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Revisiting the Effect of Household Size on Consumption Over the Life-Cycle

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Abstract

Although the link between household size and consumption has a strong empirical support, there is no consistent way in which demographics are dealt with in standard life-cycle models. We study the relationship between the predictions of the Single Agent model (the standard in the literature) versus a simple model extension where deterministic changes in household size and composition affect optimal consumption decisions. We provide theoretical results comparing both approaches and quantify the differences in predictions across models.

Keywords: Consumption, Life-Cycle Models

JEL classification: D12, D91, E21, J10

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1 Introduction

Humps in household consumption are closely related to changes in its size and composition over the life-cycle.[1] However, the use of *Single Agent* or *bachelor* models is common place. For example, a standard approach in quantitative macroeconomics entails extracting per-adult equivalent consumption facts from household survey data and use them as targets to be replicated by *Single Agent* models, which for consistency are calibrated with per-adult equivalent income. Some recent papers in this vein include Heathcote, Storesletten, and Violante (2008) who assess the welfare effects of a rise in wage dispersion and the welfare gains of completing markets and eliminating income risk; Low and Pistaferri (2010) decompose changes in income risk using consumption data based on the predictions of a life-cycle model; Fernández-Villaverde and Krueger (2010) investigate the role of consumer durables for life-cycle consumption patterns.

There are numerous ways in which household consumption choices might differ from individual ones (e.g., because of two individuals choosing instead of one, the presence of children, uncertainty about household's compositional changes, etc.). Probably, the simplest way, henceforth labeled as the *Demographics* model, has been introduced by Attanasio, Banks, Meghir, and Weber (1999): household size and composition change deterministically over the life-cycle and affect consumption/savings choices in a unitary household model. Various specifications of that model have been used to study different questions in the literature: the welfare effects of different bankruptcy laws in Livshits, MacGee, and Tertilt (2007), the effects of German reunification on savings behavior in Fuchs-Schündeln (2008), and the optimality of day care subsidies in Domeij and Klein (2012) .

The objective of this paper twofold: first, we ask whether the *Single Agent* model is an innocuous shortcut of the *Demographics* model. Second, we document the implications of changing the relative importance of the size of economies of scale compared to the utility gains from consuming in households of different size/composition on an array of predictions of the *Demographics* model.

Using a two period version of our model, we show theoretically that the *Single Agent* model produces in general different predictions of per-adult equivalent consumption than the *Demographics* model. In terms of consumption growth rates, changes in family size shift the relative price of consumption across periods through the effect of economies of scale in consumption and direct

[1] See for example, Banks, Blundell, and Preston (1994), Deaton and Paxson (1994a), Attanasio and Weber (1995), Attanasio, Banks, Meghir, and Weber (1999), Gourinchas and Parker (2002), Browning and Ejrnæs (2009) among others.

preferences over household size in the *Demographics* model.[2,3] In the *Single Agent* model the relative price of consumption does not change because (by construction) household size does not change. Only when these two effects cancel each other out in the *Demographics* model, i.e. the relative price of consumption does not change, the predicted consumption growth rate is the same in the two model setups. This result is in general only true without income uncertainty. With uninsurable income uncertainty an additional channel unfolds its impact. An increase in household size between two periods, decreases the effective interest rate households face: next period assets plus capital gains have to be shared with more people. This in turn increases the resources required to provide a given level of per-adult equivalent consumption insurance in the *Demographics* model, which drives an additional wedge between the predictions of both models.

We then quantify the differences in predictions in a standard model of life-cycle consumption with income uncertainty and incomplete markets, similar to Storesletten, Telmer, and Yaron (2004). In this general framework we embed both the *Demographics* and *Single Agent* models. The models are calibrated using information on income and household composition from the US economy.

Using numerical simulations, we confirm our theoretical predictions for the case when the relative price of consumption across periods is unchanged by demographics in the *Demographics* model. The *Single Agent* model predicts a significantly flatter per-adult equivalent consumption profile over the life-cycle. In how far these differences affect the qualitative and quantitative conclusions in a given application (e.g. evaluating the welfare consequences of a policy reform) depends on the specific setup and cannot be generalized. Our exercise however makes clear that the *Single Agent* model is not an innocuous shortcut of the simplest model allowing demographics to affect choices, even under the most favorable conditions.

When we move away from the case above, we find significant differences for the different specifications of the *Demographics* model used in the literature (and in turn, to the *Single Agent* model). We document differences in (i) per-adult equivalent consumption life-cycle profiles, (ii) the degree of consumption smoothing with respect to contemporaneous shocks to income and (iii) welfare implications of a change in a policy that affects borrowing in the economy. For (ii) we follow Blundell, Pistaferri, and Preston (2008) and Kaplan and Violante (2010) and calculate model implied *insur-*

[2] These direct preferences over household size can be interpreted as how much the household values the utility of each member in a given period, or reflect the altruism of the household head towards other household members.

[3] The relative price change is analogous to the effect of the product of the discount factor and the gross interest rate in the standard Euler equation.

ance coefficients, statistics that relate income shocks to changes in consumption; for (iii), we use the model in Livshits, MacGee, and Tertilt (2007), to compare welfare under different bankruptcy laws. For all the exercises, we find that model predictions for statistics usually derived from standard incomplete market models depend crucially on the interaction between the degree of economies of scale in the household and how the utility of per-adult equivalent consumption of each household member is valued. For example, in the setup in Livshits, MacGee, and Tertilt (2007) the Continental European-style bankruptcy law is associated with higher welfare than the US-style bankruptcy law if households put little weight on each household member's utility. This result reverses as more weight is placed on each household member's utility holding the economies of scale in consumption fixed.

Although there is no clear indication of how to specify the *Demographics* model in applied work, some evidence can be extracted when we compare our setup with the preference structure estimated in Attanasio, Banks, Meghir, and Weber (1999). Conditional on their parameter estimates and a given equivalence scale, the implied weight on each household member's utility can be backed out. We do this for the equivalence scales considered in Fernández-Villaverde and Krueger (2007) and find that the specification of the *Demographics* model where household size changes do not change the relative price of consumption can be supported for low (old OECD scale) to medium (Square Root scale) economies of scale.

The structure of the paper is as follows: in Section 2 we discuss the preference structure and optimization problem for the *Demographics* and *Single Agent* model, and present theoretical predictions in a stylized two period framework. In Section 3 we layout the model used to quantify these theoretical predictions. Section 4 presents our main quantitative results. In Section 5 we conduct an identification exercise and we conclude in the last section.

2 A Two Period Model

2.1 Setup

Households live for two periods. Household size is normalized to one in the first period ($N_1 = 1$, e.g. a young person living alone) and increases deterministically in the second period ($N_2 > 1$, e.g. a child is born). For the theoretical analysis we only need a change of household size between the two

periods. The quantitative analysis will feature differences in household size and composition, i.e. a distinction between the number of adults and children in the household, and a realistic life-cycle length and demographic profile. Households receive income $\mathbb{Y}_1$ in the first period and $\mathbb{Y}_2$ in the second period. We first consider the case when $\mathbb{Y}_2$ is deterministic and introduce income uncertainty later onwards. Households can borrow up to the natural borrowing constraint at an interest rate r. For ease of exposition, we set the interest rate r to zero and the discount factor to one. While this obviously affects some of the analytical expressions we derive, it does not alter the qualitative findings regarding the differences between the *Demographics* and *Single Agent* model. Finally, we restrict our attention to utility functions that satisfy the Inada conditions and are strictly concave in consumption.

2.2 *Demographics* Model

In our benchmark model household size N_t affects the marginal utility of consumption, i.e. $u(\mathbb{C}_t^D, N_t)$. In particular, we assume a unitarian framework in which a household decision maker allocates household consumption $\mathbb{C}_t^D$ optimally to the two periods

$$\max_{\mathbb{C}_1^D, \mathbb{C}_2^D} U = u(\mathbb{C}_1^D, N_1) + u(\mathbb{C}_2^D, N_2) \tag{1}$$

subject to

$$\mathbb{C}_1^D + \mathbb{C}_2^D = \mathbb{Y}_1 + \mathbb{Y}_2. \tag{2}$$

We specify the utility function as

$$u(\mathbb{C}_t^D, N_t) = \delta(N_t) u\left(\frac{\mathbb{C}_t^D}{\phi(N_t)}\right). \tag{3}$$

There is no private consumption. The equivalence scale $\phi(N_t)$ transforms household consumption into per-adult equivalent consumption, is normalized to one for $N_t = 1$ and increases in household size by a factor smaller than one.[4] $\delta(N_t)$ can be best interpreted as aggregating up the individual

[4]Formally, $\phi_t(1) = 1$ and $\frac{\partial \phi_t(N_t)}{\partial N_t} \in (0, 1)$. The three mechanisms through which household size affects the intra-temporal rate of transformation between expenditures and consumption services, and that are captured partially through equivalence scales, are family/public goods, economies of scale, and complementarities, see e.g. Lazear and Michael (1980). As a concrete example, consider the widely used OECD equivalence scale which is given by $\phi_{OECD} = 1 + 0.7(N_{ad} - 1) + 0.5N_{ch}$ with N_{ad} being the number of adults and N_{ch} the number children in the

utilities from per-adult equivalent consumption $u\left(\frac{\mathbb{C}_t^D}{\phi(N_t)}\right)$ of all household members, or alternatively, as a parameter reflecting altruism towards other household members by the household decision maker. E.g. if the household planner assigns each household member i (including herself) a weight $\bar{\delta}_i \in [0,1]$ then $\delta(N_t) = \sum_{i=1}^{N_t} \bar{\delta}_i$.[5] While there are certainly more elaborate models of the household,[6] we use this formulation as it nests various specifications used in recent contributions in quantitative macroeconomics. Livshits, MacGee, and Tertilt (2007), Attanasio, Low, and Sanchez-Marcos (2008) set $\delta(N_t) = 1$ such that in each period the household planer maximizes per-capita utility or alternatively the household planer does not have any altruism towards the remaining household members. Fuchs-Schündeln (2008) and Laitner and Silverman (2012) use $\delta(N_t) = \phi(N_t)$, Heathcote, Storesletten, and Violante (2012) use the number of adults in the household and Domeij and Klein (2012) the total number of household members. Attanasio, Banks, Meghir, and Weber (1999), and Gourinchas and Parker (2002) employ CRRA preferences and use a more general taste shifter

$$u(\mathbb{C}_t, N_t) = \exp(\xi_1 N_{ad,t} + \xi_2 N_{ch,t}) \frac{\mathbb{C}_t^{1-\alpha}}{1-\alpha}, \tag{4}$$

which for given preference parameters α, ξ_1, ξ_2 and a specific equivalence scale maps directly into a corresponding δ. In fact, in Section 5 we will use the preference parameter estimates from Attanasio, Banks, Meghir, and Weber (1999) to back out δ for various equivalence scales.

household. This can be interpreted as follows: it takes \$1.7 of consumption expenditures to generate the same level of welfare out of consumption for a two adult household that \$1 achieves for a single member household. Lewbel (1997) provides an extensive survey with a particular emphasis on definition, identification, estimation and use of equivalence scales. The scales used in actual policy making are rather simple weighting schemes as e.g. the OECD scale, see Citro and Michaels (1995) for a lengthy discussion. We follow the macroeconomic literature on income and consumption inequality and use these latter equivalence scales.

Two recent macroeconomic papers took a different approach. Salcedo, Schoellman, and Tertilt (2012) develop a theory and quantitative model of household formation in which equivalence scales are an endogenous outcome of the household size, and the expenditure shares on private and public consumption goods. As households get richer they become smaller and spend more on private goods which decreases the "measured" economies of scale. Hong and Ríos-Rull (2012) estimate household-type specific equivalence scales in the context a life-cycle model with stochastic changes in household size and composition using life-insurance holdings.

[5] We impose the following properties on δ_t: $\delta_t(1) = 1$ and $\frac{\partial \delta_t(N_t)}{\partial N_t} \in [0,1]$. In contrast to the equivalence scale, we allow that an additional household member may increase δ_t by one or not at all.

[6] See for example, Greenwood, Guner, and Knowles (2003), Mazzocco, Ruiz, and Yamaguchi (2007), or Hong and Ríos-Rull (2007).

2.3 *Single Agent* Model

A more common approach is to assume that households consist only of a single member in any period. At least since the empirical work by Attanasio and Weber (1993) and Attanasio and Browning (1995) it is well understood that household size changes are important for understanding the patterns of household consumption over the life-cycle. A response to these findings is to clean household consumption data for household size and household composition, i.e. demographic, effects. One popular approach is to transform total household consumption in the data into a per-adult equivalent consumption by the division with an equivalence scale. The predictions of the *Single Agent* model are then compared to the empirical per-adult equivalent consumption, see e.g. Blundell, Low, and Preston (2011), Heathcote, Storesletten, and Violante (2008) or Low and Pistaferri (2010).[7] To ensure consistency between the model and the data, income fed into the model is cleaned as well for household size and household composition effects. One popular approach, in particular in the inequality literature, is to divide household income by the same equivalence scale used for consumption as done by Cutler and Katz (1992), Krueger and Perri (2006), Blundell, Low, and Preston (2011), Meyer and Sullivan (2010), and the 2010 special issue of the Review of Economic Dynamics (Krueger, Perri, Pistaferri, and Violante (2010)). One alternative is to use only the household heads income as done e.g. by Heathcote, Storesletten, and Violante (2008), Low and Pistaferri (2010), and Kaplan (2012). We adjust income with the factor $\kappa(N_t)$ as a stand-in for these different empirical strategies to obtain a per-adult equivalent income. In all considered cases $\kappa(N_t)$ is normalized to one for a household of size one and the household chooses per-adult equivalent consumption c_t^S to solve the following optimization problem

$$\max_{c_1^S, c_2^S} U = u\left(c_1^S\right) + u(c_2^S) \tag{5}$$

subject to

$$c_1^S + c_2^S = \frac{\mathbb{Y}_1}{\kappa_1(N_1)} + \frac{\mathbb{Y}_2}{\kappa_2(N_2)}. \tag{6}$$

The key distinctive feature between the two setups is how demographics affect the optimization

[7] There are obviously other methods to create per-adult equivalent information from household data. One alternative method is to estimate household size/composition effects directly from micro data using least squares regressions, see e.g. Aguiar and Hurst (2009). Although studying heterogeneity in household size/composition (which is a pre-requisite to understand the regression methodology) is beyond the scope of our paper, this approach generates adjustments that can be trivially converted to an ad-hoc equivalence scales.

problem. The *Single Agent* model abstracts from household (changes) inside the model, but is 'calibrated' in a fashion that controls for these effects outside the model. Household effects enter only via the budget constraint. In contrast, in the *Demographics* model household size changes affect utility directly but not via the budget constraint.

It is straightforward to make the optimal consumption allocations comparable. The *Single Agent* model directly predicts per-adult equivalent consumption (in our notation the lower case letter c) because the household receives a per-adult equivalent income. In the *Demographics* model, household consumption (in our notation an upper case letter $\mathbb{C}$) is predicted which can be easily transformed into per-adult equivalent consumption by deflating with the equivalence scale ϕ_t, i.e. $\frac{\mathbb{C}_1^D}{\phi_1} = \frac{\mathbb{C}_1^D}{1}$ and $\frac{\mathbb{C}_2^D}{\phi_2}$.

The objective of the following paragraphs is to state the conditions under which the *Single Agent* model can replicate the per-adult equivalent consumption choices obtained from the *Demographics* model.[8]

2.4 Consumption Levels

Proposition 1. *Life-time per-adult equivalent consumption in the* Demographics *model and* Single Agent *model coincide if and only if* $\kappa_2 = \frac{\phi_2}{1-(\phi_2-1)\frac{\mathbb{Y}_1}{\mathbb{Y}_2}\left(1-\frac{\mathbb{C}_1^D}{\mathbb{Y}_1}\right)}$.

This result is straightforward to show. Life-time per-adult equivalent consumption in the *Demographics* model $\mathfrak{c}_D$ is given by

$$\mathfrak{c}_D = \mathbb{C}_1^D + \frac{\mathbb{C}_2^D}{\phi_2} = \mathbb{C}_1^D + \frac{\overbrace{\mathbb{Y}_1 + \mathbb{Y}_2 - \mathbb{C}_1^D}^{=\mathbb{C}_2^D}}{\phi_2} = \frac{\phi_2 - 1}{\phi_2}\mathbb{C}_1^D + \frac{\mathbb{Y}_1 + \mathbb{Y}_2}{\phi_2} \tag{7}$$

while in the *Single Agent* model life-time per-adult equivalent consumption $\mathfrak{c}_S$ equals life-time per-adult equivalent income (see also Equation (6)):

$$\mathfrak{c}_S = c_1^S + c_2^S = \mathbb{Y}_1 + \frac{\mathbb{Y}_2}{\kappa_2}. \tag{8}$$

[8] Note that this is different from the exercise in Attanasio, Banks, Meghir, and Weber (1999). They emphasize the importance of demographics, via affecting the marginal utility, for explaining the life-cycle profile of household consumption.

Equating Equations (7) and (8) yields the critical value of κ_2 stated in Proposition 1. Although in quantitative work life-time (per-adult equivalent) consumption is not a statistic of primary interest, we care implicitly about it when conducting welfare comparisons between different economic environments. If e.g. a linear income tax rate is raised to finance wasteful government consumption, households in the economy with the lower, pre-reform life-time per-adult equivalent consumption suffer a larger welfare loss with concave utility.

The critical value of κ_2 in Proposition 1 has an intuitive interpretation. One can directly see from Equations (7) and (8) that when the household in the *Demographics* model optimally chooses to neither save nor borrow ($\mathbb{C}_1^D = \mathbb{Y}_1$), the life-time per-adult equivalent consumption levels in both setups are the same if and only if $\kappa_2 = \phi_2$. Now consider the case when the household in the *Demographics* model optimally chooses to be a borrower ($\mathbb{C}_1^D > \mathbb{Y}_1$). Assume for a moment that κ_2 is still equal to ϕ_2 instead of the critical value stated in Proposition 1. In the *Demographics* model only period two consumption — which in this example is smaller than period two income — is deflated by the equivalence scale in the calculation of life-time per-adult equivalent consumption. This directly implies a higher life-time per-adult equivalent consumption level in the *Demographics* model compared to the *Single Agent* model which is determined only by life-time per-adult equivalent income. To undo this, κ_2 has to be decreased below ϕ_2 in order to increase life-time per-adult equivalent income and thus life-time per-adult equivalent consumption in the *Single Agent* model.

The derivation and implications of Proposition 1 only depend on the choice of δ_2 in so far that it determines $\mathbb{C}_1^D$ and thus, for a given $\mathbb{Y}_1$ and $\mathbb{Y}_2$, the relationship between the two per-adult equivalent consumption levels.[9]

2.5 Consumption Growth Rate

Proposition 2. *The per-adult equivalent consumption growth rates in the* Demographics *model and* Single Agent *model are the same if and only if* $\delta_2 = \phi_2$.

This result can be immediately read of from the two Euler equations for the *Demographics*

[9] Another interesting implication arises in the presence of heterogeneity in the timing of household income, i.e. $\frac{y_1^A}{y_2^A} \neq \frac{y_1^B}{y_2^B}$. Even for the same life-time household income $y_1^A + y_2^A = y_1^B + y_2^B$, life-time per-adult equivalent incomes differ in the *Single Agent* but not in the *Demographics* model. This implies an artificial inequality in life-time per-adult equivalent consumption in the former model that is not present in the latter.

model (9) and *Single Agent* model (10):

$$u'(\mathbb{C}_1^D) = \frac{\delta_2}{\phi_2} u'\left(\frac{\mathbb{C}_2^D}{\phi_2}\right) \tag{9}$$

$$u'(c_1^S) = \quad u'(c_2^S). \tag{10}$$

In both first-order conditions only per-adult equivalent consumption appears. Equation (10) implies no per-adult equivalent consumption growth in the *Single Agent* model, i.e. $c_1^S = c_2^S$ which in the *Demographics* model occurs if and only if $\delta_2 = \phi_2$. If $\delta_2 > \phi_2 \;\Rightarrow\; \mathbb{C}_1^D = \frac{\mathbb{C}_1^D}{\phi_1} < \frac{\mathbb{C}_2^D}{\phi_2}$, or in words: per-adult equivalent consumption grows over time, while the opposite is true for $\delta_2 < \phi_2$.[10] The intuition behind this result can be best explained when decomposing the benefit of consuming one additional unit of household consumption in the second period in the *Demographics* model which

1. is associated with the marginal utility of per-adult equivalent consumption in period two $\left[u'\left(\frac{\mathbb{C}_2^D}{\phi_2}\right)\right]$
2. accrues to all household members reflected through the multiplication by the weighting or altruism factor $[\delta_2]$
3. has to be divided by the equivalence scale $[\phi_2]$ because each household member does not get the full unit to consume but only the fraction $\frac{1}{\phi_2}$.

As an example, consider the case of $\delta_2 = N_2 > \phi_2$. The larger household size in period two provides an incentive to allocate more consumption to period two. The household enjoys a larger utility from consuming because each unit of per-adult equivalent consumption is weighted by $\delta_2 = N_2$. However, in period two every unit of consumption has to be shared with more people which is reflected through the division with the equivalence scale ϕ_2. This in turn reduces the incentive to allocate more consumption to period two although the presence of economies of scale lowers this cost. Relative to period one, the absolute loss in consumption in period two is nevertheless outweighed by the fact that each household member enjoys extra per-adult equivalent consumption.

[10] Note that in the *Demographics* model *household* consumption may nevertheless increase even if $\delta_2 < \phi_2$. E.g., with CRRA preferences this is the case as long as the coefficent of relative risk aversion is larger than $1 - \frac{\ln\delta_2}{\ln\phi_2}$.

The ratio $\frac{\delta_2}{\phi_2}$ has a similar interpretation as $\beta(1+r)$ in standard Euler equations: it changes the effective discount factor in the Euler equation, or alternatively, it changes the relative price of per-adult equivalent consumption between two periods whenever there is a change in household size. This 'demographic' channel is absent in the *Single Agent* model and only irrelevant in the *Demographics* model if $\delta_2 = \phi_2$. In this case, the two modeling approaches yield the same predictions for the per-adult equivalent consumption profiles. In Section 5 we discuss some empirical estimates for δ_2.

Note that this result is completely independent from κ_2 which is used to construct per-adult equivalent income used to calibrate the *Single Agent* model. This is of course not true for the level of consumption which depends on κ_2, as shown in the previous subsection.

2.6 Consumption Growth Rates and Income Uncertainty

This section introduces income uncertainty, the empirically more relevant case, in the most simplistic fashion. Period two income can take two values: $\mathbb{Y}_{2,l}$ with probability p_l and $\mathbb{Y}_{2,h}$ with probability $p_h = 1 - p_l$, where $\mathbb{Y}_{2,h} > \mathbb{Y}_{2,l}$. Households are (as before) only allowed to borrow what can be repaid for sure, i.e. at most $\mathbb{Y}_{2,l}$. To highlight the key distinction between the *Demographics* and *Single Agent* model in the presence of income uncertainty, we consider the more general case of a non-zero interest rate and a discount factor β unequal to one. The implied Euler equations for the *Demographics* model (11) and *Single Agent* model (12) are given by:

$$u'\left(\mathbb{C}_1^D\right) = \beta(1+r)\frac{\delta_2}{\phi_2}\sum_{i=l,h} p_i u'\left(\overbrace{\left[\mathbb{Y}_1 - \mathbb{C}_1^D\right]\frac{1+r}{\phi_2} + \frac{\mathbb{Y}_{2,i}}{\phi_2}}^{=\mathbb{C}_2^D/\phi_2}\right) \tag{11}$$

$$u'\left(c_1^S\right) = \beta(1+r)\sum_{i=l,h} p_i u'\left(\underbrace{\left[\mathbb{Y}_1 - c_1^S\right](1+r) + \frac{\mathbb{Y}_{2,i}}{\kappa_2}}_{=c_2^S}\right) \tag{12}$$

where period two consumption in each Euler equation has been replaced with the respective life-time budget constraint from the *Demographics* (13) and *Single Agent* model (35):

$$\mathbb{C}_1^D + \frac{\mathbb{C}_2^D}{1+r} = \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{1+r} \qquad \forall\ i = l, h \tag{13}$$

$$c_1^S + \frac{c_2^S}{1+r} = \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{(1+r)\kappa_2} \ \forall\ i = l, h \tag{14}$$

The standard pre-multiplication factor $\beta(1+r)$ in the Euler equations (11) and (12) is not introducing any differences between the two model setups. The replacement of period two per-adult equivalent consumption with the respective life-time budget constraint provides the key insight. While in both models period two per-adult equivalent income (i.e. deflated with the respective equivalence scale) shows up in the marginal utility on the right hand side, period one assets (i.e. period one income less period one consumption) are multiplied with two different effective interest rates. The return to savings is only $\frac{1+r}{\phi_2}$ in the *Demographics* model whereas it is $1+r$ in the *Single Agent* model. When household size is increasing, it is more expensive (cheaper) to save (borrow) in the *Demographics* model compared to the *Single Agent* model because of the lower effective interest rate payments received (to be paid). This generates differences in the resources required to provide insurance against the low income shock and drives an additional wedge between the two models independent of the ratio of δ and ϕ.[11]

In the subsequent analysis, we return to our simplified setting with $r = 0$ and $\beta = 1$. Again, this simplification is only made for the ease of exposition and does not alter the qualitative results regarding the differences between the two model setups.

Proposition 3. *If there is income uncertainty and the optimal per-adult equivalent consumption growth rates in the* Demographics *and* Single Agent *model are the same, then* $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1^D}$.

The detailed proof of Proposition 3 is given in Appendix A.1. It presumes that the Euler equations Equations (11) and (12) are satisfied (same per-adult equivalent consumption growth rate) and that the budget constraints hold with equality (ensures feasibility and thus optimality).[12]

[11] The difference in the effective interest rates in the two models is also present if there is no income uncertainty but does not affect the choice of consumption growth.

[12] This implies that per-adult equivalent consumption growth rates are the same in *each* state of the world. We follow this strategy since it buys us sharp analytical conditions to ensure this equality. If we rather opted for equality of the *expected* per-adult equivalent consumption growth rate, we would not be able to provide analytical results.

Therefore Proposition 3 constitutes only a necessary condition, i.e. there exist at most one κ_2 for which the optimal per-adult equivalent consumption profiles in the two models are the same.

Before discussing a sufficient condition, we briefly provide the intuition for the critical value of κ_2 in Proposition 3 takes: If the household in the *Demographics* model neither saves nor borrows, κ_2 equals ϕ_2. If the household in the *Demographics* model is a saver, the household receives a lower effective interest rate than the household in the *Single Agent* model. This makes it (as described above) relatively more expensive to provide insurance because of the lower return to savings. To counteract this, κ_2 has to decrease below ϕ_2. Lowering κ_2 has two opposing effects: first, it increases period two per-adult equivalent income in any state with a positive income; second, it increases income uncertainty in period two. Key is that income uncertainty relative to expected life-time per-adult equivalent income increases. This can be seen from the coefficient of variation:

$$CV^S = \frac{S.D.(\mathbb{Y}_1 + \mathbb{Y}_{2,i}/\kappa_2)}{\mathbb{Y}_1 + \sum_{i=l,h} p_i \mathbb{Y}_{2,i}/\kappa_2} = \frac{S.D.(\mathbb{Y}_{2,i})}{\kappa_2 \mathbb{Y}_1 + \sum_{i=l,h} p_i \mathbb{Y}_{2,i}}. \tag{15}$$

Put differently, if the household in the *Demographics* model saves and faces a lower effective return on these savings, the household in the *Single Agent* model has to be endowed with a more risky income process (relative to a choice of $\kappa_2 = \phi_2$).[13]

The two next propositions, partly building on Proposition 3, are together the counterpart of Proposition 2 for the case of income uncertainty. To obtain analytical solutions, we need to restrict our attention to utility functions whose first derivative u' is homogenous of degree q.[14]

Proposition 4. *If u' is homogenous of degree q and $\delta_2 \neq \phi_2$, then the optimal per-adult equivalent consumption growth rates in the* Demographics *and* Single Agent *model are* **never** *the same.*

Assume to the contrary, the optimal growth rates would be the same, i.e.

$$c_1^S = \eta \mathbb{C}_1^D \text{ and } c_{2,i}^S = \eta \frac{\mathbb{C}_{2,i}^D}{\phi_2} \ \forall\ i = l, h \text{ with } \eta > 0. \tag{16}$$

Plugging allocation (16) in the Euler equation (12) for the *Single Agent* model and using the

[13]Note that for $\mathbb{Y}_1$=0, the critical value for κ in Proposition 3 is not defined. Put differently, in this case the per-adult equivalent consumption growth rates will never be the same. In both models, the household has to borrow but the in the *Demographics* model the effective interest rate is smaller. Varying κ does however not change relative income risk any longer if $\mathbb{Y}_1 = 0$, see Equation (15).

[14]This includes CRRA preferences, the most commonly used utility function in the quantitative macro literature. The following results however also apply to quadratic utility, although the first derivative is not homogenous.

homogeneity assumption yields

$$\eta^q u'\left(\mathbb{C}_1^D\right) = \eta^q \sum_{i=l,h} p_i u'\left(\frac{\mathbb{C}_{2,i}^D}{\phi_2}\right). \tag{17}$$

Comparing Equation (17) and the Euler equation in the *Demographics* model (11), it is obvious that both cannot hold jointly at the allocation (16) if $\delta_2 \neq \phi_2$.[15] It is important to mention that Proposition 4 is independent of the choice of κ_2. This underpins the claim made above: Proposition 3 constitutes only a necessary condition in the sense that there exists at most one κ_2 for which the per-adult equivalent consumption growth rates in the two models are the same. In fact, Proposition 4 is an example where no such κ_2 exists. We will now state a sufficient condition under which the per-adult equivalent consumption profiles in the two models are the same.

Proposition 5. *If u' is homogenous of degree q, $\delta_2 = \phi_2$ and $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1^D}$, then the optimal per-adult equivalent consumption growth rates from the* Demographics *model and the* Single Agent *model are the same.*

The detailed proof of Proposition 5 is given in Appendix A.2. Building on Proposition 3, Proposition 5 is a sufficient condition for the per-adult equivalent consumption growth rates in the two models being the same. Propositions 4 and 5 together are the counterpart of Proposition 2 for the case of income uncertainty. However, it takes more than only $\delta_2 = \phi_2$ for the *Single Agent* model to generate the same per-adult equivalent consumption growth rates as in the *Demographics* model: first, homogeneity of u'; second, a specific value of κ_2 which depends on the optimal consumption choice in the *Demographics* model.[16]

From this discussion, it is clear that the predictions of both models are generically different and that it takes a very convoluted combination of parameters to align them. Here we discussed results for a two period model, but these parameter restrictions are highly dependent on model characteristics (model periods, type of income, etc.) and no general rule exists for every single case. This is specifically true in an environment where household size changes non-monotonically over the life-cycle, e.g. as we typically see in the data with an increase in the early part of the life-cycle and decrease later in life.

[15] Recall that we have returned to the simplified setting with $r = 0$ and $\beta = 1$.

[16] In fact, Propositions 4 and 5 are a special case of Proposition 2 and also hold without income uncertainty.

3 Quantitative Model

3.1 Basic Setup

We now move from the two period model to a full life-cycle model, which emulates basic facts of the US economy, in terms of earnings processes and family size and composition. Here we present a standard incomplete markets life-cycle model, which follows closely the one in Storesletten, Telmer, and Yaron (2004) and Kaplan and Violante (2010), to investigate how the predictions of the theoretical section generalize to a more realistic setting, which cannot be solved analytically.

For simplicity we abstract from population growth and general equilibrium effects. In the model, households start their economic life in period t_0 with zero assets.[17] During their working life until period t_w they receive a stochastic income in every period. There is no labor supply choice. From period $t_w + 1$ onwards households are retired and have to live from their accumulated savings during working life and social security benefits. Life ends with certainty at age T but there is an age specific survival probability ζ_t. Households do not leave bequests and cannot die with debt. Households have access to a risk-free bond a which pays the interest rate r and can borrow at the same interest rate up to the natural borrowing constraint, i.e. an age specific level $a_{min,t}$ of debt that they can repay for sure. As in Storesletten, Telmer, and Yaron (2004), we introduce actuarially fair annuity markets so that savings of the dying population are redistributed equally among the surviving members of their cohort.

In the *Demographics* model, household size changes over the life-cycle deterministically as in Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002) and is homogenous

[17] While it is per se interesting to compare the implications for the equilibrium interest rate in the different setups, we refrain from doing so because we want to document the (mean of the) individual behavior between different model specifications in the same economic environment. Differences in the equilibrium interest rate essentially tilt the respective per-adult equivalent consumption profile.

across all households. The maximization problem is given by

$$\max_{\{a_{t+1}\}_{t=t0}^{T-1}} E_0 \sum_{t=t0}^{T} \left(\Pi_{j=t_0}^{j=t} \zeta_j\right) \beta^{t-t_0} \ \ \delta_t u\left(\frac{c_t}{\phi_t}\right) \tag{18}$$

$$\text{subject to} \tag{19}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + (1-\tau)y_t \quad \forall\ t \leq t_w \tag{20}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + p(\bar{y}) \qquad \forall\ t_w < t \leq T \tag{21}$$

$$a_{t+1} \geq a_{min,t} \tag{22}$$

where δ and ϕ are functions of household size and its composition ($N_{ad,t}$ and $N_{ch,t}$) over the life-cycle.[18] Pre labor tax income y_t is stochastic during the working life, i.e. as long as $t \leq t_w$, and is given by the following process:

$$\ln\ y_t = \varrho_t + \epsilon^F + z_t + \epsilon_t^{Tr} \tag{23}$$

where ϱ_t is an age-dependent, exogenous experience profile (common to all individuals), ϵ^F is a fixed effect drawn by households at the beginning of economic life from a normal distribution with mean zero and variance σ_F^2, z_t is a permanent shock to labor income, with

$$z_t = \rho z_{t-1} + \epsilon_t^P \text{ with } \epsilon_t^P \sim N(0, \sigma_P^2) \tag{24}$$

and $\epsilon^{Tr} \sim N(0, \sigma_{Tr}^2)$ is a transitory shock. Finally, we assume that the social security system is financed through linear labor taxes τ.

During retirement (for $t_w < t \leq T$) households receive age independent social security contributions $y_t = p(\bar{y})$ that depend on the realization of income over the working life: $\bar{y} = \frac{1}{t_w - t_0 + 1} \sum_{j=t_0}^{t_w} y_j$.

The Euler equation to this problem is given by

$$\frac{\delta_t}{\phi_t} u'\left(\frac{c_t}{\phi_t}\right) = \beta(1+r)\frac{\delta_{t+1}}{\phi_{t+1}} E_t\left[u'\left(\frac{c_{t+1}}{\phi_{t+1}}\right)\right] \tag{25}$$

The structure of the *Single Agent* problem is very similar. Demographics do not affect the marginal

[18] Note that in contrast to Section 2, we now denote all variables with lower case letters.

utility of consumption while income y_t is deflated by household size through equivalence scales κ_t:

$$\max_{\{a_{t+1}\}_{t=t0}^{T-1}} E_0 \sum_{t=t0}^{T} \left(\Pi_{j=t_0}^{j=t} \zeta_j\right) \beta^{t-t_0} \; u\left(c_t\right) \quad \text{subject to} \tag{26}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + (1-\tau)\frac{y_t}{\kappa_t} \quad \forall\, t \leq t_w \tag{27}$$

$$c_t + a_{t+1} \leq \frac{a_t(1+r)}{\zeta_t} + \frac{p(\bar{y})}{\kappa_t} \qquad \forall\, t_w < t \leq T \tag{28}$$

$$a_{t+1} \geq a_{\min,t}, \tag{29}$$

with y_t following the same process as for the *Demographics* model given by Equations (23) and (24).[19] The Euler equation to this problem is given by

$$u'\left(c_t\right) = \beta(1+r)E_t\left[u'\left(c_{t+1}\right)\right] \tag{30}$$

In line with studies that investigate jointly income and consumption inequality, see e.g. Cutler and Katz (1992), Krueger and Perri (2006), Meyer and Sullivan (2010), and the 2010 special issue of the Review of Economic Dynamics, we use the same equivalence scale for computing per-adult equivalent income and per-adult equivalent consumption ($\kappa_t = \phi_t$).

3.2 Quantitative Features of the Model

A model period is one year. Agents start life at age 25, retire when 65 and live until age 95 after which they die with certainty. The common profile for survival probabilities comes from the National Center for Health Statistics.[20] To maintain comparability across models, we keep some parameters fixed:[21] we pick common choices from the literature and set the interest rate at 2%, the discount factor β at 0.96 and work with CRRA preferences ($\alpha = 2$)

$$u = \delta(N_{ad,t}, N_{ch,t}) \frac{\left(\frac{c_t}{\phi(N_{ad,t}, N_{ch,t})}\right)^{1-\alpha}}{1-\alpha}. \tag{31}$$

[19] For consistency with the *Demographics* model, the pension in the *Single Agent* model is based on the realizations of household income and not per-adult equivalent income over the life-cycle and turned in per-adult equivalent income through the division by κ during retirement.

[20] http://www.cdc.gov/nchs/data/lifetables/life90_2acc.pdf

[21] Alternatively, we could have calibrated each model to match a certain target, e.g. setting the discount factor β to replicate the empirical wealt-income ratio. Holding preferences constant instead of certain properties of the partial equilibrium economy reflects more the spirit of our theoretical exercise.

Of course, for the *Single Agent* model δ and ϕ are not part of the utility function. In the next section, we use as a benchmark the square root scale $\phi_t^{SQR} = \sqrt{N_{ad} + N_{ch}}$. This scale is now used by the OECD and almost identical to the 'Mean' scale in Fernández-Villaverde and Krueger (2007) which is their preferred choice.[22] Our results are similar when we use other equivalence scales, so we skip them from the presentation.[23]

As for utility weights, we remain agnostic and compare three cases: (i) $\delta_t = 1$ represents the case when households do not value household size (as e.g. in Livshits, MacGee, and Tertilt (2007), Attanasio, Low, and Sanchez-Marcos (2008)); (ii) $\delta_t = N_t = N_{ad,t} + N_{ch,t}$ (as e.g. in Domeij and Klein (2012)) is the opposite, since households always enjoy having more members; and (iii) an intermediate case when $\delta_t = \phi_t$ (as e.g. in Fuchs-Schündeln (2008) and Laitner and Silverman (2012)), i.e., we let the utility weight take the same value as the equivalence scale.

For income profiles, we use data from the Current Population Survey, from 1984 to 2003,[24] in particular the March supplements for years 1985 to 2004, given that questions about income are retrospective. We use total wage income (deflated by CPI-U, leaving amounts in 2000 US dollars). We construct total household income $W_{i\tau}$ for household i observed in year τ, as the sum of individual incomes in the household for all households with at least one full time/full year worker. The latter is defined as someone who worked more than 40 hours per week and more than 40 weeks per year and earned more than \$2 per hour. To get life-cycle profiles, we estimate the following regression:

$$\log W_{i\tau} = D_{i\tau}^{age} \varrho^{age} + X_{i\tau}\gamma + \epsilon_{i\tau} \tag{32}$$

where $\phi_{i\tau}$ is an equivalence scale, $D_{i\tau}^{age}$ represents a set of age dummies of the head of household, ϱ^{age} and γ are estimated coefficients and ε are estimation errors.[25] From this estimation, we

[22] Table 1 in Fernández-Villaverde and Krueger (2007) summarizes six representative equivalence scales used in the empirical consumption literature. The 'Mean' scale in their paper is just the mean over these six scales.

[23] Results using different equivalence scales are available upon request.

[24] The choice of sample period is to maintain comparability with the data used in Fernández-Villaverde and Krueger (2007) to analyze different equivalence scales.

[25] We also control for cohort effects and time effects by introducing birth year and year dummies in $X_{i\tau}$. Since year dummies are perfectly collinear with age and birth cohort dummies, we follow Fernández-Villaverde and Krueger (2007) and Aguiar and Hurst (2009) and include normalized year dummies instead, such that for each year τ

$$\sum_\tau \gamma_\tau = 0 \quad \text{and} \quad \sum_\tau \tau\gamma_\tau = 0$$

where $\{\gamma_\tau\}$ are the coefficients associated to these normalized year dummies. This procedure was initially proposed by Deaton and Paxson (1994b). To compare life-cycle profiles across different cohorts/time periods, we normalize the

Table 1: Stochastic Income Process:

ρ	σ_F^2	σ_P^2	σ_{Tr}^2
0.9989	0.2105	0.0166	0.0630

are interested in the regression coefficients associated with age dummies of the household head (experience profiles ϱ_t in the model). In our exercise below, we smooth such profiles, using a quartic polynomial.

To parameterize the income process in (24) we pick the parameter estimates from Storesletten, Telmer, and Yaron (2004) (see Table 1). We discretize this calibrated process using the Rouwenhorst method, using 7, 5 and 2 points for the permanent, transitory and fixed effect components respectively. This methodology is specially suited for our case, given the high persistence of the process (see the discussion in Kopecky and Suen (2010)).

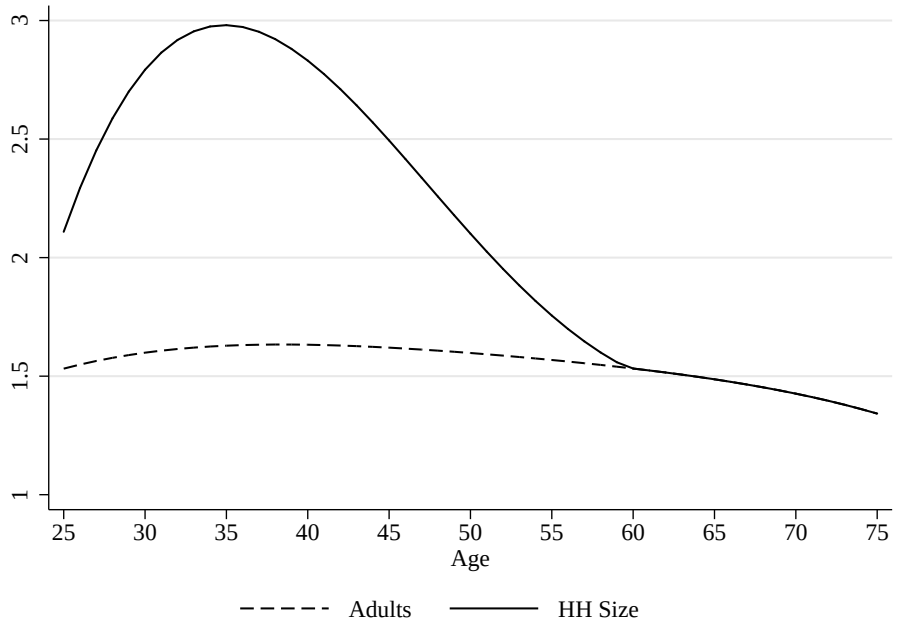
After age 65, agents receive social security payments, which we model to mimic the existing payment schedule in the U.S. We use the same function as Storesletten, Telmer, and Yaron (2004) and Kaplan and Violante (2010). We re-scale $p(\cdot)$ so that the average replacement rate (pension payments over average life-cycle wages) equals 0.45 and compute a value for τ so that the social security system is fully funded through a linear labor income tax.

To compute profiles for family size and composition, we use the March supplements of the CPS for the years 1984 to 2003. For each household, we count the number of adults (individuals age 17+) and the number of children: individuals age 16 or less who are identified as being the 'child' of an adult in the household. We compute two separate profiles: one for number of adults and one for number of children. As above, we run dummy regressions to extract life-cycle profiles, where the considered age is that of the head (irrespective of gender) and control for cohort and year effects (as described in Footnote 25). After extracting these life-cycle profiles, we smooth them using a cubic polynomial in age, and restrict the number of children to zero after age 60, see Figure 1.[26]

estimated coefficients associated to age dummies by adding the effect of a particular cohort/time. More specifically, we picked the cohort corresponding to the median age at the last observed year.

[26]The smoothed life-cycle profiles for adults, children and deterministic income can be downloaded from the authors' websites: `pareto.uab.es/schoi/` or `www.public.asu.edu/~abick/`

Figure 1: Profiles for Household Size and Composition



Note: HH size corresponds to number of adults and children. Both profiles are constructed using data from the CPS, 1983-2003.

4 Results

We solve all models by backwards induction using the endogenous grid method as described by Carroll (2006) and Barillas and Fernndez-Villaverde (2007), using a grid of 101 points for assets. We then simulate one hundred thousand life-cycles and compute aggregate statistics of the variables of interest. In the paragraphs below, we relate our theoretical results to two quantitative exercises which differ in the considered value of the utility weight δ.

4.1 *Demographics* model with $\delta = \phi$ vs. *Single Agent* model

In Section 2 we derived within a two period model the following predictions: first, per-adult equivalent consumption levels will only be the same between the *Demographics* and *Single Agent* model for a unique income deflator κ. This result was independent from the relationship between κ and ϕ. Second, without income uncertainty the *Single Agent* model predicts exactly the same

per-adult equivalent consumption growth rate as the *Demographics* model if $\delta = \phi$, independent of the choice of income deflator κ. Third, this latter result is no longer true once income uncertainty is introduced. In this case it requires again a unique income deflator κ. With multiple periods, two different income shocks and multiple realizations of each, we can neither establish the existence of such a κ nor derive analytic solutions. Nevertheless, the following quantitative exercise addresses all three of these theoretical predictions: we compare the predictions of the *Demographics* model for the case of $\delta = \phi$ with the *Single Agent* model using $\kappa = \phi$ and with the *Single Agent* model using an estimated κ.[27] Specifically, we consider a general deflator

$$\hat{\kappa} = (1 + \xi_1(N_{ad} - 1) + \xi_2 N_{ch})^{\xi_3}$$

where we choose the parameter vector $\{\xi\}$ such that the average per-adult equivalent consumption predicted by a *Single Agent* model using $\hat{\kappa}$ matches that of its *Demographics* model counterpart at each age as close as possible.[28,29]

Figure 2 shows the mean per-adult equivalent consumption for our three models. The first observation is that the *Demographics* model predicts higher mean per-adult equivalent consumption at each age than the *Single Agent* model specified with $\kappa = \phi$. The predictions of the *Single Agent* model with the estimated κ are very close to the *Demographics* model. In order to do so, the per-adult equivalent income in the *Single Agent* model must be increased which can only be achieved through a lower income deflator κ (at least on average). The last column of Table 2 shows the average of the respective κ over working life, i.e. $\frac{1}{(tw-t_0+1)} \sum_{t=t_0}^{tw} \kappa(N_{ad,t}, N_{ch,t})$, whereas the first three columns list the respective parameter values.

Figure 3 compares the per-adult equivalent consumption growth rates between the three models which are very close to each other. Note that after retirement (from age 66 onwards) the consumption growth rates are exactly the same as predicted in Proposition 2 for the case without income uncertainty. Over the working life however, differences in the growth rates accumulate such that the ratio of per-adult equivalent consumption at the peak age relative to age 25 is nearly 4.4 percentage

[27] To facilitate the comparison, in this subsection we consider a quantitative model without probabilistic death (everyone survives before their last age, at which point they exit the economy for sure) and we remove social security.

[28] Note that this specification nests the square root scale and the old OECD scale. Furthermore, in Section 2 we compared the *Single Agent* and *Demographics* model on the individual level while we refer to means here.

[29] Hong and Ríos-Rull (2012) estimate househod-type specific equivalence scales in the context a life-cycle model with stochastic changes in household size and composition using life-insurance holdings.

Figure 2: Mean of log per-adult equivalent consumption

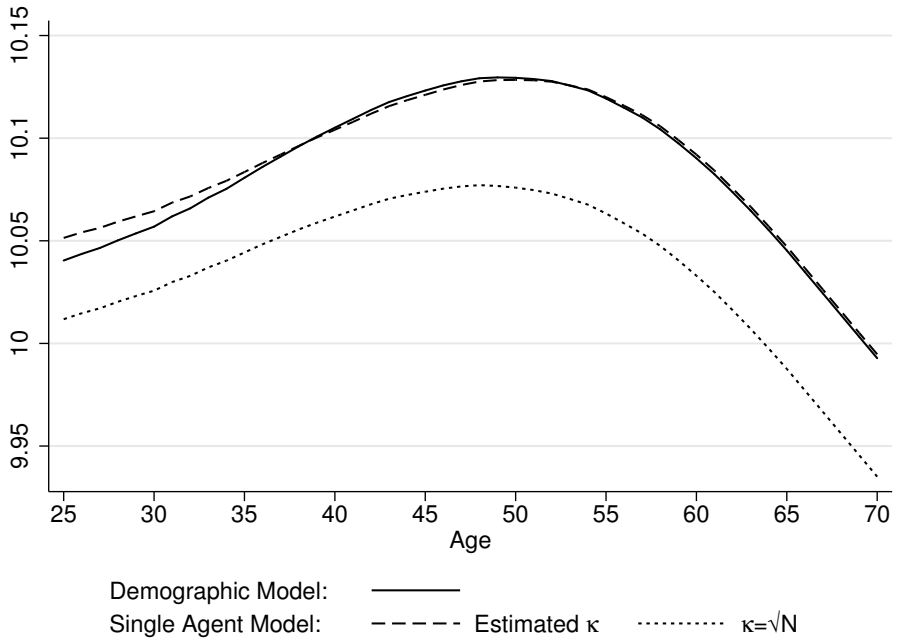


Table 2: Estimated Income Equivalence scale:
$\kappa = (1 + \xi_1(N_{ad} - 1) + \xi_2 N_{ch})^{\xi_3}$

	ξ_1	ξ_2	ξ_3	Life-Cycle Average
Square Root Scale	1.00	1.00	0.50	1.54 (0.18)
$\hat{\kappa}$	0.23	0.44	1.00	1.47 (0.23)

points or 15% lower in the *Single Agent* model with $\kappa = \phi$ than in the *Demographics* model (see Table 3). The lower estimate of $\hat{\kappa}$ does not only increase mean income in the *Single Agent* model but as argued before, in the context of Proposition 3, increases relative income risk. This translates into a steeper per-adult equivalent consumption in the *Single Agent* model with $\hat{\kappa}$ relative to $\kappa = \phi$. However, even there per-adult equivalent consumption increases less strongly until the peak age than in the *Demographics* model.

Summing up, even under the assumption that the relative price of per-adult equivalent consumption does not change over time, the predictions between the *Demographics* and *Single Agent*

Figure 3: Per-adult equivalent consumption growth rates

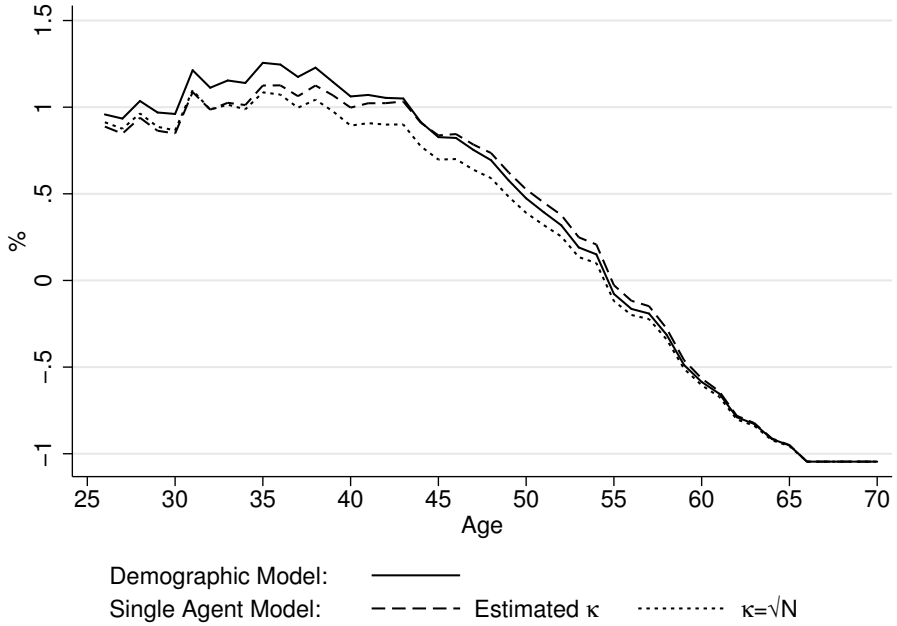


Table 3: Life-cycle per-adult equivalent consumption

	Peak age $\tilde{t}$	$c_{\tilde{t}}/c_{25} - 1$
Demographics	54	29.4 %
$\boldsymbol{\kappa = \sqrt{N}}$	54	25.0 %
$\boldsymbol{\hat{\kappa}}$	54	27.8 %

model differ. In line with theoretical predictions from the two-period exercise, we find that: life-time per-adult equivalent consumption differs between the two setups; per-adult equivalent consumption growth rates are the same once income uncertainty is resolved (during retirement); per-adult equivalent consumption growth rates are different in the presence of income uncertainty (during working life). In how far these differences affect the qualitative and quantitative conclusions in a given application (e.g. evaluating the welfare consequences of a policy reform) depends on the specific setup and cannot be generalized. Our exercise however makes clear that the *Single Agent* model is not an innocuous shortcut of the simplest model allowing demographics to affect choices, even under the most favorable conditions. Furthermore, while we are able to find an income deflator κ that aligns the predictions of the two models (at least for the mean), this deflator is not useful

for other applications of the *Single Agent* model: the estimates depend on the predictions of the *Demographics* model and are different for different calibrations of it (e.g. if β is varied) or changes in the economic environment (e.g. an increase in income uncertainty).

4.2 Case 2: Different Specifications of δ in the the *Demographics* Model

In what follows we concentrate on different specifications of the *Demographics* model, their implications and predictions.[30] The different choices of the utility weight imply different relative prices of per-adult equivalent consumption between two periods when household size and composition change whereas the effective interest rates are the same across all specifications. By inspecting Figure 4 (where we also plot the empirical household size for reference), we see that the value of the utility weight is very important for determining the size and shape of per adult equivalent consumption over the life-cycle. When $\delta = 1$ (as e.g. in Livshits, MacGee, and Tertilt (2007) and Attanasio, Low, and Sanchez-Marcos (2008)), per adult equivalent consumption actually decreases in the early part of the life-cycle, tracking the increase in household size in the opposite direction.[31] In this case, consuming when household size is large is costly because it has to be shared among a lot of members which is however not valued by the household. When $\delta = N$ equals household size N (as e.g. in Domeij and Klein (2012)), the consumption profile tracks household size more closely, since households want to allocate more consumption to periods when their size is bigger.

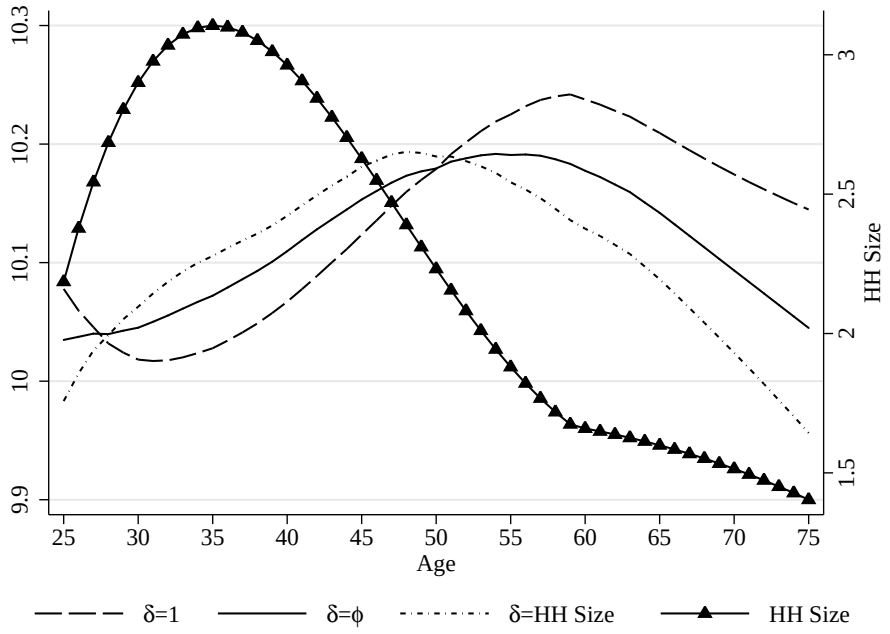
The assumption of $\beta(1 + r) < 1$ in conjunction with the presence of uncertainty creates a humped shaped per-adult equivalent consumption profile. The steepness of per-adult equivalent consumption earlier in life depends on the values of δ and these patterns are robust to changes in the values of β and r. Related to the steepness of the profiles, the different utility weights imply different incentives to delay consumption, and different ages at which per-adult equivalent consumption peaks.

For $\delta = N$, consumption peaks around age 46 and is 21% larger than at age 25; the opposite scenario occurs when $\delta = 1$ and households want to delay per-adult equivalent consumption as much as possible to allocate it to periods with small households, which produce a peak age around 60 along with a per-adult equivalent consumption only 16% larger than at age 25. The intermediate

[30] For this section we return to the full specification of the model (i.e., with survival probabilities and a social security system).

[31] Note that *household* consumption is nevertheless increasing for $\delta = 1$.

Figure 4: Mean of log per-adult equivalent consumption



case of $\delta = \phi = \sqrt{N}$, has a peak age at 55.[32] While differences in the peak age of per-adult equivalent consumption can be equalized between the models through differences in the discount factor, the very distinct patterns over the entire life-cycle prevail.

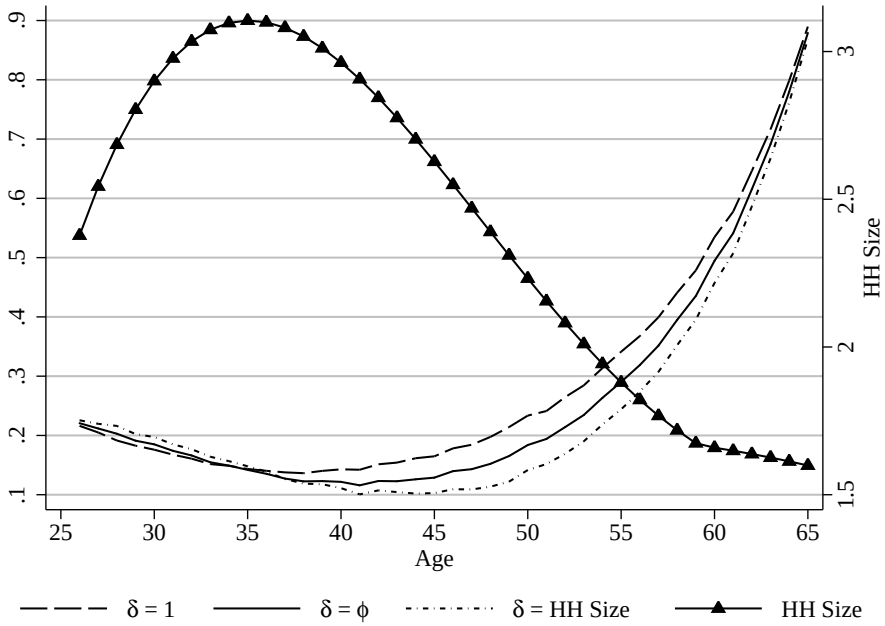
4.2.1 Insurance in Incomplete Markets

Our results show that particular parameterizations of the *Demographics* model have first order effects on the level and timing of per-adult equivalent consumption. In this section we ask a question of central importance in standard incomplete market models: by how much does the degree of consumption smoothing implicit in the calibrated models change across different specifications of household preferences?

Below we compute insurance coefficients for each model, a measure of how much consumption comoves with income shocks. The results in Kaplan and Violante (2010) suggest that the predictions from the standard incomplete markets model under natural borrowing constraints are close to the empirical estimates presented in Blundell, Pistaferri, and Preston (2008). For our exercise, in each

[32] The difference between the per-adult equivalent consumption profile for the *Demographics* model when $\delta = \phi$ does not move much when survival probabilities and the social security system are introduced or not.

Figure 5: Insurance coefficients across models (permanent income shocks)



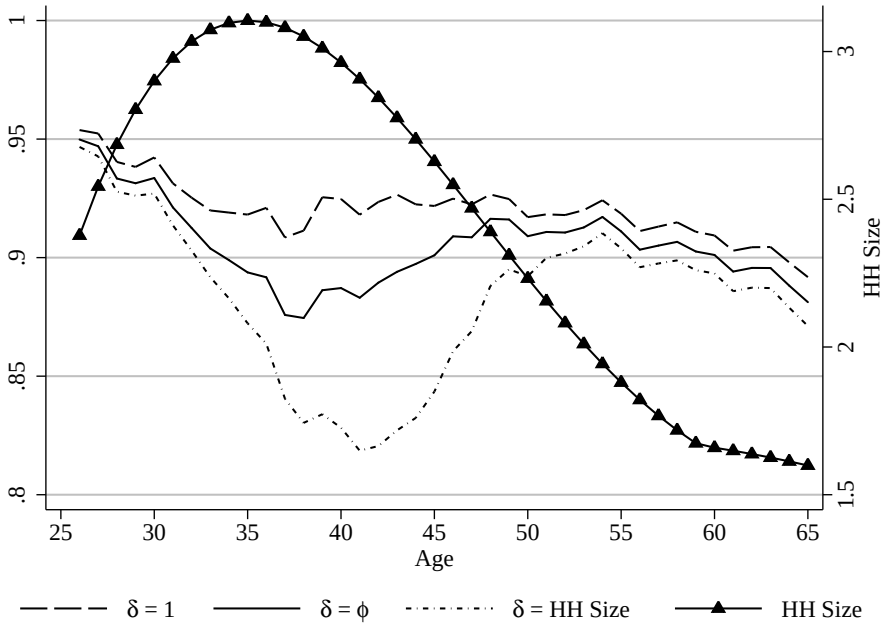
model we calculate the following statistic

$$\psi_t^x = 1 - \frac{cov(\Delta \log(c_t/\phi_t), \epsilon_t^x)}{var(\epsilon_t^x)}$$

where c_t/ϕ_t is per-adult equivalent consumption at age t and $x = \{P, Tr\}$, so we calculate the contemporaneous correlation of changes in consumption and permanent (P) and transitory (Tr) shocks to labor income. The higher the comovement between consumption and unexpected changes in income (represented by the shocks ϵ^x), the lower is the implied value of the insurance coefficient.

In Figure 5, we plot the insurance coefficients against age for the permanent income shock. As a reference, we also plot the household size (right axis). The overall shape of these profiles is in line with findings by Kaplan and Violante (2010), where the increasing nature of this coefficient responds mainly to the accumulation of assets by agents over the life-cycle: with more assets, agents can better insulate their consumption when there are unexpected, permanent shocks to their labor income. From the figure we also see that the different *Demographics* models imply similar shapes for the profile of insurance coefficients, but the differences are as large as 10 percentage points (between ages 45 to 55). On the one hand, we have the case where $\delta = 1$, households do not value (per-adult

Figure 6: Insurance coefficients across models (transitory income shocks)



equivalent) consumption much when they are large, producing higher asset accumulation early on, which in turn allows them to smooth per-adult equivalent consumption more later in the life-cycle. When the utility weight is equal to the household size, we get the opposite effect: consumption is optimally allocated to periods when the size of the household is large, producing less savings and allowing less self insurance against permanent income shocks later in life.

Figure 6 shows that the same effects materialize for the transitory income shocks, with the maximum difference between the three choices for δ being closer to the time when household size peaks. Again the general pattern looks similar to Kaplan and Violante (2010).

A more traditional statistic than the insurance coefficient is the cross-sectional variance of log (per-adult equivalent) consumption. Naturally differences in the insurance coefficients map into differences in these variances as can be seen in Figure 8 in Appendix B. From age 40 onwards, the variances diverge. For $\delta = 1$, the case with the highest insurance coefficient for both type of shocks, the increase in the cross-sectional variance is the smallest, while the opposite is true for $\delta =$ household size.

4.2.2 Welfare Comparisons: An Example

Livshits, MacGee, and Tertilt (2007) (henceforth LMT) conduct a quantitative analysis of two different consumer bankruptcy arrangements: a US-style system where debtors can fully discharge their debt via bankruptcy without seizure of future earnings (labeled FS, for 'Fresh Start') and a continental European-style where bankruptcy does not discharge debt but only restructures a consumer's debt payments and limits the amount of (future) earnings that can be garnished (NFS, or 'No Fresh Start'). LMT show that the welfare comparison between the two bankruptcy regimes is sensitive both to the nature and magnitude of idiosyncratic shocks and the life-cycle profile of household size. This latter aspect provides the link to our analysis.

The basic setup in LMT is the same as in the previous section: a life-cycle model where households face permanent and transitory income shocks, household size changes deterministically with age and the utility function is specified as in Equation (31) with $\delta_t = 1$ and $\phi_t \approx$ Square Root Scale.[33] However, in their setup, households have the option to default on household debt taking as given a bankruptcy rule.[34] The default option gives households the possibility to lower the face value of their debt via bankruptcy. This provides a greater insurance against bad income and expenditure realizations and hence increases the household's ability to smooth consumption across states. On the other hand, the limited ability to commit to future debt repayment limits the ability to smooth consumption over time. These opposing forces are present in different degrees on both the FS and NFS regimes. Thus, there is no clear a priori welfare ranking between the two.[35]

In LMT's benchmark scenario, welfare (measured as the equivalent consumption variation) in the FS economy exceeds welfare relative to the NFS economy marginally, by around 0.06%. This welfare comparison however, is amongst other aspects sensitive to life-cycle considerations. In particular, in one experiment Livshits, MacGee, and Tertilt (2007) keep household size constant over the life cycle. Implicitly, this corresponds to setting $\delta_t = \phi_t^{1-\alpha} < 1$ in (31) as in both cases

[33] LMT do not exactly use the Square Root scale but the mean scale from Fernández-Villaverde and Krueger (2007) which is very close to the Square Root Scale.

[34] Their model also introduces an additional iid expenditure shock (e.g., uninsured medical bills, divorce costs, or unplanned children) and financial intermediaries charge individual specific interest rates for debtors which depend on the household's age, borrowing level and current (but not future) income. These interest rates are determined in equilibrium given an exogenous risk-free savings (set to 4%) interest rate and zero expected profits for financial intermediaries.

[35] Intuitively, the more generous bankruptcy option under FS (a fully discharge of all debt and no seizure of future earnings) provides more insurance against bad states of nature. But it also implies (in equilibrium) tighter borrowing constraints which in turn allow less consumption smoothing over time.

the (marginal) utility of consumption does not any longer change with household size. The gains of moving to FS from NFS are actually *negative* (-0.24% versus the figure before of 0.06%), as the welfare reducing effects of tighter borrowing constraints implied by the more generous default option becomes more important.

We conduct a further experiment, leaving everything unchanged except that we set $\delta_t = \phi_t$ implying that consumption allocated to periods with a large household size increases (relative to the case of $\delta = 1$). Put differently, consumption is smoothed less over time which reinforces the benefits from FS: welfare gains in this case are now 0.21%. At first glance, this exercise does not provide any new insights other than an additional quantification of the welfare comparison. The interesting aspect relates to the debt-earnings ratio in the US economy of 8.4%, the only calibration target in LMT. Their benchmark model ($\delta_t = 1$) yields a value of 8.42% and our extension ($\delta_t = \phi_t$) of 8.37%.[36] If LMT would have calibrated the model right from the start with $\delta_t = \phi_t$, the only non-externally calibrated parameter (the marginal rate of garnishment γ) would have been essentially the same whereas the welfare comparison with NFS would have been much higher. This highlights the importance of the choice of δ, for which neither LMT nor the other papers using a utility function as specified in Equation (31) provide any justification. We will address this issue the next section.

5 Discussion on Identification

Given the previous analysis, the choice of the utility weight δ_t may have stark quantitative implications. This raises the question of which δ_t to choose for numerical work. The only piece of empirical evidence we are aware of is Attanasio, Banks, Meghir, and Weber (1999) (henceforth ABMW). They introduce a general taste shifter to capture the impact of household size on the marginal utility of per-adult equivalent consumption (see Equation (4)). For a given coefficient of relative risk aversion, this specification of the utility function coincides with the one used in our

[36] Similarly, the default rate which LMT discuss in the context of the model's performance is 0.709% and 0.704%, respectively. The corresponding values for $\delta_t = \phi_t^{1-\alpha}$ are 8.8% and 0.711%.

exercise (Equation (31)), if the following relationship holds:[37]

$$\exp(\zeta_1[N_{ad}-1]+\zeta_2 N_{ch}) = \frac{\delta(N_{ad},N_{ch})}{\phi(N_{ad},N_{ch})^{1-\alpha}}. \tag{33}$$

ABMW log-linearize the Euler equation implied by Equation (4) to estimate ζ_1, ζ_2, and α from CEX data.[38] With those estimates at hand, one can back out the δ_t that solves Equation (33) for a given household size and composition, and a given equivalence scale ϕ_t (henceforth labeled as δ_{ABMW}). We consider the seven equivalence scales discussed in Fernández-Villaverde and Krueger (2007).[39] Figure 7 plots the ratio of the two (with δ_{ABMW} in the denominator) against the equivalence scales ordered from the lowest economies of scale (OECD) to the highest economies of scale (Nelson). Figure 7a considers the case of a two adults, two children household. To give a concrete example: for the OECD (NAS) scale $\delta = \phi$ is 1.2 as large as (nearly identical to) the corresponding δ implied by the ABMW estimates. Overall, $\delta = \phi$ and $\delta = N_{ad}$ (number of adults in the household) most closely resemble the δ implied by the estimates in ABMW for the equivalence scales featuring low to medium economies of scale (OECD to DOC), whereas for the equivalence scales featuring high economies of scale (LM and Nelson) $\delta = N_{ad} + N_{ch}$ (household size) stands out. Figure 7b shows the life-cycle averages over each ratio using the household size and composition profiles from our quantitative analysis in Section 4, i.e. $\frac{1}{71}\sum_{t=25}^{95} \frac{\delta_t(N_{ad,t},N_{ch,t})}{\delta_{\text{ABMW},t}(N_{ad,t},N_{ch,t})}$. Deviations from the ABMW estimates of δ are smaller for all cases; $\delta = N_{ad}$ delivers a close fit across all equivalence scales, whereas $\delta = \phi$ only for those scales featuring low to medium economies of scale.

6 Conclusions

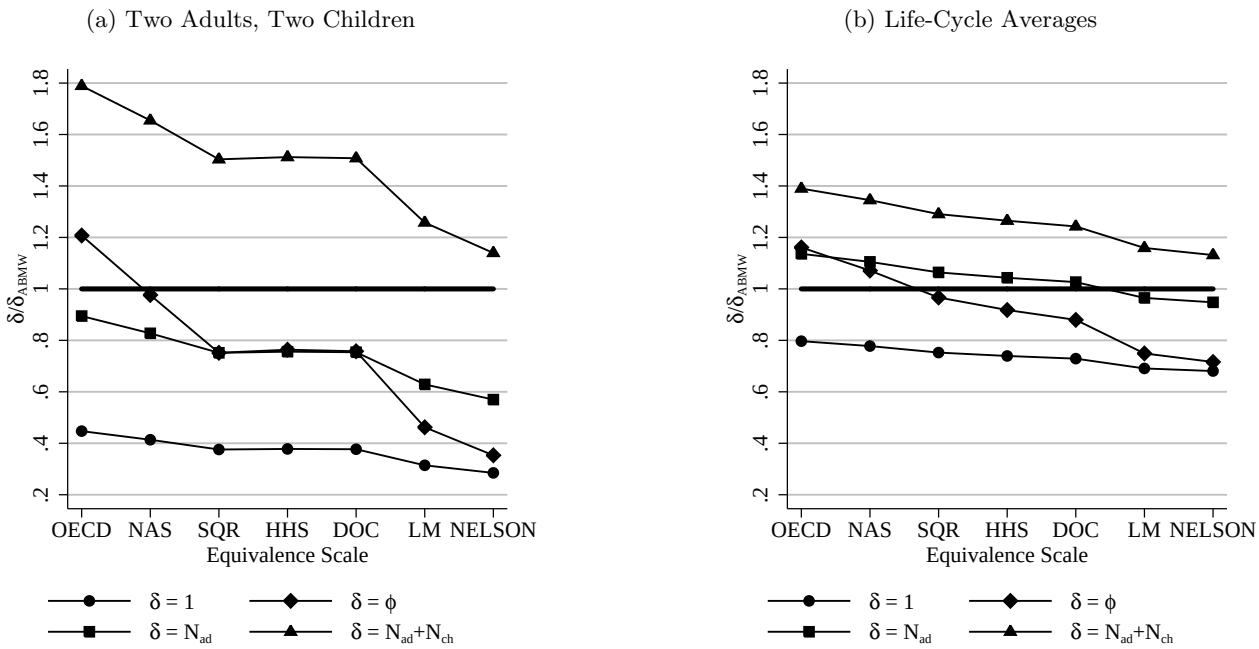
In this paper we compare two widely used versions of the life-cycle model of consumption: in the *Single Agent* model household size is constant over the life-cycle and the model is calibrated with a per-adult equivalent income. To the contrary, the *Demographics* model is calibrated using household income and household size changes deterministically over the life-cycle and impacts the marginal utility of consumption. The objective of this paper is twofold: first, we ask whether the

[37] We normalize the taste shifter from ABMW to one for households of size one.

[38] This assumes that households can borrow up to the natural borrowing constraint. The coefficient estimates are: $\zeta_1 = 0.71, \zeta_2 = 0.34$ and $\alpha = 1.57$.

[39] We use the Square Root Scale instead of the 'Mean' scale in Fernández-Villaverde and Krueger (2007) as the two are almost identical.

Figure 7: Ratio $\frac{\delta}{\delta_{\text{ABMW}}}$ for Various Utility Weights and Equivalence scales



Single Agent model is an innocuous shortcut of the *Demographics* model. Second, we document the implications of changing the relative importance of the size of economies of scale versus the utility gains from consuming in households of different size/composition on an array of predictions of the *Demographics* model.

Using a two period setup, we show theoretically that the *Single Agent* model produces in general different predictions of per-adult equivalent consumption than the *Demographics* model. In the *Demographics* model changes in family size shifts the relative price of consumption across periods through the effect of economies of scale in consumption and direct preferences over household size. This channel is by construction absent in the *Single Agent* model. When the *Demographics* model is specified such that this relative price does not change, the two approaches predict the same per-adult equivalent consumption profiles. However, this result breaks down if income uncertainty is introduced. In the *Demographics* model, changes in household size also change the effective interest rate which affects the resources required to provide a given level of per-adult equivalent consumption insurance.

We then perform a quantitative exercise where we setup a standard model of life-cycle consumption with income uncertainty and incomplete markets, similar to the one in Storesletten, Telmer, and Yaron (2004). We first confirm that under the assumption that in the *Demographics* model the relative price of consumption does not change with household size (the $\delta = \phi$ case), income uncertainty drives an additional and quantitatively non-negligible wedge between the predictions of the *Single Agent* and *Demographics* model. In how far these differences affect the qualitative and quantitative conclusions in a given application (e.g. evaluating the welfare consequences of a policy reform) depends on the specific setup and cannot be generalized. Our exercise however makes clear that the *Single Agent* model is not an innocuous shortcut of the simplest model allowing demographics to affect choices, even under the most favorable conditions. Furthermore, while we can specify the *Single Agent* model to align its predictions with the *Demographics* model in terms of mean per-adult equivalent consumption the characterization of these specifications is dependent on the predictions of the *Demographics* model and are different for different calibrations of it (e.g. if β is varied) or changes in the economic environment (e.g. an increase in income uncertainty).

Next, we investigate the implications for different specifications of the *Demographics* model. For all the exercises, we find that model predictions depend crucially on the interaction between the degree of economies of scale in the household and how the utility of per-adult equivalent consumption of each household member is valued. For example, following in the setup by Livshits, MacGee, and Tertilt (2007) the Continental European-style bankruptcy laws are associated with higher welfare than the US-style bankruptcy law only if households put little weight on each household member's utility and this result reverses as more weight is placed on each household member's utility.

Finally, we compare our setup with the preference structure estimated in Attanasio, Banks, Meghir, and Weber (1999) and discuss how to choose preference parameters for quantitative work. For the most commonly used equivalence scales, as e.g. the OECD scale, with low to medium economies of scale, the empirical estimates suggest that *Demographics* models with a utility weight equal to the equivalence scale or the number of adults in the household are closer to the data.

References

Aguiar, M., and E. Hurst (2009): "Deconstructing Lifecycle Expenditure," Working paper, University of Rochester and University of Chicago.

Attanasio, O., H. Low, and V. Sanchez-Marcos (2008): "Explaining Changes in Female Labor Supply in a Life-Cycle Model," *American Economic Review*, 98(4), 1517–1542.

Attanasio, O. P., J. Banks, C. Meghir, and G. Weber (1999): "Humps and Bumps in Lifetime Consumption," *Journal of Business & Economic Statistics*, 17, 22–35.

Attanasio, O. P., and M. Browning (1995): "Consumption over the Life Cycle and over the Business Cycle," *The American Economic Review*, 85(5), pp. 1118–1137.

Attanasio, O. P., and G. Weber (1993): "Consumption Growth, the Interest Rate and Aggregation," *Review of Economic Studies*, 60(3), 631–49.

——— (1995): "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey," *Journal of Political Economy*, 103, 1121–1157.

Banks, J., R. Blundell, and I. Preston (1994): "Life-cycle expenditure allocations and the consumption costs of children," *European Economic Review*, 38(7), 1391 – 1410.

Barillas, F., and J. Fernndez-Villaverde (2007): "A generalization of the Endogenous Grid Method," *Journal of Economic Dynamics & Control*, 31, 2698–2712.

Blundell, R., H. Low, and I. Preston (2011): "Decomposing changes in income risk using consumption data," IFS Working Paper 04/26, Institute for Fiscal Studies.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption Inequality and Partial Insurance," *American Economic Review*, 98, 1887–1921.

Browning, M., and M. Ejrnæs (2009): "Consumption and Children," *Review of Economics and Statistics*, 91(1), 93–111.

Carroll, C. D. (2006): "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems," *Economic Letters*, 91, 312–320.

Citro, C., and R. Michaels (eds.) (1995): *Measuring Poverty: A New Approach*. National Academy Press.

Cutler, D. M., and L. F. Katz (1992): "Rising Inequality? Changes in the Distribution of Income and Consumption in the 1980's," *The American Economic Review*, 82(2), pp. 546–551.

Deaton, A., and C. Paxson (1994a): "Intertemporal Choice and Inequality," *Journal of Political Economy*, 102(3), pp. 437–67.

Deaton, A., and C. H. Paxson (1994b): "Saving, Growth and Aging in Taiwan," in *Studies in the Economics of Aging*, ed. by D. A. Wise. Chicago University Press for the National Bureau of Economic Research.

Domeij, D., and P. Klein (2012): "Should day care be subsidized?," *Review of Economic Studies*, forthcoming.

Fernández-Villaverde, J., and D. Krueger (2007): "Consumption over the Life Cycle: Facts from Consumer Expenditure Survey Data," *Review of Economics and Statistics*, 89(3), 552–565.

——— (2010): "Consumption and Saving over the Life Cycle: How Important are Consumer Durables?," *Macroeconomic Dynamics*, 15, 725–770.

Fuchs-Schündeln, N. (2008): "The Response of Household Saving to the Large Shock of German Reunification," *American Economic Review*, 98(5), 1798–1828.

Gourinchas, P.-O., and J. A. Parker (2002): "Consumption over the life cycle," *Econometrica*, 70, 47–89.

Greenwood, J., N. Guner, and J. A. Knowles (2003): "More on Marriage, Fertility and the Distribution of Income," *International Economic Review*, 44(3), 827–826.

Heathcote, J., K. Storesletten, and G. L. Violante (2008): "Insurance and opportunities: A welfare analysis of labor market risk," *Journal of Monetary Economics*, 55(3), 501 – 525.

——— (2012): "Consumption and Labor Supply with Partial Insurance: An Analytical Framework," Mimeo.

Hong, J. H., and J.-V. Ríos-Rull (2007): "Social security, life insurance and annuities for families," *Journal of Monetary Economics*, 54(1), 118 – 140, Carnegie-Rochester Conference Series on Public Policy: Economic Consequences of Demographic Change in a Global Economy April 21-22, 2006.

——— (2012): "Life Insurance and Household Consumption," *American Economic Review*, forthcoming.

Kaplan, G. (2012): "Inequalit and the Lifecycle," *Quantitative Economics*, forthcoming.

Kaplan, G., and G. L. Violante (2010): "How Much Consumption Insurance Beyond Self-Insurance?," *American Economic Journal: Macroeconomics*, 2(4), 53–87.

Kopecky, K., and R. Suen (2010): "Finite State Markov-chain Approximations to Highly Persistent Processes," *Review of Economic Dynamics*, 13(3), 701–714.

Krueger, D., and F. Perri (2006): "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory.," *Review of Economic Studies*, 73(1), 163 – 193.

Krueger, D., F. Perri, L. Pistaferri, and G. L. Violante (2010): "Cross-sectional facts for macroeconomists," *Review of Economic Dynamics*, 13(1), 1 – 14, Special issue: Cross-Sectional Facts for Macroeconomists.

Laitner, J., and D. Silverman (2012): "Consumption, Retirement and Social Security: Evaluating the Efficiency of Reform that Encourages Longer Careers," *Journal of Public Economics*, 96, 615–634.

Lazear, E. P., and R. T. Michael (1980): "Family Size and the Distribution of Real Per Capita Income," *The American Economic Review*, 70(1), pp. 91–107.

Lewbel, A. (1997): "Consumer Demand Systems and Household Equivalence Scale," in *Handbook of Applied Econometric*, ed. by M. H. Pesaran, and P. Schmidt, vol. II. Oxford: Blackwell Publishers Ltd.

Livshits, I., J. MacGee, and M. Tertilt (2007): "Consumer Bankruptcy: A Fresh Start," *The American Economic Review*, 97(1), pp. 402–418.

Low, H., and L. Pistaferri (2010): "Disability Risk, Disability Insurance and Life Cycle Behavior," NBER Working Papers 15962, National Bureau of Economic Research, Inc.

Mazzocco, M., C. Ruiz, and S. Yamaguchi (2007): "Labor Supply, Wealth Dynamics and Marriage Decisions," Mimeo.

Meyer, B., and J. X. Sullivan (2010): "Empirical Paper: Consumption and Income Inequality in the U.S. Since the 1960s," Working paper, University of Chicago and University of Notre Dame.

Salcedo, A., T. Schoellman, and M. Tertilt (2012): "Families as roommates: Changes in U.S. household size from 1850 to 2000," *Quantitative Economics*, 3, 133–175.

Storesletten, K., C. I. Telmer, and A. Yaron (2004): "Consumption and risk sharing over the life cycle," *Journal of Monetary Economics*, 51(3), pp. 609–633.

Appendix

A Two Period Model

A.1 Proof of Proposition 3

The per-adult equivalent consumption profiles in the two approaches can only be the same if for these allocations the following condition holds

$$c_1^S = \eta \mathbb{C}_1^D \text{ and } c_{2,i}^S = \eta \frac{\mathbb{C}_{2,i}^D}{\phi_2} \ \forall \ i = l, h \text{ with } \eta > 0. \tag{34}$$

Without loss of generality, assume that $\{\mathbb{C}_1^D, \mathbb{C}_{2,l}^D, \mathbb{C}_{2,h}^D\}$ also satisfies the budget constraint (13) in the *Demographics* model, i.e. this allocation is optimal. The allocation (34) in the *Single Agent* model can however also only constitute an optimum if for the low and high income shock the respective budget constraints

$$c_1^S + c_{2,i}^S = \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \ \forall \ i = l, h \tag{35}$$

hold with equality, i.e. the allocation is feasible and no resources are wasted. Replacing condition (34) into the budget constraints of the *Single Agent* model (35) and replacing $\mathbb{C}_{2,i}^D$ with the respective budget constraint from the *Demographics* model yields after some reformulations

$$\mathbb{C}_1^D = \frac{1}{\phi_2 - 1} \frac{1}{\eta} \left[(\phi_2 - \eta) \mathbb{Y}_1 + \left(\frac{\phi_2}{\eta \kappa_2} - 1 \right) \mathbb{Y}_{2,i} \right] \ \forall \ i = l, h. \tag{36}$$

Equation (36) has to hold for the low and high period two income realization which can only be the case if

$$\eta = \frac{\phi_2}{\kappa_2}. \tag{37}$$

Using condition (37) we can solve Equation (36) for the κ_2 for which the per-adult equivalent consumption in the two approaches are the same:

$$\kappa_2 = 1 + (\phi_2 - 1) \frac{\mathbb{C}_1^D}{\mathbb{Y}_1}. \tag{38}$$

A.2 Proof of Proposition 5

Without loss of generality, assume that $\{\mathbb{C}_1^D, \mathbb{C}_{2,l}^D, \mathbb{C}_{2,h}^D\}$ satisfies the Euler equation (11) and the budget constraint (13) in the *Demographics* model. Recall that we our results are derived for $r = 0$ and $\beta = 1$. Hence, the allocation in the *Demographics* model is optimal. We will show that a consumption allocation for the *Single Agent* model exists that implies the same per-adult equivalent consumption profile as the *Demographics* model, satisfies the Euler equation (12) and for $\kappa_2 = 1 + (\phi_2 - 1) \frac{\mathbb{C}_1^D}{\mathbb{Y}_1}$ the budget constraint in the *Single Agent* model. Since we assume a strictly concave utility function this allocation has to be the unique optimum in the *Single Agent* model.

The profile in the *Single Agent* model is the same as in the *Demographics* model if

$$c_1^S = \eta \mathbb{C}_1^D \text{ and } c_{2,i}^S = \eta \frac{\mathbb{C}_{2,i}^D}{\phi_2} \ \forall\ i = l, h \text{ with } \eta > 0. \tag{39}$$

Plugging allocation (39) in the Euler equation (12) for the *Single Agent* model and using the homogeneity assumption yields

$$\eta^q u'\left(\mathbb{C}_1^D\right) = \eta^q \sum_{i=l,h} p_i u'\left(\frac{\mathbb{C}_{2,i}^D}{\phi_2}\right). \tag{40}$$

Comparing Equation (40) and the Euler equation in the *Demographics* model (11), it is obvious that both hold jointly at the allocation (39) if $\delta_2 = \phi_2$.

The next step is to show that for $\kappa_2 = 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1}$ the budget constraint in the *Single Agent* model holds with equality for the allocation (39) for the low and high income shock. We start with the budget constraint in the *Demographics* model which has to hold for the low ($i = l$) and high ($i = h$) income:

$$\mathbb{C}_1^D + \mathbb{C}_2^D = \mathbb{Y}_1 + \mathbb{Y}_{2,i} \tag{41}$$

Now plug allocation (39) into Equation (41). The derivation of the critical value of κ_2 relied on $\eta = \frac{\phi_2}{\kappa_2}$ which we us as well in this step:

$$\begin{aligned} \frac{1}{\eta} c_1^S + \frac{\phi_2}{\eta} c_{2,i}^S &= \mathbb{Y}_1 + \mathbb{Y}_{2,i} \\ \frac{\kappa_2}{\phi_2} c_1^S + \kappa_2 c_{2,i}^S &= \mathbb{Y}_1 + \mathbb{Y}_{2,i} \\ \frac{c_1^S}{\phi_2} + c_{2,i}^S &= \frac{\mathbb{Y}_1}{\kappa_2} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \end{aligned} \tag{42}$$

Now plug allocation (39) in the critical value of κ_2 and solve for κ_2 (again using $\eta = \frac{\phi_2}{\kappa_2}$):

$$\begin{aligned} \kappa_2 &= 1 + (\phi_2 - 1)\frac{\mathbb{C}_1^D}{\mathbb{Y}_1} \\ \kappa_2 &= 1 + (\phi_2 - 1)\frac{1}{\eta}\frac{c_1^S}{\mathbb{Y}_1} \\ \kappa_2 &= 1 + (\phi_2 - 1)\frac{\kappa_2}{\phi_2}\frac{c_1^S}{\mathbb{Y}_1} \\ \kappa_2\left(1 - \frac{\phi_2 - 1}{\phi_2}\frac{c_1^S}{\mathbb{Y}_1}\right) &= 1 \\ \kappa_2 &= \frac{1}{1 - \frac{\phi_2 - 1}{\phi_2}\frac{c_1^S}{\mathbb{Y}_1}} \end{aligned} \tag{43}$$

Inserting (43) in (42) for $\frac{\mathbb{Y}_1}{\kappa_2}$ yields after a few reformulations the budget constraint for low ($i = l$)

and high ($i = h$) period two income realization in the *Single Agent* model (compare (35)):

$$
\begin{aligned}
\frac{c_1^S}{\phi_2} + c_{2,i}^S &= \frac{\mathbb{Y}_1}{\kappa_2} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \\
\frac{c_1^S}{\phi_2} + c_{2,i}^S &= \mathbb{Y}_1 \left(1 - \frac{\phi_2 - 1}{\phi_2} \frac{c_1^S}{\mathbb{Y}_1}\right) + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \\
\frac{c_1^S}{\phi_2} + c_{2,i}^S &= \mathbb{Y}_1 - \mathbb{Y}_1 \frac{\phi_2 - 1}{\phi_2} \frac{c_1^S}{\mathbb{Y}_1} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \\
\frac{c_1^S}{\phi_2} + c_{2,i}^S &= \mathbb{Y}_1 - c_1^S + \frac{c_1^S}{\phi_2} + \frac{\mathbb{Y}_{2,i}}{\kappa_2} \\
c_1^S + c_{2,i}^S &= \mathbb{Y}_1 + \frac{\mathbb{Y}_{2,i}}{\kappa_2}
\end{aligned}
\tag{44}
$$

B Figures

Figure 8: Variance of log per-adult equivalent consumption