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Alcantud, José Carlos R. and Laruelle, Annick

Universidad de Salamanca, Spain, University of the Basque Country  
(UPV/EHU) and IKERBASQUE

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# To approve or not to approve: this is not the only question\*

José Carlos R. Alcantud<sup>†</sup> and Annick Laruelle<sup>‡§</sup>

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## 1 Abstract

This paper deals with electing candidates. In elections voters are frequently offered a small set of actions (voting in favor of one candidate, voting blank, spoiling the ballot, and not showing up). Thus voters can express neither a negative opinion nor an opinion on more than one candidate. Approval voting partially fills this gap by asking an opinion on all candidates. Still the choice is only between approval and non approval. However non approval may mean disapproval or just indifference or even absence of sufficient knowledge for approving the candidate. In this paper we characterize the disapproval voting rule, a natural extension of approval voting that distinguishes between indifference and disapproval.

## 2 Introduction

In polls many citizens express some dissatisfaction with politicians. Usual ways to voice this disaffection in elections are absenteeism, spoiled or blank vote, or voting for an unviable candidate. No legitimate and explicit negative option is generally provided to electors. There are some exceptions. In the State of Nevada voters can express their disapproval of all official candidates with the “none of the above candidate” option (Arcelus, Mauser and Spindler, 1978). In 1987 the deputies elections were reorganized in the former Soviet Union. Under the new rule, voters crossed off the names of those against whom they wished to vote (Hahn, 1988). The same rule is used in some Chinese

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<sup>†</sup>Universidad de Salamanca, Campus Miguel de Unamuno, E-37007 Salamanca, Spain; jcr@usal.es

<sup>‡</sup>Bridge, Fundamentos del Análisis Económico I, University of the Basque Country (UPV/EHU), Avenida Lehendakari Aguirre, 83, E-48015 Bilbao, Spain; a.laruelle@ikerbasque.org.

<sup>§</sup>IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain.

village elections (Zhong and Chen, 2002). For some other historical examples of rules that include negative options, see Kang (2010).

If all voters have a clear-cut (i.e., positive or negative) opinion on each candidate, the previous rule is equivalent to approval voting: candidates who are not crossed off the list are the approved candidates.

However it is not rare that voters are indifferent or even lack the sufficient knowledge to express a positive or a negative opinion on some candidates. When indifference exists, the two rules differ because the non disapproved candidates consist of the approved candidates and those who leave the voter indifferent.

Both rules can be extended to include indifference as a third explicit option. The new rule, that we refer to as disapproval voting, requires voters to cast a positive, negative or null vote on each candidate. In case of a blank or spoiled vote the vote is considered as null. The disapproval rule selects the candidates who obtain the largest difference between the number of positive votes and the number of negative votes.

Earlier references have studied this rule. Felsenthal (1989) compares it with approval voting from a voter's point of view. Hillinger (2004, 2005) advocates for its use for general elections. Lepelley and Smaoui (2012) study some of its properties, while Baujard and Igersheim (2011) test it in a frame-field experiment during the French presidential elections. However the literature has not provided a characterization of the rule. We focus on providing one such characterization.

Fishburn (1978a, 1978b) characterizes approval voting with the help of four axioms. Alós-Ferrer (2006) shows that one axiom (neutrality) is unnecessary and provides a characterization based on faithfulness, cancellation and consistency. Faithfulness requires that if the society consists of one individual, his or her approved candidate(s) is (are) selected. Cancellation requires that whenever all candidates receive the same number of approvals, the full set of candidates is selected. Consistency requires that whenever there are common selected candidates for two disjoint societies, those candidates that were selected for both of the original societies are exactly the candidates that are selected for the joint society. In this paper we provide a characterization of the disapproval rule in similar terms. However we make note that the adapted version of cancellation to our context with three-option ballots is not sufficient for our purpose, and a stronger (under consistency) requirement is introduced in its place, namely a 'compensation' property. This requires that if all candidates receive as many approvals as disapprovals, then the full set of candidates is selected.

This paper is organized as follows. Section 3 introduces our model. Section 4 provides a characterization of the disapproval rule and the discussion about the necessity of replacing 'cancellation' by 'compensation'. Then Section 5 gives some concluding remarks.

### 3 Ternary ballots electing rules

Henceforth we refer to the following setting. There is a set  $C$  composed of  $c$  candidates and there is an electorate  $N$  of  $n$  voters. Generic candidates are denoted by  $x, y, z$ , and generic voters by  $i, j, k$ . Voters are asked to cast a (ternary) ballot where, for each candidate, they answer the question: Would each of the following candidates be a good president/head of .../ etc.? <sup>1</sup> Each voter can either answer “yes”, or “no”, or “indifferent, abstain or do not know”. Voters are warned that when a candidate receives a blank vote, the latter option is marked by default.

We proceed to describe two different ways to model the relevant information from the recount of these ballots. The first one, namely vote profiles, captures the gross collection of the possible outcomes of the voting. The second one, namely vote response profiles, is more efficient (but possibly, less natural at first glance) because it aggregates equal ballots so that the identities of the voters are lost. Ballot aggregation functions are the formal expression of voting rules. Consequently we express our characterization of the disapproval rule/ballot aggregation function in terms of vote response profiles. We stress that for the case of anonymous electing rules where the identities of the voters do not matter, both approaches are equivalent.

#### 3.1 Vote profiles of ternary ballots

Voter  $i$ 's ternary ballot can be represented by a 3-partition of the set of candidates  $\mathcal{C}_i(C) = (C_i^+(C), C_i^0(C), C_i^-(C))$ , where  $C_i^+(C)$  are the candidates whom  $i$  approves,  $C_i^-(C)$  are the candidates whom voter  $i$  disapproves, and  $C_i^0(C)$  are the remaining candidates (i.e., those who either leave voter  $i$  indifferent, or on whom voter  $i$  does not emit an opinion, or on whom voter  $i$  admits ignorance). If voter  $i$  does not show up then we set  $C_i^0(C) = C$ . The respective cardinals of  $C_i^+(C)$ ,  $C_i^0(C)$ , and  $C_i^-(C)$  are denoted by  $c_i^+(C)$ ,  $c_i^0(C)$ , and  $c_i^-(C)$ . When the set of candidates is clear from the context we just write  $\mathcal{C}_i = (C_i^+, C_i^0, C_i^-)$  and  $c_i^+, c_i^0, c_i^-$ . Most rules (as the classical plurality rule) impose  $c_i^+ \leq 1$  for all voters, but we are not bound by such restriction.

A *vote profile of ternary ballots* gives the summary of all ternary ballots cast by all voters. Henceforth we omit the reference to ternary ballots for simplicity. A vote profile is denoted by  $\mathcal{C}(N, C)$ , with  $\mathcal{C}(N, C) = (\mathcal{C}_1(C), \dots, \mathcal{C}_n(C))$  if  $N = \{1, 2, \dots, n\}$ . When  $N = \{i\}$  then  $\mathcal{C}(\{i\}, C) = \mathcal{C}_i(C)$  abusing notation. We assume that at least a

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<sup>1</sup>Therefore, the question to be confronted by a voter must not suggest “how do you compare the candidates,” but instead it must mean “how do you evaluate each candidate.” Balinski and Laraki (2011, p. 30). The very different questions posed in two 2007 polls (see above, Table 2.20): “Would each of the following candidates be a good President of France?” and “Do you personally wish each of the following candidates to win the presidential election?” The first poses an absolute question, the second a relative one. The first invites an evaluation, the second suggests a contrast. The answers are, in consequence, completely different. Significantly, the first question elicited a “yes” for the four major candidates considerably more in keeping with their Good or better grades in the 2007 majority judgment experience than did the second question. Balinski and Laraki (2011, p. 42)

candidate receives a positive vote: <sup>2</sup> there always exists at least a pair formed by a candidate  $x$  and a voter  $i$  such that  $x \in C_i^+(C)$ . Fishburn and Brams (1978, p. 833) define unconcerned voters as voters who are indifferent among all candidates. In this vein we say that voter  $i$  is *unconcerned* if  $C_i^0(C) = C$ .

We use the following operations: merging two disjoint electorates and suppressing a candidate from the list of candidates. Consider two disjoint electorates  $N_1 = \{1, 2, \dots, N_1\}$  and  $N_2 = \{N_1 + 1, N_1 + 2, \dots, N_1 + N_2\}$  with vote profiles  $\mathcal{C}(N_1, C)$  and  $\mathcal{C}(N_2, C)$ . We define the vote profile on the merged electorate  $N_1 \cup N_2 = \{1, \dots, N_1 + N_2\}$  as  $\mathcal{C}(N_1 \cup N_2, C) = (\mathcal{C}(N_1, C), \mathcal{C}(N_2, C))$ , which presumes that the evaluation of one electorate is not affected by the other electorate. Suppose now that a vote profile  $\mathcal{C}(N, C)$  is given and we drop a candidate  $x$  from  $C$ . When voters do not modify their opinion with respect to the other candidates under this circumstance, it turns out that  $\mathcal{C}_i(C \setminus \{x\}) = (C_i^+(C) \setminus \{x\}, C_i^0(C) \setminus \{x\}, C_i^-(C) \setminus \{x\})$  for each  $i$ . Thus the vote profile that results from dropping candidate  $x$  is  $\mathcal{C}(N, C \setminus \{x\}) = (\mathcal{C}_1(C \setminus \{x\}), \dots, \mathcal{C}_n(C \setminus \{x\}))$ .

**Definition 1** *A ballot aggregation function on vote profiles is a correspondence  $W$  that assigns a non-empty set of candidates to every vote profile.*

Unless otherwise stated, we refer to *anonymous* ballot aggregation functions on vote profiles. These are ballot aggregation functions on vote profiles that are invariant under any permutation of the voters. Formally speaking:

**Definition 2** *A ballot aggregation function on vote profiles  $W$  is anonymous when for each bijective map  $\sigma : N \rightarrow N$ , if  $\mathcal{C}(N, C) = (\mathcal{C}_1(N, C), \dots, \mathcal{C}_N(N, C))$  and  $\mathcal{C}'(N, C) = (\mathcal{C}_{\sigma(1)}(N, C), \dots, \mathcal{C}_{\sigma(N)}(N, C))$  then  $W(\mathcal{C}(N, C)) = W(\mathcal{C}'(N, C))$ .*

## 3.2 Vote response profiles of ternary ballots

We now reinterpret the specification of our model in line with the inspiring Alós-Ferrer (2006).

Let  $\mathcal{T}$  denote the set of all ternary ballots  $T = (C^+, C^0, C^-)$  on the set  $C$  of candidates. A vote profile naturally induces a *vote response profile of ternary ballots*  $\pi : \mathcal{T} \rightarrow \mathbb{N}$ . Henceforth we omit the reference to ternary ballots for simplicity. We interpret  $\pi(T)$  as the number of voters that cast ballot  $T$ . Therefore one cannot recover the original vote profile (of ternary ballots) from its induced vote response profile (of ternary ballots). Because we are going to maintain anonymity throughout, that fact does not handicap our alternative, more concise notation. The class of all vote response profiles is denoted by  $\Pi$ , thus we are allowing for variable electorate size. Obviously the number of voters in the electorate is  $\sum\{\pi(T) : T \in \mathcal{T}\}$ . Every  $T \in \mathcal{T}$  can be identified with  $\pi^T$  such that  $\pi^T(T) = 1$ ,  $\pi^T(T') = 0$  otherwise. Summing up two vote response profiles corresponds to merging the vote profiles of two disjoint societies.

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<sup>2</sup>In particular we do not allow that all voters cast abstain/negative votes for all candidates. With this requirement we avoid stating that all candidates must be elected in case that everyone votes against all candidates.

**Definition 3** *A ballot aggregation function on vote response profiles is a correspondence  $W$  that assigns a non-empty set of candidates to every  $\pi \in \Pi$ .*

Each  $W$  given by Definition 3 is anonymous in a sense alike to Definition 2, since vote response profiles capture the number of ballots of each type without any reference to specific voters. Furthermore, (anonymous) ballot aggregation functions on vote profiles are redefined from Definition 3 because vote profiles induce vote response profiles, thus we simply refer to ballot aggregation functions in the following.

The following tallies are defined for each vote response profile  $\pi \in \Pi$  and  $x \in C$ :  $n^+(x, \pi) = \sum\{\pi(T) : T = (C^+, C^0, C^-) \in \mathcal{T}, x \in C^+\}$ ,  $n^-(x, \pi) = \sum\{\pi(T) : T = (C^+, C^0, C^-) \in \mathcal{T}, x \in C^-\}$ , and  $n^0(x, \pi) = \sum\{\pi(T) : T = (C^+, C^0, C^-) \in \mathcal{T}, x \in C^0\}$ . We gather  $n(x, \pi) = (n^+(x, \pi), n^0(x, \pi), n^-(x, \pi))$ . When  $\pi$  is induced by a vote profile  $\mathcal{C}(N, C)$  then we also write  $n^+(x, \mathcal{C}(N, C)) = n^+(x, \pi)$ ,  $n^0(x, \mathcal{C}(N, C)) = n^0(x, \pi)$ ,  $n^-(x, \mathcal{C}(N, C)) = n^-(x, \pi)$ , and  $n(x, \mathcal{C}(N, C)) = n(x, \pi)$ .

## 4 A characterization of the dis&approval voting rule

The dis&approval rule is the ballot aggregation function where  $W(\pi)$  is the subset of  $C$  formed by the candidates that maximize  $v_H(x, \pi) = n^+(x, \pi) - n^-(x, \pi)$ .

It is easy to check that the dis&approval rule satisfies the three axioms that characterize the approval rule (v. Alós-Ferrer, 2006) when they are extended to the ternary ballots framework in the following way. Consistency can be adapted without changes. It requires that whenever there are common selected candidates for two disjoint societies, those candidates that were selected for both of the original societies are exactly those who are selected for the joint society. Formally:

**Consistency:** for each  $\pi, \pi' \in \Pi$ , if  $W(\pi) \cap W(\pi') \neq \emptyset$  then  $W(\pi + \pi') = W(\pi) \cap W(\pi')$ .

In the binary ballots framework cancellation requires that whenever all candidates receive the same number of approvals, the full set of candidates is selected. In order to adapt it to the ternary ballots framework, we require that whenever all candidates receive the same number of approvals and the same number of disapprovals, then the full set of candidate is selected. Formally:

**AD-cancellation:** for each  $\pi, \pi' \in \Pi$ , if for all  $x \in C$  it is true that  $n^+(x, \pi) = n^+(x, \pi')$  and  $n^-(x, \pi) = n^-(x, \pi')$  then  $W(\pi) = W(\pi')$ .

In the binary ballots framework faithfulness requires that if the society consists of one individual then his or her approved candidate(s) is (are) selected. In the ternary ballots framework we have to state what happens in case the individual approves no candidate. Under such circumstance, in our version all candidates are selected if

all candidates are disapproved, otherwise the candidates for whom the individual is indifferent are selected.<sup>3</sup> Formally:

**(0, -1)-faithfulness:** for all  $T = (C^+, C^0, C^-) \in \mathcal{T}$ ,

$$W(\pi^T) = W(T) = \begin{cases} C^+ & \text{if } C^+ \neq \emptyset, \\ C & \text{if } C^- = C, \\ C^0 & \text{otherwise.} \end{cases}$$

Nonetheless these three properties, namely, consistency, *AD*-cancellation and (0, -1)-faithfulness, do not characterize the dis&approval voting. Although they are implied by this rule, they do not uniquely determine it. We demonstrate this in Example 1 below:

**Example 1** Define  $W^*(\pi)$  as the subset of  $C$  formed by the candidates that sequentially maximize  $u(x, \pi) = (n^+(x, \pi) - n^-(x, \pi), n^+(x, \pi))$  in the following sense: In order for  $x$  to be elected not only it must maximize  $n^+(x, \pi) - n^-(x, \pi)$  over  $C$ , but also it must maximize  $n^+(x, \pi)$  over the set of maximizers of  $n^+(x, \pi) - n^-(x, \pi)$ . In words: among the candidates that have the highest difference approvals vs. disapprovals, only the candidates that have the highest number of approvals are selected. Clearly this is not the dis&approval rule.

However this expression defines a ballot aggregation function that satisfies consistency, *AD*-cancellation and (0, -1)-faithfulness. *AD*-cancellation and (0, -1)-faithfulness are trivial. To check for consistency, suppose  $\pi, \pi' \in \Pi$  and  $W^*(\pi) \cap W^*(\pi') \neq \emptyset$ . Let us denote  $A_1(x) = n^+(x, \pi) - n^-(x, \pi)$  and  $B_1(x) = n^+(x, \pi') - n^-(x, \pi')$ . Observe that  $n^+(x, \pi + \pi') = n^+(x, \pi) + n^+(x, \pi')$  and  $n^-(x, \pi + \pi') = n^-(x, \pi) + n^-(x, \pi')$ . Therefore if we let  $C_1(x) = n^+(x, \pi + \pi') - n^-(x, \pi + \pi')$ , then  $C_1(x) = A_1(x) + B_1(x)$ .

$W^*(\pi)$ , respectively  $W^*(\pi')$ , denote the non-empty set of candidates that maximize  $A_1(x)$ , respectively  $B_1(x)$ , and whose evaluations by  $n^+(x, \pi)$ , respectively  $n^+(x, \pi')$ , are highest among the maximizers of  $A_1(x)$ , respectively  $B_1(x)$ . By assumption  $W^*(\pi) \cap W^*(\pi') \neq \emptyset$ . Now some tedious but straightforward computations show that the maximizers of  $C_1(x)$  whose evaluation by  $n^+(x, \pi + \pi')$  is highest coincides with  $W^*(\pi) \cap W^*(\pi')$ . This boils down to  $W^*(\pi + \pi') = W^*(\pi) \cap W^*(\pi')$ .

In order to characterize the dis&approval rule we replace cancellation with an alternative property, that we refer to as *AD*-compensation. It states that if all candidates receive the same number of approvals than of disapprovals then the set of selected candidates is the whole set of candidates. Formally:

**AD-compensation:** for each  $\pi \in \Pi$ ,  $n^+(x, \pi) = n^-(x, \pi)$  for all  $x \in C$  implies  $W(\pi) = C$ .

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<sup>3</sup>This is in line with the role of this property in the characterization by Alós-Ferrer (2006), as explained in his Footnote 3.

Lemma 1 below demonstrates that AD-compensation is stronger than cancellation in the presence of consistency. To prove it we introduce some additional notation that we also use along the remaining of this section. We let  $T_I = (\emptyset, C, \emptyset)$  and with each  $T = (C^+, C^0, C^-) \in \mathcal{T}$  we associate  $T^\pm = (C^-, C^0, C^+)$ . Observe that if  $W$  is a ballot aggregation function that satisfies AD-compensation then  $W(T + T^\pm) = C$  for all  $T \in \mathcal{T}$ .

**Lemma 1** *If a ballot aggregation function  $W$  satisfies AD-compensation and consistency then it satisfies AD-cancellation.*

**Proof.** Suppose  $\pi, \pi' \in \Pi$  are such that for all  $x \in C$  it is true that  $n^+(x, \pi) = n^+(x, \pi')$  and  $n^-(x, \pi) = n^-(x, \pi')$ . With each  $\pi \in \Pi$  we associate  $\pi^\pm \in \Pi$  by the expression  $\pi^\pm(T) = \pi(T^\pm)$  for all  $T \in \mathcal{T}$ . AD-compensation yields  $W(\pi + \pi^\pm) = C = W(\pi' + \pi^\pm)$ . Consistency yields  $W(\pi) = W(\pi + \pi' + \pi^\pm) = W(\pi')$ .  $\square$

We are in a position to prove our main result:

**Theorem 1** *The dis&approval rule is the only ballot aggregation function satisfying AD-compensation,  $(0, -1)$ -faithfulness, and consistency.*

**Proof.** We need to prove that if  $W$  is a ballot aggregation function that satisfies AD-compensation,  $(0, -1)$ -faithfulness, and consistency then it coincides with the dis&approval rule. Lemma 1 assures that AD-cancellation holds true, hence the distribution of votes other than approved/disapproved does not affect the outcomes of  $W$ . In particular, for each  $\pi \in \Pi$  and  $T \in \mathcal{T}$

$$\text{if } T = (\emptyset, C, \emptyset) \text{ then } W(\pi) = W(\pi + T) \quad (1)$$

which just means that voters that neither approve of nor disapprove of any candidate do not affect the outcomes of  $W$ .

**Step 1.** For any  $\pi \in \Pi$  and  $T_i = (C_i^+, C_i^0, C_i^-) \in \mathcal{T}$ ,  $i = 1, 2$ , such that  $C_1^+ \cap C_2^+ = \emptyset = C_1^- \cap C_2^-$ ,

$$W(\pi + T_1 + T_2) = W(\pi + T)$$

where  $T = (C_1^+ \cup C_2^+ - (C_1^- \cup C_2^-), C^0, C_1^- \cup C_2^- - (C_1^+ \cup C_2^+)) = (C^+, C^0, C^-)$  with  $C^0 = C - (C^+ \cup C^-)$ . Observe that  $W(T + T^\pm) = C$ , and also  $W(T_1 + T_2 + T^\pm) = C$  by virtue of AD-compensation. Now consistency implies

$$W(\pi + T_1 + T_2) = W(\pi + T_1 + T_2 + T + T^\pm)$$

$$W(\pi + T) = W(\pi + T + T_1 + T_2 + T^\pm)$$

Combining both results we get the desired equality.

We now fix an arbitrary  $\pi \in \Pi$ .



**Step 2.** Let  $\pi' \in \Pi$  be such that

- (a) for all  $x \in C$ ,  $n^+(x, \pi) = n^+(x, \pi')$  and  $n^-(x, \pi) = n^-(x, \pi')$ , and
- (b) if  $\pi'(T = (C^+, C^0, C^-)) > 0$  then  $|C^+ \cup C^-| = 1$ .

To construct  $\pi' \in \Pi$  we take apart every ternary ballot cast under  $\pi$  into ballots where exactly one candidate is either approved of or disapproved of, the other candidates going into the other category.

Then  $AD$ -cancellation ensures  $W(\pi) = W(\pi')$ .

**Step 3.** There is  $\bar{\pi} \in \Pi$  such that

- (a) for all  $x \in C$ , either  $n^+(x, \bar{\pi}) = 0$  or  $n^-(x, \bar{\pi}) = 0$
- (b) for all  $x \in C$ ,  $n^+(x, \bar{\pi}) = n^+(x, \pi) - n^-(x, \pi)$  when  $n^+(x, \bar{\pi}) > 0$ , and  $n^-(x, \bar{\pi}) = n^-(x, \pi) - n^+(x, \pi)$  when  $n^-(x, \bar{\pi}) > 0$
- (c) if  $\bar{\pi}(T = (C^+, C^0, C^-)) > 0$  then  $|C^+ \cup C^-| = 1$
- (d)  $W(\pi) = W(\pi') = W(\bar{\pi})$ .

To prove this we proceed iteratively for the candidates in  $C$ .

For a fixed candidate  $x$ , suppose firstly  $K = n^+(x, \pi') \geq n^-(x, \pi') = k$ . If  $k = 0$  then we are done with this candidate. Otherwise  $\pi' = \hat{\pi} + T_1 + \dots + T_K + T_{K+1} + \dots + T_{K+k}$  with the following structure: the ternary ballots  $T = (C^+, C^0, C^-)$  for which  $\hat{\pi}(T) > 0$  verify both  $x \notin C^+ \cup C^-$  and  $|C^+ \cup C^-| = 1$ ; and the ternary ballots  $T_i = (C_i^+, C_i^0, C_i^-)$  verify  $C_i^+ = \{x\}$  and  $C_i^- = \emptyset$  when  $i = 1, \dots, K$ ,  $C_i^- = \{x\}$  and  $C_i^+ = \emptyset$  when  $i = K + 1, \dots, K + k$ . Using Step 1

$$W(\pi') = W(\hat{\pi} + T_2 + \dots + T_K + T_{K+2} + \dots + T_{K+k} + T_l)$$

and now Eq. (1) yields  $W(\pi') = W(\hat{\pi} + T_2 + \dots + T_K + T_{K+2} + \dots + T_{K+k})$ . After  $k$  steps we reach  $W(\pi') = W(\hat{\pi} + T_{k+1} + \dots + T_K)$  and we carry forward the profile in the right-hand-side (which is simply  $\hat{\pi}$  when  $K = k$ ) in order to continue our iteration with the next candidate. To that purpose we observe that  $n^+(y, \pi') = n^+(y, \hat{\pi})$  and  $n^-(y, \pi') = n^-(y, \hat{\pi})$  if candidate  $y$  has not been considered yet, and also that the profile that we carry forward consists of ternary ballots  $T = (C^+, C^0, C^-)$  with the property that  $|C^+ \cup C^-| = 1$ , and satisfies requirement (b) for  $x$ . Secondly, if  $K < k$  then the argument is alike the case that we have considered.

Proceeding with all candidates in succession we prove the claim.

**Step 4.** In order to conclude that  $W$  coincides with the disapproval rule we proceed to prove that  $W(\bar{\pi})$  is the set of maximizers of  $n^+(x, \pi) - n^-(x, \pi)$ .

In case that there is  $x$  with  $n^+(x, \bar{\pi}) > 0$ , then for  $i = 1, \dots, \max\{n^+(x, \bar{\pi}) : x \in C\}$  we let  $C_i^+ = \{x \in C : n^+(x, \bar{\pi}) \geq i\}$  thus  $C_1^+ \supseteq C_2^+ \supseteq \dots$ , and we select  $T_i^+ = (C_i^+, C - C_i^+, \emptyset)$ .

In case that there is  $x$  with  $n^-(x, \bar{\pi}) > 0$ , for  $i = 1, \dots, \max\{n^-(x, \bar{\pi}) : x \in C\}$  we let  $C_i^- = \{x \in C : n^-(x, \bar{\pi}) \geq i\}$  thus  $C_1^- \supseteq C_2^- \supseteq \dots$ , and we select  $T_i^- = (\emptyset, C - C_i^-, C_i^-)$ . Observe that when  $n^-(x, \bar{\pi}) \geq t > 0$  for each candidate  $x$ , one has  $W(T_1^-) = \dots = W(T_t^-) = C$ .

In case that there is  $x$  with  $n^+(x, \bar{\pi}) = n^-(x, \bar{\pi})$ , we select  $T_0^- = (\emptyset, C, \emptyset)$  thus  $W(T_0^-) = C$ .

The profile  $\tilde{\pi}$  where each of these possible ballots is selected by exactly one voter verifies  $W(\bar{\pi}) = W(\tilde{\pi})$  since  $n^+(x, \bar{\pi}) = n^+(x, \tilde{\pi})$  and  $n^-(x, \bar{\pi}) = n^-(x, \tilde{\pi})$  throughout thus *AD*-cancellation applies. We conclude by showing that  $W(\tilde{\pi})$  is the set of maximizers of  $n^+(x, \pi) - n^-(x, \pi)$ :

*Case 4.1:* if there is  $x$  with  $n^+(x, \bar{\pi}) > 0$  then for  $\alpha = \max\{n^+(x, \bar{\pi}) : x \in C\}$  one has  $C_\alpha^+ \subseteq C_i^+ = W(T_i^+)$  for each possible  $i$ , and also  $C_\alpha^+ \subseteq W(T_i^-)$  for each possible  $i$  by using requirement (a) on  $\tilde{\pi}$  where necessary. Consistency assures that  $W(\tilde{\pi}) = C_\alpha^+$ , which is the set of maximizers of  $n^+(x, \pi) - n^-(x, \pi)$  using requirement (b) on  $\tilde{\pi}$ .

*Case 4.2:* if  $n^+(x, \bar{\pi}) = n^+(x, \tilde{\pi}) = 0$  and  $n^-(x, \bar{\pi}) = n^-(x, \tilde{\pi}) = 0$  for all  $x \in C$  then  $n^+(x, \pi) - n^-(x, \pi)$  is constantly 0. *AD*-compensation assures  $W(\pi) = C$  as the dis&approval rule claims.

*Case 4.3:* if Cases 4.1 and 4.2 do not hold then for  $\beta = \min\{n^-(x, \bar{\pi}) = n^-(x, \bar{\pi}) - n^+(x, \bar{\pi}) : x \in C\} \geq 0$  one has  $W(T_\beta^-) = C$  (both when  $\beta = 0$  and when  $\beta > 0$ ), and  $W(T_{\beta+1}^-) \subseteq W(T_{\beta+2}^-) \subseteq \dots$  for each possible ternary ballot whenever  $\beta < \max\{n^-(x, \bar{\pi}) : x \in C\}$ . When  $\beta = \max\{n^-(x, \bar{\pi}) : x \in C\}$  we have  $W(\tilde{\pi}) = C$ , otherwise consistency assures  $W(\tilde{\pi}) = W(T_{\beta+1}^-)$ , which is the set of maximizers of  $n^+(x, \pi) - n^-(x, \pi)$  by requirement (b) on  $\tilde{\pi}$ .  $\square$

## 5 Conclusion

The properties in our characterization of the dis&approval rule suffice to discuss on its virtues and faults, but of course they do not exhaust the list of its relevant attributes. They have further implications like *AD*-cancellation and others, and knowing them helps to understand the normative behavior of the rule. It satisfies neutrality: the names of the candidates do not matter. It also satisfies positive and negative unanimity: if all voters vote in favor (resp., against) of some candidate(s), the candidate(s) should (resp., should not) be selected. Independence of irrelevant alternative holds true too: if a non selected candidate is removed from the list of candidate and this does not affect the voters' ballots for the remaining candidates, then the selected candidates do not change. We can name other more technical properties, like tally-simplicity: when all tallies coincide for each pair of candidates then all candidates are elected.

Furthermore, dis&approval voting satisfies some practical properties advanced by Brams and Fishburn (2005, p. 461) as arguments in favor of approval voting. Approval voting gives more flexible options than plurality voting, increases voter turnout, gives minority candidates their proper due, and is as eminently practicable. Dis&approval voting still enriches the options offered by approval voting by allowing voters to explicitly express disagreement with some (or all) candidates. If voters are better able

to express their preferences they are more likely to vote. Moreover the electoral absenteeism due to the dissatisfaction with politicians would be reduced under disapproval voting. As under approval voting, minority candidates will not suffer. If supporters are allowed to vote for several candidates, they will not be tempted to desert a candidate who is weak in the polls. Finally, disapproval voting is also simple for voters to understand and use. In fact, online practices show that a negative vote is as natural as a positive vote (see for instance the *thumb up* or *thumb down* vote in Community Question Answering sites such as Yahoo! Answers).

## References

- [1] Alós-Ferrer, C., 2006, A simple characterization of approval voting, *Social Choice and Welfare* **27**, 621-625.
- [2] Arcelus, F., G. Mauser and Z. A. Spindler, 1978, The right to vote no: revising the voting system and resuscitating the F-Y voter, *Public Choice* **33**, 67-83.
- [3] Baujard, A., and Igersheim, H., 2011, Framed-field Experiment on Approval Voting and Evaluation Voting. Some Teachings to Reform the French Presidential Electoral System. In: Bernard Dolez, Bernard Grofman and Annie Laurent (eds), In Situ and Laboratory Experiments on Electoral Law Reform French Presidential Elections. Studies in Public Choice Serie 1, vol. 25, 69-89.
- [4] Brams, S. J., P. C. Fishburn, 2005, Going from theory to practice: the mixed success of approval voting. *Social Choice and Welfare* **25** , 457-474.
- [5] Felsenthal, D. S., 1989, On combining approval with disapproval voting, *Behavioral Science* **34**, 53-60.
- [6] Fishburn, P. C., 1978a, Symmetric and consistent aggregation with dichotomous voting. In: Laffont, J.-J. (ed), Aggregation and revelation of preferences. North Holland, Amsterdam, 201-218.
- [7] Fishburn, P. C., 1978b, Axioms for approval voting: direct proof, *Journal of Economic Theory* **19**, 180-185.
- [8] Hahn, J., 1988, An experiment in competition: The 1987 elections to the local soviets, *Slavic Review* **47**, 434-447.
- [9] Hillinger, C., 2004, Voting and the cardinal aggregation of judgments, *Discussion Paper 2004-09*, University of Munich.
- [10] Hillinger, C., 2005, The case for utilitarian voting, *Homo Oeconomicus* **22**, 295-321.

- [11] Kang, M. S., 2010, Voting as veto, *Michigan Law Review* **108**, 1221-1282.
- [12] Lepelley, D., and H. Smaoui, 2012, Le système de vote par note à trois niveaux: étude axiomatique d'un nouveau mode de scrutin. Mimeo: paper presented at the conference: XXIX Journées de microéconomie appliquée, Brest, June 2012.
- [13] Zhong, Y. and J. Chen, 2002, To vote or not to vote: an analysis of peasants' participation in Chinese village elections, *Comparative Political Studies* **35**; 686-712.