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The Real Exchange Rate and Real Interest Differentials: The Role of the Trend-Cycle Decomposition*

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Abstract

We propose an alternative model and method to reconcile the puzzling feature in the relationship between the real exchange rate and real interest rate differentials. Our simple two-country model with pre-set prices, along with firms' misperception about the future exchange rate, implies that the real exchange rate follows an ARIMA(0,1,p) process. This allows us to compute the exact Beveridge-Nelson decomposition, which is a model-consistent decomposition. In accordance with our model, unit roots in the real exchange rates are found; and statistical inference is partially found to be affirmative regarding the link between the real exchange rate detrended by the Beveridge-Nelson decomposition and corresponding real interest differentials.

JEL Classification Number: F31, F41.

Keywords: Trend-Cycle Decomposition, Real Interest Parity, Sticky Price Model, Beveridge-Nelson Decomposition.

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1 Introduction

Finding a relationship between real exchange rates and real interest rate differentials is one of the important topics in the field of international macroeconomics. Despite the fact that the theory suggesting such a relationship requires fairly unrestrictive conditions, the vast majority of empirical studies have been unable to discover it.[1]

One of the few exceptions is Baxter (1994), who first considers the decomposition of the real exchange rates in the frequency domain.[2] She then shows: (1) As standard theory suggests that the transitory components of the real exchange rate are correlated with interest rates, if the real exchange rate has a random walk component; (2) The transitory components extracted in several different ways, indeed, produce much stronger empirical connections with real interest differentials than does the real exchange rate itself. In addition to the permanent components in the real exchange rate, subsequent studies, including Nakagawa (2002) and Mark and Moh (2005), develop more sophisticated empirical models that allow for non-linearity in the time series process of the real exchange rate.

In this paper, however, to move forward with Baxter's (1994) emphasis on the importance of the trend-cycle decomposition, we consider the following questions: What is an exchange rate model that: (i) consists of both permanent and transitory components; (ii) has a relationship only between the transitory component of the real exchange rate and real interest differentials; and (iii) allows us to decompose the real exchange rate into those two components in certain way(s)? Having such model(s) and real exchange rates detrended in a model-consistent manner, we shall be able to provide a more decisive answer as to whether or not there is a statistically significant link between real exchange rates and real interest differentials.

Using the pricing-to-market with a preset pricing model, we show that the real exchange rate may have a unit root. If a unit root actually exists, then the Beveridge-Nelson (1982, hereafter BN) decomposition is a model-consistent decomposition; and the BN transitory (cycle) components

[1] In the earlier stage of this study, neither Campbell and Clarida (1987) nor Meese and Rogoff (1988) found a statistically significant link. See Edison and Merick (1999) for different empirical approaches to this problem.

[2] The business cycle frequency components of the data are extracted by the Baxter and King (1999) Band-pass filter.

and the expected future sum of the real interest differentials should be linked. Furthermore, our empirical results confirm most of our models predictions: unit roots in the real exchange rates and stationary real interest differentials. The relationship between the real exchange rate and the real interest differentials is, however, only partially supported.

The paper is structured as follows. Section 2 reviews the main question together with the appropriate decomposition method. Section 3 presents our model that is based on Devereux and Engel (2002) with misperception of price-setting firms; showing that we may describe the log of the real exchange rate as an ARIMA(0,1,p) process. Our main results, the BN decomposition and its statistical inference are given in Section 4. Section 5 concludes.

2 Detrending Problem

Theoretically, the relationship between the real interest rate and real interest rate differential is given by the so-called real interest parity:

$$E_t \left[q_{t+1} \right] - q_t = r_t - r_t^* \tag{1}$$

where q_t is the log of (hereafter variables denoted lower case are logged variables) the real exchange rate; r_t and r_t^* are real interest rates in Home and Foreign countries respectively; and E_t denotes the expectation conditional on information available at time t. A great majority of studies, however, have failed to find this relationship (1). It is important to note that an iterative substitution of equation (1) becomes

$$q_t = \lim_{k \to \infty} E_t q_{t+k} - \sum_{j=0}^{\infty} E_t \left(r_{t+j} - r_{t+j}^* \right). \tag{2}$$

Most of the previous studies assume $\lim_{k \to \infty} E_t q_{t+k} = 0$, meaning that there is no unit root in q_t. Had there been a unit root component in q_t, the first term on the right hand side of (2) would be the trend component of the BN decomposition:[3]

[3] Baxter (1994) also points this out. Another example is Asea and Reinhart (1996), who examine the link in African coutries' data by utilizing the BN decomposition. They, however, compute the BN decomposition without modeling the real exchange rate and real interest differentials, as opposed to our paper whose model is fully described below. In effect, Asea and Reinhart (1996) estimate more general ARIMA(q,1,p) models to find the BN cycle of the real exchange rate; whereas our model predicts (as we shall see later) ARIMA(0,1,p) models are the model-consistent

$$q_t = q_t^{trend} + q_t^{cycle}, \tag{3}$$

where q_t^{trend} and q_t^{cycle} are respectively the BN trend and cycle components. Hence, when a random walk component exists in the real exchange rate, an appropriate way to search for the real exchange rate-real interest differentials relationship would be: (i) to compute the *exact* BN decomposition,[4] along with computing the expected sum of future real interest differentials, which is the second term on the right hand side of equation (2); and (ii) to test the link therein.

3 The Model

Our model is based on that of Devereux and Engel (2002), the pricing-to-market with a preset pricing model. There are two countries, say, "Home" and "Foreign" on a unit interval. Home lies between 0 and n, while Foreign lies between n and 1. There are: (i) households, (ii) firms, (iii) governments in both countries; while (iv) foreign exchange dealers exist only in Home.

3.1 Households

Households in each country maximize their (identical) preference:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\rho} C_t^{1-\rho} + \chi \ln \frac{M_t}{P_t} - \frac{\kappa}{2} L_t^2 \right] \tag{4}$$

where β is the intertemporal discount factor; χ and κ are constants; M_t is nominal money balance; P_t is the level of prices in terms of Home currency; L_t is labor supply; and C_t is a consumption index defined as

$$C_t = \left[n^{1/\omega} C_{ht}^{(\omega-1)/\omega} + (1-n)^{1/\omega} C_{ft}^{(\omega-1)/\omega} \right]^{\omega/(\omega-1)}, \tag{5}$$

time series process for the real exchange rate.

[4] The BN decomposition is computed exactly when the time-series process of q_t, such as an ARIMA (p,1,q), is known (see Morley 2002). Otherwise, approximate BN decomposition can be computed in several different ways (See Newbold 1990).

$$C_{ht} = \left[n^{-1/\lambda} \int_0^n C_{ht}(i)^{(\lambda-1)/\lambda} \, di \right]^{\lambda/(\lambda-1)} \tag{6}$$

$$C_{ft} = \left[(1-n)^{-1/\lambda} \int_n^1 C_{ft}(i)^{(\lambda-1)/\lambda} \, di \right]^{\lambda/(\lambda-1)}, \tag{7}$$

where $\lambda > 1$. Variables C_{ht} and C_{ft} represent consumption of goods produced in Home and Foreign, respectively. Let the price index be

$$P_t = \left[n P_{ht}^{1-\omega} + (1-n) P_{ft}^{1-\omega} \right]^{1/(1-\omega)}$$

where the Home currency price of the Home goods is

$$P_{ht} = \left[\frac{1}{n} \int_0^n P_{ht}(i)^{1-\lambda} \, di \right]^{1/(1-\lambda)},$$

and the Home currency price of the Foreign goods is

$$P_{ft} = \left[\frac{1}{1-n} \int_n^1 P_{ft}(i)^{1-\lambda} \, di \right]^{1/(1-\lambda)}.$$

Then, the demand functions are given by

$$\begin{aligned} C_{ht}(i) &= \frac{1}{n} \left[\frac{P_{ht}(i)}{P_{ht}} \right]^{-\lambda} C_{ht}, \;\; C_{ft}(j) = \frac{1}{1-n} \left[\frac{P_{ft}(j)}{P_{ft}} \right]^{-\lambda} C_{ft} \\ C_h &= n \left[\frac{P_{ht}}{P_t} \right]^{-\omega} C_t, \;\; C_f = (1-n) \left[\frac{P_{ft}}{P_t} \right]^{-\omega} C_t. \end{aligned}$$

Households in Home have the budget constraint:

$$P_t C_t + \delta_t B_{t+1} + M_t = W_t L_t + \Pi_t + \Pi_t^f + M_{t-1} + T_t + B_t \tag{8}$$

where B_t is a home currency denominated riskless bond that pays one unit of Home currency in time t; δ_t is the price of bond B_{t+1}; W_t is wage; Π_t are profits from firms in Home; T_t is transfer from government; and Π_t^f represents the payments from foreign exchange dealers, which we shall explain in the following section. Having only riskless bonds in this economy, the structure of asset markets is incomplete. Indeed, such a structure opens up the possibility of the non-stationary real exchange rate.[5] The first order conditions of the Home household include

$$\delta_t = \beta E_t \left[\frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}} \right] \tag{9}$$

and

$$1 = \chi \frac{P_t C_t^{\rho}}{M_t} + \beta E_t \left[\frac{P_t C_t^{\rho}}{P_{t+1} C_{t+1}^{\rho}} \right]. \tag{10}$$

Denoting variables with asterisk as the foreign variables, Foreign households maximize their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\rho} C_t^{*1-\rho} + \chi \ln \frac{M_t^*}{P_t^*} - \frac{\kappa}{2} L_t^{*2} \right] \tag{11}$$

where C_t^* is the Foreign composite consumption index and P_t^* is the Foreign price level in terms of the Foreign currency.

Similar to Home households, they have the budget constraint:

$$P_t^* C_t^* + \delta_t^* B_{ft+1}^* + M_t^* = W_t^* L_t^* + \Pi_t^* + M_{t-1}^* + T_t^* + B_{ft}^*, \tag{12}$$

where B_{ft}^* is a foreign currency denominated riskless bond. Note that there are no foreign exchange dealers in Foreign; thus we assume that their profits do not appear in the Foreign households' budget constraint. The first order conditions for Foreign households include

$$\delta_t^* = \beta E_t \left[\frac{P_t^* C_t^{*\rho}}{P_{t+1}^* C_{t+1}^{*\rho}} \right]. \tag{13}$$

[5] If markets were complete, the real exchange rate would be

$$\frac{S_t P_t^*}{P_t} = \left[\frac{P_0^* C_0^{*\rho} S_0}{P_0 C_0^{\rho}} \right] \frac{C_t^{\rho}}{C_t^{*\rho}}.$$

3.2 The Governments

The governments of both Home and Foreign issue monies, and pay or receive transfers. The government budget constraints, both Home and Foreign are, therefore, given by

$$M_t = M_{t-1} + T_t, \quad M_t^* = M_{t-1}^* + T_t^*.$$

The money supplies follow

$$\frac{M_{t+1}}{M_t} = \varepsilon_{t+1}, \quad \frac{M_{t+1}^*}{M_t^*} = \varepsilon_{t+1}^* \tag{14}$$

where ε_{t+1} and ε_{t+1}^* are log-normally distributed random variables with the same mean and variance: We assume

$$\log \varepsilon_{t+1} \sim i.i.d.Normal\left(0, \sigma_\varepsilon^2\right), \quad \log \varepsilon_{t+1}^* \sim i.i.d.Normal\left(0, \sigma_\varepsilon^{*2}\right),$$

and $\sigma_\varepsilon = \sigma_\varepsilon^*$.

3.3 Foreign Exchange Dealers

By assumption, Home households are not allowed to buy and sell Foreign currency denominated bonds directly. Instead, foreign exchange dealers do this on behalf of Home households; and they make payments to (receipts from, if foreign exchange dealers suffer losses) Home households.[6] At time t, foreign exchange dealers buy foreign assets by spending $\delta_t^* S_t B_{ht+1}^*$ worth of Home currency; and in the following period, the bonds will pay $S_{t+1} B_{ht+1}^*$, where S_t is the nominal exchange rate (the Home price of Foreign currency); and B_{ht+1}^* is a Foreign currency denominated bond held by Home residents. Thus, together with the discount factor $Z_t = \beta P_t C_t^\rho / \left(P_{t+1} C_{t+1}^\rho\right)$, the maximization problem for the foreign exchange dealer is

$$\max_{B_{ht+1}^*} E_t \left[Z_t S_{t+1} B_{ht+1}^* - \delta_t^* S_t B_{ht+1}^* \right],$$

[6] Unlike Devereux and Engel (2002), the assumption that foreign exchange dealers are only in Home country is not a crucial assumption for our model.

and its first order condition is

$$E_t \left[Z_t \frac{S_{t+1}}{S_t} \right] = \delta_t^* . \tag{15}$$

Given the assumption of the money supply (14), equations (13) and (10), and whose Foreign counterpart (15) implies

$$E_t \left[\frac{1}{\varepsilon_{t+1}^*} \right] = E_t \left[\frac{S_{t+1}}{\varepsilon_{t+1} S_t} \right] . \tag{16}$$

By the log-linearlization around the non-stochastic steady state, we have[7]

$$E_t \left[s_{t+1} \right] = s_t . \tag{17}$$

3.4 Firms: Price Setting Rule and Misperception

A monopolistically competitive firm i, which locates Home (between 0 and n) produces one unit of good i using one unit of labor input. Similarly, firm j locating Foreign (between n and 1) produces good j by the same technology. In our model, regardless of the location of a firm, it can sell its products both in Home and Foreign. However, we assume that all firms have to set their prices in terms of the currency that is used in the local market. For example, a Home firm selling its product in the Foreign market sets its price in Foreign currency; and a Foreign firm selling its product in the Home market has to set its price in terms of Home currency. Assuming that prices have to be set one period in advance, firm i sets its price as:

$$P_{ht}(i) = \frac{\lambda}{\lambda - 1} \frac{E_{t-1} \left[W_t C_{ht} \right]}{E_{t-1} \left[C_{ht} \right]}, \quad P_{ht}^*(i) = \frac{\lambda}{\lambda - 1} \frac{E_{t-1} \left[W_t C_{ht}^* \right]}{E_{t-1} \left[S_t C_{ht}^* \right]}, \tag{18}$$

and Foreign firm j sets its price as

$$P_{ft}(j) = \frac{\lambda}{\lambda - 1} \frac{E_{t-1} \left[W_t^* S_t C_{ft} \right]}{E_{t-1} \left[C_{ft} \right]}, \; P_{ft}^*(j) = \frac{\lambda}{\lambda - 1} \frac{E_{t-1} \left[W_t^* C_{ft}^* \right]}{E_{t-1} \left[C_{ft}^* \right]}. \tag{19}$$

In addition to the benchmark model of Devereux and Engel (2002), we allow for firms' "mis-

[7] $s_t = \ln S_t$: Lower case letters are log deviations from the steady state.

perception." That is, firms' *own expectation* about the next period's exchange rate may not be the *conditional expectation* of the exchange rate. Suppose that firms are not well-informed about foreign exchange markets, hence they may assume the existence of "noise traders."[8] Not only is the distribution of disturbances that can be caused by such traders unknown to firms, but also firms at time-t-1 do not know if such noise traders will exist at time-t. In this case, the best firms can do when they set their prices is to expect the next period's exchange rate by using the (realized) current and past exchange rates.[9]

To simplify this argument, consider the case in which firms expect the time-t exchange rate given the information available at $t-1$ to be S_{t-1}^{a}, where a is a constant. Due to the fact that the exchange rate follows a random walk process in our model, $a = 1$ corresponds to the case in which firms set their prices based upon the conditional expectation. Consider an extreme case, where $a = 0$. This implies that firms expect the exchange rate to always be at the level of a non-stochastic steady state ($S = 1$). Since it is reasonable to generalize this assumption so that the firms' expectation about the time t exchange rate relies on exchange rates of current and past $p-1$ periods (where p is a fixed number), we denote the logarithm of the firms' expectation regarding the exchange rate at time t as

$$s_t^e = a(L)\, s_{t-1} \tag{20}$$

where

$$a(L) = a_1 + a_2 L + \cdots + a_p L^{p-1}$$

and L is the lag operator (i.e., $LX_t = X_{t-1}$ for any variable X). Remarkably, it is possible to show that the log of the nominal exchange rate s_t, still follows the random walk process:

$$s_t = s_{t-1} + \eta_t \tag{21}$$

where η_t is innovation.[10]

[8] Devereux and Engel (2002) also assume noise traders, but their assumption is that foreign exchange dealers take into account the effect of noise traders when foreign exchange dealers maximize their objective function.

[9] We assume that the degrees of misperception by Home and Foreign firms are the same.

[10] This innovation is a linear combination of monetary shocks in Home and Foreign (the natural logs of ε_t and ε_t^*).

3.5 The Real Exchange Rate and Real Interest Rates

3.5.1 The Real Exchange Rate

The real exchange rate is defined as

$$Q_t = \frac{S_t P_t^*}{P_t},$$

and its logarithm is

$$q_t = s_t + p_t^* - p_t. \tag{22}$$

Allowing for the firms' misperception, as an example, when firms set their prices according to their expectations (20), it is possible to show that the log of the real exchange rate is an ARIMA (0,1,p) process:

$$\Delta q_t = \eta_t - a_1\eta_{t-1} - a_2\eta_{t-2} - \cdots - a_p\eta_{t-p}. \tag{23}$$

In contrast to Devereux and Engel's (2002) stationary real exchange rate, ours is non-stationary unless $a(1) = 1$ (without misperception in price setting firms).[11] This is due to the fact that the firms' misperception prevents prices (Home and Foreign) and the nominal exchange rate from cointegrating. Though both our model and that of Devereux and Engel (2002) have a non-stationary nominal exchange rate, prices in the latter are also non-stationary due to the rational expectations of firms. Therefore, the real exchange rate (the linear combination of those three variables) exhibits stationarity.

3.5.2 The Real Interest Differential

The (gross) real interest rates in Home and Foreign are defined as

$$R_t = \frac{P_t}{\delta_t P_{t+1}}, \quad R_t^* = \frac{P_t^*}{\delta_t^* P_{t+1}^*},$$

By assumption (in equation 14), two shocks are normally distributed: mutually and serially uncorrelated. Therefore, we have

$$\eta_t \sim i.i.d.Normal\left(0, \sigma_\eta^2\right)$$

[11] Note that the *ex-ante* purchasing power parity (PPP) holds in this case.

respectively. Since nominal interest rates in both countries are equal ($\delta_t = \delta_t^*$), the ratio of the real interest rates is given by

$$\frac{R_t}{R_t^*} = \frac{P_{t+1}^* P_t}{P_{t+1} P_t^*}. \tag{24}$$

One can show that the log-linearization of equation (24) (i.e. the real interest differential) under the firms' misperception (20) is

$$r_t - r_t^* = -a_1\eta_t - a_2\eta_{t-1} - \cdots - a_p\eta_{t-p+1} \tag{25}$$

where r_t and r_t^* are the logarithms of R_t and R_t^*, respectively.[12]

3.5.3 Real Interest Parity

The expected growth rate of the real exchange rate in our preset-pricing model is then

$$\begin{aligned} E_t\left[\frac{Q_{t+1}}{Q_t}\right] &= E_t\left[\frac{S_{t+1}P_{t+1}^* P_t}{S_t P_{t+1} P_t^*}\right] \\ &= E_t\left[\frac{S_{t+1}}{S_t}\right]\frac{R_t}{R_t^*}, \end{aligned} \tag{26}$$

and its log linearization yields real interest parity, stated in equation (1). It is important to note that even under the firms' misperception (20), equations (23) and (25) imply real interest parity to hold.

4 Empirical Results

Our plan to discover and test the relationship between real exchange rates and real interest differentials is the following. First, we confirm the existence of unit roots in real exchange rates and the non-existence of unit roots in real interest differentials. Then, the BN cycles of real exchange rates and the corresponding expected sum of future real interest differentials are computed in a way that is consistent with our model. The link between these two variables is then formally tested.[13]

[12] Due to the pre-set prices, there is no distinction between *ex-ante* and *ex-post* real interest rates in our model.

[13] From equations (23) and (25), one may think that we can test the link between those two variables by ordinary least squares (OLS). However, as the appendix explains, OLS is an inappropriate method to measure the link.

The quarterly data from the first quarter of 1973 through the third quarter of 2009 are utilized for Canada (CA), Japan (JA), Switzerland (SW), the United Kingdom (UK), and the United States (US). Our data for exchange rates are from the Federal Reserve Board (for the US and Canadian dollars) and *International Financial Statistics* (the rest of the exchange rates); data for consumer price indices (CPI) are from International Financial Statistics; and data for short-term and long-term interest rates are taken from the Federal Reserve Board (Secondary market rates of 3-month treasury bills are used for the short-term rate; and the 10-year treasury note yield is used for the long-term rate), the Bank of Canada (short-term interest rates) and the OECD *Economic Outlook* (short-term and long-term interest rates are used for the rest).[14]

4.1 Unit Root Tests

Many empirical studies indicate inconclusive results as to whether real exchange rates have a unit root. More specifically, depending on the unit root test being used, test results vary widely (e.g. Edison and Merick, 1999). One explanation for such a perplexity can be attributed to finite sample properties of unit root tests. In particular, (i) finite sample properties are largely different across the tests; and (ii) finite sample properties depend heavily on the true data generating process (DGP) under the null and alternative hypotheses. Therefore, when researchers have a specific model in mind, it is practically more reasonable to use the most suitable tests for the model, rather than agnostically use as many tests as they can.

In our model, the real exchange rate may have a unit root, together with moving-average (MA) errors. For such a case, it is well known since Schwert (1989) that popular unit root tests, such as the Phillips-Perron test (Phillips and Perron 1988), suffer from large size distortions. Put differently, when the Phillips-Perron test is used for the process that actually has a unit root and MA errors, then the test often mistakenly rejects the null hypothesis of a unit root, thereby indicating that the

[14] Real interests are constructed as follows. First we compute the (ex-post quarterly) inflation rate:

$$\pi_t = (p_t - p_{t-1})$$

where p_t is the (logarithm of) consumer price index (CPI, quarterly) at t. Then, the real interest rate for Home is computed as

$$r_t = i_t - \pi_{t+1};$$

where i_t is the nominal interest rate for one quarter (i.e., the annual nominal interest rate is divided by four).

process being tested is stationary.[15]

To evade the potential problem of erroneous rejection, following Ng and Perron (2001, 2002), we implement three types of unit root tests that have less size distortion and higher power than others under the presence of MA terms. Those are: Modified Phillips-Perron (MZ_α) tests, the Augmented Dickey-Fuller (ADF) test, and the Elliott, Rothenberg and Stock (1996) feasible point optimal (PT) test. All the series are detrended by the generalized least square (GLS) method with a constant as a regressor; and the length of the lags is selected by the modified Akaike information criteria (MAIC). The results reported in Table 1 indicate a unanimous conclusion: we fail to reject the null hypothesis of the unit root for US-CA, US-JP, US-SW and US-UK.

Having size corrected, we focus on the power of the tests. Our simulations (in the appendix) reveal that when the DGP is an MA(1) process without a unit root, as we model for real interest differentials, the size-corrected unit root tests have relatively low power. Instead, the KPSS test (Kwiatkowski, Phillips, Schmidt and Shin, 1992), with no unit root as a null, behaves considerably well under the null hypothesis of a stationary MA(1) (i.e., it has good size) as well as under the alternative of an ARIMA(0,1,1) (i.e., it has good power). After the power of the unit root tests is taken into account, the stationarity of the Switzerland-US real exchange rate is doubted; therefore, we exclude it from further investigation.

Table 2 reports the results of the unit root tests for real interest differentials. In our model, the real interest differentials are stationary; and they follow MA processes. The unit root tests for both the short-term and long-term real interest rate differentials, in general, fail to reject the null hypothesis. However, not being able to reject the null hypothesis does not necessarily mean the test's null hypothesis is correct: This is due to the fact that the test has low power, as Table A-1 in the appendix shows. Moreover, the KPSS test – which appears to do a good job for our purposes, from Table A-1 panels (a) and (b) – does not reject the null hypothesis of stationarity for all the short-term and long-term real interest differentials.[16]

What is the conclusion of the unit root tests? Based on the tests we have conducted, it is

[15] Size distortions are more serious when errors follow a negative MA process. See Maddala and Kim (1998) and Stock (1994), for example.

[16] Another justification for using the KPSS test for real interest differentials is that our null hypothesis should be an MA process without a unit root, as our model predicts.

meaningful to compute the BN decomposition for the real exchange rates of US-CA, US-JP and US-UK pairs. Then, it is meaningful to compare the BN cycle component of those real exchange rates with their corresponding real interest differentials, in order to investigate the link predicted by our model.

4.2 BN Cycles of the Real Exchange Rate and the Expected Sum of Future Real Interest Differentials

The BN decomposition of the real exchange rate and future real interest differentials are obtained as follows. First, we choose the length of lags, p, for the real exchange rate, using the Akaike Information Criterion (AIC) and Schwartz Bayesian Information Criterion (BIC) [17] of the estimated ARIMA(0,1,p) process. Then, given the estimated parameters in the ARIMA(0,1,p) and its fitted residuals, the BN cycle of the real exchange rate is computed exactly.[18] Similarly, an ARIMA(0,0,p^*) process of the real interest differentials is estimated,[19] where p^* is chosen by AIC, BIC, or is set to p-1. Finally, the expected sum of future real interest differentials, $D_t \equiv E_t \sum_{j=0}^{\infty} \left(r_{t+j} - r^*_{t+j} \right)$, are computed.

Tables 3 and 4 report the MA estimation of the real exchange rates and real interest differentials, respectively. The first order MA coefficients and the estimated standard deviations of the errors are all statistically significant. Given the fact that the MA roots are far from -1, and the fact that the estimated standard deviations of the errors are significantly different from zero, the estimated results are consistent with the conclusions from the unit roots tests, as well as with our model.

Selected p and p^* are summarized in Table 5. It is interesting that for the US-UK pair, the BIC chooses p^*=4 for the short-term real interest differentials. This number coincides with p-1, when the AIC chooses the length of the lags for the same pair's real exchange rate. In fact, this is the

[17] An ARIMA(0,1,p) is estimated by maximum likelihood estimation method. The likelihood function is computed exactly by the Kalman filter.

[18] See the appendix for a detailed computation.

[19] Equation (25) is estimated by an MA(p^*) model:

$$(r_t - r^*_t) = u_t + \theta_1 u_{t-1} + \cdots + \theta_{p^*} u_{t-p^*}.$$

See appendix A-1-2.

very case that we assume in our model.

Figures 1 to 7 compare the BN cycle components of the real exchange rate and the expected sum of future real interest differentials.[20] Keep in mind from equations (2) and (3) that q_t^{cycle} and D_t should be negatively correlated, although the figures do not explicitly show that they are negatively correlated. Still, the BN detrended real exchange rates and the corresponding real exchange differentials are moving more closely together, compared to the growth rates of the real exchange rates (Δq_t) and the real interest differentials ($r_t - r_t^*$) shown in Figure 8. Considering the fact that earlier studies of this literature tried to connect those two variables (to be more precise, Δq_{t+1} and $r_t - r_t^*$), the detrending substantially improves the resemblance of the two variables.

Table 6 measures sample standard deviations of real exchange rates and real interest differentials, together with that of the BN-detrended real exchange rates and corresponding real interest differentials. From this, it is quite obvious that the real exchange rates are almost five times more volatile than the real interest rate differentials (first three columns from the left). Thanks to the BN decomposition and computed expected future sums of real exchange differentials, this difference in the volatilities becomes insignificant, or even negative. For example, the standard deviation of the BN detrended real exchange rate for the US-UK pair is 2.03 when the AIC is used. This is strikingly smaller than the standard deviation of the first difference of the real exchange rate, 5.03. Also, when the expected future sum is computed with the BIC (or p-1) -chosen number of lags, the real interest differentials for this pair is increased to 2.12 from 1.17. The standard deviation of interest differentials that is almost four times smaller than that of the real exchange rate turns out to be larger than the standard deviation of the real exchange rate, due to the model-consistent decomposition.

4.3 Statistical Inference

Under the null hypothesis, which states that the cyclical components of q_t and the real interest differentials, D_t are not correlated, the estimated cross-autocorrelation function $\widehat{\rho}_{q^{cycle}D}(j)$ has an

[20] Both series are multiplied by 100 to represent percentage deviations from the BN trend (real exchange rates) and percentage deviation from the mean (real interest differentials).

asymptotic distribution (see Brockwell and Davis 1991[21]) of

$$\sqrt{T}\widehat{\rho}_{q^{cycle}D}(j) \xrightarrow{d} Normal\left(0, \sum_{h=-\infty}^{\infty} \rho_{q^{cycle}}(h)\,\rho_D(h)\right) \tag{27}$$

where $\xrightarrow{d}$ denotes convergence in distribution; $\rho_{q^{cycle}}(h)$ and $\rho_D(h)$ are autocorrelation functions of the cyclical components of q_t and D_t, respectively.

In order to compute $\rho_{q^{cycle}}(h)$ and $\rho_D(h)$, which are unknowns, we utilize the estimated ARIMA errors of q_t and $(r_t - r_t^*)$.[22] Clearly, our test encompasses the one for the contemporaneous correlation, namely, testing whether or not $\rho_{q^{cycle}D}(0) = 0$.

Figures 9 to 15 show the cross-autocorrelations together with the 95 percent bounds based on the distribution (27). In particular, it is worth mentioning that the cross- autocorrelations around $j = 0$, which correspond to the contemporaneous correlation, are negative and significant for the US-UK pair. The cross-autocorrelation at $j = 1$ for "uk q1l4" in Figure 14 (The real exchange rate and real interest differentials are estimated by an ARIMA (0,1,1) and an MA(4), respectively.) is -0.1829, and the 95 percent bound is -0.166. This result indicates that the BN cycle of the real exchange rate and the expected future real interest differentials may not be independent, and are, indeed, negatively correlated (with one lag), as our model predicts. Other results, however, are still puzzling. For instance, as Figures 10 to 13 present, the US-JP pair has a positive and significant cross-autocorrelation at $j = 0$ or its vicinity.

Having found only a weak link between the real exchange rate and real interest differentials (for the US-UK pair), our next question is whether the relationship has remained stable over time. In other words, can we find sub-samples, for which the link between the two variables is stronger than that in the full sample? To answer this question, Figure 16 demonstrates rolling 10 year moving-average estimates of $\rho_{q^{cycle}D}(0)$ together with its 95 percent bounds (box). Once again, the US-UK pair of an ARIMA(0,1,5) real exchange rate and an MA(4) real interest differentials are consistent

[21] Let two series be $X_{1t} = \sum_{j=-\infty}^{\infty} a_j z_{1j}$ and $X_{2t} = \sum_{j=-\infty}^{\infty} b_j z_{2j}$ where $\{z_{1j}\}$ and $\{z_{2j}\}$ are independent. Then, the asymptotic distribution of the estimated cross-autocorrelation function is $\sqrt{T}\widehat{\rho}_{12}(j) \xrightarrow{d} Normal\left(0, \sum_{h=-\infty}^{\infty} \rho_{11}(h)\,\rho_{22}(h)\right)$ where $\rho_{lm}(h) = E\left[\left(X_{lt} - E\left[X_{lt}\right]\right)\left(X_{mt-h} - E\left[X_{mt-h}\right]\right)\right]$, $l, m = 1, 2$.

[22] Brockwell and Davis (1991) recommend the use of the fitted residual of X_{1t} and X_{2t} in calculating $\rho_{11}(j)$ and $\rho_{22}(j)$.

with our model, in the sense that the BIC chosen lag-length is indeed p-1: one lag less than the MA components in the real exchange rate ("uk q5s4" and "uk q5l4" in Figure 16). Including such model-consistent combinations, the 10 year windows centered in 1990 and earlier indicate significant negative relationships for both the short-term and long-term real interest differentials, regardless of the lengths of lags that are used to compute q^{cycle} and D_t (there is no significant link after 1990, however).[23]

5 Conclusion

Following Baxter's (1994) influential paper, we focused on the trend-cycle decomposition issue in the real exchange rate. In particular, we developed a model which allows for a more flexible time series process of the real exchange rate, namely the non-stationary nature of a process that is widely supported by a vast majority of empirical studies. It is important to note that the non-stationarity of the real exchange rate in our model comes from the presetting price rule of monopolistically competitive firms, together with their possible misperceptions. Thanks to the well-specified time series model, we can accurately compute the cycle components of the BN decomposition that theoretically correspond to the sum of the future expected values of real interest differentials.

However, the model presented in this paper is by no means the only one that causes a no-stationary real exchange rate. For example, as argued in Mark (2001), the Balassa-Samuelson model (Balassa 1964; Samuelson 1964) with non-stationary productivity shocks predicts a non-stationary real exchange rate. More recently, a seminal paper by Engel and West (2005) predicts a "random walk like" real exchange rate, when the value of the discount factor is near unity.

Our model is partly empirically supported. First, after taking into account the time-series process of real exchange rates, the most appropriate statistical tests are in favor of the existence of unit root components in real exchange rates, with one seeming exception, for which a unit root is strongly doubted. As for the real interest differentials, in accordance with our model, all of them are found to be stationary. Secondly, as theory implies, a rigorous statistical test constructed from

[23] Although not reported here, rolling 10 years moving-average estimates of $\rho_{q^{cycle}D}(0)$ for the US-CA and US-JP pairs are not promising at all for any 10 year windows.

the asymptotic process of the cross-autocorrelation function confirms the link between the BN cycle of the real exchange rate and the expected future sum of real interest differentials for the pair of US-UK (especially before 1990). Yet, some cases, including US-Japan, remain as a puzzle: not only is an insignificant correlation found, but a significant positive relationship is also found.

A possible extension of this paper may include developing a model in which the misperception parameter, $a(1)$ changes over time so that the real exchange rate sometimes follows the random walk process; while other times it exhibits a stationary variable. Such a model would reconcile the debate over non-stationarity or non-linearity of the real exchange rate.

A-1 Appendix

A-1.1 The Beveridge-Nelson decomposition

This is based on Morley (2002). We can estimate ARIMA(0,1,p) model using the following Kalman filter. (The mean of Δq_t is subtracted before estimation.)

$$\Delta q_t = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} \Delta q_t \\ \eta_t \\ \eta_{t-1} \\ \vdots \\ \eta_{t-p+1} \end{bmatrix}}_{V_t}$$

$$\underbrace{\begin{bmatrix} \Delta q_t \\ \eta_t \\ \eta_{t-1} \\ \vdots \\ \eta_{t-p+1} \end{bmatrix}}_{V_t} = \underbrace{\begin{bmatrix} 0 & a_1 & a_2 & \cdots & a_p \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} \Delta q_{t-1} \\ \eta_{t-1} \\ \eta_{t-2} \\ \vdots \\ \eta_{t-p} \end{bmatrix}}_{V_{t-1}} + \underbrace{\begin{bmatrix} \eta_t \\ \eta_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\boldsymbol{\eta}_t}$$

and the variance-covariance matrix of $\boldsymbol{\eta}_t$ is given by

$$\boldsymbol{\eta}_t \sim Normal\,(0, \Omega)\,,$$

where

$$\Omega = \begin{bmatrix} \sigma_\eta^2 & \sigma_\eta^2 & 0 & \cdots & 0 \\ \sigma_\eta^2 & \sigma_\eta^2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 0 & 0 \end{bmatrix};$$

and σ_η^2 is the variance of η_t. Then, the BN trend can be computed by

$$q_t^{BNtrend} = \lim_{k\to\infty} E_t q_{t+k} = q_t + \lim_{k\to\infty} E_t \sum_{j=1}^{k} \Delta q_{t+j}$$

where

$$\lim_{k\to\infty} E_t \sum_{j=1}^{k} \Delta q_{t+j} = H\left(I - F\right)^{-1} F V_{t|t}$$

thus, the BN cycle is

$$q_t^{BNcycle} = -H\left(I - F\right)^{-1} F V_{t|t}$$

where $V_{t|t}$ is the filtered state of V_t.

A-1.2 The expected future sum of the real interest differentials

Using the same technique, we can set the equation in the state space form:

$$
(r_t - r_t^*) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{h} \underbrace{\begin{bmatrix} (r_t - r_t^*) \\ u_t \\ u_{t-1} \\ \vdots \\ u_{t-p+1} \end{bmatrix}}_{v_t}
$$

$$
\begin{bmatrix} (r_t - r_t^*) \\ u_t \\ u_{t-1} \\ \vdots \\ u_{t-p+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \theta_1 & \theta_2 & \cdots & \theta_p \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}}_{f} \begin{bmatrix} \left(r_{t-1} - r_{t-1}^*\right) \\ u_{t-1} \\ u_{t-2} \\ \vdots \\ u_{t-p} \end{bmatrix} + \begin{bmatrix} u_t \\ u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}.
$$

(The mean of $r_t - r_t^*$ is subtracted before estimation.) With the estimated parameters, the expected sum of future real interest differentials can be computed by

$$
D_t \equiv E_t \sum_{j=0}^{\infty} \left(r_{t+j} - r_{t+j}^*\right) = h\,(I - f)^{-1}\, v_{t|t}.
$$

A-1.3 OLS and the t-test as not desirable method

It is shown that the OLS is not an appropriate way to gauge the link between the real exchange rate and real interest differentials. Two problems arise: (i) the estimator is a biased estimator; (ii) the asymptotic variance of the estimator depends on the degree of misperception. From equations (23) and (25), one can set the regression equation as

$$\Delta q_{t+1} = b_1 + b_2 \left(r_t - r_t^*\right) + \xi_{t+1} \tag{A.1}$$

for $t = 1, 2, \ldots, T$. Note that $\xi_{t+1} = \eta_{t+1}$. In order to understand the problems about the OLS estimator $\widehat{b}_2$ of (A.1), let us denote:

$$\begin{aligned}
y &= \begin{bmatrix} \Delta q_2 & \Delta q_3, & \cdots & \Delta q_{T+1} \end{bmatrix}' \\
X &= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ -a(L)\,\eta_1 & -a(L)\,\eta_2 & \cdots & -a(L)\,\eta_T \end{bmatrix}' \\
\beta &= \begin{bmatrix} b_1 & b_2 \end{bmatrix}' \\
\xi &= \begin{bmatrix} \eta_2 & \eta_2, & \cdots & \eta_{T+1} \end{bmatrix}'.
\end{aligned}$$

Then, the OLS estimator is given by

$$\widehat{\beta} = \left(X'X\right)^{-1} X'y.$$

- Bias

The OLS estimator is not necessarily unbiased: The right hand side of

$$E\left[\widehat{\beta}\right] - \beta = E\left[\left(X'X\right)^{-1} X'\xi\right]$$

is not necessarily equal to 0. This is due to the fact that the strict exogenousity, i.e., ξ_{t+1} is not uncorrelated with all *leads* and lags of $(r_t - r_t^*)$, does not hold. To show the practical importance, Figure 17 plots bias $(b_2 - 1)$ as a function of $a(1)$.[24] Clearly, one can see that a large bias is associated with serious misperceptions (small $a(1)$).

- Problem of the asymptotic variance

Note that the OLS estimator is a consistent estimator. Let

$$p \lim_{T\to\infty} \frac{X'X}{T} = \begin{bmatrix} 1 & 0 \\ 0 & \sigma_\eta^2 \alpha^2 \end{bmatrix} \equiv A \text{ and} \tag{A.2}$$

$$p \lim_{T\to\infty} \frac{\xi'\xi}{T} = \sigma_\eta^2 \tag{A.3}$$

where

$$\alpha^2 = a_1^2 + \cdots + a_p^2$$

Together with the fact that $E[x_t \xi_t] = 0$ for all t, where $x_t = [1, -a(L)\eta_{t-1}]$, the Mann-Wald theorem guarantees:

$$\frac{X'\xi}{\sqrt{T}} \xrightarrow{d} Normal\left(0, \sigma_\eta^2 A\right).$$

Hence,

$$\sqrt{T}\left(\widehat{\beta} - \beta\right) \xrightarrow{d} Normal\left(0, \sigma_\eta^2 A^{-1}\right). \tag{A.4}$$

The problem is that the asymptotic variance of $\widehat{b}_2$ (i.e., the (2,2) element of $\sigma_\eta^2 A^{-1}$) solely depends on $1/\alpha^2$: For example, given $p = 1$, The smaller the a_1 (a more serious misperception),

[24] The data generating process is

$$y_t = a\eta_{t-1} + \eta_t$$

where $\eta_t \sim i.i.d.Normal(0,1)$. We compute the bias of $\widehat{b}$ for 200 values of a from 0.01 to 1. The sample sizes are: T=50, 100, and 200 (10,000 replications). Figure 14 shows the mean bias of the replications.

the larger is the asymptotic variance of $\widehat{b}_2$. Since a large asymptotic variance prevents one from estimating b_2 accurately, OLS should not be used for our purpose.

In summary, the OLS estimator is not appropriate for measuring the link between the real exchange rate and real interest differentials. This is why a model-consistent detrending method is necessary; and also is why previous studies have often failed to find such a link.

A-1.4 Simulations

In order to investigate the power and size of the unit root tests used in our model, we conduct the following simulations.

A-1.4.1 M-tests (Ng and Perron 2001)

The data generating process (DGP) is

$$\begin{aligned} y_t &= d_t + u_t \\ u_t &= \alpha u_{t-1} + v_t \\ v_t &= e_t + \theta e_{t-1}, \end{aligned}$$

where

$$d_t = 0; \quad e_t \sim N(0,1).$$

We consider two cases: (1) unit root case $\alpha = 1$, which corresponds to the real exchange rate in our model; and (2) stationary case $\alpha = 0$, which corresponds to the real interest differentials in our model. Note that case (1) is simply a replication of Ng and Perron (2001).

From Table A-1, important observations are the following: (1) From Panel-a, the size of unit root tests with GLS detrending is good. The KPSS test has a good power, almost for all the values of θ. (2) Panel-b exhibits that the power of the unit root tests is rather low, especially for negative values of θ. On the other hand, the KPSS has a good size regardless of the values of θ.

A-1.4.2 KPSS test

The model is

$$\begin{aligned} y_t &= \delta t + \beta_t + \varepsilon_t \\ \beta_t &= \beta_{t-1} + u_t \end{aligned}$$

where ε_t is a stationary process and $u_t \sim i.i.d.\left(0, \sigma_u^2\right)$. Then, under the null hypothesis of $\sigma_u^2 = 0$, the asymptotic distribution of

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{s^2}$$

is known. Here,

$$\begin{aligned} s^2 &= T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{l=1}^{\widetilde{q}} w_{l\widetilde{q}} \sum_{t=l+1}^{T} e_t e_{t-l}; \\ S_t &= \sum_{t=1}^{T} e_t; \\ w_{l\widetilde{q}} &= 1 - \frac{l}{\widetilde{q}+1} \quad \text{for } l \leq \widetilde{q}; \end{aligned}$$

e_t is the residual from the regression of y_t on constant; and $\widetilde{q}$ is the bandwidth.

A-1.4.3 Choice of $\widetilde{q}$ – Newey and West (1994)

First, we set

$$L = int\left[4\left(\frac{T}{100}\right)^{2/9}\right],$$

where T is the length of the data; and $int\,[\cdot]$ denotes the integer part of the argument, $\cdot$.

Next, we compute

$$\begin{aligned} s(1) &= 2T^{-1}\sum_{l=1}^{L} l \sum_{t=l+1}^{T} e_t e_{t-l} \\ s(0) &= T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{l=1}^{L}\sum_{t=l+1}^{T} e_t e_{t-l}; \end{aligned}$$

and

$$\gamma = 1.1447\left(\frac{s(1)}{s(0)}\right)^{2/3} T^{1/3}.$$

This is analogue to Andrews' (1991) plug-in method:

$$\gamma = 1.1447\alpha(1)^{1/3} T^{1/3}$$

when

$$\alpha(1) = \left(\frac{s(1)}{s(0)}\right)^2$$

As noted in Newey and West (1994), when we use an integer $\widetilde{q}$, i.e., $\widetilde{q} = int[\gamma]$, then the weight is

$$w_{l\widetilde{q}} = 1 - \frac{l}{\widetilde{q}+1};$$

but if we use the real bandwidth, $\widetilde{q} = \gamma$, then

$$w_{l\widetilde{q}} = 1 - \frac{l}{\widetilde{q}}.$$

In our analysis, we use the integer case.

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Table 1: Unit Root Tests: Real Exchange Rates

	MZa	ADF	MPT	KPSS
US-CA	-3.45	-1.28	7.10	0.548^{*}
US-JP	-2.95	-1.05	8.09	0.598^{*}
US-SW	-3.69	-1.17	6.70	0.313
US-UK	-7.77	-1.88	3.39	0.572^{*}
5% Critical Value	-8.10	-1.98	3.17	0.463
1% Critical Value	-13.8	-2.58	1.78	0.739

Notes: 1) The lag length is chosen by the Modified Akaike Information Criteria (MAIC) with $kmax = 12(T/100)^{1/4}$ where T is the sample size. 2) GLS detrending uses $\overline{\alpha} = 1-7/T$ as an autoregressive parameter. 3) An asterisk (*) indicates that one can reject the null hypothesis at a 5% level. For more details, see the appendix.

Table 2: Unit Root Tests: Real Interest Differentials

	MZa	ADF	MPT	KPSS
US-CA (Short)	-1.64	-1.06	12.50	0.175
(Long)	-9.18^{*}	-2.65^{**}	2.96^{*}	0.187
US-JP (Short)	0.37	-0.57	46.23	0.092
(Long)	0.14	-0.35	40.64	0.101
US-UK (Short)	-4.05	-2.64^{**}	6.05	0.450
(Long)	-11.45^{*}	-3.64^{**}	2.16^{*}	0.263
5% Critical Value	-8.10	-1.98	3.17	0.463
1% Critical Value	-13.8	-2.58	1.78	0.739

Notes: 1) The lag length is chosen by the Modified Akaike Information Criteria (MAIC) with $kmax = 12(T/100)^{1/4}$ where T is the sample size. 2) GLS detrending uses $\overline{\alpha} = 1-7/T$ as an autoregressive parameter. 3) The asterisks (*) and (**) indicate that one can reject the null hypothesis at the 5% and 1% levels, respectively. 4) “Long”and “Short”are Long-term and Short-term interest rate differentials, respectively. For more details, see the appendix.

Table 3: MA Estimation of Real Exchange Rates

	a_1	a_2	a_3	a_4	a_5	σ	μ	LL	Criterion
US-CA	0.446^{**} (0.085)					0.024^{**} (0.001)	-0.001 (0.003)	337.180	AIC, BIC
US-JP	0.310^{**} (0.082)	-0.023 (0.087)	0.174 (0.101)	0.236^{**} (0.093)		0.049^{**} (0.003)	0.004 (0.007)	232.927	AIC
US-JP	0.305^{**} (0.089)					0.050^{**} (0.003)	0.003 (0.005)	228.919	BIC
US-UK	0.262^{**} (0.080)	0.012 (0.090)	-0.067 (0.080)	0.089 (0.098)	-0.252 (0.109)	0.047^{**} (0.003)	0.002 (0.004)	238.688	AIC
US-UK	0.242^{**} (0.081)					0.049^{**} (0.003)	0.002 (0.005)	233.537	BIC

Notes: 1) LL stands for the value of the log-likelihood. 2) σ is the standard deviation of the error term. 3) Standard errors are given in parentheses. 4) An asterisk (*)indicates that the parameter is significant at a 5% level; and (**) indicates that the parameter is significant at a 1% level.

Table 4: MA Estimation of Real Interest Rate Differentials

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	σ	μ	LL	Criterion
US-CA(S)	0.427^{**} (0.067)										0.006^{**} (0.000)	-0.003^{**} (0.001)	545.496	BIC
US-CA(S)	0.432^{**} (0.082)	0.128 (0.090)	0.173^{*} (0.088)	0.195^{**} (0.072)							0.006^{**} (0.000)	-0.003^{**} (0.001)	550.248	AIC
US-CA(L)	0.368^{**} (0.076)										0.005^{**} (0.000)	-0.002^{*} (0.001)	553.650	BIC
US-CA(L)	0.365^{**} (0.084)	0.038 (0.098)	0.137 (0.089)	0.176^{*} (0.074)							0.005^{**} (0.000)	-0.002^{*} (0.001)	556.732	BIC
US-JP(S)	0.027 (0.095)	0.270^{**} (0.062)	0.098 (0.102)								0.009^{**} (0.001)	-0.000 (0.001)	480.391	p-1
US-JP(S)	0.096 (0.148)	0.312^{**} (0.086)	0.200^{*} (0.080)	0.335^{**} (0.085)							0.009^{**} (0.000)	-0.000 (0.001)	487.963	BIC
US-JP(S)	0.168^{*} (0.082)	0.295^{**} (0.089)	0.254^{**} (0.094)	0.247^{*} (0.113)	0.038 (0.116)	0.157 (0.100)	-0.206^{*} (0.095)	0.265^{**} (0.099)			0.008^{**} (0.000)	-0.000 (0.001)	493.509	AIC
US-JP(L)	0.001 (0.139)	0.261^{**} (0.063)	0.145 (0.138)								0.009^{**} (0.001)	0.002^{*} (0.001)	481.749	p-1
US-JP(L)	0.108 (0.121)	0.292^{**} (0.084)	0.221^{**} (0.080)	0.387^{**} (0.084)							0.008^{**} (0.001)	0.002 (0.001)	492.164	BIC
US-JP(L)	0.146 (0.081)	0.269^{**} (0.086)	0.254^{**} (0.086)	0.297^{**} (0.106)	0.060 (0.106)	0.194^{*} (0.091)	-0.086 (0.114)	0.368^{**} (0.102)			0.008^{**} (0.000)	0.002 (0.002)	499.385	AIC
US-UK(S)	0.263^{**} (0.084)	0.153 (0.092)	-0.009 (0.083)	0.350^{**} (0.069)							0.010^{**} (0.001)	-0.003^{**} (0.001)	459.780	BIC,p-1
US-UK(S)	0.197^{*} (0.083)	0.273^{**} (0.082)	0.085 (0.084)	0.516^{**} (0.085)	-0.212^{*} (0.097)	0.019 (0.095)	0.135 (0.086)	0.394^{**} (0.077)	0.037 (0.082)	-0.217^{*} (0.090)	0.010^{**} (0.001)	-0.004 (0.085)	469.908	AIC
US-UK(L)	0.154 (0.082)	0.045 (0.088)	-0.073 (0.077)	0.372^{**} (0.070)							0.010^{**} (0.001)	0.000 (0.001)	470.541	BIC,p-1
US-UK(L)	0.111 (0.081)	0.127 (0.082)	-0.085 (0.085)	0.392 (0.087)	-0.232^{**} (0.090)	0.044 (0.093)	0.047 (0.083)	0.165 (0.095)	-0.156 (0.092)	-0.212^{*} (0.085)	0.009^{**} (0.001)	0.000 (0.001)	478.776	AIC

Notes: 1) LL stands for the value of the log-likelihood. 2) σ is the standard deviation of the error term. 3) Standard errors are given in parentheses. 4) An asterisk (*) indicates that the parameter is significant at a 5% level; and (**) indicates that the parameter is significant at a 1% level. 5) Entries below Criterion" are criteria used to choose the length of lags. $p-1$ means that the length of lags is $p-1$; where p is the length of lags used to compute the BN decomposition of the real exchange rate. (p for the real exchange rate is selected by the AIC.)

Table 5: Summary of the Selected Length of Lags

Real Exchange Rates			Real Interest Differentials			
	AIC	BIC		AIC	BIC	$p-1$
US-CA	1	1	(Short)	4	1	0
			(Long)	4	1	0
US-JP	4	1	(Short)	8	4	$3, 0$
			(Long)	8	4	$3, 0$
US-UK	5	1	(Short)	10	4	$4, 0$
			(Long)	10	4	$4, 0$

Notes: 1) $p-1$ means that the length of lags is $p-1$; where p is the length of lags used to compute the BN decomposition of the real exchange rate. (p for the real exchange rate is selected by the AIC.)

Table 6: Standard Deviations

	Δq	r_{short}	r_{long}	q^{cycle}_{AIC}	q^{cycle}_{BIC}	D^{short}_{AIC}	D^{short}_{BIC}	$D^{short}_{p-1,AIC}$	$D^{short}_{p-1,BIC}$	D^{long}_{AIC}	D^{long}_{BIC}	$D^{long}_{p-1,AIC}$	$D^{long}_{p-1,BIC}$
US-CA	2.56	0.64	0.59	1.07	1.07	1.26	0.87	0.64	0.64	1.04	0.78	0.59	0.59
US-JP	5.25	0.93	0.93	4.42	1.55	2.21	1.95	1.35	0.93	2.63	2.01	1.39	0.93
US-UK	5.03	1.17	1.07	2.03	1.09	2.72	2.12	2.12	1.17	1.25	1.83	1.83	1.07

Notes: 1) Δq is the first difference of the real exchange rate; r_{short} and r_{long} are short-term and long-term real interest differentials. 2) AIC, BIC, $p-1$ indicate the criterion used to choose the length of lags. For example, $D^{short}_{AIC,p-1}$ means the expected future sum of short-term real interest differentials, computed from an MA model with the lags of p-1, where p is the length of lags used to compute the BN decomposition of the real exchange rate; and p is selected by AIC. 3) Standard deviations are multiplied by 100, so that they indicate percentage deviations.

Table A-1: Rejection Rates of Unit Root Tests

Panel a: $\alpha = 1$ (Unit Root Case)				
θ	MZa	ADF	PT	KPSS
-0.8	0.028	0.101	0.027	0.720
-0.5	0.054	0.065	0.049	0.748
0.0	0.046	0.052	0.043	0.746
0.5	0.051	0.047	0.049	0.743
0.8	0.065	0.041	0.061	0.736

Notes: 1) MZa, ADF and PT all use GLS detrended residuals, with $\overline{\alpha} = 1 - 7/T$ as an autoregressive parameter. The lag length is chosen by the Modified Akaike Information Criteria (MAIC) with $kmax = 12(T/100)^{1/4}$ where T is the sample size. 2) KPSS test uses the Bartlett kernel to compute the long-run variance with the bandwidth proposed in Newey and West (1994). 3) Rejection rates for MZa, ADF and PT are considered as the size of the test, where the nominal size is 5%; while that for KPSS is the power of the test against the unit root alternative. 4) The rejection rates are computed from the sample size T=150 with 5,000 replications.

Panel b: $\alpha = 0$ (Stationary Case)				
θ	MZa	ADF	PT	KPSS
-0.8	0.140	0.174	0.140	0.066
-0.5	0.110	0.230	0.110	0.035
0.0	0.185	0.378	0.184	0.045
0.5	0.323	0.509	0.322	0.061
0.8	0.376	0.565	0.373	0.065

Notes: 1) MZa, ADF and PT all use GLS detrended residuals, with $\overline{\alpha} = 1 - 7/T$ as an autoregressive parameter. The lag length is chosen by the Modified Akaike Information Criteria (MAIC) with $kmax = 12(T/100)^{1/4}$ where T is the sample size. 2) The KPSS test uses the Bartlett kernel to compute the long-run variance with the bandwidth proposed in Newey and West (1994). 3) Rejection rates for MZa, ADF and PT are considered as the power of the test against the unit root alternative; while that for KPSS is the size of the test, where the nominal size is 5%. 4) The rejection rates are computed from the sample size T=150 with 5,000 replications.

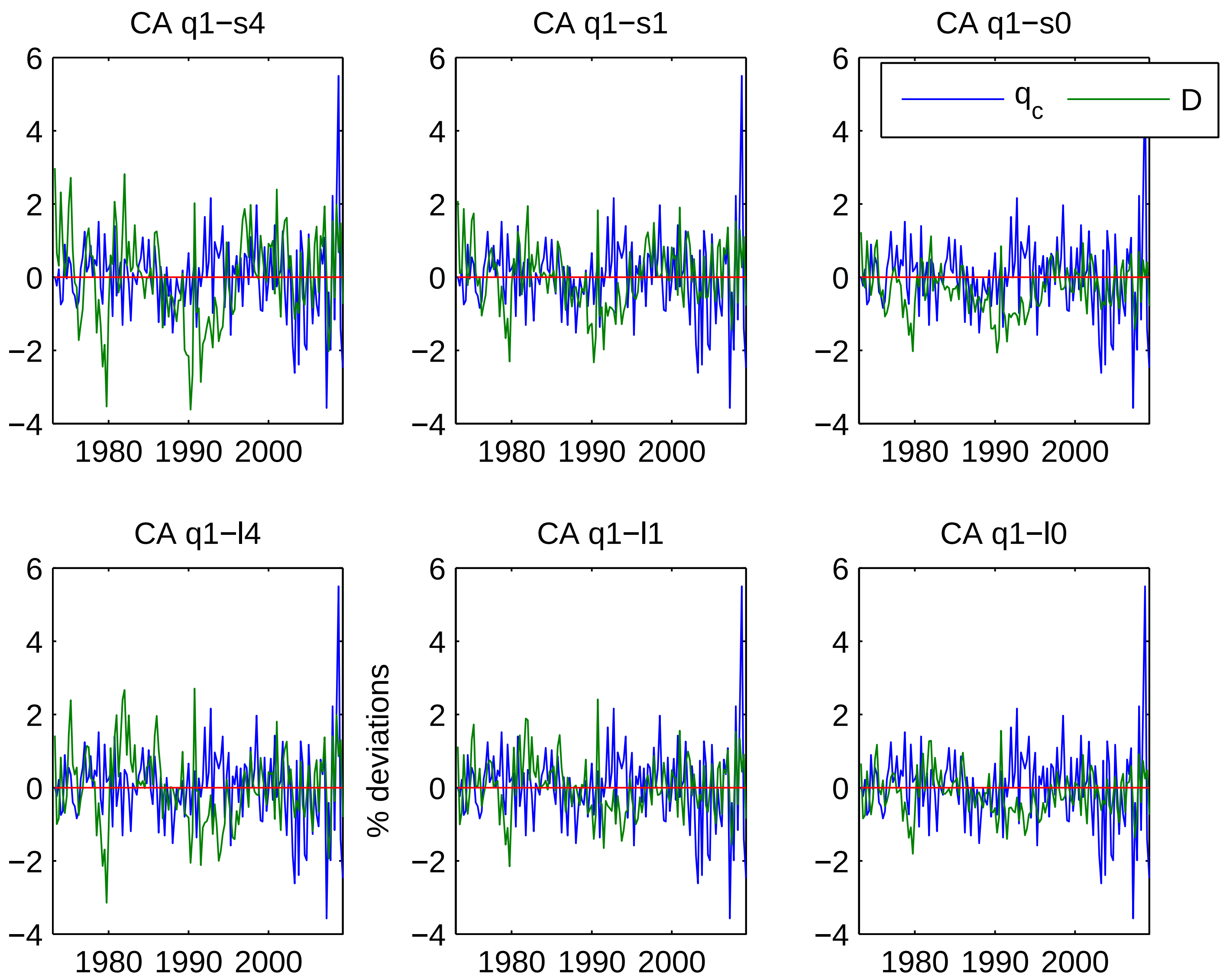

Figure 1: The detrended real exchange rates (q_c) and the expected sum of future real interest differentials (D). "CA $qi - lj$" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the long-term interest rates for the US-Canada pair.

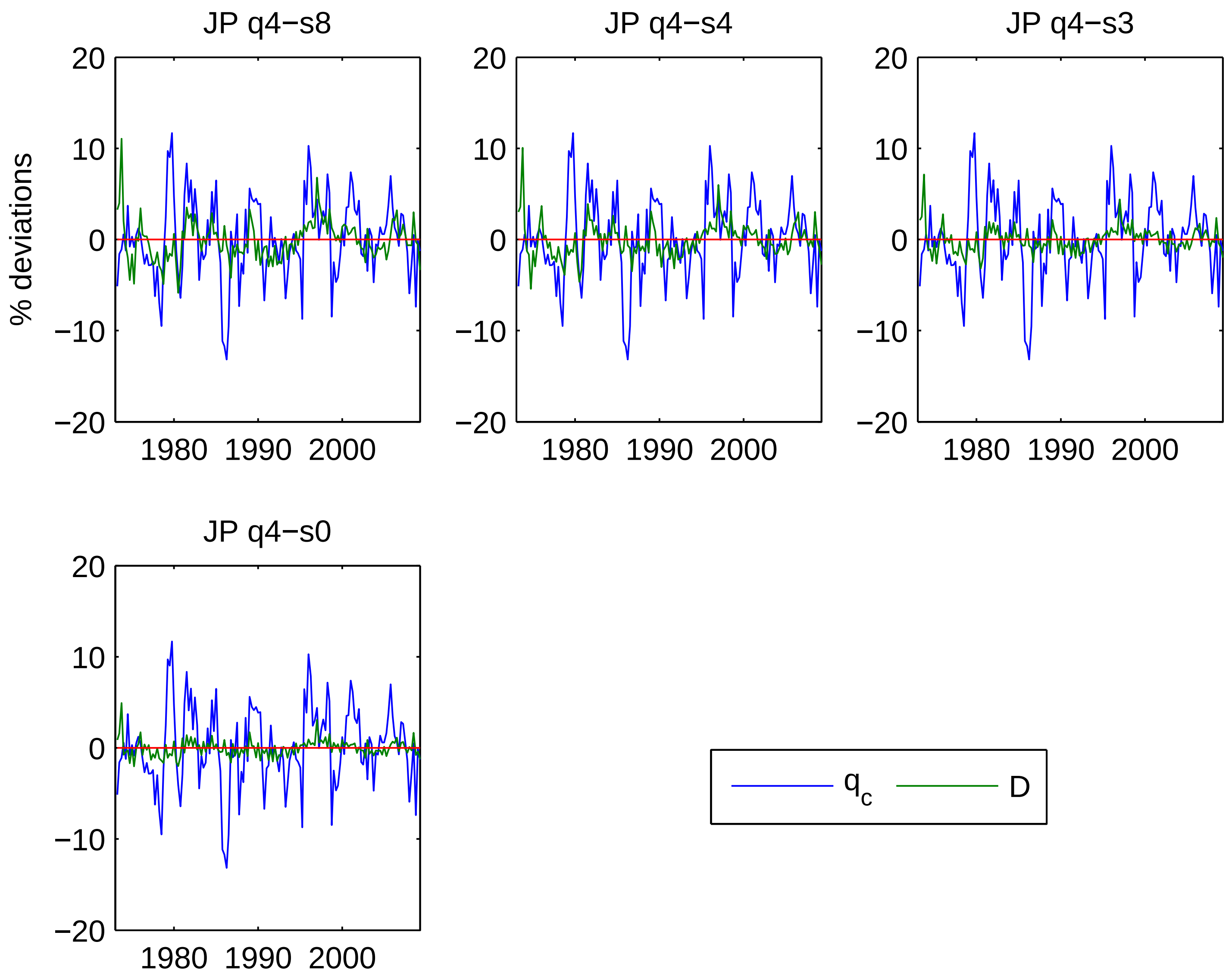

Figure 2: The detrended real exchange rates (q_c) and the expected sum of future real interest differentials (D). "JP $qi - sj$" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the short-term interest rates for the US-Japan pair.

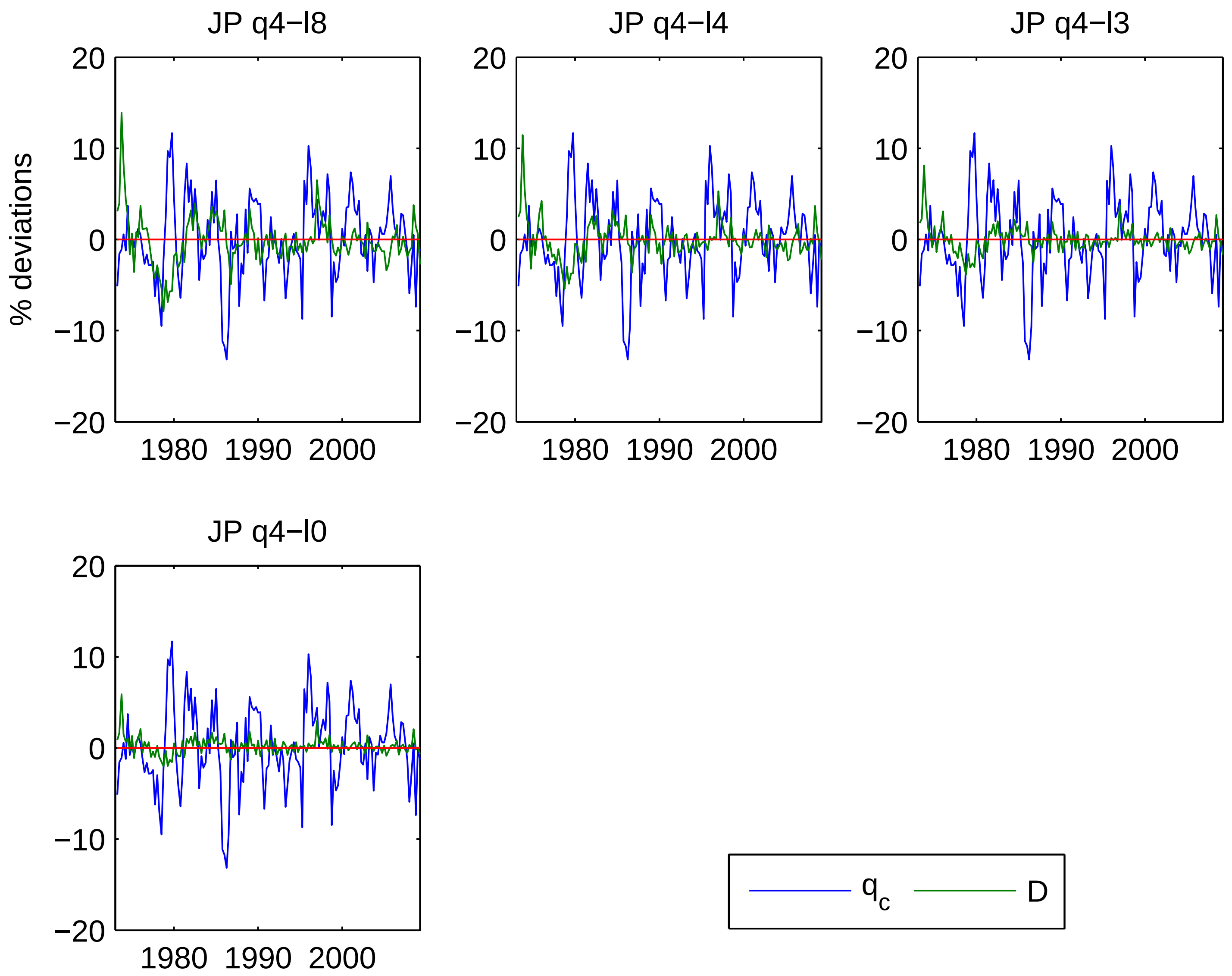

Figure 3: The detrended real exchange rates (q_c) and the expected sum of future real interest differentials (D). "JP $qi - lj$" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the long-term interest rates for the US-Japan pair.

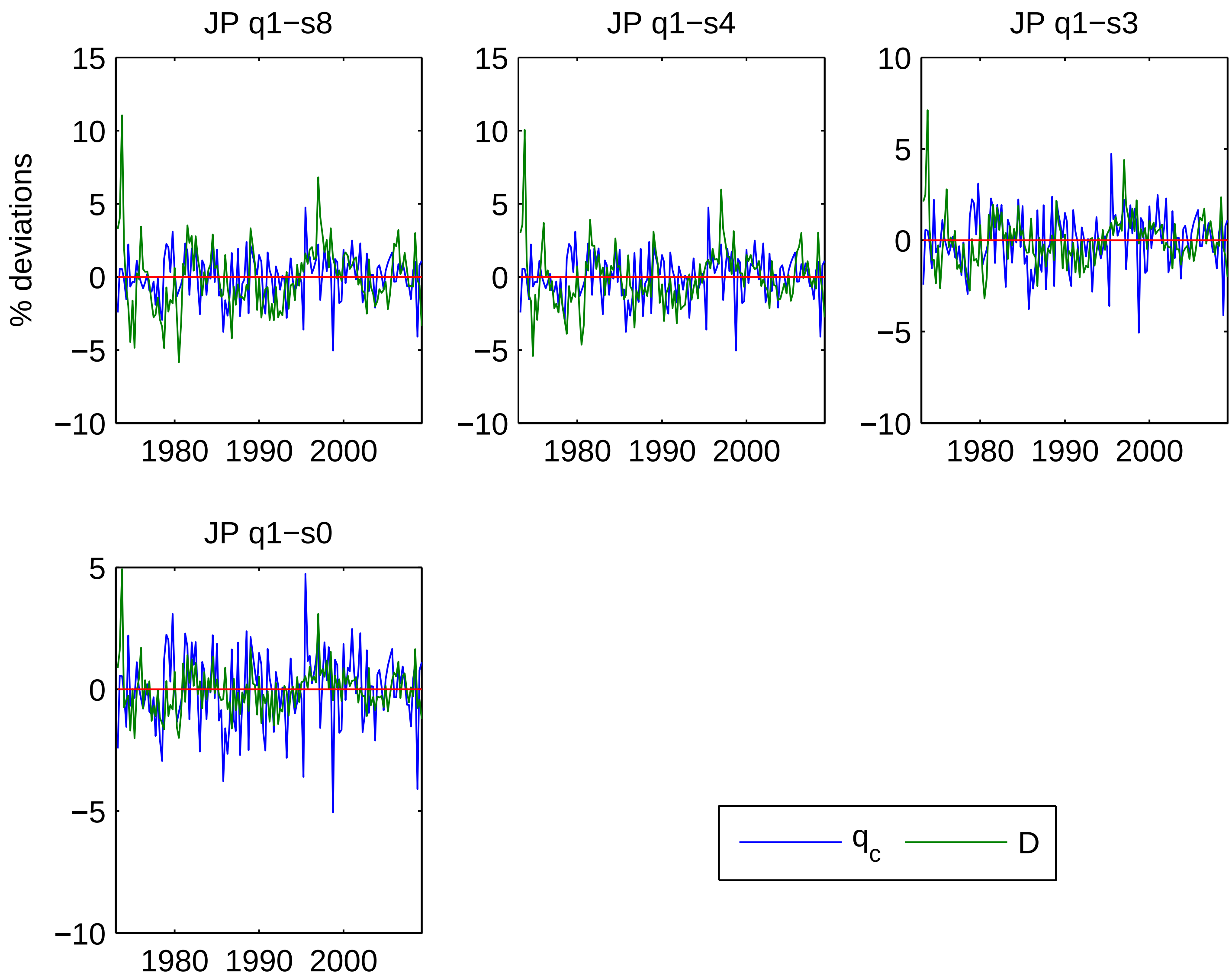

Figure 4: The detrended real exchange rates (q_c) and the expected sum of future real interest differentials (D). "JP $qi - sj$" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the short-term interest rates for the US-Japan pair.

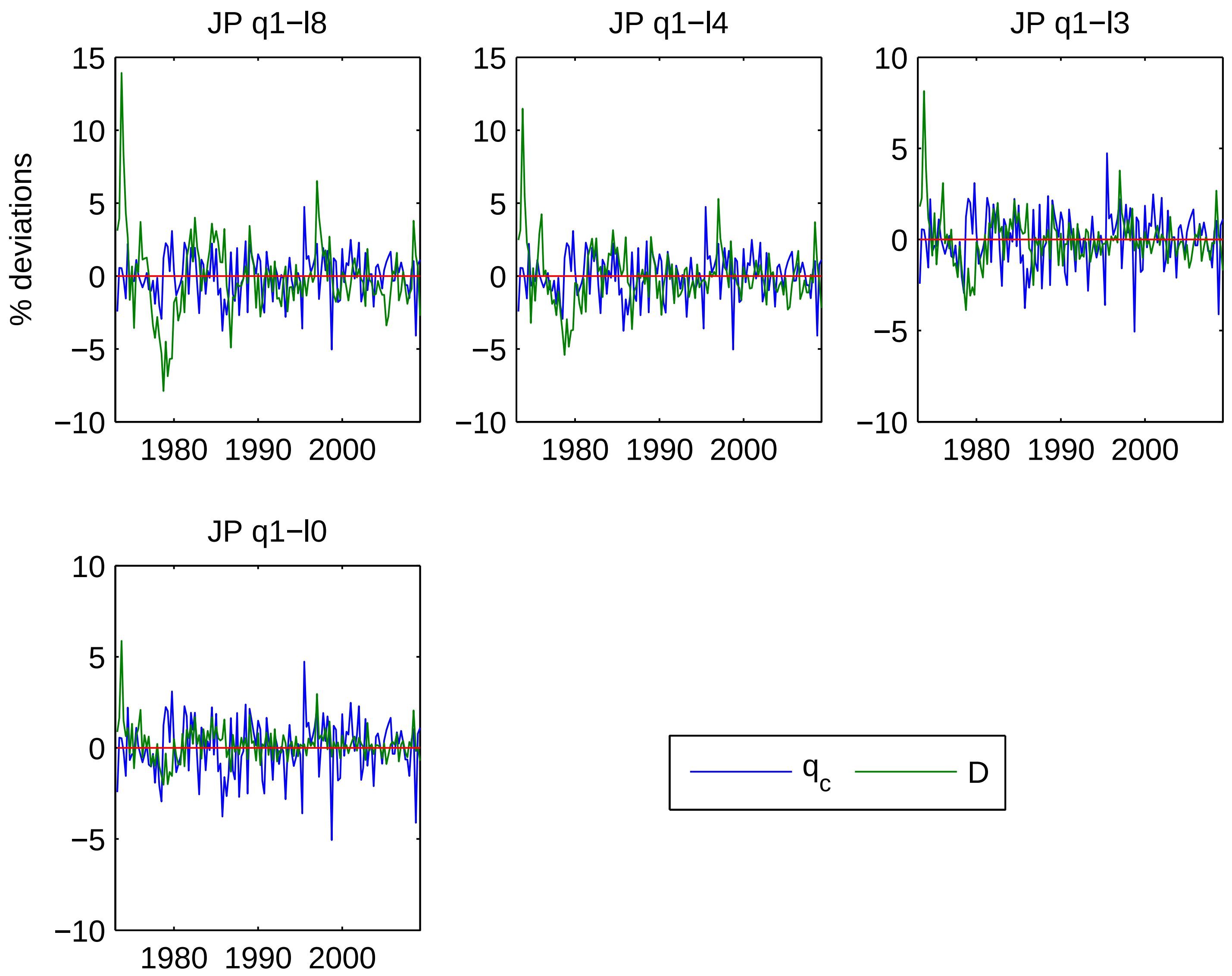

Figure 5: The detrended real exchange rates (q_c) and the expected sum of future real interest differentials (D). "JP $qi - lj$" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the long-term interest rates for the US-Japan pair.

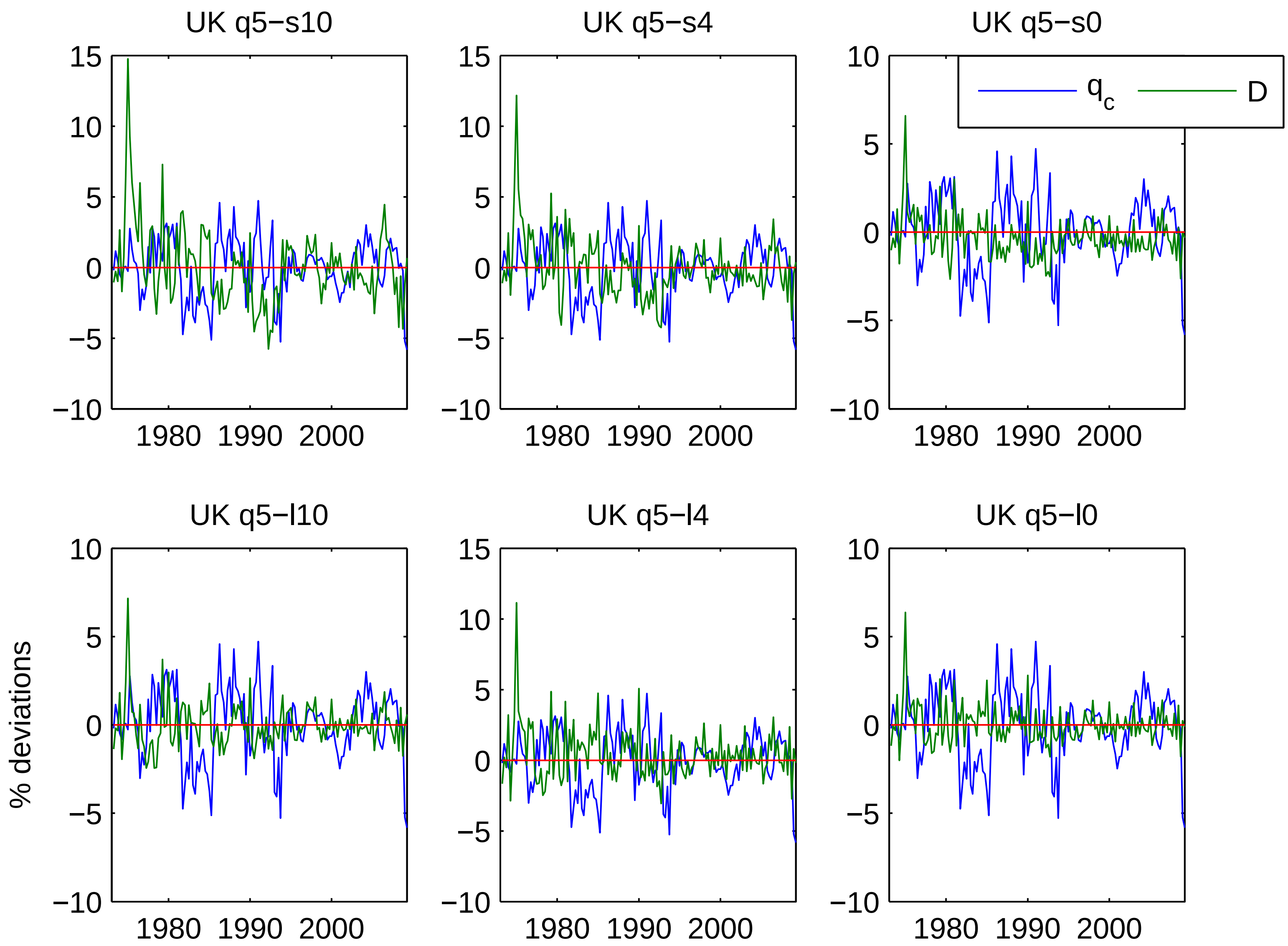

Figure 6: The detrended real exchange rates (q_c) and the expected sum of future real interest differentials (D). "UK $qi - lj$" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the long-term interest rates for the US-UK pair.

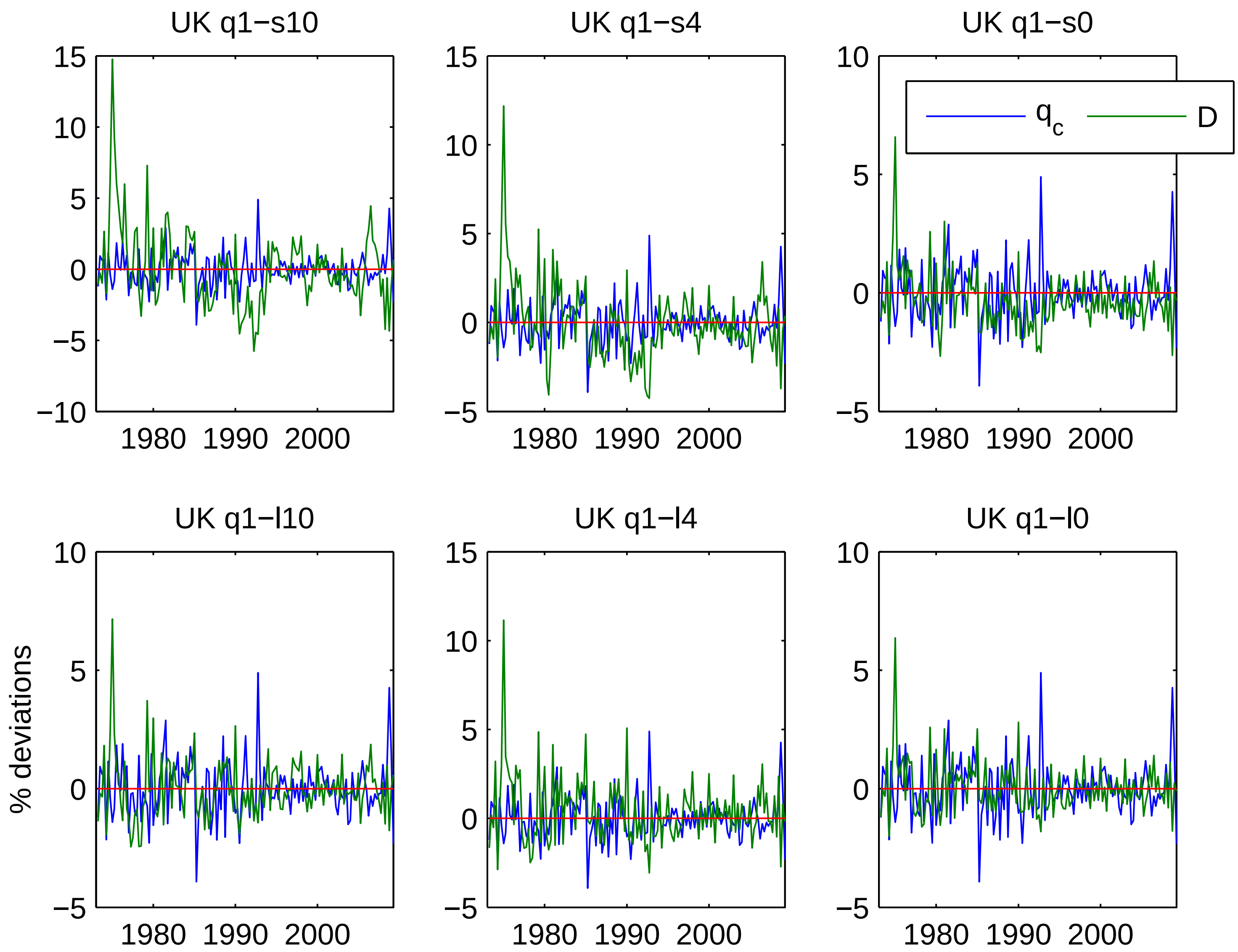

Figure 7: The detrended real exchange rates (q_c) and the expected sum of future real interest differentials (D). "UK $qi - lj$" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the long-term interest rates for the US-UK pair.

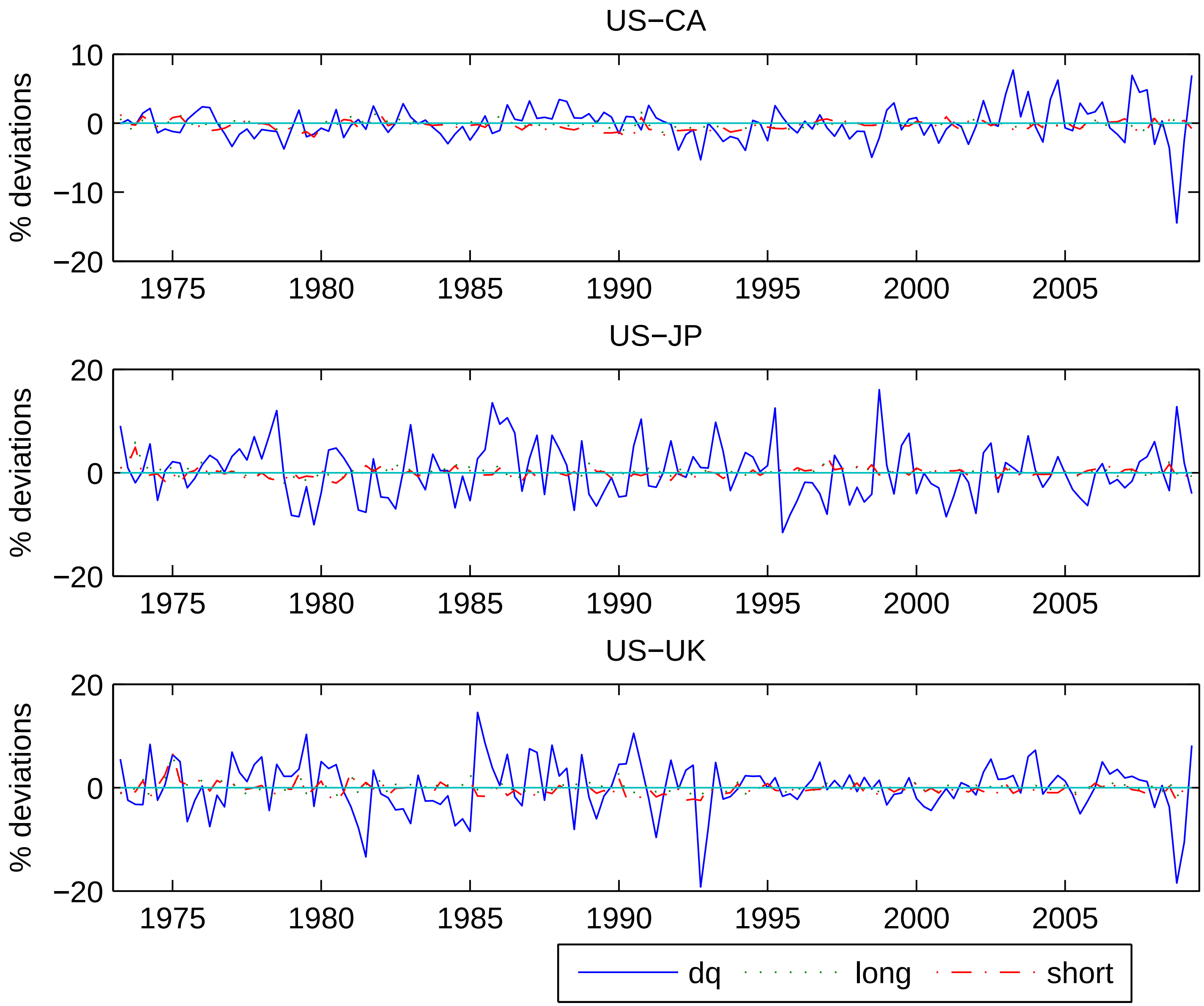

Figure 8: The real exchange rates (growth rates, dq), long-term real interest differentials (long) and short-term real interest differentials.

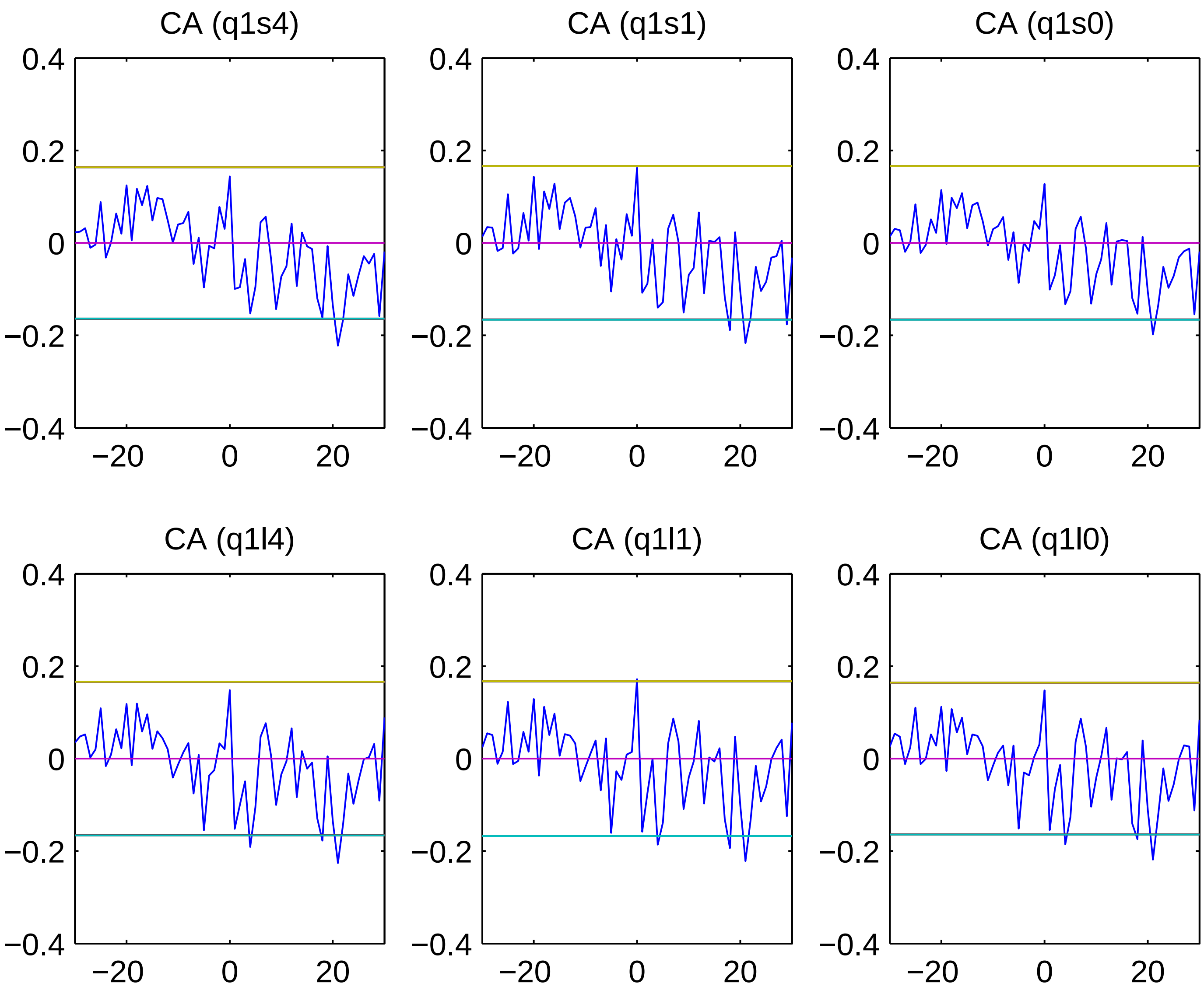

Figure 9: Cross-autocorrelates and the 95% confidence bounds. "CA($qisj$)" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the short-term interest rates, whie "CA ($qilj$)" uses the long-term real interest rates, for the US-Canada pair.

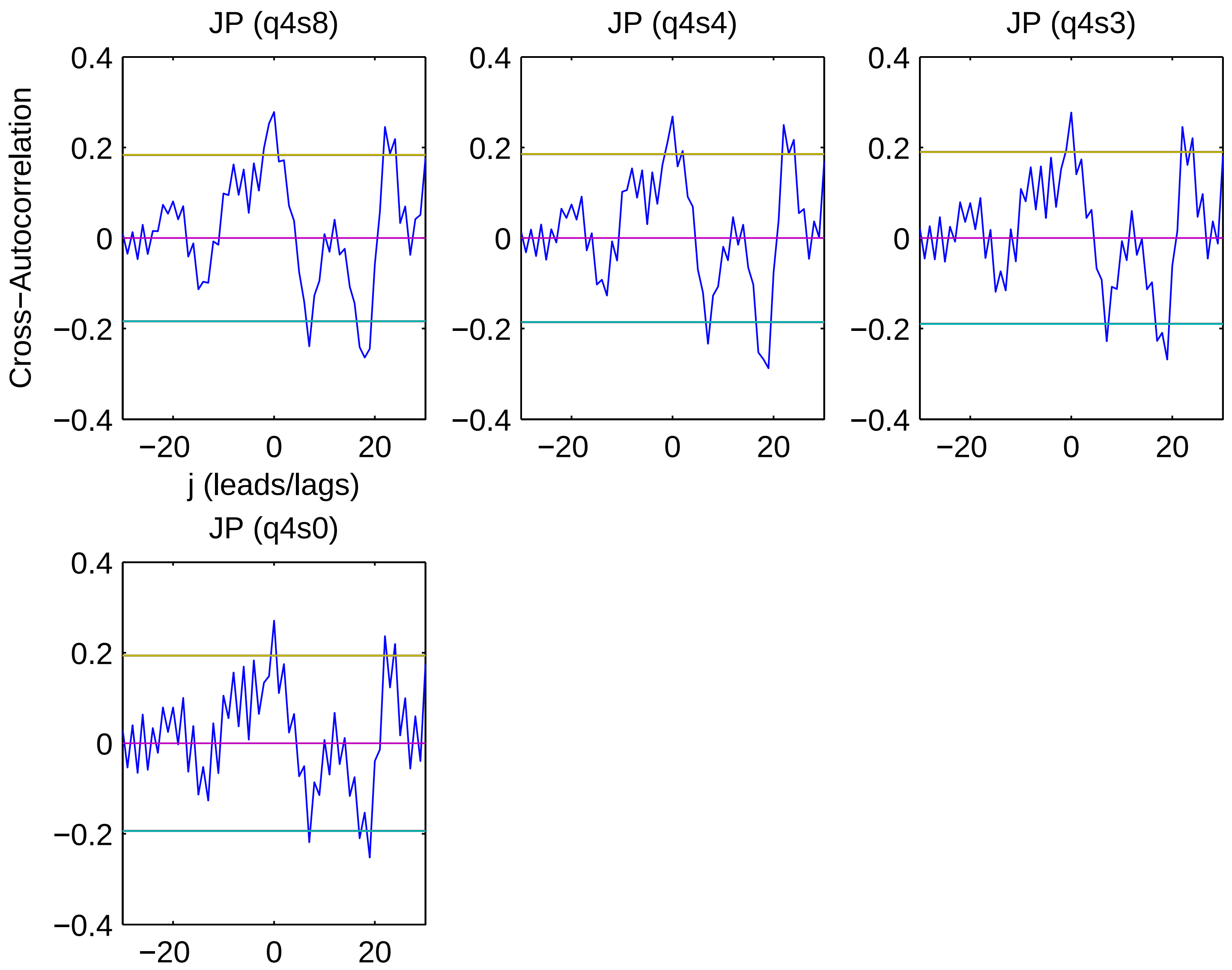

Figure 10: Cross-autocorrelates and the 95% confidence bounds. "JP($qisj$)" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the short-term interest rates for the US-Japan pair.

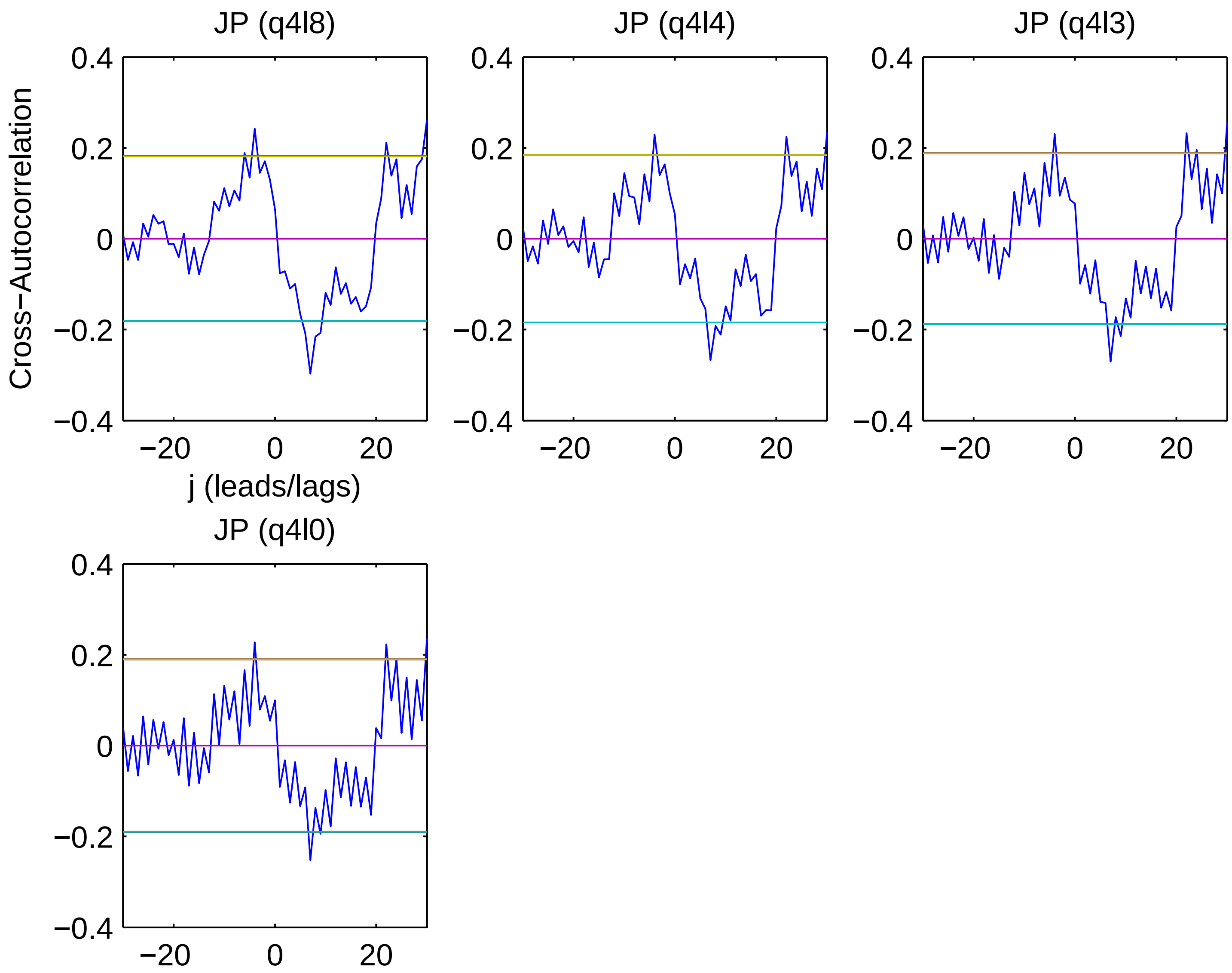

Figure 11: Cross-autocorrelates and the 95% confidence bounds. "JP($qilj$)" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the long-term interest rates for the US-Japan pair.

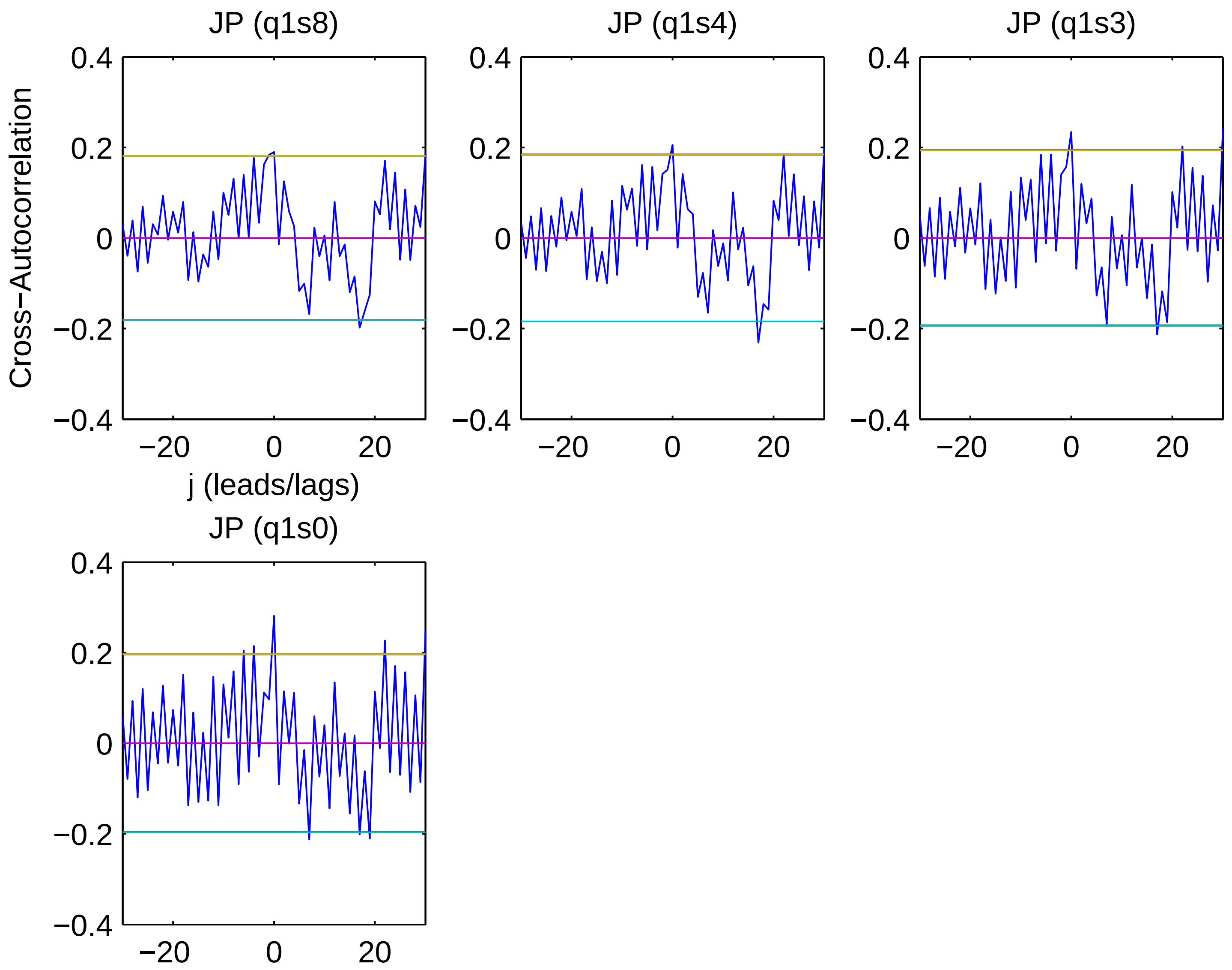

Figure 12: Cross-autocorrelates and the 95% confidence bounds. "JP($qisj$)" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the short-term interest rates for the US-Japan pair.

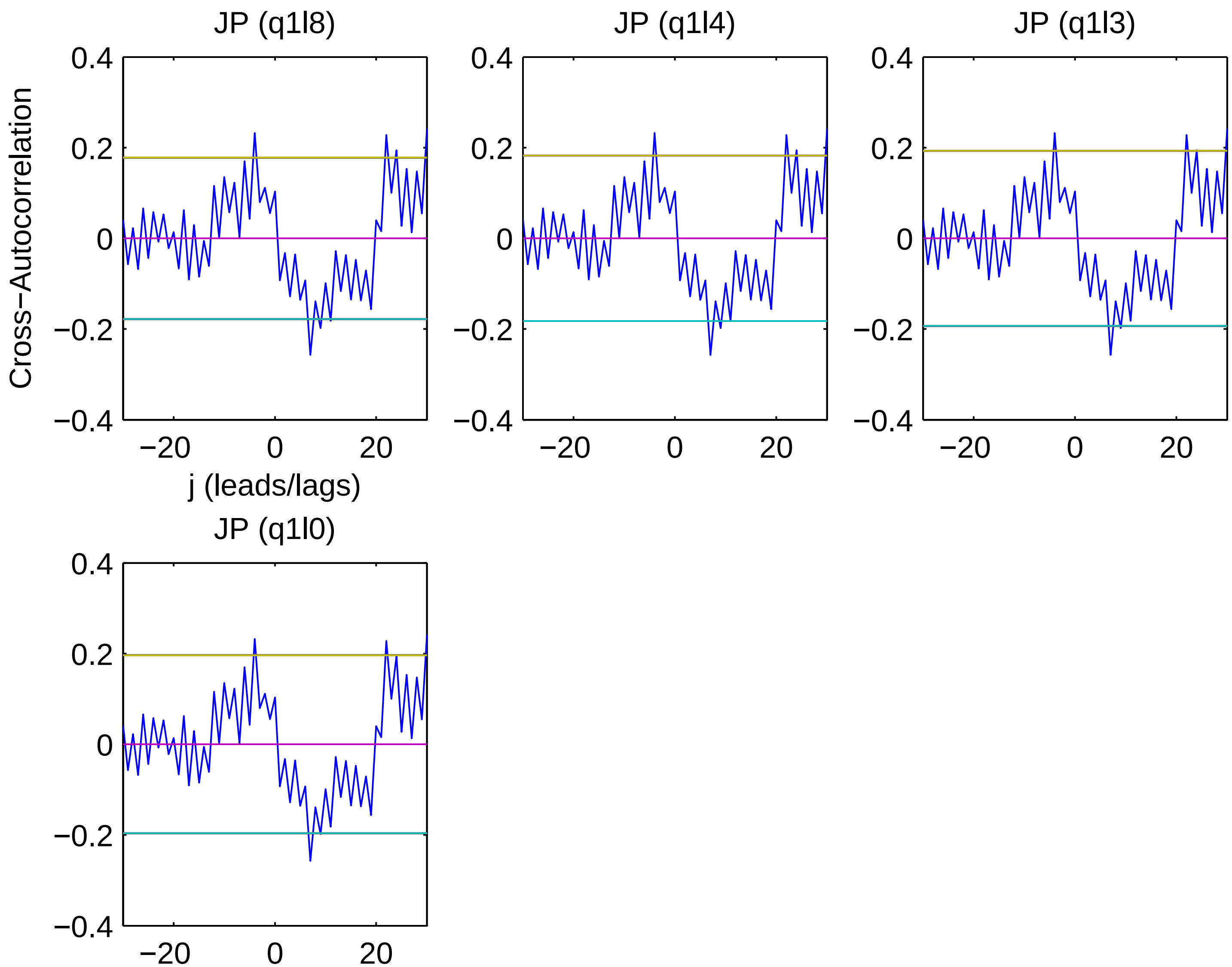

Figure 13: Cross-autocorrelates and the 95% confidence bounds. "JP($qilj$)" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the long-term interest rates for the US-Japan pair.

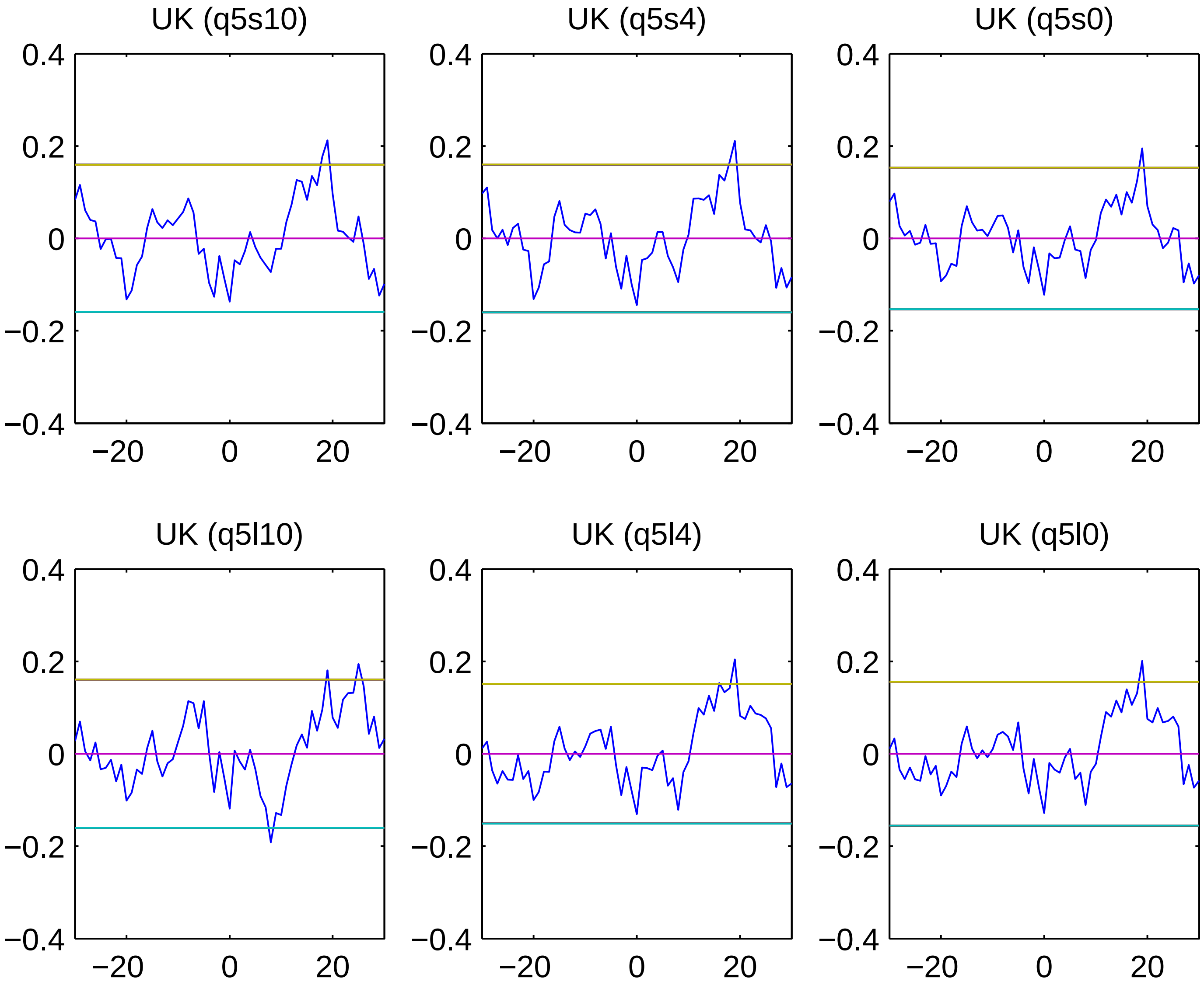

Figure 14: Cross-autocorrelates and the 95% confidence bounds. "UK($qisj$)" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the short-term interest rates for the US-UK pair.

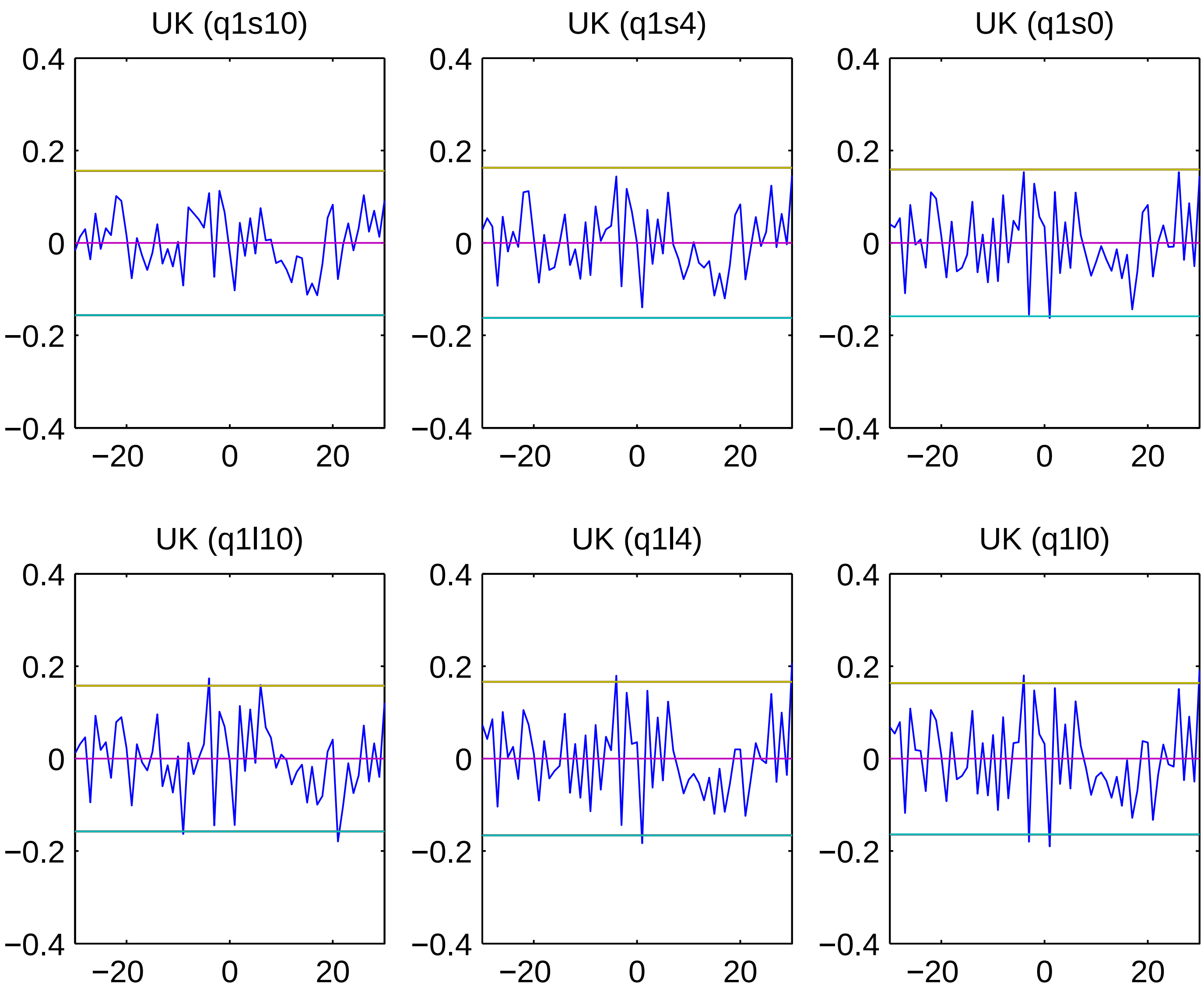

Figure 15: Cross-autocorrelates and the 95% confidence bounds. "UK($qisj$)" shows real exchange rate computed from an ARIMA(0,1,i) and the real interest differentials computed from an ARIMA(0,0,j) for the short-term interest rates for the US-UK pair.

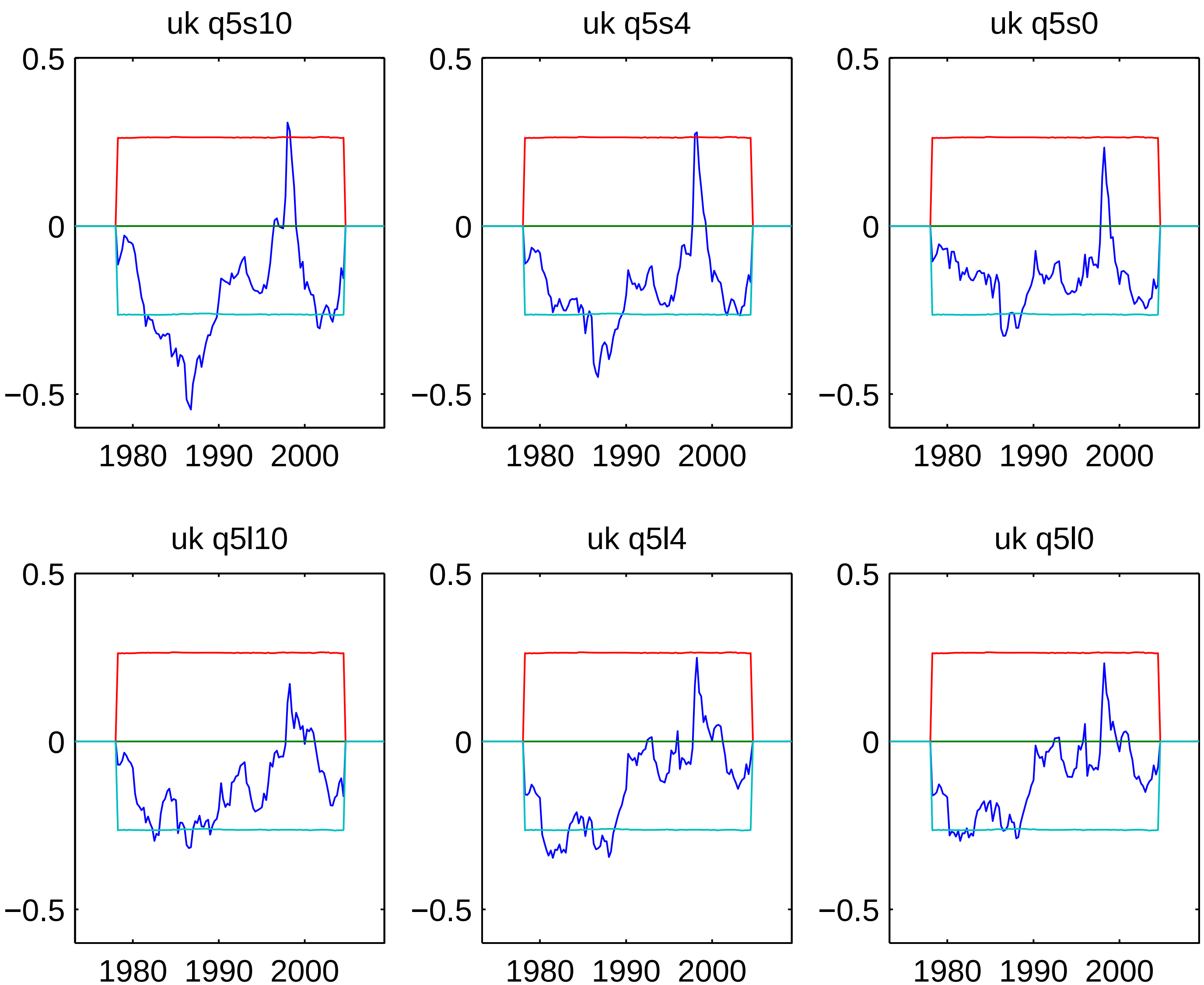

Figure 16: Rolling 10 years moving-average estimates of $\rho_{q^{cycle}D}(0)$ and its 95 percent bounds (box)

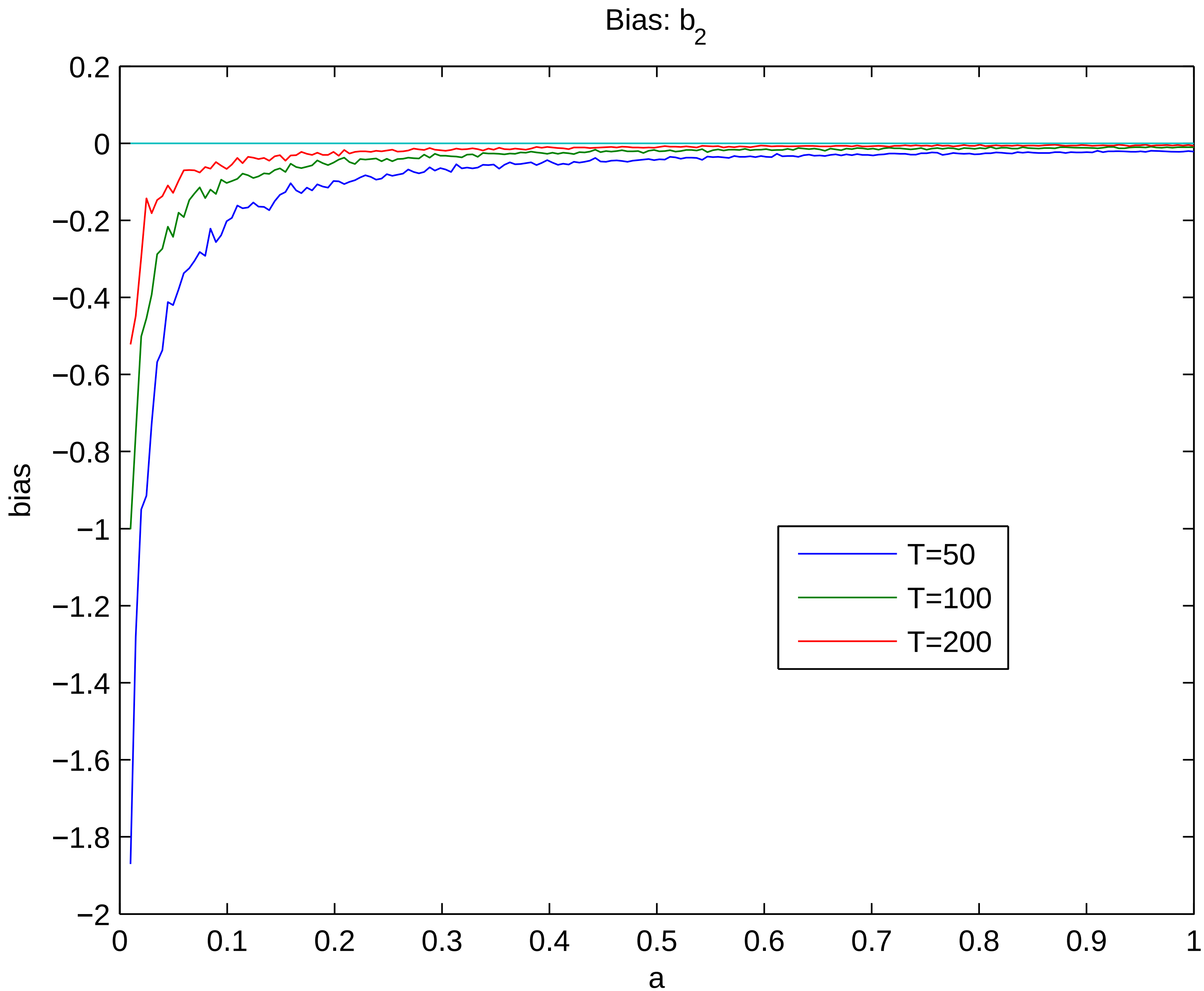

Figure 17: The estimated bias of $\widehat{b}_2$ and the misperception parameter a (1).