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Abstract

We analyze the optimal location choice of a monopolistic firm that operates two platforms on a two-sided market. We show that the optimal platform locations are equivalent to the one-sided benchmark if both sides are either restricted to single- or multi-homing. In the mixed case (one side single-homes, the other one multi-homes), the optimal platform locations are determined by the relative profitability of both market sides. Our results indicate that modeling mergers on two-sided markets with fixed locations is often inappropriate.

Keywords: two-sided markets, location choice, monopoly, merger simulation

JEL Classification: D42, K20, L12, L51

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1. Introduction

This paper analyzes the location choice of a monopolistic firm that operates on a two-sided market. The one-sided analogue to this problem is the location choice of a multi-store monopolist, which was studied by Salop (1979) and Katz (1980), and later by Hanjoul et al. (1990), Hansen, Peeters, and Thisse (1997), and Dasci and Laporte (2004). Location choice can be understood literally, and the question is, where to locate the store(s) in a specified geographical space. An alternative interpretation is the one of horizontal product differentiation.

So far, monopolistic location choice has not been analyzed in the context of two-sided markets, i.e. markets where platforms facilitate interaction between two distinct groups of agents. While there is a growing body of literature dealing with two-sided markets, the most prominent papers are Roche and Tirole (2003, 2006), Caillaud and Jullien (2003), and Armstrong (2006), who analyze monopolistic and duopolistic pricing decisions in various two-sided settings. The vast majority of the mentioned papers focus on oligopolistic frameworks, and the monopoly case is often used as a benchmark for comparing the results. However, the monopolistic behaviour is usually not explicitly analyzed in detail. Instead, it is assumed that platform locations as well as the number of platforms remain unchanged. For instance, Anderson and Coate (2005) analyze a duopolistic model of media markets where consumers' taste is modelled in a framework based on Hotelling (1929). The platform locations are supposed to be at both ends of the unit line and this assumption is retained in the monopoly case, i.e. the case where both platforms operate under joint management.
Recently, monopolistic location choice on two-sided markets has become particularly relevant, because economic literature started to explicitly analyze merger cases in two-sided frameworks. For instance, Chandra and Collard-Wexler (2009) propose a Hotelling-based model to study merger cases in the Canadian newspaper industry empirically. Ambrus and Reisinger (2006) compare monopolistic and duopolistic outcomes on media markets where agents are allowed to multi-home, i.e. they are allowed to simultaneously patronize both platforms. Leonello (2010) analyzes the welfare effects of mergers on two-sided markets. A common feature of these papers is the use of a Hotelling framework with two platforms, where the platform locations remain unchanged when comparing the case of monopoly to the case of competing platforms.

In the present paper, we study the optimal location choice of a monopolist that operates one or two platforms under various two-sided settings. The results of our paper indicate that a profit-maximizing firm will not keep the platform locations at the extremes of the Hotelling line. Instead, we find that in case of two-sided single-homing, i.e. if agents on either side of the market are restricted to patronizing one platform only, the monopolist will choose a smaller degree of product differentiation, whereas under two-sided multi-homing, the firm will choose homogeneous platforms. While these results are in line with the corresponding findings from one-sided markets, we show that in a "competitive bottleneck" scenario, i.e. if agents on one side of the market are restricted to single-homing, whereas agents on the opposite side are allowed to multi-home, the relative profitability of the market sides will determine the outcome.

The paper is constructed as follows: In Section 2, we will present the theoretical framework, whereas Section 3 analyzes the optimal location choice under two-sided single-homing. Section 4 deals with the case of two-sided multi-homing and in Section 5, we study the location and pricing decisions in a "competitive bottleneck" model. Section 6 summarizes our findings.

2. Analytical Framework

Consider a two-sided market that is served by a monopolistic firm operating $N \in \{1,2\}$ platforms. Given a specific platform $i \in N$, agents on market side $k = 1,2$ obtain an intrinsic utility v_k^i from joining this platform. In addition, there is as a network utility $\alpha_k \cdot n_{-k}^i$ resulting from the interaction with n_{-k}^i agents from the opposite market side, where α_k denotes the magnitude of the network externality. Hence, $\alpha_k > 0$ implies an additional benefit from interaction, whereas $\alpha_k < 0$ describes a negative network externality. In the special case $\alpha_k = 0$, network effects are absent.

Platforms are heterogeneous and we assume that the agents on either side of the market are uniformly distributed along the unit line. The mass of agents on each market side is normalized to one. Given a specific platform location $x_i \in [0,1]$, an agent on market side *k*, who is located at $\tilde{x}_k \in [0,1]$, has to pay transportation costs $t_k \left(x_i - \tilde{x}_k \right)$ if it holds that $0 \le \tilde{x} \le x_i$. In case that $x_i < \tilde{x}_k \le 1$, transportation costs are given by $t_k \left(\tilde{x}_k - x_i \right)$. As per usual, we suppose that $t_k > 0$. The monopolist charges the price p_k^i for joining platform *i*. As we assume perfect platform symmetry throughout the paper, we can immediately conclude that all platforms will charge an identical price on market side *k*, denoted by p_k. Given the specifications above, the utility of an agent on market side *k*, who joins platform *i*, is described by

$$U_k^i = \begin{cases} v_k + \alpha_k \cdot n_{-k}^i - p_k - t_k\left(x_i - \tilde{x}_k\right) & \text{for } 0 \le \tilde{x}_k \le x \\ v_k + \alpha_k \cdot n_{-k}^i - p_k - t_k\left(\tilde{x}_k - x_i\right) & \text{for } x < \tilde{x}_k \le 1 \end{cases},$$

The reservation utility is normalized to zero.

In order to keep the framework consistent to the literature dealing with monopolistic location choice on one-sided markets, we impose full market coverage, i.e. in the optimum the entire market has to be served. We assume that variable costs for production and for the choice of location are nil. In addition, fixed costs are also supposed to be absent.

3. Two-Sided Single-Homing

In this section we assume that agents on either side of the market are restricted to single-homing. The optimal monopolistic location choice in this case is characterized by the results given in Proposition 1.

Proposition 1: For $N=1$, the optimal platform location is $x_1^*=1/2$, while in case of $N=2$, the platforms are located at $x_1^*=1/4$ and $x_2^*=3/4$.

Proof: First, we consider the case of $N=1$. By the assumption of full market coverage, we can immediately conclude that $n_1^1=n_1^2=1$. The monopolistic platform operator will maximize her profit by setting a price, where the marginal agent is indifferent between joining the platform and using the outside option, i.e. obtaining the reservation utility. Let $\tilde{x}_k^*$ denote the location of the marginal agent. Then, it follows that for $0 \le x_1 \le 1/2$, we have $\tilde{x}_k^*=1$, while in case of $1/2 < x_1 \le 1$, we find that $\tilde{x}_k^*=0$. Since the marginal agent's utility equals the reservation utility, we know that

$$U_k\left(\tilde{x}_k^*=1\right)=v_k+\alpha_k \cdot 1 - p_k - t_k \cdot \left(1-x_1\right)=0$$

$$U_k\left(\tilde{x}_k^*=0\right)=v_k+\alpha_k \cdot 1 - p_k - t_k \cdot \left(x_1-0\right)=0.$$

Hence, the optimal monopoly price is described by

$$(1) \qquad p_k^* = \begin{cases} v_k+\alpha_k - t_k \cdot \left(1-x_1\right) & \textit{for} \quad 0 \le x_1 \le 1/2 \\ v_k+\alpha_k - t_k \cdot x_1 & \textit{for} \quad 1/2 < x_1 \le 1 \end{cases}.$$

Respecting that variable costs are equal to zero and using equation (1), the platform operator's maximization problem is characterized by

$$\max_{x_1} \Pi(x_1) = p_1^*(x_1)\cdot n_1^1 + p_2^*(x_1)\cdot n_2^1 \quad \Leftrightarrow$$

$$\max_{x_1} \Pi(x_1) = \begin{cases} v_1+v_2+\alpha_1+\alpha_2-t_1-t_2+(t_1+t_2)\cdot x_1 & \text{for } \; 0<x_1\le 1/2 \\ v_1+v_2+\alpha_1+\alpha_2-(t_1+t_2)\cdot x_1 & \text{for } \; 1/2<x_1\le 1 \end{cases},$$

yielding the first-order condition

$$\frac{\partial \Pi}{\partial x_1} = \begin{cases} t_1+t_2>0 \\ -t_1-t_2<0 \end{cases},$$

from which we can conclude that the optimal platform location is $x_1^* = 1/2$.

In case of $N=2$, we know that due to full market coverage and perfect platform symmetry, we have that $n_1^1 = n_1^2 = n_2^1 = n_2^2 = 1/2$. The platform locations are x_1 and $x_2 = 1-x_1$. For $0 \le x_1 \le 1/4$, the marginal agent's location is $\tilde{x}_k^* = 1/2$, whereas for $1/4 \le x_1 \le 1/2$, we have that $\tilde{x}_k^* = 0$. Since in the optimum the marginal agent's utility is still the reservation utility level, the monopoly price is given by

$$p_k^* = \begin{cases} v_k+\alpha_k/2-t_k/2+t_k\cdot x_1 & \text{for } \; 0\le x_1\le 1/4 \\ v_k+\alpha_k/2-t_k\cdot x_1 & \text{for } \; 1/4<x_1\le 1/2 \end{cases}.$$

Hence, the maximization problem is

$$\max_{x_1} \Pi(x_1) = \begin{cases} v_1+v_2+\dfrac{(\alpha_1+\alpha_2)}{2}-\dfrac{(t_1+t_2)}{2}+(t_1+t_2)\cdot x_1 & \text{for } \; 0\le x_i\le 1/4 \\ v_1+v_2+\dfrac{(\alpha_1+\alpha_2)}{2}-(t_1+t_2)\cdot x_1 & \text{for } \; 1/4\le x_i\le 1/2 \end{cases},$$

yielding the first-order condition

$$\frac{\partial \Pi}{\partial x_1} = \begin{cases} t_1+t_2>0 \\ -t_1-t_2<0 \end{cases},$$

from which we conclude that the optimal platform locations are $x_1^* = 1/4$ and $x_2^* = 1-1/4 = 3/4$.

(q.e.d.)

The results of Proposition 1 are entirely consistent with the findings from one-sided markets. As can be seen from the first-order conditions, the optimal location choice is only influenced by the transportation cost parameters, while there is no impact from the network externalities. This is not surprising, since the network effects do not affect the location of the marginal agent. Therefore, there is no marginal impact of x_1 on the optimal monopoly price. However, there is a quantitative influence on prices and profit, because the two-sidedness of the market affects the marginal agent's willingness to pay.

For $N=1$, the operator's profit in the optimum is given by

(2) $$\Pi^*(N=1)=v_1+v_2+\alpha_1+\alpha_2-\frac{(t_1+t_2)}{2},$$

whereas for $N=2$, we have that

(3) $$\Pi^*(N=2)=v_1+v_2+\frac{(\alpha_1+\alpha_2)}{2}-\frac{(t_1+t_2)}{4}.$$

Comparing equations (2) and (3), it is easy to verify that for $\alpha_k<0$, $\Pi^*(N=1)<\Pi^*(N=2)$, i.e. the monopolist will operate two platforms. This corresponds to economic intuition: Both market sides affect each other negatively. Offering a second platform decreases the exposition to the other market side, so that the marginal agent's willingness to pay increases, which yields higher profits. For the case of $\alpha_k>0$ and $\alpha_{-k}<0$, we obtain ambiguous results, i.e. for specific parameter sets, the monopolist would only operate one platform. The same holds for the presence of positive externalities on either side of the market, i.e. for $\alpha_k>0$.

4. Two-Sided Multi-Homing

In contrast to Section 3, we now assume that agents on both market sides are allowed to multi-home. Here, it is supposed that the agents obtain the intrinsic utility v_k^i from each platform they join. Since multi-homing requires the presence of at least two platforms, we focus on the case of $N=2$, which is visualized in Figure 1.

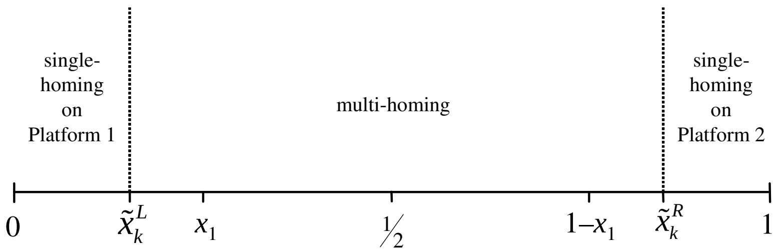

Figure 1: Platform-specific demand in case of two-sided multi-homing

Figure 1 depicts the unit line for given platform locations $(x_1, x_2=1-x_1)$. Let $\tilde{x}_k$ denote the location of an arbitrary agent. Then, due to the assumption of full market coverage and supposing that multi-homing exists in the optimum,[1] we know that for $\tilde{x}_k\in[0,x_1]$, an agent will join platform 1 in any case, while for $\tilde{x}_k\in[x_1,1]$, an agent will patronize the first

[1] If there was no multi-homing in the optimum, we would have local monopolies, but these are out of the scope of this analysis.

platform as long as the utility is at least as great as the reservation utility. If the location of the marginal agent is denoted by $\tilde{x}_k^R$, we find that

(4) $$U_k = v_k + \alpha_k \cdot n_{-k}^1 - p_k - t_k \cdot \left(\tilde{x}_k^R - x_1\right) = 0 \quad \Leftrightarrow \quad \tilde{x}_k^R = \frac{v_k + \alpha_k \cdot n_{-k}^1 - p_k + t_k \cdot x_1}{t_k}.$$

The same argument holds for platform 2, where the location of the marginal agent is denoted by $\tilde{x}_k^L$. This specific location is determined by

(5) $$U_k = v_k + \alpha_k \cdot n_{-k}^2 - p_k - t_k \cdot \left(1 - x_1 - \tilde{x}_k^L\right) = 0 \quad \Leftrightarrow \quad \tilde{x}_k^L = \frac{-v_k - \alpha_k \cdot n_{-k}^2 + p_k + t_k - t_k \cdot x_1}{t_k}.$$

From Figure 1 it is easy to see that $\tilde{x}_k^L + (1 - \tilde{x}_k^R)$ agents single-home, while the number of multi-homing agents is described by $\tilde{x}_k^R - \tilde{x}_k^L$. Since we know that the demand for each platform consists of multi-homing agents and the corresponding number of single-homing agents, we use (4) and (5) to find that

(6) $$n_k^1 = \tilde{x}_k^L + \tilde{x}_k^R - \tilde{x}_k^L = \tilde{x}_k^R = \frac{v_k + \alpha_k \cdot n_{-k}^1 - p_k + t_k \cdot x_1}{t_k},$$

(7) $$n_k^2 = 1 - \tilde{x}_k^R + \tilde{x}_k^R - \tilde{x}_k^L = 1 - \tilde{x}_k^L = 1 - \frac{-v_k - \alpha_k \cdot n_{-k}^2 + p_k + t_k - t_k \cdot x_1}{t_k}.$$

Simultaneously solving equations (6) and (7) for n_k^1 and n_k^2, yields the platform-specific demand functions that are described by

(8) $$n_1^1 = n_1^2 = -\frac{\alpha_1 \cdot (v_2 - p_2) + t_2 \cdot (v_1 - p_1) + x_1 \cdot t_2 \left(\alpha_1 + t_1\right)}{\alpha_1 \cdot \alpha_2 - t_1 \cdot t_2},$$

(9) $$n_2^1 = n_2^2 = -\frac{\alpha_2 \cdot (v_1 - p_1) + t_1 \cdot (v_2 - p_2) + x_1 \cdot t_1 \left(\alpha_2 + t_2\right)}{\alpha_1 \cdot \alpha_2 - t_1 \cdot t_2}.$$

Given (8) and (9), we are now able to analyze the optimal location choice of the monopolistic platform operator. Proposition 2 summarizes the results.

Proposition 2: If agents are allowed to multi-home, the optimal platform locations are given by $x_1^* = x_2^* = 1/2$.

Proof: Using equations (8) and (9), the monopolist's maximization problem can be formulated as

$$\max_{p_1, p_2} \Pi = p_1 \cdot \left(n_1^1 + n_1^2\right) + p_2 \cdot \left(n_2^1 + n_2^2\right) \quad \Leftrightarrow$$

$$\max_{p_1, p_2} \Pi = p_1 \cdot \left(-2 \cdot \frac{\alpha_1 \cdot (v_2 - p_2) + t_2 \cdot (v_1 - p_1) + x_1 \cdot t_2 \left(\alpha_1 + t_1\right)}{\alpha_1 \cdot \alpha_2 - t_1 \cdot t_2} \right)$$

$$+p_2 \cdot 2 \cdot \left(-\frac{\alpha_2 \cdot (v_1 - p_1) + t_1 \cdot (v_2 - p_2) + x_1 \cdot t_1 (\alpha_2 + t_2)}{\alpha_1 \cdot \alpha_2 - t_1 \cdot t_2} \right),$$

yielding the profit maximizing platform prices that are given by

$$p_1^* = \frac{(v_1 + t_1 \cdot x_1) \cdot \left((\alpha_2)^2 - 2 \cdot t_1 \cdot t_2 \right) + \alpha_2 \cdot t_1 \cdot (v_2 + t_2 \cdot x_1) + \alpha_1 \cdot \left[\alpha_2 \cdot (v_1 + t_1 \cdot x_1) - t_1 \cdot (v_2 + t_2 \cdot x_1) \right]}{(\alpha_1)^2 + 2 \cdot \alpha_1 \cdot \alpha_2 - 4 \cdot t_1 \cdot t_2},$$

$$p_2^* = \frac{(v_2 + t_2 \cdot x_1) \cdot \left((\alpha_1)^2 - 2 \cdot t_1 \cdot t_2 \right) + \alpha_1 \cdot t_2 \cdot (v_1 + t_1 \cdot x_1) + \alpha_2 \cdot \left[\alpha_1 \cdot (v_2 + t_2 \cdot x_1) - t_2 \cdot (v_1 + t_1 \cdot x_1) \right]}{(\alpha_2)^2 + 2 \cdot \alpha_1 \cdot \alpha_2 - 4 \cdot t_1 \cdot t_2}.$$

The operator's optimization problem with respect to the optimal location choice is therefore

$$\max_{x_1} \Pi = p_1^*(x_1) \cdot \left(n_1^1 \left(p_1^*(x_1) \right) + n_1^2 \left(p_1^*(x_1) \right) \right) + p_2^* \cdot \left(n_2^1 \left(p_1^*(x_1) \right) + n_2^2 \left(p_1^*(x_1) \right) \right),$$

which yields the first-order condition

(10) $$\frac{\partial \Pi}{\partial x_1} = -\frac{2 \cdot \left[2 \cdot t_1 \cdot t_2 \cdot \left(v_1 + v_2 + (t_1 + t_2) \cdot x_1^* \right) + (\alpha_1 + \alpha_2) \cdot \left(t_1 \cdot v_2 + t_2 \cdot \left(v_1 + 2 \cdot t_1 \cdot x_1^* \right) \right) \right]}{(\alpha_1)^2 + (\alpha_2)^2 + 2 \cdot \alpha_1 \cdot \alpha_2 - 4 \cdot t_1 \cdot t_2} = 0.$$

Equation (10) can easily be solved for x_1^*. It can be shown that $\partial \Pi / \partial x_1 > 0$ for $x_1^* \in [0, 1/2]$ and $v_1, v_2, t_1, t_2 > 0$. This implies that the optimal platform locations are determined by the corner solution $x_1^* = x_2^* = 1/2$.

(q.e.d.)

Considering a one-sided benchmark where multi-homing is allowed, i.e. the special case of our model where $\alpha_k = 0$, we find by using equation (10) that for $x_1^* \in [0, 1/2]$ it holds that

$$\frac{\partial \Pi}{\partial x_1} = -\frac{2 \cdot \left[2 \cdot t_1 \cdot t_2 \cdot \left(v_1 + v_2 + (t_1 + t_2) \cdot x_1^* \right) \right]}{-4 \cdot t_1 \cdot t_2} = v_1 + v_2 + (t_1 + t_2) \cdot x_1^* > 0,$$

which implies that there is no qualitative difference to the results of Proposition 2. Hence, the two-sidedness of the market does not affect the optimal location choice, so that the result is consistent with the one-sided benchmark case.

Given the optimal platform locations $x_1^* = x_2^* = 1/2$, we find that for $N = 2$, the optimal monopoly profit is described by

(11) $$\Pi^*(N = 2) = -\frac{1}{2 \cdot \left((\alpha_1)^2 + (\alpha_2)^2 - 4 \cdot t_1 \cdot t_2 \right)} \cdot$$

$$\left[t_1 \cdot \left((t_2)^2 + 4 \cdot (v_2)^2 \right) + t_2 \cdot \left((t_1)^2 + 4 \cdot (v_1)^2 \right) + 4 \cdot t_1 \cdot t_2 \cdot (v_1 + v_2) + (\alpha_1 + \alpha_2) \cdot (t_1 + 2 \cdot v_1)(t_2 + 2 \cdot v_2) \right].$$

Since for $N = 1$, the operator's profit is still determined by equation (2),[2] we can conclude by comparing (2) and (11) that $\Pi^*(N=1) < \Pi^*(N=2)$, i.e. in case of two-sided multi-homing, the monopolist will in any case operate two platforms. This result is not surprising, since we had assumed that agents obtain the intrinsic utility twice if they join both platforms, which ceteris paribus implies a strong incentive for multi-homing and a higher willingness to pay if two platforms are available. It is easy to construct a robustness check that grants the intrinsic utility only once. This weakens the incentive for multi-homing. One could also assume that repeated interaction is meaningless, i.e. if two multi-homing agents interact on one platform, there is no additional network effect from interacting on the second platform. The only remaining incentive to multi-home is the interaction with single-homing agents. The results of our robustness checks indicate that the optimal platform locations are consistent with our findings from Proposition 2. However, comparing the profits for $N = 1$ and $N = 2$ yields ambiguous results.

5. Competitive Bottlenecks

In this section, we will analyze a "competitive bottleneck" model, i.e. a framework where agents on one side of the market are allowed to single-home, while agents on the opposite side of the market are allowed to multi-home. Without loss of generality, we assume that market side 1 is the single-homing side, whereas multi-homing agents on market side 2 are still supposed to obtain intrinsic utility from each platform they join. Supposing that multi-homing is present in the optimum, we focus on the case of $N = 2$ again. The platform locations are still denoted by x_1 and $x_2 = 1 - x_1$.

Considering market side 1, we know that the marginal agent is indifferent between joining platform 1 and platform 2, i.e. it holds that $U_1^1 = U_1^2$. Let the location of the marginal agent be denoted by $\tilde{x}_1^{IND}$. Then, we know that

$$v_1 + \alpha_1 \cdot n_2^1 - p_1 - t_1 \cdot \left(\tilde{x}_1^{IND} - x_1\right) = U_1^1 = U_1^2 = v_1 + \alpha_1 \cdot n_2^2 - p_1 - t_1 \cdot \left(1 - x_1 - \tilde{x}_1^{IND}\right),$$

which implies that $\tilde{x}_1^{IND}$ is given by

$$\tilde{x}_1^{IND} = \frac{\alpha_1 \cdot n_2^1 - \alpha_1 \cdot n_2^2 + t_1}{2 \cdot t_1}.$$

Hence, we know that the platform-specific demand on the single-homing side is characterized by

[2] If there is just one platform, agents cannot multi-home.

(12) $$n_1^1 = \tilde{x}_1^{IND} = \frac{\alpha_1 \cdot n_2^1 - \alpha_1 \cdot n_2^2 + t_1}{2 \cdot t_1},$$

(13) $$n_1^2 = 1 - \tilde{x}_1^{IND} = 1 - \frac{\alpha_1 \cdot n_2^1 - \alpha_1 \cdot n_2^2 + t_1}{2 \cdot t_1}.$$

Analogue to Section 4, the platform-specific demand on the multi-homing side is determined by those agents that are located at $\tilde{x}_2^R$ and $\tilde{x}_2^L$. Following our analysis from Section 4, these locations are now given by

(14) $$U_2 = v_2 + \alpha_2 \cdot n_1^1 - p_2 - t_2 \cdot \left(\tilde{x}_2^R - x_1\right) = 0 \quad \Leftrightarrow \quad \tilde{x}_2^R = \frac{v_2 + \alpha_2 \cdot n_1^1 - p_2 + t_2 \cdot x_1}{t_2},$$

(15) $$U_2 = v_2 + \alpha_2 \cdot n_1^2 - p_2 - t_2 \cdot \left(1 - x_1 - \tilde{x}_2^L\right) = 0 \quad \Leftrightarrow \quad \tilde{x}_2^L = \frac{-v_2 - \alpha_2 \cdot n_1^2 + p_2 + t_2 - t_2 \cdot x_1}{t_2}.$$

Using equations (14) and (15), we find that

(16) $$n_2^1 = \tilde{x}_2^L + \tilde{x}_2^R - \tilde{x}_2^L = \tilde{x}_2^R = \frac{v_2 + \alpha_2 \cdot n_1^1 - p_2 + t_2 \cdot x_1}{t_2},$$

(17) $$n_2^2 = 1 - \tilde{x}_2^R + \tilde{x}_2^R - \tilde{x}_2^L = 1 - \tilde{x}_2^L = 1 - \frac{-v_2 - \alpha_2 \cdot n_1^2 + p_2 + t_2 - t_2 \cdot x_1}{t_2},$$

which allows us to compute the platform-specific demand functions by simultaneously solving equations (12), (13), (16), and (17) for n_k^1 and n_k^2. Hence, we have that

(18) $$n_1^1 = n_1^2 = 1/2,$$

(19) $$n_2^1 = n_2^2 = -\frac{-\alpha_2 + 2 \cdot p_2 - 2 \cdot v_2 - 2 \cdot t_2 \cdot x_1}{2 \cdot t_2}.$$

Given (18) and (19), we are now able to analyze the price-setting behaviour of the monopolistic platform operator, which is more complex than in the previous sections. On market side 1, the optimal monopoly price is determined by the marginal agent. Recalling our findings from Section 3, we know that the marginal agent's location, $\tilde{x}_1^*$, is given by

$$\tilde{x}_1^* = \begin{cases} 1/2 & for \;\; 0 \le x_1 \le 1/4 \\ 0 & for \;\; 1/4 \le x_1 \le 1/2 \end{cases}.$$

Respecting that in the optimum the marginal agent's utility equals her reservation utility and using equation (19), we find that the optimal monopoly price on market side 1 is given by

(20) $$p_1^* = \begin{cases} \dfrac{\alpha_1 \cdot (\alpha_2 - 2 \cdot p_2 + 2 \cdot v_2) - t_1 \cdot t_2 + 2 \cdot t_2 \cdot v_1 + 2 \cdot t_2 \cdot x_1 \cdot (\alpha_1 + t_1)}{2 \cdot t_2} & for \;\; 0 \le x_1 \le 1/4 \\ \dfrac{\alpha_1 \cdot (\alpha_2 - 2 \cdot p_2 + 2 \cdot v_2) + 2 \cdot t_2 \cdot v_1 + 2 \cdot t_2 \cdot x_1 \cdot (\alpha_1 - t_1)}{2 \cdot t_2} & for \;\; 1/4 \le x_1 \le 1/2 \end{cases}.$$

Therefore, the monopolist's optimization problem with respect to market side 2 is formally described by

$$\max_{p_2} \Pi = p_1^* \cdot \underbrace{\left(n_1^1 + n_1^2\right)}_{=1} + p_2 \cdot \left(n_2^1\left(p_2\right) + n_2^2\left(p_2\right)\right),$$

which by using equations (18), (19), and (20) yields that

$$p_2^* = \begin{cases} 1/4 \cdot (-\alpha_1 + \alpha_2 + 2 \cdot v_2 + 2 \cdot t_2 \cdot x_1) & \text{for } 0 \le x_1 \le 1/4 \\ 1/4 \cdot (-\alpha_1 + \alpha_2 + 2 \cdot v_2 + 2 \cdot t_2 \cdot x_1) & \text{for } 1/4 < x_1 \le 1/2 \end{cases}. \tag{21}$$

Given equations (18), (19), (20), and (21), the location choice problem can be formulated as

$$\max_{x_1} \Pi = p_1^*\left(x_1\right) + p_2^*\left(x_1\right) \cdot \left(n_2^1\left(p_2^*\left(x_1\right)\right) + n_2^2\left(p_2^*\left(x_1\right)\right)\right)$$

leading to the first-order condition

$$\frac{\partial \Pi}{\partial x_1} = \begin{cases} 1/2 \cdot (\alpha_1 + \alpha_2 + 2 \cdot (t_1 + v_2 + t_2 \cdot x_1)) = 0 & \text{for } 0 \le x_1 \le 1/4 \\ 1/2 \cdot (\alpha_1 + \alpha_2 + 2 \cdot (-t_1 + v_2 + t_2 \cdot x_1)) = 0 & \text{for } 1/4 \le x_1 \le 1/2 \end{cases}.$$

Hence, we find that

$$x_1^{\min} = \begin{cases} \dfrac{-\alpha_1 - \alpha_2 - 2 \cdot t_1 - 2 \cdot v_2}{2 \cdot t_2} & \text{for } 0 \le x_1 \le 1/4 \\ \dfrac{-\alpha_1 - \alpha_2 + 2 \cdot t_1 - 2 \cdot v_2}{2 \cdot t_2} & \text{for } 1/4 \le x_1 \le 1/2 \end{cases}. \tag{22}$$

Equation (22) is a unique minimum solution, because it can be shown that $\partial^2 \Pi / \partial x_1^2 \left(x_1^{\min}\right) > 0$. Therefore, the optimal location must be a corner solution. We summarize the findings in Proposition 3.

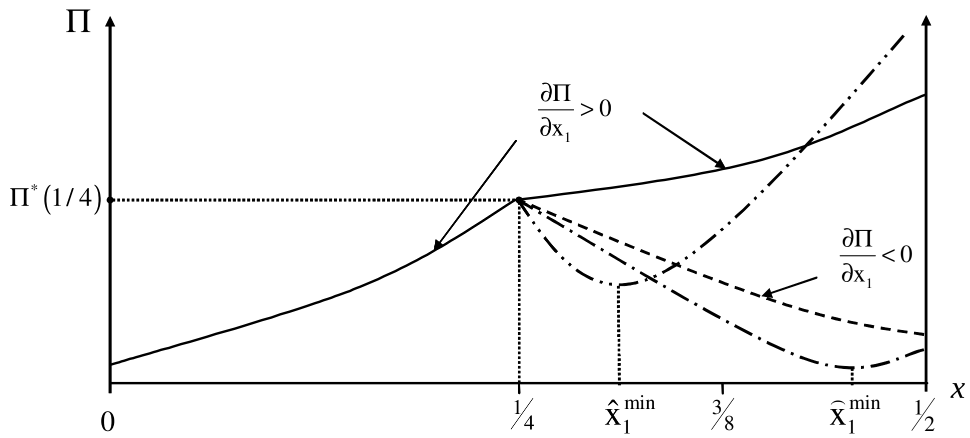

Figure 2: Optimal location choice in a "competitive bottleneck"-framework

Proposition 3: On a "competitive bottleneck"-market, the optimal monopolistic location choice is either $x_1^* = 1/4$ and $x_2^* = 3/4$, or $x_1^* = x_2^* = 1/2$ (or there is indifference between both solutions).

Proof: For $0 \le x_1 \le 1/4$, it can be shown that $\partial\Pi/\partial x_1 > 0$, i.e. we would have $x_1^{\min} < 0$, which implies that the optimal location is given by the corner solution $x_1^* = 1/4$. For $1/4 \le x_1 \le 1/2$, we have to separately analyze four different cases that are visualized in Figure 2: i) In case that $x_1^{\min} < 1/4$, we find that $\partial\Pi/\partial x_1 > 0$, which implies that the profit-maximizing location of platform 1 is $x_1^* = 1/2$. This case is depicted by the black solid curve in Figure 2. ii) For $x_1^{\min} \in (1/4, 3/8)$, we know that due to the symmetry of $\Pi(x_1)$,[3] we have that $x_1^* = 1/2$, because $\Pi(1/2) > \Pi(1/4)$. In Figure 2, this case corresponds to the example of $\hat{x}_1^{\min}$. iii) If we have that $x_1^{\min} \in (3/8, 1/2)$, we can conclude from the profit function's symmetry that $x_1^* = 1/4$, because $\Pi(1/4) > \Pi(1/2)$. This situation is visualized by $\widehat{x}_1^{\min}$ in Figure 2. iv) For $x_1^{\min} > 1/2$, it is easy to show that $\partial\Pi/\partial x_1 < 0$, which implies that $x_1^* = 1/4$. In Figure 2, this case is reflected by the dashed curve.

In case that $x_1^{\min} = 3/8$, we obviously find that the monopolist is indifferent between $x_1^* = 1/4$ and $x_1^* = 1/2$, since both locations yield the same profit.

(q.e.d.)

We find that the optimal platform locations are determined by the location of $x_1^{\min}$, which is described by equation (22) and depends on the relation of the exogenous parameters $\alpha_1, \alpha_2, v_2, t_1$ and t_2. Using (22), comparative statics for the case of $1/4 \le x_1 \le 1/2$ reveal that

$$\frac{\partial x_1^{\min}}{\partial \alpha_1}, \frac{\partial x_1^{\min}}{\partial \alpha_2}, \frac{\partial x_1^{\min}}{\partial t_2}, \frac{\partial x_1^{\min}}{\partial v_2} < 0 \text{ as well as } \frac{\partial x_1^{\min}}{\partial t_1} > 0,$$

which reflects the economics behind our findings: A marginal increase in t_1 has a negative impact on p_1^*, while demand on market side 1 is fixed. This reduces the marginal revenue from an increase of x_1, since we have that

$$\frac{\partial^2 \Pi}{\partial x_1 \partial t_1} = -1 < 0,$$

which implies that the monopolist has an incentive to shift x_1 to the left. In terms of equation (22), an increase of t_1 leads to an increase of $x_1^{\min}$, making $x_1^* = 1/4$ more likely to occur. On the other hand, any increase in the profitability of market side 2, i.e. any increase of α_2, t_2 or v_2, shifts the location of $x_1^{\min}$ to the left. Therefore, in this case $x_1^* = 1/2$ becomes more likely. Obviously, the optimal platform location depends on the relative profitability of the

[3] It can be shown that for the relevant set of parameters, the profit function is symmetric with respect to the unique minimum solution. The corresponding Mathematica code is available from the authors upon request.

two market sides: If the single-homing market side is relatively more profitable with respect to the optimal platform location, we have $x_1^{\min} \in (3/8, 1/2)$ and therefore $x_1^* = 1/4$, which corresponds to the solution of the two-sided single-homing case. In case that market side 2 is more profitable, we find the opposite, i.e. the outcome is consistent with our findings from Section 4.

Comparing the results of Proposition 3 to a hypothetical one-sided benchmark with $\alpha_k = 0$, we find by analyzing equation (22) that the optimal location choice is qualitatively affected by the two-sidedness of the market. While there is no difference for the case of $0 \le x_1 \le 1/4$, we find that for $1/4 \le x_1 \le 1/2$, it may happen that the optimal platform location in the benchmark case is $x_1^* = 1/4$, whereas for $\alpha_k \neq 0$, we could have $x_1^* = 1/2$ (or vice versa). This result is consistent with our interpretation of Proposition 3, because the network externalities crucially influence the relative profitability of the two market sides.

In case that the optimal location choice is $x_1^* = 1/4$, the profit is

$$\text{(23)} \qquad \Pi^*_{x_1^*=1/4}(N=2) = \frac{(\alpha_1)^2 + (\alpha_2)^2 + t_2 \cdot (-2 \cdot t_1 + t_2 + 8 \cdot v_1 + 2 \cdot v_2)}{8 \cdot t_2} + \frac{4 \cdot (v_2)^2 + \alpha_2 \cdot (t_2 + 4 \cdot v_2) + \alpha_1 \cdot (2 \cdot \alpha_2 + t_2 + 4 \cdot v_2)}{8 \cdot t_2},$$

whereas for $x_1^* = 1/2$, we obtain

$$\text{(24)} \qquad \Pi^*_{x_1^*=1/2}(N=2) = \frac{(\alpha_1)^2 + (\alpha_2)^2 + t_2 \cdot (-4 \cdot t_1 + t_2 + 8 \cdot v_1 + 4 \cdot v_2)}{8 \cdot t_2} + \frac{4 \cdot (v_2)^2 + 2 \cdot \alpha_2 \cdot (t_2 + 2 \cdot v_2) + 2 \cdot \alpha_1 \cdot (\alpha_2 + t_2 + 2 \cdot v_2)}{8 \cdot t_2}.$$

Since for $N = 1$, we have the single-homing solution of Section 2, the operator's profit in this case is given by equation (2). Comparing equation (23) or (24) with equation (2), it can be shown that for $\alpha_k < 0$, $\Pi^*(N=1) < \Pi^*(N=2)$. If $\alpha_k > 0$ and $\alpha_{-k} < 0$ or $\alpha_k > 0$, we obtain ambiguous results, i.e. the monopolist will operate either one or two platforms.

6. Conclusions and Implications

This paper analyzes the monopolistic location choice on a two-sided market. To some extent, the results of our analysis are consistent with the findings from one-sided markets: In Sections 3 and 4 we found that the optimal location choice under two-sided single-homing and two-sided multi-homing corresponds to the one-sided benchmark. While in case of two-sided single-homing, optimal platform locations are at 1/4 and 3/4, the monopolist chooses perfectly homogeneous platforms in the optimum under two-sided multi-homing. The two-sidedness of the market only has a quantitative impact on optimal prices and profit, whereas the optimal location choice is not affected.

To our knowledge, this paper is the first one to analyze the monopolistic location choice in a competitive bottleneck scenario, since there is no one-sided equivalent to this setting. When comparing the results of our two-sided framework to a hypothetical one-sided benchmark, we found that the optimal platform locations were crucially determined by the relative profitability of the two-market sides and hence, by the network externalities. In case that the single-homing side is more profitable, the optimal location choice corresponds to the case of two-sided single-homing. If the multi-homing side is relatively more profitable, we find the opposite result. It is also possible that the platform operator is indifferent between both strategies.

In addition, our paper contributes to the economic literature dealing with mergers on two-sided markets: Our results imply that two-sided merger simulations with fixed platform locations (at arbitrary locations or at their competitive benchmarks), do not correctly describe the economic behaviour under monopoly. In other words, models that assume fixed locations (usually at the ends of the Hotelling line), exogenously impose an additional restriction that potentially reduces the profit under “joint-management”. The same argument holds for the number of platforms: Our results indicate that the monopolistic platform operator may voluntarily decide to close down one platform. Choosing the correct monopolistic benchmark is basal for merger assessment as it affects welfare comparisons and therefore conclusions and policy implications.

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