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Abstract
This paper reopens the subject of currency preferences while modeling the exchange rates among three major currencies - the US dollar, the euro and the Japanese yen. The exchange rate model presented in this paper includes not only traditional determinants of bilateral exchange rates but incorporates third-currency effects in addition. The obtained estimation results are interpreted from the perspective of possible currency substitution and complementarity relationships. We find evidence of currency complementarity between the yen and the euro, and currency substitution of the dollar for both the euro and the yen. The estimated third-currency effects are consistent with our findings on currency substitution and complementarity among the three major currencies.

Keywords: Exchange Rate Modeling; Currency Substitution; Currency Complementarity; Third-Currency Effects

JEL Classification: F31, F36, F42

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1 Introduction

Recent studies of empirical models of exchange rates, see e.g. Cheung et al. (2003a, 2003b), reveal that models of the nineties do not do any better job than those of the seventies, Meese and Rogoff (1983), in terms of their forecasting ability relative to a naïve random walk. One strand of the literature thus focused on models in which the exchange rate is thought of as an asset price; see e.g. Engel and West (2005). Such models are then able to generate random walk behavior of exchange rates under plausible assumptions and to some extent demonstrate existence of reverse causality from exchange rates to fundamentals. Nevertheless, one of the conclusions Engel and West (2005) make is that there are likely to be other unobserved factors affecting the exchange rate.

One candidate factor is third-currency effects. The third-currency effects are defined as effects of fundamentals associated with currency three (e.g. the Japanese yen) on a bilateral exchange rate between currency one (the U.S. dollar) and two (the euro). Brandt et al. (forthcoming), Kingston and Melecky (2007), MacDonald and Marsh (2004), and Nucci (2003) find significant evidence of third-currency effects on exchange rates. Hodrick and Vassalou (2002) demonstrate third-country effects for bond yields, in addition. Brandt et al. start from an identity postulating that exchange rates depreciate by the difference between domestic and foreign growth of marginal utility. They argue that the growth of marginal utility implied by the equity premia requires much more variation in exchange rates to what we observe. This indicates significant correlation of marginal utilities across countries. Hence, the third-currency effects may be significant components of the pricing kernels for
exchange rates. Kingston and Melecky offer theoretical justification of third-currency effects based on non-separable currency preferences. Their empirical findings are in line with such theory. MacDonald and Marsh look at currency spillovers in a setup of three major currencies similar to that introduced in this paper. They find significant spillovers from third-currencies fundamentals. Further, if the spillover effects are incorporated in the short-run models of exchange rate dynamics they are able to beat a random-walk forecast of the exchange rates over relatively short horizons. They do not consider the effect of currencies’ expected returns on exchange rates, though, and do not relate their results to the subject of currency preferences, as we do in this paper. Similarly, Nucci finds some evidence that the term structures of forward premiums of third currencies contain significant information for exchange rate forecasting in addition to information in the currency’s own forward premium. Hodrick and Vassalou find that their multi-country models help to explain exchange rate dynamics better than their two-country counterparts and that the third-country effects appear to be significant in most cases considered.

This paper reopens the subject of currency preferences while modeling the exchange rates among three major currencies - the US dollar, the euro and the Japanese yen. The exchange rate model presented in this paper includes not only traditional determinants of bilateral exchange rates but incorporates third-currency effects in addition. The obtained estimation results are interpreted from the perspective of possible currency substitution and complementarity relationships and are new to the literature. The hypotheses of possible currency substitution or complementarity raised with respect to the three major currencies are tested using an
approach similar to that employed by Brittain (1981), and Kingston and Melecky (2007). We find evidence of currency complementarity between the yen and the euro and currency substitution of the dollar for both the euro and the yen. Further, the degree of currency substitution between the yen and the dollar appears to be stronger than the degree of currency substitution between the euro and the dollar. The third-currency effects are to a large extent consistent with our findings on the degree of currency substitution and complementarity among the three major currencies.

The paper is organized as follows. Section two outlines the model and its properties. Section three describes the data and its organization, presents unit root tests, lag-length selection tests and co-integration tests. In section four the estimations are carried out and discussed. In addition, section four contains tests and discussion of currency substitution and complementarity, and third-currency effects. Section six concludes.

2 The Model

Consider uncovered interest parity (UIP) for each of the three major currencies (the U.S. dollar, the euro, and the Japanese yen) expressed in levels of the current spot rates:

\[
\begin{bmatrix}
    s_{1t} \\
    s_{2t} \\
    s_{3t}
\end{bmatrix}
= E_t \begin{bmatrix}
    s_{1t+h} \\
    s_{2t+h} \\
    s_{3t+h}
\end{bmatrix}
+ \begin{bmatrix}
    \delta s_{1t} - \delta s_{2t} \\
    \delta s_{2t} - \delta s_{3t} \\
    \delta s_{3t}
\end{bmatrix}
\]
where \( s_t \) is the spot rate (in logs) defined in terms of the direct quotation, i.e. domestic currency per units of foreign currency, and \( i_t \) and \( i_t^* \) are the domestic and foreign interest rates corresponding to the period \( h \) over which UIP is defined.

Assume further that the endogenous system of three major currencies is governed by a single common factor, \( f_t \):

\[
\begin{bmatrix}
    s_{1t} \\
    s_{2t} \\
    s_{3t}
\end{bmatrix} = \mathbf{J} f_t + \begin{bmatrix}
    u_{1t} \\
    u_{2t} \\
    u_{3t}
\end{bmatrix} \tag{2}
\]

where \( \mathbf{J} \) is a \( 3 \times (3 - r) \) matrix of coefficients and \( \mathbf{u}_t \) is a vector of shocks. The number \( r \) is given as a difference between the number of the spot rate series (three) and the number of common factors to be determined empirically by estimating \( r \), i.e. the number of co-integrating vectors in the system. A common factor is thus thought of as being equivalent to the common permanent component (the common trend).

The common factor is assumed to be a function of a set of arbitrage conditions that eliminate any opportunities of a risk free return. The set of arbitrage conditions is the vector of UIP conditions described in (1).

Consider now a representative agent without any national habits who consumes in the three major currencies. The agent thus holds a portfolio of three currencies at each point in time\(^1\). Without imposing any assumptions regarding separability of consumption in the three currencies, possible dependencies among consumptions in the three currencies may exist\(^2\). These dependencies if significant can be characterized into relationships of currency substitution or currency

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\(^1\) Some currencies can have a zero weight in the portfolio as the corner solution.

\(^2\) Kingston and Melecky (forthcoming) talk about atemporally non-separable currency preferences in this regard.
complementarity (see e.g. Brittain, 1981; Boyer and Kingston, 1987; or Kingston and Melecky, 2007). Note that presence of the latter can imply differing effects of fundamentals on an exchange rate. Furthermore, interdependencies among the three currencies give rise to possible third-currency effects, i.e. the effect of third-currency fundamentals, underlying the value of a given currency, on a bilateral exchange rate between the remaining two currencies. The optimal holdings of a currency can thus depend on (i) the situation in the other exchange rate markets (cross-currency exchange rates), (ii) expected yields in all three currencies, and (iii) equilibrium values of the exchange rates. If no asymmetries and market inefficiencies exist, such as different trade volumes, spreads, or information disclosure and its frequency, the pieces of information (i)-(iii) would be equivalent.

We assume that the rational expectations of future exchange rates reflect the optimal current and future currency holdings\(^3\). We assume that the information set, on which the expectations are conditioned, contains (i) the remaining two bilateral spot rates, completing the current information from exchange rate markets, (iii) the expected yields on all three currencies over the period \(t\) to \(t + h\), and (iii) the three relevant purchasing power parity (PPP) conditions toward which the exchange rates are bounded to gravitate (at least in the long run). In order to investigate the presence of third-currency effects due to atemporally non-separable consumption in the three major currencies we allow the expected yields and price levels to interplay with all three exchange rates. In an unrestricted form the conditional expectation can thus be expressed as:

\[^3\text{See also MacDonald (2000) and the method of deriving the behavioral equilibrium exchange rate.}\]
where $p_t$ is a vector of price levels in a given economy. $E_t i_{t+h}$ is a vector of expected yields in the three currencies defined here as $E_t i_{t+h} = r_t - i_t$ in accord with expectation hypothesis. Assuming the function $g(\cdot)$ is linear we can write $E_t s_{t+1}$ as:

$$E_t s_{t+1} = \Phi_t s_t + \Phi_2 (r_t - i_t) + \Phi_3 p_t$$

where the $\Phi$’s are $3 \times 3$ coefficient matrices all elements of which are unrestricted.

Putting together (1), (2) and (4) gives:

$$(I - J \Phi_1) \begin{bmatrix} s_{t+1} \\ s_{2t} \\ s_{3t} \end{bmatrix} = J \Phi_2 \begin{bmatrix} r_{1t} - i_{1t} \\ r_{2t} - i_{2t} \\ r_{3t} - i_{3t} \end{bmatrix} + J \Phi_3 \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} + J \begin{bmatrix} i_{1t}^* - i_{1t} \\ i_{2t}^* - i_{2t} \\ i_{3t}^* - i_{3t} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}$$

where $A = (I - J \Phi_1)$, $B = -(J + J \Phi_1)$, $C = J \Phi_2$ and $D = J \Phi_3$.

We look at the system in (5) in its unrestricted form. (5) allows for the effect of interest rates $i_t$ and $r_t$, linked to assets with maturities $h$ and $2h$, on the exchange rates. Also, price levels associated with each currency are allowed to affect all exchange rates.

### 3 Data Description and Empirical Tests

The data spans the period from January 1983 to February 2004 and are obtained from the IMF’s International Financial Statistics. The starting date of the sample is similar to that of Marsh and MacDonald (2004) which has been chosen to exclude the period of high volatility of interest rates at the beginning of the 80’s. The vector of the spot exchange rates, $s_t$, comprises of logs of the EUR/USD, JPY/USD and EUR/JPY bilateral exchange rates. However, preceding the introduction of the euro
in January 1999, the euro bilateral exchange rates are approximated by its DEM counterparts. The vector of prices in the three countries, \( p_t \), is approximated by logs of the consumption price indexes (CPIs). The two vectors of interest rates \( i_t \) and \( r_t \) are approximated by three-month and six-month LIBOR rates on the corresponding currencies. The horizon over which the UIP is defined is thus effectively set to \( h = 3 \). We have experimented with horizons of 1, 3 and 6 months as the interest rate parity is found to hold better over longer horizons (see Chinn and Meredith, 2004). Based on conventional information criteria we found the horizon \( h = 3 \) performing best in our system. The LIBOR rates are employed as they are likely to be more consistent across currency markets than corresponding national interest rates due to differing money market regulations and tax treatments.

### 3.1 Unit Root Tests

We test for the order of integration of the series by employing the ADF-GLS test proposed by Elliott, Rothenberg, and Stock (1996). This test improves on the low power of the conventional ADF test in finite samples by estimating the coefficients on the deterministic variables in the test specification prior to the estimation of the coefficient of interest. The results are reported in table 1:

**** Table 1 Here ****

All the variables appear to be integrated to order one, I(1). Although there are good theoretical grounds suggesting that the interest rates should be stationary we should treat them as I(1) if they appear to behave as such\(^4\). The only difficulty in

\(^4\) Note that presence of trends in the interest rates can be theoretically justified by a time-varying inflation target and long-lasting disinflation (see e.g. Erceg and Levin, 2003).
determining the order of integration is experienced when dealing with the U.S. \textit{cpi} series which is found to be integrated to an order even higher than I(2). When applying other tests, e.g. ADF, KPSS, Ng-Perron, PP, the results are mixed as well. Since the conventional ADF test strongly rejects the hypothesis that the U.S. \textit{cpi} series is integrated to order higher than I(1), the series is regarded as I(1) further on.

### 3.2 Lag-Length Selection

The lag length $p$ for the system in equation (5) is determined in encompassing manner by applying Hannan and Quinn’s log iterated criterion (HQC). Paulsen (1984) shows that HQC along with the Schwartz information criterion (SIC) is a weakly consistent measure for determining the true lag order in the presence of unit roots (stochastic trends). Jacobson’s (1990) Monte-Carlo study suggests that HQC shows greater accuracy in choosing the true lag compared to SIC. We thus use HQC to determine the lag order for the system in (5) considering a maximum of nine lags. The results are reported in table 2:

**** Table 2 Here ****

### 3.3 Co-integration Test

Consider again equation (2). To determine the number of common factors (common trends) $n - r$ we perform co-integration tests. The co-integration tests should reveal how many co-integrating vectors $r$ there are in the system. We use the Johansen’s trace and maximum-eigenvalue co-integration rank tests (see Johansen, 1995) to estimate the number of co-integration vectors in system (5). The results of the tests
are shown in table 3:

**** Table 3 Here ****

Based on the results in table 3 we conclude that there are \( r = 2 \) co-integrating vectors in the system. The three exchange rates among the three major currencies are thus driven by \( n - r = 1 \) common factor, the common permanent component.

4 Identification and Estimation

4.1 The Estimation Method

The system of equations described in (5) is estimated using the autoregressive distributed lag approach (ARDL) to co-integration due to Pesaran and Shin (1995) and Pesaran et al. (1996). The error-correction form of the ARDL model is given by an equation where the dependent variable in first differences is regressed on the lagged values of the dependent and independent variables in levels and first differences:

\[
\Delta y_t = \phi y_{t-1} + \beta x_{t-1} + \sum_{j=1}^n \delta_j \Delta y_{t-j} + \sum_{j=0}^p \gamma_{i,j} \Delta x_{t-j} + \varepsilon_t
\]  

where \( y_t \) is a \( n \times T \) matrix of endogenous variables, \( x_t \) is a \( k \times T \) matrix of observations on the weakly exogenous variables and deterministic variables. The latter includes a constant and a shift dummy variable corresponding to the introduction of the euro in January 1999\(^5\). \( \Delta y_{t-j} \) and \( \Delta x_{t-j} \) are the \( j \)-period lagged values of \( \Delta y_t \equiv y_t - y_{t-1} \) and \( \Delta x_t \equiv x_t - x_{t-1} \), respectively, and \( \varepsilon_t \equiv (\varepsilon_1, \ldots, \varepsilon_T)' \) is a

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\(^5\) The dummy thus enters the co-integration space, however, its inclusion does not change the maximal eigenvalue and trace statistics to an extent that would change the inference on the number of present co-integrating relationships.
$T \times 1$ vector of residuals. The disturbances $\varepsilon_i$ are assumed to be independently distributed and independent of the regressors. The lag length $p$ is chosen as described in section 3 and $q$ is set to zero. Further, the underlying $ARDL(p,q)$ model is assumed to be stable, which ensures that $\phi < 0$, and thus there exists a long-run relationship between $y_t$ and $x_t$, defined as:

$$y_t = -(\beta' / \phi)x_t + \eta_t$$

(7)

where $\eta_t$ is a stationary process. The coefficient standard errors are obtained using the familiar delta method.

### 4.2 Identification

In this section we proceed to identification of the system described in (5). Since the co-integration tests indicate presence of two co-integration vectors, we need to impose a set of two restrictions on each of the two co-integrating vectors in order to identify them. In addition, the two sets of restriction are required to be independent of each other. The equation-by-equation approach, which is less efficient but more robust with respect to misspecification errors (see Johansen, 2005), requires only normalization of the vectors for estimation. This enables us to select the second restriction for each vector based on maximization of some information criterion, such as adjusted R-squared.

Before we get to this, we need to choose the common currency denominator for the two vectors. We do not take the most common route of choosing the U.S. dollar for this purpose (see e.g. MacDonald and Marsh, 2004), and choose the euro as the unifying currency to examine our system from an alternative angle. Our choice is
also motivated by the rising role of the euro in the international monetary system (see e.g. Kenen, 2005)\textsuperscript{6}.

The types of restrictions considered as the possible second identification restriction for each vector are (i) exclusion restrictions, where coefficient of a given variable is restricted to zero, and (ii) joint restrictions, where the two coefficients on exchange rates other than the dependent variable are restricted to be equal. Based on the results reported in the first two rows of table A1 in the Appendix, we impose the following restrictions on matrix $A$ in (5):

$$A_{\text{eur}} = \begin{bmatrix} 1 & 0 & \beta_1 \\ \beta_2 & 1 & \beta_2 \end{bmatrix}$$  \hspace{1cm} (8)

It means that we use an additional exclusion restriction to normalization to identify the first vector, and an additional joint restriction to identify the second vector. To make sure that the two estimated co-integrating vectors do not merely represent stationarity of the yield curves we have tested the significance of other variables in each co-integrating vector via joint zero restrictions, which were rejected.

In estimation the RHS variables may be to some extent endogenous with respect to the exchange rate, however, at least asymptotically the possible endogeneity bias should be eliminated by faster convergence of the coefficients on the co-integrating I(1) variables to their true values. Nevertheless, we carry out too checks to support our estimates. First we lagged all the RHS variables one period and carried the estimation again. The results were similar to those obtained earlier. Second, we tested how strong is the error-correction mechanism of the RHS series

\textsuperscript{6} We have checked our results against the standard of using the U.S. dollar as unifying currency. The results do not contradict those obtained with the euro and are available from the author.
towards the equilibrium represented by the two co-integrating vectors. Results of this joint weak exogeneity test support the assumption that the error-correction mechanism of the RHS variables in (5) is, in general, weak. This can be expected at least in the case of the U.S. and the euro area where both economies show rather low pass-through of exchange rates to prices and the output gaps, and are usually modeled as closed economies in the literature (see Cho and Moreno, 2006, for the U.S.; and Smets and Wouters, 2003, for the euro area).

Finally, to ground the assumption of purchasing parity into our estimates we test the restrictions implied by PPP for each of the two co-integrating vectors. Recall at this point that in section 2 we have assumed that at least in the long run the exchange rates gravitate to their underlying purchasing power parities. The results of those tests are shown in third row of table A1 in the Appendix. The PPP restriction is not rejected only in the case of the EUR/JPY exchange rate.

### 4.3 Estimation Results

We use the identification restrictions discussed in section 4.2 and estimate the system in (5) using the ARDL method. Further, we employ the general-to-specific approach and restrict the coefficients of insignificant determinants to zero. The resulting estimates are provided in table 4:

**** Table 4 Here ****

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We interpret the estimation results in table 4 raising some hypotheses of possible currency substitution or complementarity relationships between the dollar, the euro and the yen, and try to verify these hypotheses later in section 4.4.

Consider first the column for eur/jpy. The euro tends to appreciate against the yen at times when the Japanese yen appreciates relative to the dollar as implied by the positive coefficient of 0.62 on jpy/usd. The results further suggest that an increasing expected return on the euro, i.e. $E_i^t r_{i,t+k} = r_i^{cur} - i_i^{cur}$, has rather an insignificant economic effect on the eur/jpy exchange rate, see the small difference between the coefficients attached to “r currency 1” and “i currency 1” in the first column of table 4. On the other hand, the increasing expected yield on the yen, “currency 2”, makes the eur/jpy exchange rate appreciate. This result can be explained under currency complementarity between the euro and the yen by assuming that the marginal utility of consumption in the yen increases less than the marginal utility of consumption in the euro when the expected return on the yen increases. The third-currency effect of an increasing expected return on the dollar, “currency 3”, appears to cause depreciation of the eur/jpy exchange rate. As marginal utility of consumption in dollars increases, the marginal utility of consumption in both the euro and the yen decreases. For the increasing expected return on dollar to cause depreciation of the eur/jpy rate the marginal utility of consumption in euro has to be decreasing at a lower rate than the one in yen. This line of explanation postulates that the dollar is currency substitute for both the euro and the yen where stronger currency substitution is likely to take place between the yen and the dollar. The increase in the U.S. price level increases the marginal utility
of consumption in dollars and under currency substitution results in depreciation of both the euro and the yen. Should the euro and the yen both be currency substitutes for the dollar, only a higher degree of currency substitution between the dollar and the yen relative to the degree of currency substitution between the dollar and the euro can explain our finding of the eur/jpy rate appreciation in response to an increasing price level in the U.S. The dummy associated with the introduction of the euro is significantly negative, i.e. that the eur/jpy rate appreciated with respect to the dem/jpy rate. The latter is consistent with the 1.95583 rate at which the Deutsche mark is changed for the euro.

Consider now the column for eur/usd. The eur/usd rate is estimated to be affected by the eur/jpy and jpy/usd rates equally so that the effect from the cross-exchange rates is effectively zero. This is due to the fact that when the EUR/JPY and JPY/USD rates are multiplied there is no additional information to that already contained in the EUR/USD rate. This may imply that the EUR/USD rate is the most efficient exchange rate, supposedly as it is the most traded one. An increasing expected return on the euro \( E_{t} r_{t+h}^{e_{1}} = r_{t}^{e_{1}} - i_{t}^{e_{1}} \), seems to result in appreciation of the eur/usd rate as the marginal utility of consumption in euros increases – the difference between the coefficient on “\( r \) currency 1” and “\( i \) currency 1” in the second column of table 4 is negative. The effect of rising expected return on the dollar seems to have an economically negligible effect on the eur/usd rate, the difference between “\( r \) currency 2” and “\( i \) currency 2” is small. On the other hand, an increasing return on the Japanese yen has an appreciating effect on the eur/usd rate consistent with our hypothesis of currency complementarity between the euro
and the yen, and currency substitution between the dollar and the yen. For the latter to hold, we would expect to find a positive coefficient on the price level in Japan and a negative coefficient attached to the U.S. price level. Instead we find the opposite in both cases. The dummy for emergence of the euro appears to be insignificant in the eur/usd vector supposedly because the introduction of the euro was already priced in the exchange rate well in advance.

In addition to the interpretation of the results we want to raise the question of how important are the third-currency effects in general. This is due to the fact that many would argue that if exchange rate markets are efficient and symmetric all the third-currency effects should be arbitraged away. In general, the estimates seem to deliver significant evidence on the importance of third-currency effects, i.e. changes in fundamentals associated with currencies that do not appear in the quotation of bilateral exchange rates. The results of the F-tests reported in table A1 in the Appendix support this claim. The null hypothesis for the test in the case of both vectors is that the coefficients attached to the third-currency fundamentals are equal to zero.

The hypotheses concerning currency preferences can be summarized as follows:

**Hypothesis 1:** The U.S. dollar acts as a currency substitute for both the euro and the yen.

**Hypothesis 2:** There exists a currency complementarity relationship between the euro and the yen.

**Hypothesis 3:** The degree of currency substitution between the yen and the dollar is larger than the degree of currency substitution between the euro and the dollar.
In order to validate these hypotheses we will now investigate in a greater
detail possible currency substitution or currency complementarity relationships
among the dollar, the euro and the yen using an approach recently applied in the
literature.

4.4 Currency Substitution and Complementarity

As discussed earlier, the marginal utility of consumption in a given currency
increases (i) when there is a relative decrease in the amount of currency used to
finance currency-specific consumption, i.e. the relevant money supply. Also, the
marginal utility increases with (ii) the growing number of transactions that need to
be performed in the currency, i.e. approximately as GDP in the country of the
currency grows. Finally, the marginal utility increases as (iii) the average size of a
single transaction rises, i.e. as the price level in the country of the currency rises. The
marginal utility of consumption in a given currency can thus be conveniently
expressed using respective money velocity. It can be defined in logs as

\[ v_t = p_t + y_t - m_t \]

where \( y_t \) is the log of output, \( p_t \) is the log of the price level and \( m_t \)
the log of money supply.

Brittain (1981) introduced an informal graphical test for currency
substitution. Such a graphical tool has been recently augmented by Kingston and
Melecky (2007) to inspect both currency substitution and complementarity. A
comparison of velocities of monetary aggregates associated with the currencies of
interest is the focal point here. Currencies should be considered elements of
internationally diversified portfolios. *Ceteris paribus*, an increase money velocity of
(marginal utility of consumption in) one currency induces portfolio reallocation according to relative characteristics of its individual elements: the velocity of currency substitutes falls and the velocity of currency complements rises, according to the degree of substitution or complementarity between given currencies. Considering e.g. the euro and the US dollar, rising velocity of US money contrasted with falling velocity of euro area money would suggest that there is substitution between the euro and the dollar. In the same manner, a significant co-movement of the latter money velocities would suggest currency complementarity between the euro and the dollar. Using this approach Brittain (1981) finds evidence of currency substitution between the dollar and the Deutsche mark which can imply currency substitution between the dollar and the euro that we hypothesize. Further, Kingston and Melecky (2007) find some evidence of currency substitution between the euro and the dollar, and currency complementarity between the dollar and the yen. They estimate the correlations between velocities of M0 and M1 money (in first differences) for each currency pair to draw their conclusions. Although economically interesting their estimates are not statistically significant.

To overcome the low informative contents of the M0 and M1 money velocities we look at a broader measure of money velocity, the quasi-money velocity. Money velocity is calculated as $v_i = gdpt_i - mq_t$ where $gdpt$ is the log of nominal GDP and $mq$ is the log of the quasi money supply in a given country. Plots of the quasi-money velocities in the U.S., the euro area and, Japan are shown in figure 1:

**** Figure 1 Here ****
To investigate whether there are significant relationships of currency substitution or complementarity among the dollar, the euro and the yen, we regress the respective velocities against each other in accord with the quotations of the investigated exchange rates, i.e. EUR/JPY, EUR/USD and JPY/USD. We use the fully modified OLS (FMOLS) estimator due to Phillips and Hansen (1990) which corrects for the small-sample bias emerging when possibly endogenous variables with a unit root are consistently estimated by OLS. The results are summarized below:

\[ v_t^{EU} = 0.792 v_t^{JAP} - 2.197 \]  
\[ v_t^{EU} = -0.538 v_t^{US} - 0.535 \]  
\[ v_t^{JAP} = -0.936 v_t^{US} - 3.550 \]  

Recall now the hypotheses 1-3 presented at the end of section 4.3 and let us see whether the estimation results in (9) validate them. “Hypothesis 1” postulates that the dollar is a currency substitute for both the euro and the yen. This requires that the money velocities in the euro area and Japan move in an opposite direction to the money velocity in the U.S. and this is indeed what we find - see the negative coefficients attached to \( v_t^{US} \) and \( v_t^{US} \) in the second and third line of (9). “Hypothesis 2” assumes that the euro and the Japanese yen are currency complements. This requires co-movement of the velocities in the euro area and Japan, again this is what we find – viz. the positive coefficient attached to \( v_t^{JAP} \) in the first equation of (9). And finally, “Hypothesis 3” proposes that the degree of currency substitution between the yen and the dollar is larger than the degree of currency substitution between the euro and the dollar. This requires that once money velocity in the U.S.
moves up money velocity in the euro area moves down to a lesser extent than the money velocity in Japan. Our estimates support this hypothesis as the magnitude of the coefficient attached to $v_i^{US}$ in the second equation of (9) is smaller than the magnitude of the coefficient on $v_i^{US}$ in the third equation of (9).

In the next step we relate money velocity to the expected return on a currency which is used in its unrestricted form in the estimation. Assume the money demand in logs is of this form:

$$m_t = p_t + y_t - (r_t - i_t) = p_t + y_t - E_t^i_{t+1}$$

where $(r_t - i_t)$ represents the opportunity cost of holding money, $m_t$. Using the definition of money velocity introduced earlier we can derive that

$$v_t \equiv p_t + y_t - m = E_t^i_{t+1}$$

As utilized earlier when interpreting the estimation results in section 4.3, money velocity (the marginal utility of consumption in a given currency) co-moves with the expected return on the currency. The latter relationship completes the link from money velocities to interest rates and exchange rates within the system described in (5). Our estimation results can be to a large extent explained by the established relationships of currency substitution and complementarity among the three major currencies. This not only relates to the results within the conventional bilateral elements of our model, but also extends to the third-currency effects imbedded in the model.

The obtained results can be to a certain degree compared with those of MacDonald and Marsh (2004) and Hodrick and Vassalou (2003). MacDonald and Marsh (pp. 105, table 2, column 1 and 2) find, judging by the signs of the coefficients
on the interest rates, that the Deutsche mark and the U.S. dollar appear to be substitutes as in our case. On the other hand, the Japanese yen and the dollar appear to be rather complements than substitutes. Since they do not report how significant the coefficient estimates are the results for the Japanese yen seem indecisive owing to the relatively small magnitude of the estimate. Hodrick and Vassalou examine the short-run dynamics of exchange rates and work with restricted interest rate differentials. They find (pp. 1284, table 2) some evidence of currency complementarity between the Japanese yen and the Deutsche mark as we do.

5 Conclusion

Exchange rates among the three major currencies, the U.S. dollar, the euro and the Japanese yen, have been estimated in this paper. The employed exchange rate model includes, in addition to determinants related to the pair of currencies of an exchange rate, the third-currency effects. The considered determinants of an exchange rate include other cross-currency exchange rates, interest rates and prices. We interpret the estimation results using reasoning based on possible currency substitution and currency complementarity relationships. This interpretation looks the international monetary system into a useful perspective and is new to the literature. We find that the U.S. dollar acts as a currency substitute for both the euro and the Japanese yen, and that the euro and the yen act as currency complements. Further, the degree of currency substitution between the yen and the dollar appears to be stronger than the degree of currency substitution between the euro and the dollar. Our findings also include evidence of significant third-currency effects within the system of three major
currencies. The impacts of third-currency effects are also in line with our findings of substitution and complementarity among the three major currencies.

Both private and public asset and liability portfolios are often denominated in a range of currencies. Hence, our findings can be exploited in areas of asset and liability management that focus on foreign currency risk, since the established relationships of currency substitution and complementarity can help in forming judgments about possible future trends in exchange rates and thus narrow the scope of scenario analyses. Further, presence of currency substitution and complementarity can have important implications for covariances between currencies as outlined in this paper. Finally, the established structure concerning the major three currencies can be utilized in estimation of risks (variance) associated with exchange rates between the currencies.

Appendix

**** Table A1 Here ****
References


**Figures**

**Figure 1**  Comparison of Money Velocities

- $v(\text{eu})$
- $v(\text{us})$
- $v(\text{jap})$
### Tables

#### Table 1  Unit Root Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit root test: <strong>ADF-GLS</strong></th>
<th>Levels</th>
<th>1st differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>eur/usd</td>
<td>C,T,0[-1.4102]</td>
<td>C,0[-16.6871]***</td>
<td></td>
</tr>
<tr>
<td>jpy/usd</td>
<td>0[-1.4627]</td>
<td>0[-15.4114]***</td>
<td></td>
</tr>
<tr>
<td>eur/jpy</td>
<td>C[0.2355]</td>
<td>C[-12.3874]***</td>
<td></td>
</tr>
<tr>
<td>p_eu</td>
<td>C,T,1 [-0.3495]</td>
<td>C,0[-13.0747]***</td>
<td></td>
</tr>
<tr>
<td>p_jap</td>
<td>C,12[-0.1252]</td>
<td>11[-1.9855]***</td>
<td></td>
</tr>
<tr>
<td>p_us</td>
<td>C,T,12[-1.5212]</td>
<td>C,11[-0.2542]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>C,1[-7.0513]***</td>
<td></td>
</tr>
<tr>
<td>i_eu</td>
<td>C,2[-0.4318]</td>
<td>1[-9.1717]***</td>
<td></td>
</tr>
<tr>
<td>i_jap</td>
<td>C,3[-0.2509]</td>
<td>2[-8.1040]***</td>
<td></td>
</tr>
<tr>
<td>i_us</td>
<td>C,8[-0.7128]</td>
<td>7[-5.8260]***</td>
<td></td>
</tr>
<tr>
<td>r_eu</td>
<td>C,2[-0.5511]</td>
<td>1[-8.8249]***</td>
<td></td>
</tr>
<tr>
<td>r_jap</td>
<td>C,0[0.3223]</td>
<td>0[-16.7137]***</td>
<td></td>
</tr>
<tr>
<td>r_us</td>
<td>C,8[-0.6070]</td>
<td>7[-5.9078]***</td>
<td></td>
</tr>
</tbody>
</table>

**, *** - indicates rejection of the null at 5% and 1% significance level respectively.

#### Table 2  Lag Length Selection for Endogenous Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Selected Lag Length in Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>eur/usd</td>
</tr>
<tr>
<td>eur/usd</td>
<td>1</td>
</tr>
<tr>
<td>jpy/usd</td>
<td>2</td>
</tr>
<tr>
<td>eur/jpy</td>
<td>1</td>
</tr>
</tbody>
</table>

#### Table 3  Co-integration Rank Tests

<table>
<thead>
<tr>
<th>H0</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>1 % Critical Value</th>
<th>Max-Eigenvalue Statistic</th>
<th>1 % Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>r==0</td>
<td>0.246496</td>
<td>100.5539*</td>
<td>35.65</td>
<td>71.88729*</td>
<td>25.52</td>
</tr>
<tr>
<td>p &lt;= 1</td>
<td>0.093381</td>
<td>28.66663*</td>
<td>20.04</td>
<td>24.90030*</td>
<td>18.63</td>
</tr>
<tr>
<td>p &lt;= 2</td>
<td>0.014719</td>
<td>3.766328</td>
<td>6.65</td>
<td>3.766328</td>
<td>6.65</td>
</tr>
</tbody>
</table>

* denotes rejection of the hypothesis at the 1 % significance level.
Table 4  Estimation of the Identified System

<table>
<thead>
<tr>
<th>Variable</th>
<th>eur/jpy</th>
<th>eur/usd</th>
</tr>
</thead>
<tbody>
<tr>
<td>eur/jpy</td>
<td>-1</td>
<td>0.6491</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0984)***</td>
</tr>
<tr>
<td>eur/usd</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>jpy/usd</td>
<td>0.6201</td>
<td>0.6491</td>
</tr>
<tr>
<td></td>
<td>(0.0462)***</td>
<td>(0.0984)***</td>
</tr>
<tr>
<td>i currency 1</td>
<td>0.0293</td>
<td>0.3402</td>
</tr>
<tr>
<td></td>
<td>(0.0082)***</td>
<td>(0.1124)***</td>
</tr>
<tr>
<td>i currency 2</td>
<td>0.1883</td>
<td>0.34018</td>
</tr>
<tr>
<td></td>
<td>(0.1063)*</td>
<td>(0.1124)***</td>
</tr>
<tr>
<td>i currency 3</td>
<td>-0.2092</td>
<td>0.058652</td>
</tr>
<tr>
<td></td>
<td>(0.0626)***</td>
<td>(0.0215)***</td>
</tr>
<tr>
<td>r currency 1</td>
<td>0</td>
<td>-0.39021</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1177)***</td>
</tr>
<tr>
<td>r currency 2</td>
<td>-0.2351</td>
<td>0.31697</td>
</tr>
<tr>
<td></td>
<td>(0.1065)**</td>
<td>(0.1157)***</td>
</tr>
<tr>
<td>r currency 3</td>
<td>0.2311</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.0624)***</td>
<td></td>
</tr>
<tr>
<td>p currency 1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p currency 2</td>
<td>-1</td>
<td>1.4220</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.5199)***</td>
</tr>
<tr>
<td>p currency 3</td>
<td>-0.5747</td>
<td>-2.3852</td>
</tr>
<tr>
<td></td>
<td>(0.0485)***</td>
<td>(1.1911)**</td>
</tr>
<tr>
<td>D99</td>
<td>-0.5227</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.0269)***</td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>0</td>
<td>7.7909</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.8187)**</td>
</tr>
</tbody>
</table>

*currency 1* is the currency quoted in the numerator of the relevant bilateral exchange rate, *currency 2* is the currency quoted in the denominator and *currency 3* is the currency that does not appear in the bilateral quote. Standard errors are in the parentheses. *, **, *** - stands for the significance at the 10 %, 5 % and 1 % level, respectively. ----- shows that a given variable is insignificant and was eliminated from the regression.
Table A1  Hypotheses Tests

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>dependant variable</th>
<th>dependant variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>eur/jpy</td>
<td>eur/usd</td>
</tr>
<tr>
<td>Exclusion Restriction</td>
<td>-0.034241[0.973]</td>
<td>6.2085[0.000]</td>
</tr>
<tr>
<td></td>
<td>1.9093[0.057]</td>
<td>-3.0339[0.003]</td>
</tr>
<tr>
<td>Joint Restriction</td>
<td>na</td>
<td>na</td>
</tr>
<tr>
<td></td>
<td>0.29548 [0.587]</td>
<td>0.29548 [0.587]</td>
</tr>
<tr>
<td>PPP</td>
<td>2.6534 [0.265]</td>
<td>5.1712 [0.017]</td>
</tr>
<tr>
<td>Third-Currency Effect</td>
<td>293.2757 [0.000]</td>
<td>47.5558 [0.000]</td>
</tr>
</tbody>
</table>

In the case of exclusion restrictions the reported value is the t-ratio in the case of the joint restriction it is the value of the F-test. Corresponding p-values are in the square brackets. *Dependent variable* denotes the exchange rate upon which we normalize. The restrictions on each vector are displayed in rows.