Delegation to potentially uninformed agent

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Abstract

We consider a delegation problem with a potentially uninformed agent when the principal cannot use monetary payments. If the bias between the principal and the agent is large, then the optimal delegation set is an interval. When the bias is small or medium however, the optimal delegation set is no longer connected. It can be one of two types: with an interval and low option, the other with two intervals. In all cases the agent has less discretion. However, in the case of medium biases the principal delegates in a wider range than in the case of informed agent. In all cases the agent will be given more freedom if he is more informed.

Introduction

Consider a situation in which a principal delegates decisions to an agent but limits the agent's discretion. The rationale for delegation is that the agent is better informed than the principal, though in reality the agent may not be competent. For example, in the decision making process within a firm, a CEO delegates investment decisions within certain limits to a manager who may or may not have the appropriate information. Such a situation is likely to occur with a junior manager, when it is prohibitively costly to collect information, or when the agent observes the realization of the pilot project to acquire information. The principal does not know the outcome of the pilot. When designing the delegation limits, the principal takes into account that the agent may be uninformed.

This paper studies a delegation problem when the agent with some positive probability does not have private information and the principal cannot use monetary payments. Instead the principal selects a set of actions from which the agent is required to choose - a delegation set. The principal faces a trade-off between her desire to use the agent's information and exerting control over decisions. We show that if the bias between the principal and the agent is sufficiently large, the optimal delegation set is an interval. When the bias is small or medium, the optimal delegation set is no longer connected. For small biases it consists of two intervals. For medium biases it consists of an interval and low option. The principal wants to exert more control over the uninformed agent compared to the informed agent. Therefore, in all cases there are fewer choices available for the agent. However, in the case of medium biases the range of the delegation set is larger than in the

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case of the informed agent. The principal distorts the delegation set downwards in order to benefit, at least partially, from the information in the low-end of the distribution. In all cases the more informed the agent, the more freedom given.

The constrained delegation framework has recently become popular to analyze the variety of economic situations: the limits placed on Central Bank's monetary or exchange rate policy, price limits in regulation, the House regulations on policies that a delegated committee may choose, tariff levels etc. This literature was pioneered by Holmström (1977) and (1984) who proves under general conditions that there exist an optimal delegation set. Holmström (1984) and Armstrong (1994) assume that the optimal delegation set is an interval. Following this tradition most of the literature focused on interval delegation with informed agent. Melumad and Shibano (1991), Martimort and Semenov (2006), Alonso and Matouschek (2008), Kovac and Mylovanov (2009) and Amador and Bagwell (2011) present different sufficient conditions under which the optimal delegation set is an interval. Frankel (2012) shows that in the case of multiple decisions, half-space analog of an interval delegation set is optimal in the case of a normal distribution of types and quadratic payoffs. Non-interval delegation was considered in Melumad and Shibano (1991) who presented condition on payoff functions of the principal and the agent in order to have non-interval optimal delegation set. The ideal decisions of the principal should be higher than the ideal decisions for the agent for some states and lower for others. Martimort and Semenov (2006) and Kovac and Mylovanov (2009) pointed out conditions on distributions of types when the optimal delegation set can be non-interval. This paper shows that with conventional payoffs and distributions, non-connectedness of the optimal delegation set may arise if the agent may be uninformed.

In the literature on signaling games Austen-Smith (1994) established that the possibility of an uninformed sender makes information transmission possible for a wider range of conflicts between the receiver and the sender compare to the cases when the sender is informed. The reason for this arising is different from ours; the low - type sender pools with the uninformed sender. This leads to a more favorable action for the low-type sender which makes it is easier for high - type sender to separate himself than in the case when the receiver is sure that the sender is informed. Lewis and Sappington (1993) consider the optimal contract with the possibility of ignorance when the principal may use transfers to elicit information. The payoff of the principal does not depend on the information parameter (private values). In this framework there is always a discontinuity in the optimal output. The optimal contract exhibits pooling and when costs are high the output is lower than in the standard second best.

The Model

A principal (she) delegates the making of a decision $d \in \mathbb{R}$ to an agent (he). The payoffs of the principal and the agent are $V_P(d,\theta)$ and $V_A(d,\theta)$ correspondingly, where the state of the world $\theta \in \Theta = [\underline{\theta}, \overline{\theta}]$ is drawn from the distribution with non-atomic, continuous density function $f(\theta)$ and cumulative density function $F(\theta)$. The principal does not observe θ. The game is as follows:

1. The principal chooses a compact delegation set $D \subset \mathbb{R}$.
2. With probability $1-p$ the agent learns the state θ, with probability p the agent remains uninformed. The probability p is a common knowledge, but the principal is unaware if the agent is informed.

3. The agent chooses $d \in D$.

A delegation set D is optimal if it maximizes the principal's expected payoff among all compact sets. Denote by $d_0 \in D$ the choice of the non-informed agent and by D_{inf} the set of choices for the informed agent, $D = \{D_{\text{inf}}, d_0\}$. If the agent learns that the state is θ then his payoff is $V_A(d(\theta), \theta)$, where

$$d(\theta) = \arg\max_{d \in D_{\text{inf}}} V_A(d, \theta)$$

and the expected payoff of the uninformed agent is $\int_\Theta V_A(d_0, \theta)\, dF(\theta)$. The principal's expected payoff is denoted by $V(D)$:

$$V(D) = p \int_\Theta V_P(d_0, \theta)\, dF(\theta) + (1-p) \int_\Theta V_P(d(\theta), \theta)\, dF(\theta).$$

We denote the first (uninformed) part of the principal's payoff as $V_0(D)$ and the second (informed) part as $V_{\text{inf}}(D)$ so that $V(D) = V_0(D) + V_{\text{inf}}(D)$. The choices of the agent maximize his payoff in the delegation set. If the agent is informed that the state is $\theta \in \Theta$ then

$$V_A(d(\theta), \theta) \geq V_A(d(\theta'), \theta) \text{ for all } \theta, \theta' \in \Theta. \tag{1}$$

When the agent does not observe the state, he prefers d_0 to any other outcome $d(\theta') \in D_{\text{inf}}$:

$$\int_\Theta V_A(d_0, \theta)\, dF(\theta) \geq \int_\Theta V_A(d(\theta'), \theta)\, dF(\theta) \text{ for all } \theta, \theta' \in \Theta. \tag{2}$$

Finally, if the agent observes the state θ, he prefers $d(\theta)$ to d_0 if

$$V_A(d(\theta), \theta) \geq V_A(d_0, \theta). \tag{3}$$

We focus on quadratic specifications of payoffs,

$$V_A(d, \theta) = -\frac{1}{2}(d-\theta)^2 \text{ and } V_P(d, \theta) = -\frac{1}{2}(d-\theta-b)^2,$$

where the parameter $b \geq 0$ is the bias between the principal and the agent. The bias b is a common knowledge. The delegation sets have the form $D = \left\{ \left(\bigcup_{i=1}^{n} D_i \right), d_0 \right\}$, where $D_i = \left[\underline{d}_i, \overline{d}_i\right], i = 1, ..., n$ are closed intervals, $\underline{d}_i \leq \overline{d}_i < \underline{d}_{i+1} \leq \overline{d}_{i+1}$ for all $i = 1, ..., n-1$. The choice of the informed agent θ is the following: if $\theta \in D_i = \left[\underline{d}_i, \overline{d}_i\right]$ then $d(\theta) = \theta$ and $V_A(d(\theta), \theta) = 0$. If $\theta \in \left(\overline{d}_i, \frac{\overline{d}_i + \underline{d}_{i+1}}{2}\right)$ then $d(\theta) = \overline{d}_i$ and $V_A(d(\theta), \theta) = -\frac{1}{2}\left(\overline{d}_i - \theta\right)^2$. If $\theta \in \left(\frac{\overline{d}_i + \underline{d}_{i+1}}{2}, \underline{d}_{i+1}\right)$ then $d(\theta) = \underline{d}_{i+1}$ and $V_A(d(\theta), \theta) = -\frac{1}{2}\left(\underline{d}_{i+1} - \theta\right)^2$. The fixed decision $E(\theta) + b$ maximizes the payoff of uninformed agent. The expected payoff of the principal is

$$V(D) = V_0(D) + V_{\text{inf}}(D) = -\frac{p}{2} \int_{\underline{\theta}}^{\overline{\theta}} (d_0 - \theta - b)^2\, dF(\theta) - \frac{1-p}{2} \left\{ \int_{\underline{\theta}}^{\underline{d}_1} (\underline{d}_1 - \theta - b)^2\, dF(\theta) + \right.$$

$$+ \sum_{i=1}^{n} \int_{\underline{d}_i}^{\overline{d}_i} b^2 dF(\theta) + \sum_{i=1}^{n} \int_{\overline{d}_i}^{\frac{\overline{d}_i + \underline{d}_{i+1}}{2}} (\underline{d}_i - \theta - b)^2\, dF(\theta) +$$

$$+\sum_{i=1}^{n}\int_{\frac{\overline{d}_i+\underline{d}_{i+1}}{2}}^{\underline{d}_{i+1}}\left(\underline{d}_{i+1}-\theta-b\right)^2 dF(\theta)+\int_{d_n}^{\overline{\theta}}\left(\overline{d}_n-\theta-b\right)^2 dF(\theta)\Bigg\}.$$

The distribution of states satisfy the following

Assumption 1 *The function $F(\theta)$ is log-concave and for all $\theta \in \Theta$ the function $f(\theta) - bf'(\theta)$ is positive for all $\theta \in \Theta$ and weakly decreasing in θ.*

Martimort and Semenov (2006) show that if the agent is always informed, $F(\theta)$ is log-concave and $f(\theta) - bf'(\theta) \geq 0$ for all θ then the optimal delegation set is an interval. Examples of distributions satisfying this Assumption are uniform distributions and exponential distributions.

Remark 1 *The delegation problem has the equivalent mechanism design formulation. Denote by $\widetilde{\Theta}$ the expanded state space $\Theta \cup \{\{\varnothing\}\}$, where $\{\varnothing\}$ corresponds to the state when the agent is uninformed. For each state $\theta \in \widetilde{\Theta}$ the principal chooses a decision $q(\theta) \in D$, where $q(\theta)$ is a measurable function from $\widetilde{\Theta}$ to $\mathbb{R}$. The corresponding delegation set is $D = \{q(\theta)\}_{\theta\in\Theta'} = \{q(\theta), q(\{\varnothing\})\}_{\theta\in\Theta}$.*

Remark 2 *We can consider more general utility for the agent: $V_A = v_A(d-\theta,\theta)$, where v_A is a single-peaked function given θ. With appropriate changes, the results also can be extended to generalized quadratic payoff for the principal $V_P = -r(\theta)(d-y_P(\theta))^2$, where $y_P(\theta)$ is the ideal policy for the principal (see also Alonso and Matouschek, 2008).*

Results

The delegation is valuable if the principal benefits from delegating decision - making to the agent instead of choosing the decision by herself. Our first result establishes the possible types of the optimal delegation sets when the delegation is valuable.

Proposition 1 *If $p < 1$ and $b < \overline{\theta} - E(\theta)$ then the delegation is valuable and the optimal delegation set is one of the following types:*

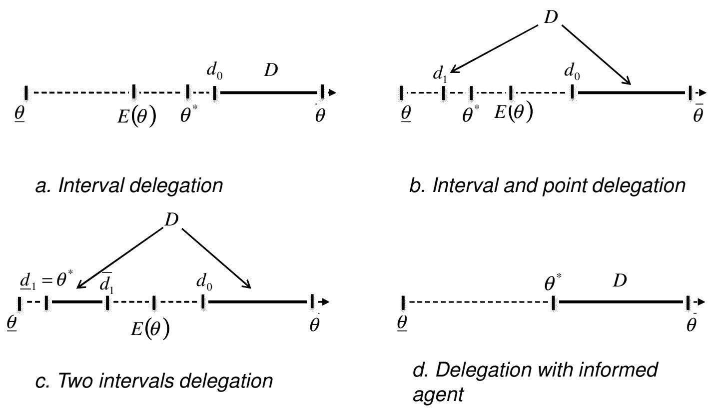
1. Interval delegation: $D = \left[d_0, \overline{\theta}\right]$, where $d_0 \leq E(\theta) + b$;

2. Interval and point delegation: $D = \{d_1\} \cup \left[d_0, \overline{\theta}\right]$, where $E(\theta) \leq d_0 < E(\theta) + b$ and $d_1 + d_0 = 2E(\theta)$;

3. Two - intervals delegation: $D = \left[\underline{d}_1, \overline{d}_1\right] \cup \left[d_0, \overline{\theta}\right]$, where $\overline{d}_1 \leq E(\theta) \leq d_0 < E(\theta) + b$ and $\overline{d}_1 + d_0 = 2E(\theta)$.

The optimal delegation sets are presented on Figure 1 a-c. The interesting feature of delegation sets of type 2 and 3 is that they are not connected. In case 2 d_1 and d_0 are equidistant from $E(\theta)$. In case 3 $\overline{d}_1$ and d_0 are equidistant from $E(\theta)$. The principal want to limit the choice of uninformed agent. This agent has the ideal policy $E(\theta)$. This policy is too low for the principal who wants in this case the execution of the policy $E(\theta) + b$. Hence, she introduces a gap in the delegation set.

Figure 1. Optimal delegation sets



Proof: Delegation is valuable if the payoff of uninformed principal $-\frac{1}{2}\int_{\underline{\theta}}^{\overline{\theta}} (E(\theta)-\theta)^2 dF(\theta)$ is smaller than $max_D V(D)$. Consider an interval delegation set $\widetilde{D} = \left[E(\theta)+b, \overline{\theta}\right]$ .Then $V(\widetilde{D}) - \left(-\frac{1}{2}\int_{\underline{\theta}}^{\overline{\theta}} (E(\theta)-\theta)^2 dF(\theta)\right) = \frac{1}{2}\int_{E(\theta)+b}^{\overline{\theta}} (E(\theta)-\theta-b)(E(\theta)-\theta+b)\, dF(\theta) > 0$. Thus, delegation is valuable.

We will prove the Proposition 1 in few Lemmas. Denote by $d(\theta^-) = \lim_{\theta' \to \theta - 0} d(\theta')$ and by $d(\theta^+) = \lim_{\theta' \to \theta + 0} d(\theta')$.

Lemma 3 $d_0 \in \{d^-(E(\theta)), d^+(E(\theta))\}$.

Proof. We re-write (2) as

$$(d(\theta') - d_0)(d(\theta') + d_0 - 2E(\theta)) \geq 0$$

and (3) as

$$(d(\theta') - d_0)(d(\theta') + d_0 - 2\theta') \leq 0.$$

Summing up these inequalities we obtain $(d(\theta') - d_0)(\theta' - E(\theta)) \geq 0$. Hence, for any $\varepsilon > 0$ we have $d(E(\theta)+\varepsilon) \geq d_0 \geq d(E(\theta)-\varepsilon)$. Taking the limit we obtain $d_0 \in \{d^-(E(\theta)), d^+(E(\theta))\}$. ■

By this Lemma, if there exist k such that $E(\theta) \in D_k$ then $d_0 = d(E(\theta))$. If on the other hand for some i we have $E(\theta) \in \left(\overline{d}_i, \underline{d}_{i+1}\right)$ then d_0 is either $\overline{d}_i$ or $\underline{d}_{i+1}$.

We introduce $\varepsilon-$ transformation $D(\varepsilon)$ of the set D on the interval (D_k, D_{k+1}), where $D_k = \left[\underline{d}_k, \overline{d}_k\right]$ and $D_{k+1} = \left[\underline{d}_{k+1}, \overline{d}_{k+1}\right]$. Consider the sets $D'_k = \left[\underline{d}_k, \overline{d}_k + \varepsilon\right]$ and $D'_{k+1} = \left[\underline{d}_{k+1} - \varepsilon, \overline{d}_{k+1}\right]$ such that $\overline{d}_k + \varepsilon < \underline{d}_{k+1} - \varepsilon$. The derivative of $V_i(D(\varepsilon))$ with respect to ε evaluated at $\varepsilon = 0$ is greater than zero if

$$b(2F(\theta') - F(\theta'-\Delta) - F(\theta'+\Delta)) - \int_{\theta'-\Delta}^{\theta'} F(\theta)d\theta + \int_{\theta'}^{\theta'+\Delta} F(\theta)d\theta \geq 0. \tag{4}$$

The Assumption 1 provides a sufficient condition for (4). Indeed, consider the function

$$\pi(t) = b(2F(\theta') - F(\theta' - t) - F(\theta' + t)) - \int_{\theta'-t}^{\theta'} F(\theta)d\theta + \int_{\theta'}^{\theta'+t} F(\theta)d\theta,$$

where $t \in [0, \Delta]$. We have $\pi(0) = 0$ and $\pi'(t) = b(f(\theta' - t) + f(\theta' + t)) - F(\theta' - t) + F(\theta' + t)$. Then $\pi''(0) = 0$ and $\pi''(t) > 0$.

Lemma 4 *The optimal delegation set has at most two disjoint intervals.*

Proof. Assume first that there exist i such that $d_0 \in \left\{\overline{d}_i, \underline{d}_{k+1}\right\}$. If for $k < i$ there are disjoint sets $D_k = \left[\underline{d}_k, \overline{d}_k\right]$ and $D_{k+1} = \left[\underline{d}_{k+1}, \overline{d}_{k+1}\right]$ then we take $\varepsilon-$ transformation $D(\varepsilon)$ of D in the interval (D_k, D_{k+1}). Because $d_0 \notin D_k \cup D_{k+1}$ we have $V_0(D) = V_0(D(\varepsilon))$ and, therefore, $V(D(\varepsilon)) > V(D)$, which contradicts the optimality of D. The case $d_0 = \underline{d}_{i+1}$ is similar. If $d_0 = d(E(\theta))$ then using the same considerations we show that D is an interval. ■

In order to compare the delegations sets in Proposition 1 with the delegation set when the agent is always informed ($p = 0$) we remind the well-known result (see Martimort and Semenov, 2006).

Lemma 5 *If $p = 0$ then the optimal delegation set is an interval $D^* = \left[\theta^*, \overline{\theta}\right]$, where θ^* is implicitly defined by*

$$\theta^* = b + E(\theta \mid \theta \leq \theta^*). \tag{5}$$

Proof. By Lemma 4 the optimal delegation set is an interval, $D = \left[\theta_1, \overline{\theta}\right]$. The principal's expected profit is

$$V(D) = V_{\text{inf}}(D) = -\int_{\underline{\theta}}^{\theta_1} (\theta_1 - \theta - b)^2 dF(\theta) - \int_{\theta_1}^{\overline{\theta}} b^2 dF(\theta).$$

The first-order condition for θ_1 leads to (5). ■

The optimal delegation set with informed agent is depicted in Figure 1 d. Lemma 3 shows that $d_0 \in \{d^-(E(\theta)), d^+(E(\theta))\}$. We can now prove that $d_0 = d^+(E(\theta)$.

Lemma 6 $d_0 = d^+(E(\theta) \geq \theta^*$.

Proof. a) Let $D = \{D_{\text{inf}}, d_0\}$ is the optimal delegation set. We prove first that $d_0 \geq \theta^*$. Assume to the contrary that $d_0 < \theta^*$. Consider the optimal delegation set D^* defined in Lemma 5. The set $D' = D^* \cup \left\{\widehat{\theta}\right\}$ is feasible for the program $(\mathcal{P})$. Because of the optimality of D we have $V_0(D) + V_{\text{inf}}(D) \geq V_0(D') + V_{\text{inf}}(D')$. By Lemma 5 we have $V_{\text{inf}}(D') \geq V_{\text{inf}}(D)$. Therefore it must be that $V_0(D') \leq V_0(D)$. This leads to $(d_0 - \theta^*)(d_0 + \theta^* - 2E(\theta) - 2b) \leq 0$. Suppose that $d_0 < \theta^*$ then we have $d_0 + \theta^* \geq 2E(\theta) + 2b$ or $\theta^* > b + E(\theta)$, which contradicts (5). Therefore, $d_0 \geq \theta^*$.

b) We prove now that $d_0 = d^+(E(\theta)$. Assume to the contrary that $d_0 = d^-(E(\theta) < d^+(E(\theta)$. Consider delegation sets $D^- = \{D_{\text{inf}}, d^-(E(\theta))\}$ and $D^+ = \{D_{\text{inf}}, d^-(E(\theta))\}$. Since $V_{\text{inf}}(D^+) = V_{\text{inf}}(D^-)$ we have

$$V(D^-) - V(D^+) = V_0(D^-) - V_0(D^+) = \left(d^+(E(\theta)) - d^-(E(\theta))\right)\left(\frac{d^+(E(\theta)) + d^-(E(\theta))}{2} - E(\theta) - b\right).$$

Thus $V(D^-) > V(D^+)$ if and only if $\frac{d^+(E(\theta))+d^-(E(\theta))}{2} > E(\theta)+b$. In this case, since $d_0 = d^-(E(\theta)) > \theta^*$, we have for all $\theta \in [d^-(E(\theta)), d^+(E(\theta))]$, $\theta > \theta^*$. Then if we consider the delegation set $\widetilde{D} = \{[d^-(E(\theta)), d^+(E(\theta))] \cup D_{\text{inf}}, E(\theta)\}$ we have $V_i(\widetilde{D}) > V_i(D)$. New uninformed option $E(\theta) > d_o$ thus, $V_0(\widetilde{D}) > V_0(D)$. Contradiction. Hence, $d_0 = d^+(E(\theta))$. ■

From Lemma 4 and Lemma 6 it follows that the optimal contract belongs to one of types 1-3 of Proposition 1. Note first that for types 2 and 3 $\frac{d^+(E(\theta))+d^-(E(\theta))}{2} \leq E(\theta)$. Indeed, if $\frac{d^+(E(\theta))+d^-(E(\theta))}{2} > E(\theta)$ then $d_0 < E(\theta)$ and $d_0 \geq \theta^*$. As in Lemma 6 *b)* we obtain a strict improvement. Thus, $\frac{d^+(E(\theta))+d^-(E(\theta))}{2} \leq E(\theta)$.

Lemma 7 *If $p < 1$ then $d_0 < E(\theta) + b$.*

Proof. See Appendix. ■

Consider a set $D_{\text{inf}} = [\theta_1, \theta' - \Delta] \cup [\theta' + \Delta, \theta_2]$. We introduce $\delta-$ transformation D^δ of D_{inf}: $D^\delta = [\underline{\theta}, \theta' - \Delta + \delta] \cup [\theta' + \Delta + \delta, \overline{\theta}]$. The derivative of $V_{\text{inf}}(D^\delta)$ with respect to δ evaluated at $\delta = 0$ is positive when

$$f(\theta' + t) - bf'(\theta' + t) \leq f(\theta' - t) - bf'(\theta' - t) \text{ for all } t \in [0, \Delta].$$

This condition is satisfied by Assumption 1.

We now can prove the Proposition 1. By Lemma 3 there are two possibilities; either the optimal delegation set is an interval $D = \{[\theta_1, \theta_2], E(\theta)\}$, or it has the form $D = \{[\theta_1, \theta' - \Delta] \cup [\theta' + \Delta, \theta_2], \theta' + \Delta\}$. Note that $\theta_2 = \overline{\theta}$. If $\theta_2 < \overline{\theta}$, then the delegation sets $D = \{[\theta_1, \overline{\theta}], E(\theta)\}$ and $D = \{[\theta_1, \theta' - \Delta] \cup [\theta' + \Delta, \overline{\theta}], \theta' + \Delta\}$ strictly dominate respective delegation sets.

If the optimal delegation set is an interval then it is of the type 1. Assume that $D = ([\theta_1, \theta' - \Delta] \cup [\theta' + \Delta, \overline{\theta}], \theta' + \Delta)$ and $\frac{d^+(E(\theta))+d^-(E(\theta))}{2} \neq E(\theta)$. Because $d_0 < E(\theta)+b$ by Lemma 7 there exist $\delta > 0$ such that $d_0 + \delta \leq E(\theta)+b$. Then the $\delta-$ transformation of $[\theta_1, \theta' - \Delta] \cup [\theta' + \Delta, \overline{\theta}]$ improves both V_0 and V_{inf}. Contradiction. Therefore in both cases 2 and 3 the intervals of delegation are equidistant from $E(\theta)$. Note that when the intervals of delegation are equidistant from $E(\theta)$ we cannot improve by introducing the $\delta-$ transformation because d_0 changes from $d_0 = d^+(E(\theta)$ to $d_0 = d^-(E(\theta)$ which is further away from $E(\theta) + b$ and, therefore, the component V_0 sharply decreases. ■

The parameters of delegation sets in Proposition 1 satisfy the following inequalities:

Corollary 1 *If $p > 0$ for corresponding types in Proposition 1 we have*

1. $d_0 > \theta^*$,
2. $d_1 \leq \theta^* < d_0$,
3. $d_1 = \theta^*$.

Proof. See Appendix. ■

For the delegation set of the type 1 in Proposition 1 if $p > 0$ we have D is strictly contained in D^*. Because of the positive probability of the agent being uniformed the information is less important for the principal than when the agent is always informed and she wants to exert more control over decisions when the agent is uninformed. Thus, she moves the boundary of delegation closer to the optimal policy of the uninformed agent, $E(\theta)+b$. Thus, the principal gives less discretion to the agent. This is generally true for all types of the delegation sets. For the type 2, the delegation set consists of the set $\left[d_0, \overline{\theta}\right]$ and the option $d_1 \leq \theta^*$. The principal wants to use information below d_0, but she cannot delegate to medium types because she wants to limit the choice of uninformed agent. Thus, she has to distort the delegation set even below the optimal boundary for the informed agent case θ^*. For the delegation set of the type 3, the principal makes use of information held by upper and lower tails of distribution still exerting control for medium range.

We consider now the comparative statics with respect to the bias. The main goal is to determine which type of the delegation set is optimal for different ranges of biases. Lets fix the bias b and consider the optimal delegation set D of type i in Proposition 1. By this Proposition the optimal delegation set of type i is uniquely determined by the outcome d_0. Define $V_i\left(d_0, b\right)$ - the corresponding expected payoff of the principal and $d_0^{(i)}\left(b\right)$ as the maximizer of $V_i\left(d_0, b\right) : d_0^{(i)}\left(b\right) = \arg\max_{d_0} V_i\left(d_0, b\right)$. Define also b_1 as the solution of

$$2E(\theta) - d_0^{(2)}\left(b_1\right) = \widehat{\theta}$$

and b_2^* is defined by

$$b_2^* = \frac{1}{F\left(E(\theta)\right)} \int_{\underline{\theta}}^{E(\theta)} F\left(\theta\right) d\theta. \tag{6}$$

The bias b_2^* is such that the optimal delegation set for the informed agent is $\left[E(\theta), \overline{\theta}\right]$. We have the following:

Proposition 2 *If $p \in (0,1)$ then there exist b_1 and $b_2, b_2 > b_2^* > b_1$ such that:*

a) (Interval delegation) if $b \geq b_2$ then the optimal delegation set is of type 1;

b) (Interval and point delegation) if $b \in \left[b_1, b_2\right)$ then the optimal delegation set is of type 2;

c) (Two - intervals delegation) if $b \in \left[0, b_1\right)$ then the optimal delegation set is of type 3.

Proof (See Appendix) ■

When the bias b is large then the delegation set with uninformed agent is $D = \left[d_0, \overline{\theta}\right] \subset D^* = \left[\theta^*, \overline{\theta}\right]$ and $d_0 > \theta^*$ for $p > 0$. Of course, when the bias is sufficiently large, then the optimal delegation set is a point $\{E(\theta)+b\}$ regardless of the agent is informed or not. If the principal faces only uninformed agent she wants to implement $E(\theta)+b$. When there is a non-zero probability of the agent being informed, the principal restricts the optimal delegation set towards this optimal decision. For medium levels of bias, the same principle applies and $d_0 > \theta^*$. However, the principal gives the low types the option $d_1 = 2E(\theta) - d_0$. She cannot neglect these types as the case of large biases because the low types are now more aligned with the principal. The principal faces additional trade-off between delegating to informed types and controlling the uninformed agent. This moves the policy d_0 towards $E(\theta)+b$. Correspondingly, the option d_1 optimally is below θ^*. If $d_1 > \theta^*$ then the principal can do better by adding extra interval of delegation $\left[\theta^*, d_1\right]$. This exactly happens when the bias is small. Note that the principal delegates in the same range as for informed agent problem, but now she excludes intermediate choices. By doing this she is able to move the choice of uninformed agent towards his ideal point.

Holmström (1984) and Alonso and Matouschek (2008) have shown that the agent will be given more freedom when he is more informed. For this purpose they use the normally distributed state of the world: $\theta \sim N(\mu, \sigma^2)$ and consider the comparative statics of interval delegation when σ^2 decreases. Another way to test this hypothesis is to check if the optimal delegation sets are larger when the probability $1-p$ of the agent being informed increases.

Proposition 3 *If $D(p_k), k = 1, 2$ are the optimal delegation set when the probability not being uninformed are $p_1 < p_2$, then $D(p_1) \supsetneqq D(p_2)$.*

Indeed, then the agent is more informed, then informed component of principal's payoff is relatively more important - it has relatively more weight. As Figure 1 shows in this case the optimal delegation set is closer to the set $D^* = [\theta^*, \overline{\theta}]$.

Uniform example: Consider uniformly distributed types $\theta \sim Uni[0,1]$. In this case the optimal delegation set with informed agent is $D^* = [2b, \overline{\theta}]$ and delegation is valuable when $b < \frac{1}{2}$. All three types of delegation sets are determined by corresponding d_0. We have

$$d_0^{(1)} = b - \frac{p}{1-p} + \sqrt[2]{b^2 + \frac{p}{(1-p)^2}},$$

$$d_0^{(2)} = b - \frac{p}{1-p} + \sqrt[2]{b^2 + \frac{p}{(1-p)^2} + \frac{1}{2} - 2b} \text{ and}$$

$$d_0^{(3)} = 1 - \frac{1 - \sqrt[2]{p(4b + p - 4bp)}}{2(1-p)}.$$

Note that if $b \to 0$ then $d_0^{(3)} \to \frac{1}{2} = E(\theta)$.

Proofs

Proof of Lemma 7. If the optimal delegation set D has type 2, $D = (\{2\theta' - d_0\} \cup [d_0, \overline{\theta}], d_0)$ we must have $\widehat{\theta} \geq 2\theta' - d_0$. If $\widehat{\theta} < 2\theta' - d_0$ then consider the set $\widetilde{D} = \left([\widehat{\theta}, 2\theta' - d_0] \cup [d_0, \overline{\theta}], d_0\right)$. This set of type 3 improves V_{inf} leaving V_0 unchanged.

Consider the first-order condition with respect to d_0 for a set $D = (\{2\theta' - d_0\} \cup [d_0, \overline{\theta}], d_0)$ of type 2. The derivative $\frac{\partial V_0}{\partial d_0} = -(d_0 - E(\theta) - b)$. In order to prove that $d_0 < E(\theta) + b$ we have to show that $\frac{\partial V_{\text{inf}}}{\partial d_0} < 0$, where

$$V_{\text{inf}} = -\frac{1-p}{2}\left[\int_{\underline{\theta}}^{\theta'} (2\theta' - d_0 - \theta - b)^2 \, dF(\theta) + \int_{\theta'}^{d_0} (d_0 - \theta - b)^2 dF(\theta) + \int_{d_0}^{\overline{\theta}} b^2 dF(\theta)\right].$$

The derivative

$$\frac{\partial V_{\text{inf}}}{\partial d_0} = -\frac{1-p}{2}\left[2bF(\theta') - bF(d_0) - \int_{\underline{\theta}}^{\theta'} F(\theta)d\theta + \int_{\theta'}^{d_0} F(\theta)d\theta\right] =$$

$$-\frac{1-p}{2}\left[2bF(\theta') - bF(d_0) - \int\limits_{\underline{\theta}}^{2\theta'-d_0} F(\theta)d\theta - \int\limits_{2\theta'-d_0}^{\theta'} F(\theta)d\theta + \int\limits_{\theta'}^{d_0} F(\theta)d\theta\right]$$

Since $\widehat{\theta} \geq 2\theta' - d_0$ and the function $F(\theta)$ is log-concave we have by (4)

$$\frac{\partial V_i}{\partial d_0} \leq -\frac{1-p}{2}\left[2bF(\theta') - bF(d_0) - bF(2\theta' - d_0) - \int\limits_{2\theta'-d_0}^{\theta'} F(\theta)d\theta + \int\limits_{\theta'}^{d_0} F(\theta)d\theta\right] < 0.$$

For a contract of type 3 we have immediately

$$\frac{\partial V_3}{\partial d_0} = -\frac{1-p}{2}\left[2bF(\theta') - bF(d_0) - bF(2\theta' - d_0) - \int\limits_{2\theta'-d_0}^{\theta'} F(\theta)d\theta + \int\limits_{\theta'}^{d_0} F(\theta)d\theta\right] < 0.$$

■

Proof of Corollary 1. 1. If $d_0 < \widehat{\theta}$ we can consider the delegation set $\widehat{D}$. Since we have $\widehat{\theta} < E(\theta)+b$, the set $\widehat{D}(\theta)$ strictly improves V_0 and $V_{\inf}$. Note that $q_0 = \widehat{\theta}$ only if $p = 0$.

2. If $d_1 > \widehat{\theta}$, then we can consider the set $\widetilde{D} = \left(\left[\widehat{\theta}, d_1\right] \cup \left[d_0, \overline{\theta}\right], d_0\right)$ which has the same uninformed decision d_0 and strictly improves $V_{\inf}$.

3. Taking the derivative with respect to d_1 we obtain $d_1 = \widehat{\theta}$. ■

Proof of Proposition 2. The payoff functions corresponding to each of types 1-3 are given by

$$V_1(d_0, b) = -\frac{p}{2}\int_{\underline{\theta}}^{\overline{\theta}} (d_0 - \theta - b)^2 \, dF(\theta) - \frac{1-p}{2}\left[\int\limits_{\underline{\theta}}^{d_0} (d_0 - \theta - b)^2 dF(\theta) + \int\limits_{d_0}^{\bar{\theta}} b^2 dF(\theta)\right].$$

$$V_2(d_0, b) = -\frac{p}{2}\int_{\underline{\theta}}^{\overline{\theta}} (d_0 - \theta - b)^2 \, dF(\theta) - \frac{1-p}{2}\left[\int\limits_{\underline{\theta}}^{E(\theta)} (2E(\theta) - d_0 - \theta - b)^2 dF(\theta) + \right.$$
$$\left.\int\limits_{E(\theta)}^{d_0} (d_0 - \theta - b)^2 dF(\theta) + \int\limits_{d_0}^{\bar{\theta}} b^2 dF(\theta)\right]$$

$$V_3(d_0, b) = -\frac{p}{2}\int_{\underline{\theta}}^{\overline{\theta}} (d_0 - \theta - b)^2 \, dF(\theta) - \frac{1-p}{2}\left[\int\limits_{\underline{\theta}}^{\widehat{\theta}} (\widehat{\theta} - \theta - b)^2 dF(\theta) + \right.$$
$$\left.\int\limits_{\widehat{\theta}}^{2E(\theta)-d_0} b^2 dF(\theta) + \int\limits_{2E(\theta)-d_0}^{E(\theta)} (2E(\theta) - d_0 - \theta - b)^2 dF(\theta) + \int\limits_{E(\theta)}^{d_0} (d_0 - \theta - b)^2 dF(\theta) + \int\limits_{d_0}^{\bar{\theta}} b^2 dF(\theta)\right]$$

Consider $b > b_2^*$, then using the first-order condition $\frac{\partial V_1}{\partial d_0}(d_0^{(1)}(b), b) = 0$ and (6) we have

$$\frac{\partial V_2}{\partial d_0}(d_0^{(1)}) = F(E(\theta)) \left[\frac{\int_{\underline{\theta}}^{E(\theta)} F(\theta)\, d\theta}{F(E(\theta))} - b \right] < 0.$$

For any fixed b, $V_2(d_0, b)$ is a convex function of d_0 and $\frac{\partial V_2}{\partial d_0}(d_0^{(2)}(b), b) = 0$. Hence $d_0^{(1)}(b) > d_0^{(2)}(b)$ for all $b \in [b_2^*, b_{\max}]$.

Consider $V_{\inf}(b) = V_{\inf}(d_0^{(i)}(b), b)$ as functions of b. Using the Envelope theorem the derivative of $V_1(b)$ is

$$\frac{dV_1(b)}{db} = b(1-p)(1 - F(d_0^{(1)})) \tag{7}$$

and

$$\frac{dV_2(b)}{db} = b(1-p)\left[-2\left(\left(E(\theta) - d_0^{(2)}(b) \right) F(E(\theta)) - \left(bF(E(\theta)) - \int_{\underline{\theta}}^{E(\theta)} F(\theta)\, d\theta \right) \right) (1 - F(d_0^{(2)})) \right].$$

By (6) we have $\frac{dV_1(b)}{db} > \frac{dV_2(b)}{db}$ for all $b \in [b_2^*, b_{\max}]$. Note that $V_1(d_0^{(1)}(b_2^*), b_2^*) = V_2(d_0^{(1)}(b_2^*), b_2^*)$. For $b \geq b_2^*$ consider the delegation set of type 1 $D(b) = \left(\left[d_0^{(1)}(b), \overline{\theta} \right], d_0^{(1)}(b) \right)$ and of type 2 $D'(b) = \left(\left(2E(\theta) - d_0^{(1)}(b) \right) \cup \left[d_0^{(1)}(b), \overline{\theta} \right], d_0^{(1)}(b) \right)$. We have $V_0(D) = V_0(D')$. Hence

$$V_2(D'(b), b) - V_1(D(b), b) = \frac{1-p}{2} \int_{\underline{\theta}}^{E(\theta)} \left[(d_0^{(1)}(b) - \theta - b)^2 - (2E(\theta) - d_0^{(1)}(b) - \theta - b)^2 \right] f(\theta) d\theta =$$

$$2(1-p) \int_{\underline{\theta}}^{E(\theta)} (d_0^{(1)}(b) - E(\theta))(E(\theta) - \theta - b) f(\theta) d\theta = 2p(d_0^{(1)}(b) - E(\theta)) \int_{\underline{\theta}}^{E(\theta)} (E(\theta) - \theta - b) f(\theta) d\theta.$$

Note that for $b = b_2^*$ we have $\int_{\underline{\theta}}^{E(\theta)} (E(\theta) - \theta - b_2^*) f(\theta) d\theta = 0$. Thus $V_2(d_0^{(2)}(b_2^*), b_2^*) \geq V_2(d_0^{(1)}(b_2^*), b_2^*) = V_1(d_0^{(1)}(b_2^*), b_2^*)$. Therefore there exist $b_2 \geq b_2^*$ such that $V_2(d_0^{(2)}(b_2), b_2) = V_1(d_0^{(1)}(b_2), b_2)$. Similarly we establish that $V_3(d_0^{(3)}(b), b) > V_2(d_0^{(2)}(b), b)$ for $b \in [0, b_1)$. ∎

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