Multiproduct search

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Abstract

This paper presents a sequential search model where consumers look for several products from multiproduct firms. In a multiproduct search market, both consumer behavior and firm behavior are different from the single-product case. For example, a consumer may return to previously visited firms before running out of options, and prices can decrease with search costs. The framework is extended by allowing firms to use bundling strategies. Bundling tends to reduce the intensity of consumer search, which can soften competition and reverse the usual welfare assessment of competitive bundling in a perfect information setting. Applications to countercyclical pricing, loss leader pricing, and endogenous retail market structure are also discussed.

Keywords: consumer search, oligopoly, multiproduct pricing, countercyclical pricing, bundling
JEL classification: D11, D43, D83, L13

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1 Introduction

Consumers often look for several products on a given shopping trip. For example, during ordinary grocery shopping they buy food, drinks and household products; in high street shopping they purchase clothes, shoes and other goods; in the Christmas season they look for several presents. Sometimes a consumer seeks electronic combinations such as computer, printer and scanner; when furnishing a house they need several furniture items; when going on holiday or attending a conference they book both flights and hotels; and new parents look for many baby products. On the other side of the market, there are many multiproduct firms such as supermarkets, department stores, electronic retailers, and travel agencies which often supply most of the products a consumer is searching for in a particular shopping trip. Usually the shopping process also involves non-negligible search costs. Consumers need to reach the store, find out each product's price and how suitable they are, and then may decide to visit another store in pursuit of better deals. In effect, in many cases a consumer chooses to shop for several goods together to save on search costs.

Despite the ubiquity of multiproduct search and multiproduct firms, the search literature has been largely concerned with single-product search markets. In part, this is because a multiproduct search model is less tractable than a single-product one. In this paper, I develop a tractable model for multiproduct search markets, and show that a multiproduct search market exhibits some qualitatively different properties compared to the single-product case. I then argue that multiproduct search has rich market implications, and the developed framework can be used to address interesting economic issues such as countercyclical pricing, the welfare impact of competitive bundling, loss leader pricing, and endogenous retail market structure.

The basic framework of this paper is a sequential search model in which consumers look for several products and care about both price and product suitability. Each firm supplies all relevant products, but each product is horizontally differentiated across firms. By incurring a search cost, a consumer can visit a firm and learn all product and price information. In particular, the cost of search is incurred jointly for all products (i.e., there are economies of scale in search), and the consumer does not need to buy all products from the same firm (i.e., they can mix and match after sampling at least two firms if firms allow them to do so). Both features are realistic and important in multiproduct search markets.

In the basic model, I assume that firms use linear pricing strategies (i.e., set separate prices for each product). A distinctive feature of consumer behavior in multiproduct search is that a consumer may return to buy from previously visited firms before running out of options. By contrast, in a standard single-product sequential search model, a consumer never returns to earlier firms before sampling all firms. As far as pricing is concerned, with multiproduct consumer search, if a firm lowers one product's price, this will induce more consumers who are visiting it to terminate search and buy some other products as well. That is, a reduction of one product's price also boosts the demand for the firm's other products. I term this the *joint search effect*. As a result, even physically independent products are priced like complements.

Due to the joint search effect, prices can decline with search costs in a multiproduct search market. When search costs increase, the standard effect is that consumers will become more reluctant to shop around, which will induce firms to raise their prices. However, in a multiproduct search market, higher search costs can also strengthen the joint search effect and make the products in each firm more like complements, which will induce firms to lower their prices. When the latter effect dominates prices will fall with search costs.

Another prediction is that firms in a multiproduct search environment tend to set lower prices than in a single-product search environment. This is for two reasons: first, due to economies of scale in search, consumers on average sample more firms in the multiproduct search case than in the single-product search case, which tends to increase each product's own-price elasticity; second, multiproduct search causes the joint search effect, which gives rise to the complementary pricing phenomenon and so increases products' cross-price elasticities. This observation can provide a possible explanation for the phenomenon of countercyclical pricing—prices of many retail products drop during high-demand periods such as weekends and holidays.[1] During high-demand periods, it is often the case that a higher proportion of consumers become multiproduct searchers (e.g., many households conduct their weekly grocery shopping during weekends), and so retailers have incentives to reduce their prices.

In multiproduct markets, bundling is a widely observed pricing strategy. Bundling is often used as a price discrimination or entry deterrence device.[2] This paper argues that in a search environment, bundling has a new function: it can discourage consumers from exploring rivals' deals. This is because bundling reduces the anticipated benefit from mixing-and-matching after visiting another firm. This search-discouraging effect works against the typical pro-competitive effect of competitive bundling in a perfect information scenario.[3] When search costs are relatively high the new effect can be such that bundling benefits firms and harms consumers.[4] Therefore, our findings indicate that assuming away information frictions may significantly distort the welfare

[1]See relevant empirical evidence documented in, for example, Warner and Barsky (1995), MacDonald (2000), and Chevalier, Kashyap, and Rossi (2003).

[2]See, for instance, Adams and Yellen (1976), and McAfee, McMillan, and Whinston (1989) for the view of price discrimination, and Whinston (1990) for the view of entry deterrence.

[3]Matutes and Regibeau (1988), Economides (1989), and Nalebuff (2000) studied competitive pure bundling, and Matutes and Regibeau (1992), Anderson and Leruth (1993), Thanassoulis (2007), and Armstrong and Vickers (2010) studied competitive mixed bundling. The main insight emerging from all these works is that compared to linear pricing, bundling (whether pure or mixed) has a tendency to intensify price competition, and under the assumptions of unit demand and full market coverage (which are also retained in this paper) it typically reduces firm profits and boosts consumer welfare.

[4]In different settings, Carbajo, de Meza and Seidmann (1990) and Chen (1997) argue that (asymmetric) bundling can create "vertical" product differentiation between firms, thereby softening price competition.

assessment of bundling.[5]

Related literature. Since the seminal work by Stigler (1961), there has been a vast literature on search, but most papers focus on single object search (see, for example, Diamond, 1971, Burdett and Judd, 1983, and Stahl, 1989 for consumer search models). There is a small branch of literature that investigates the optimal stopping rule in multiproduct search (Burdett and Malueg, 1981, Carlson and McAfee, 1984, and Gatti, 1999).[6] They have emphasized the similarity between single-product and multiproduct search in the sense that in both cases the stopping rule often features the static reservation property. However, I argue that despite this similarity, consumer search behavior still exhibits substantial differences between the two cases.

More importantly, the above works do not consider an active supply side. McAfee (1995) studies a multiproduct search model with endogenous prices. It extends Burdett and Judd (1983) to the multiproduct case. Each product is homogenous across stores, and by incurring a search cost a consumer can learn price information from a random number of stores. In particular, there are consumers who learn information from only one store. As a result, similar to Burdett and Judd (1983), firms adopt mixed pricing strategies, reflecting the trade-off between exploiting less informed consumers and competing for more informed consumers. However, multiproduct search generates multiple types of (symmetric) equilibria. In particular, there is a continuum of equilibria in which firms randomize prices on the reservation frontier such that one product's price decline must be associated with the rise of some other prices.[7] The model offers interesting insights, but both the multiplicity of equilibria and the complication of equilibrium characterization restrict its applicability. This paper develops an alternative multiproduct search framework with differentiated products, where the symmetric equilibrium is unique and prices are deterministic.

Lal and Matutes (1994) also present a multiproduct search model where two firms locate at the two ends of a Hotelling city and each product is otherwise homogenous across firms. Each consumer needs to pay a location-specific cost to reach firms and discover the price information. Their setting is, however, subject to the Diamond paradox: each firm charges the monopoly prices and no consumers participate in the

[5] The European Commission has recently branded all bundled financial products as anti-competitive and unfair. One of the main reasons is that the practice reduces consumer mobility. See the consultation document "On the Study of Tying and Other Potentially Unfair Commercial Practices in the Retail Financial Service Section", 2009.

[6] In Burdett and Malueg (1981) and Carlson and McAfee (1984), consumers search for serval products among a large number of multiproduct stores that supply homogenous products and set random prices according to an exogenous price distribution. The former mainly deals with the case with free recall, and the latter deals with the case with no recall. Gatti (1999) considers a more general setting in which consumers search for prices to maximize a general indirect utility function.

[7] In the other type of equilibria, firms randomize prices over the acceptance set (*not* just on its border). They are, however, qualitatively similar to the single-product equilibrium in the sense that the marginal price distribution for each product is the same as in the single-product search case, and so is the profit from each product.

market. Lal and Matutes show that firms can avoid the market collapse by advertising prices of a subset of products. In one type of equilibrium, each firm advertises a low price (even below marginal cost) of one product to persuade consumers to visit the store (i.e., loss leading occurs), but charges the monopoly price for the other unadvertised product. In equilibrium, consumers do not search beyond the first visited firm and so two-stop shopping never happens. In my model with product-level differentiation, consumers search for both low prices and high product suitability, and an equilibrium with an active market exists even without advertising and two-stop shopping takes place as we often observe in the real market.[8]

In terms of the modelling approach, this paper is built on the single-product search model with differentiated products (Weitzman, 1979, Wolinsky, 1986, and Anderson and Renault, 1999). Recently, this framework has been adopted to study various economic issues.[9] Compared to the homogeneous product search model, models with product differentiation often better reflect consumer behavior in markets that are typically characterized by nonstandardized products. Moreover, they avoid the well-known modelling difficulty suggested by Diamond (1971), who shows that with homogeneous products and positive search costs (no matter how small) all firms will charge a monopoly price and no consumers search beyond the first sampled firm. In search models with product differentiation, there are some consumers who are ill-matched with their initial choice of supplier and then search further, so that the pro-competitive benefit of actual search is present.

The rest of the paper is organized as follows: Section 2 presents the basic model with linear pricing and analyzes consumer search behavior. Section 3 characterizes equilibrium linear prices and conducts comparative statics analysis, and an application to countercyclical pricing is also discussed. Section 4 studies bundling in a search market and examines its welfare impact relative to linear pricing. Section 5 concludes, and discusses other applications to loss leader pricing and endogenous retail market structure. Omitted details are presented in the Appendix.

[8] Ellison (2005) uses Lal and Matutes's framework to study add-on pricing by assuming that the base product's price information is perfect while the add-on's price information is not. (In the end of this paper, I discuss a related variant in which consumers only need to search for one product's information.) Shelegia (2012) studies a multiproduct version of Varian (1980) in which for exogenous reasons one group of consumers visits only one store while the other visits two. Rhodes (2011) proposes a multiproduct monopoly model in which each consumer knows her private valuations for all products but needs to incur a cost to reach the firm and learn prices. He shows that selling multiple products can solve the Diamond hold-up problem, which would unravel the market in a single-product case with inelastic consumer demand.

[9] See, for instance, Armstrong, Vickers, and Zhou (2009) and Zhou (2011) for the market implications of prominence and non-random consumer search, Bar-Isaac, Caruana, and Cunat (2012) for how the decline of search costs affects product design, and Haan and Moraga-González (2011) for attention-grabbing advertising.

2 A Model of Multiproduct Search

There are a large number of consumers in the market with measure normalized to one. Each consumer is looking for two products (e.g., two furniture items, or clothes and shoes), and they have unit demand for each product. There are $n \geq 2$ multiproduct firms in the market, each supplying both products at a constant marginal cost, which is normalized to zero.

Each product is horizontally differentiated across firms. For example, different firms may supply different brands of furniture or clothes and shoes with different styles, and consumers have idiosyncratic tastes. Specifically, a consumer's valuations for the two products in each firm are randomly drawn from a common joint cumulative distribution function $F(u_1, u_2)$ defined on $[\underline{u}_1, \overline{u}_1] \times [\underline{u}_2, \overline{u}_2]$ which has a continuous density $f(u_1, u_2)$. The match utilities are realized independently across firms and consumers (but a consumer may have correlated match utilities for the two products in the same firm). For simplicity, I assume that the two products are neither complements nor substitutes, in the sense that a consumer obtains an additive utility $u_1 + u_2$ if product i has a match utility u_i, $i = 1, 2$. Let $F_i(u_i)$ and $H_i(u_i|u_j)$ denote the marginal and conditional distribution functions; $f_i(u_i)$ and $h_i(u_i|u_j)$ denote the marginal and conditional densities.

I assume that all consumers buy both products in equilibrium, i.e., the market is fully covered.[10] (This is the case, for example, when consumers have no outside options or when they have large basic valuations for each product on top of the above match utilities.) Consumers do not need to purchase both products from the same firm. This possibility of multi-stop shopping is realistic and also important for our model. Otherwise, the model would degenerate to a single-product one with a composite product with match utility $u_1 + u_2$. In the basic model, firms use linear pricing strategies and charge a separate price for each product.

Initially consumers are assumed to have imperfect information about the (actual) prices firms are charging and match utilities of all products.[11] But they can gather information through a sequential search process: by incurring a search cost $s \geq 0$, a consumer can visit a firm and find out both of its prices (p_1, p_2) and both of its match utilities (u_1, u_2). At each firm (except the last one), the consumer faces the following options: stop searching and buy both products (maybe from firms visited earlier), or buy one product and keep searching for the other, or keep searching for both products. The cost of search is assumed to be independent of the number of products a consumer is looking for, which reflects economies of scale in search. Finally, I suppose that consumers have free recall/return, i.e., there are no extra costs in buying products from

[10] The assumption of full market coverage is often adopted in oligopoly models. Our main insights carry over to the case without this assumption (though the analysis will become more involved).

[11] If consumers purchase products frequently, they may know both price and product information before search. However, in reality both prices and product variants in many retailers change over time, such that imperfect information might be still a plausible presumption.

a previously visited store.

The timing is as follows: Firms set prices simultaneously first, and then consumers start to search without observing firms' actual prices (though they hold the equilibrium belief about firms' pricing strategy). After visiting each firm, consumers decide whether to stop searching or not. Both consumers and firms are assumed to be risk neutral. I focus on symmetric equilibria in which firms set the same prices and consumers sample firms in a random order (and without replacement).[12] The equilibrium concept I use is the perfect Bayesian equilibrium. Each firm sets its prices to maximize profits, given its expectation of consumers' search behavior and other firms' pricing strategy. Consumers search optimally, to maximize their expected surplus, given the match utility distribution and their rational beliefs about firms' pricing strategy. At each firm, even after observing off-equilibrium offers, consumers hold the equilibrium belief about the unsampled firms' prices.[13]

I have made several simplifying assumptions to make the model tractable, and it is useful to discuss them at this point.

Economies of scale in search. The assumption that the search cost is independent of the number of products a consumer is seeking is an approximation when the search cost is mainly for learning the existence of a seller or for reaching the store. In the other polar case where the cost of search is solely from product inspection and so totally divisible among products, the multiproduct search problem degenerates to two separate single-product search problems.[14] In reality, most situations are in between (a typical shopping process involves a fixed cost for reaching the store and also variable in-store search costs for inspecting each product). Our simplification is made both for analytical convenience and for highlighting the difference between multiproduct and single-product search.

Free recall. Free recall is often assumed in the consumer search literature. It could be appropriate, for instance, when a consumer can phone previously visited firms (e.g., furniture stores) to order the products she decides to buy, or when shopping online a consumer can leave the browsed websites open. In most consumer markets, however, there are positive returning costs. I choose to assume free recall both for tractability, and for facilitating comparison with the single-product search model in Wolinsky (1986)

[12] As usual in search models, there exists an uninteresting equilibrium where consumers expect all firms to set very high prices and do not participate in the market at all, and so firms have no incentive to reduce their prices. I do not consider this equilibrium further. The issue of possible asymmetric equlibria will be discussed later.

[13] Notice that in our setting there are no correlated economic shocks (e.g., aggregate cost shocks) across firms and so their pricing decisions are independent of each other.

[14] The case with divisible inspection costs among products will be non-trivial if consumers have to one-stop shop (e.g., due to bundling). After inspecting a product at one firm, a consumer needs to decide whether to continue to inspect the other product at the same firm or to inspect products in other firms. (This case is also equivalent to a search problem with multi-attribute products and separable inspection costs among attributes.)

and Anderson and Renault (1999) (both of which assume free recall).

Independent products. In reality the products that a consumer is seeking in a particular shopping trip are rarely independent. In many circumstances (e.g., when shopping for clothes and shoes), they are more or less intrinsic complements in the sense that a higher valuation for one product increases the consumer's willingness to pay for the other (e.g., the utility function takes the form of $u_1 + u_2 + \lambda u_1 u_2$ with $\lambda > 0$). As I discuss later, considering complementary valuations of this kind will complicate analysis but not generate important new insights. In addition, given the assumption of full market coverage, the two products in my model can also be regarded as perfect complements in the sense that consumers can obtain positive utility only by consuming both of them together.

2.1 The optimal stopping rule

I first derive the optimal stopping rule (which has been proved in Burdett and Malueg, 1981, or Gatti, 1999 in a price search scenario). The first observation is that given the indivisible search cost and free recall, a consumer will never buy one product first and keep searching for the other. Hence, at any store (except the last one) the consumer faces only two options: stop searching and buy both products (one of which may be from a firm visited earlier), or keep searching for both.

Denote by

$$\zeta_i(x) \equiv \int_x^{\overline{u}_i} (u_i - x) dF_i(u_i) = \int_x^{\overline{u}_i} [1 - F_i(u_i)] du_i \tag{1}$$

the expected incremental benefit from sampling one more product i when the maximum utility of product i so far is x and all product i have the same price. (The second equality is from integration by parts.) Then the optimal stopping rule when all firms charge the same prices is as follows:

Lemma 1 *Suppose prices are linear and symmetric across firms. Suppose the maximum match utility of product i observed so far is z_i and there are firms left unsampled. Then a consumer will stop searching if and only if*

$$\zeta_1(z_1) + \zeta_2(z_2) \le s \ . \tag{2}$$

The left-hand side of (2) is the expected benefit from sampling one more firm given the pair of maximum utilities so far is (z_1, z_2), and the right-hand side is the search cost. This stopping rule seems "myopic" at the first glance, but it is indeed sequentially rational. It can be understood by backward induction. When in the penultimate firm, it is clear that (2) gives the optimal stopping rule because given (z_1, z_2) the expected benefit from sampling the last firm is $\mathbb{E}[\max(0, u_1 - z_1) + \max(0, u_2 - z_2)]$, which equals the left-hand side of (2). (Note that I did not assume u_1 and u_2 are independently distributed. The separability of the incremental benefit in (2) is because of the additive utility function and the linearity of the expectation operator.) Now step back and consider the situation when the consumer is at the firm before that. If (2) is violated, sampling one more firm is always desirable. By contrast, if (2) holds and if the consumer continues to search, the updated maximum match utility pair will be no worse than (z_1, z_2) no matter what she will find at the next firm, and so she will stop searching there. This implies that if (2) holds, the benefit from keeping searching is the same as sampling just one more firm. Expecting that, the consumer should actually cease her search now.

Figure 1 below illustrates the optimal stopping rule. A is the set of (z_1, z_2) which satisfies (2) and let us refer to it as the *acceptance set*. Then when there are no price differences across firms, a consumer will stop searching if and only if the maximum utility pair so far lies within A. Let B be the complement of A.

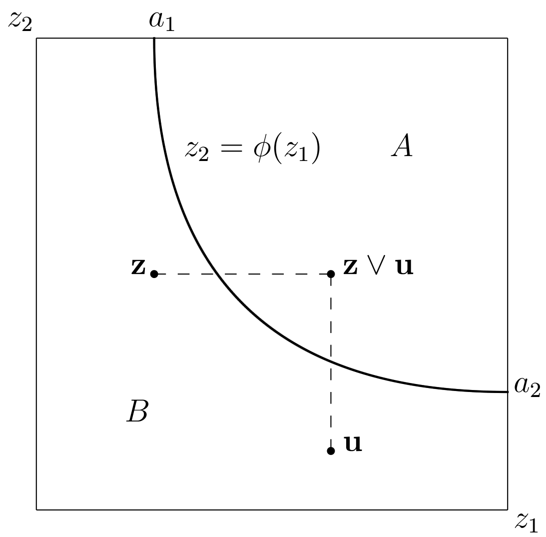

Figure 1: The optimal stopping rule in multiproduct search

Define the border of A as $z_2 = \phi(z_1)$ (i.e., $(z_1, \phi(z_1))$ satisfies (2) with equality) and call it the *reservation frontier*. The reservation frontier is decreasing and convex. From the definition of $\phi(\cdot)$, one can check that

$$\phi'(z_1) = -\frac{1 - F_1(z_1)}{1 - F_2(\phi(z_1))} < 0 ,$$

and so $\phi(\cdot)$ is a decreasing function. Then it is also easy to see that $\phi'(z_1)$ increases with z_1, i.e., $\phi(\cdot)$ is convex.[15]

[15] If we consider two intrinsic complements, the reservation frontier may no longer be decreasing. For example, when the utility function takes the form of $u_1 + u_2 + \lambda u_1 u_2$ with $\lambda > 0$, one can check that in the duopoly case, for instance, the reservation frontier satisfies

$$\frac{1}{2}(1 - u_1)^2 + \frac{1}{2}(1 - u_2)^2 + \frac{\lambda}{4}\left[(u_1 - u_2)^2 + (1 - u_1 u_2)^2\right] = s ,$$

and it is not monotonically decreasing. This is because now finding a better matched product 1 may strictly increase a consumer's incentive to find a better matched product 2. When there are more than

Notice that a_i on Figure 1 is just the reservation utility level when the consumer is only searching for product i. (If all product i have the same price, a consumer will stop searching if and only if the maximum utility so far is no less than a_i.) It solves

$$\zeta_i(a_i) = s \ , \tag{3}$$

and satisfies $\phi(a_1) = \overline{u}_2$ and $\phi(\overline{u}_1) = a_2$. This is because when the maximum possible utility of one product has been achieved, the consumer will behave as if she is only searching for the other product.

Search behavior comparison. It is useful to compare consumer search behavior between single-product and multiproduct search. The early literature has emphasized that in both cases (given additive utilities in the multiproduct case) the optimal stopping rule possesses the static reservation property. Despite this similarity, consumers' search behavior exhibits some important differences between the two cases, which have not been discussed before.

In single-product search with perfect recall, the stopping rule is characterized by a reservation utility a. When a consumer is already at some firm (except the last one), she will stop searching if and only if the current product has a utility greater than a. Previous offers are irrelevant because they must be worse than a (otherwise the consumer would not have come to this firm). As a result, a consumer never returns to previously visited firms until she finishes sampling all firms. In particular, if there are an infinite number of firms, the consumer never exercises the recall option.

However, in multiproduct search, a consumer's stopping rule depends on *both* the current firm's offer $\mathbf{u}$ and the best offer so far $\mathbf{z}$. This can be seen from the example indicated in Figure 1, where the current offer $\mathbf{u}$ lies outside the acceptance set A but the consumer will stop searching because $\mathbf{z} \vee \mathbf{u} \in A$ (where $\vee$ denotes the "join" of two vectors). When she stops searching, she will go back to some previous firm to buy product 2. This has two implications for the demand analysis. First, with multiproduct search, a firm's price adjustment will not only affect a consumer's search decision at this firm, but will also affect her search decisions at subsequent firms if she leaves this firm. Second, a consumer often returns to a previously visited firm to buy one product even if there are firms left unsampled. This is true even if there are an infinite number of firms.

These differences will complicate the demand analysis in multiproduct search. Moreover, unlike the single-product search case, considering an infinite number of firms does not achieve any simplicity. In effect, with multiproduct search, the simplest case is when there are only two firms. Hence, in the following analysis, I mainly deal with the duopoly case. As I will discuss in section 5.1, such a simplification does not lose the most important insights concerning firm pricing in a multiproduct search market. (A detailed analysis of the general case with more than two firms is provided in the online

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two firms, considering intrinsic complements will even render the optimal stopping rule non-stationary (see Gatti, 1999, for a related discussion).

supplementary document at https://sites.google.com/site/jidongzhou77/research.)

3 Equilibrium Prices

3.1 The single-product benchmark

To facilitate comparison, I first report some results from the single-product search model (see Wolinsky, 1986 and Anderson and Renault, 1999 for an analysis with n firms). Suppose the product in question is product i, and the unit search cost is still s. Then the reservation utility level is a_i defined in (3), and it decreases with s (i.e., a higher search cost will make consumers less willing to search on). In the following analysis, I mainly focus on the case with a relatively small search cost:

$$s < \zeta_i(\underline{u}_i) \Leftrightarrow a_i > \underline{u}_i \text{ for both } i = 1, 2 \ . \tag{4}$$

(Remember that $\zeta_i(\underline{u}_i)$ is the expected benefit from sampling another product i when the current one has the lowest possible match utility.) This condition ensures an active search market even in the single-product case.

The symmetric equilibrium price p_i^0 in the duopoly case is then determined by

$$\frac{1}{p_i^0} = \underbrace{f_i(a_i)[1 - F_i(a_i)] + 2\int_{\underline{u}_i}^{a_i} f_i(u)^2 du}_{\alpha_0} \ . \tag{5}$$

It follows that p_i^0 increases with the search cost s (or decreases with a_i) if

$$f_i(a_i)^2 + f_i'(a_i)[1 - F_i(a_i)] \geq 0 \ .$$

This condition is equivalent to an increasing hazard rate $f_i/(1 - F_i)$. Then we have the following result (Anderson and Renault, 1999 have shown this result for an arbitrary number of firms):

Proposition 1 *Suppose the consumer is only searching for product i and the search cost condition (4) holds. Then the equilibrium price defined in (5) increases with search costs if the match utility has an increasing hazard rate $f_i/(1 - F_i)$.*

3.2 Equilibrium prices in multiproduct search

I now turn to the multiproduct search case. Let (p_1, p_2) be the symmetric equilibrium prices. For notational convenience, let (u_1, u_2) be the match utilities of firm I, the firm in question, and (v_1, v_2) be the match utilities of firm II, the rival firm. In the symmetric equilibrium, for a consumer who samples firm I first, her reservation frontier $u_2 = \phi(u_1)$ is determined by

$$\zeta_1(u_1) + \zeta_2(\phi(u_1)) = s \ , \tag{6}$$

which simply says that the expected benefit of sampling firm II is equal to the search cost. Note that $\phi(u_1)$ is only defined for $u_1 \in [a_1, \overline{u}_1]$ (see Figure 2 below). But for convenience, let us extend its domain to all possible values of u_1, and stipulate $\phi(u_1) > \overline{u}_2$ for $u_1 < a_1$.

Instead of writing down the demand functions and deriving the first-order conditions for the equilibrium prices directly, I use the following economically more illuminating method. Starting from an equilibrium, suppose firm I unilaterally decreases p_2 by a small ε. How does this adjustment affect firm I's profits? Let us focus on the first-order effects. First, firm I suffers a loss from those consumers who only buy product 2 from it because they are now paying less. Since in equilibrium half of the consumers buy product 2 from firm I (remember the assumption of full market coverage), this loss is $\varepsilon/2$. Second, firm I gains from boosted demand: (i) For those consumers who visit firm I first, they will be more likely to stop searching since they hold equilibrium beliefs that the second firm is charging the equilibrium prices. Once they stop searching, they will buy *both* products from firm I immediately. (ii) For those consumers who eventually sample both firms, they will be more likely to buy product 2 from firm I due to the price reduction. In equilibrium, the loss and gain should be such that firm I has no incentive to deviate, which generates the first-order condition for p_2.

Now let us analyze in detail the two (first-order) gains from the proposed small price reduction. The first gain is from the effect of the price reduction on consumers' search decisions. How many consumers who sample firm I first will stop searching because of the price reduction? (Note that the consumers who sample firm II first hold equilibrium beliefs and so their stopping decisions remain unchanged.)

Denote by $\phi(u_1|\varepsilon)$ the new reservation frontier. Since reducing p_2 by ε is equivalent to increasing u_2 by ε, $\phi(u_1|\varepsilon)$ solves

$$\zeta_1(u_1) + \zeta_2(\phi(u_1|\varepsilon) + \varepsilon) = s \ ,$$

so $\phi(u_1|\varepsilon) = \phi(u_1) - \varepsilon$ according to the definition of $\phi(\cdot)$. That is, the reservation frontier moves downward everywhere by ε, and the stopping region A expands (i.e., more consumers buy immediately at firm I) as illustrated in Figure 2 below.[16] For a small ε, the number of consumers who originally continued to search but now cease searching and buy immediately at firm I (i.e., the probability measure of the shaded area between $\phi(u_1)$ and $\phi(u_1|\varepsilon)$ in Figure 2) is

$$\frac{\varepsilon}{2}\int_{a_1}^{\overline{u}_1} f(u, \phi(u))du \ . \tag{7}$$

(Remember that half of the consumers sample firm I first. The integral term is the line integral along the reservation frontier in the u_1 dimension.)

[16] More precisely, $\phi(a_1|\varepsilon) = \overline{u}_2 - \varepsilon$ and so the reservation frontier has a small vertical segment at $u_1 = a_1$. But this does not affect our analysis as ε is small.

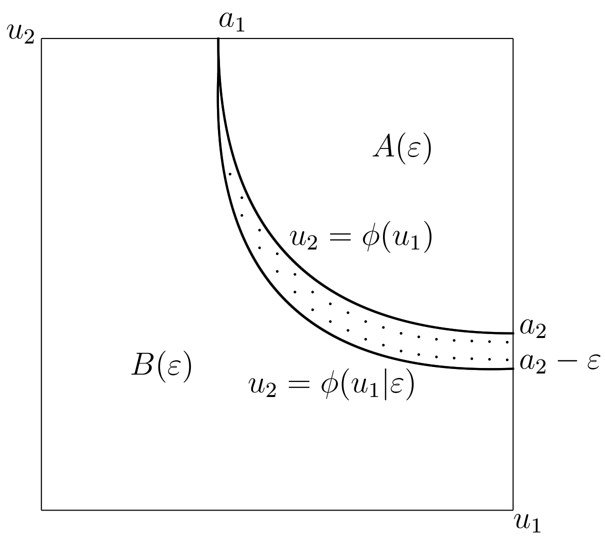

Figure 2: Price deviation and the stopping rule

What is firm I's net benefit from these marginal consumers? Realize that these marginal consumers now buy both products from firm I for sure, while before the price deviation they only bought each product from firm I with some probability less than one (i.e., when they search on but find worse products at firm II). To be specific, consider a marginal consumer on the reservation frontier with match utilities $(u_1, \phi(u_1))$. If she chooses to sample firm II, she will find a worse product 1 at firm II (i.e., $v_1 < u_1$) with probability $F_1(u_1)$, in which case she will return to firm I and buy its product 1. Similarly, if she continues to sample firm II, she will find a worse product 2 at firm II (i.e., $v_2 < \phi(u_1)$) with probability $F_2(\phi(u_1))$, in which case she will return to firm I and buy its product 2. Hence, the net benefit from inducing this marginal consumer to cease searching is $p_1[1 - F_1(u_1)] + p_2[1 - F_2(\phi(u_1))]$. We then sum this benefit over all marginal consumers on the reservation frontier. By using (7), this total benefit is

$$\frac{\varepsilon}{2}\int_{a_1}^{\overline{u}_1} \{p_1[1 - F_1(u)] + p_2[1 - F_2(\phi(u))]\} f(u, \phi(u))du \ . \tag{8}$$

The second gain is from those consumers who sample both firms. They will now more likely buy product 2 from firm I due to the price reduction. Consider first a consumer who visits firm I first and finds match utilities $(u_1, u_2) \in B(\varepsilon)$. She will then continue to visit firm II, but will return to firm I and buy its product 2 if $v_2 < u_2 + \varepsilon$. The probability of that event is $F_2(u_2 + \varepsilon) \approx F_2(u_2) + \varepsilon f_2(u_2)$. So the small price adjustment increases the probability that this consumer buys product 2 from firm I by $\varepsilon f_2(u_2)$. Then the total increased probability from all such consumers is $\frac{\varepsilon}{2}\int_{B(\varepsilon)} f_2(u_2)dF(\mathbf{u}) \approx \frac{\varepsilon}{2}\int_B f_2(u_2)dF(\mathbf{u})$. (Since $B(\varepsilon)$ converges to B as $\varepsilon \to 0$, we can discard all higher order effects.) Similarly, one can show that the increased probability that those consumers who sample firm II first and then come to firm I buy product 2 at firm I is $\frac{\varepsilon}{2}\int_B f_2(v_2)dF(\mathbf{v})$. Adding them together gives us the second gain, which is

$$p_2\varepsilon\int_B f_2(u_2)dF(\mathbf{u})\ . \tag{9}$$

In equilibrium, the (first-order) loss $\varepsilon/2$ from the small price reduction should be equal to the sum of the two (first-order) gains in (8) and (9). This yields the first-order condition for p_2:

$$\begin{aligned} 1 \;=\; & \underbrace{2p_2\int_B f_2(u_2)dF(\mathbf{u}) + p_2\int_{a_1}^{\overline{u}_1}[1-F_2(\phi(u))]f(u,\phi(u))du}_{\text{standard effect}} \\ & + \underbrace{p_1\int_{a_1}^{\overline{u}_1}[1-F_1(u)]f(u,\phi(u))du}_{\text{joint search effect}}\ . \end{aligned} \tag{10}$$

The first two terms on the right-hand side capture the standard effect of a product's price adjustment on its own demand: reducing p_2 increases demand for product 2. (This is similar to the right-hand side of (5) in the single-product search case.)

The last term, however, captures a new feature of the multiproduct search model: when firm I reduces its p_2, more consumers who sample it first will stop searching and buy *both* products, which increases the demand for its product 1 as well. This makes the two products supplied by the same firm like complements even if they are physically independent.[17] This effect occurs because each consumer is searching for two products and the cost of search is incurred jointly for them, and so I refer to it as the *joint search effect* henceforth.

Also notice that the size of the joint search effect (which determines the degree of "complementarity" between the two products in each firm) relies on the mass of marginal consumers on the reservation frontier, i.e., (7). It depends not only on the density function f but also on the "length" of the reservation frontier as indicated in Figure 2. For example, in the uniform distribution case, when the search cost increases, the reservation frontier becomes longer such that the mass of marginal consumers rises and thus the two products in each firm become more like complements. As we shall see below, this observation plays an important role in firms' pricing decisions.

Similarly, one can derive the first-order condition for p_1:

$$\begin{aligned} 1 \;=\; & 2p_1\int_B f_1(u_1)dF(\mathbf{u}) + p_1\int_{a_2}^{\overline{u}_2}[1-F_1(\phi^{-1}(u))]f(\phi^{-1}(u),u)du \\ & + p_2\int_{a_2}^{\overline{u}_2}[1-F_2(u)]f(\phi^{-1}(u),u)du\ , \end{aligned} \tag{11}$$

[17] Notice that the complementarity caused by the joint search cost is different from intrinsic complementarity. If information is perfect and the two products are intrinsic complements, then reducing the price of a firm's one product will not influence consumers' decisions of where to buy the other product. Hence, considering a perfect information setting with intrinsic complements cannot reproduce the main results in this paper.

where ϕ^{-1} is the inverse function of ϕ. The first two terms on the right-hand side reflect the standard effect of adjusting price p_1, and the last term captures the joint search effect. I summarize the results in the following lemma:[18]

Lemma 2 *Under the search cost condition (4), the first-order conditions for p_1 and p_2 to be the equilibrium prices are given in (10) and (11).*

Both (10) and (11) are linear equations in prices, and the system of the two prices has a unique solution. Thus, the symmetric equilibrium, if it is characterized by the first-order conditions, will be unique. Notice that if firms ignored the joint search effect, then the pricing problem would be separable between the two products. A special case is when $s = 0$ (so $a_i = \overline{u}_i$ and B equals the whole match utility domain). Then the effect of a price adjustment on consumer search behavior (i.e., (8)) disappears, and the first-order conditions simplify to

$$\frac{1}{p_i} = 2\int_{\underline{u}_i}^{\overline{u}_i} f_i(u)^2 du \ . \tag{12}$$

In this case, the multiproduct model yields the same equilibrium prices as the single-product model.

In the following analysis, I will often rely on the case with two symmetric products. Slightly abusing the notation, let the one-variable functions $F(\cdot)$ and $f(\cdot)$ denote the common marginal distribution function and density function, respectively. Let a be the common reservation utility in each dimension. In particular, with symmetric products, we have $f(u_1, u_2) = f(u_2, u_1)$ and the reservation frontier satisfies $\phi(\cdot) = \phi^{-1}(\cdot)$, i.e., it is symmetric around the 45-degree line in the match utility space. If p is the equilibrium price of each product, then both (10) and (11) simplify to

$$\begin{aligned}\frac{1}{p} &= \underbrace{2\int_B f(u_i)f(u_i,u_j)d\mathbf{u} + \int_a^{\overline{u}} [1-F(\phi(u))]f(u,\phi(u))du}_{\text{standard effect: }\alpha} \\ &\qquad + \underbrace{\int_a^{\overline{u}} [1-F(u)]f(u,\phi(u))du}_{\text{joint search effect: }\beta} \ .\end{aligned} \tag{13}$$

[18] One can also derive the first-order conditions by calculating the demand functions directly. For example, when firm I unilaterally deviates to $(p_1 - \varepsilon_1, p_2 - \varepsilon_2)$, the demand for its product 1 is

$$\frac{1}{2}\int_{\underline{u}_1}^{\overline{u}_1} [1 - H_2(\phi(u_1|\boldsymbol{\varepsilon})|u_1)(1-F_1(u_1+\varepsilon_1))]\, dF_1(u_1) + \frac{1}{2}\int_{\underline{u}_1}^{\overline{u}_1} H_2(\phi(v_1)|v_1)(1-F_1(v_1-\varepsilon_1))dF_1(v_1) \ ,$$

where $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2)$, $\phi(u_1|\boldsymbol{\varepsilon}) = \phi(u_1+\varepsilon_1) - \varepsilon_2$ is the reservation frontier associated with the deviation, and $H_i(\cdot|\cdot)$ is the conditional distribution function. Consumers who sample firm I first will buy its product 1 if they stop searching immediately or if they continue to search but find firm II's product 1 is a worse deal. Consumers who sample firm II first will purchase firm I's product 1 if they come to firm I and find firm I's product 1 is a better deal. The deviation demand for product 2 is similar.

Discussion: the second-order conditions and asymmetric equilibria. In our multi-product search model, it is difficult to investigate the second-order conditions in general. In the online supplementary document, I show that in the case of symmetric products and independent match utilities, each firm's profit function is locally concave around the price defined in (13) under fairly general conditions. In the examples with uniform and exponential distribution (which are used for illustration below), it can be numerically verified that a firm's profit function is globally quasi-concave, and thus the first-order conditions are sufficient for the equilibrium prices.

A related issue is the possible existence of a type of asymmetric equilibrium where firms put different products on sale. For example, in the case with symmetric products, one firm may charge price p_L for its product 1 and price $p_H > p_L$ for its product 2, and the other firm sets prices in the opposite way. However, as shown in the online supplementary document, this type of equilibrium cannot be sustained at least when the two symmetric products have independent match utilities and f is logconcave.

For illustration of the equilibrium prices, I present two examples:

The uniform example: Suppose u_1 and u_2 are independent, and $u_i \sim U[0,1]$. Then $\zeta_i(x) = (1-x)^2/2$. So $a = 1 - \sqrt{2s}$, and condition (4) requires $s \leq 1/2$. The reservation frontier satisfies

$$(1-u)^2 + (1-\phi(u))^2 = 2s \ ,$$

so the stopping region A is a quarter-disk with radius $\sqrt{2s}$. Then (13) implies

$$p = \frac{1}{2 - (\frac{1}{2}\pi - 1)s} \ , \tag{14}$$

where $\pi \approx 3.14$ is the mathematical constant. (The standard effect is $\alpha = 2 - s\pi/2$, and the joint search effect is $\beta = s$.)[19]

The exponential example: Suppose u_1 and u_2 are independent, and $f_i(u_i) = e^{-u_i}$ for $u_i \in [0, \infty)$. Then $\zeta_i(x) = e^{-x}$. So $e^{-a} = s$, and the search cost condition (4) requires $s \leq 1$. The reservation frontier satisfies

$$e^{-u} + e^{-\phi(u)} = s \ ,$$

so $\phi(u)$ is one branch of a hyperbola. Then (13) implies

$$p = \frac{1}{1 + \frac{1}{6}s^3} \ .$$

(The standard effect is $\alpha = 1$, and the joint search effect is $\beta = s^3/6$.)

The price increases with search costs in the uniform example, but it decreases with search costs in the exponential example. As I will explain below, the result that prices can decline with search costs is not exceptional in the multiproduct search model.

[19] The first term in (13) is $2\int_B d\mathbf{u}$, so it equals two times the area of region B, i.e., $2(1 - s\pi/2) = 2 - s\pi$. The second term in (13) is $\int_a^1 [1 - \phi(u)]du$, which is the area of region A and so equals $s\pi/2$. The joint search effect is $\beta = \int_a^1 (1-u)du = s$ according to the definition of a.

3.3 Price and search cost

This section investigates how prices vary with search costs. When search costs rise, there are two effects: First, consumers will become more reluctant to shop around, and so fewer of them will sample both firms (i.e., the region of B shrinks). This always induces firms to raise their prices. Second, when search costs rise, the mass of marginal consumers on the reservation frontier also changes. If the number of marginal consumers increases with search costs (which is true, for instance, in the uniform distribution case where the reservation frontier becomes "longer" as search costs rise in the permitted range of (4)), firms have an incentive to reduce prices. This incentive is further strengthened in the multiproduct case due to the joint search effect: stopping a marginal consumer from continuing to search can boost demand for both products. The final prediction depends on which effect dominates.

I introduce the following regularity condition:

$$\frac{h_i(u_i|u_j)}{1-F_i(u_i)} \text{ increases with } u_i \text{ for any given } u_j \ . \tag{15}$$

In particular, if the two products have independent match utilities, this condition is just the standard regularity condition of increasing hazard rate in the single-product case.

In the following, I focus on the case with two symmetric products, and so the equilibrium price p is given in (13).[20] One can see that p increases with search costs if and only if $\frac{\partial\alpha}{\partial s}+\frac{\partial\beta}{\partial s}<0$, where α is the standard effect and β is the joint search effect as defined in (13). As the following proposition indicates, $\frac{\partial\alpha}{\partial s}<0$ if the regularity condition (15) holds. This means that if the joint search effect were absent, prices would increase with search costs under the condition (15), similar as in the single-product scenario.

However, taking into account the joint search effect can qualitatively change the picture. As indicated in the following proposition, the joint search effect β can vary with s in either direction. If $\frac{\partial\beta}{\partial s}<0$, the joint search effect will reinforce the standard effect such that the price increases with search costs even faster. Conversely, if $\frac{\partial\beta}{\partial s}>0$ (e.g., when the conditional density is weakly decreasing), the joint search effect will mitigate or even overturn the usual relationship between price and search costs. As a result, the regularity condition (15) is not enough to ensure that prices increase with search costs in our model. The following result gives a new condition (all omitted proofs can be found in Appendix A):

[20] Product asymmetry is another force that could influence the relationship between prices and search costs. Intuitively, when one product has a lower profit margin than the other, the joint search effect from adjusting its price is stronger (i.e., reducing its price can induce consumers to buy the other more profitable product). Then this product's price may go down with the search cost. For example, when product 1 has a match utility uniformly distributed on $[0,1]$ and product 2 has a match utility uniformly distributed on $[0,4]$, one can show that p_1 decreases while p_2 increases with s when $s\leq 1/2$.

Proposition 2 *Suppose the search cost condition (4) holds, and the two products are symmetric. Then p defined in (13) increases with search costs if and only if*

$$\underbrace{\int_a^{\overline{u}} \frac{f(\phi(u))}{1-F(\phi(u))} \{f(u)h(u|\phi(u)) + [1-F(u)]h'(u|\phi(u))\} du}_{-\frac{\partial \alpha}{\partial s}} > \underbrace{f(a,\overline{u}) - \int_a^{\overline{u}} h'(u|\phi(u))f(\phi(u))du}_{\frac{\partial \beta}{\partial s}} . \tag{16}$$

If the two products further have independent match utilities, a sufficient condition for (16) is that the marginal density $f(u)$ is (weakly) increasing.

The condition (16), however, can be easily violated (such that prices decrease with search costs) even under the regularity condition of (15).[21] For example, in the exponential example with independent match utilities, $\frac{\partial \alpha}{\partial s} = 0$ and $\frac{\partial \beta}{\partial s} > 0$, and so the opposite of (16) holds. Other simple examples include the distribution with decreasing density $f(u) = 2(1-u)$ and the logistic distribution $f(u) = e^u/(1+e^u)^2$ when search costs are relatively small. As we have argued, this surprising result is due to the joint search effect, the new economic force in our multiproduct search model.

If firms supply (and consumers need) more products, the joint search effect could have an even more pronounced impact such that prices fall with search costs more likely. In Appendix A, I extend the two-product model to the case with m products. In particular, in the uniform case with m symmetric products, the equilibrium price p has a simple expression:

$$\frac{1}{p} = \underbrace{2 - \frac{V_m(\sqrt{2s})}{2^m}}_{\text{standard effect}} + \underbrace{\frac{(m-1)V_m(\sqrt{2s})}{2^{m-1}\pi}}_{\text{joint search effect}} , \tag{17}$$

where $s \in [0, 1/2]$ and $V_m(r)$ is the volume of an m-dimensional sphere with radius r.[22] One can check that p increases with s if and only if $m < 1+\pi/2 \approx 2.6$. Therefore, when consumers search for more than two products, even in the uniform example, prices start to decrease with search costs.

Discussion: large search costs. Our analysis so far has been restricted to relatively small search costs such that it is even worthwhile to search for one good alone. I now discuss the case with larger search costs beyond condition (4). (In some circumstances,

[21] If the opposite of (15) holds, the left-hand side of (16) is negative, and meanwhile h' must be negative such that the right-hand side is positive. Thus, (16) will fail to hold and so the price p will decrease with search costs. This is not surprising, because even in the single-product search case (5) implies that the equilibrium price decreases with search costs if the match utility has a decreasing hazard rate. What is more surprising in the multiproduct search model is that prices can decrease with search costs even under the regularity condition.

[22] The volume of an m-dimensional sphere with radius r is $V_m(r) = \frac{(r\sqrt{\pi})^m}{\Gamma(1+m/2)}$, where $\Gamma(\cdot)$ is the Gamma function. One can show that for any fixed r, $\lim_{m\to\infty} V_m(r) = 0$. Then as m goes to infinity, p will approach the perfect information price 1/2 in expression (12). This is because for a fixed search cost, if each consumer is searching for a large number of products, they will almost surely sample both firms.

a consumer conducts multiproduct search because it is not worthwhile to search for each good separately.) As we shall see later, this discussion will also be useful for understanding the results in the bundling case. For simplicity, let us focus on the case of symmetric products. Suppose the condition (4) is violated such that $s > \zeta_i(\underline{u})$ and $a < \underline{u}$. (But s cannot exceed $2\zeta_i(\underline{u})$ to ensure an active search market.) Then the reservation frontier is shown in Figure 3 below, where $c = \phi(\underline{u})$.

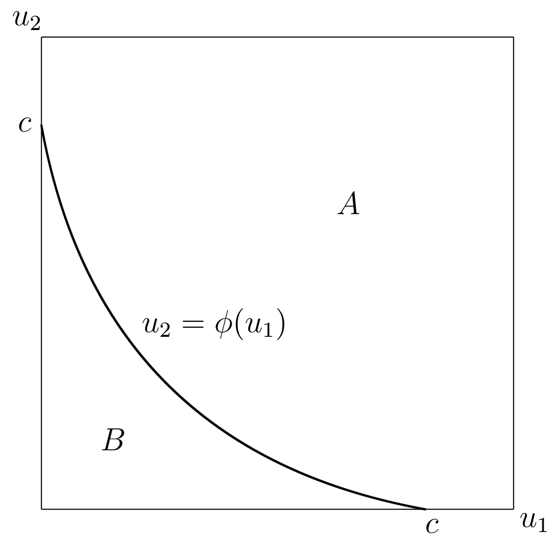

Figure 3: The optimal stopping rule with large search costs

The key difference between this case and the case of small search costs is that now the reservation frontier becomes "shorter" as search costs go up. This feature has a significant impact on how prices vary with search costs. For example, in the uniform case, a higher search cost now leads to fewer marginal consumers on the reservation frontier, which provides firms with a greater incentive to raise prices. (In this case, the joint search effect strengthens the usual relationship between prices and search costs.) I show in the online supplementary document that if the two products are symmetric and they have independent match utilities, and if search costs are relatively high such that $\zeta_i(\underline{u}) < s < 2\zeta_i(\underline{u})$, then the equilibrium price p increases with search costs if each product's match utility has a monotonic density and a (weakly) increasing hazard rate.

3.4 Price comparison with single-product search

As the end of this section, I compare the multiproduct search prices in section 3.2 with the single-product search prices in section 3.1, and discuss one empirical implication of the comparison result.

Proposition 3 *Suppose the search cost condition (4) and the regularity condition (15) hold. Then $p_i \le p_i^0$, $i = 1, 2$, i.e., each product's price is lower in multiproduct search than in single-product search. (The equality holds when $s = 0$.)*

This result is intuitive. In our model, there are economies of scale in search (i.e., searching for two products is as costly as searching for only one), so more consumers are willing to sample both firms in multiproduct search, which intensifies the price competition. On top of that, the joint search effect gives rise to a complementary pricing problem and induces firms to further lower their prices. For example, in the uniform case with $s = 0.1$, the multiproduct search price in (14) is 0.51, lower than the single-product search price 0.64 by 20%. (Notice that $p^0 = 1/(2 - \sqrt{2s})$ in the single-product case.) It is worth emphasizing that even if economies of scale in search are weak (e.g., when single-product search is less costly than multiproduct search), the joint search effect can still induce substantial price reduction. For instance, in the uniform case, if single-product search is half as costly as two-product search (i.e., if the single-product search cost is $s/2$), then the single-product search price becomes $1/(2 - \sqrt{s})$. The multiproduct search price is still significantly lower than that. For example, when $s = 0.1$, the new single-product search price is 0.59, and the multiproduct search price 0.51 is still lower than it by 13.5%.

As documented in Warner and Barsky (1995), MacDonald (2000), Chevalier, Kashyap, and Rossi (2003) and others, prices of many retail products fall during demand peaks such as holidays and weekends. (All these paper use data from multiproduct retailers such as supermarkets and department stores.) This phenomenon is termed *countercyclical pricing*. A simple extension of the multiproduct search model can provide a possible explanation for this phenomenon.[23] Suppose there are now both single-product and multiproduct searchers in the market. Suppose a higher proportion of consumers become multiproduct searchers during high-demand periods such as weekends and holidays. (For example, many households conduct their weekly grocery shopping during weekends, and more people buy multiple gifts in Christmas season.[24]) Then Proposition 3 immediately implies that market prices will decline.

For illustration, I consider an example where there are two symmetric products and each product is needed by a consumer with a probability $\theta \in [0, 1]$. (Our basic model corresponds to $\theta = 1$.) Suppose the need for each product occurs independently across products and consumers. Then there are three groups of consumers in the market: a fraction of θ^2 of consumers are searching for both products, a fraction of $2\theta(1 - \theta)$ of consumers are searching for only one product, and the rest need none of them. A demand rise can be reflected by an increase of θ.

[23] There are of course other possible explanations for countercyclical pricing. For example, it may be due to the dynamic interaction among competing retailers, who are more likely to have a price war during demand booms (Rotemberg and Saloner, 1986). It may also be because retailers advertise price information more intensely during high-demand periods, or because more low-income consumers who are usually more price sensitive enter the market.

[24] Another possible justification is that the demand fluctuations may also arise endogenously: anticipating firms' pricing pattern, consumers may strategically accumulate their demand for various products and shop intensively during low-price periods, which in turn justifies firms' pricing strategies.

The equilibrium price for each product in this extended model is given by

$$\frac{1}{p} = (1-\theta)\alpha_0 + \theta(\alpha+\beta) \ ,$$

where α_0 is the right-hand side of (5), and α and β are the standard effect and the joint search effect in multiproduct search, respectively. (Conditional on a consumer buying one product, this consumer is a single-product searcher with probability $1-\theta$ and a multiproduct searcher with probability θ.) Proposition 3 implies that $\alpha_0 < \alpha + \beta$, so p decreases with θ. This result is due to both $\alpha_0 < \alpha$ (which reflects the economies of scale in search) and $\beta > 0$ (which reflects the joint search effect).

Warner and Barsky (1995) have suggested an explanation based on consumer search for countercyclical pricing, though they did not develop a formal search model. Their idea is wholly based on economies of scale in search, while my model suggests that even if economies of scale in search are weak, the joint search effect can still induce multiproduct firms to reduce their prices substantially. In effect, one argument in Chevalier, Kashyap, and Rossi (2003) against Warner and Barsky's explanation is that they did not find clear evidence that consumers become significantly more price elastic during peak-demand periods. However, they only consider each product (category)'s own-price elasticity. According to our model, the cross-price elasticity due to the joint search effect may play an important role in multiproduct retailers' pricing decisions. Taking that into account may enhance the explanatory power of a search model for countercyclical pricing.

4 Bundling in Search Markets

Bundling is a widely used multiproduct pricing strategy. One adopted form, termed *pure bundling*, is that the firm sells several products in a package (e.g., software suites, TV program packages, and music albums), and none of them is available for individual purchase. The other form, termed *mixed bundling*, is that alongside each separately available product, a package is sold at a discount relative to the components. For example, retailers such as electronic stores, travel agencies and online book shops often offer a customer a discount if she buys more than one product from the same store. Another related example in the retail market is that the shipping fee is often independent of the number of products (e.g., furniture items) in the same order.

The existing literature on competitive bundling assumes perfect information on the consumer side (i.e., consumers know all price and product information). However, in many circumstances where firms use bundling strategies, imperfect information and consumer search are clearly relevant and could have a significant influence on firms' incentive to bundle and the welfare impacts of bundling. This section intends to fill this gap by allowing firms to adopt bundling strategies in the multiproduct search model.

To illustrate the main insights in a simple way, I focus on the case of pure bundling. (The case of mixed bundling is more complicated to analyze, but the main results derived in the pure bundling case hold qualitatively there. See the online supplementary document for the details.) I assume that when firms bundle, consumers buy only one of the two bundles, i.e., they will not buy both bundles to mix and match. This is the case, for instance, when pure bundling introduces the compatibility problem, or when the bundle price is so high that it is not worthwhile to buy both bundles.[25]

Bundling and search incentive. I first examine how bundling might affect consumers' search incentive. In the linear pricing case, given match utilities (u_1, u_2) at firm I, the expected benefit from sampling firm II is

$$\mathbb{E}\left[\max\left(0, \sum_{i=1}^{2}(v_i - u_i), v_1 - u_1, v_2 - u_2\right)\right] \ . \tag{18}$$

(This merely rewrites the left-hand side of (6). The expectation operator is over (v_1, v_2).) If both products at firm II are a worse match, the consumer will return and buy at firm I and so the gain from the extra search will be zero; if both products at firm II are a better match, the consumer will buy at firm II and gain $\sum_{i=1}^{2}(v_i - u_i)$; if only product i at firm II is the better match, she will mix and match and the gain will be $v_i - u_i$.

Suppose now both firms adopt the pure bundling strategy and charge the same prices. Then the expected benefit from sampling firm II becomes

$$\mathbb{E}[\max(0, \sum_{i=1}^{2}(v_i - u_i))] \ , \tag{19}$$

since the opportunity to mix and match has been completely ruled out. This benefit is clearly smaller than (18). Therefore, compared to linear pricing, bundling reduces consumers' search incentive and induces them to stop at the first sampled firm more often.[26]

When both firms bundle, consumers in effect face a single-product search problem: firm I offers a composite product with a match utility $U = u_1 + u_2$, and firm II offers another one with an independent match utility $V = v_1 + v_2$. Both U and V belong to $[\underline{U} = \underline{u}_1 + \underline{u}_2, \overline{U} = \overline{u}_1 + \overline{u}_2]$. Let $G(\cdot)$ and $g(\cdot)$ denote their common cdf and pdf, respectively. Denote by b the reservation utility level in this search problem. It satisfies

$$\int_{b}^{\overline{U}} (U - b)dG(U) = s \ . \tag{20}$$

The left-hand side is the expected benefit from sampling the second bundle given the first one has a match utility b. Hence, in a symmetric equilibrium a consumer will visit

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[25] For example, in the uniform example below, when the search cost is relatively high, the bundle price is greater than 1. Then even if firm I's products have match utilities $(1, 0)$ and firm II's products have match utilities $(0, 1)$, it is not worthwhile to buy both bundles.

[26] In the case of mixed bundling, the joint-purchase discount, which is the difference between the sum of the two stand-alone prices and the bundle price, acts as a cost of mixing-and-matching and so the benefit from sampling one more firm is also reduced.

the second firm if and only if the first bundle has a match utility $u_1 + u_2$ below b. Since pure bundling reduces consumers' search incentive, the acceptance set expands, i.e., $b < u_1 + \phi(u_1)$ for any $u_1 \in [a_1, \overline{u}_1]$.

Figure 4 below illustrates this change of the consumer stopping rule, where the linear line is the reservation frontier in the pure bundling case and the new acceptance set is A plus the shaded area.

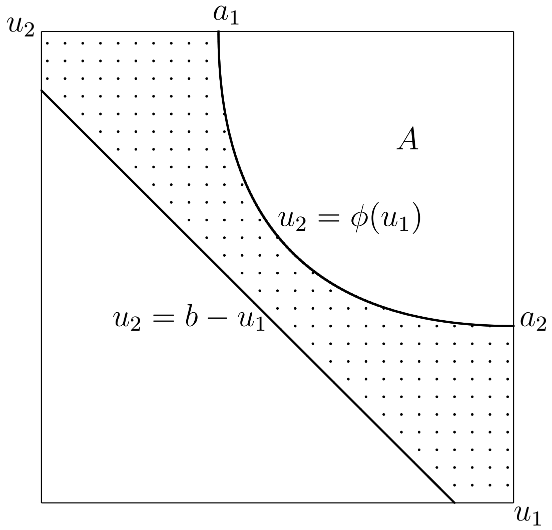


Figure 4: The optimal stopping rule: linear pricing *vs* pure bundling

As I will demonstrate below, this search-discouraging effect of bundling may make firms compete less aggressively and reverse the usual welfare impact of competitive bundling.

Incentive to bundle. Starting from the linear pricing equilibrium, does a firm have a unilateral incentive to introduce bundling in a search environment? Suppose that firms choose prices and whether to bundle products simultaneously,[27] and both choices are unobservable to consumers until they reach the store.

When a firm unilaterally introduces pure bundling, it will make more consumers who visit it first stop searching and buy immediately. (For these consumers, the situation is now actually equivalent to both firms adopting the bundling strategy.) But bundling will also exclude some consumers who continue to search and would otherwise come back and buy a single product. The following result shows that with costly search, the first positive effect always dominates.

Proposition 4 *In the duopoly model with costly search ($s > 0$), starting from the linear pricing equilibrium, each firm has a unilateral incentive to introduce the pure bundling strategy.*

Proof. Each firm earns $(p_1 + p_2)/2$ in the linear pricing equilibrium. Now consider the following deviation: firm I unilaterally bundles its products and sells the bundle at

[27] In the retail market, bundling strategies are often as easy to adjust as pricing strategies.

a price $p_1 + p_2$. (Before they reach firm I, consumers still hold the equilibrium belief that firm I is setting linear prices.) Since this deviation does not change the bundle price, it suffices to show that more than half of the consumers will buy the bundle from firm I after the deviation. Notice that if both firms bundle and charge the same price, each will have demand 1/2. So it suffices to show that more consumers will buy the bundle from firm I in the deviation case than in the pure bundling case with the bundle price $p_1 + p_2$.

Consider firm I's two demand sources: (a) For those consumers who sample firm I first, they will act as in the pure bundling case. So the demand from them is the same as in the pure bundling case. (b) For those consumers who sample firm II first, they will adopt the stopping rule in the linear pricing case. So compared to the pure bundling case, more consumers (i.e., those on the shaded area in Figure 4) will come to firm I. (This argument relies on costly search. If the search cost is zero, all consumers who sample firm II first will come to firm I in either case.) But once they arrive at firm I and find out its bundling strategy, they will make choices as in the pure bundling case. Hence, in the deviation case, the demand from those consumers who sample firm II first is greater than in the pure bundling case. Combining (a) and (b) proves the result. ■

It is also easy to see that starting from the situation where both firms bundle (and consumers believe so), no firm has a unilateral incentive to unbundle. This is simply because given consumers' beliefs and the rival's bundling strategy, unilaterally unbundling has no impact at all on the market. Therefore, if firms choose prices and their bundling strategies simultaneously, the only symmetric equilibrium is a bundling equilibrium.

Comparison with linear pricing. When firms bundle, we have in effect a single-product search problem. Let P be the symmetric equilibrium bundle price. Then, similar to (5), P is determined in

$$\frac{1}{P} = g(b)[1 - G(b)] + 2\int_{\underline{U}}^{b} g(U)^2 dU \ , \tag{21}$$

where the reservation utility b has been defined in (20). P increases with search costs provided that $U = u_1 + u_2$ has an increasing hazard rate (which is true, for example, when each u_i has an increasing hazard rate and is independent from u_j, as shown by Miravete, 2002).

When information is perfect, Matutes and Regibeau (1988), Economides (1989), and Nalebuff (2000) have shown in a two-dimensional Hotelling setting (with unit demand and full market coverage) that pure bundling typically lowers price (and profit) and boosts consumer welfare. This is mainly because pure bundling makes a price reduction doubly profitable, thereby intensifying price competition, and the price reduction is large enough to outweigh the restriction of choices available with pure bundling.

The same argument applies in our setting when the search cost is zero. Suppose the two products are symmetric. Then from (13) and (21) we can see that at $s = 0$ (so

$a = \overline{u}$ and $b = \overline{U}$) pure bundling results in a lower bundle price ($P < 2p$) if and only if

$$\int_{\underline{u}}^{\overline{u}} f(u)^2 du < 2 \int_{\underline{U}}^{\overline{U}} g(U)^2 dU \ . \tag{22}$$

With independent match utilities, one can check that this condition holds for a large range of distributions such as uniform, normal and logistic. But it does not always hold. For instance, as we will see below, in the exponential case the equality of (22) holds. (However, as implied by Proposition 5 below, if consumers buy a large number of products, pure bundling always leads to a lower bundle price than linear pricing in the perfect information case.)

When search is costly, the pro-competitive effect of pure bundling still applies to those consumers who sample both firms. However, pure bundling weakens consumers' search incentive and so reduces the number of informed consumers in the first place, which tends to soften price competition. The net effect hinges on the relative importance of these two forces. Intuitively, when the search cost is higher, there will be fewer fully informed consumers, and then the first effect will be less important and pure bundling may lead to a higher bundle price. This intuition is confirmed in our two examples:

The uniform example: Suppose u_1 and u_2 are independent, and $u_i \sim U[0,1]$. To facilitate the comparison with linear pricing, we keep the search cost condition $s \leq 1/2$. One can show that $G(U) = U^2/2$ and $g(U) = U$ if $U \in [0,1]$, and $G(U) = 1 - (2-U)^2/2$ and $g(U) = 2 - U$ if $U \in [1,2]$. According to (20), the reservation utility b satisfies $(2-b)^3/6 = s$ if $s \in [0, 1/6]$ (so $b \geq 1$ in this case), and $1 - b + b^3/6 = s$ if $s \in [1/6, 1/2]$ (so $b < 1$ in this case). Then (21) implies

$$P = \begin{cases} \dfrac{1}{\frac{4}{3} - s} & \text{if } s \in [0, \frac{1}{6}) \\[2ex] \dfrac{1}{\frac{1}{6}b^3 + b} & \text{if } s \in [\frac{1}{6}, \frac{1}{2}] \end{cases} .$$

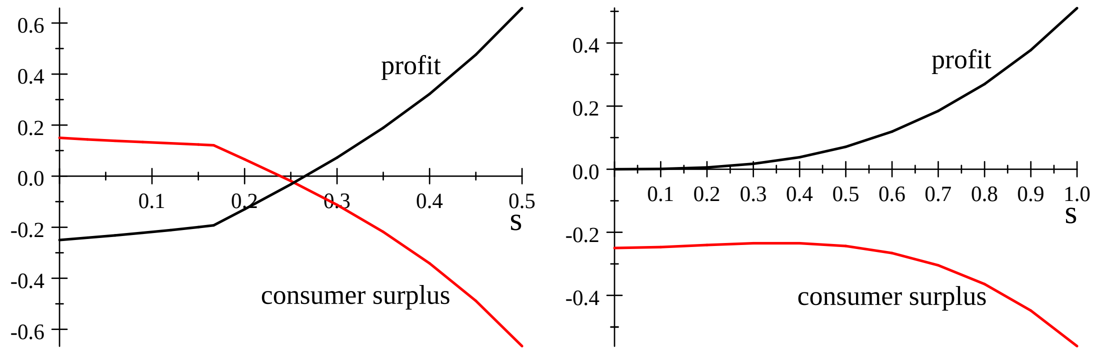
One can check that P increases with s, but the speed is much faster when $s \geq 1/6$. (The upward sloping curve in Figure 5(a) below depicts how $P - 2p$ varies with search costs.) This is because in the range of $s \in [0, 1/6)$, $b > 1$ and so as s increases, the reservation frontier gets "longer" (i.e., there are more marginal consumers on the frontier), which mitigates firms' incentive to raise prices. By contrast, after s exceeds $1/6$, $b < 1$ and so the reservation frontier gets "shorter" as s increases, which strengthens firms' incentive to raise prices. In other words, when the reservation frontier is still getting longer in the linear pricing case, it already starts to get shorter in the bundling case. In particular, when the search cost exceeds about 0.26, the bundle price is higher in the pure bundling case than in the linear pricing case.

The exponential example: Suppose u_1 and u_2 are independent, and $f_i(u_i) = e^{-u_i}$ for $u_i \in [0, \infty)$. Then $G(U) = 1 - (1+U)e^{-U}$ and $g(U) = Ue^{-U}$. (Note that U has a strictly increasing hazard rate, though u_i has a constant one.) According to (20), the reservation utility b satisfies $(2+b)e^{-b} = s$. Substituting G and g into (21) yields

$$P = \frac{2}{1-e^{-2b}} \ ,$$

which increases with s and is always greater than the bundle price $2p$ in the linear pricing case (except $P = 2p$ at $s = 0$). (The upper curve in Figure 5(b) depicts how $P - 2p$ varies with search costs in this example.) With pure bundling, as s increases the reservation frontier always gets "shorter" in the exponential case, which explains why pure bundling reverses the relationship between price and search costs.

Now let us examine the welfare impact of pure bundling relative to linear pricing. The first observation is that total welfare—defined as the sum of industry profit and consumer surplus—must fall with bundling. With the assumption of full market coverage, consumer payment is a pure transfer and so only the match efficiency (including search costs) matters. Bundling reduces efficiency because it not only results in insufficient consumer search (i.e., too few consumers search beyond the first sampled firm due to bundling) but also rules out the opportunity to mix and match for the consumers who sample both firms. This general result holds no matter whether information frictions exist or not.



(a): uniform example (b): exponential example

Figure 5: The welfare impact of pure bundling

However, how bundling affects industry profit and consumer surplus depends on the size of search frictions. First, each firm earns a higher profit whenever pure bundling leads to a higher bundle price (given the assumption of full market coverage). Hence, given that total welfare always falls with bundling, consumers must become worse off if the bundle price rises in the pure bundling case. But things are less clear when the bundle price falls because consumers also end up consuming less well matched goods in the pure bundling case. In the uniform example, as indicated by the downward sloping curve in Figure 5(a) which represents the impact of pure bundling on consumer surplus relative to linear pricing, pure bundling benefits consumers when search costs are lower than about 0.24, but it harms consumers when search costs exceed that threshold. In the exponential case, pure bundling always harms consumers since it (weakly) raises the bundle price for any search cost level. This is indicated by the lower curve in Figure 5(b). (Calculating consumer surplus directly in our multiproduct search framework is complicated. In Appendix B, I develop a more efficient indirect method.)

In sum, in a search environment pure bundling can generate a significant competition-relaxing effect such that relative to linear pricing it can benefit firms and harm consumers, in contrast to the perfect information case.[28]

Nevertheless, this search-based effect is most pronounced when the number of goods a consumer is looking for is relatively small. If a consumer is looking for a large number of goods and if the search cost is fixed, she will almost surely sample both firms, and the situation will be close to the perfect information case. Then, as the following result shows, the pro-competitive effect of pure bundling will dominate.

Proposition 5 *For given search costs, if each firm supplies (and each consumer needs) a large number of symmetric products with independent match utilities, then compared to linear pricing, pure bundling leads to a lower bundle price and so lower industry profits, and it benefits consumers if f is logconcave.*

This result is not trivial, and it has not been noticed in the existing literature. Pure bundling leads to lower prices, but it also lowers match efficiency. What I show in the proof is that the bundle price increases with the number of products much slower in the pure bundling case than in the linear pricing case, such that the price effect eventually dominates the match effect and consumers become better off.

5 Concluding Discussion

This paper has two contributions: First, it developed a tractable multiproduct search framework and showed how consumers and firms may behave differently compared to the single-product search case. In particular, the presence of the joint search effect can induce prices to decline with search costs even in regular cases. Second, the multiproduct search framework has been used to address economic issues such as countercyclical pricing and bundling, and new insights emerged. For instance, compared to the perfect information scenario, the welfare assessment of competitive bundling can be reversed in a search environment.

[28] A more extreme example is when the two products are symmetric but have perfectly negatively correlated match utilities. Then in the pure bundling case, the two bundles are in effect homogenous. With perfect information, we have Bertrand competition and price will be equal to marginal cost, which is better than linear pricing for consumers; while with costly search, we have the Diamond paradox in which all consumers stop at the first sampled firm (if the first search is costless) and the price will be the monopoly price, which is of course worse than linear pricing for consumers.

In the remainder of this section, I discuss the case with an arbitrary number of firms and the case with costly recall, and I also discuss two other applications of the model.

5.1 More firms

Considering an arbitrary number of firms entails a more intricate analysis (see the online supplementary document for the details). But the main insights from the duopoly case survive.

When there are more than two firms, if a firm is not at the first position of a consumer's search process, the consumer may go back to a previous firm to buy one product when she ceases searching. Hence, the joint search effect is weakened, but it does not vanish. For instance, in the exponential example with more than two firms, prices still declines with s due to the joint search effect.

Compared to linear pricing, if the maximum utilities so far are fixed, bundling still reduces the benefit from sampling more firms, and so it has a tendency to restrain consumers' search incentive. However, with more than two firms, bundling also restricts the opportunity of mixing-and-matching among previous offers and thus lowers the maximum utilities so far. This tends to increase consumers' search incentive. The final effect depends on which force dominates. But numerical simulations suggest that the new force is relatively weak. For example, in the uniform case with $n = \infty$, pure bundling reduces consumer search intensity when s is greater than about 0.03; and in the exponential case with $n = \infty$, pure bundling reduces consumer search intensity for all s permitted in condition (4). Numerical calculations in the two examples also suggest that pure bundling lead to a higher bundle price and lower consumer surplus when s is relatively large.

5.2 Costly recall

In many cases (e.g., in the case of high street shopping), returning to a previously visited store is costly. The consumer search literature, however, often assumes free recall for tractability.[29] Another justification for the assumption of free recall is that, if we consider an infinite number of firms in the single-product search case, consumers never return to previously visited firms and the optimal stopping rule is always stationary, independent of the recall assumption. But as I have argued before, in the multiproduct search case, returning occurs even if there are an infinite number of firms and so the recall assumption matters for the stopping rule.

When recall is costly, the optimal stopping rule in multiproduct search has a new feature: when one product is a good match and the other is a bad match, a consumer may buy the well-matched product first (to avoid paying the returning cost) and then

[29] Janssen and Parakhonyak (2010) studies the optimal stopping rule in the price search case with a single product and costly recall. They find that when there are more than two (but a finite number of) firms, the stopping rule is non-stationary and depends on the historical offers in an intricate way.

continue to search for the other. As a result, each firm will (endogenously) face both single-product searchers (who have bought one product from some previous firm) and multiproduct searchers (who have not bought any product). The main difficulty in dealing with the case with costly recall is that the optimal stopping rule does not have a simple characterization (even in the duopoly case), which hinders demand analysis.

Although an analysis with costly recall is difficult, I anticipate that the joint search effect should survive given that there still exist multiproduct searchers in the search process. However, the effect of bundling on consumer search could be different. For instance, in the polar case with no recall, since consumers cannot return to mix and match anyway, bundling does not reduce consumers' search incentive. However, in a more reasonable case where recall is costly but not impossible the search-discouraging effect of bundling, though reduced, will persist.

5.3 Search costs *vs* shopping costs

Search costs usually mean the costs that are incurred to find and evaluate a new option when information is initially imperfect. The literature sometimes also considers shopping costs that are not related to information search. For example, in a multiproduct case, even if information is perfect, there may still exist shopping costs (e.g., the costs of paying several bills) when the customer sources supplies from more than one firm (see, for instance, Klemperer, 1992, and Armstrong and Vickers, 2010). This kind of search unrelated shopping costs also induce customers to one-stop shop more likely, and can cause a similar effect as the joint search effect in this paper: they render two independent products in each firm complements and so have a tendency to intensify price competition. But compared to search costs, this kind of shopping costs are often less important in the retail market.

In addition, there is also an essential difference between search costs and search unrelated shopping costs: search costs always have an anti-competitive effect by reducing the number of firms a consumer considers in the market. However, how shopping costs affect competition may crucially depend on whether consumers face information frictions and need to conduct costly information search. If information is initially perfect, shopping costs are pro-competitive due to the complementary pricing issue. While if there is costly information search, shopping costs can work in the opposite way by reducing the benefit from mixing and matching and so dampening consumers' search incentive. (Notice that the search unrelated shopping cost is similar to the joint-purchase discount in mixed bundling. When it is sufficiently large, it works as the pure bundling strategy.)

5.4 Other applications

In this part, I briefly discuss two other applications of the multiproduct search model which deserve separate research.

Search and loss leader pricing. Retailers often adopt the loss leader pricing strategy: they sell some products at very low prices (sometimes even below marginal costs) to attract consumers to visit and then make money back from other products with high profit margins. A variant of our multiproduct search model can be used to study this pricing strategy.

Suppose consumers know some products' match utilities and prices before search, but they need to visit firms to find out the information of other products. This is probably the case, for example, when some products are frequently purchased while others are not, or when some products are advertised by firms, or when firms deliberately hide the information of some products (e.g., add-ons). Specifically, suppose that product i's information is perfect but product j's information is not. For simplicity, let us consider the case where the two products have independent match utilities (and so observing u_i and v_i does not provide any new information about u_j and v_j). Suppose that each firm simultaneously chooses P_i, the observable price of product i, and p_j, the unobservable price of product j. Then one can derive the first-order conditions for P_i and p_j in the symmetric equilibrium:[30]

$$P_i = \underbrace{\frac{1}{2\int_{\underline{u}_i}^{\overline{u}_i} f_i(u)^2 du}}_{\text{perfect info price}} - \underbrace{[1 - F_j(a_j)]^2 p_j}_{\text{loss leading effect}}$$

and

$$\frac{1}{p_j} = f_j(a_j)[1 - F_j(a_j)] + 2\int_{\underline{u}_j}^{a_j} f_j(u)^2 du \ ,$$

where a_j is the reservation utility in the single-product search case as defined in (3).

Notice that the unobservable price p_j is the same as in the single-product search case (since consumers will search as in a single-product case once they arrive at the first firm), while the observable price P_i equals the price in the perfect information case adjusted by a loss leading effect. Reducing the price of product i will attract more consumers to visit first, and thus it will increase not only the demand for product i but also the demand for product j. This is similar to the joint search effect in our base model. In our base model, when a firm reduces a product's price (privately), it can cause some consumers who are already in the store to cease searching; but when this price reduction is public, it can increase the store traffic in the first place. This cross-price effect, referred to as the *loss leading effect*, is more pronounced than the joint search effect in our base model. As a result, firms will compete intensely in the observable price to attract consumers such that loss leading (i.e., $P_i < 0$) can occur.[31]

Under the regularity condition of increasing hazard rate, one can show that (i) the observable price P_i decreases while the unobservable price p_j increases with search costs,

[30] The proofs of all analytical statements in the discussion are available upon request.

[31] In the base model, even with asymmetric products, I did not find any examples in which the joint search effect is strong enough such that loss leading occurs.

and (ii) a sufficient condition for loss leading is that product j has a (weakly) higher profit margin than product i in the perfect information situation (i.e., $\int_{\underline{u}_j}^{\overline{u}_j} f_j(u)^2 du \leq \int_{\underline{u}_i}^{\overline{u}_i} f_i(u)^2 du$) and search costs are sufficiently high (but the search market is still active). For example, in the uniform distribution example with two symmetric products, we have $P_i = \frac{1}{2} - \frac{2s}{2-\sqrt{2s}}$ and $p_j = \frac{1}{2-\sqrt{2s}}$. The former is negative when s is greater than about 0.3. In effect, there is evidence that in the grocery market, for instance, loss leaders are usually staples such as milk and dairy, alcohol, bread and bakery products that consumers purchase repeatedly and regularly. This is consistent with our prediction: for these frequently purchased products, consumers may know their product and price information, and they also usually exhibit little product differentiation across retailers such that their profit margins are low in a perfect information setting.

If we use advertising to justify the perfect information of loss leaders (as Lal and Matutes, 1994, did), a few interesting questions deserve further investigation: Starting from the base model without advertising, do firms have incentives to advertise? If so, what products (if products are asymmetric) will firms choose to advertise, and what is the welfare impact of advertising? (Notice that my model has a meaningful benchmark without advertising, which will make welfare comparison more interesting.)

Search and retail market structure. In the retail market, large multiproduct retailers such as department stores often coexist with smaller competitors such as specialist shops which have narrower product ranges. What is the value of being a large multiproduct retailer? A large retailer may enjoy economies of scale in operations, and may also have an advantage in bargaining with manufacturers. But from the demand side, one important advantage of being a multiproduct retailer is to provide consumers with the convenience of one-stop shopping. The multiproduct search framework developed in this paper can be modified to study competition between large and small retailers, and it can also be used to study endogenous retail market structure.

Consider, for example, a market with three firms: one firm supplies two products (say, clothes and shoes), while the other two firms are single-product shops (say, one is a clothes shop, and the other is a shoe shop). Suppose consumers' costs of reaching any firm are identical for all firms. Then in the setting with symmetric products and independent match utilities, it can be shown that under the regularity condition of increasing hazard rate, there is an equilibrium in which the multiproduct firm charges lower prices than the two single-product firms, and consumers visit the multiproduct firm first.[32] This result indicates that all else equal, a multiproduct retailer might have an incentive to charge lower prices than its smaller competitors.

The multiproduct search model can also be extended to discuss endogenous retail

[32] Consumer search order reveals information about their preferences: a consumer will visit a single-product shop only if she is unsatisfied with the product in the multiproduct shop. This gives the single-product shops extra monopoly power and induces them to charge higher prices. This logic is similar to Armstrong et al. (2009) and Zhou (2011) which study non-random search in a single-product search scenario.

market structure. Suppose there are initially four single-product retailers in the market, two supplying shoes and the other two supplying clothes. Each product is horizontally differentiated across its two suppliers. Suppose conglomerate merger between two shops supplying different products is possible and costless (but horizontal merger between two shops selling the same product is not permitted, for example, because of antitrust reason). Retailers make their publicly observable merger choices before they engage in price competition. In such an extended model, it can be shown, for example, in the uniform distribution case that asymmetric market structure with a big and two small retailers arises when search costs are relatively low, while for high search costs, two big multiproduct stores emerge in equilibrium. (If conglomerate merger involves some (not too high) fixed costs, the fragmented market structure with four single-product shops is an equilibrium outcome when the search cost is sufficiently small.) This suggests that the size of search friction could be an important determinant of the retail market structure.

Appendix A

Proof of Proposition 2: In the case of symmetric products, from (13) we know the standard effect is

$$\begin{aligned}\alpha &= 2\int_B f(u_i)dF(\mathbf{u}) + \int_a^{\overline{u}} [1-F(\phi(u))]f(u,\phi(u))du \\ &= \int_{\underline{u}}^{\overline{u}} \left\{ 2\int_{\underline{u}}^{\phi(u)} f(u_i)h(u_i|u)du_i + [1-F(\phi(u))]h(\phi(u)|u) \right\} dF(u) \ .\end{aligned}$$

(Note that for $u < a$, $\phi(u)$ is independent of a and $1-F(\phi(u)) = 0$.) Using the notation

$$\lambda(x|u) \equiv f(x)h(x|u) + [1-F(x)]h'(x|u) \ , \tag{23}$$

we have

$$\begin{aligned}\frac{d\alpha}{ds} &= \int_a^{\overline{u}} \frac{d\phi(u)}{ds}\lambda(\phi(u)|u)dF(u) = \int_a^{\overline{u}} \frac{\phi'(u)}{1-F(u)}\lambda(\phi(u)|u)dF(u) \\ &= -\int_a^{\overline{u}} \frac{f(\phi(x))}{1-F(\phi(x))}\lambda(x|\phi(x))dx \ .\end{aligned}$$

The second step used

$$\frac{d\phi(u)}{ds} = -\frac{1}{1-F(\phi(u))}, \quad \phi'(u) = -\frac{1-F(u)}{1-F(\phi(u))} \ , \tag{24}$$

which are both derived from the definition of $\phi(\cdot)$ in (6). The last step is from changing the integral variable from u to $x = \phi(u)$ and using the symmetry of $\phi(\cdot)$. Notice that the regularity condition (15) implies $\lambda(x|u) > 0$ for any given u. So $\frac{d\alpha}{ds} < 0$ under (15).

The joint search effect is $\beta = \int_a^{\overline{u}} [1 - F(u)] f(u, \phi(u)) du$, and so

$$\begin{aligned}
\frac{d\beta}{ds} &= f(a, \overline{u}) - \int_a^{\overline{u}} \frac{d\phi(u)}{ds} [1 - F(u)] h'(\phi(u)|u) f(u) du \\
&= f(a, \overline{u}) - \int_a^{\overline{u}} [-\phi'(u)] h'(\phi(u)|u) f(u) du \\
&= f(a, \overline{u}) - \int_a^{\overline{u}} h'(x|\phi(x)) f(\phi(x)) dx \ .
\end{aligned}$$

The first step used $\frac{da}{ds} = -1/[1 - F(a)]$, the second step used (24), and the last step is again from changing the integral variable from u to $x = \phi(u)$. Therefore, $p = 1/(\alpha + \beta)$ increases with s if and only if $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$ or the condition (16) in the main text holds.

Now suppose the two products have independent valuations and the marginal density satisfies $f'(u) \geq 0$. Then

$$\begin{aligned}
-\frac{d\alpha}{ds} &= \int_a^{\overline{u}} \frac{f(\phi(x))}{1 - F(\phi(x))} \{ f(x)^2 + [1 - F(x)] f'(x) \} dx \\
&\geq \int_a^{\overline{u}} \frac{f(\phi(x))}{1 - F(\phi(x))} f(x)^2 dx \geq \frac{f(a)}{1 - F(a)} \int_a^{\overline{u}} f(x)^2 dx \\
&\geq \frac{f(a)^2}{1 - F(a)} \int_a^{\overline{u}} f(x) dx = f(a)^2 \ ,
\end{aligned}$$

and

$$-\frac{d\beta}{ds} = \int_a^{\overline{u}} f'(x) f(\phi(x)) dx - f(a) f(\overline{u}) \geq f(a)[f(\overline{u}) - f(a)] - f(a) f(\overline{u}) = -f(a)^2 \ .$$

Therefore, $\frac{d\alpha}{ds} + \frac{d\beta}{ds} \leq 0$, i.e., p increases with s.

The case with m products: I first present the first-order conditions for the linear pricing case with m products. Let $\mathbf{u}_{-i} \equiv (u_j)_{j \neq i} \in \mathbb{R}^{m-1}$. In a symmetric equilibrium, without loss of generality the reservation frontier can be defined as $u_m = \phi(\mathbf{u}_{-m})$, where $\phi(\mathbf{u}_{-m})$ satisfies

$$\sum_{i=1}^{m-1} \zeta_i(u_i) + \zeta_m(\phi(\mathbf{u}_{-m})) = s \ .$$

As in the two-product case, let A denote the acceptance set and B denote its complement. Suppose firm II sticks to the equilibrium prices, and firm I lowers p_m by a small ε. Following the same logic as in the two-product case, the first-order condition for p_m is

$$\begin{aligned}
1 &= \underbrace{2p_m \int_B f_m(u_m) dF(\mathbf{u}) + p_m \int_{A_{-m}} [1 - F_m(\phi(\mathbf{u}_{-m}))] f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m})) d\mathbf{u}_{-m}}_{\text{standard effect}} \\
&\quad + \underbrace{\sum_{i=1}^{m-1} p_i \int_{A_{-m}} [1 - F_i(u_i)] f(\mathbf{u}_{-m}, \phi(\mathbf{u}_{-m})) d\mathbf{u}_{-m}}_{\text{joint search effect}} \ , \quad (25)
\end{aligned}$$

where A_{-m} is the projection of A on an $(m-1)$-dimensional hyperplane with a fixed u_m.

Now consider the uniform case with m symmetric products and independent match utilities. Then the first integral in (25) measures the volume of solid B, and thus it equals one minus the volume of solid A. Since A is $1/2^m$ of an m-dimensional sphere with radius $\sqrt{2s}$, we get

$$1-\frac{V_m(\sqrt{2s})}{2^m} \, .$$

(See the expression for $V_m(\cdot)$ in footnote 22.) The second integral equals

$$\int_{A_{-m}} [1-\phi(\mathbf{u}_{-m})]d\mathbf{u}_{-m} = \frac{V_m(\sqrt{2s})}{2^m} \, ,$$

since it just measures the volume of A. Finally, the third integral equals

$$\int_{A_{-m}} (1-u_1)d\mathbf{u}_{-m} = \frac{V_m(\sqrt{2s})}{2^{m-1}\pi} \, . \tag{26}$$

(This equality has no straightforward geometric interpretation. See its proof below.) Then (17) in the main text follows.

Proof of (26): For $m=2$, $A_{-m}=[a,1]$ and (26) is easy to be verified. Now consider $m \geq 3$. Let $A_{-1,m}(u_1)$ be a "slice" of A_{-m} at u_1. Then we have

$$\int_{A_{-m}} (1-u_1)d\mathbf{u}_{-m} = \int_a^1 (1-u_1)\left(\int_{A_{-1,m}(u_1)} d\mathbf{u}_{-1,m}\right) du_1 \, .$$

Since $A_{-1,m}(u_1)$ is $1/2^{m-2}$ of an $(m-2)$-dimensional sphere with radius $r=\sqrt{2s-(1-u_1)^2}$, the internal integral term equals

$$\frac{V_{m-2}(r)}{2^{m-2}} = \frac{\pi^{(m-2)/2} r^{m-2}}{2^{m-2}\Gamma(m/2)} \, ,$$

where $\Gamma(\cdot)$ is the Gamma function. Hence,

$$\begin{aligned} \int_{A_{-m}} (1-u_1)d\mathbf{u}_{-m} &= \frac{\pi^{(m-2)/2}}{2^{m-2}\Gamma(m/2)} \times \int_a^1 (1-u_1)\left(\sqrt{2s-(1-u_1)^2}\right)^{m-2} du_1 \\ &= \frac{\pi^{(m-2)/2}}{2^{m-2}\Gamma(m/2)} \times \frac{\left(\sqrt{2s}\right)^m}{m} = \frac{V_m(\sqrt{2s})}{2^{m-1}\pi}. \end{aligned}$$

The second step used $a = 1-\sqrt{2s}$ and the fact that the integrand is the derivative of $\frac{1}{m}(\sqrt{2s-(1-u_1)^2})^m$ with respect to u_1. The last step used the expression for $V_m(\cdot)$ and the fact $x\Gamma(x)=\Gamma(x+1)$.

Proof of Proposition 3: Let us consider product 2. (The proof for product 1 is similar.) From (10), we have

$$\begin{aligned} \frac{1}{p_2} &> 2\int_B f_2(u_2)dF(\mathbf{u}) + \int_{a_1}^{\overline{u}_1} [1-F_2(\phi(u_1))]f(u_1,\phi(u_1))du_1 \\ &= \int_{\underline{u}_1}^{\overline{u}_1} \left\{ 2\int_{\underline{u}_2}^{\phi(u_1)} f_2(u_2)h_2(u_2|u_1)du_2 + [1-F_2(\phi(u_1))]h_2(\phi(u_1)|u_1) \right\} dF_1(u_1) \, . \end{aligned}$$

Under the regularity condition (15), the curly-bracket term is an increasing function of $\phi(u_1)$. Since $\phi(u_1) \geq a_2$, it is greater than

$$2\int_{\underline{u}_2}^{a_2} f_2(u_2)h_2(u_2|u_1)du_2 + [1 - F_2(a_2)]h_2(a_2|u_1) \ .$$

Realizing $\int_{\underline{u}_1}^{\overline{u}_1} h_2(x|u_1)dF_1(u_1) = f_2(x)$, we have

$$\begin{aligned}
\frac{1}{p_2} &> \int_{\underline{u}_1}^{\overline{u}_1} \left\{ 2\int_{\underline{u}_2}^{a_2} f_2(u_2)h_2(u_2|u_1)du_2 + [1 - F_2(a_2)]h_2(a_2|u_1) \right\} dF_1(u_1) \\
&= 2\int_{\underline{u}_2}^{a_2} f_2(u_2)^2 du_2 + [1 - F_2(a_2)]f_2(a_2) = \frac{1}{p_2^0} \ .
\end{aligned}$$

Proof of Proposition 5: For a given search cost, when the number of products goes to infinity, consumers will always sample both firms. Therefore, we only need to prove the result in the perfect information scenario.

Suppose each firm supplies m products, and each product's match utility is distributed independently according to a cdf $F(\cdot)$ and has a mean μ and variance σ^2. When m is large, by applying the *central limit theorem*, the match utility of the bundle distributes (approximately) according to a normal distribution $N(m\mu, m\sigma^2)$, so

$$g(U) \approx \frac{1}{\sqrt{2m\pi}\sigma} \exp\left[-\frac{1}{2}\frac{(U - m\mu)^2}{m\sigma^2}\right] \ .$$

When $s = 0$, one can check that (21) implies

$$\frac{1}{P} = 2\int_{-\infty}^{\infty} g(U)^2 dU \approx \frac{1}{\sqrt{m\pi}\sigma} \ .$$

That is, the bundle price P rises at the speed of $\sqrt{m}$. However, in the linear pricing case with $s = 0$ we have

$$\frac{1}{mp} = \frac{2}{m}\int_{-\infty}^{\infty} f(u)^2 du \ .$$

So the bundle price mp rises at the speed of m. Hence, when $s = 0$, $P < mp$ for a sufficiently large m. (This generalizes Nalebuff (2000)'s observation in the Hotelling model with a uniform distribution.)

Now turn to consumer surplus. Denote by v the consumer surplus in the linear pricing case with $s = 0$. Then the expected surplus from each product is

$$\frac{v}{m} = \mathbb{E}[\max(u_i, v_i)] - p \ .$$

Denote by V the consumer surplus in the pure bundling case with $s = 0$. Then

$$\frac{V}{m} = \mathbb{E}\left[\max\left(\frac{1}{m}\sum_{i=1}^m u_i, \frac{1}{m}\sum_{i=1}^m v_i\right)\right] - \frac{P}{m} \ .$$

As P rises with m at the speed of $\sqrt{m}$, P/m tends to zero as $m \to \infty$. On the other hand, the expectation term tends to μ. So

$$\lim_{m\to\infty} \frac{V}{m} = \mu \ .$$

Therefore, when $s = 0$ and m is large, pure bundling improves consumer welfare if $\mathbb{E}[\max(u_i, v_i)] - \mu < p$. With linear pricing, consumers enjoy better matched goods (which is reflected by the left-hand side) but they also pay more (which is reflected by the right-hand side). Using $1/p = 2\int_{\underline{u}}^{\overline{u}} f(u)^2 du$, this condition can be written as $\int_{\underline{u}}^{\overline{u}} udF(u)^2 - \int_{\underline{u}}^{\overline{u}} udF(u) < 1/(2\int_{\underline{u}}^{\overline{u}} f(u)^2 du)$. By integration by parts, it simplifies to

$$\left(\int_{\underline{u}}^{\overline{u}} F(u)[1-F(u)]du\right)\left(\int_{\underline{u}}^{\overline{u}} f(u)^2 du\right) < \frac{1}{2} \ .$$

This is further equivalent to

$$\left(\int_0^1 \frac{t(1-t)}{f(F^{-1}(t))}dt\right)\left(\int_0^1 f(F^{-1}(t))dt\right) < \frac{1}{2} \tag{27}$$

by changing the integral variable from u to $t = F(u)$. (27) holds if f is logconcave by invoking the following lemma.[33]

Lemma 3 *Suppose $\varphi : [0,1] \to \mathbb{R}$ is a nonnegative function such that $\int_0^1 \frac{\varphi(t)}{t(1-t)}dt < \infty$, and $h : [0,1] \to \mathbb{R}$ is a concave pdf. Then*

$$\int_0^1 \frac{\varphi(t)}{h(t)}dt \le \max\left(\int_0^1 \frac{\varphi(t)}{2t}dt, \int_0^1 \frac{\varphi(t)}{2(1-t)}dt\right) \ .$$

Let $\varphi(t) = t(1-t)$ and

$$h(t) = \frac{f(F^{-1}(t))}{\int_0^1 f(F^{-1}(t))dt} \ .$$

Since $f(F^{-1}(t))$ is concave if and only if f is logconave, the defined $h(t)$ is indeed a concave pdf. (The integral in the denominator is finite since $f(F^{-1}(t))$ is nonnegative and concave.) Then the lemma implies that the left-hand side of (27) is no greater than 1/4.[34] (For the exponential density $f(x) = e^{-x}$, it equals 1/4.)

Proof. Since h is a concave pdf, it is a mixture of triangular distributions and admits a representation of the form

$$h(t) = \int_0^1 h_\theta(t)\mu(\theta)d\theta \ ,$$

[33] I am grateful to Tomás F. Móri in Budapest for helping me to prove this lemma.

[34] Our result is not tight. However, if f is non-logconcave, it is easy to find counterexamples. For instance, (27) fails to hold for a power distribution $F(x) = x^k$ with k close to 1/2, or for a Weibull distribution $F(x) = 1 - e^{-x^k}$ with a small $k \in (0,1)$.

where $\mu(\cdot)$ is a pdf defined on $[0,1]$, $h_1(t) = 2t$, $h_0(t) = 2(1-t)$, and for $0 < \theta < 1$

$$h_\theta(t) = \begin{cases} 2\dfrac{t}{\theta} & \text{if } 0 \le t < \theta \\ \\ 2\dfrac{1-t}{1-\theta} & \text{if } \theta \le t \le 1 \end{cases} .$$

(See, for instance, Example 5 in Csiszár and Móri, 2004.)

By Jessen's inequality we have

$$\frac{1}{h(t)} = \frac{1}{\int_0^1 h_\theta(t)\mu(\theta)d\theta} \le \int_0^1 \frac{1}{h_\theta(t)}\mu(\theta)d\theta .$$

Then

$$\int_0^1 \frac{\varphi(t)}{h(t)}dt \le \int_0^1 \varphi(t)\left(\int_0^1 \frac{1}{h_\theta(t)}\mu(\theta)d\theta\right)dt = \int_0^1 \left(\int_0^1 \frac{\varphi(t)}{h_\theta(t)}dt\right)\mu(\theta)d\theta \le \sup_{1\le\theta\le1}\int_0^1 \frac{\varphi(t)}{h_\theta(t)}dt .$$

Notice that

$$\int_0^1 \frac{\varphi(t)}{h_\theta(t)}dt = \frac{\theta}{2}\int_0^\theta \frac{\varphi(t)}{t}dt + \frac{1-\theta}{2}\int_\theta^1 \frac{\varphi(t)}{1-t}dt .$$

This is a convex function of θ, because its derivative is

$$\frac{1}{2}\int_0^\theta \frac{\varphi(t)}{t}dt - \frac{1}{2}\int_\theta^1 \frac{\varphi(t)}{1-t}dt ,$$

which is increasing in θ. Hence, its maximum is attained at one of the endpoints of the domain $[0,1]$. This completes the proof. ■

Appendix B: Calculating Consumer Surplus

In our search model (especially in the case of linear pricing and the case of mixed bundling analyzed in the supplementary document), it is complicated to calculate consumer surplus directly. Here I develop a more efficient indirect method (which also carries over to the case with more than two firms).

For any given symmetric price vector $\mathbf{p}$ (which can be a linear or bundling pricing scheme) and search cost s, consumer surplus is

$$v(s|\mathbf{p}) = \sup_{\sigma\in\Sigma}\left[U(\sigma|\mathbf{p}) - s\cdot t(\sigma)\right] , \tag{28}$$

where Σ is the (well-defined) set of all possible stopping rules, $U(\sigma|\mathbf{p})$ is the expected match utility minus payment if the consumer chooses a particular stopping rule σ, and $t(\sigma)$ is the expected search times. Let $\sigma(s|\mathbf{p})$ be the optimal stopping rule associated with $\mathbf{p}$ and s. Since the objective function in (28) is linear in s, $v(s|\mathbf{p})$ is convex in s and so is differentiable almost everywhere. Then the envelope theorem implies that

$$v'(s|\mathbf{p}) = -t(\sigma(s|\mathbf{p})) \equiv -\hat{t}(s|\mathbf{p}) .$$

If $\mathbf{p}$ is an equilibrium price vector, then $\hat{t}(s|\mathbf{p})$ is just the corresponding equilibrium number of searches. (In the duopoly case, it equals two minus the measure of the stopping region.) We can then decompose consumer surplus into two parts:

$$v(s|\mathbf{p}) = v(0|\mathbf{p}) - \int_0^s \hat{t}(x|\mathbf{p})dx \ , \tag{29}$$

where the first term captures the surplus when the information is perfect (but given prices $\mathbf{p}$), and the second term reflects the inefficiency caused by imperfect information and costly search.

We can apply the general formula (29) to any case discussed in this paper. For example, in the linear pricing case with two firms, $v(0|\mathbf{p}) = \sum_{i=1}^{2} (\mathbb{E}[\max (u_i, v_i)] - p_i)$, and the optimal stopping rule is independent of p and so $\hat{t}(x) = 2 - A(x)$, where $A(x)$ is the measure of the acceptance set when the search cost is x. In the pure bundling case, $v(0|\mathbf{p}) = \mathbb{E}[\max (U, V)] - P$ and $\hat{t}(x) = 1 + G(b(x))$.

References

Adams, W., and J. Yellen (1976): "Commodity Bundling and the Burden of Monopoly," *Quarterly Journal of Economics*, 90(3), 475–498.

Anderson, S., and L. Leruth (1993): "Why Firms May Prefer Not to Price Discriminate via Mixed Bundling," *International Journal of Industrial Organization*, 11(1), 49–61.

Anderson, S., and R. Renault (1999): "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model," *RAND Journal of Economics*, 30(4), 719–735.

Armstrong, M., and J. Vickers (2010): "Competitive Non-linear Pricing and Bundling," *Review of Economic Studies*, 77(1), 30–60.

Armstrong, M., J. Vickers, and J. Zhou (2009): "Prominence and Consumer Search," *RAND Journal of Economics*, 40(2), 209–233.

Bar-Isaac, H., G. Caruana, and V. Cunat (2012): "Search, Design, and Market Structure," *American Economic Review*, 102(2), 1140–1160.

Burdett, K., and K. Judd (1983): "Equilibrium Price Dispersion," *Econometrica*, 51(4), 955–969.

Burdett, K., and D. Malueg (1981): "The Theory of Search for Several Goods," *Journal of Economic Theory*, 24(3), 362–376.

Carbajo, J., D. de Meza, and D. Seidmann (1990): "A Strategic Motivation for Commodity Bundling," *Journal of Industrial Economics*, 38(3), 283–298.

Carlson, J., and P. McAfee (1984): "Joint Search for Several Goods," *Journal of Economic Theory*, 32(2), 337–345.

Chen, Y. (1997): "Equilibrium Product Bundling," *Journal of Business*, 70(1), 85–103.

Chevalier, J., A. Kashyap, and P. Rossi (2003): "Why Don't Prices Rise During Periods of Peak Demand? Evidence from Scanner Data," *American Economic Review*, 93(1), 15–37.

Csiszár, V., and T. Móri (2004): "The Convexity Method of Proving Moment-Type Inequalities," *Statistics and Probability Letters*, 66(3), 303–313.

Diamond, P. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3(2), 156–168.

Economides, N. (1989): "Desirability of Compatibility in the Absence of Network Externalities," *American Economic Review*, 79(5), 1165–1181.

Ellison, G. (2005): "A Model of Add-on Pricing," *Quarterly Journal of Economics*, 120(2), 585–637.

Gatti, R. (1999): "Multi-Commodity Consumer Search," *Journal of Economic Theory*, 86(2), 219–244.

Haan, M., and J. Moraga-González (2011): "Advertising for Attention in a Consumer Search Model," *Economic Journal*, 121(552), 552–579.

Janssen, M., and A. Parakhonyak (2010): "Oligopolistic Search Markets with Costly Second Visits," mimeo, Vienna.

Klemperer, P. (1992): "Equilibrium Product Lines: Competing Head-to-Head May Be Less Competitive," *American Economic Review*, 82(4), 740–755.

Lal, R., and C. Matutes (1994): "Retail Pricing and Advertising Strategies," *Journal of Business*, 67(3), 345–370.

MacDonald, J. (2000): "Demand, Information, and Competition: Why Do Food Prices Fall at Seasonal Demand Peaks?," *Journal of Industrial Economics*, 48(1), 27–45.

Matutes, C., and P. Regibeau (1988): "Mix and Match: Product Compatibility Without Network Externalities," *RAND Journal of Economics*, 19(2), 221–234.

——— (1992): "Compatibility and Bundling of Complementary Goods in a Duopoly," *Journal of Industrial Economics*, 40(1), 37–54.

McAfee, P. (1995): "Multiproduct Equilibrium Price Dispersion," *Journal of Economic Theory*, 67(1), 83–105.

McAfee, P., J. McMillan, and M. Whinston (1989): "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values," *Quarterly Journal of Economics*, 104(2), 371–383.

Miravete, E. (2002): "Preserving Log-Concavity Under Convolution: Comment," *Econometrica*, 70(3), 1253–1254.

Nalebuff, B. (2000): "Competing Against Bundles," in *Incentives, Organization, and Public Economics*, ed. by G. Myles, and P. Hammond. Oxford University Press.

Rhodes, A. (2011): "Multiproduct Pricing and the Diamond Paradox," mimeo, Oxford.

Rotemberg, J., and G. Saloner (1986): "A Supergame-Theoretic Model of Price Wars during Booms," *American Economic Review*, 76(3), 390–407.

Shelegia, S. (2012): "Multiproduct Pricing in Oligopoly," *International Journal of Industrial Organization*, 30(2), 231–242.

Stigler, G. (1961): "The Economics of Information," *Journal of Political Economy*, 69(3), 213–225.

Thanassoulis, J. (2007): "Competitive Mixed Bundling and Consumer Surplus," *Journal of Economics & Management Strategy*, 16(2), 437–467.

Varian, H. (1980): "A Model of Sales," *American Economic Review*, 70(4), 651–659.

Warner, E., and R. Barsky (1995): "The Timing and Magnitude of Retail Store Markdowns: Evidence from Weekends and Holidays," *Quarterly Journal of Economics*, 110(2), 321–352.

Weitzman, M. (1979): "Optimal Search for the Best Alternative," *Econometrica*, 47(3), 641–654.

Whinston, M. (1990): "Tying, Foreclosure, and Exclusion," *American Economic Review*, 80(4), 837–859.

Wolinsky, A. (1986): "True Monopolistic Competition as a Result of Imperfect Information," *Quarterly Journal of Economics*, 101(3), 493–511.

Zhou, J. (2011): "Ordered Search in Differentiated Markets," *International Journal of Industrial Organization*, 29(2), 253–262.