Productivity shocks and Optimal Monetary Policy in a Unionized Labor Market Economy

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Abstract

In this paper we analyze a general equilibrium dynamic stochastic New Keynesian model characterized by labor indivisibilities, unemployment and a unionized labor market. The presence of monopoly unions introduces real wage rigidities in the model. We show that as in Blanchard Galì (2005) the so called "divine coincidence" does not hold and a trade-off between inflation stabilization and the output stabilization arises. In particular, a productivity shock has a negative effect on inflation, while a reservation-wage shock has an effect of the same size but with the opposite sign. We derive a welfare-based objective function for the Central Bank as a second order Taylor approximation of the expected utility of the economy's representative household, and we analyze optimal monetary policy under discretion and under commitment. Under discretion a negative productivity shock and a positive

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exogenous wage shock will require an increase in the nominal interest rate. An operational instrument rule, in this case, will satisfy the Taylor principle, but will also require that the nominal interest rate does not necessarily respond one to one to an increase in the efficient rate of interest. The model is calibrated under different monetary policy rules and under the optimal rule. We show that the correlation between productivity shocks and employment is strongly influenced by the monetary policy regime. The results of the model are consistent with a well known empirical regularity in macroeconomics, i.e. that employment volatility is relatively larger than real wage volatility.

JEL codes: E24, E32, E50, J23, J51

1 Introduction

In the last ten years, the Dynamic Stochastic New Keynesian () model has emerged as an important paradigm in macroeconomics and as a useful framework for the study of monetary policy. Most of the models proposed so far along this line of research, however, are based on the standard competitive model and completely ignore the role that trade unions play in determining wages and employment conditions in many countries. If this is probably an acceptable (although very strong) simplification for countries, like the U.S. where, in the year 2002, only about 15% of workers were covered by collective contract agreements, it becomes instead problematic for other countries such as France, Italy or Sweden where the percentage of workers covered by collective contracts is above 84%.[1] Given that wage bargaining may introduce significant distortions in the functioning of a modern economy and have an impact on its behavior at the aggregate level, the study of unionized labor markets and of the consequences of these markets for monetary policy becomes of crucial importance if one wants to understand the functioning of many important economies around the world.

The purpose of this paper is to propose a model where wages are the result of a contractual process between unions and firms and where, at the same time, the movements of the rate of unemployment are explicitly accounted for. In order to evaluate movements of labor along the extensive margin, we assume, as in Hansen [36] and Rogerson and Wright [46], that labor supply is indivisible and that workers face a positive probability to remain unemployed. As in the recent papers of Maffezzoli [39] and Zanetti [58], wages are set by unions according to the popular monopoly-union model introduced by Dunlop [18] and Oswald [41]; differently from these papers, however, we use a more general formulation of the unions' objective function,[2] i.e. we assume that unions maximize a Stone-Geary utility function as in Dertouzos and Pencavel [16], Pencavel [42] and, more recently, by De la Croix et al. [15] and Raurich and Sorolla [45].

This paper, therefore, contributes to a literature which has recently tried to improve on the "standard" NK model by focusing on the behavior of the labor market. Models characterized by labor market frictions and price staggering, where labor is allowed to move not only along the intensive margin

[1] More precisely, the number of persons covered by collective agreeements over total employment was 94.5% in France in 2003, 84.1% in Italy in the year 2000 and 85.1% in Sweden in the year 2000. For a complete set of data on union coverage on the various countries see Lawrence and Ishikawa [38].

[2] As we will see later on, the unions' objective function assumed by these two authors is a special case of the utility function we consider in this model.

but also the extensive margin, have been proposed, among others, by Chéron and Langot [10], Walsh [54] [55], Trigari [52], [53], Moyen and Sahuc [40] and Andres et al. [2]. All these models show that search and matching frictions improve the ability of the standard New Keynesian model to replicate the dynamics of inflation and unemployment by explaining, in particular, the persistence of output and the sluggishness of inflation. More recently Christoffel and Linzert [13] and Blanchard and Galì [6] [7] have proposed models characterized by both labor market frictions and real wage rigidities. Blanchard and Galì [6] show that, if real wages are assumed to adjust slowly, what they define as the "divine coincidence" does not hold any more: for a central bank pursuing, as a policy objective, the level of output that would prevail under flexible prices is not equivalent to pursuing the efficient level of output, in which case a trade-off between inflation stabilization and output gap stabilization arises. Blanchard and Galì [7] use a model with search and matching frictions and sluggish real wages, and show that a policy trade-off does not only pertain to the output gap, but also to the rate of unemployment.

Our model differs from previous studies in several respects. First, as in Zanetti [58] we abstract from the search and matching frictions based on the Mortensen-Pissarides [43] model, and we concentrate on the consequences of union behavior, studying in particular the implications of microfounded real wage rigidities. Differently from Zanetti, however, we propose a simpler model without human and physical capital accumulation and we are able, therefore, to study analytically the optimal interest rate rule a Central Bank should implement in our unionized economy. Morever, by assuming Rogerson and Wright [46] indivisible labor model, we are able to analyze unemployment in a simple and tractable way which allows us to establish an inverse relationship between unemployment and the output gap. The model is capable of producing a series of interesting results.

First, it shows that productivity shocks and reservation wage shocks give rise to a significant policy trade-off between stabilizing inflation and stabilizing unemployment, and in this respect it provides a way to overcome an important shortcoming of the NK model, i.e. its inability to account for the significant challenges that exogenous changes in technology represent for monetary policy in the real world. According to the "standard" NK model, in fact, an optimal monetary policy that stabilizes output around its flexible price equilibrium also produces zero inflation,[3] so that stabilizing inflation implies automatically an optimal response to a productivity shocks.

[3] This is shown quite clearly, for example, by Galì, Lopez Salido and Valles [25].

This, however, not only is at odds with the historical accounts[4] and the widespread perception of financial markets, but there is also some recent empirical evidence indicating that, in most countries, central banks have actively responded to technology shocks, increasing or decreasing the nominal interest rate.[5] What is interesting, in our model, is that this result is not the consequence of some kind of exogenous real wage inertia, as in Blanchard and Galì [6], but is simply the consequence of monopolistic unions pursuing a well defined contractual strategy in the labor market. In our economy, in fact, a productivity slowdown, i.e. a negative productivity shock, tends to lower efficient output but, since unions will keep real wages constant, the level of output that would prevail under price flexibility (that we define as "natural" output) decreases even more, so that the difference between efficient output and "natural" output increases. Since in sticky price models inflation depends on marginal costs and, in turn, marginal costs depend on the difference between "natural" output and actual output, then a Phillips curve, correctly defined as depending on the gap between efficient output and actual output, will depend on productivity shocks, and a trade-off between inflation stabilization and output gap stabilization arises.

Second, we show that a policy trade-off for the central bank arises not only in response to technology shocks, but also in response to exogenous wage push shocks. If the unions' reservation wage is subject to exogenous changes, and these changes tend to be persistent over time, then a welfare maximizing central bank must again face the problem of whether to accommodate these shocks with a easier monetary policy. Our model therefore provides a convenient framework to address important normative issues such as, for

[4]In this respect the debate on the Fed's monetary policy during governor Greenspan tenure is quite instructive. There is a lot of anectodical evidence that the Fed has spent large efforts in understanding the increase in productivity growth that has characterized the American economy since the mid 1990s. The success of monetary policy in this period has been attributed by important commentators (Woodward 2000) to the ability of the Fed to respond to exogenous technological progress.

[5]Galì et al. [24], analyzing a 4 variable SVAR model where thechnology shocks were identified by assuming a unit root in labor productivity, found that the Fed, in the post-Volcker period, di not change the nominal interest rate in response to productivity shocks, but in a recent paper, Francis, Owyang and Theodoru [22] used the same methodology to analyze monetary policy for the G7 countries and found a wide range of variation in the behavior of different countries: while France, Japan, the United Kingdom (after the break in monetary regime) seem to have reacted to a technology shock by increasing nominal interest rates, the U.S. (before and after Vocker), Canada, Germany, the U.K (pre-break) and Italy, seem to have reacted instead by lowering interest rates. A policy of no-reaction to a productivity shock consistent with the prescriptions of the standard DNK model, therefore, does not seem to have been widely adopted by the major industrialized countries.

example, the optimal behavior of central banks in periods characterized by labor market turmoil and wage shocks.

Third, we are able to derive, also in a model with non separable utility and indivisible labor, the objective function of the central bank as a second order Taylor approximation of the expected utility of the representative household, and we show that, when the economy is hit by technology and wage shocks, monetary policy presents some interesting peculiarities relative to the standard case. If the central bank that cannot commit to future policy actions (optimal discretionary policy), optimality requires a decrease (increase) in the interest rate following a positive (negative) productivity shock and an increase in the interest rate following a reservation wage shock. An optimal instrument rule that implements such policy can be expressed as an interest rate reacting to the expected rate of inflation and to the efficient rate of interest.

In this model monetary policy satisfies the Taylor principle, i.e. the nominal interest rate must be raised more than proportionally with respect to the expected rate of inflation. The response of the nominal interest rate to the "efficient" rate of interest, however, is not one to one like in the standard model: if the persistence of the technological shock is greater than the persistence of the reservation wage shock, the nominal interest rate will increase less than proportionately to an increase in the efficient rate of interest. This is due to the fact, unlike what happens in the standard model, the central bank must react, at the same time, both to a technology shock and to a reservation wage shock.

A fourth, important result is that the model is able to account for a well known stylized fact in macroeconomics, i.e. the relatively smooth behavior of wages and the relatively volatile behavior of unemployment over the business cycle. When the level of unemployment that the economy achieves under an optimal discretionary policy is written as a function of the relevant shocks, an exogenous wage shock will in general induce a movement both in the real wage and in the rate of unemployment; a productivity shock, instead, will induce a movement in the rate of unemployment, but not in the real wage. An economy frequently hit by exogenous changes in technology will show, therefore, a strong variability in the rate of unemployment without experiencing, at the same time, significant movements in the real wage.[6]

Following Zanetti, we calibrate the model using parameters of the Euro area. We first consider the dynamics of our model when the central bank uses

[6]Also Gertler and Trigari [32] propose a model where wages and unemployment move consistently with the observed data. They achieve this result, however, by introducing exogenous multiperiod wage contracts.

a Taylor-type rule similar the one estimated for EU by Smets and Wouters [57]. The model's impulse response functions are similar to those observed in the data, both for the indivisible labor model with walrasian labor markets and the one with unionized labor markets. In this last case, however, the reaction of the relevant variables to productivity and wage shocks is stronger and more persistent. Interestingly, the model is consistent with the recent findings of Galì [23], [27], [29], and Francis et al. [22] [21] who show that a negative productivity shock has a positive impact on the labor input. In our model this last result can be obtained also using the simpler rule originally proposed by Taylor [51], and this is in clear contrast with the search and matching model of Blanchard Galì [7] who, using this rule to approximate the optimal monetary policy, find a negative correlation between productivity and unemployment.

We then calibrate the unionized labor market model using the optimal discretionary monetary policy and we show that such policy may be replicated by a simple Taylor rule. A rule that is capable to deliver impulse response functions similar to the ones implied by the optimal rule, however, implies that the reaction to inflation is quite high relative to the most popular estimates, and a smaller reaction to the output gap. Moreover, unlike the previous case, under the optimal policy rule the correlation between productivity shock and employment is positive. This seems to suggest that the negative reaction of a unemployment to a negative productivity shock found in the data may be due, rather than to some "structural" characteristic of the economy to the central bank pursuing a too accomodating monetary policy.

The paper is organized as follows. In Section 2 we discuss the standard indivisible labor model (hereafter, IL model) in a New Keynesian framework. In Section 3 we develop the the monopoly union model (hereafter, UM model) in detail. In Section 4 we study optimal monetary policy in the UM model. Section 4 we calibrate the MU model under the optimal rule and some simpler policy rules.

2 The Indivisible Labor Model

2.1 Households

We consider an economy populated by many identical, infinitely lived worker-households each of measure zero. Households demand a Dixit, Stiglitz [17] composite consumption bundle produced by a continuum of monopolistically competitive firms. In each period households sell labor services to the firms. As in Hansen [36], Rogerson [46] and Rogerson and Wright [47], for each household the alternative is between working a fixed number of hours and not working at all. We assume that agents enter employment lotteries, i.e. sign, with a firm, a contract that commits them to work a fixed number of hours, that we normalize to one, with probability N_t. The contract itself is being traded, so a household gets paid whether it works or not which implies that the firm is providing complete unemployment insurance to the workers. Since all households are identical, all will choose the same contract, i.e. the same N_t. However, although households are ex-ante identical, they will differ ex-post depending on the outcome of the lottery: a fraction N_t of the continuum of households will work and the rest $1 - N_t$ remains unemployed. The allocation of individuals to work or leisure is determined completely at random by a lottery, and lottery outcomes are independent over time.

Before the lottery draw, the expected intratemporal utility function is:

$$\frac{1}{1-\sigma} N_t \left[C_{0,t} \upsilon\left(0\right)\right]^{1-\sigma} + \frac{1}{1-\sigma}\left(1-N_t\right)\left[C_{1,t}\upsilon\left(1\right)\right]^{1-\sigma} \tag{1}$$

where $C_{0,t}$ is the consumption level of employed individuals, $C_{1,t}$ is the consumption of unemployed individuals, N_t is the ex-ante probability of being employed and $\upsilon(\cdot)$ is the utility of leisure. Since the utility of leisure of employed individuals $\upsilon(0)$ and the utility of leisure of unemployed individuals $\upsilon(1)$ are positive constants, we assume $\upsilon(0) = \upsilon_0$ and $\upsilon(1) = \upsilon_1$. As in King and Rebelo [35], we assume $\upsilon_0 < \upsilon_1$.

If asset market are complete, households can insure themselves against the risk of being unemployed. Under perfect risk sharing we have:

$$C_{0,t}^{-\sigma} \upsilon_0^{1-\sigma} = C_{1,t}^{-\sigma} \upsilon_1^{1-\sigma} \tag{2}$$

which implies that the marginal utilities of consumption are equal for employed and unemployed individuals. Defining the average consumption level as:

$$C_t = N_t C_{0,t} + \left(1-N_t\right) C_{1,t} \tag{3}$$

and given (2), equation (1) can be rewritten as:

$$\frac{1}{1-\sigma} C_t^{1-\sigma}\left[N_t \upsilon_0^{\frac{1-\sigma}{\sigma}} + \left(1-N_t\right)\upsilon_1^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}. \tag{4}$$

This allows us to write the life-time expected intertemporal utility function of a representative household as:

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\sigma}\left[C_t \phi\left(N_t\right)\right]^{1-\sigma}, \tag{5}$$

where $\sigma > 1$, and $0 < \beta < 1$ is the subjective discount rate and where the function

$$\phi(N_t) = \left[N_t \upsilon_0^{\frac{1-\sigma}{\sigma}} + (1 - N_t) \upsilon_1^{\frac{1-\sigma}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}} \tag{6}$$

can be interpreted as the disutility of employment for the representative household. The elasticity of $\phi(N_t)$ with respect to its argument is given by $\xi_\phi = \frac{\phi_N(N_t)}{\phi(N_t)} N < 0$. Under perfect risk sharing all individual incomes are pooled, so the flow budget constraint of the representative household is given by:

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + \Pi_t - T_t \tag{7}$$

where P_t is the corresponding consumption price index (CPI) and W_t is the wage rate. Notice that here a worker is paid according to the probability that it works, not according to the work it does; in other words, the firm is automatically providing full employment insurance to the households. The purchase of consumption goods, C_t, is financed by labor income, profit income Π_t, and a lump-sum transfers T_t from the Government. Households have access to a financial market, where nominal bonds are exchanged. We denote by B_t the quantity of nominally riskless one period bonds carried over from period $t-1$, and paying one unit of the numéraire in period t. The maximization of (5) subject to (7) gives the following::

$$1 = \beta R_t E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{\phi(N_t)}{\phi(N_{t+1})} \right)^{1-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{8}$$

$$\frac{W_t}{P_t} = -C_t \frac{\phi_N(N_t)}{\phi(N_t)} \tag{9}$$

where equation (8) is the standard consumption Euler equation and (9) gives us the supply of labor of the representative household.

2.2 The Finished Goods-Producing Sector

The representative finished goods-producing firm uses $Y_t(j)$ units of each intermediate good $j \in [0,1]$ purchased at a nominal price $P_t(j)$ to produce Y_t units of the finished good with the constant returns to scale technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{10}$$

where θ is the elasticity of substitution across intermediate goods. Profit maximization yields the following set of demands for intermediate goods:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t \tag{11}$$

Perfect competition and free entry drives the finished good-producing firms' profits to zero, so that from the zero profit condition we obtain:

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{12}$$

which defines the aggregate price index of our economy.

2.3 The Intermediate Goods-Producing Sector

We abstract from capital accumulation and assume that there is a continuum of intermediate good-producing firms $j \in (0,1)$ which hire N_t units of labor from the representative household and produce $Y_t(j)$ units of the intermediate good using the following technology:

$$Y_t(j) = A_t N_t(j)^{\alpha} \tag{13}$$

where A_t is an exogenous productivity shock. We assume that the $\ln A_t \equiv a_t$ follows follows the autoregressive process

$$a_t = \rho_a a_{t-1} + \hat{a}_t \tag{14}$$

where $\rho_a < 1$ and $\hat{a}_t$ is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_a.
Before choosing the price of its goods, a firm chooses the level of $N_t(j)$ which minimizes its costs, solving the following costs minimization problem:

$$\min_{\{N_t\}} TC_t = (1-\tau) W_t N_t(j)$$

subject to (13), where τ represents an employment subsidy to the firm[7]. The first order condition with respect to $N_t(j)$ is given by:

$$(1-\tau)\frac{W_t}{P_t} = MC_t(j)\,\alpha \frac{Y_t(j)}{N_t(j)}, \tag{15}$$

where $MC_t(j)$ represents firm's j marginal costs. The assumption of decreasing return to scale technology, which is in line with a non-competitive intermediate good sector has important implication on the optimal price-settings rule, and then on the derivation of the traditional Phillips curve.[8]

[7] We assume that the subsidy is covered by a lump sum tax in that the Government runs always a balanced budget.

[8] In fact, as shown in Sbordone [49] and in Galì et al. [31], it should be taken into account that marginal costs are no longer common across firms (see appendix A5 or).

The aggregate real marginal costs are defined as:

$$MC_t = \frac{(1-\tau)}{\alpha}\frac{W_t}{P_t}\frac{N_t}{Y_t}, \tag{16}$$

equation (15) implies,

$$MC_t = MC_t(j)\frac{Y_t(j)}{N_t(j)}\frac{N_t}{Y_t}. \tag{17}$$

2.4 Market clearing

Equilibrium in the goods market of sector j requires that the production of the final good be allocated to expenditure, as follows:

$$Y_t(j) = C_t(j) \tag{18}$$

considering (10) then

$$Y_t = C_t \tag{19}$$

which represents the economy resource constraint. Defining as X the steady state value of a generic variable X_t and by $x_t = \ln X_t - \ln X$ the log-deviation of the variable from its steady state value, then a linear first order approximation of the resource constrained around the steady state is given by:

$$y_t = c_t \tag{20}$$

Since the net supply of bonds, in equilibrium is zero, equilibrium in the bonds market, instead, implies

$$B_t = 0. \tag{21}$$

Labor market clearing implies

$$N_t = \int_0^1 N_t(j)\,dj \tag{22}$$

given equation (11), (13) and (22) the aggregate production function can be expressed as

$$D_t Y_t = A_t N_t^{\alpha} \tag{23}$$

where

$$D_t = \left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\theta}{\alpha}} dj\right]^{\alpha} \tag{24}$$

is a measure of price dispersion. Given that in a neighborhood of a symmetric equilibrium and up to a first order approximation $D_t \simeq 1$, log-linearizing equation (23) we obtain,

$$y_t = a_t + \alpha n_t. \tag{25}$$

2.5 The First Best Level of Output

The efficient level of output can be obtained by solving the problem of a benevolent planner that maximizes the intertemporal utility of the representative household, subject to the resource constraint and the production function. This problem is analyzed in the Appendix A1, where we show that the efficient supply of labor, in our economy, is given by:

$$\frac{\phi_N(N_t)}{\phi(N_t)}N_t = -\alpha. \tag{26}$$

Log-linearizing (26), and considering (25), we obtain[9]

$$y_t^{Eff} = a_t. \tag{27}$$

2.6 The Flexible Price Equilibrium and the Natural Output

Equilibrium in the labor market is obtained by equating (9) and (16). Substituting (19), this implies

$$-Y_t\frac{\phi_N(N_t)}{\phi(N_t)} = \frac{1}{(1-\tau)}\alpha MC_t\frac{Y_t}{N_t} \tag{28}$$

Under flexible prices, all firms set their prices equal to a constant markup over marginal cost. Assuming that firms mark-up, μ_t^P is constant, under the flexible price-equilibrium firms real marginal costs are constant at their steady state level and therefore given by:

$$MC_t = \frac{1}{1+\mu^P}. \tag{29}$$

Considering now the log-linearization of (28) we obtain[10],

$$mc_t = \left[1 + \frac{\phi_N(N)}{\phi(N)}N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right]n_t \tag{30}$$

Considering that $mc_t = 0$,then $n_t = 0$ and, from the aggregate production function, we have that under the flexible price equilibrium:

$$y_t^f = a_t \tag{31}$$

[9] See appendix A2.

[10] See appendix A3.

Taking the difference between the log-linearized flexible and efficient output we obtain:

$$y_t^{Eff} - y_t^f = 0 \tag{32}$$

As in the standard NK model, when labor market is frictionless the difference between the efficient output (its first best) coincides with its flexible price equilibrium level (its second best) that we have defined as the natural level of output.[11] In other words, what Blanchard and Galì [6] call "the divine coincidence" will hold, since any policy that stabilizes output around its natural level, will stabilize it also around its efficient level. Notice that, in this model, we have left the subsidy τ as parametric in order to show that the divine coincidence holds for any possible value of the subsidy. As in the standard case, also in this model an optimal subsidy could be set in order to eliminate the constant distortion induced by monopolistic competition.

2.7 The Phillips Curve

Firms choose $P_t(j)$ in a staggered price setting à la Calvo-Yun [8]. In the appendix A4 we show that, in our decreasing return to scale economy, the solution of the firm's problem is given by:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_\alpha mc_t \tag{33}$$

where $\lambda_\alpha = \frac{(1-\psi)(1-\beta\psi)}{\psi} \frac{\alpha}{\alpha+\theta(1-\alpha)}$ and ψ is the probability with which firms reset prices.[12]

Given (25), (30) and (31), marginal costs can be rewritten in terms of the gap between actual and natural output,

$$mc_t = \left(1 + \frac{\phi_N(N)}{\phi(N)} N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right)\left(y_t - y_t^f\right) \tag{34}$$

so that, equation (33) can be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \left(1 + \frac{\phi_N(N)}{\phi(N)} N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right) x_t \tag{35}$$

where

$$x_t = y_t - y_t^f \tag{36}$$

[11] Our result is equivalent to the one of Blanchard Galì, where they consider (log) real marginal costs instead of log-deviation from the steady state, and therefore the difference between the efficient and the flexible price output is constant and not zero.

[12] For details see Appendix A5 or Sbordone [49] and Galì et al. [31].

is the output gap with respect to the natural rate of output. As in the standard case there is no trade-off between output stabilization and inflation stabilization, since a central bank that sets inflation to zero will immediately stabilize output.
It is worth noticing that because of decreasing returns technology the output gap coefficient, λ_a, of the New Keynesian Phillips curve, is lower than the traditional coefficient found with constant return to scale. This means that the elasticity of inflation to output gap is lower than in the case of constant returns, which is consistent with the empirical estimates. In fact, inflation does not seems to respond strongly to the output gap.[13]

3 The Monopoly Union Model

As in the previous subsection the individual labor supply is indivisible. Each firm is endowed with a pool of households from which it can hire. In fact, as in Maffezzoli [39] and Zanetti [58], firms hire workers from a pool composed of infinitely many households so that the individual household member is again of measure zero. Since each household supplies its labor to only one firm, which can be clearly identified, workers try to extract some producer surplus by organizing themselves into a firm-specific trade union. The economy is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiate with a single union $i \in (0,1)$, which is too small to influence the outcome of the market. Unions negotiate the wage on behalf of their members.

Once unions are introduced in the analysis, two important issues arise: what is the objective function of the union and what are the variables subject to bargaining. Both these questions have been extensively investigated by the literature, although no conclusive agreement has been reached on the issue.[14] The problem of identifying an appropriate maximand for the union dates back to Dunlop [18] and Ross [48]; since then the debate has revolved over the relative importance of economic considerations (basically how employers respond to wage bargaining) and political considerations in the determination of union wage policy. For political considerations we intend how the preferences of workers, the preference of union leaders and market constraints interact in determining a union's objective.

One approach often followed in the literature is the "utilitarian" approach pioneered by Oswald [41] which consists on assuming that all workers are

[13] See for example Galì et al ([31]) and Sbordone ([49]).

[14] For an extensive survey of unions model see Farber [20], and, more recently, Kaufman [33].

equal and that the union simply maximizes the sum of workers' utility, defined over wages. Although simple and appealing because coherent with a standard economic approach, this formulation of unions' utility does not allow for political considerations.[15] An alternative approach, initially proposed by Dertouzos and Pencavel [16] and Pencavel [42] and, more recently, reproposed by De la Croix et al. [15] and Raurich and Sorolla [45], is to assume that unions maximize a modified Stone-Geary utility function of the form:

$$V\left(\frac{W_t}{P_t}\right) = \left(\frac{W_t\,(i)}{P_t} - \frac{W_t^r}{P_t}\right)^{\gamma} N_t\,(i)^{\varsigma} \tag{37}$$

The relative value of γ and ς is an indicator of the relative importance of wages and employment in in the union's objective function.[16] The reservation wage W_t^r is the absolute minimum wage the union can tolerate. This reservation wage has many possible interpretations. One possible interpretation is that W_t^r is the opportunity wage of the workers (Pencavel 1984) since it is unlikely that a union can survive if it negotiates a wage below such level. Another possible interpretation is that W_t^r is what Blanchard and Katz [5] define as an "aspiration wage", i.e. a wage that workers have come to regard as "fair". Unions' reservation wage is generally unobservable and therefore hard to model. As in De la Croix[17] [15], however, we assume that:

$$\frac{W_t^r}{P_t} = \frac{\bar{W}}{P_t} e^{\varepsilon_t^w} \tag{38}$$

with

$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \hat{\varepsilon}_t^w \tag{39}$$

where $\rho_w < 1$ and ε_t^w is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_w. If the real reservation

[15] As it can be easily verified, if unions set wage to simply maximize agents' utility, the wage schedule would be similar to the labor supply in the IL model, with the difference that the wage would be a constant mark-up over the marginal rate of substitution. In this case, wages would fully respond to technology shocks and no significant trade-off between inflation and unemployment (output gap) would emerge. Therefore, assuming that the union leader has this type of objective function is a very simple and realistic way to obtain endogenous real wage rigidities.

[16] The objective function we consider is closed to the one suggested by the Ross tradition. In fact, for different parameters values, the union's objective function is almost equivalent to the one of a union which maximizes his income or his membership, as for example in Skatun [50] and in Booth [4].

[17] In the model of De la Croix et al. [15] the real reservation wage is a weighted sum of a constant term and of the past real wage. In order not to add further ad-hockery to the model, we chose not to include past real wages. Nevertheless adding these to equation (38) would leave the results unchanged. A technical appendix is avalable upon request.

wage is constant, $\hat{\varepsilon}_t^w = 0$. The fact that the reservation wage is subject to persistent shocks is meant to capture the exogenous wage shocks, often associated with political and social factors that have often characterized industrialized economies, especially in Europe.[18] Notice that the Stone-Geary utility function we consider in this model not only is appealing, both for its ability to approximate the actual behavior of unions and for its flexibility and tractability, but also for its generality. If we set for example $\gamma = 1$, $\varsigma = 1$ and $\hat{\varepsilon}_t^w = 0$, maximizing (37) is equivalent to maximize the unions' objective function assumed by Maffezzoli [39] and Zanetti [58] in their recent papers.

The bargaining process we consider here is in the tradition of the "right to manage" models. In particular, we follow the popular "monopoly union" model first proposed by [18] and Oswald [41], where the employment rate and the wage rate are determined in a non-cooperative dynamic game between unions and firms. We restrict the attention to Markov strategies, so that in each period union and firm solve a sequence of independent static games. Each union behaves as a Stackelberg leader and each firm as a Stackelberg follower. Once the wage has been chosen, each firm decides the employment rate along its labor demand function. Even if unions are large at the firm level, they are small at the economy level, and therefore they take the aggregate wage as given. The ex-ante probability of being employed is equal to the aggregate employment rate and the allocation of union members to work or leisure is completely random and independent over time. Finally, as in the previous IL economy, we assume that workers are able to perfectly insure themselves against the possibility of being unemployed. This result can either be obtained through the lottery mechanism previously described or by assuming, as in Maffezzoli [39] and Zanetti [58] that, in order to impede workers from leaving the Union, the Union pursues a redistributive goal, acting as a substitute for competitive insurance market. Insurance is supplied under zero-profit condition and is therefore actuarially fair. The problem of the firm is the same as in the IL model. From the first order conditions of the union's maximization problem with respect to $W_t(i)$, given that in this model the labor demand elasticity with respect to the real wage $\frac{1}{1-\alpha}$ is constant, we obtain:

$$\frac{W_t(i)}{P_t} = \frac{\varsigma}{\varsigma - \gamma(1-\alpha)} \frac{W_t^r}{P_t} e^{\varepsilon_t^w}. \tag{40}$$

[18] We consider both these two alternative in order to show that the our results on the endogenous inflation unemployment (output) trade-off is not qualitatively influenced by the fact that the reservation wage shock is an exogenous shock. Moreover, to our knowledge this is the first attempt to study how the optimal interest rate rule should react in response to more than one supply shock.

The technology shock has no effect on the real wage rate chosen by the monopoly union. Since $\frac{1}{1-\alpha} > 1$, we see that the real wage rate is always set above the reservation wage.

3.1 Households

If the union offers actuarially fair insurance, household will again perfectly share the risk to be unemployed. The model is quite similar to the IL model except for the fact that now households, in solving their problem, take N_t as given, since the supply of labor is determined by the maximization problem of the monopoly union. The maximization of utility function (5) subject to budget constraint (7) gives the same Euler equation as in the Walrasian model, which is given by equation (8).

3.2 The Flexible Price Equilibrium and the Natural Level of Output

Given that both intermediate goods and finished goods producing firm problem are the same as in the previous problem, the aggregate labor demand function is again given by equation (16). Equating (16) and (40), we obtain:

$$\frac{1}{\alpha}\frac{W_t^r}{P_t} = \frac{1}{(1-\tau)}\alpha MC_t \frac{Y_t}{N_t}. \tag{41}$$

also in this case τ guarantees that the MU model steady state is equal to the pareto-efficeint one (see appendix A4 for details).
Since under flexible prices all firms set their prices as a constant markup over marginal costs, which is given by equation (29), we can rewrite equation (41) as:

$$\frac{1}{\alpha}\frac{W_t^r}{P_t}e^{\varepsilon_t^w} = \frac{1}{(1-\tau)}\alpha \frac{1}{1+\mu^P}\frac{Y_t}{N_t} \tag{42}$$

Considering now the log-linearization of (41) we obtain the following expression for real marginal costs

$$mc_t = n_t - y_t + \varepsilon_t^w \tag{43}$$

where w_t^r is the logarithm of the real reservation wage. Solving (25) for n_t and substituting in (43), we get:

$$mc_t = \varepsilon_t^w + \frac{1-\alpha}{\alpha}y_t - \frac{1}{\alpha}a_t \tag{44}$$

Considering that $mc_t = 0$, substituting in (44) and solving for y_t we find an expression for the flexible-price level of output, which we define as the natural rate of output for our unionized economy:

$$y_t^f = \frac{1}{1-\alpha} a_t - \frac{\alpha}{1-\alpha} \varepsilon_t^w \tag{45}$$

Recalling now that the efficient level of output, for our economy with indivisible labor, is given by equation (27) we immediately see that the difference between natural output and efficient output of the unionized economy is given by

$$y_t^{Eff} - y_t^f = -\frac{\alpha}{1-\alpha} a_t + \frac{\alpha}{1-\alpha} \varepsilon_t^w . \tag{46}$$

Unlike what happens in the walrasian model, this difference is not constant, but is a function of the relevant shocks that hit the economy. In this model therefore, as in Blanchard and Galì [6] stabilizing the output gap - the difference between actual and natural output - is not equivalent to stabilizing the *welfare relevant* output gap - the gap between actual and efficient output. In other words, what Blanchard and Galì call "the divine coincidence" will not hold, since any policy that brings the economy to its natural level is not necessarily an optimal policy. It is worth noticing that even if the real reservation wage is constant, the flexible price equilibrium output is different from the efficient one, since:

$$y_t^{Eff} - y_t^f = -\frac{\alpha}{1-\alpha} a_t .$$

3.3 The IS-Curve

In order to obtain the IS curve we start by log-linearizing[19] around the steady state the Euler equation (8) as:

$$c_t = E_t \{c_{t+1}\} - \frac{1-\sigma}{\sigma} \frac{\phi_N (N) N}{\phi (N)} E_t \{\Delta n_{t+1}\} - \frac{1}{\sigma} (\hat{r}_t - E_t \{\pi_{t+1}\}) \tag{47}$$

with $\hat{r}_t = r_t - \varrho$, where $r_t = \ln R_t$ and $\varrho = -\ln \beta$ which is the steady state interest rate all the variables without a subscript are taken at their steady

[19] In order to loglinearize $\phi(N_t)^{1-\sigma}$ we first log-linearize the term N_t obtaining $\phi [N(1+n_t)]^{1-\sigma}$ .Applying a first order Taylor expansion, we obtain

$$\phi [N(1+n_t)]^{1-\sigma} = \phi (N)^{1-\sigma} + (1-\sigma) \phi (N)^{-\sigma} \phi_N (N) N n_t$$

state levels. Given that optimal subsidy setting implies $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$, we can then rewrite equation (47) as

$$c_t = E_t\{c_{t+1}\} + \frac{\alpha(1-\sigma)}{\sigma}E_t\{\Delta n_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}). \tag{48}$$

Given the economy resource constraint (20) and the production function (25), the Euler equation (48) can be written as:

$$y_t = E_t\{y_{t+1}\} - (1-\sigma)E_t\{\Delta a_{t+1}\} - (r_t - E_t\{\pi_{t+1}\}) \tag{49}$$

which represents the IS equation of our simple economy. Given (27) and (36) the IS equation can be rewritten in terms of the output gap as

$$x_t = E_t x_{t+1} + \sigma E_t\{\Delta a_{t+1}\} - (r_t - E_t\{\pi_{t+1}\}).$$

where, in the IL model, we define $x_t = y_t - y_t^{Eff}$ as the output gap with respect to the efficient equilibrium output. The efficient rate of interest, instead, can be expressed as:

$$\hat{r}_t^e = \sigma E_t\{\Delta a_{t+1}\} = \sigma E_t\left\{\Delta y_{t+1}^{Eff}\right\} = -\sigma(1-\rho_a)a_t. \tag{50}$$

Given (**??**), the definition of the efficient equilibrium output (27) and that for the efficient interest rate (50) the IS relation can be rewritten as

$$x_t = E_t\{x_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - r_t^e). \tag{51}$$

Note that (51) relates the output gap rate to current and anticipated deviations of the real interest rate from its efficient counterpart.

3.4 The Phillips Curve

As in the Walrasian case, firms choose $P_t(j)$ in a staggered price setting à la Calvo-Yun [8] and the Phillips curve is again given by (33). Given (43) and (45), marginal costs can be rewritten in terms of the gap between actual output and its natural level,

$$mc_t = \frac{1-\alpha}{\alpha}\left(y_t - y_t^f\right) \tag{52}$$

so that, equation (33) can now be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha}\left(y_t - y_t^f\right) \tag{53}$$

Given the relationship between efficient and natural output, (see eq. (46)), equation (53) can finally be expressed as:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha} x_t - \lambda_a a_t + \lambda_a \varepsilon_t^w \tag{54}$$

We can now state:

Result 1. *In a unionized labor market economy the "divine coincidence" does not hold, i.e., stabilizing inflation is not equivalent to stabilizing the output gap defined as the deviation of output from the efficient output. A positive (negative) productivity shock has a negative (positive) effect on inflation, while a cost push shock has an effect of the same size but with the opposite sign on inflation.*

This result depends on the existence of a real distortion in the economy, beside the one induced by monopolistic competition, and the nominal distortion caused by firms' staggered price setting. When a productivity shock hits the economy, efficient output, given by equation (27), increases by the same amount. Natural output instead (i.e., the level of output that would prevail in a flexible price equilibrium) increases more than proportionally so that the difference between efficient output and natural output decreases. This is due to the fact that in a unionized economy, following a productivity shock, real wages remain constant and therefore do not offset the effects of the shock on real marginal cost (see equation (52)).

Because of staggered price adjustment we know that inflation is proportional to real marginal costs which, in turn, because of monopolistic competition (see equation (53)) are proportional to the difference between actual and natural output. As we will see in the following paragraphs, a Central Bank pursuing an optimal monetary policy will decide to stabilize the distance between output and its efficient level. If the difference between efficient and flexible output were constant, as in the standard model with Walrasian labor markets, stabilizing the gap between actual and natural output would be equivalent to stabilizing the gap between actual and efficient output. In this case, stabilizing the output gap with respect to the natural output would be sufficient to stabilize inflation. In our unionized economy, instead, the natural level of output differs from the efficient level because of productivity and cost-push shocks. As it is evident from equation (54), if the Central Bank stabilizes output around the efficient level, inflation will be completely vulnerable to productivity and cost-push shocks; in other words the output gap is no longer a sufficient statistics for the effect of real activity on inflation.

One interesting aspect of this model is that we are able to express the Phillips curve in its more traditional form, i.e. in terms of unemployment. From equations (25), (27) and (??) we obtain in fact that

$$n_t = \frac{x_t}{\alpha} \tag{55}$$

Expressing the rate of unemployment as $U_t = 1 - N_t$ and log linearizing around the steady state we obtain

$$u_t = -\frac{\eta}{\alpha} x_t \tag{56}$$

where $\eta = \frac{N}{1-N}$. We can therefore rewrite the Philllips curve as

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\lambda_a (1-\alpha)}{\eta} u_t - \lambda_a a_t + \lambda_a \varepsilon_t^w. \tag{57}$$

The relationship between unemployment and the output gap that we find in this model, therefore, allows us to consider, indifferently, the output gap and the unemployment rate as policy objectives for the central bank.

4 Optimal Monetary Policy

In the appendix A5 we show that also for the non-separable preferences assumed in our framework, consumers' utility can be approximated up to the second order by a quadratic equation of the kind:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2} E_t \sum_{t=0}^{\infty} \left[\pi^2_{H,t+k} + \frac{\lambda_a}{\theta \sigma} x^2_{t+k} \right] + \bigcirc \left(\|\alpha\|^3 \right) \tag{58}$$

where $\tilde{U}_{t+k} = U_{t+k} - \bar{U}_{t+k}$ is the deviation of consumers' utility from the level achievable in the frictionless equilibrium, and θ is the elasticity of substitution between intermediate goods, which are used as input in the final good sector. Notice that, the relative weights assigned to inflation and to the output gap are linked to the structural parameters reflecting preferences and technology. In this paragraph we will consider optimal monetary policy under discretion, i.e. when the Central Bank cannot credibly commit in advance to a future policy action or a sequence of future policy actions; the case of constrained commitment, i.e. when the Central Bank is committed to follow a well specified policy rule, is analyzed in Appendix A8.

When optimal monetary policy is discretionary, policy makers choose in each period the value to assign to the policy instrument, that here we assume to be the short-term nominal interest rate $\hat{r}_t$. In order to do so, the Central

Bank maximizes the welfare-based loss function (58), subject to the Phillips curve (54), and (51).
The first order conditions imply:

$$x_t = -\frac{1-\alpha}{\alpha}\theta\sigma\pi_t. \tag{59}$$

Substituting into (54) we obtain :

$$\pi_t = \frac{1}{\Omega}\left(\beta E_t \pi_{t+1} - \lambda_a a_t + \lambda_a \varepsilon_t^w\right) \tag{60}$$

where $\Omega = 1 + \lambda \left(\frac{1-\alpha}{\alpha}\right)^2 \theta\sigma$.
Iterating forward (60),

$$\pi_t = -\frac{\lambda_a}{\Omega} E_t \sum_{i=0}^{\infty} \left(\frac{\beta}{\Omega}\right)^i \left(a_{t+i} - \varepsilon_{t+i}^w\right) \tag{61}$$

Given that,

$$E_t\left\{a_{t+i+1}\right\} = \rho_a^i a_t \qquad \text{and} \qquad E_t\left\{\varepsilon_{t+i+1}^w\right\} = \rho_w^i \varepsilon_t^w$$

(61) can be rewritten as,

$$\pi_t = -\frac{\lambda_a}{\Omega - \beta\rho_a} a_t + \frac{\lambda_a}{\Omega - \beta\rho_w} \varepsilon_t^w \tag{62}$$

The optimal level of inflation can be implemented by the Central Bank by setting the nominal interest rate. The interest rate rule can be obtained by substituting (59), (62) and (62) one period ahead, into the IS curve (51), we obtain:

$$\begin{aligned} \hat{r}_t^* = &-\left[\left[1 + \left(\frac{1-\rho_a}{\rho_a}\right)\left(\frac{1-\alpha}{\alpha}\right)\theta\sigma\right]\left(\frac{\lambda_a\rho_a}{\Omega - \beta\rho_a}\right) + \sigma\left(1-\rho_a\right)\right] a_t + \\ &+ \left[1 + \left(\frac{1-\rho_w}{\rho_w}\right)\left(\frac{1-\alpha}{\alpha}\right)\theta\sigma\right]\left(\frac{\lambda_a\rho_w}{\Omega - \beta\rho_w}\right)\varepsilon_t^w \end{aligned} \tag{63}$$

We can therefore state

Result 2. *Under discretion an optimal monetary policy requires a decrease in the nominal interest rate following a positive productivity shock and an increase in the nominal interest rate following a positive reservation wage shock.*

An interest rate rule that implements such optimal policy, can be found using (59) and (62). In this case we obtain:

$$\hat{r}_t^* = \left[1 + \left(\frac{1-\rho_w}{\rho_w}\right)\frac{1-\alpha}{\alpha}\theta\sigma\right] E_t\pi_{t+1} + \left[1 + \frac{(\rho_w - \rho_a)}{\rho_w (1-\rho_a)}\frac{\lambda_a \theta}{\Omega - \beta\rho_a}\frac{1-\alpha}{\alpha}\right]\hat{r}_t^e. \tag{64}$$

In Appendix A6 we show that under rule (64) equilibrium is determinate. Assuming, as a particular case $\rho_a = \rho_w = \rho$, equation (64) becomes

$$\hat{r}_t^* = \left[1 + \left(\frac{1-\rho}{\rho}\right)\frac{1-\alpha}{\alpha}\theta\sigma\right] E_t\pi_{t+1} + r_t^e \tag{65}$$

which is equivalent on assuming that the real reservation wage $\hat{\varepsilon}_t^w = 0$. We can now state:

Result 3. *Optimal monetary policy under discretion requires a more than proportional increase in the nominal interest rate following an increase in the expected rate of inflation. However, an increase in the efficient rate of interest implies a proportional increase in the nominal interest rate if and only if $\rho_a = \rho_w = 1$. Otherwise an increase in the efficient rate implies a more than proportional increase in the nominal interest rate if $\rho_w > \rho_a$ and a less than proportional increase if $\rho_w < \rho_a$.*

As in the standard NK model, optimality requires that the Central Bank responds to increasing inflationary expectations by raising more than proportionally nominal interest rates. In other words, also for our unionized economy, the Taylor principle applies. The optimal response of the nominal interest rate to an increase in the efficient rate of interest, instead, is different from the one that is usually obtained in the "standard" NK model.
Notice that (54) and (56) together imply

$$u_t = -\frac{\eta(1-\alpha)\theta\sigma\lambda_a}{\alpha^2(\Omega - \beta\rho_a)}a_t + \frac{\eta(1-\alpha)\theta\sigma\lambda_a}{\alpha^2(\Omega - \beta\rho_w)}\varepsilon_t^w. \tag{66}$$

Given the log-linearization of equation (40), we can now state

Result 4. *Under an optimal discretionary monetary policy a productivity shock will induce a change in the rate of unemployment without affecting the real wage rate.*

This result is quite important since it is consistent with a well known fact in macroeconomics, i.e. the relatively smooth behavior of wages along the business cycle together with the relatively volatile behavior of unemployment.

In this simple model, wages move only when there is a shock in the reservation wages of households. Productivity shocks imply some degree of volatility in unemployment while real wages remain constant. Wages, in the simple set up we consider in this paper, are probably too rigid, as we assume that all markets are unionized. Nevertheless, the model makes an interesting point, i.e. that the behavior of monopoly unions, in itself, is able to generate a dynamics of wages and unemployment that is roughly consistent with the one typically observed in the real economy.

5 Calibration

In our calibration exercise, aimed at illustrating the qualitative properties of our model, we start by following the work of Zanetti [58], who also studies a unionized model with labor market indivisibilities and unemployment. With respect to Zanetti's model, ours is somewhat simpler, since it does not allow for human and physical capital accumulation; as is well known in the literature, however, the advantage of proposing a simpler model is that we can easily study optimal monetary policy and therefore provide a benchmark to evaluate actual monetary policy. As in Zanetti [58], the variables of the model are calibrated using data from the Euro area.

The model is calibrated on quarterly frequencies and the value of each parameter is described in table 1. For the parameters describing preferences, we set the elasticity of intertemporal substitution at $\sigma = 2$. The output elasticity of labor, $\alpha = 0.72$, is based on the estimate of Christoffel et al. [14]. The discount factor β, the Calvo parameter φ, and the elasticity of substitution among intermediate goods θ are set at values commonly found in the literature (for example in Galì [24]). In particular we set $\beta = 0.99$, $\varphi = 0.75$, which implies an average price duration of one year, and finally $\theta = 6$, which is consistent with a 10 percent markup in the steady state. The persistence of the technology shock ρ_a is set as in Amato and Laubach [3], i.e. $\rho_a = .93$. The persistence of the wage shock is assumed to be as the persistence of a cost-push shock, i.e., equal to 0.7 as estimated by Ireland [37]. As discussed in Zanetti [58] $N = 0.61$.

In this exercise, the Central Bank is assumed to follow the following Taylor-type monetary policy rule:

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1 - \phi_r) [\phi_\pi \pi_{t-1} + \phi_x x_{t-1}] \tag{67}$$

As advocated by Carlstrom and Fuerst [9], we employ lagged values for output and inflation because it can be considered consistent with the information set of the Central Bank at time t. The parameters of the Taylor rule follow the estimates of Smets and Wouters [57] for the Euro area. In particular, the degree of interest rate smoothing is set at $\phi_r = 0.9$, the nominal interest rate response to inflation at $\phi_\pi = 1.658$ and the response to output at $\phi_x = 0.148$.

We evaluate the dynamics of the Walrasian model and the dynamics of the unionized model in response to a negative productivity shock. In figure (1) we plot the response of the interest rate, inflation, output, the output gap and unemployment to a one unit standard deviation negative productivity shock in the unionized economy (solid line) and in the walrasian labor market economy (dashed line). Our model, although simpler, produces results that are very similar to the ones found by Zanetti [58], and this suggests that adding physical and human capital accumulation does not sensibly change the dynamics of the model.

A negative productivity shock, in both the walrasian and the unionized economies it causes the output gap to rise, total output to decline, the nominal interest rate and inflation to rise and unemployment (employment) to decline (rise). The difference between the responses of the walrasian and the unionized economies lies not on the sign of the effect, but on the size of the effect: in the unionized economy the response of the main variables to a productivity shock is amplified and all the variables in the unionized economy are characterized by a higher degree of persistence.

The fact that unemployment experiences a large decline after a negative productivity shock, as shown in figure (1), is extremely interesting. A negative comovement between productivity shocks and various measures of the labor input has been recently found in the empirical literature, among others, by Galì [23], [27], [29], and by Francis et al. [22] [21].[20]. This empirical result has given rise to an important debate[21], since the standard RBC model is not able to replicate it, and has been used to cast serious doubts on the relevance of productivity shocks in explaining business fluctuations. In contrast, our model seems to suggest that technology shocks might still be a driving force of the business cycle if the economy is characterized by labor indivisibilities and sticky prices and the central bank follows a standard Taylor-type rule.

This result is robust to the inclusion of alternative formulations of the central bank reaction function: a positive correlation between productivity and unemployment is also obtained when using the simpler rule originally proposed by Taylor [51]. In figure (2) we consider the dynamics of our unionized economy under this rule and we show that when a negative productivity shock hits the economy, unemployment decreases even if the nominal interest

[20] See Galì and Rabanal [26] for a survey.

[21] For alternative estimations of the effect of productivity shocks on the labor input see Christiano et al. [12].

rate increases more than proportionally than the inflation rate.[22] We use this formulation of our model to provide a direct comparison with the Blanchard Galì [7] model, characterized by search frictions and real wage rigidity. In that model, when monetary policy follows the simple rule proposed by Taylor [51], which is in turn used to approximate the optimal interest rate rule, the correlation between productivity and unemployment is negative. Differently from ours, the Blanchard Galì [7] model is not able to replicate the empirical findigs of Galì [23], [27], [29] and others.

In figure (3) we describe the response of the main variables of the unionized model under the optimal discretionary rule (64). In this case a one standard deviation negative productivity shock requires a 0.3% initial increase in the nominal interest rate of 0.3% and an increase in inflation of almost 0.13%. Initially output gap falls by 1% and the rate of unemployment has an initial increase of about 2 percentage points, while output decrease by 2%. An optimal monetary policy, therefore, will take into account the trade-off that exists between inflation stabilization and output stabilization and will require some degree of accomodation: as a response to a productivity shock output will decrease and inflation will increase.

In order to evaluate to what extent Taylor rules found in the empirical literature on monetary policy compare relatively to the optimal monetary policy analyzed in this paper, we report, in figure (4), the results of an exercise aimed at replicating the optimal policy through a simple Taylor rule. We found that a rule that approximates quite well the optimal monetary policy (i.e. that achieves a response of the major variables quite close to the one achieved by our economy under the optimal discretional monetary policy) is given by

$$i_t = 2.5\pi_t + 0.05x_t \tag{68}$$

Notice that this rule implies a stronger response to inflation and a weaker response to the output gap than the ones found in the literature. It is also worth noticing that, differently from what happens under the Taylor rules considered above, under the optimal policy rule negative productivity shocks have a negative effect on unemployment. This result, together with the one described in figures (1) and (2), suggests that the negative correlation between productivity and employment found in the data by Galì and others might be the result of a monetary policy too accomodating with respect to inflation, rather than consequence of some "structural" characteristic of the

[22] As we can see, in this case the inflation rate goes up by almost 0.2%, while the nominal interest rate increases more than 0.3%. Output falls by almost 1% and the output gap increases by almost 0.04%, while unemployment falls by almost 0.08%.

economy.

In figures (5), (6) and (7) we show the responses of the interest rate, output and unemployment to a one standard deviation shock to the reservation wage under the rule estimated by Taylor [51], under the optimal rule and under the rule which mimic the optimal one. The responses are similar to the ones found for the productivity shocks, although, in the case of wage push shocks, the correlation between output and unemployment is always negative.

6 Conclusions

We have considered in this paper a NK model where labor is indivisible and where wages are set by monopoly unions. We found that, with respect to the standard NK framework, our model gives a more satisfactory description of the reality of modern industrialized economies, especially of those where collective bargaining dominates the labor market. In a unionized economy, significant trade-offs between stabilizing inflation and stabilizing unemployment arise in response to technology and exogenous wage shocks. Because of real wage rigidity, an optimizing central bank must respond to negative (positive) technology shocks by increasing (decreasing) the interest rate and, similarly, must respond to exogenous increases in unions' reservation wage with an interest rate increase. Interestingly, if we consider an optimal instrument rule, an optimizing central bank not only will increase the interest rate more than proportionately in response to an increase in future expected inflation, but will also react to increases in the natural rate that are not necessarily one to one. The model is also capable of accounting for the greater volatility of unemployment relative to the wage volatility that is usually found in the data. Moreover, once calibrated on Euro-area parameters, with the addition of an exogenous interest rate rule, our model is consistent with the positive correlation between technology shocks and the labor input found in the data. This correlation, however, becomes negative once the optimal discretionary rule is included in the model, suggesting that this correlation might be explained by the nature of monetary policy.

Even though, for the sake of simplicity, many other market imperfections, like search and matching frictions and firing costs are absent from our model, and therefore only some of the characteristics of European labor markets are taken into account, the model provides a coherent framework for the analysis of monetary policy in countries where unions play an important role. Obviously, there are many possible extensions to this model that could provide a deeper understanding of the relationship between monetary policy and different institutional settings in the labor market; we leave however these challenges to future research.

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A Technical Appendix

A.1 The Ramsey Problem

We consider a social planner which maximizes the representative household utility subject to the economy resource constraint and production function as follows:

$$\max_{N_t} U\left(C_t, N_t\right) = \frac{1}{1-\sigma} C_t^{1-\sigma} \phi\left(N_t\right)^{1-\sigma} \tag{A1}$$

$$s.t.$$

$$C_t = Y_t \tag{A2}$$

$$Y_t = A_t N_t^{\alpha} \tag{A3}$$

Substituting the constraint into the utility function the problem is:

$$\max_{N} \frac{1}{1-\sigma} \left(A_t N_t^{\alpha}\right)^{1-\sigma} \phi\left(N_t\right)^{1-\sigma} \tag{A4}$$

the first order condition requires

$$\left(A_t N_t^{\alpha}\right)^{-\sigma} \alpha \frac{Y_t}{N_t} \phi\left(N_t\right)^{1-\sigma} = -\left(A_t N_t^{\alpha}\right)^{1-\sigma} \phi\left(N_t\right)^{-\sigma} \phi_N\left(N_t\right) \tag{A5}$$

simplifying

$$Y_t \frac{\phi_N\left(N_t\right)}{\phi\left(N_t\right)} = -\alpha \frac{Y_t}{N_t} \tag{A6}$$

Multiplying both sides of equation for $\frac{N_t}{Y_t}$ we find

$$\frac{\phi_N\left(N_t\right)}{\phi\left(N_t\right)} N_t = -\alpha \tag{A7}$$

and

$$\frac{U_N}{U_C} \frac{N_t}{Y_t} = \frac{\phi_N\left(N_t\right)}{\phi\left(N_t\right)} N_t = -\alpha \tag{A8}$$

A.2 Derivation of the Efficient Output

We consider the Ramsey solution (A17)

$$\phi_N\left(N_t\right) N_t = -\alpha \phi\left(N_t\right) \tag{A9}$$

in order to find an equation for the efficient output we first log-linearizing equation (A9) around the steady state, which implies

$$[\phi_N(N) + \phi_{NN}(N) N n_t] N(1 + n_t) = -\alpha(\phi(N) + \phi_N(N) N n_t) \quad \text{(A10)}$$

which can be rewritten as

$$\phi_N(N) N + \phi_N(N) N n_t + \phi_{NN}(N) N^2 n_t = -\alpha(\phi(N) + \phi_N(N) N n_t) \quad \text{(A11)}$$

Considering the steady state equation

$$\phi_N(N) N_t = -\alpha\phi(N) \quad \text{(A12)}$$

and collecting terms in n_t we obtain

$$\left(1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\phi(N)} \left(\frac{\phi_N(N) N_t}{\phi(N)}\right)^{-1}\right) n_t = 0 \quad \text{(A13)}$$

given that $1 + \frac{\phi_N(N)N_t}{\phi(N)} + \frac{\phi_{NN}(N)N^2}{\alpha\phi(N)}\left(\frac{\phi_N(N)N_t}{\phi(N)}\right)^{-1} \neq 0$ we require,

$$n_t = 0 \quad \text{(A14)}$$

and then from the aggregate production function we obtain equation (27) in the text.

A.3 Derivation of the Flexible Price Equilibrium Output in the IL Model

Let us rewrite equation (28) as:

$$\phi_N(N_t) N_t = -\frac{\alpha}{(1-\tau)} MC_t \phi(N_t) \quad \text{(A15)}$$

at the steady state becomes,

$$\phi_N(N) N = -\frac{1}{(1-\tau)} \alpha MC \phi(N) \quad \text{(A16)}$$

Then log-linearizing,

$$[\phi_N(N) + \phi_{NN}(N) N n_t] N(1 + n_t) = -\frac{\alpha MC}{(1-\tau)}(1 + mc_t)[\phi(N) + \phi_N(N) N n_t] \quad \text{(A17)}$$

considering the steady state equation (A16) we have,

$$mc_t = \left(1 + \frac{\phi_N(N) N}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\phi(N)\alpha}\right) n_t \quad \text{(A18)}$$

given equation (A14) and considering the aggregate production function we obtain equation (31) in the text.

A.4 Optimal Subsidy in the MU Model

In this economy, when firms can revise their price at each time, beside the distortion created by monopolistic competition and firms' markup we have a distortion created by the monopoly union wage setting. We assume that, at the steady state, the government uses the employment subsidy to the firms τ, to bring steady state output to its efficient level, i.e. to the level at which

$$\frac{\phi_N(N)}{\phi(N)}N = -\alpha.$$

Since in the unionized economy labor market equilibrium is given by:

$$\frac{1}{\alpha}\frac{W^r}{P} = \frac{1}{(1-\tau)}\alpha\frac{1}{1+\mu^P}\frac{Y}{N},$$

if the government sets a subsidy such that

$$-\frac{W^r}{P}(1-\tau)(1+\mu^P)\frac{N}{Y} = \frac{\phi_N(N)}{\phi(N)}N\alpha$$

which implies

$$\tau^* = \left[1 + \frac{1}{1+\mu^P}\alpha\frac{\phi_N(N)}{\phi(N)}N\frac{Y}{N}\frac{P}{W^r}\right].$$

A.5 Derivation of the Phillips Curve

Following Calvo [8] we assume that each firm may reset its price with probability $1-\varphi$ each period, independently from the time elapsed since the last adjustment. This means that each period a measure $1-\varphi$ of firms reset their price, while a fraction φ of them keep their price unchanged. The law of motion of the aggregate price is given by:

$$\ln P_t = \varphi \ln P_{t-1} + (1-\varphi)\ln P_t^* \tag{A19}$$

which implies

$$\pi_t = (1-\varphi)\ln\left(\frac{p_t^*}{P_{t-1}}\right) \tag{A20}$$

where $\ln P_t^*$ denotes the (log) price set by a firm i adjusting its price in period t. Under Calvo [8] price-setting structure $p_{t+k}(i) = p_t^*$ with probability φ^k for $k = 0, 1, 2, ...$, hence firms have to be forward-looking.

Given that the individual firm technology is characterized by decreasing return to scale, the optimal price setting rule should take into account that marginal cost is no longer common across firms. In particular, in the neighborhood of the zero inflation steady state, we have the following price-setting rule:

$$\ln P_t^*(i) = \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{mc_{t,t+k}^n\right\} \tag{A21}$$

where $mc_{t,t+k}^n$ is the log-linearized nominal marginal cost in period $t+k$ of a firm which last set its price in period t. Considering the equation of real marginal cost and the one of the aggregate production function,

$$\begin{aligned} MC_{t,t+k} &= (1-\tau)\frac{(W_{t+k}/P_{t+k})}{\alpha\,(Y_{t,t+k}/N_{t,t+k})} \\ &= MC_{t+k}\frac{(Y_{t+k}/N_{t+k})}{(Y_{t,t+k}/N_{t,t+k})} \\ &= MC_{t+k}\left(\frac{Y_{t+k}}{Y_{t,t+k}}\right)^{\frac{1-\alpha}{\alpha}} \\ &= MC_{t+k}\left(\frac{P_t^*}{P_{t+k}}\right)^{\theta\frac{1-\alpha}{\alpha}} \end{aligned} \tag{A22}$$

taking the logs

$$\ln MC_{t,t+k} = \ln MC_{t+k} - \theta\frac{1-\alpha}{\alpha}\ln\left(\frac{P_t^*}{P_{t+k}}\right) \tag{A23}$$

Considering that all firms resetting prices in period t will choose the same price P_t^+ we can rewrite equation (A21) as,

$$\begin{aligned} \ln\left(\frac{P_t^*(i)}{P_{t-1}}\right) &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^n - \ln P_{t-1}\right\} \\ &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^n\right\} + \\ &\quad + \sum_{k=0}^{\infty}(\beta\varphi)^k\left\{\pi_{t+k}\right\} \end{aligned} \tag{A24}$$

substituting equation (A5) which can be rewritten as

$$\begin{aligned} \ln\left(\frac{P_t^*(i)}{P_{t-1}}\right) &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^n - \theta\frac{1-\alpha}{\alpha}\ln\left(\frac{P_t^*}{P_{t+k}}\right)\right\} \\ &\quad + \sum_{k=0}^{\infty}(\beta\varphi)^k\left\{\pi_{t+k}\right\} \end{aligned} \tag{A25}$$

then

$$\ln P_t^*(i) - \ln P_{t-1} = \mu^P + \beta\varphi E_t\left\{\ln P_{t+1}^* - \ln P_t\right\} + (1-\beta\varphi)\ln MC_t \quad \text{(A26)}$$

Combining (A26) with (A19) we obtain

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_\alpha mc_t \quad \text{(A27)}$$

as in the text.

A.6 The Welfare-Based Loss Function

A second-order Taylor expansion of the period utility around the efficient equilibrium yields,

$$\begin{aligned} U_t = \bar{U}_t + \bar{U}_{\bar{C},t}\bar{C}_t\tilde{C}_t + \frac{1}{2}\bar{U}_{\bar{C}\bar{C},t}\bar{C}_t^2\tilde{C}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 + \\ + \bar{U}_{\bar{C}\bar{N},t}\bar{C}_t\bar{N}_t\tilde{C}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right) \end{aligned} \quad \text{(A28)}$$

where the generic $\tilde{X} = \ln\left(X/\bar{X}_t\right)$ denotes log-deviations from the efficient equilibrium and $\bar{X}_t$ denotes the value of the variable under efficient equilibrium. Moreover, we denote as $\bar{x}_t = \ln\left(\frac{\bar{X}_t}{X}\right)$.

Considering the flexible prices economy resource constraint,

$$\begin{aligned} U_t \quad = \quad & \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\tilde{Y}_t + \frac{1}{2}\bar{U}_{\bar{Y}\bar{Y},t}\bar{Y}_t^2\tilde{Y}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 + \\ & + \bar{U}_{\bar{Y}\bar{N},t}\bar{Y}_t\bar{N}_t\tilde{Y}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right) \end{aligned} \quad \text{(A29)}$$

Collecting terms yields

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \begin{bmatrix} \tilde{Y}_t + \frac{\bar{U}_{\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t + \frac{1}{2}\frac{\bar{U}_{\bar{Y}\bar{Y},t}}{\bar{U}_{\bar{Y},t}}\bar{Y}_t\tilde{Y}_t^2 + \\ + \frac{1}{2}\frac{\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t^2 + \frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}}\tilde{Y}_t\tilde{N}_t \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A30)}$$

Considering that, $\frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}} = \frac{\phi_{\bar{N},t}\left(\bar{N}_t\right)\bar{N}_t}{\phi\left(\bar{N}_t\right)} = -(1-\sigma)\,\alpha$, we have,

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \begin{bmatrix} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ + \frac{1}{2}\left[\frac{\phi_{NN}\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)} - \sigma\left(\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A31)}$$

It can be shown that $\frac{\phi_{NN,t}\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)} = \frac{2\sigma-1}{\sigma}\left(\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\right)^2$, hence

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left[\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\left(\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A32}$$

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\,\alpha\tilde{Y}_t\tilde{N}_t + \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2 \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A33}$$

We now take a first-order expansion of the term $\bar{U}_{\bar{Y},t}\bar{Y}_t$ around the steady state.

$$\begin{aligned} \bar{U}_{\bar{Y},t}\bar{Y}_t &= U_Y\left(1 + (1-\sigma)\,\bar{y}_t + (1-\sigma)\frac{\phi_N(N)}{\phi(N)}N\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right) \\ &= U_Y\left(1 + (1-\sigma)\,\bar{y}_t - (1-\sigma)\,\alpha\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right) \end{aligned} \tag{A34}$$

$$\frac{\phi_N\left(\bar{N}_t\right)}{\phi\left(\bar{N}_t\right)}\bar{N}_t = \frac{\phi_N(N)}{\phi(N)}N + \Gamma_n\bar{n}_t + + \bigcirc\left(\|\alpha\|^2\right) \tag{A35}$$

where $\Gamma_n = \left(\frac{\phi_N(N)N}{\phi(N)} + \frac{\phi_{NN}(N)N^2}{\phi(N)} - \frac{\phi_N(N)^2N^2}{\phi(N)^2}\right)$

$$\left(\frac{\phi_N\left(\bar{N}_t\right)\bar{N}_t}{\phi\left(\bar{N}_t\right)}\right)^2 = \left(\frac{\phi_N(N)\,N^2}{\phi(N)}\right)^2 + \Lambda_n\bar{n}_t + \bigcirc\left(\|\alpha\|^2\right) \tag{A36}$$

where $\Lambda_n = 2\left(\frac{\phi_N(N)\phi_{NN}(N)N}{\phi(N)^2} + \left(\frac{\phi_N(N)N}{\phi(N)}\right)^2 - \left(\frac{\phi_N(N)}{\phi(N)}\right)^3 N\right)$
given that $\bar{n}_t = 0$, and that $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$, substituting (A35) and (A36) into the Welfare function,

$$U_t = \bar{U}_t + U_Y\left(1 + (1-\sigma)\,\bar{y}_t\right)\left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - \alpha(1-\sigma)\,\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2 \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A37}$$

Given the aggregate production function and that the log-deviations of the price dispersion index $-d_t = \tilde{Y}_t - \alpha\tilde{N}_t$ are of second-order, and that:

$$\tilde{Y}_t^2 = \alpha^2\tilde{N}_t^2 \qquad n_t\alpha\tilde{N}_t = n_t\tilde{Y}_t \qquad y_t\alpha\tilde{N}_t = y_t\tilde{Y}_t \qquad \tilde{Y}_t\alpha\tilde{N}_t = \tilde{Y}_t^2$$

considering only terms up to the second-order we have:

$$U_t = \bar{U}_t + U_Y\left[\begin{array}{c} \tilde{Y}_t - \tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\,\tilde{Y}_t^2 \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\tilde{Y}_t^2 \end{array} \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A38}$$

$$\begin{aligned}\tilde{U}_t &\equiv U_t - \bar{U}_t = -U_Y \left\{ d_t + \frac{1}{2}\left(\frac{2\sigma - 1}{\sigma} - 2\right)\tilde{Y}_t^2 \right\} + \bigcirc\left(\|\alpha\|^3\right) \\ &= U_t - \bar{U}_t = -U_Y \left\{ d_t - \frac{1}{2\sigma}\tilde{Y}_t^2 \right\} + \bigcirc\left(\|\alpha\|^3\right) \qquad \text{(A39)}\end{aligned}$$

As proven by Galì and Monacelli [28], the log-index of the relative-price distortion is of second-order and proportional to the variance of prices across firms, which implies that:

$$d_t = \ln\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right) di\right) = \frac{\theta}{2} var_i \left\{ p_t(i) + \bigcirc\left(\|\alpha\|^3\right)\right\} \qquad \text{(A40)}$$

proof Galì and Monacelli [28].
As shown in Woodford [56], this means that

$$\sum_{t=0}^{\infty} \beta^t var_i \{p_t(i)\} = \frac{1}{\lambda_a} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \qquad \text{(A41)}$$

where $\lambda = (1-\psi)(1-\psi\beta)/\psi$.
Finally, denoting the output gap $\tilde{Y}_t$ as in the standard way x_t, the Welfare-Based loss-function can be written as,

$$\begin{aligned}W_t &= E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_Y}{2} E_t \sum_{t=0}^{\infty} \left[\frac{\theta}{\lambda_a}\pi_{t+k}^2 + \frac{1}{\sigma}x_{t+k}\right] + \bigcirc\left(\|\alpha\|^3\right) \\ &= E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_Y}{2} E_t \sum_{t=0}^{\infty} \left[\frac{\theta}{\lambda_a}\pi_{t+k}^2 + \frac{1}{\sigma}x_{t+k}\right] + \bigcirc\left(\|\alpha\|^3\right) \qquad \text{(A42)}\end{aligned}$$

A.7 Stability and Determinacy in the Reduced Form Dynamic System

Our model can be expressed in the following reduced form:

$$x_t = E_t x_{t+1} - \left[r_t - E_t\pi_{t+1} - \hat{r}_t^n\right] \qquad \text{(A43)}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha} x_t + \frac{\lambda_a}{\sigma(1-\rho_a)}\hat{r}_t^n + \lambda_a \varepsilon_t^w \qquad \text{(A44)}$$

the model is completed adding the optimal instrument rule interest rate which, under discretion is given by:

$$\hat{r}_t^* = \Phi_\pi E_t \pi_{t+1} + \Phi_r \hat{r}_t^n \qquad \text{(A45)}$$

where $\Phi_\pi = 1 + \left(\frac{1-\rho_w}{\rho_w}\right)\frac{1-\alpha}{\alpha}\theta\sigma$ and $\Phi_r = 1 + \frac{(\rho_a - \rho_w)}{\sigma(1-\rho_a)}\frac{\lambda\theta}{\Omega - \beta\rho_a}$.

In order to verify if the optimal policy can guarantee the uniqueness of the equilibrium we combine the IS the AS and the optimal interest rate with the natural interest rate equation, so that we can write the following reduced dynamic system:

$$\begin{aligned} x_t &= E_t x_{t+1} + (1-\Phi_\pi) E_t \pi_{t+1} + (1-\Phi_r) r_t^n & \text{(A46)} \\ \pi_t &= \lambda_a \frac{1-\alpha}{\alpha} E_t x_{t+1} + \left[\lambda_a \frac{1-\alpha}{\alpha} (1-\Phi_\pi) + \beta \right] E_t \pi_{t+1} + \\ & + \left(1 + \frac{\lambda_a}{\sigma (1-\rho_a)} - \Phi_r \right) r_t^n + \lambda_a \varepsilon_t^w & \text{(A47)} \end{aligned}$$

which can be rewritten as

$$\varpi_t = A_1 E_t \varpi_{t+1} + A_2 u_t \tag{A48}$$

where $\varpi_t = E_t [x_t; \pi_t]^T$ and $A_2^T = \begin{bmatrix} 1-\Phi_r & 0 \\ 1 + \frac{\lambda}{\sigma(1-\rho_a)} - \Phi_r & \lambda_a \end{bmatrix}$, $u_t = [r_t^n, \varepsilon_t^w]^T$ while the transition matrix is given by:

$$A_1 = \begin{bmatrix} 1 & 1-\Phi_\pi \\ \lambda_a \frac{1-\alpha}{\alpha} & \lambda_a \frac{1-\alpha}{\alpha} (1-\Phi_\pi) + \beta \end{bmatrix} \tag{A49}$$

Given that ϖ_t is a 2-vector of non-predetermined endogenous variable, rational expectation equilibrium is determinate if and only if the matrix A_1 has both eigen values outside the unit circle, which occurs if and only if[23]:

$$\begin{aligned} \det A_1 &< 1, & \text{(A50)} \\ |-trA_1| &< 1 + \det A_{1.} & \text{(A51)} \end{aligned}$$

Notice that, in our case:

$$\det A_1 = \lambda_a \frac{1-\alpha}{\alpha} (1-\Phi_\pi) + \beta - \lambda_a \frac{1-\alpha}{\alpha} (1-\Phi_\pi) = \beta < 1 \tag{A52}$$

$$|-trA_1| = 1 + \lambda_a \frac{1-\alpha}{\alpha} (1-\Phi_\pi) + \beta < 1 + \beta \quad \text{given that} \quad \Phi_\pi > 1 \tag{A53}$$

which implies that the rational expectations equilibrium of our model under an optimal discretionary policy is determinate.

[23] See Proposition 1 in the Appendix of Woodford [56].

The optimal instrument rule commitment is given by:

$$\hat{r}_t^{c*} = \Phi_\pi^c E_t \pi_{t+1} + \Phi_r^c \hat{r}_t^n \tag{A54}$$

where $\Phi_\pi^c = 1 + \left(\frac{1-\rho}{\rho}\right) \frac{\alpha}{1-\alpha} \frac{\theta}{1-\beta\rho}$ and $\Phi_r^c = 1$.
Under commitment matrix A_1 becomes:

$$A_1 = \begin{bmatrix} 1 & 1 - \Phi_\pi^c \\ \lambda_a \frac{1-\alpha}{\alpha} & \lambda_a \frac{1-\alpha}{\alpha} (1 - \Phi_\pi^c) + \beta \end{bmatrix} \tag{A55}$$

Notice that

$$\det A_1 = \lambda_a \frac{1-\alpha}{\alpha} (1 - \Phi_\pi^c) + \beta - \lambda_a \frac{1-\alpha}{\alpha} (1 - \Phi_\pi^c) = \beta < 1 \tag{A56}$$

$$|-trA_1| = 1 + \lambda_a \frac{1-\alpha}{\alpha} (1 - \Phi_\pi^c) + \beta < 1 + \beta \quad \text{given that} \quad \Phi_\pi^c > 1 \tag{A57}$$

which implies that also under constrained commitment the optimal interest rule can guarantee the uniqueness of the equilibrium.

A.8 Constrained Commitment

Let us assume that the Central Bank follows a rule for the target variable x_t which depends on the fundamental shocks w_t^r and r_t^n. In order to obtain an analytical solution we assume the following feedback rule equation

$$x_t^c = \omega \left(a_t - \varepsilon_t^w\right) \qquad \forall t$$

and we also assume

$$\rho_a = \rho_w = \rho$$

where $\omega > 0$ is the coefficient of the feedback rule and the variable x_t^c is the value of x_t conditional on commitment to the policy.
Before solving the Central Bank problem under constrained commitment, we iterate forward the Phillips curve (54) and we obtain:

$$\pi_t^c = \left(1 - \frac{1-\alpha}{\alpha}\omega\right) \frac{\lambda_a}{1-\beta\rho} \left(\varepsilon_t^w - a_t\right)$$

which, can be rewritten as:

$$\pi_t^c = \lambda_a \frac{1-\alpha}{\alpha} \frac{1}{1-\beta\rho} x_t^c - \frac{\lambda_a}{1-\beta\rho} \left(a_t - \varepsilon_t^w\right)$$

Notice that, in this case, a one percent contraction of x_t^c reduces π_t^c by the amount $\lambda_a \frac{1-\alpha}{\alpha}\frac{1}{1-\beta\rho_w}$, while under discretion, reducing x_t by one percent only produces a fall in π_t of $\lambda_a \frac{1-\alpha}{\alpha} < \lambda_a \frac{1-\alpha}{\alpha}\frac{1}{1-\beta\rho_w}$. As in the case analyzed by Clarida, Galì and Gertler [11], the Central bank will enjoy an improved trade off, due to the fact that commitment to a policy rule affects expectations on the future course of the output gap.

We can now write the problem of the Central Bank under constrained commitment as follows:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2}\left[(\pi_t^c)^2 + \frac{\lambda_a}{\theta\sigma}(x_t^c)^2\right] E_t \sum_{i=0}^{\infty}\left(\frac{\varepsilon_{t+i}^w - a_{t+i}}{\varepsilon_t^w - a_t}\right)^2$$

subject to equation Phillips rule under commitment. The first order conditions imply the following optimal rule:

$$x_t^c = -\frac{1-\alpha}{\alpha}\theta\sigma\frac{1}{1-\beta\rho}\pi_t^c$$

Since $\frac{1-\alpha}{\alpha}\theta\sigma\frac{1}{1-\beta\rho} < \frac{1-\alpha}{\alpha}\theta\sigma$ this implies that commitment to a rule makes it optimal, for the central bank, to induce a greater contraction of output in response to an increase in inflation. Substituting the optimal rule into the Phillips curve and iterating forward we obtain:

$$\pi_t = -\frac{\lambda_a}{\Omega^c(1-\beta\rho)}(a_t - \varepsilon_t^w)$$

where $\Omega^c = 1 + \lambda\left(\frac{1-\alpha}{\alpha}\frac{1}{1-\beta\rho}\right)^2\theta\sigma > \Omega$. The interest rate rule can be obtained by substituting the last equation and the optimal rule, into the IS curve (51), in which case we obtain:

$$r_t^{c*} = -\left[\left[1 + \left(\frac{1-\rho}{\rho}\right)\left(\frac{1-\alpha}{\alpha}\right)\frac{\theta\sigma}{1-\beta\rho}\right]\left(\frac{\lambda_a\rho}{\Omega^c(1-\beta\rho)}\right) + \sigma(1-\rho)\right]a_t +$$
$$+\left[1 + \left(\frac{1-\rho}{\rho}\right)\left(\frac{1-\alpha}{\alpha}\right)\frac{\theta\sigma}{1-\beta\rho}\right]\left(\frac{\lambda_a\rho}{\Omega^c(1-\beta\rho)}\right)\varepsilon_t^w$$

Using equation the optimal rule, the one of the Phillips curve and the one of the IS-curve we find the following optimal instrument rule:

$$\hat{r}_t^{c*} = \left[1 + \left(\frac{1-\rho}{\rho}\right)\frac{1-\alpha}{\alpha}\frac{\theta\sigma}{1-\beta\rho}\right]E_t\pi_{t+1} + \hat{r}_t^e.$$

Since $\frac{1}{1-\beta\rho} > 1$, we have the following

Result 4. *Under commitment to a simple feedback rule, when $\rho_a = \rho_w = \rho$, an optimal interest rule requires that, in reacting to an increase in expected inflation, the nominal interest rate must be increased more than in the case of discretion .*

Tables and Figures

Table 1.

Parameters	value	Description
β	0.99	Discount factor
σ	2	Coefficient of relative risk adversion
θ	11	Elasticity of substitution across intermediate goods
α	0.72	Output elasticity to labor input
φ	0.75	Probability of goods price non-adjustment
ρ_w	0.7	Persistence of the reservation wage shock
σ_w	1	Standard deviation of the reservation wage shock
ρ_a	0.93	Persistence of the reservation productivity shock
σ_a	1	Standard deviation of the reservation productivity shock
N	0.61	Steady state employment

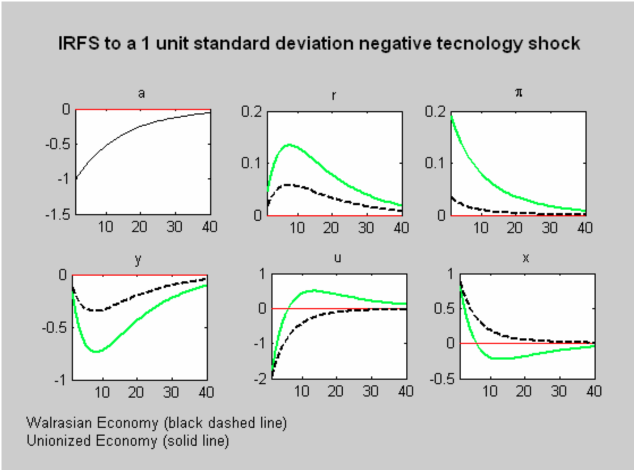


Figure 1:

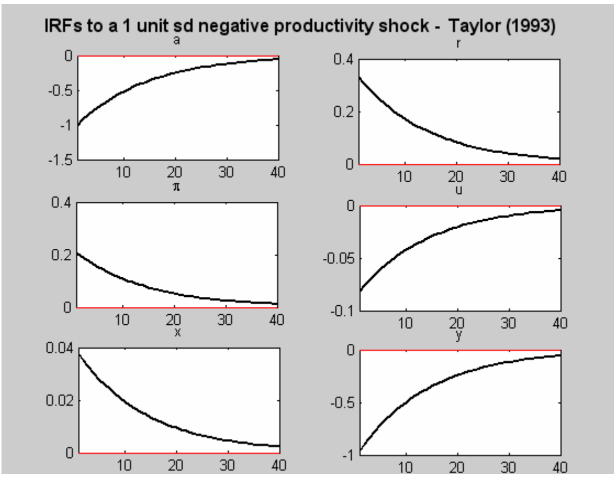


Figure 2:

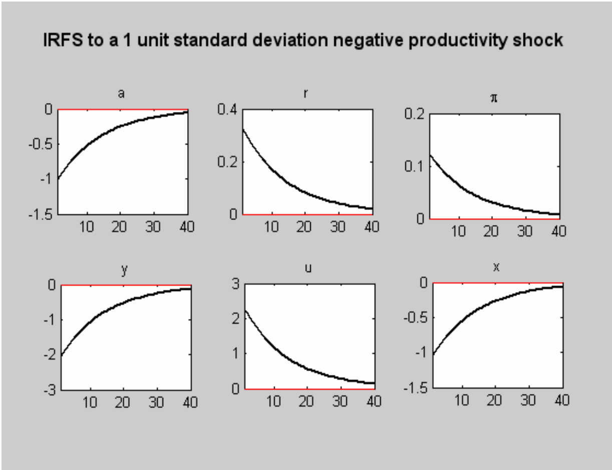


Figure 3:

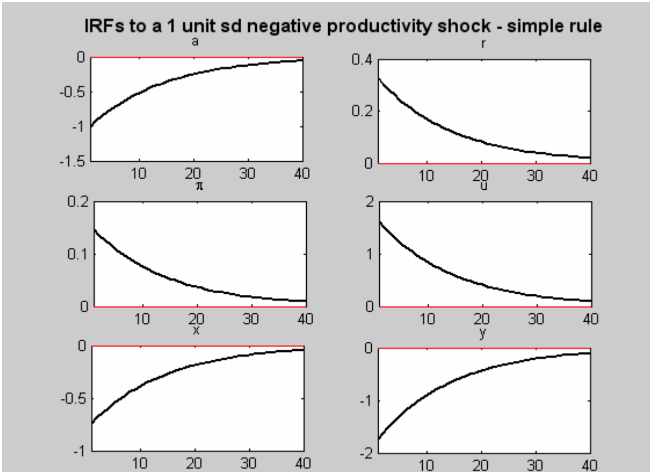
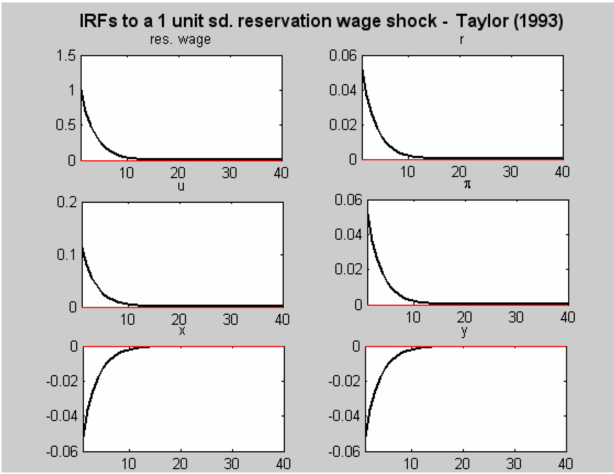


Figure 4:



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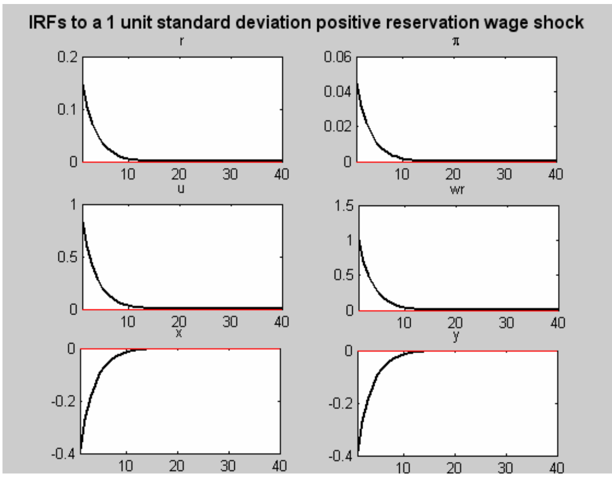

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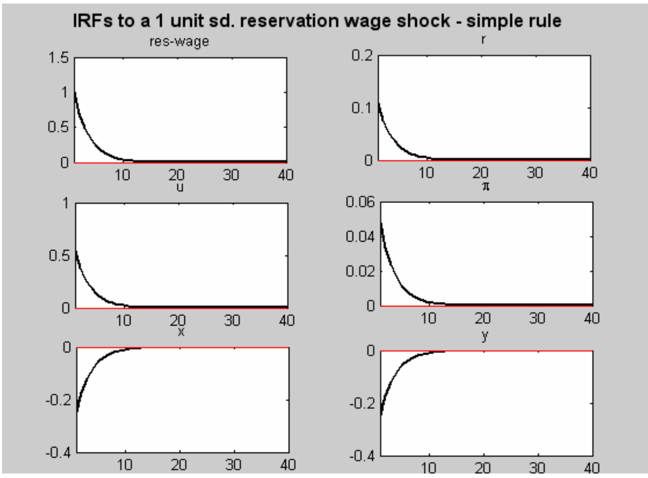
IRFs to a 1 unit sd. reservation wage shock - simple rule
res-wage
r
u
π
x
y
1.5
1
0.5
0
0.2
0.1
0.06
0.04
0.02
-0.2
-0.4
10
20
30
40

Figure 7: