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Abstract

We analyze whether relative risk aversion varies with wealth. We first derive theoretical predictions on how risky shares respond to wealth fluctuations in a portfolio choice model with both external habits and time-varying labor income. Our analytical results indicate that: (1) for each household, there are two channels through which the risky share responds to wealth fluctuations, the habit channel and the income channel; (2) across households, there are heterogeneous responses through the habit channel: those who experience large negative income shocks reduce their share of risky assets; and (3) two potential mis-identification problems arise when both the heterogeneity in responses through the habit channel and the income channel are ignored. We then test the theoretical predictions with data from the Panel Study of Income Dynamics. Contrary to the existing literature, our empirical results show evidence of relative risk aversion varying with wealth over time after correcting those two mis-identification problems.

Keywords: Time-varying relative risk aversion; Habit formation preference; Micro data; Portfolio choice.

JEL classification: E21; E24; D91.

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1 Introduction

The assumption that agent's relative risk aversion varies with wealth is appealing because it provides an important mechanism that helps explain numerous economic phenomena. One popular way of generating time-varying relative risk aversion (hereafter TVRRA) with wealth is to assume habit formation preferences. Macroeconomic models with habit formation preferences have been used to explain a variety of stylized macroeconomic facts that are hard to explain using theoretical models with standard constant relative risk aversion preferences. These facts include the equity premium [see, Constantinides (1990), Jermann (1998), Campbell and Cochrane (1999), Boldrin et al. (2001), and others], the excess sensitivity of consumption to income [see, for example, Boldrin et al. (2001)], the equity home bias [see, among others, Shore and White (2002)], the hump-shaped response of aggregate variables to monetary shocks [Fuhrer (2000), Uribe (2002), Christiano et al. (2005)], and countercyclical markups [Ravn et al. (2006)]. However, despite the mounting literature that uses habit formation preferences, thus automatically embodying a TVRRA assumption, there are only a few papers that test the theoretical implications of TVRRA with micro-data.

In this paper, we derive theoretical predictions of TVRRA on the relation between risky shares and wealth and test these predictions with data from the Panel Study of Income Dynamics (PSID). To derive the theoretical predictions between risky shares and wealth, we build a discrete-time portfolio choice model with external habits and time-varying labor income.[1] Our emphasis on time-varying labor income is motivated by two empirical facts of the PSID data. Fact 1: the majority of households in the PSID data receive labor income. And Fact 2: a large portion of households in the PSID data experience large income shocks. For example, about 40% of the households in the sample received income below 30% of their

[1]We generate TVRRA by assuming habits. Thus, we test the theoretical predictions of TVRRA by examining how risky shares respond to wealth fluctuations through the habit channel. Here the habit channel means the response of risky shares to wealth simply because of the existing of habit. We provide a mathematical presentation of the habit channel in Section 2.3.1.

average income over time.[2] Our model with time-varying labor income captures those two empirical observations in the data. Mostly importantly, our model modifies the existing theoretical predictions about the relation between risky shares and wealth in a non-trivial way, which we will discuss below.

Our first contribution is theoretical. We establish the theoretical predictions of TVRRA on the relation between risky shares and wealth. Our theoretical predictions are different from the existing prediction which argues that, when it becomes richer, the household will increase its risky share if the household has decreasing relative risk aversion in wealth [Brunnermeier and Nagel (2008)]. Instead, our analytical solution changes such a conventional prediction in two dimensions. First, our closed-form solution suggests that for each household its risky share responds to wealth fluctuations through two channels: the habit channel and the income channel. When the household does not experience large negative income shocks, the risky share, as wealth accumulates, will increase through the habit channel and decrease through the income channel.[3] Thus, a mis-identification problem may arise when the response through both the habit channel and the income channel is ascribed to the response through the habit channel alone. We call this an internal mis-identification problem. The second bias arises due to the heterogeneity in households' income shocks. Specifically, households with large income drops are likely to decrease their risky shares responding to wealth accumulations through the habit channel, while households without large negative income shocks will increase their risky shares through that channel when they become richer. Thus, households who have decreasing relative risk aversion in wealth may adjust their risky shares to wealth fluctuations in opposite ways in the presence of heterogenous income shocks. As a result, an external mis-identification problem may arise when estimating over samples in which heterogenous households are pooled together.

Our second contribution is empirical. We show strong evidence of TVRRA which is in line

[2]Section 4.1.1 provides more details.

[3]We show this point analytically in Section 2.3.1.

with the theoretical predictions. To facilitate the discussion, we define three different forms of TVRRA implications (hereafter TVRRAI): the strong form, the semi-strong form, and the weak form.[4] If the portfolio choice model with habits considers neither the aforementioned two channels for each household, nor the aforementioned heterogeneous responses through the habit channel across households, we label the key theoretical implication(s) from such a model as the strong form of TVRRAI. The strong form implies that households whose preferences could be represented by habit formation will, independent of their income flows, increase their risky shares when their wealth increases, as discussed in Brunnermeier and Nagel (2008). If the portfolio choice model with habits considers the two channels but ignores the heterogeneity, we label the key theoretical implication(s) from such a model as the semi-strong form of TVRRAI. The semi-strong form implies that after controlling for the response through the income channel, the response through the habit channel should be positive. At last, if the portfolio choice model with habits considers both the two channels and the heterogeneity, we label the key theoretical implication(s) from such a model as the weak form of TVRRAI. The weak-form implies that after controlling for the response through the income channel and the impact of large negative income shocks, the response through the habit channel in the group in which households experienced large negative income shocks should be lower than that in the other group in which households did not experience large negative income shocks.

We empirically test the semi-strong form and the weak form of TVRRAI and compare the results with the strong form tested in Brunnermeier and Nagel (2008). We find evidence of the weak-form of TVRRAI and no evidence of the semi-strong form of TVRRAI. First, in the semi-strong form of TVRRAI, the identification scheme builds on a model that does consider the two channels but ignores the heterogeneity so that the test corrects the internal but not the external mis-identification problem, our estimates are statistically insignificant. This contrasts to Brunnermeier and Nagel (2008), who test the strong-form of TVRRAI and find

[4]When a model imposes less restrictions, we say the derived theoretical implication is stronger.

significant negative responses. This comparison is in line with our theory, which states that controlling for the response through the income channel will increase the estimated response through the habit channel.[5] Second, in the weak form of TVRRAI, if the identification scheme builds on a model that considers both the two channels and the heterogeneity so that the test corrects both the internal and the external mis-identification problems, our estimates are both economically and statistically significant. We consider our empirical results as strong and clear evidence of TVRRA. Furthermore, our empirical results highlight the importance of isolating the impacts of time-varying labor income in carrying out empirical tests of time-varying relative risk aversion.

Our paper is related to several strands of the existing literature. First, our paper contributes to the literature about the impact of habit formation preferences on households' portfolio choices. Existing theoretical models with habit formation that abstract from labor income imply a positive relation between wealth and risky shares [Constantinides (1990), Campbell and Cochrane (1999), etc.]. Our theoretical contribution here is that we show, analytically, how labor income affects such a relation. On the empirical side, Brunnermeier and Nagel (2008), on the contrary, find a negative relation between risky shares and wealth in the data. Our empirical contribution here is that we show that estimates which ignore the two aforementioned mis-identification problems are likely to be biased down as is predicted in our theory. Put together, by carefully controlling for the impact of labor income on the relation between risky shares and wealth, we provide a mechanism that is able to reconcile the seeming conflict between the existing theoretical predictions of TVRRA from a model with habit formation preferences and the existing empirical findings.[6]

Second, our paper is related to the existing studies that test key theoretical predictions

[5] We show this point analytically in Section 2.3.1.

[6] It is worth mentioning that our mechanism is one of many possible ones that may be able to reconcile the seeming conflict. For example, one mechanism could be to control for the impact of inertia in portfolio adjustments as proposed in Calvet et al. (2009). They estimate the Brunnermeier–Nagel regression on their Swedish data set, with which they can control for inertia, and they find a positive relation when they estimate the regression by instrumental variables.

implied by habit using micro-data, which find mixed evidence of habit formation preferences. For example, Dynan (2000) rejects habit preference using US consumption data and Chiappori and Paiella (2011) reject habit preference using Italian data. On the contrary, Ravina (2007) provides evidence of habit persistence in household consumption choices using panel data on U.S. credit-card account holders. Carrasco et al. (2007) estimate the intra-temporal marginal rate of substitution using Spanish consumption panel data and find strong support of habit. Cappelletti (2012) uses Italian data and finds that, after controlling for the decision to enter and leave the risky asset market, wealth fluctuations do help to explain changes in portfolio allocations. Recently, Brunnermeier and Nagel (2008) investigate how households' portfolio allocations change in response to wealth fluctuations and find a negative link between risky shares and wealth, a finding which may be interpreted as evidence of a combination of the lack of TVRRA and the inertia in portfolio adjustments. Calvet et al. (2009) estimate the Brunnermeier–Nagel regression on Swedish data and find a positive relation after controlling for inertia.

Lastly, our paper is also related to the literature on how income affects households' portfolio choices. For example, Wachter and Yogo (2010) show in a life-cycle model that risky shares fall in normalized cash-on-hand (which corresponds to wealth in our paper) and rises in permanent income even if households have decreasing relative risk aversion in wealth. Our results instead indicate that the impact of *temporary* income also matters in terms of theoretical predictions of TVRRA on the relation between risky shares and wealth. First of all, our results show that controlling for the impact through the income channel may help explain the negative link found in Brunnermeier and Nagel (2008). Second, further controlling for the heterogeneous responses through the habit channel across households provides us evidence of TVRRA.

The rest of the paper is organized as follows. Section 2 presents the model and provides testable implications. Section 3 briefly explains the data, variables, and the sample selection. Section 4 presents the empirical results on both the weak form of TVRRAI and the semi-strong Form of TVRRAI. And Section 5 concludes.

2 Model, Solution, Biases, and Testable Implications

In this section, we present the benchmark model, derive the analytic solution to risky share, discuss the importance of time-varying income, and derive testable implications of TVRRA. The theoretical model is a highly stylized portfolio choice model with a time-varying labor income and external habits. We consider the model for several reasons. First, the model delivers clear testable predictions of TVRRA on how risky shares respond to wealth fluctuations. Second, our model captures the realistic feature that the majority of the US households do receive time-varying labor income. Third, the majority of macroeconomics models, if they assume habit formation preferences, contain these two elements.

2.1 The Benchmark Model

In this model, a household carries wealth, W_t, from the last period, and receives labor income, Y_t, in the current period. Note that the household receives time-varying labor income in our model. This is different from the model in Brunnermeier and Nagel (2008) in which there is no labor income. The household chooses consumption C_t and the share of wealth $W_t+Y_t-C_t$ invested in the risky asset, α_t, to maximize

$$\max_{\{C_t,\, \alpha_t\}_{t=0}^{\infty}} \qquad U = \mathbb{E}\sum_{t=0}^{\infty} \delta^t \frac{(C_t - X)^{1-\sigma}}{1-\sigma},$$

where $\mathbb{E}$ denotes the unconditional expectation operator, δ denotes the subjective discount factor, and X denotes the external habit. It is worth mentioning that there are many functional forms to choose to represent habits. We choose the simplest form that enables us to generate time-varying relative risk aversion implications on how risky shares respond to wealth fluctuations. Alternatively, X can be interpreted as a constant subsistence level or a consumption commitment as in Chetty and Szeidl (2007). In Section 2.2.2, we show that when habits do not change dramatically over time, time-varying external habits do not bring much additional insight about how habit formation preferences affect the relation between risky shares and wealth. For this very reason and given that we focus on how income affects the relation between risky shares and wealth, we assume constant, instead of time-varying, external habits in the benchmark model.

In order to obtain the analytical solution, we make the following assumption on labor income

$$(Y_{t+1} - Y) = \kappa (Y_t - Y), \tag{2.1}$$

where Y denotes the steady state of labor income. κ is a parameter whose value is within $(-1, 1)$. Our specification modifies the standard assumption that income follows an AR(1) process. Such a process enables us, when there is no uncertainty in the income process, to derive a close-form solution, which will deliver clear theoretical predictions.

The household can invest in two securities: a risky asset with return R_t and a risk-free asset with return R_f. As a result, the household's wealth at the beginning of period $t+1$ is given by

$$W_{t+1} = (1 + R_{p,t+1}) (W_t + Y_t - C_t),$$

where $R_{p,t+1} = \alpha_t (R_t - R_f) + R_f$ denotes the return to the household's wealth portfolio.

2.2 The Analytical Solution

It is not straightforward to derive the analytical solution to the risky share in our model. The key idea behind the strategy to derive the analytical solution is as follows: transform the original model into one which has an analytical solution; and back up the solution to the risky share in the original model since both the original model and the transformed model should have the same amount of wealth invested in the risky asset. To successfully implement this strategy, we need to impose additional restrictions such as Eq. (2.1). With some additional restrictions and by following four steps, we derive the analytical solution to the risky share in our benchmark model.

2.2.1 Step 1: Adding A Fixed Labor Income Flow

In this step, we introduce a constant labor income flow and a constant external habit into Samuelson (1969). We define this model as Model 1. The household's optimization problem becomes to choose C_t and the share of $W_t - C_t + Y$ invested in the risky asset, α_t^1, to maximize

$$\begin{aligned} \max_{\{C_t,\, \alpha_t^1\}} \quad & U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X)^{1-\gamma}}{1-\gamma}, \\ \text{s.t.} \quad & W_{t+1} = (1 + R_{p,t+1}) (W_t - C_t + Y). \end{aligned}$$

To derive the solution, we first divide the total household post-consumption wealth into two parts: $W_t + Y - C_t - \frac{X-Y}{R_f}$ and $\frac{X-Y}{R_f}$. For the first part, the household invests a fraction $\tilde{\alpha}_t^1$ in the risky asset and the rest in the risk-free asset. The return to this (partial) wealth portfolio, $W_t + Y - C_t - \frac{X-Y}{R_f}$, is then given by $\tilde{R}_{p,t+1}^1 = \tilde{\alpha}_t^1 (R_t - R_f) + R_f$. The second part, the remaining $\frac{X-Y}{R_f}$, is 100% invested in the risk-free asset. The law of motion of wealth is given by

$$W_{t+1} = \left(1 + \tilde{R}_{p,t+1}^1\right) \left(W_t - C_t + Y - \frac{X-Y}{R_f}\right) + (1 + R_f) \frac{X-Y}{R_f}.$$

To transform the model, we then define some auxiliary variables: $\tilde{W}_t^1 = W_t - (X - Y) - \frac{X-Y}{R_f}$ and $\tilde{C}_t^1 = C_t - X$. With the auxiliary variables, Model 1 can be rewritten as

$$\begin{aligned} \max_{\{\tilde{C}_t^1,\, \tilde{\alpha}_t^1\}_{t=0}^{\infty}} \quad & U = \mathbb{E} \sum_{t=0}^{\infty} \delta^t \frac{\left(\tilde{C}_t^1\right)^{1-\gamma}}{1-\gamma}, \\ \text{s.t.} \quad & \tilde{W}_{t+1}^1 = \left(1 + \tilde{R}_{p,t+1}^1\right) \left(\tilde{W}_t^1 - \tilde{C}_t^1\right). \end{aligned}$$

The transformed model is identical to the one studied in Samuelson (1969). Under the condition that the expected returns and volatility are constant, the share of the risky asset, $\tilde{\alpha}_t^{1*}$, to the above transformed optimization problem is constant [Samuelson (1969)], and it is 1, that is $\tilde{\alpha}_t^{1*} = \alpha \approx 1$.

Finally, since both Model 1 and the transformed one give the same answer to the amount of wealth invested in the risky asset, it is true that

$$\begin{aligned}\alpha_t^{1*}(W_t - C_t + Y) &= \tilde{\alpha}_t^{1*} \times \left(W_t - C_t + Y - \frac{X-Y}{R_f}\right) \\ \Rightarrow \alpha_t^{1*} &= \tilde{\alpha}_t^{1*} \times \frac{W_t - C_t + Y - \frac{X-Y}{R_f}}{W_t - C_t + Y} \approx 1 - \frac{X-Y}{(W_t - C_t + Y)\, R_f}.\end{aligned}$$

In particular, if we set $Y_t = 0$, we will get the same model as in Brunnermeier and Nagel (2008) and the solution to the risky share, α_t^1, will be simplified to

$$\alpha_t^{1*} = \alpha\left[1 - \frac{X}{(W_t - C_t)\, R_f}\right] \approx 1 - \frac{X}{(W_t - C_t)\, R_f}, \tag{2.2}$$

This is also the same as that in Brunnermeier and Nagel (2008).

2.2.2 Step 2: Time-varying Habits

In this step, we allow time-varying habits and set $Y_t = 0$ in the benchmark model. By doing so, we obtain Model 2. In this case, the household's optimization problem is to maximize

$$\begin{aligned}\max_{\{C_t,\, \alpha_t^2\}_{t=0}^{\infty}} \quad & U = \mathbb{E}\sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}, \\ \text{s.t.} \quad & W_{t+1} = (1 + R_{p,t+1})(W_t - C_t),\end{aligned}$$

To obtain an analytical solution, we impose the following restrictions on habits

$$(X_{t+1} - X) = \eta\,(X_t - X).$$

Even though our specification is restrictive, such a process enables us to derive the close-form solution.

We then divide the total household post-consumption wealth into two parts: $W_t - C_t - \frac{X_t - X}{Z+R_f}$ and $\frac{X_t - X}{Z+R_f}$, where $Z = \frac{1+R_f}{\eta} - (1+R_f)$. In the first part, the household invests a fraction $\tilde{\alpha}_t^2$ in the risky asset and the rest in the risk-free asset. The return to this (partial) wealth portfolio is $\tilde{R}_{p,t+1}^2 = \tilde{\alpha}_t^2 (R_t - R_f) + R_f$. The second part, the remaining $\frac{X_t - X}{Z+R_f}$, is 100% invested in the risk-free asset. The law of motion of wealth can thus be written as

$$
\begin{aligned}
W_{t+1} &= \left(1 + \tilde{R}_{p,t+1}^2\right)\left(W_t - C_t - \frac{X_t - X}{Z + R_f}\right) + (1+R_f)\frac{X_t - X}{Z+R_f}, \\
&= \left(1 + \tilde{R}_{p,t+1}^2\right)\left\{W_t - (X_t - X) - \frac{X_t - X}{Z + R_f} - [C_t - (X_t - X)]\right\} \\
&\quad + \frac{Z + 1 + R_f}{Z + R_f}\left(X_{t+1} - X\right).
\end{aligned}
$$

Third, we define some auxiliary variables: $\tilde{W}_t^2 = W_t - (X_t - X) - \frac{X_t - X}{Z+R_f}$ and $\tilde{C}_t^2 = C_t - X_t + X$. And Model 2 can be rewritten as

$$
\begin{aligned}
\max_{\{\tilde{C}_t^2,\, \tilde{\alpha}_t^2\}_{t=0}^{\infty}} &= \mathbb{E}\sum_{t=0}^{\infty}\delta^t \frac{\left(\tilde{C}_t^2 - X\right)^{1-\gamma}}{1-\gamma}, \\
\text{s.t.} \qquad & \tilde{W}_{t+1}^2 = \left(1 + \tilde{R}_{p,t+1}^2\right)\left(\tilde{W}_t^2 - \tilde{C}_t^2\right).
\end{aligned}
$$

This transformed model is the same as the special case of Model 1 when $Y = 0$. As is shown in Eq. (2.2), the share of the risky asset in the transformed model is given by

$$
\tilde{\alpha}_t^{2*} = \alpha\left[1 - \frac{X}{\left(\tilde{W}_t^2 - \tilde{C}_t^2\right)R_f}\right] \approx 1 - \frac{X}{\left(W_t - C_t - \frac{X_t - X}{Z+R_f}\right)R_f},
$$

where the approximation sign comes from $\alpha \approx 1$.

Finally, since the original model and the transformed model should have the same solution to the amount of wealth invested in risky assets, we have

$$\alpha_t^{2*}\left(W_t - C_t\right) = \tilde{\alpha}_t^{2*}\left(\tilde{W}_t^2 - \tilde{C}_t^2\right) = \tilde{\alpha}_t^{2*}\left[\left(W_t - C_t\right) - \frac{X_t - X}{Z + R_f}\right].$$

And the solution to the risky share in Model 2 is given by

$$\alpha_t^{2*} = \left[1 - \frac{X}{\left(W_t - C_t - \frac{X_t - X}{Z + R_f}\right) R_f}\right]\left[1 - \frac{X_t - X}{\left(W_t - C_t\right)\left(Z + R_f\right)}\right]. \tag{2.3}$$

From Eq. (2.3), the necessary and sufficient condition for the last term, $\left[1 - \frac{X_t - X}{(W_t - C_t)(Z + R_f)}\right]$, to be positive, is $((X_t - X) < (W_t - C_t) * (Z + R_f))$. That is to say, in the case that habits do not dramatically deviate above from the mean, we have the conventional wisdom: households with habit formation preferences will increase their risky shares when they accumulate wealth. This is the same theoretical prediction as in Brunnermeier and Nagel (2008) which assume time-invariant habits. In other words, allowing time-varying habits will not fundamentally change the positive prediction in the literature if habits move slowly around the mean. For this reason, we do not impose any additional restriction in our regression equations to control for the time-varying habits. However, if X_t is far above X, it becomes possible the response of risky shares to wealth accumulations through the habit channel becomes negative. It seems intuitive to argue that the chance that the response becomes negative increases over longer time horizons.

2.2.3 Step 3: Time-varying Labor Income and Fixed Habits

With all the results we have obtained in steps 1–2, we are now ready to derive the analytical solution to the risky share in the benchmark model. To proceed, we define $\tilde{X}_t^3 = X - Y_t$,

$\tilde{X}^3 = X - Y$, and $\tilde{C}_t^3 = C_t - Y_t$. We obtain

$$\tilde{X}_{t+1}^3 - \tilde{X}^3 = \kappa\left(\tilde{X}_t^3 - \tilde{X}^3\right),$$
$$C_t - X = \tilde{C}_t^3 + Y_t - X = \tilde{C}_t^3 - \tilde{X}_t^3.$$

And we can transform the benchmark model into a model without income but with time-varying habits

$$\max_{\{\tilde{C}_t^3, \tilde{\alpha}_t^3\}_{t=0}^{\infty}} U = \mathbb{E}\sum_{t=0}^{\infty}\delta^t \frac{\left(\tilde{C}_t^3 - \tilde{X}_t^3\right)^{1-\gamma}}{1-\gamma},$$
$$\text{s.t. } \tilde{W}_{t+1}^3 = \left(1 + \tilde{R}_{p,t+1}^3\right)\left(\tilde{W}_t^3 - \tilde{C}_t^3\right).$$

where $\tilde{W}_{t+1}^3 = W_{t+1}$ and $\tilde{W}_t^3 = W_t$. This transformed model is the same model as Model 2 and the law of motion of $\tilde{X}_t^3$ is also the same as the one imposed in Model 2. Thus, Under the condition that expected return and the standard deviation of R_t are constant and there are no income shocks, i.e., $\eta_t \equiv 0$, the solution to the risky share in the transformed model is given by

$$\tilde{\alpha}_t^{3*} = \left[1 - \frac{\tilde{X}^3}{\left(\tilde{W}_t^3 - \tilde{C}_t^3 - \frac{\tilde{X}_t^3 - \tilde{X}^3}{Z + R_f}\right) R_f}\right]\left[1 - \frac{\tilde{X}_t^3 - \tilde{X}^3}{\left(\tilde{W}_t^3 - \tilde{C}_t^3\right)(Z + R_f)}\right],$$

where Z is defined as $Z = \frac{1+R_f}{\kappa} - (1 + R_f)$.

Finally, since the original model and the transformed model should have the same solution to the amount of wealth invested in risky assets, we have the following

$$\alpha_t^*(W_t - C_t + Y_t) = \tilde{\alpha}_t^{3*}\left(\tilde{W}_t^3 - \tilde{C}_t^3\right) \Rightarrow \alpha_t^* = \tilde{\alpha}_t^{3*},$$
$$\Rightarrow \alpha_t^* = \left[1 - \frac{X - Y}{\left(W_t - C_t + Y_t + \frac{Y_t - Y}{Z + R_f}\right) R_f}\right]\left[1 + \frac{Y_t - Y}{(W_t - C_t + Y_t)(Z + R_f)}\right]. \quad (2.4)$$

Equation (2.4) provides an analytical solution that enables us to discuss how the risky share responds to post consumption wealth and how time-varying labor income and habits affect the response.

2.3 Biases and Two Mis-identifications

In this section, we show that estimates of habit may be biased down if time varying labor income is ignored. This is because in this case, those estimates are subject to two mis-identification problems.

2.3.1 Two Channels and An Internal Mis-identification

For each household, its risky share responds to its wealth accumulation through two channels, the habit channel and the income channel. A mis-identification problem arises when the response through two channels is ascribed as the response through the habit channel. We label this mis-identification problem as the internal mis-identification problem. To see this, we set $Y_t \equiv Y$ and Eq. (2.4) reduces to

$$\alpha_t^* = \underbrace{\left[1 - \frac{X}{(W_t - C_t + Y)\, R_f}\right]}_{\text{The habit channel:}+} + \underbrace{\left[\frac{Y}{(W_t - C_t + Y)\, R_f}\right]}_{\text{The income channel:}-}. \tag{2.5}$$

The sign "+" ("-") means that the risky share will increase (decrease) when the post-consumption wealth increases. It is clear that α_t^* respond to the change of $W_t - C_t$ in two channels: the habit channel ("+") and the income channel ("-").[7] Intuitively, adding a constant stream of labor income in case of a constant habit is mathematically isomorphic (in terms of the asset allocation implications) to reducing the habit by a constant. As a result, ignoring the impact of labor income will bias down the estimates that are used to

[7]One thing worth mentioning is, even though α_t^* is decreasing in post consumption wealth through the second channel, $\partial\alpha_t^*/\partial Y$ is still positive: the higher labor income the household has, the larger the α^* will be.

make judgment about the time-varying relative risk aversion implications.

From Eq. (2.5), we obtain the following core regression equation

$$\Delta\alpha_t^* \approx (\rho - \theta Y)\,\Delta w_t + \varepsilon_t, \tag{2.6}$$

where Δ denotes the first-order difference, $\rho = \frac{XW}{(W+Y)^2 R_f}$ and $\theta = \frac{W}{(W+Y)^2 R_f}$ where W denotes the average of wealth, $w_t \equiv \log(W_t - C_t)$, and ε_t follows identical, independent, normal distribution and is uncorrelated with Δw_t. ρ is the parameter that catches the response of risky shares to wealth fluctuations through the habit channel. The detail is in Appendix 6.1.

If we remove labor income from the previous model, i.e., ignoring the impact of income on the relation, the core regression equation and the corresponding ordinary least squares (hereafter OLS) estimate are, respectively, given by

$$\begin{aligned}\Delta\alpha_t &\approx \rho\Delta w_t + \varepsilon_t,\\ \tilde{\rho} &= \left[(\Delta w_t)'(\Delta w_t)\right]^{-1}(\Delta w_t)'(\Delta\alpha_t)\,.\end{aligned}$$

However, if Eq. (2.6) is correctly specified, we will have

$$\mathbb{E}(\tilde{\rho}) = \left[(\Delta w_t)'(\Delta w_t)\right]^{-1}(\Delta w_t)'(\rho - \theta Y)\,\Delta w_t = \rho - \theta Y \le \rho. \tag{2.7}$$

Thus a mis-identification problem will arise when the estimate of $(\rho - \theta Y)$ is ascribed as the estimate of ρ. From Eq. (2.7), if labor income has a strong impact, i.e., θY is large, $\tilde{\rho}$, may be close to zero or negative even though the true value of ρ is still positive. Thus, the fact that $\tilde{\rho}$ is close to zero or negative does not necessarily imply that micro-data does not support the TVRRA assumption, because it does not necessarily mean that ρ is negative or zero. Put it differently, ignoring the response through the income channel biases down the estimate of ρ.

2.3.2 Heterogeneity and An External Mis-identification

Households with different income shocks may be heterogenous in terms of the responses of their risky shares to wealth fluctuations through the habit channel. When households do not have large negative income shocks, they increase their risky shares as the optimal response to wealth accumulations through the habit channel. However, when households have large negative income shocks, they may decrease their risky shares as the optimal response to wealth accumulations through the habit channel. To see the heterogenous responses, note that when Y is time-varying, the response of α_t^* through the habit channel is given by

$$\alpha \underbrace{\left[1 - \frac{X}{\left(W_t - C_t + Y_t + \frac{Y_t - Y}{Z + R_f}\right) R_f}\right] \left[1 + \frac{Y_t - Y}{(W_t - C_t + Y_t)(Z + R_f)}\right]}_{\text{The habit channel}}. \tag{2.8}$$

Expression (2.8) shows that if Y_t is far below Y, it is likely that the sum in the second parenthesis becomes negative. This implies that the response through the habit channel can be negative. In this case, the conventional wisdom, that risky shares are increasing in wealth through the habit channel, may break down in the presence of large negative income shocks.

Thus, there could be two different groups of households. Households in the first group have large negative income shocks and respond negatively, in terms of adjusting their risky shares, to wealth accumulations through the habit channel. Households in the second group do not have large negative income shocks and they respond positively. If we run regressions with a sample that pools the two heterogenous groups together, the associated estimate is mis-identified and it is likely to be insignificant. We label this mis-identification problem as the external mis-identification problem.

2.4 Testable Predictions

We have shown that incorporating time-varying labor income matters in terms of identification. We now derive empirical tests of the theoretical predictions of TVRRA by controlling the response through the income channel for each household and/or the heterogeneous responses through the habit channel across households, i.e., testing the weak form and the semi-strong form of TVRRAI.

2.4.1 Weak Form of TVRRAI

Given the aforementioned internal and external mis-identification problems, we design the following test to examine the weak form of TVRRAI. We divide households in each subsample into two groups: households in the first group, $i = 1$, experienced large negative income shocks and households in the second group, $i = 2$, did not experience large negative income shocks. Second, for each group, we obtain an estimate of $\rho^i, i = 1, 2$. Third, our testable hypothesis for habit formation preference is,

$$\rho^2 - \rho^1 \quad > \quad 0. \tag{2.9}$$

Instead of imposing $\rho^2 > \rho^1 > 0$, we test the difference of ρ's across groups in each subsample. The main reason for not testing $\rho^i > 0$ is as follows. As we have discussed in Section 2.2.2, when X_t dramatically deviates from its mean, the response of risky shares to wealth fluctuations through the habit channel would be negative even if households have not experienced large negative income shocks. In our empirical exercise, the test of the weak form of TVRRAI uses the $k = 5$ subsample.[8] During the five-year interval between any two observations for each household, it is possible that habits have changed a lot. Because of this consideration of large changes of habits over 5 years, we test Inequality (2.9). Nevertheless, our testable hypothesis will still be reasonable in this case as long as habits across households change roughly in the same fashion over time.

[8] The notation of $k = 5$ means that any two observations for the same household in the sample are 5 years apart. We explain in detail in Section 3.

2.4.2 Semi-strong Form of TVRRAI

Given the internal mis-identification problem, we run regression to obtain the estimates of the response through the habit channel, ρ. We consider the following testable hypothesis to test the semi-strong form of TVRRAI

$$\rho > 0. \tag{2.10}$$

Mathematically, without considering the impact of large negative income shocks (or as in the case of $Y_t \equiv Y$), our analysis in Section 2.3.1 indicates that $X > 0$ implies that $\rho > 0$. That is to say, a positive estimate of ρ suggests that habit formation preferences are in line with portfolio choice data. This is the same as in Brunnermeier and Nagel (2008): intuitively, an increase in wealth, for example, should lead to a temporary decrease in relative risk aversion and an increase of the risky share if households have habit formation preferences.

3 Variables, Data, and Sample Selection

Here we give a brief introduction about variables, data, and sampling. Variables are defined in the standard way. In particular, risk-free assets are defined as the sum of cash-like assets and holdings of bonds. We consider two different measures of risk share. In the first measurement, liquid assets are given by the sum of risk-free assets and the holdings of stocks and mutual funds. Subtracting other liabilities from liquid assets yields liquid wealth. As a result, risky assets is liquid wealth minus risk-free assets, and risky share is the holdings of risky assets divided by liquid wealth. The second measurement uses financial wealth, which is the sum of liquid wealth, equity in a private business, and home equity. Accordingly, risky assets include the holdings of stocks and mutual funds, equity in a private business, and home equity, and risky share is the holdings of risky assets divided by financial wealth. Lastly, income in our paper is represented by labor income of households.

The PSID data set contains many household characteristics annually after 1997 and households' asset holdings in years 1984, 1989, 1994, 1999, 2001, and 2003. Thus, time-series data about asset holdings are either 2-year apart or 5-year apart. Hence, we divide the data into two subsamples: the 1984-1999 ($k = 5$) subsample and the 1999-2003 ($k = 2$) subsample. We select households who hold at least \$10,000 liquid wealth or at least \$10,000 financial wealth in the last period, $t - k$. In addition, we require that the martial status of the family unit head remained unchanged from $t - k$ to t and that no assets were moved in or out as a consequence of a family member moving into or out of family unit. Table 1 provides some summary statistics of three key variables, liquid wealth, financial wealth, and income.

4 Empirical Analysis

In this section, we present the empirical results. We first investigate our benchmark model with time-varying labor income that considers both the habit channel and the income channel, and the heterogeneity (the weak form of TVRRAI). We then explore the model that considers both channels but with constant income (semi-strong form of TVRRAI).

4.1 Weak Form of TVRRAI

4.1.1 Methodology

To test our hypothesis, Inequality (2.9), we have to compare households' current income to their long run average in order to divide households in each subsample into two groups. The 1999-2003 ($k = 2$) subsample does not provide such data to calculate the averages. As a result, we only test the weak form of TVRRAI with the 1984-1999 ($k = 5$) subsample by dividing the households in that subsample into two groups. In the first group, $i = 1$, households' current income is below a threshold ratio of their time-series averages. The remaining households enter the second group, $i = 2$. In the benchmark exercise, we set the threshold ratio at 30%. In the sensitivity analysis in in Appendix 1, we change the ratio from 20% to 50%.

For each group, we estimate the following equation

$$\Delta_k \alpha^i_{t,j} \;=\; \beta^i q^i_{t-k,j} + \gamma^i \Delta_k h^i_{t,j} + \rho^i \Delta_k w^i_{t,j} - \vartheta^i y^i_{t,j} \Delta_k \left(w^i_{t,j}\right) + \varepsilon^i_{t,j}, i = 1, 2. \qquad (4.1)$$

where $q^i_{t-k,j}$ is a vector of household characteristics and the fixed time effects for household j in the i-th group. For example, it includes a broad range of variables related to the life cycle, background, and financial situation of the household. The vector $\Delta_k h^i_{t,j}$ contains variables that capture major changes in household characteristic or asset ownership for the i-th group. For example, it includes: changes in family size, changes in the number of children, and sets of dummies for house ownership, business ownership, and nonzero labor income at t and $t - k$. The inclusion of these additional variables serves the purpose of controlling for some important econometric issues, such as life-cycle effects and preference shifters, and idiosyncratic versus aggregate wealth changes.

We set $y_t = \log(Y_t)$. We use labor income in our empirical analysis of both liquid risky shares and financial risky shares. Note that, labor income may not correspond to income in our portfolio choice model if wealth means liquid wealth. Since we do not have the data for the right choice of income in line with liquid wealth, our results associated with liquid risky shares should be interpreted with caution.

Finally, comparing to Eq. (10) in Brunnermeier and Nagel (2008), we introduce the term $y_t \times \Delta(w_t)$ in Eq. (4.1) in order to get the estimate of ρ. Since the additional term is the only difference between Eq. (4.1) and Eq. (10) in Brunnermeier and Nagel (2008), all econometric issues except the instruments that have been addressed in Brunnermeier and Nagel (2008) are handled in the same way.

4.1.2 Regression Results

We focus on reporting results on the response of risky shares to wealth fluctuations.[9] The main results about the weak form of TVRRAI are presented in Tables 2 and 3. In Table 2, we report the first-stage TSLS estimates. In Table 3, we report two OLS estimates, OLS1 and OLS2, and the second stage TSLS estimate for both liquid risky shares and financial risky shares. The difference between OLS1 and OLS2 is that OLS2 includes "Asset composition controls" in the control variables. In particular, for the liquid asset share, asset composition controls include: the labor income/liquid wealth ratio interacted with age, the business wealth/liquid wealth ratio, and the housing wealth/liquid wealth ratio. For the financial asset share, asset composition controls consist only of the labor income/financial wealth ratio interacted with age. It immediately follows that a big portion of households in our subsample suffered large negative income shocks. According to Table 2, 573 out of 1362 households in the 1984-1999 subsample, 42 percent, had the current income below 30% of their time-average income.

From Table 3, we see that with the OLS estimates, the responses of both financial and liquid risky shares to wealth fluctuations in the first group are smaller than the corresponding responses in the second group and the differences are, in general, statistically significant. For example, the difference between the responses of liquid risky shares, ρ's, across groups is 0.144 percentage points if we use the OLS1 estimate. This finding provides some evidence of TVRRA in the households' portfolio choice data. Note that the OLS2 estimate associated with liquid risky shares is not statistically different from zero. We do not interpret this insignificant result as evidence against habit formation preferences. The reason is that our practice of using labor income to denote the income from sources other than liquid wealth seems to be problematic.

[9] For results about the stock market participation, please see Brunnermeier and Nagel (2008).

Strong evidence comes from the two OLS estimates associated with financial risky shares.[10] Both estimates associated with the bottom group are statistically significant. They are economically significant as well: one is -0.967 percentage points and the other is -0.681 percentage points. According to our model, if we use financial risky shares, y_t should denote the income coming from sources other than financial assets. It seems reasonably to argue that labor income is the right income to use. Since the relation between financial risky shares and labor income in our empirical results is in line with the theoretical model, we believe that two OLS estimates, both economically significant and statistically significant, provide some evidence of TVRRA in the PSID data.

Nevertheless, it is well known that PSID data, micro-data from surveys, about wealth and risky shares contain measurement errors and OLS estimates are thus inconsistent. To address such an issue, we do two-stage least squares regressions. The identification requirement is that the instruments, IVs, are (partially) correlated with $\Delta_k w_t$, but not the error terms. Brunnermeier and Nagel (2008) choose quantile dummies for income growth from $t-k$ to t (similar to Dynan (2000), in a different application), and inheritance receipts (as in Meer et al. (2003)) between $t-k$ and t as instruments. One reason is, as they argue, that these instruments are based upon survey questions that are different from those for the components of w_t. Hence, it is reasonable to assume that the elements of IV are uncorrelated with measurement errors. Even though the choice of those instruments does not reject that the instruments are uncorrelated with the regression residual, they are, in general, weak instruments when we run our regressions. For example, the Cragg-Donald Wald F statistics associated with the liquid risky shares and the financial asset shares in our TSLS regressions using the the first group data are 2.863 and 5.825, respectively. In both cases, we cannot reject the hypothesis of weak instruments at 25%. In the TSLS regressions using the the second group data, the Cragg-Donald Wald F statistics associated with the liquid risky

[10]The OLS estimates associated with the top group are not significantly different from zero. Thus, we focus on the estimates associated with the bottom group in which households suffered large negative income shocks.

shares and the financial risky shares are 10.871 and 6.439, respectively. We can only reject the null hypothesis of weak instruments at 20% in the liquid risky shares case but not the financial risky shares case. Unfortunately, the latter case is clearly the relevant case because the relation between financial risky shares and labor income is in line with the theoretical model as we have discussed.

To deal with the weak instrument issue, we choose different instruments. In particular, our instrument, IV, is the difference between the growth rate of the household's labor income and the growth rate of the household's liquid assets. For the formal definition of the instrument, please see the note under Table 2. The results in the table show that the instrument has a significant partial correlation with changes in log liquid wealth and changes in log financial wealth. The partial R^2 of the instrument is close to 1, which suggests that the instrument explains a large fraction of variation in wealth changes. The instrument is highly significant, with p-values smaller than 0.001 for each of the specifications. To see this, note that F statistics are larger than the rule of thumb of ten suggested by Staiger and Stock (1997).

The second stage TSLS regression results are reported in Table 3. The difference between the response of liquid risky shares to wealth fluctuations in the first group is 1.369 percentage points smaller than the corresponding response in the second group. The difference associated with financial risky shares is even larger, 6.341 percentage points. Both differences are statistically significant and economically significant. Note that we have only one endogenous regressor and one instrument in any of our regression specifications. Our specifications are exactly identified and the p-values associated with the over-identification tests are always almost zero. In addition, we observe F statistics above Stock and Yogo weak identification critical values, rejecting the hypothesis that the IV is weak. In particular, the Cragg-Donald Wald F statistics associated with the first group data are substantially larger than the 10% Stock-Yogo weak ID test critical value; and those associated with the second group data are substantially larger than the 15% Stock-Yogo weak ID test critical value. Given that our instrument is strong and the differences are both statistically significant and economically significant, we argue that, overall, the TSLS results do provide evidence of TVRRA since they are in line with the theoretical predictions about relative risk aversion of a portfolio choice model with habits.

As addressed in Brunnermeier and Nagel (2008), it is to be expected that the TSLS estimator will lose precision compared with the OLS estimator. It is not clear that the TSLS estimator will be closer to the true parameter in a mean-squared error sense. In realizing this, we argue that if both OLS estimates and the TSLS estimate show the difference as predicted by the portfolio choice model with habits, it is then reasonable to argue that there is evidence of TVRRA in the PSID data. Our results associated with the financial risky shares could thus be viewed as evidence of TVRRA. Still, note that the use of labor income when wealth refers to liquid wealth may be an issue as we have discussed before.

In our sensitivity analysis, we change the 30% threshold value from 20% to 50% and we obtain the similar evidence of TVRRA (Fig. 1). In each panel, the horizontal axis represents the value we set for the threshold ratio that is used to divide the subsample into two groups. In our empirical exercise, all ρ^2's are not significantly different from zero. Thus, we set them to be zero and the vertical axis in Fig. 1 represents $-\rho^1$. In the figure, OLS1 denotes the the value of ρ^1 associated with our first OLS estimates in our tables; OLS2 denotes the the value of ρ^1 associated with our second OLS estimates in our tables; and TSLS denotes the the value of ρ^1associated with our TSLS estimates in our tables. The results in panels (a)-(c) hold at the 10% significant confidence interval and the results in panels (b)-(d) hold at the 5% significant confidence interval.

In summary, our hypothesis essentially implies that controlling for the response through the income channel for each household and the heterogeneous responses through the habit channel across households will help generate a positive increase of ρ from the $i = 1$ group to the $i = 2$ group. Since our empirical results confirm such a hypothesis, we argue, in terms of the testable theoretical predictions, that the weak form of habit formation preferences is supported by the PSID data. Nevertheless, it is worth mentioning that our conclusion builds on a strong assumption that habits across households change roughly in the same fashion over time.

4.2 Semi-strong Form of TVRRAI

To test the prediction, Inequality (2.10), we estimate the following equation for both subsamples

$$\Delta_k \alpha_{t,j} \;=\; \beta q_{t-k,j} + \gamma \Delta_k h_{t,j} + \rho \Delta_k w_{t,j} - \vartheta y_{t,j} \Delta_k w_{t,j} + \varepsilon_{t,j}, \tag{4.2}$$

We deal with the instruments in the same way as we have done in the discussion of the weak form of TVRRAI. We use labor income in our empirical analysis of both liquid risky shares and financial risky shares. Since we do not have to calculate the averages in testing the semi-strong form, we can use both the 1984-1999 subsample and the 1999-2003 subsample.

The main results are in Table 4. In general, we find no response of risky shares to wealth fluctuations. For example, the liquid risky share decreases with liquid wealth in the 1984-1999 subsample and has no response in the 1999-2003 subsample. The financial risky share presents no response to financial wealth in both subsamples, except the TSLS estimate associated with the 1984-1999 subsample. In contrast, Brunnermeier and Nagel (2008) find generally negative response of the financial risky share to the wealth fluctuations. To facilitate comparison, we present the test results about the strong form of TVRRAI in Table 5 in which we replicate those in Tables 4 and 5 in Brunnermeier and Nagel (2008). In particular, we estimate the following equation for both subsamples

$$\Delta_k \alpha_{t,j} \;=\; \beta q_{t-k,j} + \gamma \Delta_k h_{t,j} + \rho \Delta_k w_{t,j} + \varepsilon_{t,j}, \tag{4.3}$$

Note that our TSLS estimates are quite different from their TSLS estimates. The main reason is that we use a different instrumental variable from theirs. We have discussed this point in detail above.

This comparison shows that controlling for the response through the income channel raises the estimate of ρ, confirming the implication of our theoretical model with constant labor income that omitting the impact of labor income channel biases downward the estimate of ρ.

5 Conclusion

In this paper, we introduce time-varying labor income, an empirically important element, into a portfolio choice model with external habits. The key theoretical contribution of our paper is that our analytical solution adds the following new insights to the literature: (1) risky shares respond to wealth fluctuations through two channels, habit and labor income; (2) depending on whether they experience large negative income shocks or not, households response differently through the habit channel; and (3) an internal mis-identification problem arises if the two channels are considered as one channel, while an external mis-identification problem arises if the heterogeneous responses across households are ignored.

Accordingly, we test the semi-strong form and the weak form of TVRRAI. Our empirical contribution is that we find evidence of the weak form of TVRRAI. Our positive evidence of the weak form of TVRRAI is clear evidence of TVRRA in the household level data. Our refined results provide some confidence with respect to the use of habit formation preferences in macro models. Even though our results reject the semi-strong form of TVRRAI, in line with the rejection of the strong form of TVRRAI in the literature, our acceptance of the weak form shows the importance of controlling for the internal and external mis-identification problems. In addition, our analysis shows some potential of bridging the gap between the success of macro models with habits and the previous negative evidence in micro data by using more realistic theoretical models to identify the estimation.

Questions still remain. First, the effect of inertia on portfolio adjustments remains unchanged from those in Brunnermeier and Nagel (2008), which casts reasonable doubt on the soundness of TVRRA. Thus, the strong asset allocation inertia identified in Brunnermeier and Nagel (2008) remains an interesting and not well-understood phenomenon. Second, new data have been issued. It is of interest to check the robustness with additional data and this is on our future research agenda. Last, the relation between risky shares and wealth is indeed highly nonlinear (see Eq. (2.4)). Our estimates from linear regressions may still be biased. In a separate project, we develop non-linear estimates. By overcoming the biased associated with the linear estimates when the underlying relation is highly nonlinear, we further explore what additional insights we can obtain about the time-varying relative risk aversion.

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Table 1: Summary Statistics

Variables	Mean	Tenth percentile	Median	Ninetieth percentile	N
All households, 1984-1999 (k=5)					
Liquid wealth	158,213	1,063	53,774	351,004	3,262
Financial wealth	454,783	37,860	206,608	871,875	3,262
Income	93,216	25,051	73,417	160,810	3,262
All households, 1999-2003 (k=2)					
Liquid wealth	203,697	16,981	63,843	402,042	3,005
Financial wealth	503,547	39,477	224,952	914,796	3,005
Income	100,238	27,215	77,011	174,170	3,003

Notes: Our summary statistics are slightly different from those in Brunnermeier and Nagel (2008). The reason for the discrepancy is that we corrected several typos in their program.

Table 2: First Stage Regressions: Weak Form of TVRRAI

	$k = 5$ (1984–1999)			
	Δ_k log liquid wealth$_t$		Δ_k log financial wealth$_t$	
	Bottom 30%	Top 70%	Bottom 30%	Top 70%
Instrumental Variable				
IV	-.031**	.013**	-.013**	-.009**
	(.005)	(.003)	(.002)	(.002)
Explanatory variables:				
Preference shifters	√	√	√	√
Life-cycle controls	√	√	√	√
Year-region FE	√	√	√	√
F statistics of instrument	36	15	27	13
[p-value]	[.00]	[.00]	[.00]	[.00]
N	523	766	573	786

Notes: Define $hdlabinc5$ and $lhdlabinc5$ as the labor income in the current period and in the past period, fw and lfw as the liquid wealth in the current period and in the past period, and $svodbt$ and $lsvodbt$ as the dollar value of other debts in the current period and in the past period. The other debts are defined in the same way as in Brunnermeier and Nagel (2008), which comprise nonmortgage debt such as credit card debt and consumer loans. As a result, $(fw + svodbt)$ and $(lfw + lsvodbt)$ denote liquid assets in the current period and in the past period, respectively. The instrumental variable, IV, is given by

$$IV = \log(labfw/llabfw),$$

where $labfw = hdlabinc5/(fw + svodbt)$ and $llabfw = lhdlabinc5/(lfw + lsvodbt)$. Heteroskedasticity- and autocorrelation-robust standard errors are used to judge the significance of estimates. ** denotes the estimate is statistically significantly different from 0 at the 5% significance level and * denotes that the estimate is statistically different from 0 at the 10% significance level.

Table 3: Changes in Risky Shares: Weak Form of TVRRAI

	$k = 5$ (1984–1999)					
	Bottom 30%			Top 70%		
	OLS1	OLS2	TSLS	OLS1	OLS2	TSLS
Dependent variable:						
Proportion of liquid wealth invested in stocks and mutual funds						
Explanatory variables[a]:						
Δ_k log liquid wealth$_t$	-.304*	-.268	-.786	-.160	-.205	.714
	(.176)	(.202)	(.610)	(.163)	(.173)	(1.357)
Asset composition controls		√			√	
Preference shifters	√	√	√	√	√	√
Life-cycle controls	√	√	√	√	√	√
Year-region FE	√	√	√	√	√	√
Adj. R^2	.11	.11	–	.09	.10	–
Overidentification test	–	–	[.00]	–	–	[.00]
N	496	496	565	688	688	766
Dependent variable:						
Proportion of financial wealth invested in stocks, mutual funds, equity in a private business, and home equity						
Explanatory variables[a]:						
Δ_k log financial wealth$_t$	-.967**	-.681*	-4.649**	-.244	-.374	1.692
	(.440)	(.370)	(1.606)	(.472)	(.601)	(3.882)
Asset composition controls		√			√	
Preference shifters	√	√	√	√	√	√
Life-cycle controls	√	√	√	√	√	√
Year-region FE	√	√	√	√	√	√
Adj. R^2	.15	.17	–	.20	.21	–
Overidentification test	–	–	[.00]	–	–	[.00]
N	502	502	573	704	704	786

Notes: Heteroskedasticity- and autocorrelation-robust standard errors are used to judge the significance of estimates. ** denotes the estimate is statistically significantly different from 0 at the 5% significance level and * denotes that the estimate is statistically different from 0 at the 10% significance level. The difference between OLS1 and the OLS2 is that OLS2 includes "Asset composition controls" in the control variables. In particular, asset composition controls for the liquid asset share include: the labor income/liquid wealth ratio interacted with age, the business wealth/liquid wealth ratio, and the housing wealth/liquid wealth ratio. For the financial asset share, asset composition controls consist only of the labor income/financial wealth ratio interacted with age. The benchmark regression equation is given by

$$\Delta_k \alpha^i_{t,j} \quad = \quad \beta^i q^i_{t-k,j} + \gamma^i \Delta_k h^i_{t,j} + \rho^i \Delta_k w^i_{t,j} - \vartheta^i y^i_{t,j} \Delta_k \left(w^i_{t,j}\right) + \varepsilon^i_{t,j}, i = 1, 2.$$

Table 4: Changes in Risky Shares: Semi-Strong Form of TVRRAI

	k = 5 (1984–1999)			k = 2(1999–2003)		
	OLS1	OLS2	TSLS	OLS1	OLS2	TSLS
Dependent variable:						
Proportion of liquid wealth invested in stocks and mutual funds						
Explanatory variables[a]:						
Δ_k log liquid wealth$_t$	-.223**	-.216*	–	.043	.007	–
	(.114)	(.121)	–	(.108)	(.109)	–
Asset composition controls		√			√	
Preference shifters	√	√	–	√	√	–
Life-cycle controls	√	√	–	√	√	–
Year-region FE	√	√	–	√	√	–
Adj. R^2	.05	.05	–	.05	.05	–
Overidentification test	–	–	–	–	–	–
N	1,184	1,184	–	1,348	1,348	–
Dependent variable:						
Proportion of financial wealth invested in stocks, mutual funds, equity in a private business, and home equity						
Explanatory variables[a]:						
Δ_k log financial wealth$_t$	-.514	-.465	-2.073	-.390	-.393	-1.685
	(.347)	(.406)	(1.748)	(.248)	(.245)	(2.199)
Asset composition controls		√			√	
Preference shifters	√	√	√	√	√	√
Life-cycle controls	√	√	√	√	√	√
Year-region FE	√	√	√	√	√	√
Adj. R^2	.13	.13	–	.10	.10	–
Overidentification test	–	–	[.00]	–	–	[.00]
N	1,206	1,206	1,359	1,379	1,379	1,561

Notes: Heteroskedasticity- and autocorrelation-robust standard errors are reported in parentheses, and p-values in brackets. ** denotes the estimate is statistically significantly different from 0 at the 5% significance level and * denotes that the estimate is statistically different from 0 at the 10% significance level. The difference between the OLS1 and the OLS2 is that OLS2 includes "Asset composition controls" in the control variables. In particular, asset composition controls for the liquid asset share include: the labor income/liquid wealth ratio interacted with age, the business wealth/liquid wealth ratio, and the housing wealth/liquid wealth ratio. For the financial asset share, asset composition controls consist only of the labor income/financial wealth ratio interacted with age. The regression equation is given by

$$\Delta_k \alpha_{t,j} \quad = \quad \beta q_{t-k,j} + \gamma \Delta_k h_{t,j} + \rho \Delta_k w_{t,j} - \vartheta y_{t,j} \Delta_k w_{t,j} + \varepsilon_{t,j}.$$

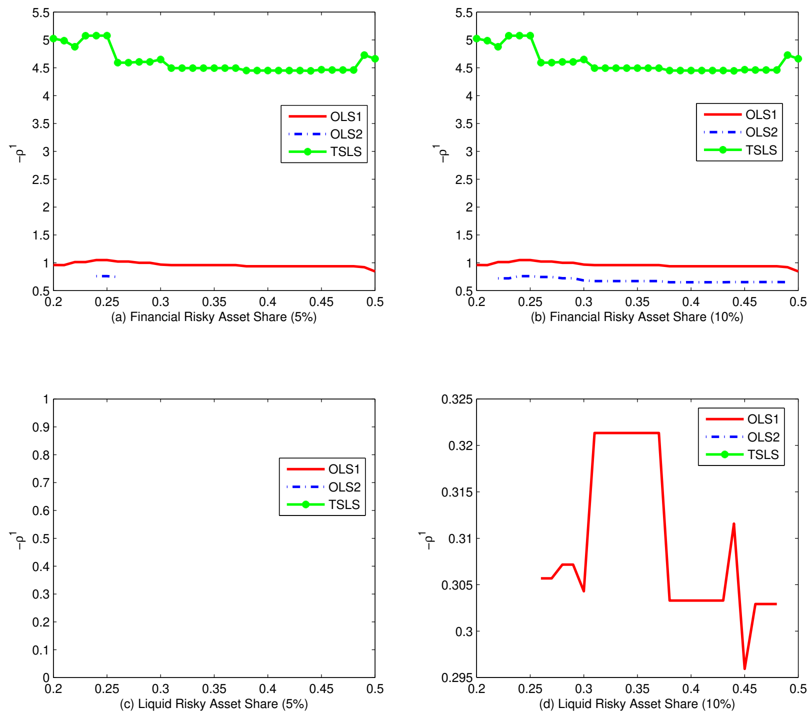
Table 5: Changes in the risky shares: Strong Form of TVRRAI: The Brunnermeier and Nagel (2008) Results

	k = 5 (1984–1999)			k = 2(1999–2003)		
	OLS1	OLS2	TSLS	OLS1	OLS2	TSLS
Dependent variable:						
Proportion of liquid wealth invested in stocks and mutual funds						
Explanatory variables:						
Δ_k log liquid wealth$_t$	-.014**	-.009	-.018	.023**	.017	.039**
	(.006)	(.009)	(.013)	(.011)	(.015)	(.014)
Asset composition controls		$\surd$			$\surd$	
Preference shifters	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Life-cycle controls	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Year-region FE	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Adj. R^2	.05	.05	–	.04	.05	–
Overidentification test	–	–	[.00]	–	–	[.00]
N	1,236	1,236	1,397	1,454	1,454	1,648
Dependent variable:						
Proportion of financial wealth invested in stocks, mutual funds, equity in a private business, and home equity						
Explanatory variables:						
Δ_k log financial wealth$_t$	-.161**	-.172*	-.164**	-.108**	-.103**	-.159**
	(.059)	(.091)	(.025)	(.031)	(.036)	(.028)
Asset composition controls		$\surd$			$\surd$	
Preference shifters	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Life-cycle controls	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Year-region FE	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Adj. R^2	.16	.16	–	.09	.09	–
Overidentification test	–	–	[.00]	–	–	[.00]
N	1,260	1,260	1,427	1,487	1,487	1,687

Notes: Table 5 replicates Tables 4 and 5 in Brunnermeier and Nagel (2008). Heteroskedasticity- and autocorrelation-robust standard errors are reported in parentheses, and p-values in brackets. ** denotes the estimate is statistically significantly different from 0 at the 5% significance level and * denotes that the estimate is statistically different from 0 at the 10% significance level. The difference between the OLS1 and the OLS2 is that OLS2 includes "Asset composition controls" in the control variables. In particular, asset composition controls for the liquid asset share include: the labor income/liquid wealth ratio interacted with age, the business wealth/liquid wealth ratio, and the housing wealth/liquid wealth ratio. For the financial asset share, asset composition controls consist only of the labor income/financial wealth ratio interacted with age. The regression equation is given by

$$\Delta_k \alpha_{t,j} \quad = \quad \beta q_{t-k,j} + \gamma \Delta_k h_{t,j} + \rho \Delta_k w_{t,j} + \varepsilon_{t,j}.$$

Figure 1: Sensitivity Analysis Results



Notes: The horizontal axis represents the value we set for the threshold ratio that is used to divide the sample into two groups. The vertical axis represents the difference between ρ^2 and ρ^1. In particular, if ρ^i is not statistically different from zero, we set it at zero. OLS1 denotes the differences associated with our first OLS estimates in our tables; OLS2 denotes the differences associated with our second OLS estimates in our tables; and TSLS denotes the differences associated with our TSLS estimates in our tables. The results in panels (a)-(b) hold at the 10% significant confidence interval and the results in panels (c)-(d) hold at the 5% significant confidence interval. Panels (a)-(b) present the results associated with financial risky shares. Panels (c)-(d) present the results associated with liquid risky shares.

6 Appendix

6.1 The Derivation of Eq. (2.6)

Define $\tilde{W}_t = W_t - C_t$ and $w_t = \log(\tilde{W}_t)$, we can rewrite Eq. (2.5) as

$$\alpha_t = 1 - \frac{X - Y}{(W_t - C_t + Y)\, R_f} = 1 - f(w_t).$$

where $f(w_t) = \frac{X-Y}{(e^{w_t}+Y)R_f}$. We approximate $f(w_t)$ around a point, for example, the mean of wealth w, up to the first order and obtain

$$\begin{aligned} f(w_t) &\approx f(w) + \left[\frac{df(w_t)}{dw_t}|_{w_t=w}\right](w_t - w) \\ &= f(w) + \frac{(X-Y)e^w}{(e^w+Y)^2 R_f} w - \frac{(X-Y)e^w}{(e^w+Y)^2 R_f} w_t \\ &= f(w) + \Theta w - \Theta w_t, \\ \Rightarrow \alpha_t &\approx 1 - [f(w) + \Theta w - \Theta w_t] \\ &= [1 - f(w) - \Theta w] + \Theta w_t, \end{aligned}$$

where $\Theta = \frac{(X-Y)e^w}{(e^w+Y)^2 R_f}$. Note the values of $f(w)$, Θ, and w do not depend on time.

Take the first-order difference (over time) and we get

$$\begin{aligned} \Delta\alpha_t &\approx \Theta\Delta w_t = \frac{Xe^w}{(e^w+Y)^2 R_f}\Delta w_t - \frac{Ye^w}{(e^w+Y)^2 R_f}\Delta w_t \\ &= \rho\Delta w_t - \theta Y \Delta w_t, \end{aligned}$$

where $\rho = \frac{Xe^w}{(e^w+Y)^2 R_f}$ and $\theta = \frac{e^w}{(e^w+Y)^2 R_f}$. This is exactly Eq. (2.6).