of the paper is four-fold. First, we estimate multivariate conditional volatility for stock market returns using several recent models of multivariate conditional volatility. Second, we investigate the importance of volatility spillovers on the conditional variance across the three developed markets. Third, we focuse on the forecasting performance of the multivariate conditional volatility models under study for the last 250 days of the data set. Forecast comparison is based on four different loss functions including the mean squared error, the mean absolute error, the mean absolute percentage error and the logarithm loss error. Fourth, we propose a wavelet-based multi-resolution analysis in order to combine between traditional multivariate conditional volatility models and wavelet decomposition. The combination of wavelet decomposition and dynamic conditional correlation models was introduced to analyze the comovements and volatility spillovers between three developed markets on multi-scale framework, based on behavior of investors. Finally, we compare the performance of wavelet-based multivariate conditional volatility model against the traditional one for one-day ahead forecast.

The structure of the paper is organized as follows: Section 2 presents the data used for the empirical analysis and the multivariate conditional volatility methodology. Section 3 reports the empirical results under raw data. Wavelet analysis is discussed in section 4. Finally, concluding remarks are stated in section 5.

[2] Hansen et al. (2003), Hansen and Lunde (2005), Becker and Clements (2008) and others, showed that evaluation of univariate volatility forecast is well understood, while for an applied point of view ther are no clear guidelines available on model evaluation and selection in multivariate setting (see Laurent et al. (2011)).

2. Methodology and empirical specifications

2.1. Data

Our data on stock market prices consist of the S&P 500, FTSE 100 and NIKKEI 225 composite indices for U.S., U.K. and Japan. We collect daily data over the period from January 01, 2003 to February 04, 2011. Indices are obtained from DataStream. We use daily data in order to retain a high number of observations to adequately capture the rapidity and intensity of the dynamic interactions between markets.

Returns of market i (index i) at time t are computed as $r_{i,t} = \log(P_{i,t}/P_{i,t-1}) \times 100$, where $P_{i,t}$ and $P_{i,t-1}$ are the closing prices for day t and $t-1$, respectively.

2.2. Descriptive statistics

The summary statistics of the data are given in Table 1. In panel A (Table 1), for each return series, the mean value is close to zero. For each return series the standard deviation is larger than the mean value. Each return series displays a small amount of skewness and large amount of kurtosis and the returns are not normally distributed.

In panel B (Table 1), unconditional correlation coefficients in stock market index returns indicate strong pairwise correlations. The correlation between S&P 500 and FTSE 100 is positive and larger than the correlation between NIKKEI 225 and FTSE 100. This could be due to the high trade share between the two markets.

Table 1
Descriptive statistics of stock market returns.

Panel A: Descriptive statistics.

	Mean	Max	Min	Std.dev	Skewness	Kurtosis	Jarque-Bera
FTSE 100	0.0199	9.384	-9.265	1.252	-0.0914	9.019	7180.19*
S&P 500	0.0187	10.957	-9.469	1.314	-0.256	11.410	11508.39*
NIKKEI 225	0.0097	13.234	-12.111	1.544	-0.466	8.889	7048.35*

Panel B: Unconditional (market return) correlation matrix

	FTSE 100	S&P 500	NIKKEI 225
FTSE 100	1.000	0.55	0.34
S&P 500		1.000	0.11
NIKKEI 225			1.000

Notes: * denotes the rejection of the null hypothesis of normality at the 1% level of the Jarque-Bera test. The data frequency is daily and covers the period from 01 January 2003 to 04 February 2011.

The results of the unit root tests for all sample of level prices (in logs) and returns in each market are summarized in Table 2. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests are used to explore the existence of unit roots in individual series. The results show that all returns are stationary.

Table 2
Unit root tests.

Panel A: Stock market level prices (in log)					
Prices	ADF test			Phillips-Perron test	
	None	Constant	Constant and trend	Constant	Trend
FTSE 100	0.724	-1.870	-1.920	-1.847	-1.866
S&P 500	0.644	-1.908	-1.879	-1.951	-1.912
NIKKEI 225	0.247	-1.744	-1.807	-1.670	-1.733
Panel B: Stock market returns					
Returns	ADF test (t-statistic)			Phillips-Perron test	
	None	Constant	Constant and trend	Constant	Trend
FTSE 100	**-35.171**	**-35.177**	**-35.171**	**-49.933**	**-49.924**
S&P 500	**-37.754**	**-37.758**	**-37.754**	**-52.593**	**-52.587**
NIKKEI 225	**-34.817**	**-34.811**	**-34.833**	**-46.359**	**-46.382**

Notes: Entries in bold indicate that the null hypothesis is rejected at 1% level.

2.3. Model specifications

The econometric specification used in our study has two components. To model the stock market return we used a vector autoregression (VAR). To model the conditional variance we used a multivariate GARCH model.

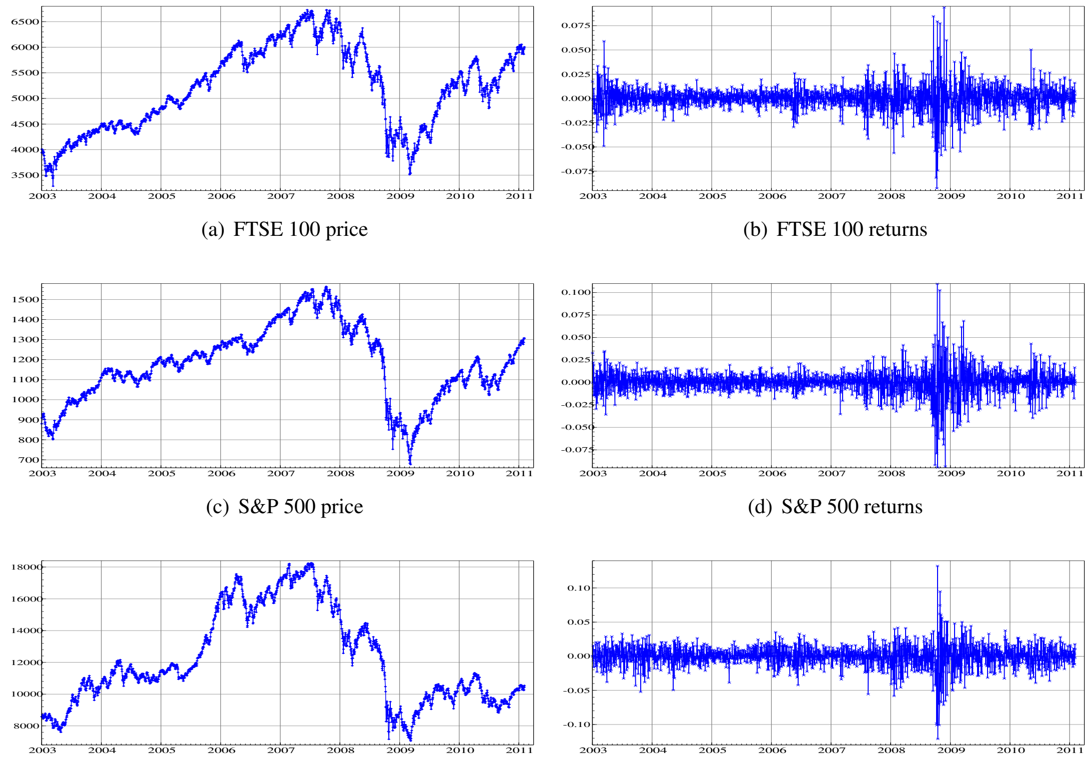
A VAR of order p, where the order p represents the number of lags, that includes N variables can be written as the following form:

$$Y_t = \Phi_0 + \sum_{i=1}^{p} \Phi_i Y_{t-i} + \varepsilon_t, \quad t = 1, \ldots, T \tag{1}$$

where $Y_t = (Y_{1t}, \ldots, Y_{Nt})^{'}$ is a column of observations on current values of all variables in the model,Φ_i is $N \times N$ matrix of unknown coefficients, Φ_0 is a column vector of deterministic constant terms, $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Nt})^{'}$ is a column vector of errors.[3] Our basic VAR will have the three stationary variables, first log differences of FTSE 100, S&P 500 and NIKKEI 225 stock market prices (will be defined in empirical section). We focused on the modelling of multivariate time-varying volatilities. The most widely used model is DCC one of Engle (2002) which captures the dynamic of time-varying conditional correlations, contrary to the benchmark CCC model (Bollerslev (1990)) which retains the conditional correlation constant.[4]

[3]Following Brooks (2002), the main advantage of the VAR is that there is no need to specify which variables are the endogenous variables and which are the explanatory variables because in the VAR, all selected variables are treated as endogenous variables. That is, each variable depends on the lagged values of all selected variables and helps in capturing the complex dynamic properties of the data. Note that selection of appropriate lag length is crucial. If the chosen lag length is too large relative to the sample size, the degrees of freedom will be reduced and the standard errors of estimated coefficients will be large. If the chosen lag length is too small, then the selected lags in the VAR analysis may not be able to capture the dynamic properties of the data. The chosen lag length should be free of the problem of serial correlation in the residuals.

[4]The CCC specification can be presented as:
$H_t = D_t R D_t$, where, $D_t = diag\sqrt{h_{i,t}}$ is a diagonal matrix with square root of the estimated univariate GARCH variances on the diagonal. R is



(a) FTSE 100 price

(b) FTSE 100 returns

(c) S&P 500 price

(d) S&P 500 returns

(e) NIKKEI 225 price

(f) NIKKEI 225 returns

Figure 1 Time series plots of FTSE 100, S&P 500 and NIKKEI 225 stock market indices. We plot the daily level stock market indices (left panel) and corresponding returns (right panel) in the period January 01, 2003 to February 04, 2011.

The specification of the DCC model is as follows:

$$r_t = \mu + \sum_{s=1}^{p} \Phi_s r_{t-s} + \varepsilon_t, \quad t = 1, \ldots, T, \quad \varepsilon_t \,|_{\Omega_{t-1}} \sim \mathcal{N}(0, H_t), \tag{2}$$

$$\varepsilon_t = (\varepsilon_{UK,t}, \varepsilon_{US,t}, \varepsilon_{JP,t})' = H_t^{1/2} z_t, \quad z_t \sim \mathcal{N}(0, \mathbf{I_3}), \tag{3}$$

$$H_t = \mathrm{E}(\varepsilon_t \varepsilon_t' \,|_{\Omega_{t-1}}), \tag{4}$$

where, r_t is a 3×1 vector of the stock market index return, ε_t is the error term from the mean equations of stock market indices (Equation 2), z_t is a 3×1 vector of $i.i.d$ errors and H_t is the conditional covariance matrix. Equation 2

the time-invariant symmetric matrix of the correlation returns with $\rho_{ii} = 1$.

In CCC model, the conditional correlation coefficients are constant, but conditional variances are allowed to vary in time.

can be re-written as follows:

$$\begin{bmatrix} r_{UK,t} \\ r_{US,t} \\ r_{JP,t} \end{bmatrix} = \begin{bmatrix} \mu_{UK} \\ \mu_{US} \\ \mu_{JP} \end{bmatrix} + \sum_{s=1}^{p} \begin{bmatrix} \phi_{11}^{s} & \phi_{12}^{s} & \phi_{13}^{s} \\ \phi_{21}^{s} & \phi_{22}^{s} & \phi_{23}^{s} \\ \phi_{31}^{s} & \phi_{32}^{s} & \phi_{33}^{s} \end{bmatrix} \begin{bmatrix} r_{UK,t-s} \\ r_{US,t-s} \\ r_{JP,t-s} \end{bmatrix} + \begin{bmatrix} \varepsilon_{UK,t} \\ \varepsilon_{US,t} \\ \varepsilon_{JP,t} \end{bmatrix}$$

To represent the Engle's (2002) DCC-GARCH model for the purpose of this study, let $r_t = (r_{UK,t}, r_{US,t}, r_{JP,t})^{'}$ be a 3×1 vector of stock market returns, such that, r_{UK}, r_{US} and r_{JP} are the returns of FTSE 100, S&P 500 and NIKKEI 225 indices, respectively: $r_t \mid_{\Omega_{t-1}} \sim \mathscr{N}(0, H_t)$.

The DCC-GARCH specification of the covariance matrix, H_t, can be written as:

$$H_t = D_t R_t D_t \tag{5}$$

where $D_t = \text{diag}\left(\sqrt{h_{UK,t}}, \sqrt{h_{US,t}}, \sqrt{h_{JP,t}}\right)$ is 3×3 diagonal matrix of time-varying standard deviation from univariate GARCH models; i.e. $h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$, i=UK, US, JP, and $R_t = \{\rho_{ij}\}$ is the time-varying conditional correlation matrix.

The estimation procedure of DCC-GARCH model is based on two stages. In the first stage, a univariate GARCH model is estimated. In the second step, the vector of standardized residuals $\eta_{i,t} = r_{i,t} / \sqrt{h_{i,t}}$ is employed to develop the DCC correlation specification as follows:

$$R_t = \text{diag}\left(q_{11,t}^{-1/2}, \ldots, q_{33,t}^{-1/2}\right) Q_t \text{diag}\left(q_{11,t}^{-1/2}, \ldots, q_{33,t}^{-1/2}\right) \tag{6}$$

where $Q_t = (q_{ij_t})$ is a symmetric positive define matrix. Q_t is assumed to vary according to a GARCH-type process:

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 \eta_{t-1} \eta_{t-1}^{'} + \theta_2 Q_{t-1} \tag{7}$$

The parameters θ_1 and θ_2 are scalar parameters to capture the effects of previous shocks and previous dynamic conditional correlation on current dynamic conditional correlation. The parameters θ_1 and θ_2 are positive and $\theta_1 + \theta_2 < 1$. $\bar{Q}$ is 3×3 unconditional variance matrix of standardized residuals $\eta_{i,t}$. The correlation estimators of equation 7 are of the form:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}.$$

In the DCC model the choice of $\bar{Q}$ is not obvious as Q_t is neither a conditional variance nor correlation. Although $E(\eta_{t-1}\eta_{t-1}^{'})$ is inconsistent for the target since the recursion in Q_t does not have a martingale representation.[5] Aielli (2008) proposed the *corrected* Dynamic Conditional Correlation (*c*DCC) to evaluate the impact of both the lack of consistency and the existence of bias in the estimated parameters of the DCC model of Engle (2002). He showed that the bias depends on the persistence of the DCC dynamic parameters.[6]

In order to resolve this issue, Aielli (2008) introduces the *c*DCC model, which have the same specification as the DCC model of Engle (2002), except of the correlation process Q_t is reformulated as follows:

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \eta_{t-1}^{*} \eta_{t-1}^{*'} + \theta_2 Q_{t-1} \tag{8}$$

where $\eta_t^{*} = \text{diag}\{Q_t\}^{1/2}\eta_t$.

To investigate the asymmetric properties of stock market returns we introduce the conditional asymmetries in variance. Cappiello et al. (2006) estimate several asymmetric versions of the dynamic conditional correlation models. The version which we use is based on the following specification:

$$\begin{aligned}
&r_t = \mu + \textstyle\sum_{s=1}^{p} \Phi^s r_{t-s} + \varepsilon_t, \quad t = 1,\ldots,T, \quad \varepsilon_t\,|_{\Omega_{t-1}} \sim \mathcal{N}(0,H_t)\\
&H_t = D_t R_t D_t \quad \text{where} \quad D_t = \text{diag}\left(\sqrt{h_{1,t}}, \sqrt{h_{2,t}}, \sqrt{h_{3,t}}\right)\\
&h_{it} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_i \mathbb{1}_{\{\varepsilon_{i,t-1}<0\}} \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1} \quad \text{for} \quad i = UK, US, JP\\
&R_t = Q_t^{*-1} Q_t Q_t^{*-1} \quad \text{where} \quad Q_t^{*} = \text{diag}\left\{\sqrt{q_{11,t}}, \sqrt{q_{22,t}}, \sqrt{q_{33,t}}\right\}\\
&Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \eta_{t-1}\eta_{t-1}^{'} + \theta_2 Q_{t-1}.
\end{aligned}$$

3. Empirical results

In this section we initially employed a vector autoregressive (VAR) model to examine the relationship among stock market returns of the three developped countries. Our model is estimated on set of stationary variable. These variables are returns in stock market prices for the United Kingdom (U.K), United States (U.S) and Japan.

Table 3 reports the findings of the VAR(8) model (lags is selected by AIC criterion).

[5] see Aielli (2008) for further details.

[6] Aielli (2008) showed that the lack of consistency of the three-step DCC estimator depends strictly on the persistence of the parameters driving the correlation dynamics and on the relevance of the innovations. The bias is an increasing function of both θ_1 and $\theta_1+\theta_2$. The parameter estimates obtained from fitting DCC models are small, and close to zero for θ_1 and close to unit for $\theta_1+\theta_2$.

3.1. *Conditional variance and volatility analysis*

This subsection presents the empirical results from symmetric and asymmetric multivariate models. In the first step the univariate GARCH(1,1) model for each stock market is fitted. We model the conditional variance as a GARCH(1,1), EGARCH(1,1) and TGARCH(1,1). In the second step the symmetric multivariate GARCH(1,1) models, such as; Constant Conditional Correlation (CCC), the Dynamic Conditional Correlation (DCC), the *corrected* Dynamic Conditional Correlation (*c*DCC) and the asymmetric multivariate GARCH(1,1) models, such as; aCCC, aDCC and a-*c*DCC are fitted.

Symmetric and asymmetric univariate GARCH analysis. Table 4 reports the model estimates (panel A) and related diagnostic tests (panel B) for the three models and for the three stock markets. Firstly, panel A of Table 4 shows that the parameters in the conditional variance equations are all statistically significant, except for the α's in the EGARCH model for the three markets. The estimated value of β (GARCH effect) is close to unity (in all models the estimated values are greater than 0.90) and is significant at the 1% level for each model. This indicates a high degree of volatility persistence in the U.K, U.S and Japan stock market returns. Secondly, results given by the GARCH(1,1) model assume that positive and negative shocks will have the same influence in conditional volatilities forecasts. In order to identify the asymmetry in conditional volatilities, we fitted the univariate EGARCH(1,1) and TGARCH(1,1) models. This asymmetry is generally reffered to as a "leverage" and a "Threshold" effects. The EGARCH(1,1) model captures this "leverage" asymmetry and the TGARCH(1,1) captures this "threshold" asymmetry.[7]

The results showed that the asymmetric parameter γ or δ is positive and significantly different from zero at the 5% level in the EGARCH(1,1) model, indicating that U.K, U.S and Japan stock markets exhibit a leverage effect with positive shocks (good news) and significantly different from zero at 1% level in the TGARCH(1,1) model, identifying that the U.K, U.S and Japan stock markets exhibit a threshold effect with positive effect.

To select the adequate model for our data, we compare between the three models using three criteria; such as the Log-likelihood, Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC). We showed that the asymmetric GARCH model; TGARCH(1,1) has a superior goodness of fit for the data employed. For instance, AIC in the TGARCH(1,1) is lower than in GARCH(1,1) and EGARCH(1,1) models.

[7] The EGARCH (Exponential GARCH) model of Nelson (1991) is formulated in terms of the logarithm of the conditional variance, as in the EGARCH(1,1) model,

$$\log(h_t) = \omega + \alpha \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} + \gamma \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta \log(h_{t-1}).$$

The parametrization in terms of logarithms has the obvious advantage of avoiding non-negativity constraints on the parameters.

The TGARCH (Threshold) GARCH model or GJR-GARCH model defined by Glosten et al. (1993) and Zakoian (1994) augments the GARCH model by including an additional ARCH term conditional on the sign of the past innovation,

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \delta \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\varepsilon_{t-1}^2 + \beta h_{t-1}.$$

Table 3
VAR diagnostic test.

	FTSE 100 [p-value]	S&P 500 [p-value]	NIKKEI 225 [p-value]
Panel A: Univariate diagnostic test			
Q(12)	3.8854 [0.421]	5.4819 [0.241]	3.2173 [0.522]
ARCH test	85.192 [0.000]	73.997 [0.000]	53.224 [0.000]
JB test	811.82 [0.000]	1732.9 [0.000]	964.66 [0.000]
Std.dev	0.0114	0.0128	0.0123
Panel B: Multivariate diagnostic test			
Q(12) [p-value]	39.6760 [0.309]	AIC	-18.523
JB test [p-value]	2841.6 [0.000]	SIC	-18.321
Log-likelihood	19561.61		

The VAR was estimated using eight lags (the lag was selected using the AIC criterion). p-values show the statistical significance of the results.

Table 4
GARCH parameter estimates stock market indices for raw data.

	United Kingdom			United States			Japan		
	GARCH	EGARCH	TGARCH	GARCH	EGARCH	TGARCH	GARCH	EGARCH	TGARCH
Panel A: model estimates.									
ω	0.0078**	-9.2720^{*}	0.0101*	0.0112**	-9.0095^{*}	0.0114**	0.0218**	-8.9373^{*}	0.0244**
α	0.0894*	1.0788	0.0080	0.0707*	2.3070	-0.0078	0.0753*	0.1645	0.0449*
β	0.9055*	0.9594*	0.9267*	0.9190*	0.9616*	0.9416*	0.9101*	0.9306*	0.9091*
γ or δ		0.1266*	0.1064*		0.0651	0.1055*		0.2179*	0.0562**
Log-likelihood	6875.8	6848.3	**6900.5**	6764.4	6734.5	**6797.5**	6497.1	6468.2	**6504.5**
AIC	-6.5331	-6.5050	**-6.5556**	-6.4272	-6.3969	**-6.4577**	-6.1731	-6.1437	**-6.1792**
SIC	-6.5250	-6.4916	**-6.5449**	-6.4191	-6.3834	**-6.4469**	-6.1650	-6.1303	**-6.1685**
Panel B: diagnostic test for standardized residuals.									
$Q(24)$	21.1221	18.6651	19.0830	39.6830	34.7351	40.3153	22.6904	18.6922	25.6935
p-value	0.6315	0.7696	0.7475	0.0231	0.0724	0.0197	0.5381	0.7681	0.3688
$Q_s(24)$	34.6176	37.0246	33.1795	37.9826	45.9568	31.3504	19.1083	35.0669	14.5893
p-value	0.0424	0.0235	0.0593	0.0184	0.0020	0.0891	0.6386	0.0381	0.8792

Notes: The estimates are produced by the univariate GARCH(1,1), univariate EGARCH(1,1) and TGARCH(1,1) models. The univariate variance estimates are introduced as inputs in the estimation of the CCC, DCC and *c*DCC models. The estimated coefficient ω denotes the constant of the variance equation, α represents the ARCH term, β is the GARCH coefficient, γ and δ are the asymmetric effects.
***, ** and * indicate significance at 10%, 5% and 1%, respectively. $Q(24)$ and $Q_s(24)$ respectively represent the Ljung-Box Q statistics of order 24 computed on the standardized residuals and squared standardized residuals. Value of the estimated coefficient ω is multiplied by 10^4 for the EGARCH model. Values in bold indicate the selected model.

Panel B of Table 4 depicts the Ljung-Box statistics computed on the standardized residuals and squared standardized residuals. We showed that all the $Q(24)$ and $Q_s(24)$ values don't reject the null hypothesis of no serial correlation in the standardized residuals and squared standardized residuals at 1% level.

Symmetric and asymmetric dynamic conditional correlation analysis. In the second step the symmetric and asymmetric multivariate GARCH(1,1) models were estimated in order to investigate the constant and time-varying conditional correlation in the stock markets under study. To fit these models, the standardized residuals of the univariate GARCH(1,1) models specification (discussed in the first step of our study) was employed for the estimation of the symmetric models; CCC, DCC, *c*DCC and asymmetric models; aCCC, aDCC, a-*c*DCC. The DCC and *c*DCC estimates of the conditional correlations between the volatilities of the FTSE 100, S&P 500 and NIKKEI 225 returns are

Table 5
CCC diagnostic under raw data

	Log-likelihood: -7233.3	AIC: 6.8872	SIC: 6.9194
Diagnostic test for standardized residuals.			
	FTSE 100	S&P 500	NIKKEI 225
$Q(12)$	25.9838	17.2378	13.8918
p-value	0.0107	0.14086	0.3076
$Q_s(12)$	26.1324	17.1204	13.3307
p-value	0.0102	0.1451	0.3454

The Constant Conditional Correlation (CCC) model of Bollerslev (1990) assumes that the conditional variance for each return, h_{it}, i=UK, US, JP, follows a univariate GARCH process. The specification is as

$$
\begin{aligned}
& r_t = \Phi_0 + \textstyle\sum_{s=1}^{p} \Phi_s r_{t-s} + \varepsilon_t, \quad \varepsilon_t \mid_{\Omega_{t-1}} \sim \mathcal{N}(0, H_t) \\
& \varepsilon_t = H_t^{1/2} z_t, \quad z_t \sim \mathcal{N}(0, \mathbf{I_3}) \\
& H_t = \mathrm{E}(\varepsilon_t \varepsilon_t^{'} \mid_{\Omega_{t-1}}),
\end{aligned}
$$

where, $r_t = \left(r_{UK,t}, r_{US,t}, r_{JP,t}\right)^{'}$ is the vector of stock market index returns, $\varepsilon_t = \left(\varepsilon_{UK,t}, \varepsilon_{US,t}, \varepsilon_{JP,t}\right)^{'}$ is the error term from the mean equation of stock market indices (Equation 2), z_t is a 3×1 vector of $i.i.d$ errors and H_t is the conditional covariance matrix, which satisfies the following equation:

$$H_t = D_t R D_t$$

where $D_t = diag\left(h_{UK,t}^{1/2}, h_{US,t}^{1/2}, h_{JP,t}^{1/2}\right)$, and $R = \left\{\rho_{ij}\right\}$, for $i, j = UK, US, JP$, is the unconditional (time-invariant) correlation matrix. The off-diagonal elements of the conditional covariance matrix are given by

$$H_{ij} = h_{it}^{1/2} h_{jt}^{1/2} \rho_{ij}, i \neq j$$

***, ** and * indicate significance at 10%, 5% and 1%, respectively. $Q(12)$ and $Q_s(12)$ respectively represent the Ljung-Box Q statistics of order 12 computed on the standardized residuals and squared standardized residuals.

Table 6
Constant conditional correlation estimates under raw data.

Stock market returns	FTSE 100	S&P 500	NIKKEI 225
FTSE 100	1	0.5692* (0.0149)	0.2583* (0.0212)
S&P 500		1	0.1751* (0.0222)
NIKKEI 225			1

The table summarizes the estimated invariant correlations between the U.K., U.S. and Japan stock markets, as they are produced by the CCC model. Values in $(\cdot)$ are standard errors. ***, ** and * indicate significance at 10%, 5% and 1%, respectively.

given in tables 7 and 8. Results showed that all coefficients are statistically significant at 1% and 5% levels.

For the CCC model (Table 5 and 6), the correlation between FTSE 100 and S&P 500, FTSE 100 and NIKKEI 225 and S&P 500 and NIKKEI 225 are each positive and statistically significant at 1% level, and the highest correlation is between FTSE 100 and S&P 500 followed by the correlation between FTSE 100 and NIKKEI 225. This indicates the positive comovement between the three markets. For instance, we found that the co-movements between U.K and U.S are higher than the co-movements between U.S and Japan. However, we remarked that estimated constant conditional correlation coefficients of the sample stock markets do not seem to be informative on dynamic linkages and co-movements between the abovementioned markets. To evaluate the dynamic (pairwise) correlation structure of U.K., U.S. and Japan stock markets, we employed the DCC and *c*DCC models in trivariate framework.

For the DCC and *c*DCC models (Tables 7 and 8), the estimated parameters θ_1 and θ_2 capture the effect of lagged standardized shocks; $\eta_{t-1}\eta_{t-1}^{'}$, and $\eta_{t-1}^{*}\eta_{t-1}^{*'}$ and lagged dynamic conditional correlations; Q_{t-1}, on current dynamic conditional correlations, respectively. We remarked that these parameters are significant, and this statistical significance in each market indicates the presence of time-varying stock market correlations. Following the dynamic conditional variance, the three markets under study produce similar behavior and the estimated conditional variance shows a sharp spikes in the period between 2007 and 2008 (The maximum value of estimated conditional variance is in October 21, 2008 for FTSE 100 returns, in October 16, 2008 for the S&P 500 returns and in November 03, 2008 for NIKKEI 225 returns). This period is related to the U.S *subprime* financial crisis. This financial crisis led U.K, U.S and Japan capital markets to abrupt downturns, dramatically increasing systematic volatility.

We conclude that the estimates of the conditional variances based on DCC and *c*DCC models suggest the presence of volatility spillovers in the U.K, U.S and Japan stock market returns.

As shown in Figure 2, we remarked also that the dynamic conditional correlations of the three markets under study show considerable variation, and can vary from the constant conditional correlations (ρ_{UK-US}, ρ_{UK-JP} and ρ_{US-JP}) indicating that the assumption of constant conditional correlation for all shocks to returns is not supported empirically. We stated that during the period 2003-2006, correlations between U.K. and U.S. decreased (58% to 38%), as indicated in Figure 2. Whereas, after 2006 we remarked a substantial increase of correlations between U.K. and U.S., it might be due to the Afghanistan, Iraq and Liban wars and the american *subprime* financial crisis.

The diagnostic tests for standardized residuals of CCC model are shown in Table 5 and those of DCC and *c*DCC models are shown in panel B of Tables 7 and 8. Ljung-Box $Q(12)$ and $Q_s(12)$ statistics for the residuals models indicate no serial correlation in either the standardized residuals $Q(12)$ or the squared standardized residuals $Q_s(12)$, inferring that the fitted models are appropriate for the data employed.

To investigate the asymmetry in the conditional volatility we fitted the aCCC, aDCC and a-*c*DCC models. The

Table 7
DCC estimates under raw data

	θ_1	θ_2	Log-likelihood	AIC
Panel A: model estimates.				
Coefficient	0.005218*	0.992655*	-7211.0	6.8679
Std.error	0.001416	0.002210		
t-Stat	3.685000	449.1000		
p-value	0.000200	0.000000		
Panel B: diagnostic test for standardized residuals.				
	FTSE 100	S&P 500	NIKKEI 225	
$Q(12)$	21.6339	16.2283	13.9273	
p-value	0.04180	0.18100	0.30530	
$Q_s(12)$	25.7240	18.5131	13.3440	
p-value	0.01170	0.10090	0.34450	

Estimates of a symmetric version of Engle (2002) dynamic conditional correlation model are computed. The specification is as

$$
\begin{aligned}
& r_t = \Phi_0 + \textstyle\sum_{s=1}^{p} \Phi_s r_{t-s} + \varepsilon_t \quad \varepsilon_t \,|_{\Omega_{t-1}} \sim \mathcal{N}(0, H_t) \\
& \varepsilon_t = \begin{pmatrix} \varepsilon_{UK,t} \\ \varepsilon_{US,t} \\ \varepsilon_{JP,t} \end{pmatrix} = H_t^{1/2} z_t, \quad z_t \sim \mathcal{N}(0, \mathbf{I_3}) \\
& H_t = D_t R_t D_t \quad \text{where} \quad D_t = diag\left(\sqrt{h_{UK,t}}, \sqrt{h_{US,t}}, \sqrt{h_{JP,t}}\right) \\
& h_{it} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \quad i = UK, US, JP
\end{aligned}
$$

The dynamic conditional correlation $R_t = \{\rho_{ij}\}_t$ is a time-varying matrix defined as

$$
\begin{aligned}
& R_t = Q_t^{*-1} Q_t Q_t^{*-1} \quad \text{where} \quad Q_t^{*-1} = diag\left\{\sqrt{q_{11,t}}, \sqrt{q_{22,t}}, \sqrt{q_{33,t}}\right\} \\
& Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 \eta_{t-1} \eta_{t-1}^{'} + \theta_2 Q_{t-1} \quad \text{where} \quad \eta_t = D_t^{-1} \varepsilon_t.
\end{aligned}
$$

$\bar{Q}$ is the unconditional covariance matrix of η. The elements of H_t are $\{H_{ij}\}_t = \sqrt{h_{it} h_{jt}} \rho_{ij}$ where $\rho_{ii} = 1$. ***, ** and * indicate significance at 10%, 5% and 1%, respectively. $Q(12)$ and $Q_s(12$ respectively represent the Ljung-Box Q statistics of order 12 computed on the standardized residuals and squared standardized residuals.

results are reported in Table 9. In the asymmetric case, we focused on the TGARCH(1,1)-based model to model the asymmetric behavior in the conditional volatilities.[8] The asymmetric dynamic conditional correlation estimates are all significant at 1% level (as shown in Table 9). Based on the log-likelihood values and AIC criterion reported in the last two columns of Table 9, the asymmetric *c*DCC (a-*c*DCC) is supperior to the aCCC and aDCC. Moreover, as it can be seen, the log-likelihood values and AIC criteria of aDCC and a-*c*DCC are nearly equivalent, hence the choice of model needs to be made on other grounds (see the following subsection).

[8]The results from Table 4 indicate that TGARCH(1,1) model compared to GARCH(1,1) and EGARCH(1,1), achieves a significant improvement in the log-likelihood function, AIC and SIC. As shown in Table 4 the TGARCH(1,1) model has the lowest AIC and SIC, and maximize the log-likelihood value. For instance, the AIC values are -6.5556, -6.4577 and -6.1792 for the FTSE 100, S&P 500 and NIKKEI 225 returns, respectively. The log-likelihood values are 6900.5, 6797.5 and 6504.5 for the FTSE 100, S&P 500 and NIKKEI 225 returns, respectively. The apparent superiority of the TGARCH specification compared with EGARCH models is that the former is more robust to large shocks. This intuition is supported by Nelson and Foster (1994). The authors showed that the TGARCH model is consistent estimator of the conditional variance of near diffusion processes.

Table 8

corrected DCC (*c*DCC) estimates under raw data.

	θ_1	θ_2	Log-likelihood	AIC
Panel A: model estimates.				
Coefficient	0.005282*	0.992716*	-7210.5	6.8674
Std.error	0.001430	0.002047		
t-Stat	3.692000	484.8000		
p-value	0.000200	0.000000		
Panel B: diagnostic test for standardized residuals.				
	FTSE 100	S&P 500	NIKKEI 225	
$Q(12)$	21.6228	16.3030	13.9019	
p-value	0.04190	0.17770	0.30700	
$Q_s(12)$	25.6085	18.5792	13.3573	
p-value	0.01210	0.09920	0.34360	

Aielli (2008) proposed a consistent DCC (*c*DCC) model. He suggested that the DCC correlation Q_t should take a slightly different from than given in DCC model. The specification is similar to DCC one, except of Q_t which becomes

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \eta^*_{t-1}\eta^{*'}_{t-1} + \theta_2 Q_{t-1}$$
$$\eta^*_t = diag\{Q_t\}^{1/2}\eta_t$$

***, ** and * indicate significance at 10%, 5% and 1%, respectively. $Q(12)$ and $Q_s(12)$ respectively represent the Ljung-Box Q statistics of order 12 computed on the standardized residuals and squared standardized residuals.

Table 9

Asymmetric dynamic conditional correlation model under raw data.

	θ_1	θ_2	Log-likelihood	AIC	BIC
aCCC			-7169.5	6.8293	6.8696
aDCC	0.0055(0.00162)*	0.9913(0.00295)*	-7148.3	6.8111	6.8568
a-*c*DCC	0.0056(0.00162)*	0.9913(0.00273)*	**-7148.0**	**6.8108**	**6.8565**

This table presents estimates coefficients for the asymmetric CCC [aCCC] the asymmetric DCC [aDCC] model and the asymmetric *c*DCC [a-*c*DCC] model. The specification is similar to those in DCC and *c*DCC models (given in Tables 7 and 8), except of the conditional variances follow the asymmetric TGARCH model of Glosten et al. (1993). The value in parentheses are standard errors. * indicate significance at 1% level. Log-likelihood is the log-likelihood value; AIC is the Akaike information criterion. Values in bold indicate the selected model.

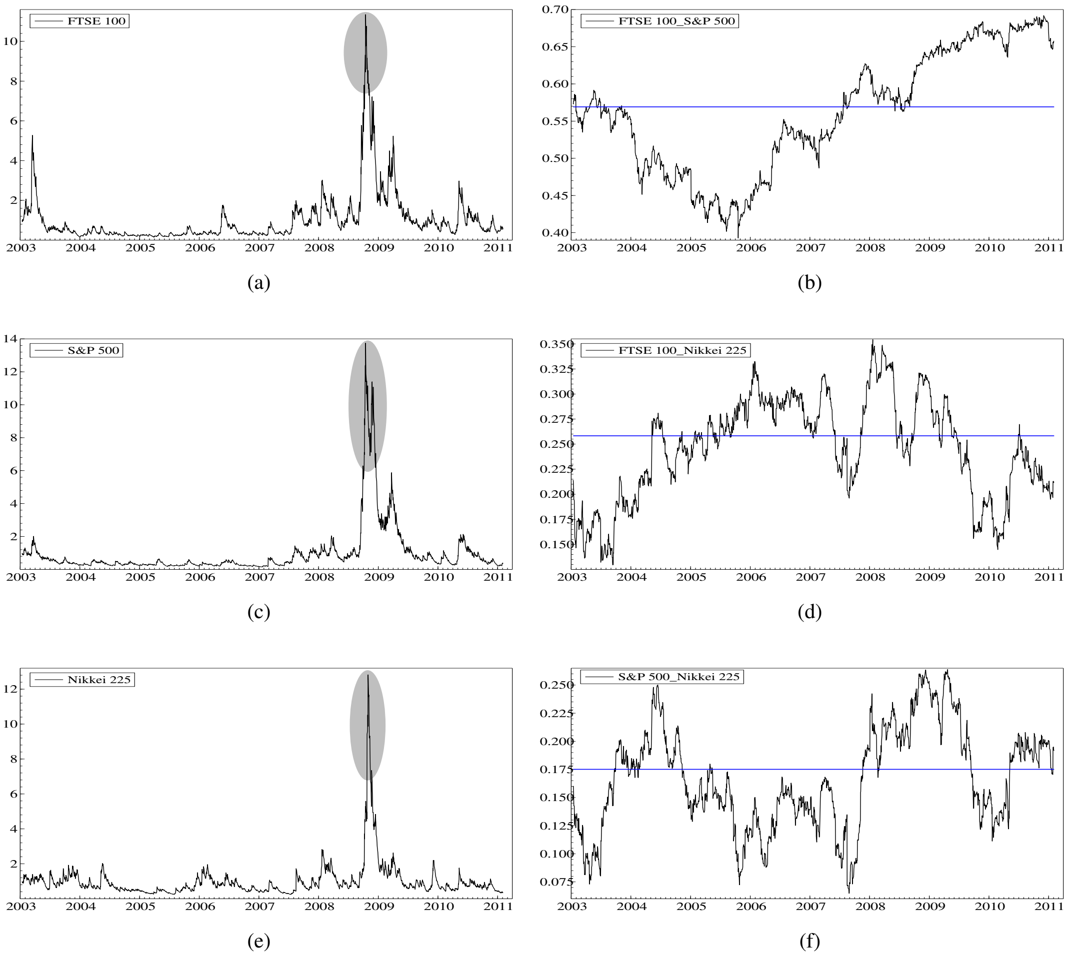


Figure 2 Time-varying conditional variances (left column) and conditional correlations (right column) from DCC model. The pictures on the left column:(a), (c) and (e) refer to conditional variances of U.K, U.S and Japan stock market indices and those on the right column:(b), (d) and (f) report conditional correlations among the same group of stock market indices. The blue line (horizontal) is the constant conditional correlation estimate.

3.2. Forecast performance evaluation

To evaluate the volatility forecasting performance of different multivariate GARCH models and to compare between them, further loss functions can be used. The popular statistical loss functions employed to assess the accuracy of competing models in the forecasting of volatilities over daily trading horizon are Mean squared Error (MSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Logarithm Loss Error (LLE). These loss metrics are expressed as follows:

$$MSE_k = \frac{1}{M}\sum_{m=1}^{M}\left(\varsigma_{m,k} - \hat{\sigma}_m^2\right)^2 \tag{9}$$

$$MAE_k = \frac{1}{M}\sum_{m=1}^{M}\left|\varsigma_{m,k} - \hat{\sigma}_m^2\right| \tag{10}$$

$$MAPE_k = \frac{1}{M}\sum_{m=1}^{M}\left|\frac{\varsigma_{m,k} - \hat{\sigma}_m^2}{\hat{\sigma}_m^2}\right| \tag{11}$$

$$LLE_k = \frac{1}{M}\sum_{m=1}^{M}\left[\log(\varsigma_{m,k}) - \log(\hat{\sigma}_m^2)\right]^2 \tag{12}$$

where M is the number of forecast data points; $\varsigma_{m,k} = \kappa' H_m \kappa$ denotes the portfolio volatility forecast generated by model k for day m; $\hat{\sigma}_m^2$ is the actual volatility on day m.

The assessment of the forecast performance of the various volatility models described in section 2.3 is based on out-of-sample one-day ahead prediction conditional variances and correlations. This forecasting study is done by first removing the last 250 observations from our sample starts from 02 January 2003 and ends 04 February 2011. A forecast of volatilities and correlations are generated for the period $n+1$, where n is the size of the first sample to be estimated starts 02 January 2003 and ends 19 February 2010 (the estimation is based on three models: CCC, DCC and *c*DCC). The second sample, starting 03 January 2003 and ending on $n+1$, is used to forecast the volatility and correlation of $n+2$ based on the estimated models (CCC, DCC and *c*DCC) for the second sample. The procedure of estimation and forecasting steps is repeated 250 times for the available sample from 02 January 2003 to 04 February 2011.[9] More specifically, we produce the 250 one-day ahead forecasts ς_m, where ς_m, $m = n+1, \ldots, n+250$ is the forecast of the conditional variance.

The actual volatility $\hat{\sigma}_m^2$ is unobservable. The use of squared returns as a proxy for actual volatility is used in the literature (see Kang et al. (2009), Sadorsky (2006) and Wei et al. (2010)). In our empirical study, the proxies of actual volatilities are as follows: for forecasts based on the unfiltered return series (original stock market return series), we proxied the actual volatility by the squared returns, r_m^2.

Table 10 presents the forecast accuracy statistics that consist of the mean of four loss functions: MSE, MAE, MAPE and LLE. In terms of these criteria and based on symmetric models, we find that DCC model produces smaller values than those produced by CCC and *c*DCC. In terms of these statistics and basing in asymmetric models, the a-*c*DCC model produces the smallest values. Moreover, we remarked that all loss functions values produced by asymmetric models are smaller than those calculated by symmetric ones. In summary, we can conclude that, in our

[9] For instance, forecasting the DCC model is as follows: the initial sample consists of the first 1862 daily observations, i.e. 02 January 2003 to 19 February 2010. The last 250 trading days constitute the sample for which we compute one-day ahead forecasts. We re-estimated the DCC model basing on the initial sample every day using a recursive method (all estimations and forecasts are computed with OxMetrics 6). After that, we computed the conditional variance and conditional correlation forecast values. These forecast values are used to compute the loss functions of the weighted portfolio defined before.

Table 10
Volatility forecasts performance.

	Symmetric models			Asymmetric models		
	CCC	DCC	*c*DCC	aCCC	aDCC	a-cDCC
MSE	0.16923	0.15971	0.15986	0.09237	0.08652	**0.08639**
MAE	0.40981	0.39822	0.39844	0.30178	0.29195	**0.29170**
MAPE	313.908	304.884	304.990	221.732	214.423	**214.199**
LLE	111.794	111.208	111.220	105.430	104.763	**104.744**

The table reports the loss functions over the 250 forecasting observations for the symmetric and asymmetric dynamic conditional correlation models. The *MSE*, *MAE*, *MAPE* and *LLE* of the weighted portfolio are computed as follows: $MSE_k = M^{-1}\sum_{m=1}^{M}\left(\varsigma_{m,k}-\hat{\mathfrak{s}}_m^2\right)^2$, $MAE_k = M^{-1}\sum_{m=1}^{M}\left|\varsigma_{m,k}-\hat{\mathfrak{s}}_m^2\right|$, $MAPE_k = M^{-1}\sum_{m=1}^{M}\left|(\varsigma_{m,k}-\hat{\mathfrak{s}}_m^2)/\hat{\mathfrak{s}}_m^2\right|$ and $LLE_k = M^{-1}\sum_{m=1}^{M}\left[\log(\varsigma_{m,k})-\log(\hat{\mathfrak{s}}_m^2)\right]^2$, where M is the number of forecast data points ($M = 250$); $\varsigma_{m,k}$ denotes the volatility forecast of the portfolio constituted of three stock market indices and generated by model k for day m, it is defined by equation 14; $\hat{\mathfrak{s}}_m^2$ is the actual portfolio volatility on day m.

forecasting study, asymmetric models produce more accurate volatility forecasts as compared to those models with symmetric conditional volatility.

3.3. Application to Value-at-Risk

In this section, we present a methodology allowing us to compute the VaR of diversified portfolio. A portfolio with weight κ_i in stock market index i has return as

$$r_t^p = \kappa' r_t \tag{13}$$

where $r_t = (r_{UK,t}, r_{US,t}, r_{JP,t})'$ denotes the vector of stock market returns of FTSE 100 ($r_{UK,t}$), S&P 500 ($r_{US,t}$) and NIKKEI 225 ($r_{JP,t}$). $\kappa' = (\kappa_{UK}, \kappa_{US}, \kappa_{JP}) = (0.27, 0.40, 0.33)$ is the weight vector of the three given indices.[10]

The portfolio variances are as follows

$$\varsigma_t = \kappa' H_t \kappa \tag{14}$$

where H_t is the conditional covariance defined in section 2.

The multivariate GARCH-based VaR estimates for q-day holding periods are computed as follows:

$$VaR_{t+l}^k(n,\alpha) = q\kappa'\mu_{t+l} + z_\alpha\sqrt{q}\varsigma_{t+l} \tag{15}$$

[10] These weights are computed using the R function portfolio.optim of the PerformanceAnalytics R package, which computes an efficient portfolio from the given return series in the mean-variance sense. The computed portfolio has the desired expected return and no other portfolio exists, which has the same mean return, but a smaller variance.

where the argument (q) of VaR is used to denote the time horizon, z_α denotes the corresponding quantile which depends on the chosen distribution. For instance, when computing a 99% VaR using normal distribution, $z_\alpha = 2.33$. $\varsigma_{t+l} = (\kappa' H_{t+l}\kappa)^{1/2}$ is the square root of the daily conditional variance forecast of the portfolio generated from model k (CCC, DCC, cDCC), made at time $t+l$. μ_{t+l} is the conditional mean forecast at time $t+l$, generated from VAR(8) model.

3.3.1. Backtesting VaR measures

In order to analyze our results implied by different time-varying volatilities models, we use the above estimation to compute one-day ahead out-of-sample VaR. We performed a backtesting analysis based on likelihood ratio test (Kupiec (1995) and Christoffersen (1998)) and dynamic quantile regression test (Engle and Manganelli (2004)).

Unconditional coverage. Kupiec (1995) developped the likelihood ratio test, LR_{uc} as follows:

Let $n_1 = \sum_{t=1}^{n_0} \mathbb{1}_t$ be the number of days over a n_0 period that the portfolio loss was larger than the VaR estimate, where

$$\mathbb{1}_{t+1} = \begin{cases} 1 & \text{if} \quad r_{t+1} < VaR_{t+1}\,|_{\Omega_t} \\ 0 & \text{if} \quad r_{t+1} \geq VaR_{t+1}\,|_{\Omega_t} \end{cases}$$

$n_1 \sim \mathscr{B}(n_0, \pi)$ is a Binomial variable representing the number of *exceedances* or *exceptions* in the sample of length n_0. The null hypothesis of the failure probability, π, is tested against the alternative hypothesis that the failure probability differs from π_0.[11] The LR_{uc} statistic is

$$LR_{uc} = -2\ln\left[(1-\pi)^{n_0-n_1}\pi^{n_1}\right] + 2\ln\left[(1-\frac{n_1}{n_0})(\frac{n_1}{n_0})n_1\right] \tag{16}$$

Asymptotically, $LR_{uc} \sim \chi^2(1)$.

Conditional coverage. Christoffersen (1998) developped a likelihood statistic to test joint assumption of unconditional coverage and independence of failures. He tests the null hypothesis of independence against the alternative of first-order Markov structure of $\{\mathbb{1}_{t+1}\}$,[12] with transition matrix

$$\Pi = \begin{pmatrix} \pi_{00} & 1-\pi_{11} \\ 1-\pi_{00} & \pi_{11} \end{pmatrix}$$

[11] The null hypothesis for Kupiec (1995) test is, $\mathbf{H}_0 : \pi = n_1/n_0 = \pi_0$, where π is the expected proportion of exceedances, which equals the desired coverage level π_0 (usually equal to 1% and 5%).

[12] The null hypothesis for Christoffersen (1998) test is, $\mathbf{H}_0 : \pi_{00} = \pi_{11}$.

where $\pi_{ii} = p(\mathbb{1}_{t+1} = i \mid \mathbb{1}_t = i), i = 0,1$. Let n_{ij} be the number of observations of $\mathbb{1}_{t+1}$ assuming value i followed by j, for $i, j = 0,1$; and $\pi_{ij} = n_{ij}/\sum_j n_{ij}$ is the corresponding probability. Hence, we have $\hat{\pi}_{01} = n_{01}/(n_{00}+n_{01})$ and $\hat{\pi}_{11} = n_{11}/(n_{10}+n_{11})$. The statistic of the test is given by

$$LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2) \tag{17}$$

where

$$LR_{ind} = -2\ln\left[(1-\pi)^{n_0-n_1}\pi^{n_1}\right] + 2\ln\left[(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}\right]$$

Asymptotically, $LR_{ind} \sim \chi^2(1)$.

Dynamic Quantile. Engle and Manganelli (2004) proposed the dynamic quantile (DQ) test to correct for the inefficiency in the conditional coverage test of Christoffersen (1998). They defined an indicator function $Hit_t(\alpha) = \mathbb{1}_{\left\{r_t < VaR_t(\alpha)|_{\Omega_{t-1}}\right\}} - \alpha$ to test the VaR of long position as follows:

$$Hit_{t+1}(\alpha) = \begin{cases} 1-\alpha & \text{if} \quad r_{t+1} < VaR_{t+1}|_{\Omega_t} \\ -\alpha, & \text{else} \end{cases} \tag{18}$$

Engle and Manganelli (2004) suggested to test jointly the following hypothesis:

$$\mathbf{H_0}: \begin{cases} \mathrm{E(Hit_{t+1}) = 0}, \quad (1) \\ \mathrm{Hit_{t+1}} \quad \text{is uncorrelated with variables included in the information set.} \quad (2) \end{cases}$$

These two tests (1) and (2) can be done using artificial regression $Hit_{t+1} = X_{t+1}\beta + u_{t+1}$, where X_{t+1} is a $N \times K$ matrix whose first column is a column ones and remaining columns are additional explanatory variables.[13]

Under $\mathbf{H_0}$, Engle and Manganelli (2004) show that the dynamic quantile test statistic is given by

$$DQ = \frac{\hat{\beta}'_{OLS}X'X\hat{\beta}_{OLS}}{\alpha(1-\alpha)} \sim \chi^2(7) \tag{19}$$

The statistical adequacy of VaR forecasts is obtained by the previous tests. It is well-known that these tests have limited power in distinguishing among various models for VaR. Using these metrics, we cannot conclude whether an adequate model is more accurate than another one. Loss functions represent an alternative approach that can be

[13] We include five lags of Hit_t and the current VaR as explanatory variables. Hence, $Hit_t = \left(\iota, Hit_{t-1}, Hit_{t-2}, Hit_{t-3}, Hit_{t-4}, Hit_{t-5}, VaR_t(\alpha)\right)$

Table 11
Daily VaR forecasts.

Model	Mean VaR α-level			
	1%	2.5%	5%	10%
Symmetric models				
CCC	1.48782	1.25373	1.05289	0.82008
DCC	1.46673	1.23595	1.03796	0.80845
*c*DCC	1.46716	1.23632	1.03827	0.80869
Asymmetric models				
aCCC	1.27590	1.07515	0.90292	0.70327
aDCC	1.25483	1.05739	0.88801	0.69166
a-*c*DCC	**1.25426**	**1.05692**	**0.88761**	**0.69135**

Notes: The table presents the out-of-sample daily VaR of the weighted portfolio consisted of three stock market indices (FTSE 100, S&P 500 and NIKKEI 225). We used quantile for normal distribution. Values in bold show the selected model for better forecasts.

used to compare models. Lopez (1998) suggested a loss function based on regulatory needs. He proposed measuring the accuracy of VaR forecasts on the basis of distance between observed return, r_t, and forecasted VaR_t values. He defined a penalty variable as follows:

$$\mathbf{LF_{t+1}} = \begin{cases} 1 + \left(r_{t+1} - VaR_{t+1} \,|_{\Omega_t} \right)^2, & \text{if} \quad r_{t+1} < VaR_{t+1} \,|_{\Omega_t} \\ 0, & \text{if} \quad r_{t+1} \geq VaR_{t+1} \,|_{\Omega_t} \end{cases}$$

In our study we computed the loss function **LF** as a sum of $\mathbf{LF_{t+1}}$, with $t = 0, \ldots, n_0$.

Table 11 depicts the out-of-sample mean daily VaR of the weighted portfolio consisted of the FTSE 100, S&P 500 and NIKKEI 225 stock market returns. We remarked that the a-*c*DCC model yields the lowest average daily VaR estimates at 1%, 2.5%, 5% and 10% levels. The backtesting analysis considers a comparison over the last 250 days (from February 19, 2010 to February 04, 2011) and focuses mainly on exceptions or violations, i.e. the number of times in which the portfolio returns underperform the VaR measure, and LR_{uc}, LR_{cc} and DQ tests statistics. We also calculated the failure probability and statistic loss functions suggested by Lopez (1998). Tables 12 and 13 present unconditional, conditional and dynamic quantile test statistics. The results showed that symmetric and asymmetric models have been rejected by the LR_{uc}, LR_{cc} and DQ tests. Hence, these models are slow at updating the VaR values when market volatility changes rapidly.

For symmetric models, at 1% significance level, CCC, DCC and *c*DCC models provide the same number of violations and failure probabilities. The same remark is shown in case of asymmetric models. The results in Tables 12 and 13 showed that all symmetric models produce the lowest number of violations and failure probabilities. For instance, we identify 13 and 23 violations and 5.22% and 9.23% p-values for the DCC model at 1% and 5% level, respectively. While, for the asymmetric DCC model we find 18 and 31 violations and 7.22% and 12.44% p-values at

Table 12
Summary results for daily VaR diagnostic tests in symmetric case.

	Exeedances	Fail. prob. (%)	LR_{uc}	LR_{ind}	LR_{cc}	DQ	**LF**
1% daily VaR							
CCC	13	5.22	22.403	94.587	116.991	93.013	**21.728**
DCC	13	5.22	22.403	94.587	116.991	93.538	22.103
*c*DCC	13	5.22	22.403	94.587	116.991	93.441	22.087
2.5% daily VaR							
CCC	18	7.22	41.188	120.452	161.640	121.155	**31.706**
DCC	18	7.22	41.188	120.452	161.640	121.685	32.149
*c*DCC	18	7.22	41.188	120.452	161.640	121.586	32.131
5% daily VaR							
CCC	21	8.43	53.960	129.689	183.649	231.252	**40.484**
DCC	23	9.23	63.003	136.722	199.726	340.902	42.967
*c*DCC	23	9.23	63.003	136.722	199.726	340.788	42.948
10% daily VaR							
CCC	33	13.25	113.481	173.093	286.574	391.790	**61.424**
DCC	35	14.05	124.484	178.564	303.049	519.173	63.944
*c*DCC	35	14.05	124.484	178.564	303.049	519.110	63.924

Notes: The table presents the evaluation of out-of-sample daily VaR for the weighted portfolio (consisted from three stock market indices) generated by symmetric models. it reports test statistics over last 250 days (February 19, 2010 to February 03, 2011). Fail. prob.: The failure probability. LR_{uc}: The LR test of unconditional coverage. LR_{ind}: The LR test of independence. LR_{cc}: The joint test of coverage and independence. DQ: The Dynamic Quantile test. **LF**: The loss function. Values in bold show the selected model for better forecasts.

1% and 5% level, respectively.

As shown in Tables 12 and 13, we also reported the **LF** criterion, which facilitates the selection of optimal VaR model for risk manager. It is clearly that CCC and aCCC models provide the lowest **LF** values at 1%, 2.5%, 5% and 10% levels. For symmetric volatilty models, the study identifies the CCC model as the best performing model for a firm, followed by *c*DCC and DCC models. For the asymmetric volatility models, we identify the aCCC model as the best performing model, followed by DCC and *c*DCC models. However, we remarked that DCC and *c*DCC models provide approximately the same performance.

Table 13
Summary results for daily VaR diagnostic tests in asymmetric case.

	Exeedances	Fail. prob. (%)	LR_{uc}	LR_{ind}	LR_{ind}	DQ	**LF**
1% daily VaR							
aCCC	18	7.22	41.188	120.452	161.640	116.833	**29.947**
aDCC	18	7.22	41.188	120.452	161.640	116.352	30.462
a-cDCC	18	7.22	41.188	120.452	161.640	116.394	30.470
2.5% daily VaR							
aCCC	21	8.43	53.960	130.774	184.734	182.956	**38.503**
aDCC	24	9.63	67.671	142.213	209.884	244.406	42.074
a-cDCC	24	9.63	67.671	142.213	209.884	244.437	42.083
5% daily VaR							
aCCC	30	12.04	97.505	162.832	260.338	361.907	**53.707**
aDCC	31	12.44	102.758	166.311	269.069	375.634	55.327
a-cDCC	31	12.44	102.758	166.311	269.069	375.646	55.338
10% daily VaR							
aCCC	37	14.85	135.752	183.844	319.597	576.566	**70.127**
aDCC	38	15.26	141.483	186.050	327.534	607.452	71.764
a-cDCC	18	15.26	141.483	186.050	327.534	607.448	71.776

Notes: The table presents the evaluation of out-of-sample daily VaR for the weighted portfolio (consisted of three stock market indices) generated by asymmetric models. it reports test statistics over last 250 days (February 19, 2010 to February 03, 2011). Fail. prob.: The failure probability. LR_{uc}: The LR test of unconditional coverage. LR_{ind}: The LR test of independence. LR_{cc}: The joint test of coverage and independence. DQ: The Dynamic Quantile test. **LF**: The loss function. Values in bold show the selected model for better forecasts.

4. Wavelet-based approach

In this section we employed a new approach, called wavelet transform.[14] We used a discrete wavelet transform (DWT), more specifically, we adopt the maximal overlap discrete wavelet transform (MODWT)[15] in a Multiresolution Analysis (MRA) framework.[16]

In our study, we sample the daily stock market returns at different scale j as follows: D_1 (2-4 days), D_2 (4-8 days), D_3 (8-16 days), D_4 (16-32 days), D_5 (32-64 days), D_6 (64-128 days), D_7 (128-256 days), D_8 (256-512 days).[17] We used Daubechies Least Asymmetric wavelet transformation of length $L = 8$ via LA(L) to obtain multiscale decomposition of the return series. The MRA yields an additive decomposition through MODWT given by

[14]The wavelet transform has two types of transform, namely, continuous wavelet transform (CWT) and discrete wavelet transform (DWT). Since most of the time series have a finite number of observations, the discrete version of wavelet transform is used in finance and economic applications. The wavelet transform decomposes a time series in terms of some elementary functions, called, wavelets; $\psi_{u,\tau}(t) = \frac{1}{\sqrt{\tau}}\psi[(t-u)/\tau]$. Where $\frac{1}{\sqrt{\tau}}$ is a normalization factor, u is the translation parameter and τ is the dilation parameter. $\psi(t)$ must fulfill several conditions (see, Gençay et al. (2002) and Percival and Walden (2000)): it must have zero mean, $\int_{-\infty}^{+\infty}\psi(t)dt = 0$, its square integrates to unity, $\int_{-\infty}^{+\infty}\psi^2(t)dt = 1$ and it should also satisfy the admissibility condition, $0 < C_\psi = \int_0^{+\infty}\frac{|\Psi(\lambda)|^2}{\lambda}d\lambda < +\infty$ where $\Psi(\lambda)$ is the Fourier transform of $\psi(t)$, that is $\Psi(\lambda) = \int_{-\infty}^{+\infty}\psi(t)e^{-i\lambda u}dt$. Following the latter condition we can reconstruct a time series $x(t)$.

[15]We use the MODWT because we can align perfectly the details from the decomposition with the original time series. In comparison with the DWT, no phase shift will result in the MODWT (Gençay et al. (2002)). Fore more information about the MODWT, please refer to Percival and Walden (2000) and Gençay et al. (2002).

[16]For more details, see Mallat (1989) and Percival and Walden (2000).

[17]We decompose our time series up to scale 8 (scale $J \leq \log_2[(T-1)/(L-1)+1]$, where T is the number of observations of stock market returns ($T = 2112$), and L is the length of the wavelet filter LA(8)). We used the *wavelets* R package for the MODWT.

$$
\begin{aligned}
X(t) &= \sum_k \tilde{s}_{j,k}\phi_{j,k}(t) + \sum_j \sum_k \tilde{d}_{j,k}\psi_{j,k}(t), \quad j=1,\ldots,J \\
&= S_J(t) + D_J(t) + D_{J-1}(t) + \ldots + D_1(t)
\end{aligned} \tag{20}
$$

where $S_J(t)$ refers to the smooth series and it represent the approximation that captures the long memory term properties, $D_j(t),\ j=1,\ldots,J$. refers to details series, which represent the contribution of frequency j to the original series.

The wavelet detail, $D_j(t)$, captures local fluctuations over the whole period of time series at each scale, while the wavelet smooth, $S_J(t)$, gives an approximation of the original series at scale J.

After computing the MODWT crystals (details and smooths) for every stock market return, and from the decomposed series $(D_1,\ldots,D_8,S_8)$ we classify the short, medium and long term series as follows: Short term $= D_1+D_2+D_3$; Medium term $= D_4+D_5+D_6$; Long term $= D_7+D_8+S_8$. This choice of time-horizon decomposition is used to classify three types of investors or traders, such as short, medium and long term ones, i.e. to analyze the behavior of investors among different time-horizons. Here the highest frequency component Short term, $D_1+D_2+D_3$ represents the short-term variations due to shocks occurring at a time scale of 2 to 16 days, it provides daily and weekly spillovers, the next component Medium term, $D_4+D_5+D_6$ represents the mid-term variations at time scale of 32 to 128 days, it defines the monthly and quarterly spillovers, and the third component Long term, $D_7+D_8+S_8$ represents the long-term variations of 256 days and more, it provides the annual spillovers. The main advantage of this classification is to decompose the risk and the volatility spillovers into three investment horizons. Therefore, we focus in three sub-spillovers. All market participants, such as regulators, traders and investors, who trade in stock markets (in our study, U.K., U.S. and Japan stock markets) make decisions over different time scales. In fact, due to the different decision-making time scales among investors, the time-varying volatilities and correlations of stock market indices will vary over the different time scales associated with those horizons (investment strategies).

In order to analyze the comovements in returns and volatilities (the analysis is based on new time series: Short term, Medium term and Long term) between the three markets defined before and to investigate the dynamics and spillover effects, we applied a trivariate dynamic conditional correlation (CCC, DCC and *c*DCC) model. We proceed the same specifications presented in section 2.3 to model the conditional mean and the conditional variances, we fitted a VAR(1)-MGARCH(1,1) to the three new series as follows:

$$
\begin{cases}
X_t = c_0 + AX_{t-1} + \upsilon_t \\
\upsilon_t = D_t z_t
\end{cases} \tag{21}
$$

where $X_t = (\text{FTSE-Short}_t, \text{S\&P500-Short}_t, \text{NIKKEI-Short}_t)'$, c_0 is a (3×1) vector of constants, A is (3×3) co-

Table 14
Unit root tests of wavelet components.

	FTSE 100		S&P 500		Nikkei 225	
	ADF test	KPSS test	ADF test	KPSS test	ADF test	KPSS test
Short term	-42.986^*	0.006^*	-46.451^*	0.019^*	-43.018^*	0.008^*
Medium term	-54.386^*	0.016^*	-52.996^*	0.013^*	-49.944^*	0.004^*
Long term	-33.796^*	0.520^*	-34.725^*	0.486^*	-47.833^*	0.648^*

Notes: The table reports results of the augmented Dickey-Fuller (Dickey and Fuller (1979)) and Kwiatkowski–Phillips–Schmidt–Shin (Kwiatkowski et al. (1992)) tests. The KPSS test contains a constant and not a time trend. While, the ADF test without constant and trend. The null hypothesis of ADF test is that a time series contains a unit root, $I(1)$ process, whereas the KPSS test has the null hypothesis of stationarity, $I(0)$ process.
* indicate the rejection of the unit root null at 1% significance level.

efficient matrix, υ_t is (3×1) vector of error term from the mean equations of decomposed series, and z_t refers to a (3×1) vector of independently and identically distributed errors.

To ensure the stationarity of our reconstructed series (Short term, Medium term and Long term series), we applied the ADF and KPSS tests to our decomposed data. As shown in Table 14 all test statistics are statistically significant at 1% level, therefore indicating stationarity.

Table 15 presents estimates from two types of conditional volatility regressions: (i) a univariate GARCH model, and (ii) a univariate EGARCH model for each market and at each time scale. A GARCH(1,1) model and EGARCH(1,1) model proved adequate for capturing conditional heteroskedasticity. As shown in Table 15 the univariate GARCH models display more statistically significant coefficients than a univariate EGARCH ones. In terms of GARCH model, the α and β coefficients (ARCH and GARCH effects, respectively) are positive and statistically significant in all stock markets, indicating highly persistent volatility dynamics in short-term horizon (lower scales/ higher frequencies).

The time scale constant conditional correlations among the three stock market returns from the CCC model are summarized in Table 16. The results indicate that the highest constant correlation between markets is shown in medium-term horizon, which corresponds to monthly and quarterly time horizons. Therefore, the comovements between the three markets are higher in medium horizon than comovements in short or long horizons. For instance, the correlations between U.K. and U.S. are as follows: 52.7%, 68.9% and 65% for short, medium and long term, respectively, and the correlations between U.S. and Japan are: 13%, 55.7% and 35.1% for short, medium and long term, respectively. Furthermore, we remark that time scale constant correlations between U.K. and U.S. are stronger than the others in all time scales.

Parameter estimates for the conditional variance-covariance equations in the DCC and *c*DCC models are reported in Table 17. Coefficients θ_1 and θ_2 reflect the ARCH and GARCH effects, respectively. The estimates parameters of the DCC and *c*DCC models are statistically significant at 1% level in all time scales, and the sum $\theta_1+\theta_2$ close

Table 15
Univariate GARCH(1,1) estimates for different time horizons.

	ω	α	β	γ	Log-likelihood	AIC
Short term						
FTSE 100	0.0076**	0.1056*	0.8896*		-2638.6	2.5027
S&P 500	0.0089**	0.0757*	0.9149*		-2740.2	2.5989
NIKKEI 225	0.0188*	0.0868*	0.8992*		-3045.3	2.8881
Medium term						
FTSE 100		0.8657*	0.1172*		574.33	-0.5419
S&P 500		0.8863*	0.0944*		658.69	-0.6268
NIKKEI 225		0.9542*	0.0014*		-156.81	0.1503
Long term						
FTSE 100		−0.0508	1.0027*	2.5384*	2950.9	-2.7907
S&P 500		−0.0608	1.0209*	2.2234*	2953.6	-2.7932
NIKKEI 225		−0.0485	1.0232*	2.2854*	2008.5	-1.8982

Notes: The table summarizes the estimated coefficients produced by the univariate GARCH(1,1) model for short and medium term horizon series and EGARCH(1,1) model for long term horizon series. The univariate variance estimates are introduced as inputs in the estimation process of the CCC, DCC and *c*DCC models. The estimated coefficient ω denotes the constant term, α and β are the ARCH and GARCH terms, respectively, in the conditional variance equations, γ is the asymmetric parameter.
The above estimates are for the sample period of 02 January 2003 to 04 February 2011. Significance levels at 1%, 5% and 10% are denoted by *, ** and ***, respectively. Log-likelihood is the logarithm maximum likelihood function value. AIC is the Akaike information criterion.

Table 16
Constant conditional correlation estimates under wavelet return series

	Short term			Medium term			Long term		
	FTSE 100	S&P 500	NIKKEI 225	FTSE 100	S&P 500	NIKKEI 225	FTSE 100	S&P 500	NIKKEI 225
FTSE 100	1.000	0.527*	0.242*	1.000	0.689*	0.570*	1.000	0.650*	0.283*
S&P 500		1.000	0.130*		1.000	0.557*		1.000	0.351*
NIKKEI 225			1.000			1.000			1.000

The table reports the estimated time scale conditional correlations between the U.K., U.S. and Japan stock markets, as they are produced by the CCC model. * indicate significance at 1% level.

to unity for the two models and at each scale, implying high persistent volatility in short, medium and long term horizons. Based on DCC model, the degree of persistence, $\theta_1 + \theta_2$, are 0.997, 0.894 and 0.995 for short, medium and long term, respectively. Based on *c*DCC model the degree of persistence is 0.997, 0.995 and 0.999 for the short, medium and long terms, respectively. The short run persistence of shocks on dynamic conditional correlations is the greatest for high scales (0.594 and 0.831), while the largest long run persistence of shocks to conditional correlations is 0.997 (0.004+0.993) for low scales. We remark also that the short run persistence shocks increases by scales, while the long run persistence shocks decreases among scales.

The scale spillover effect in volatility provide strong persistence for all components (Short term, Medium term and Long term), this phenomenon result from the trading of heterogeneous group of investors. In fact, at finest scales (Short term component), market participants are hedging strategists, speculators and market makers. Therefore, they

Table 17
Estimates of the multivariate GARCH(1,1) models (DCC and *c*DCC) for different time horizons.

	θ_1	θ_2	Log-likelihood	AIC
DCC model				
Short term	0.004457 (0.0017)*	0.993156 (0.0032)*	-8002.8	7.59530
Medium term	0.719497 (0.0066)*	0.175299 (0.0078)*	14834.1	-14.0408
Long term	0.594683 (0.0269)*	0.400597 (0.0277)*	40909.8	-38.7454
cDCC model				
Short term	0.004560 (0.0016)*	0.993199 (0.0027)*	-8002.2	7.59470
Medium term	0.763499 (0.0055)*	0.231645 (0.0056)*	15229.6	-14.4155
Long term	0.831890 (0.0242)*	0.167528 (0.0244)*	43104.5	-40.8247

Notes: This table reports the estimates parameters of DCC and *c*DCC models. Significance levels at 1%, 5% and 10% are denoted by *, ** and ***, respectively. Log-likelihood is the logarithm maximum likelihood function value. AIC is the Akaike information criterion.

trade the three markets (U.K., U.S. and Japan) simultaneously. Kim and In (2003) showed that speculators and market makers intensively trade to realize a quick profit (or minimize loss) over short time scales. In the intermediate scales (mid-horizon; Medium term component), the main traders are international portfolio managers who mainly follow index tracking trading strategies. Kim and In (2003) showed that trades typically occurs on a weekly to monthly basis, with little attention paid to daily prices. At high scales (Long term component), the main traders are central banks which operate on long-term horizons and often consider long-term economic fundamentals for their strategy.

The forecasting performance of the CCC, DCC and *c*DCC models at each scale is evaluated by comparing the four statistical loss functions defined in section 3.2. This forecasting study is based on recursive out-of-sample one-day ahead forecast of variance-covariance matrix of dynamic conditional models at each time horizon, i.e. Short term, Medium term and Long term components. The "wavelet" volatility models are estimated based on 1863 observations corresponding to the period 02 January 2003 to 19 February 2010. The variables used in the models are: Short term, Medium term and Long term components. We compare the accuracy of wavelet volatility forecasts based on:

$$\mathrm{RMSE}_k(j) = \left\{ \frac{1}{M} \sum_{m=1}^{M} \left[\varsigma_{m,k}(j) - \hat{\sigma}_m^2(j) \right]^2 \right\}^{1/2},$$

$$\mathrm{MAE}_k(j) = \frac{1}{M} \sum_{m=1}^{M} \left| \varsigma_{m,k}(j) - \hat{\sigma}_m^2(j) \right|,$$

$$\mathrm{MAPE}_k(j) = \frac{1}{M} \sum_{m=1}^{M} \frac{\left| \varsigma_{m,k}(j) - \hat{\sigma}_m^2(j) \right|}{|\hat{\sigma}_m^2(j)|},$$

and

Table 18
One-day out-of-sample volatility forecasts performance for different time horizons.

	Forecast loss functions			
	RMSE	MAE	MAPE	LLE
Panel A: Short term				
CCC	1.488636	0.725253	5.076901	8.356644
DCC	1.488659	0.723788	5.050991	8.336478
*c*DCC	1.489430	0.718159	4.958577	8.273333
Panel B: Medium term				
CCC	0.139242	0.082996	1.001749	24.867650
DCC	0.139241	0.082996	1.001749	24.865190
*c*DCC	0.139241	0.082996	1.001749	24.864380
Panel C: Long term				
CCC	0.015626	0.012123	0.999986	73.91793
DCC	0.015626	0.012123	0.999986	73.91789
*c*DCC	0.015626	0.012123	0.999986	73.91788

Notes: The table summarizes the four loss metrics estimates at different time horizons. The RMSE is defined as; $\mathrm{RMSE}_k(j) = \left\{\sum_{m=1}^{M}\left[\varsigma_{m,k}(j) - \hat{\sigma}_m^2(j)\right]^2/M\right\}^{1/2}$, the MAE is defined as; $\mathrm{MAE}_k(j) = \sum_{m=1}^{M}\left|\varsigma_{m,k}(j) - \hat{\sigma}_m^2(j)\right|/M$, the MAPE is defined as; $\mathrm{MAPE}_k(j) = \left\{\sum_{m=1}^{M}\left|\varsigma_{m,k}(j) - \hat{\sigma}_m^2(j)\right|/\left|\hat{\sigma}_m^2(j)\right|\right\}/M$ and the LLE is defined as; $\mathrm{LLE}_k(j) = \sum_{m=1}^{M}\left[\log(\varsigma_{m,k}(j)) - \log(\hat{\sigma}_m^2(j))\right]^2/M$, where, $\varsigma_{m,k}(j)$ is the wavelet volatility forecast generated by model k for day m and scale j and $\hat{\sigma}_m^2(j)$ is the actual volatility on day m at scale j.

$$\mathrm{LLE}_k(j) = \frac{1}{M}\sum_{m=1}^{M}\left[\log(\varsigma_{m,k}(j)) - \log(\hat{\sigma}_m^2(j))\right]^2.$$

where $\varsigma_{m,k}(j)$ is the wavelet volatility forecast generated by model k for day m and scale j and $\hat{\sigma}_m^2(j)$ is the actual volatility on day m at scale j. The actual wavelet volatility is not observable, therefore we define three proxies of $\hat{\sigma}_m^2(j)$, such as: (Short term)^2, (Medium term)^2 and (Long term)^2, for the short-term horizon, medium-term horizon and long-term horizon, respectively.

Table 18 summarizes the results of the one-day out-of-sample volatility forecast loss functions. We observe that, at low scales (Short term component) and in terms of MAE, MAPE and LLE criteria, the *c*DCC model provides better volatility forecasts, it has lower loss function values than CCC and DCC models. While, at intermediate scales (Medium term component) and high scales (Long term component), the three dynamic conditional correlation models show similar accuracy in one-day out-of-sample forecasts (the values of RMSE, MAE and MAPE are equals).

Table 19 presents mean WVaR estimates for each of the dynamic conditional correlation models under 99%, 97.5%, 95% and 90% confidence levels over the out-of-sample period from February 22, 2010, to February 04, 2011. This basic statistic can be shown as the preliminary understanding of average performance during the forecasting period before the implementation of backtesting tests and market risk loss criteria. As shown in Table 19, the WVaR

Table 19
Daily WVaR forecasts.

	Mean WVaR α-level			
	1%	2.5%	5%	10%
CCC model				
Short term	1.7684	1.4902	1.2514	0.9747
Medium term	0.0381	0.0321	0.0270	0.0210
Long term	0.0025	0.0021	0.0018	0.0014
DCC model				
Short term	1.7641	1.4865	1.2484	0.9723
Medium term	0.0381	0.0321	0.0270	0.0210
Long term	0.0025	0.0021	0.0018	0.0014
cDCC model				
Short term	1.7455	1.4708	1.2352	0.9620
Medium term	0.0381	0.0321	0.0270	0.0210
Long term	0.0025	0.0021	0.0018	0.0014

Notes: The table presents the out-of-sample daily VaR of the weighted portfolio across wavelet scales. We used quantile for normal distribution.

estimates produced by the *c*DCC model at short-term horizon (low scales or high frequencies) are smaller than those of CCC and DCC models in 99%, 97.5%, 95% and 90% confidence levels. For instance, at 99% confidence level, the average WVaR is 1.7455 for the cDCC model, while those of CCC and DCC are 1.7684 and 1.7641, respectively. We remark also, that WVaR at medium-term horizon (intermediate scales) and long-term horizon (high scales or low frequencies) produced by the three models are equals, for each confidence level, 99%, 97.5%, 95% and 90%. However, the mean WVaR decreases from lower scales to higher ones, such that, Short term component (low scales/ high frequencies) provides high WVaR estimates, followed by Medium term (intermediate scales) component and Long term (high scales/ low frequencies) component, at all confidence levels.

To accurately comparing the forecasting ability of the mentioned models in terms of WVaR,[18] backtesting diagnostic tests: unconditional coverage of Kupiec (1995), conditional coverage of Christoffersen (1998), and dynamic quantile of Engle and Manganelli (2004) are introduced and used for determining the accuracy of wavelet model-based VaR measurements. We used also, the regularity loss function of Lopez (1998) based on wavelet market risk (WVaR) defined before. It is defined in wavelet case as follows:

[18] The multivariate GARCH-based WVaR estimate for one-day ahead forecasts are defined as follows:

$$VaR^{k}_{t+m}(j) = \kappa' d_{t+m} + z_{\alpha}\varsigma_{t+m}(j).$$

where z_{α} denotes the normal quantile, $\kappa' d_{t+m}$ is the wavelet mean one-day ahead forecast estimate computed by forecasting VAR(1) based on wavelet components series. The wavelet weighted portfolio return is $d_t^{ptf} = \kappa' d_t$: $d_t = \left(d_{UK,t}, d_{US,t}, d_{JP,t}\right)'$ denotes the vector of wavelet return components of FTSE 100 ($d_{UK,t}$), S&P 500 ($d_{US,t}$) and NIKKEI 225 ($d_{JP,t}$), $\kappa' = (\kappa_{UK}, \kappa_{US}, \kappa_{JP})$ is the vector of weights in the portfolio. $\varsigma_t(j) = \kappa' H_t(j)\kappa$, where $H_t(j)$ is the wavelet conditional variance-covariance matrix defined in section 2.3. Briefly, the wavelet conditional mean and wavelet conditional variance-covariance estimates are calculated from forecasting the model defined in equation 21.

Table 20
Summary results for daily WVaR diagnostic tests.

		Exceedances	Fail. prob. (%)	LR_{uc}	LR_{ind}	LR_{cc}	DQ	$\mathbf{LF}(j)$
1% WVaR	*CCC*							
	Short term	7	2.81	5.533	58.791	64.325	68.020	**68.020**
	Medium term	88	35.34	490.278	310.040	800.318	7404.16	96.628
	Long term	98	39.35	571.829	329.860	901.689	9404.57	99.037
	DCC							
	Short term	7	2.81	5.533	58.791	64.325	67.173	**8.723**
	Medium term	88	35.34	490.278	310.040	800.318	7404.16	96.628
	Long term	98	39.35	571.829	329.860	901.689	9404.57	99.037
	cDCC							
	Short term	7	2.81	5.533	58.791	64.325	68.115	**8.857**
	Medium term	88	35.34	490.278	310.040	800.318	7404.16	96.628
	Long term	98	39.35	571.829	329.860	901.689	9404.57	99.037
2.5% WVaR	*CCC*							
	Short term	11	4.41	15.960	83.318	99.278	67.001	**14.978**
	Medium term	90	3.61	506.312	312.373	818.685	7600.83	98.894
	Long term	98	39.35	571.829	329.860	901.689	9404.57	99.044
	DCC							
	Short term	11	4.41	15.960	83.318	99.278	65.107	**15.003**
	Medium term	90	36.14	506.312	312.373	818.685	7600.83	98.893
	Long term	98	39.35	571.829	329.860	901.689	9404.57	99.044
	cDCC							
	Short term	11	4.41	15.960	83.318	99.278	67.182	**15.199**
	Medium term	90	36.14	506.312	312.373	818.685	7600.83	98.893
	Long term	98	39.35	571.829	329.860	901.689	9404.57	99.044
5% WVaR	*CCC*							
	Short term	16	6.42	33.264	110.467	143.731	101.254	**23.170**
	Medium term	94	37.75	538.797	314.791	853.588	7827.88	103.126
	Long term	99	39.75	580.171	330.707	910.879	9503.55	100.049
	DCC							
	Short term	16	6.42	33.264	110.467	143.731	98.500	**23.204**
	Medium term	94	37.75	538.797	314.791	853.588	7827.88	103.126
	Long term	99	39.75	580.171	330.707	910.879	9503.55	100.049
	cDCC							
	Short term	16	6.42	33.264	110.467	143.731	101.542	**23.457**
	Medium term	94	37.75	538.797	314.791	853.588	7827.88	103.126
	Long term	99	39.75	580.171	330.707	910.879	9503.55	100.049

Notes: The table presents the evaluation of out-of-sample daily WVaR for the wavelet details. it reports test statistics over last 250 days (February 19, 2010 to February 03, 2011). Exceedance: The term "exceedance" refers to an instance when portfolio losses are greater than corresponding VaR estimates in the backtest, In the literature, the terms "breaches" and "violations" are also used when referring to "exceedances". Fail. prob.: The failure probability. LR_{uc}: The LR test of unconditional coverage. LR_{ind}: The LR test of independence. LR_{cc}: The joint test of coverage and independence. DQ: The Dynamic Quantile test. $\mathbf{LF}(j)$: The loss function. The respective critical values of LR_{uc} and LR_{cc} statistics at 5% significance level are 3.84 and 5.99.

$$\mathbf{LF_{t+1}(j)} = \begin{cases} 1+\left(d_{t+1} - VaR_{t+1}(j)\,|_{\Omega_t}\right)^2, & \text{if} \quad d_{t+1} < VaR_{t+1}(j)\,|_{\Omega_t} \\ 0, & \text{if} \quad d_{t+1} \geq VaR_{t+1}(j)\,|_{\Omega_t} \end{cases} \tag{22}$$

The statistical adequacy of the WVaR forecasts computed by the three dynamic conditional correlation models is obtained by the backtesting procedure described before: we will characterize the model as an adequate one for the volatility forecasting, if the null hypothesis cannot rejected.[19] Table 20 presents the summary backtesting statistics for the daily WVaR forecasts of wavelet return series. In terms of LR_{uc}, LR_{ind}, LR_{cc} and DQ statistics, the null hypothesis

[19] rejection of the null hypothesis indicates that the computed VaR estimates are not sufficiently accurate.

are rejected at 1%, 2.5% and 5% significance level. According to these metrics, we cannot conclude whether an adequate multivariate model is more accurate than another one.[20] Column 9 of Table 20 shows the mean loss function values. A WVaR model is preferred to another one, if it yields a lower averaging loss criterion value: $\mathbf{LF(j)}$, defined as the sum of $\mathbf{LF_t(j)}$, that is,$\mathbf{LF(j)} = \sum_{t=1}^{n_0} \mathbf{LF_t(j)}$. At 99%, 97.5% and 95% confidence levels, $\mathbf{LF(j)}$ statistic indicates that DCC and *c*DCC models provide strong WVaR performance (smaller $\mathbf{LF(j)}$ values) than CCC model. We can note also, that the three dynamic conditional correlation models have the same WVaR performance at medium-term (intermediate scales) and long-term (high scales) horizons for 99%, 97.5% and 95% confidence levels.

5. Concluding remarks

The empirical analysis in the paper examined the co-movements and spillover effects in the stock market returns of three developed countries: U.K., U.S. and Japan for the period 01 January 2003 to 04 February 2011. Three multivariate conditional correlation volatility models were used, namely CCC model of Bollerslev (1990), DCC model of Engle (2002) and *c*DCC model of Aielli (2008). Empirical results show that multivariate estimates were significant for all returns in the CCC, DCC and *c*DCC models. However, these models showed evidence of volatility spillovers and asymmetric effects of negative and positive shocks of equal magnitude on the conditional variances. The statistical significance of DCC and *c*DCC estimates indicates that the conditional correlations were dynamic. In fact, the variance-covariance analysis produced useful information on the dynamic correlations between the three developed markets, and for each pairwise series, the dynamic conditional correlations vary considerably from their respective constant correlations, implying the absence of any constant correlation between the stock markets under study. The empirical findings showed that the U.K. and U.S. markets were highly correlated since the end of 2007 (the beginning of *subprime* crisis), followed by the U.K. and Japan markets and U.S. and Japan markets. These results confirm the presence of spillover effects between pairwise stock market returns. The paper also compared one-day ahead conditional volatility forecasts from the dynamic conditional correlation models used in the study, using 250 one-day out-of-sample forecasts, and showed that asymmetric *c*DCC model is preferred over the other models, according to the four used statistical loss metrics: mean squared error, mean absolute error, mean absolute percentage error and logarithm loss error.

The paper also combined the wavelet analysis and multivariate conditional volatility models to analyze the co-movements and volatility spillover effects in a multi-scale framework. Unlike the traditional multivariate dynamic

[20] The abovementioned backtesting tests focused on examining the accuracy of failure frequency and the independence of the failure process for VaR models. However, there are a large number of VaR models that can pass these statistical evaluation tests. How do risk managers choose among alternative VaR models? which model will generate fewer regulatory capital requirements and induce less oppurtunity cost of capital?. To answer these questions we use a regulatory loss function related to market risk.

conditional correlation volatility models, the wavelet-based dynamic conditional correlation approach allows one to decompose the spillover effect and co-movement into many sub-spillovers and sub-comovements on various time scales according to heterogeneous groups of traders and investors. However, wavelet analysis help investors to uncover the complex pattern of return and volatility spillovers on their own horizon, and make a good hedging strategy on their risk. The findings of wavelet analysis show that multivariate estimates were significant for all wavelet time series. Moreover, wavelet-based multivariate models highlight volatility spillovers on the conditional variance for all stock markets under study. In fact, the wavelet study successfully decomposes the total spillover into sub-spillovers, respectively for short-term, mid-term and long-term horizons. The out-of-sample forecasts over wavelet scales are evaluated using four statistical loss functions and one-day ahead wavelet VaR (WVaR) forecast accuracy. The out-of-sample forecast results showed that high scales (Long term component) provide smaller statistical loss metrics values than lower scales (Short component).

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