Unique bid auctions: Equilibrium solutions and experimental evidence

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Equilibrium Solutions and Experimental Evidence

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Abstract

Two types of auction were introduced on the Internet a few years ago and have rapidly been gaining widespread popularity. In both auctions, players compete for an exogenously determined prize by independently choosing an integer in some finite and common strategy space specified by the auctioneer. In the unique lowest (highest) bid auction, the winner of the prize is the player who submits the lowest (highest) bid, provided that it is *unique*. We construct the symmetric mixed-strategy equilibrium solutions to the two auctions, and then test them in a sequence of experiments that vary the number of bidders and size of the strategy space. Our results show that the aggregate bids, but only a minority of the individual bidders, are accounted for quite accurately by the equilibrium solutions.

1. Introduction

In a single-unit auction, an indivisible good is offered for sale to one of many potential buyers according to some commonly known rules for formulating the bidding procedure and determining the outcome of the auction (Monderer & Tennenholtz, 2000). Auctions are not lotteries. Rather, they may be considered a substitute to lotteries where the strategic uncertainty regarding the actions taken by the other bidders serves as an implicit lottery mechanism (Monderer & Tennenholtz, 2004). Much has been written recently about the dramatic increase in auction-trade volume on the Internet and the growing popularity of the auction mechanism in electronic commerce (see, e.g., Bajari & Hortaçsu, 2004). There also has been a sharp increase in the variety of online auction mechanisms that differ from the classical auctions (e.g., Ivanova-Stenzel & Sonsino, 2004). In a recent survey of the field, Ockenfels, Reiley, and Sadrieh (2007) remarked that "New features concerning the design of online auctions are proposed and discussed almost on a daily basis."

The present paper considers a new type of auction that first appeared on the Internet a few years ago and has rapidly been gaining widespread popularity particularly in Great Britain, Australia, and the US. The new feature of this type of auction, that sharply differentiates it from previous auctions, is the *uniqueness* of the winning bid. The common rule in classical auctions (e.g., first-price sealed-bid auction) is to break ties with some lottery mechanism. In contrast, in this new type of auction ties are not considered. A necessary condition for winning the auction is for the bid to be unique.

In unique bid auctions conducted on the Internet an auctioneer wishes to sell a particular good with a common valuation V, and n agents (called "bidders" or "buyers") wish to buy it. The auctioneer specifies a discrete strategy space $B=\{\underline{b}, \underline{b}+1, \underline{b}+2, \ldots, \overline{b}\}$, where $\overline{b}<V$. Then, each

bidder i, (i=1, 2, … , n) independently and anonymously submits as her bid an element of the set B. Bidders are symmetric. In the *unique lowest bid* auction, the winner is the bidder submitting the *lowest* bid, provided that it is unique (no other agent submits the same bid). In the *unique highest bid* auction, the winner is the bidder who submits the *highest* bid, provided that it is unique. In the absence of a unique lowest (highest) bid, the unique lowest (highest) bid auction terminates with no winner. If there is a winner, then she receives the good (hereafter called "prize") and pays her bid. Each bidder i pays upfront a participation fee c. Denote the winning bid, if there is one, by b^*. Then, if there is a winner, the payoffs are:

$nc+b^*-V$	for the auctioneer
$V-b^*-c$	for the winner
$-c$,	for each of the n-1 non-winners.

Probably because they cannot enforce it, auctioneers do not restrict the bidders to submit a single bid. To ensure profit, they conduct the auction (and notify potential bidders of this fact) only after the sum of the participation fees nc (or, equivalently, the total number of bids) exceeds some predetermined and commonly known threshold value. Most Internet auctions set the minimum bid $\underline{b}$ at the lowest possible denomination (e.g., 1p in Great Britain, 1¢ in the US). When the unique highest bid auction is conducted, they typically set $\bar{b} << V$.

In a typical Internet unique highest bid auction, a car valued at \$20,000 might be offered to bidders at maximum bid price of $\bar{b}$ =\$100. If the auctioneer charges \$10 participation fee per bid, then he would need at least 2,000 bids to cover the cost of the car (excluding costs of conducting the auction and processing the bids). If the auction closes with a unique highest bid of \$80, then the winner would purchase the car for 0.4% of its value and earn a profit of \$20,000-(80+10). As indicated above, in both the unique bid auctions the auctioneer is guaranteed a positive profit by pre-specifying the minimum number of bids that have to be placed before the auction is conducted. The attraction for the bidder is that she may acquire the good at well below its true value. As reported by the *Boston Globe* (02/04/2006), one of the website specializing in unique lowest bid auctions recently sold a laptop for $19, a living room suite for $43, and a Hummer SUV for less than $700. In yet another unique lowest bid auction, a bar of gold bullion worth more than 1,000 British pounds has been sold for just 1p. Noting that this successful bid might have been unprecedented, the auctioneer remarked "It is incredible that no-one else bid 1p. Every other bidder must have thought it was too obvious" [http://www.humraz.com].

Example. Consider a unique lowest bid auction with n=9, B={1, 2, … , 50}, and V=500. Order the n bids from lowest to highest, and denote the vector of n bids by $\boldsymbol{b}=(b_1, b_2, \dots , b_n)$. If $\boldsymbol{b}$=(1, 1, 1, 2, 2, 5, 7, 11, 15), then the agent bidding 5 is the winner. If $\boldsymbol{b}$=(3, 3, 6, 6, 6, 7, 7, 7, 19), then the agent bidding 19 is the winner. And if $\boldsymbol{b}$=(2, 2, 2, 4, 4, 5, 5, 5, 5), then the auction closes with no winner.

There have been arguments that unique bid auctions are more like lotteries than auctions possibly because the connection between the bid and prize values is less apparent than in the classical auctions. If, for example, V is changed from 500 to, say, 50,000, then most likely the unique bid auctions will attract many more bidders. However, for a fixed n the effect of this change on the equilibrium bids is minimal. To illustrate this, consider two unique lowest bid auctions that are only different in the value of the prize. In auction 1, B={0, 1, … , 30}, c=0, and $V_1 = 500$, whereas in auction 2, B={0, 1, … , 30}and c=0, as in auction 1, but $V_2 = 50{,}000$. Then, the equilibrium probabilities for bids 0, 1, 2, ... are

Auction 1: 0.2519, 0.2351, 0.2088, 0.1642, 0.0998, 0.0358, 0.0042; 0 otherwise.

Auction 2: 0.2518, 0.2348, 0.2086, 0.1641, 0.1000, 0.0363, 0.0046, 0.00007; 0 otherwise.

The maximum difference is 0.003. The expected values in auctions 1 and 2 are 49.0481 and 4927.2273, respectively. If $\bar{b} << V$, as it is typically the case on Internet auctions, then the bid amount paid by the winner is insignificant, and bids may be considered as placeholders that determine the winner. Referring to the previous example, it matters little (in expected value computations) whether the winner bids a few cents or a few dollars to win an expensive car.

Previous literature. The only attempt at an equilibrium analysis of unique bid auctions is by Raviv and Virag (2007), who have offered an analysis under additional assumptions and, in addition, have used empirical data provided to them by an Internet portal to test their solution. To our knowledge they are the first to provide a solution to one special case of this auction. Their analysis is based on several simplifying assumptions, namely, (i) $\bar{b} << V$; (ii) focus only on probability of winning (or a tie) rather than expected value, and (iii) repeat the auction (or return entry fee) in case of a tie. These assumptions imply they provided the exact solution for an auction with constant net payoff $V - \bar{b}$ no matter what the winning bid. The constant payoff greatly simplifies their solution procedure. In section 2 we derive a solution to a different and larger class of auctions without making these assumptions.

Also the empirical data provided to Raviv and Virag by one of the Internet portals are inappropriate to test their proposed model. Critical to their model, as to our model presented in Section 2, is the assumption that each bidder places a single bid. Consequently, in their model and in ours the number of bids is equal to the number of bidders. However, private unique bid auctions conducted on the Internet do not limit the number of bids per entrant. In fact, many of them allow for multiple bids (and multiple participation fees). Clearly, if a bidder places several bids, they will not be placed on the same value. Consequently, the multiple bids of an entrant may not be considered as independent. This is the major reason that, rather than using empirical data, we have opted to test the equilibrium solution experimentally.

Implementation. It is well known that the same auction may be implemented ("framed") in alternative ways that, in theory, are strategically equivalent (Krishna, 2002). For example, the first-price auction may be implemented in several ways, as a sealed-bid auction in which bids are placed simultaneously, or as a Dutch descending-clock auction. Bayesian Nash equilibrium theory suggests that these forms are isomorphic. In a similar way, both the unique bid auctions may be implemented in alternative ways. In what we call here "sequential implementation," the market operates like a Dutch descending-clock auction with the auctioneer calling $\bar{b}$ and then lowering the price of the good in discrete steps (minimum bid increment is normalized to 1) until reaching $\underline{b}$. The major difference from the classical "noisy" Dutch auction is that the *n* bids are not revealed until the clock reaches its minimum price $\underline{b}$. Under this implementation, the only difference between the unique lowest and unique highest bid auctions is whether the unique lowest bid or unique highest bid, respectively, wins the auction. Turocy, Watson, and Battalio (2007) introduced the "silent" Dutch descending-clock auction and studied it experimentally. One of their major findings is that framing matters: market values in the "silent" Dutch implementation generally fell between those generated by the "noisy" Dutch auction and the ones generated by the first-price sealed-bid auction. Another finding is that the two "noisy" and "silent" Dutch clock-based auctions exhibited more heterogeneity across cohorts of subjects in the level of prices and in the way prices changed over time in comparison to the sealed-bid implementation.

For another implementation, we next show that an explicit prize *V* is not necessary in order to formulate and conduct unique bid auctions. For each unique bid auction with a prize *V* there exists a strategically equivalent auction with no exogenous prize in which the winner is awarded the value of her bid. To show this, consider a unique highest bid auction with a common strategy space $B=\{\underline{b},\ \underline{b}+1,\ \underline{b}+2,\ \ldots,\ \bar{b}\}$ and prize V. Hereafter, we refer to this class of auctions as auctions with *exogenous* prizes. It is easy to see that this auction is strategically equivalent to a unique lowest bid auction with strategy space $B=\{V-\bar{b},\ V-\bar{b}+1,\ldots,\ V-\underline{b}\}$ and instead of an exogenous prize V, the player choosing the unique lowest bid is awarded the value of this bid. We refer to this class of auctions as unique lowest (UL) bid auctions with *endogenous* prizes. Similarly, consider a unique lowest bid auction with exogenous prize V and strategy space $B=\{\underline{b},\ \underline{b}+1,\ \underline{b}+2,\ \ldots,\ \bar{b}\}$. It is strategically equivalent to a unique highest bid auction with endogenous prize and strategy space $B=\{V-\bar{b},\ V-\bar{b}+1,\ldots,\ V-\underline{b}\}$ in which the player choosing the unique highest bid is awarded the value of this bid. We refer to this class of auctions as unique highest (UH) bid auctions with *endogenous* prizes. Although exogenous unique lowest (highest) bid auctions are strategically equivalent to endogenous UH (UL) auctions, it is an empirical question whether they yield the same pattern of bidding behavior.

In the experiments reported in this paper we implement the simpler UL and UH auctions in which the winner is paid her bid b^*. (In the remainder of this paper all references to unique bid auctions will be of this type.) To achieve tractability, we assume hereafter that n is fixed and commonly known before the auction commences. We also restrict each of the n bidders to submitting a single bid. To simplify the procedure, we charge no participation fee (c=0). The most similar study to ours is the first-price sealed-bid auction studied by Gneezy (2005) in which each of two bidders simultaneously selects an integer from the set $B=\{1, 2, \ldots, \bar{b}\}$. The winner choosing the lowest bid is paid a dollar amount times the integer she bids whereas the other player gets 0. The main difference is that in both the UL and UH endogenous auctions winning bids must be unique. Therefore, a unique bid auction is not guaranteed to end with a winner. In contrast, if there is a tie in the two-person auction studied by Gneezy, then the earnings are split equally between the bidders. We show below that the requirement of uniqueness of the winning bid has profound implications for the equilibrium solution of unique bid auctions.

The rest of the paper is organized as follows. Section 2 presents and discusses the equilibrium solution to the UL and UH auctions. It describes a computational procedure for constructing the symmetric mixed-strategy equilibrium to any degree of accuracy. Sections 3 and 4 describe two experiments designed to study the predictive power of the equilibrium solutions and identify deviations from equilibrium, if any. Section 3 presents an experiment designed to study bidding behavior in UL auctions under two conditions where either $|B| < n$ or $|B| > n$. In Section 4, we shift the focus to bid patterns with the same n for both the UL and UH auctions. Section 5 concludes.

2. Equilibrium Solutions

The Symmetric Mixed-strategy Equilibrium. Both the UL and UH auctions have multiple asymmetric pure-strategy equilibria. Because the players are symmetric, we only focus on symmetric mixed-strategy equilibria (SMSE). We describe a procedure that uses non-stationary Markov chains to numerically compute the SMSE for the UL endogenous auction to any desired degree of accuracy. The SMSE for the UH auction is computed in a similar way.

There are n players, n>2. Each player chooses a bid, which is a positive integer in the set $B = \{\underline{b}, \underline{b}+1, ..., \overline{b}\}$. Bids are made simultaneously and anonymously. The player making the smallest unique bid is the winner and he is paid his bid, b ($b \in B$). If there is no unique bid then no one wins. We assume no entry fee to play the game which simplifies the analysis. If we have an entry fee then we would need to refund the fee or allow players to repeat the auction in case of a tie. An auction under those assumptions will be solved as part of a future research agenda.

Denote by p_b the equilibrium probability of bidding *b*. Let one of the *n* players be a designated player. The expected payoff for this player for each bid *b* is computed and used to solve for the equilibrium probabilities $p_{\underline{b}}, p_{\underline{b}+1},..., p_{\bar{b}}$. Note that each of the $n-1$ others, as well as the designated player, independently chooses the bids according to the probabilities $p_{\underline{b}}, p_{\underline{b}+1},..., p_{\bar{b}}$.

To construct the equilibrium probabilities, we use a non-stationary Markov chain. Denote by $s_b \in S_b$ a state vector at *b* which has a length of 2. The first element specifies the number of bidders (out of the *n*-1 other bidders) making a bid greater than *b*. Then, this element takes an integer from 0 to *n*-1. The second element is used to keep track of whether there was a unique bidder less than or equal to *b*. Thus, it is assumed to take either 0 or 1. The value 0 indicates that there was no such bidder whereas 1 indicates there was at least one such bidder. For example, with $n-1=5$, one state of the game is [3 0]. This state vector indicates that among all bids exactly two are less than or equal to *b*, and none are unique (therefore, the two bidders must have placed different bids). Note that there are $2n-1$ possible states for each bid value *b*.[1]

Denote by $P(\underline{b}-1)$ a $1\times(2n-1)$ initial vector, whose elements are probabilities over possible state vectors at $\underline{b}-1$. Since $\underline{b}-1$ is not an element of the strategy space (i.e., all the $n-1$ players are only able to accept higher asking prices than $\underline{b}-1$), the probability that $s_{\underline{b}-1}=[n-1 \quad 0]$ is 1, in other words, $P_{[n-1 \quad 0]}(\underline{b}-1)=1$. Therefore,

$$P(\underline{b}-1)=[P_{[0 \quad 0]}(\underline{b}-1) \quad P_{[0 \quad 1]}(\underline{b}-1) \quad ... \quad P_{[n-2 \quad 1]}(\underline{b}-1) \quad P_{[n-1 \quad 0]}(\underline{b}-1)]=[0 \quad 0 \quad ... \quad 0 \quad 1].$$

[1] State [*n*-1 1] does not exist.

For $b \geq \underline{b}$, define a $(2n-1)\times(2n-1)$ transition matrix $P(b-1,b)$ with elements $P_{x,y}(b-1,b) = P(s_b = y \mid s_{b-1} = x)$. This gives the probability of a transition from state vector *x* at *b*-1 to state vector *y* at *b*. To illustrate such a transition, consider $s_{b-1} = [3 \quad 0]$. Then, the probability that $s_b = [2 \quad 1]$ is

$$P(s_b = [2 \quad 1] \mid s_{b-1} = [3 \quad 0]) = \binom{3}{1} h_b (1-h_b)^2,$$

where $h_b = \dfrac{p_b}{p_b + ... + p_{\bar{b}}} = \dfrac{p_b}{1-\sum_{\beta<b} p_\beta}$. h_b is the probability of bidding *b* given that the bid was at least *b*. To construct a transition matrix, all possible transitions from one state to another must be considered.[2] Then, the row vector that constitutes the probability distribution over state vectors at *b* is obtained by the following matrix multiplication:

$$P(b) = P(\underline{b}-1)P(\underline{b}-1,\underline{b})P(\underline{b},\underline{b}+1)...P(b-1,b).$$

Recall that the designated player who bids *b* will win provided that there was no unique bidder who bid less than *b*. This probability can be extracted from the row vector $P(b-1)$ by finding the probability of the state whose second component is 0. For example, $P_{[u \quad 0]}(b-1)$ is the probability that *u* players (of the $n-1$ players) bid greater than $b-1$ and there was no unique bidder who bid less than or equal to $b-1$.

Now suppose that the designated player places a bid of *b* while $s_{b-1} = [u \quad 0]$. Then, the designated player becomes the winner only if none of the *u* players bid *b*. This probability is $(1-h_b)^u$. Hence, the expected payoff of bidding *b* is

$$b\sum_{u=0}^{n-1}(1-h_b)^u P_{[u \quad 0]}(b-1).$$

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[2] It is impossible for some transitions to take place. For example, state [1 0] cannot be reached from state [2 0]. Thus, the probability of such a transition is 0.

To compute the SMSE, note that the behavior of players who bid greater than *b* does not affect the payoffs of those who bid less than or equal to *b*. Thus, the expected payoff of accepting *b* is a function of the equilibrium probabilities only through $p_{\underline{b}}, p_{\underline{b}+1}, ..., p_b$. To determine p_b, the values of $p_{\underline{b}}, p_{\underline{b}+1}, ..., p_{b-1}$ are fixed and p_b is varied. We use $E_b(p_b)$ for the designated player's expected payoff of bidding *b* given that all the other players follow the mixed strategy $p_{\underline{b}}, p_{\underline{b}+1}, ..., p_b$. Notice that E_b is strictly decreasing in p_b because the probability of a tie (i.e., losing) increases as p_b increases. This fact will be used to search for p_b.

To find the equilibrium probabilities, we use the following general result. Suppose that *E* is a true expected payoff of the game. Then, (a) $E_b \le E$ for any bid *b*, (b) $E_b = E$ if $p_b > 0$, and (c) $p_b = 0$ if $E_b < E$.[3] Since the true value of *E* is unknown, we must start with an estimate of *E*.[4] For a given value of *E*, the SMSE probabilities $p_{\underline{b}}, ..., p_{\bar{b}}$ are constructed sequentially through the following algorithm that starts at $\underline{b}$ and continues through $\bar{b}$.

1. Set a value of *E*.
2. Consider bid *b*. Given $p_{\underline{b}}, ..., p_{b-1}$, compute $E_b(0)$.
   a. If $E_b(0) \le E$, then keep $p_b = 0$ since it maximizes E_b. If $b < \bar{b}$, increase *b* by a single unit and repeat step 2. Otherwise, go to step 3.

[3] For proof of this general result, see sections 3.1.5, 3.4.2, and 3.4.3 in Vorob'ev (1977).

[4] To search for a true value of *E*, we used a root-finding technique called a *bisection method*. The bisection method works by recursively dividing in half the interval in which the true value of *E* lies. For example, if α and β are lower and upper bounds of the interval where the true value of *E* exists, then the above algorithm starts with setting *E*=(α+β)/2. If the current value of *E* is an upper (lower) bound, the interval above (below) the current value of *E* will be discarded. Then, we set a midpoint of the new interval as a value of *E* and repeat the above algorithm with the new *E*. This method guarantees convergence to the true value of *E*.

b. If $E_b(0) > E$, evaluate $E_b(1-\sum_{\beta<b} p_\beta)$, where $1-\sum_{\beta<b} p_\beta$ is the maximum feasible value of p_b.

   i. If $E_b(1-\sum_{\beta<b} p_\beta) \le E$, then, there exists p_b ($0 < p_b \le 1-\sum_{\beta<b} p_\beta$) such that $E_b(p_b) = E$ since E_b is continuous and strictly decreasing in p_b. If $b < \bar{b}$, then increase b by 1 unit and repeat step 2. Otherwise, go to step 3.

   ii. If $E_b(1-\sum_{\beta<b} p_\beta) > E$, then, the game has no solution for the given value of E. Terminate the algorithm. Since the current value of E is a lower bound, go to step 1, increase E, and repeat the algorithm.

3. If $1-\sum_{b=\underline{b}}^{\bar{b}} p_b > \varepsilon$, where ε specifies how precise the equilibrium probabilities will be, then, go to step 1, decrease E because the current value of E is an upper bound, and repeat the algorithm. Otherwise, $p_{\underline{b}},\dots,p_{\bar{b}}$ are the equilibrium probabilities.

To illustrate the equilibrium solutions to the UL and UH auctions, consider the following two endogenous auctions in which the winner, if there is one, is paid her bid. We consider groups of n=30 players and a common strategy space B={1, 2, … , 50}. The unique lowest bid auction is denoted by UL(30,50) and the unique highest bid auction by UH(30,50). Figure 1 exhibits the SMSE solutions to the two auctions, which are clearly not the mirror images of each other. First, in the equilibrium solution to the UL(30,50) auction (upper panel), each of the bids in B is chosen with a positive probability. In contrast, in the equilibrium solution to the UH(30,50) auction (lower panel) the bids 1 through 37 are never chosen at all. Second, in the equilibrium solution to the UL(30,50) auction, the probabilities p_b first increase and then decrease as the bid b increases, whereas in the solution to the UH(30,50) auction they increase monotonically in b.

Thirdly, the probability of ending the UH(30,50) auction with no winner ($p_{\text{no winner}} = 0.044$) is about 14 times as high as for the UL(30,50) auction ($p_{\text{no winner}} = 0.003$). Our computations show that the expected bids in the UL(30,50) and UH(30,50) auctions are 7.690 and 45.586, respectively (the standard deviations are 6.368 and 3.151), and the corresponding expected payoffs are 0.140 and 1.446. These results suggest that in testing the equilibrium solutions experimentally, these two types of unique bid auctions ought to be considered separately.

--Insert Fig. 1 about here--

3. Experiment 1: UL Auctions with *n*=5 Bidders

Experiment 1 was designed to test the descriptive power of the SMSE solution for the UL auction under two conditions. In both conditions, only five bidders participated in each auction. In Condition UL(5,4), n=5 and $B=\{1, 2, 3, 4\}$. This condition forces at least one tie and, as a consequence, a relatively high probability that the auction closes with no winner. In the second condition, called Condition UL(5,25), n=5 and $B=\{1, 2, \ldots, 25\}$. This condition allows each bidder a considerably wider selection of bids. It yields fewer ties in equilibrium and, consequently, a much smaller probability of closing the auction with no winner.

Method

Subjects. The subjects were 65 undergraduate and graduate students at the University of Arizona, who volunteered to participate in a computer-controlled group decision making experiment for payoff contingent on their performance. Male and female subjects participated in approximately equal numbers. Subjects interacted in groups of 5, five groups (sessions) in Condition UL(5,4) and eight other groups in Condition UL(5,25). Each session lasted approximately 1 hour. Including a $5 show-up bonus, the mean payoff for Conditions UL(5,4) and UL(5,25) was $16.50 and $16.00, respectively.

Procedure. All the sessions were conducted in the same way. The five group members were randomly seated in a large computer laboratory and handed written instructions. No communication between the subjects was possible. The subjects were instructed that the purpose of the experiment was to study "a new type of auction that has become quite popular in the Internet." Implementing a within-group design, each session included 60 identical rounds (auctions) that were structured as follows. On each round, the subject was asked to enter a bid by choosing one of the elements in the common set B. Bids were entered anonymously. The subjects were instructed that the winner would be the one entering the lowest bid, provided that it is unique. A winner would earn the value of her bid, whereas non-winners would earn nothing. No participation fee was charged. Four numerical examples of bid profiles $\boldsymbol{b}=(b_1, b_2, \dots, b_5)$ were presented and explained.

Three screens were presented on each round. The Decision Screen listed the possible bids in B and probed the subject to choose and enter one of them. The Results Screen presented all the five bids for the round, identified the winning bid (if at all), and recorded the subject's payoff for the round. Individual bidders were not identified. At any time, the subject could access a History Screen, which displayed the round number, all her previous bids from round 1 to the present round, and all the values of the previous winning bids. The experiment was self-paced.

At the end of the session, the subjects were paid in cash their cumulative earnings and dismissed from the laboratory. In equilibrium, the expected earning per auction is 0.341 for Condition UL(5,4) and 0.582 for Condition UL(5,25). Consequently, the exchange rate was set differently for the two conditions in order to equalize the mean earnings ($1.00 per 2 points and $1.00 per 3.5 points in Conditions UL(5,4) and UL(5,25), respectively).

Results

This section is organized around the observed bids, the observed auction outcomes, and the dynamics of play. We first test the null hypothesis of SMSE play on the individual and group levels. Next, we focus on the relative frequency distributions of the auction outcomes and compare them to the corresponding distributions under SMSE play. We conclude this section with the analysis of the frequencies of switches between bids and examination of individual differences. Since the equilibrium analysis yields quite different predictions for bidding behavior in Conditions UL(5,4) and UL(5,25), the two are not directly comparable to each other. Consequently, we present and discuss the results of these two auctions separately.

Condition UL(5,4)

Bids. Under the null hypothesis of SMSE play, bidders are independent of one another as well as are the 60 iterations of the same auction for each bidder. On each round, under this null hypothesis, bidders independently randomize their bids in the set *B* according to the equilibrium probabilities. This is a very powerful, easily refutable, hypothesis that leaves no scope for individual differences. Differences in frequencies of bids between subjects are due to different realizations of the SMSE probabilities, not to the probability distributions governing the choice of bids.

The equilibrium probabilities of the four bids are exhibited in the upper panel of Fig. 2. The four probabilities are almost equal to one another (0.236, 0.270, 0.251, and 0.243 for bids 1, 2, 3, and 4, respectively), implying that all four bids are chosen almost equally likely. We used the Kolmogorov-Smirnov (K-S) test (df=60) to test the hypothesis of SMSE play for each subject separately. Of the 25 individual cases, the null hypothesis could not be rejected ($p<0.05$) for 14 subjects (56%). As we show later in this section, significant deviations from equilibrium play on the individual level are mostly due to a minority of the subjects who did not switch their bids between rounds as frequently as expected. Rather, they displayed longer than expected runs of the same bid.

--Insert Fig. 2 about here--

The upper panel of Table 1 presents the relative frequency distributions of the observed bids across all 60 auctions. The distributions are shown separately by session (columns 2-6) and then combined across the five sessions (column 7). The major differences between the observed and predicted distributions concern bids 1 and 2. Using the session as the unit of analysis, Table 1 (and Fig. 2) shows that in all five sessions the mean frequency of bid 1 exceeded the theoretical value (p=0.031 by a Binomial test). Similarly, again in all five sessions, the mean frequency of bid 2 was smaller than predicted (p=0.031).

--Insert Table 1 about here--

<u>Outcomes</u>. Given the equilibrium probabilities of the four bids, it is possible to compute the resulting probabilities of the auction outcome as shown in the right-hand column of the lower panel in Table 1. Unlike the equilibrium probabilities of bids in the upper panel (column 8), the equilibrium probabilities of the auction outcome exhibit considerable disparity. In equilibrium, the auction should end with a winning bid 1 40.2 percent of the time compared to 10.4 percent for the winning bid 4. The auction should close with no winning bid 12.2 percent of the time.

Table 1 (lower panel) also displays the relative frequency distributions of the observed auction outcomes. The distributions are presented for each session separately (columns 2-6) and then combined across the five sessions (column 7). Each of the five relative frequency distributions summarizes the outcomes of 60 auctions. Once again, the K-S test was used to compare the observed and predicted CDF's. The null hypothesis of no difference between these distributions was tested separately for each session (df=60). In no case did we find evidence to reject it (p<0.05). A comparison of columns 7 and 8 of Table 1 shows that the equilibrium solution accounts for the aggregate outcome frequencies (computed across sessions) extremely well. Remarkably, the deviations from equilibrium recorded above for 11 of the 25 subjects did not result in significant differences between observed and predicted outcome distributions.

Dynamics. The K-S test results for the distribution of bids rejected the null hypothesis of SMSE play for 11 of the 25 subjects. Therefore, any systematic changes in aggregate bidding behavior over time are only due to these subjects. To characterize the deviations from SMSE, we computed for each subject separately the number of switches in bids, that we denote by w ($0 \leq w \leq 59$). A switch occurs if a subject bids $b' \in B$ on round t and $b'' \in B$ on round t+1, where t=1, 2, … , 59, and $b' \neq b''$. We then computed how many switches would be observed in the UL(5,4) auction under SMSE play. Each bidder either switches or not on each of 59 pairs of adjacent rounds, independently of the previous outcome. Therefore, the total number of switches per bidder is binomial. The probability of not switching is given by conditioning on the result of a bid: $\sum_{i=1}^{4} p_i^2 = 0.236^2 + 0.270^2 + 0.251^2 + 0.243^2 = 0.250$. Then, the expected number of switches is 59(1-0.250) = 44.25 and the associated standard deviations is $\sqrt{59(1-0.250)0.25}$ =3.326. Under SMSE play, we can compute the 0.005 and 0.995 percentiles of the distribution of w and find $P(36 \leq w \leq 52) \approx 0.99$. Figure 3 exhibits the theoretical and observed relative frequency distributions of number of switches, and displays the central 99% interval of the distribution of w. It shows that, in agreement with the K-S results for the individual distributions of bids, 14 of the 25 values of w fall within the [36, 52] interval. Of the remaining 11 subjects, 10 subjects switched their bids between consecutive rounds less frequently than expected (p<0.001 by a one-tailed Binomial test).

--Insert Fig. 3 about here—

To illustrate the bid patterns of individual subjects, Fig. 4 portrays the 60 bids of each of the five subjects in Session 1. Session 1 is typical; as the other four sessions exhibit similar bidding patterns, they are not displayed. Each individual graph plots the bids by round. Subject 1 in Fig. 4 is the only group member who switched significantly fewer times than predicted. This is due to a long run of bid 4 that started on round 18 and, with a few exceptions, continued until the end of the session.

--Insert Fig. 4 about here—

Condition UL(5,25)

Bids. The lower panel of Fig. 2 displays the SMSE probabilities of bids for Condition UL(5,25). In equilibrium, each of the 25 bids in B is chosen with a positive probability. Figure 2 shows that the equilibrium distribution of bids is positively skewed. The mode of the distribution is at bid 2. As the bid values increase from 2 to 20 the SMSE probabilities decrease, and then they slowly increase as the bid values continue increasing from 20 to 25. This particular behavior at the right tail of the probability distribution is due to the relatively small number of bidders. If a critical bidder attaches a positive probability to the event that the other n-1 group members will tie, an event that is not entirely unlikely when n=5, then it is advantageous to her to bid the maximum $\bar{b}$=25 and thereby maximize her payoff for the round. However, another bidder who may anticipate her behavior may bid $\bar{b}$-1. This line of reasoning, not strange to some of our subjects, is reflected in the right tail of the probability distribution of bids. Also depicted in the lower panel of Fig. 2 are the observed relative frequencies of bids computed across all the 60 rounds and 8 sessions.

The K-S test (df=60) was used as before to test the null hypothesis of SMSE play on the *individual* level. Unlike Condition UL(5,4), the null hypothesis was rejected for 35 (87.5%) of the 40 subjects. Notwithstanding this result, Fig. 2 seems to suggest that the *aggregate* bids are accounted for quite accurately by the SMSE probabilities. To test this hypothesis, we invoked again the K-S test (df=2400). Given the exceedingly large number of degrees of freedom, it is not surprising that the null hypothesis was rejected also on the aggregate level (D=0.039, p<0.01). The deviation from SMSE on the aggregate level, small as it may seem, resulted from subjects choosing the bids in the range 1-10 more frequently than predicted. A closer analysis of the data that focuses on the individual sessions reveals that the deviation from equilibrium on the aggregate level was largely due to a single session, Session 6, in which the subjects displayed different patterns of bidding behavior than in the other seven sessions. In particular, bids 8-25 were chosen by the members of Group 6 in only 4 (1.33%) of 300 cases. The comparable values for the other seven groups ranged between 13.67 to 36.00 percent. Clearly, Session 6 was an outlier. Once it was omitted from the analysis, the difference between the observed and predicted CDF's of bids on the aggregate level was not statistically significant by the K-S test.

Outcomes. Like Table 1 for Condition UL(5,4), Table 2 presents the observed and predicted probabilities of the auction outcomes for Condition UL(5,25). Because there are very few winning bids greater than 8, we combined all the winning bids between 9 and 25 into a single class. Thus, Table 2 presents ten outcomes, namely, winning bids 1, 2, … , 8, 9-25, and no win. Columns 2-9 show the observed relative frequencies of the outcomes by session, column 10 presents the relative frequencies of outcomes across the eight sessions, and the right-hand column displays the probabilities of the ten outcomes under SMSE.

--Insert Table 2 about here--

Comparison of columns 10 and 11 suggests very minor and non-systematic deviations between the observed and predicted probabilities. The null hypothesis of no difference between the observed and predicted CDF's of the outcomes was tested separately for each session (df=60). Once again, in complete agreement with the results for Condition UL(5,4), we found no evidence to reject the null hypothesis for any of the eight sessions. Aggregating the outcome frequencies across the sessions and testing the same null hypothesis now applied to the aggregate data also yielded non-significant results (D=0.029, df=480, p>0.05).

Dynamics. The K-S test of the null hypothesis of no difference between observed and predicted CDF's of bids was rejected for 35 of the 40 subjects. Similarly to Condition UL(5,4), we wish to determine whether deviations from SMSE play on the individual level are mostly due to under-switching. The expected number of switches was computed to be $\mu(w)$=53.376, and the standard deviation was $\sigma(w)$=2.256 with $P(47 \le w \le 58) \approx 0.99$. Figure 5 displays the theoretical and observed relative frequency distributions of the number of switches. A total of 22 out of 40 subjects switched their bids significantly less frequently than predicted. Most of them had longer than expected runs of the same bid particularly in the second part of the session. Of the remaining 18 subjects only 5 were shown before to adhere to equilibrium play. The remaining 13 subjects did, indeed, switched their bids as frequently as expected but did not choose the different bid values according to the equilibrium probabilities.

--Insert Fig. 5 about here—

Discussion

The major purpose of Experiment 1 was to test the descriptive power of the SMSE solution for the UL auction under two different conditions that had the same small number of bidders in each group but differed from each other in the number of possible bids. Condition UL(5,4) severely limited the choice to four bids and, consequently, yielded a relatively large probability of ending the auctions with no winner. In contrast, Condition UL(5,25) imposed very weak constraints on the choice of bids. The SMSE solutions for the two conditions were shown to be quite different. Our results show that the bidding patterns of the majority of the subjects in Condition UL(5,4) followed SMSE play, but even a larger majority of the subjects in Condition UL(5,25) did not. Rather, in both conditions—but more so in Condition UL(5,25)—we observe bidding patterns that vary considerably from one group member to another with a tendency of many bidders to generate longer runs of the same bid than expected. Despite the small size of the groups, aggregate bids within session are mostly accounted for quite well by the SMSE.

The two auctions studied in Experiment 1 share the same rule for determining the outcome with Internet auctions. However, they differ from Internet auctions in several important respects. First, due to a different framing of the auction in our study, the prize in Internet auctions is exogenously determined, whereas in our experiment it is endogenously determined. Second, Internet auctions do not reveal the number of bidders to the potential participants whereas in our experiment it is commonly known. Also, Experiment 1 allowed only a single bid per subject whereas the number of bids per bidder in Internet auctions is typically not regulated. We do not know how many potential bidders considered the UL auction on the Internet and decided not to bid. Finally, the number of bidders in Internet auctions is counted in the hundreds and thousands whereas in our experiment it is very small. Our purpose in the present study and in subsequent studies of unique bid auctions is to remove these differences one at a time. For this purpose, we conducted a second experiment that examined bidding behavior in both the UL and UH auctions with larger groups.

4. Experiment 2: UL and UH Auctions with *n*=10 Bidders

The purpose of Experiment 2 was to extend the previous research to two other unique bid auctions with a larger number of bidders (n=10) and same strategy space B={1, 2, … , 25} as in Condition UL(5,25). The two new auctions only differed from each other in the rule determining the outcome: unique lowest bid in Condition UL(10,25) and unique highest bid in Condition UH(10,25). Figure 6 exhibits the SMSE solutions for the two auctions (the UL (UH) auction in the upper (lower) panel). Similarly to Fig. 1, Fig. 6 shows that the equilibrium solutions to the UL(10,25) and UH(10,25) auctions are not mirror images. A heuristic explanation for this is as follows. In placing her bid, a bidder in both auctions is driven by two motives, namely, to maximize the probability of choosing a winning bid and maximize her expected payoff. Both motives operate in the same direction in the UH auction: to win the auction, the bidder wishes to place a high bid. The higher the bid she places, the higher her payoff if she wins the auction. On the other hand, these two motives operate in opposite directions in the UL auction: to win the auction, the bidder wishes to place a low bid. However, the higher the bid she places, the higher her payoff if she wins the auction. The same two forces operate in the auction studied by Gneezy, where in the case of tie the winning bidder is determined by lottery. In the auctions that he conducted, the equilibrium solution is always in pure strategies. The results of Experiment 1 reported in Section 3 show that, for groups including only five members, the subjects balanced these two conflicting motives successfully. One of the goals of Experiment 2 is to ascertain whether this finding generalizes to groups twice as large. A second and more important goal is to determine whether it holds in the case of the UH auction.

--Insert Fig. 6 about here--

Method

Subjects. The subjects who participated in Experiment 2 were 100 undergraduate and graduate students at the University of Arizona, who volunteered to take part in a group decision making experiment for payoff contingent on performance. Male and female subjects participated in nearly equal proportions. None of the subjects had participated in Experiment 1. Subjects were run in groups of 10, five groups in Condition UL(10,25) and five other groups in Condition UH(10,25). A session lasted about 75 minutes. Including a $5.00 show-up bonus, the mean payoff in Conditions UL(10,25) and UH(10,25) was $17.21 and $17.02, respectively.

Procedure. The experimental procedure was identical to the one in Experiment 1 with three minor exceptions. First, two different sets of written instructions were handed, one for the UL sessions and the other for the UH sessions. Second, the examples presented in the instructions included groups of 10 not 5 members. Third, different exchange rates were used than in Experiment 1. Including the $5 show-up bonus, the expected earning in Conditions UL(10,25) and UH(10,25) were $16.09 and $16.04, respectively. To equalize mean earnings across the two conditions, the exchange rate was set at $1.00 per 1.5 points in Condition UL(10,25) and 11 points in Condition UH(10,25).

Results

The organization of this section is similar to the one in Experiment 1. First, we present the distribution of bids and test the equilibrium solution on the individual, group, and aggregate levels. We then present the distribution of the auction outcomes and test the equilibrium solution on the group and aggregate levels. Finally, we focus on the frequencies of switches in bids in an attempt to account for the reasons, if at all, for deviation from equilibrium play. Our results show that, in agreement with Experiment 1, although individual play does not in general adhere to mixed-strategy equilibrium play, the SMSE continues to give an accurate description of the outcomes in Condition UL(10,25) on the group, and even more so on the aggregate level. This is no longer the case in Condition UH(10,25) where the unique highest bid wins the auction. Instead, bidders spread their bids across a wider range than predicted. To compare the two types of unique bid auction, we no longer separate them as in Experiment 1.

Bids. Following the same procedure as in Experiment 1, the K-S test was invoked to test the null hypothesis of SMSE play on the individual level (df=60). The null hypothesis could not be rejected for 23 of the 50 subjects in Condition UL(10,25) and 18 of the 50 subjects in Condition UH(10,25). The difference between these two percentages was not significant ($\chi^2(1)=1.03$, $p>0.3$). Assuming that both subjects within a group and rounds within a subject are independent, we next tested the same null hypothesis on the group level (df=600). The null hypothesis was now rejected for three of the five groups in Condition UL(10,25) and four of the five groups in Condition UH(10,25).

The difference between the two conditions is apparent when the distributions of bids are computed across sessions in each condition. The upper panel of Fig. 6 exhibits side by side the aggregate (over sessions) relative distributions of bids and the SMSE probability distributions of the bids for Condition UL(10,25). The lower panel displays the same results for Condition UH(10,25). Figure 6 shows that the SMSE solution describes the aggregate results for Condition UL(10,25) very well. We observe no systematic discrepancies between observed and predicted probabilities across the entire range of bids from 1 through 25. The only possible exception is bid 25 that was chosen four times as frequently as expected (compare 1.6 to 0.4 percent). However, this discrepancy is mostly due to a few subjects who chose the maximum bid a disproportionally large number of times. In contrast, we observed systematic discrepancies between observed and predicted probabilities of bids for Condition UH(10,25). In equilibrium, bids equal to or smaller than 18 should never be placed and bid 19 should be chosen only 0.1 percent of the time. In contrast, bids 1-18 were actually chosen on 3.35 percent of the rounds, and bid 19 was chosen on 3.67 percent of the rounds. In total, about 7 percent of the bids were placed in a region that, in equilibrium, should be chosen with probability of one-tenth of a percent. Figure 6 (lower panel) shows that, on the aggregate, bid values 21-25 were chosen less than predicted and bids 1-20 more than predicted. The same pattern of "stretching" the bids was observed in each of the five groups in Condition UH(10,25). This is the first systematic and significant deviation from equilibrium play on the group and aggregate levels that we have uncovered.

Outcomes. The latter finding was reflected in the difference in the size of deviation from the equilibrium prediction of the distributions of the auction outcome. Table 3 presents the observed relative frequencies and equilibrium outcome probabilities for Condition UL(10,25). It uses the same format as Table 2 for Condition UL(5,25) in Experiment 1. The observed relative frequencies are presented separately for each of the five sessions (columns 2-6) and across the sessions (column 7). The right-hand column of the table presents the equilibrium probabilities. Similarly to Condition UL(5,25) that had a smaller number of players but the same strategy space, we observe only minor and non-systematic deviations between the observed and predicted outcome CDF's. The K-S test could not reject the null hypothesis of SMSE generated outcomes (df=60) for each of the five sessions. Aggregating the outcome frequencies across the five sessions and testing the same hypothesis on the aggregate level also failed to reject the null hypothesis of no difference between the two CDF's (D=0.036, df=300, p>0.10).

--Insert Table 3 about here—

We repeated the same analyses of the auction outcomes in Condition UH(10,25) but with very different results. Table 4 presents the observed relative frequencies and equilibrium probabilities of the outcomes for Condition UH(10,25). The winning bids 1-18 are collapsed into a single class (row 2), whereas the winning bids 19 to 25 are shown separately. As before, the observed relative frequencies of the outcomes are displayed for each of the five sessions (columns 2-6) and across sessions (column 7). The right-hand column shows the respective outcome probabilities generated by equilibrium play. Table 4 shows that the major effect of spreading the bids over a wider range than predicted considerably and significantly reduced the probability of ending the auction with no winner. This discrepancy was found in each session, and it is statistically significant on the aggregate level (D=0.084, df=300, p<0.001). Considered jointly, Fig. 6 (lower panel) and Table 4 show that the effect of about 7 percent of the bids placed below the minimum value under equilibrium play was sufficiently strong to reduce the percentage of auctions with no winners from 12.4 to 0.4.

--Insert Table 4 about here—

Dynamics. The mean number of switches for Conditions UL(10,25) and UH(10,25) was computed to be 52.36 and 47.47, respectively. The corresponding standard deviations were 2.43 and 3.05. Our computations also yielded $P(46 \leq w \leq 57) \approx 0.99$ and $P(39 \leq w \leq 54) \approx 0.99$ for Conditions UL(10,25) and UH(10,25), respectively. Our results show that the number of switches for 24 of the 50 subjects in Condition UL(10,25) and 19 of the 50 subjects in Condition UH(10,25) were included in the corresponding central intervals for w. With the exception of a single subject in Condition UH(10,25) who switched his bids on 58 of 59 possible times, the number of switches for 56 subjects out of a total of 100 in both conditions is smaller than predicted. As shown above, in equilibrium, subjects should switch their bids in Condition UL(10,25) more often than in Condition UH(10,25). This, indeed, was the case (z=3.61, p<0.001, Mann-Whitney U test for large samples).

Discussion

Doubling the group size from 5 to 10, while maintaining a relatively large strategy space, did not change the results reported in Experiment 1 in any major way. Across 60 iterations of the UL(10,25) auction the bid patterns of most of the subjects did not follow SMSE play. Rather, the subjects who deviated from equilibrium play did so by switching their bids between rounds less frequently than predicted. We did not find even a single subject who placed the same bid in all 60 rounds; the lowest number of switches per subject that we recorded is 5. Rather than switching their bid on almost every round, most subjects often placed the same bid on anywhere between 2 and 8 rounds perhaps in an attempt to discover the ever changing patterns of bids across short sequences of rounds and then exploit this information by best responding with a different bid. If the auction is repeated for multiple rounds without changing group members, is it beneficial to switch often? Our results show that it is not. The correlations between the individual number of switches and the individual payoff were computed for each of the two conditions separately. Both were negative, although only the one for Condition UH(10,25) was significantly different from zero (r= -0.60, p<0.001).

On the group level, and even more so across all the groups, the equilibrium solution accounted for the distributions of bids and auction outcomes in the UL auction very well. Once again, we observe a somewhat chaotic behavior on the individual level coupled with systematic and replicable patterns of bidding on the group and aggregate levels that seem to differ very little from equilibrium play. This is no longer the case when we change the rule for winning by choosing the unique highest, rather than lowest, bid. When participating in the UH auction, our subjects deviated from the Pareto deficient equilibrium solution by occasionally bidding below the predicted minimum bid. This behavior resulted in a significant drop in the probability of ending the auction with no winner but with no effect on the mean payoff.

5. Conclusion

Presenting and exemplifying the rules for the exogenous UL and UH auctions, Wikipedia made the following claim about the optimal strategy (May 20, 2007): "Assuming there was an optimal strategy for unique bid auctions, all players would come to the same conclusion about what the optimal bet(s) should be, thereby invalidating the same strategy. Therefore, by proof of contradiction, there exists no optimal strategy for a unique bid auction in the general case" [http://en.wikipedia.org/wiki/Unique_bid_auction]. This claim ignores the solution of the auction in mixed strategies. We have constructed an equilibrium solution in mixed strategies to both auctions that maintains the symmetry between the players. Like equilibrium solutions to all other auctions, our solutions assume that the number of bidders is commonly known. Consequently, we also have to impose the restriction—not shared by Internet auctions—that each player can only place a single bid. The procedure for numerically computing the probability distribution of bids can be used with both the UL and UH auctions. Theoretically, it is not limited by the number of players or number of strategies. However, it is restricted in practice mostly by the number of players, as computation time increases exponentially in n.

To test the equilibrium solution in a wide setting, we conducted four experiments, namely, UL(5,4), UL(5,25), UL(10,25), and UH(10,25), that varied the number of bidders, number of strategies, and type of auction. In all four experiments, no participation fee was charged and no exogenous prize was introduced. Rather, the game that the subjects were asked to play was framed as a unique bid auction with no participation fee in which the winner is paid her bid. Taken together, these experiments resulted in three major findings. First, a substantial minority of the subjects generated sequences of bids across 60 iterations of the auction that did not deviate significantly from mixed-strategy equilibrium play. The major reason for deviating from equilibrium play was the inclination to repeat the same bid for short sequences of 2 to 8 rounds. Second, aggregate results in the UL auction on the group or population level in most cases did not deviate significantly from equilibrium play. Similar results of almost chaotic behavior on the individual level coupled with systematic and replicable behavior on the aggregate level that adheres to the symmetric mixed-strategy equilibrium were reported in previous studies of market entry behavior (e.g., Rapoport, Seale, & Winter, 2002; Seale & Rapoport, 2000) and arrival times in single-server queues (Rapoport et al., 2004). Thirdly, aggregate bids and outcomes in UH auctions on the group or population level did deviate significantly from equilibrium play due to a minority of bids that were placed below the values predicted to be chosen by the equilibrium solution.

These findings suggest several directions in which additional experimental research on unique bid auctions might proceed. The first direction is to test the difference between the UL and UH auctions more extensively by using different group sizes and different strategy spaces. A second direction is to frame the experimental games as auctions with exogenous prizes. The results on private value auctions reported by Turocy et al. (2007), who tested and consequently rejected the null hypothesis that alternative framings of strategically equivalent games as first-price sealed bid auctions and descending-clock Dutch auctions result in the same bidding behavior, suggest that framing of auctions matters. A third direction is to endogenize the number of bidders by charging a participation fee. We intend to pursue all three directions.

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Table 1. Observed and equilibrium probabilities of bids (upper panel) and auction outcome (lower panel): Condition UL(5,4)

Bids

Bid	**Session 1**	**Session 2**	**Session 3**	**Session 4**	**Session 5**	**Total**	**Equilibrium**
1	0.263	0.237	0.383	0.303	0.300	0.297	**0.236**
2	0.230	0.197	0.207	0.250	0.213	0.219	**0.270**
3	0.240	0.343	0.210	0.270	0.323	0.277	**0.251**
4	0.267	0.223	0.200	0.177	0.163	0.206	**0.243**

Auction Outcome

Winning bid	**Session 1**	**Session 2**	**Session 3**	**Session 4**	**Session 5**	**Total**	**Predicted**
1	0.433	0.400	0.317	0.383	0.433	0.393	**0.402**
2	0.183	0.233	0.267	0.317	0.200	0.240	**0.230**
3	0.117	0.217	0.167	0.117	0.200	0.163	**0.143**
4	0.150	0.117	0.083	0.100	0.050	0.100	**0.104**
No winner	0.117	0.033	0.167	0.083	0.117	0.103	**0.122**

Table 2. Observed and predicted probabilities of auction outcome: Condition UL(5,25)

	Session									
Winning bid	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**Total**	**Predicted**
1	0.283	0.317	0.367	0.350	0.533	0.333	0.383	0.467	0.379	**0.378**
2	0.217	0.167	0.183	0.150	0.150	0.383	0.283	0.283	0.227	**0.254**
3	0.200	0.150	0.133	0.167	0.117	0.100	0.067	0.083	0.127	**0.145**
4	0.100	0.117	0.100	0.150	0.133	0.083	0.083	0.083	0.106	**0.081**
5	0.100	0.117	0.050	0.017	0	0.083	0.017	0.017	0.050	**0.042**
6	0	0.017	0.017	0.017	0	0	0.033	0.017	0.012	**0.023**
7	0.067	0.017	0.033	0	0	0	0	0.017	0017	**0.014**
8	0	0.033	0	0	0	0	0	0	0.004	**0.010**
9-25	0.033	0.067	0.083	0.133	0.067	0	0.083	0.017	0.060	**0.050**
No winner	0	0	0.033	0.017	0	0.017	0.050	0.017	0.017	**0.001**

Table 3. Observed and predicted probabilities of auction outcome: Condition UL(10,25)

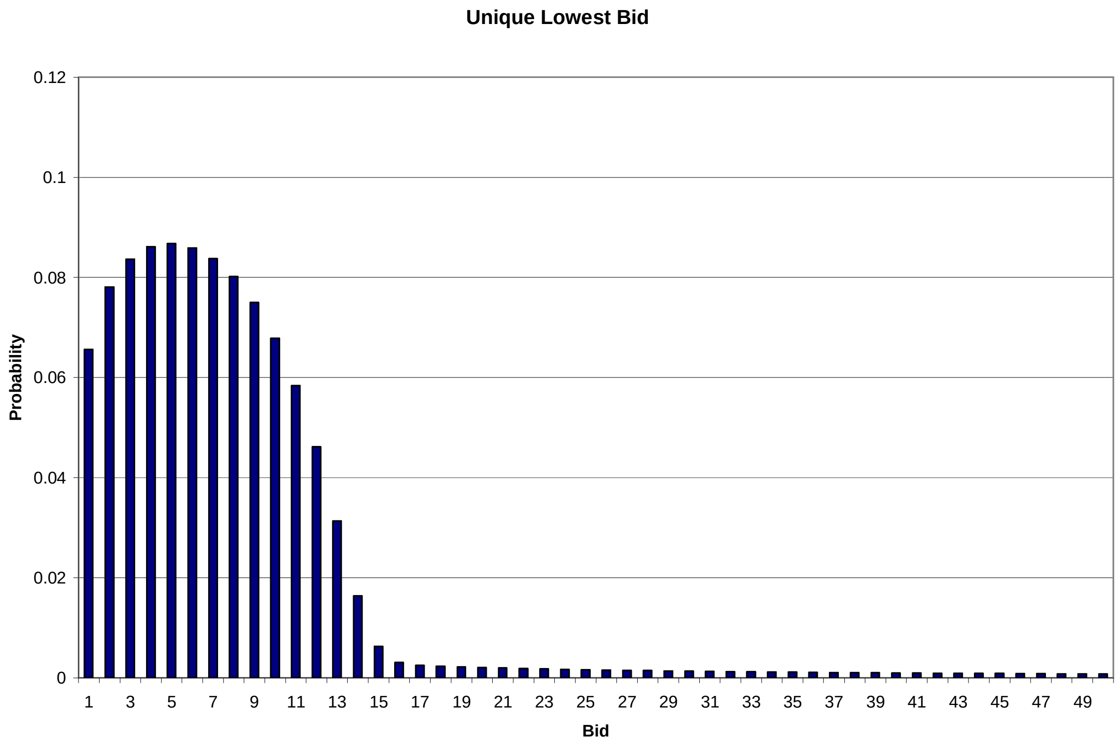
Winning bid	Session S1	S2	S3	S4	S5	Total	Predicted
1	0.300	0.417	0.317	0.300	0.383	0.343	**0.368**
2	0.217	0.183	0.200	0.217	0.233	0.210	**0.217**
3	0.133	0.167	0.133	0.150	0.117	0.140	**0.145**
4	0.117	0.117	0.117	0.083	0.133	0.113	**0.099**
5	0.100	0.033	0.100	0.117	0.083	0.087	**0.066**
6	0.033	0.067	0.017	0.033	0.017	0.033	**0.039**
7	0.033	0	0.033	0	0	0.013	**0.020**
8	0	0.017	0.033	0.033	0.017	0.020	**0.009**
9-25	0.050	0	0.033	0.050	0.017	0.037	**0.027**
No winner	0.017	0	0.017	0.017	0	0.003	**0.011**

Table 4. Observed and predicted probabilities of auction outcome: Condition UH(10,25)

	Session						
Winning bid	**S1**	**S2**	**S3**	**S4**	**S5**	**Total**	**Predicted**
1-18	0.033	0.033	0.017	0	0.017	0.020	**0**
19	0	0.050	0.033	0	0	0.017	**0.012**
20	0.067	0.033	0.117	0.083	0.083	0.077	**0.045**
21	0.100	0.083	0.033	0.133	0.017	0.073	**0.107**
22	0.200	0.167	0.100	0.167	0.150	0.157	**0.154**
23	0.167	0.117	0.217	0.217	0.133	0.170	**0.180**
24	0.200	0.267	0.233	0.133	0.333	0.233	**0.192**
25	0.200	0.217	0.200	0.200	0.250	0.213	**0.197**
No winner	0.033	0.033	0.050	0.067	0.017	0.040	**0.124**

List of Figures

Figure 1. Symmetric mixed-strategy Nash equilibrium solutions for the distributions of bids
(upper panel: Unique lowest bid; lower panel: Unique highest bid)



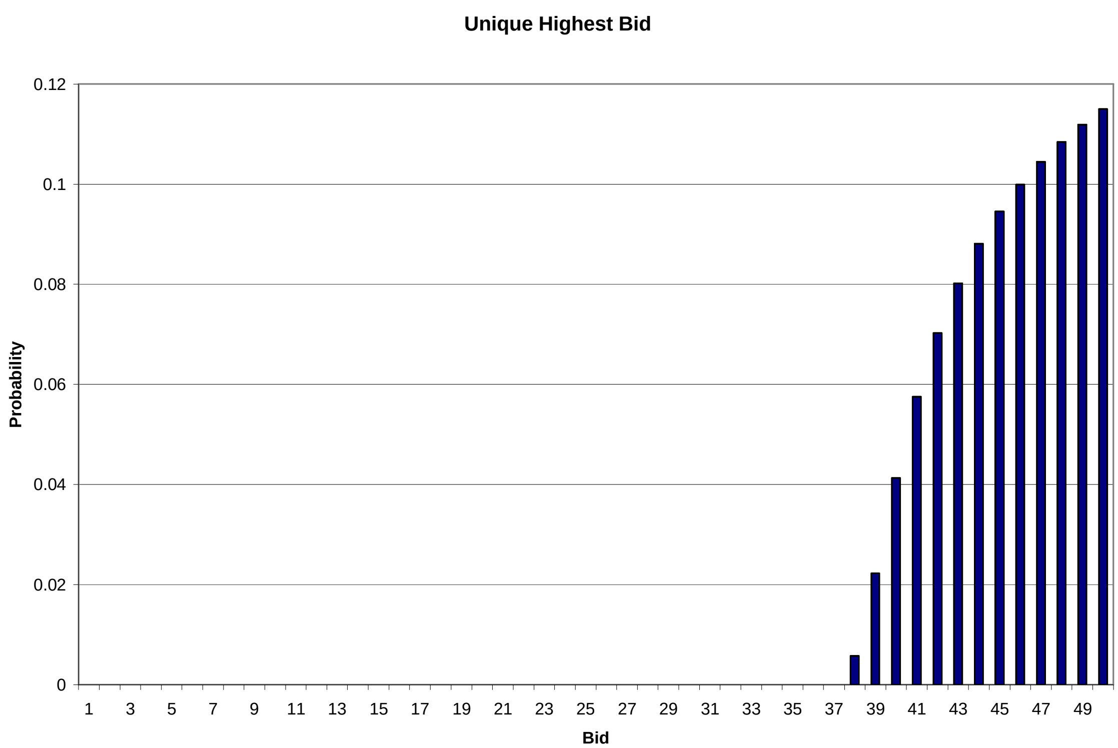
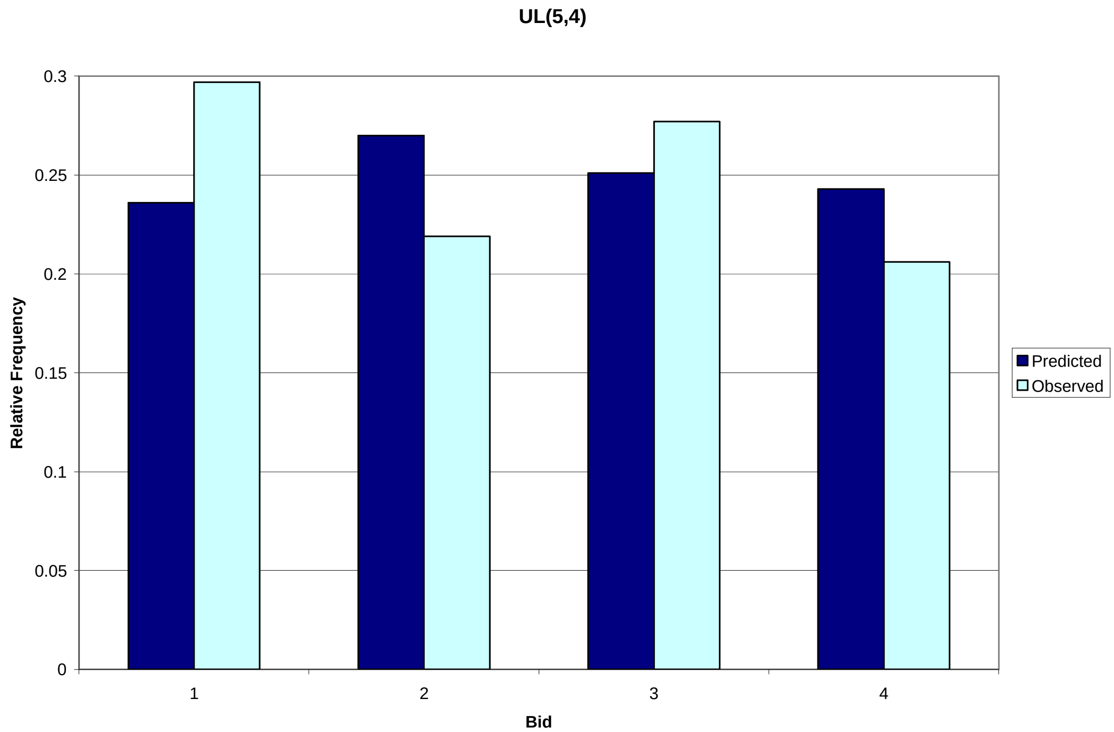


Figure 2. Predicted probabilities and observed relative frequency distributions of bids across all the subjects in Conditions UL(5,4) (upper panel) and UL(5,25) (lower panel)



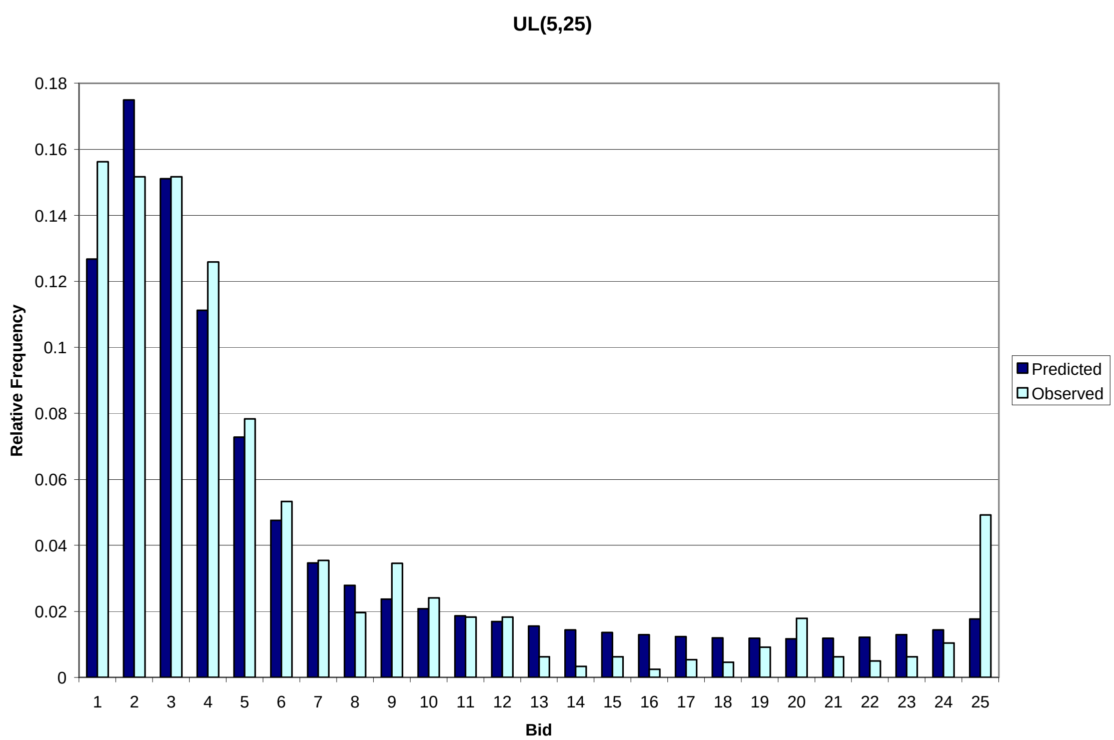


Figure 3. Predicted and observed relative frequency distributions of number of switches in Condition UL(5,4)

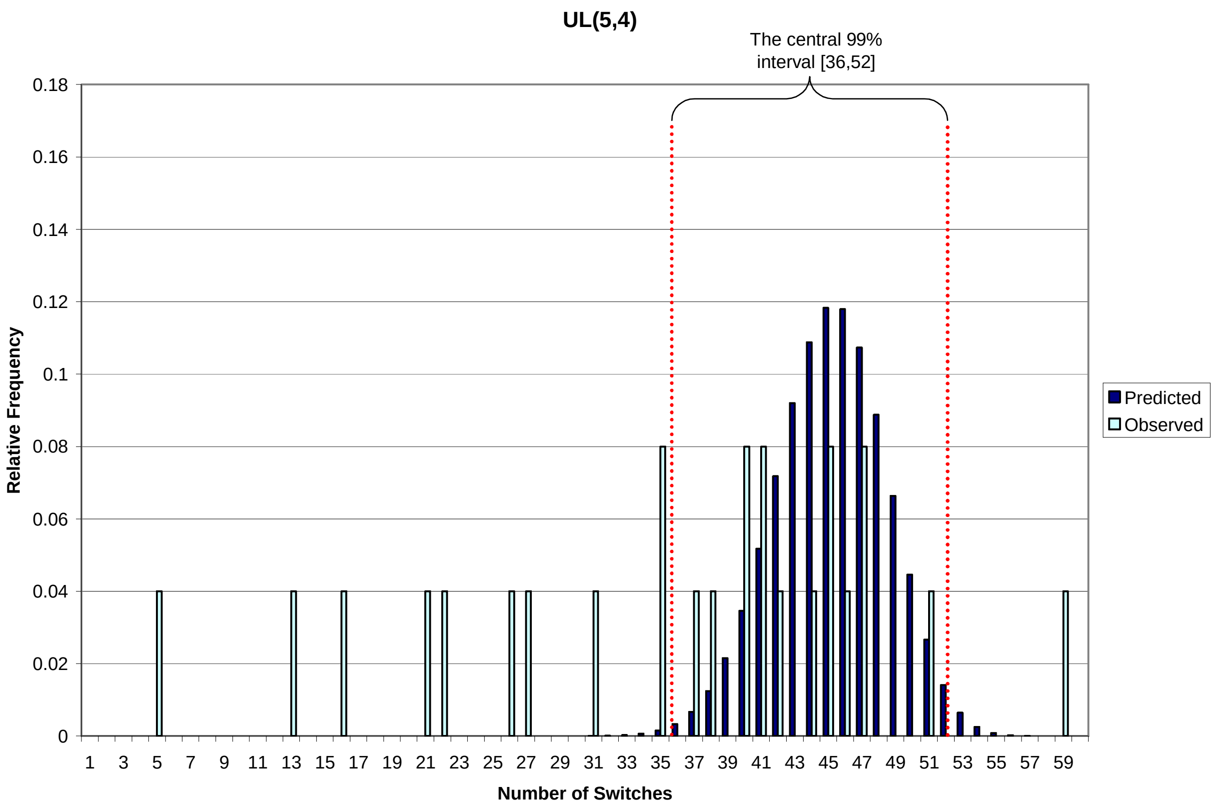


Figure 4. Individual bids by round of all five bidders in Session 1 of Condition UL(5,4).

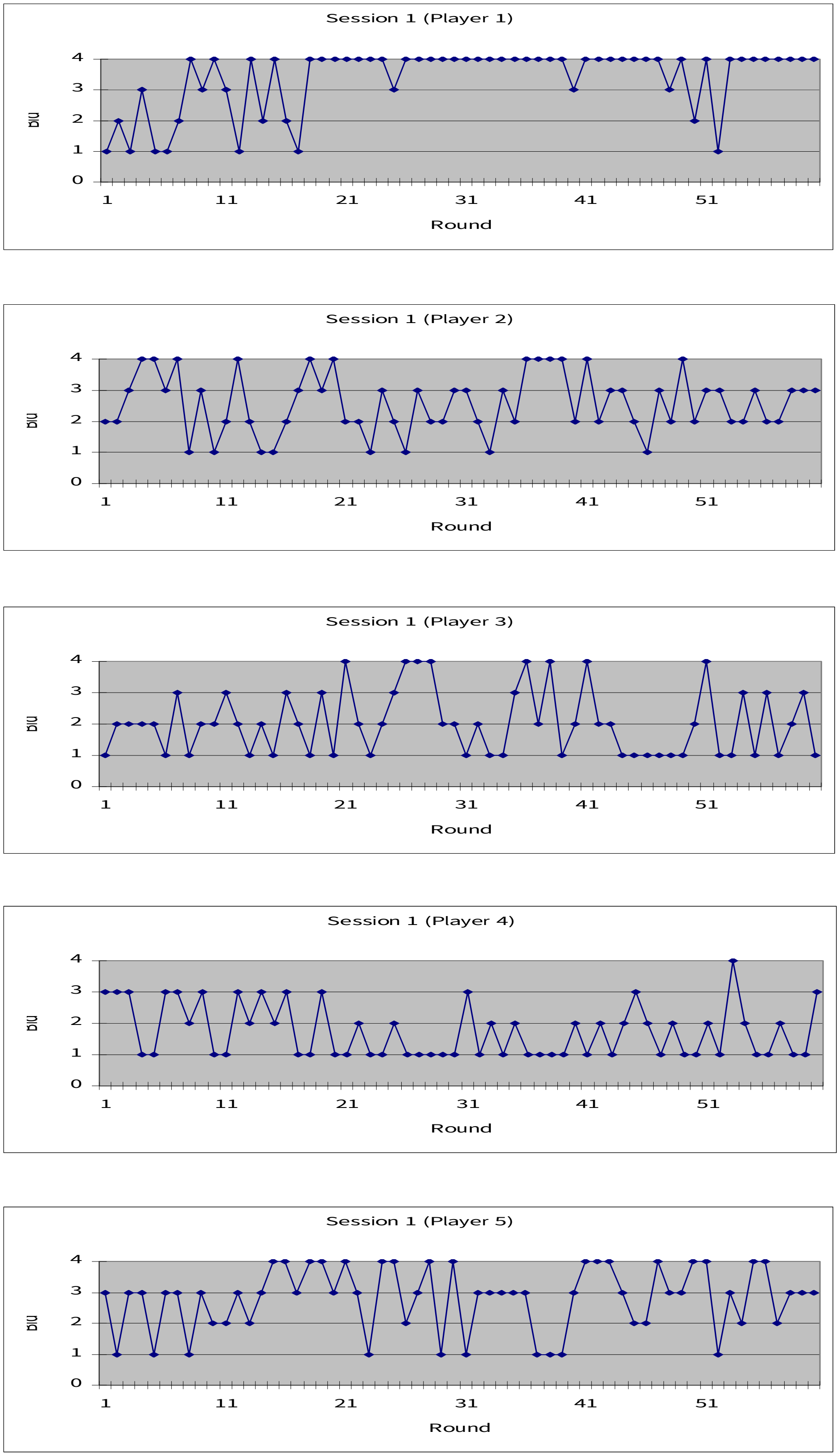


Figure 5. Predicted and observed relative frequency distributions of number of switches in Condition UL(5,25)

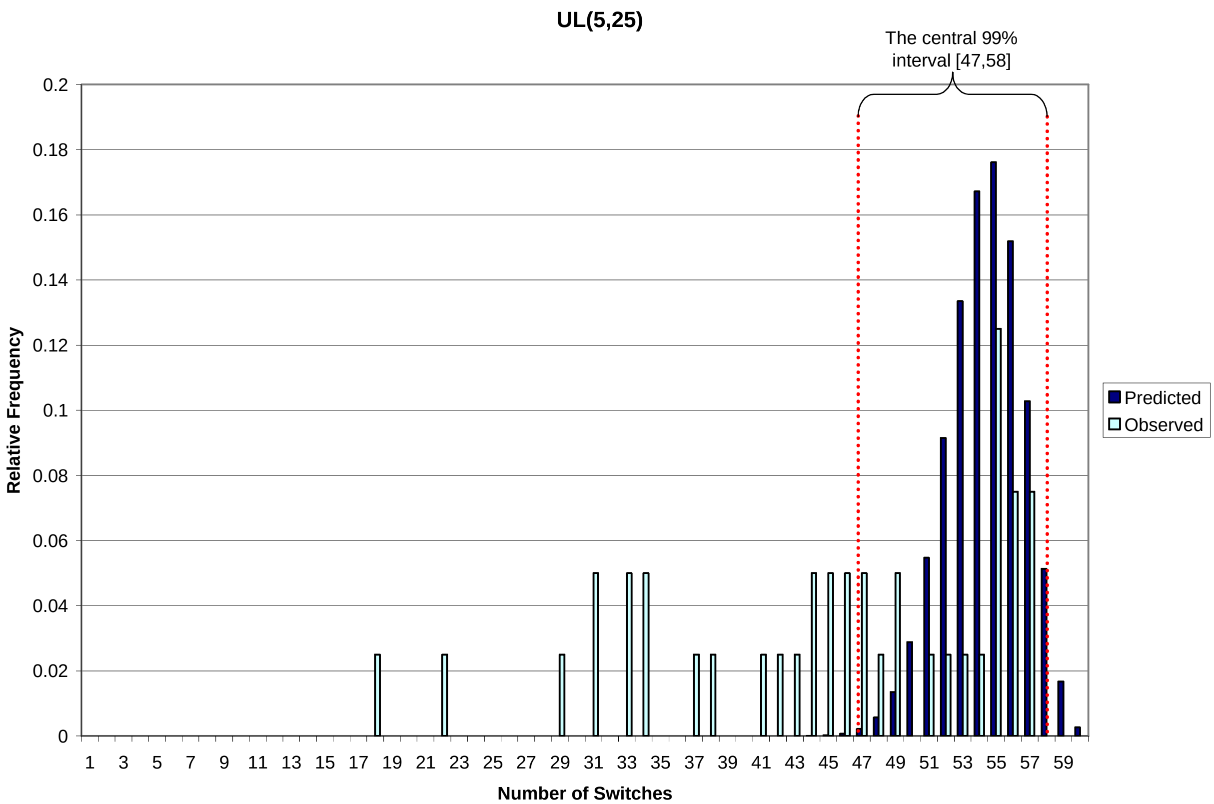


Figure 6. Predicted probabilities and observed relative frequency distributions of bids across all the subjects in Conditions UL(10,25) (upper panel) and UH(10,25) (lower panel)