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Abstract

This paper develops an information-based theory of international currency based on search frictions, private trading histories, and imperfect recognizability of assets. Using an open-economy search model with multiple competing currencies, the value of each currency is determined without requiring agents to use a particular currency to purchase a country's goods. Strategic complementarities in portfolio choices and information acquisition decisions generate multiple equilibria with different types of payment arrangements. While some inflation can benefit the country issuing an international currency, the threat of losing international status puts an inflation discipline on the issuing country. When monetary authorities interact in a simple policy game, the temptation to inflate can lead optimal policy to deviate from the Friedman rule. A calibration of the generalized model shows that for the U.S. dollar, the welfare cost of losing international status ranges from 1.3% to 2.1% of GDP per year.

Keywords: international currencies, monetary search, liquidity, information frictions
JEL Classification Codes: D82, D83, E40, E50

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1 Introduction

The U.S. dollar plays a central role in the international monetary system. The 2006 Treasury reports that nearly 60% of dollar banknotes in circulation are held abroad. Over the last half century, the dollar has also been the main currency used for foreign exchange trades and international trade invoicing.[1] Although this current arrangement is the joint outcome of choices made by private citizens and regulations by official bodies, much of the existing international macroeconomic literature treats payment arrangements as given by restricting agents to only using a particular currency. While this assumption prevents the exchange rate from being indeterminate, as in Kareken and Wallace (1981), such an approach is especially unsatisfactory if we want to understand the process by which a currency achieves international status, or how it might lose that status. In particular, what factors can cause the U.S. dollar to lose its international role? And what are the welfare consequences of such an event?

The objective of this paper is to provide a simple framework for exploring both positive and normative aspects of different international monetary systems. For that purpose, I develop an open-economy search model with multiple currencies to analyze three central issues in international economics: (*i*) the conditions under which a currency emerges as an international medium of exchange, (*ii*) optimal monetary policy in an open economy, and (*iii*) the welfare benefits of having an international currency for both the issuing country and the rest of the world. Search-theoretic models are particularly insightful at addressing international currency use since they explicitly formalize the essential role of money, rather than assuming it exogenously.[2] In this spirit, this paper differs from much of the international macro literature by letting agents choose which currencies to accept, and not fixing its role by assumption. In turn, the model provides a tight link between a currency's international role and international trade.

The model features two key ingredients that capture the fact that international monetary arrangements are the dual outcomes of choices made by private citizens and regulations by official bodies. First, payment patterns are pinned down by letting private citizens choose which currencies to accept. Before trades occur, sellers can acquire at some cost the information, or the technology,

[1]In addition to an international currency's role as a store of value and medium of exchange in cross-border transactions, which is the main focus of this paper, it can also serve as an anchor currency in exchange rate regimes; a reserve currency in official foreign exchange reserves; a vehicle currency in foreign exchange; and an invoicing currency in international trade. All these roles are correlated however and Goldberg (2011) finds that the U.S. dollar maintains a dominant role in all key functions.

[2]A case in point is Matsuyama, Kiyotaki, and Matsui (1993) and Wright and Trejos (2001) which examines the possible role for two fiat currencies in a two country world. However, restrictions on asset divisibility and portfolio holdings prevent discussion of inflation and exchange rates. Recent works (e.g. Shi (1997) and Lagos and Wright (2005)) have relaxed these restrictions; though without additional frictions, the theory predicts rate-of-return equality across currencies and cannot pin down payments or exchange rates.

in order to accept payment in a particular currency. Here, *information costs* simply reflect the costly nature of dealing with multiple currencies or administering multi-payment economies. For example, there are technological costs for installing new technologies, such as debit card devices; or, it may be costly to verify asset quality if some currencies are not perfectly recognizable, as in Kim (1996) and Lester, Postlewaite, and Wright (2012).[3] Second, *government transaction policies* are introduced in order to account for the fact that payment outcomes also reflect choices made by official bodies. Historically, a currency will not become international unless there is a centralized institution that favors its use. This is often achieved in practice by announcing legal tender status or only accepting domestic money for tax payments.[4] The basic idea is that by simply accepting a particular currency in its own trades, governments may induce private agents to do the same.

In the baseline two-country, two-currency model, agents trade locally and internationally under different market structures, as in Lagos and Wright (2005). The frictions in this environment are decentralized exchanges, private trading histories, and imperfect recognizability of assets.[5] Each country issues one currency and is defined by two features: citizens in each country receive transfers of domestic currency and meet each other more frequently than they meet foreigners. Trade entails exchanging local goods for a portfolio of currencies, with no restrictions on which monies can be used between private citizens. Since what sellers accept depend on what buyers hold, and vice versa, complementarities in the trading environment lead to multiple equilibria where zero, one, or two international monies can emerge. Network externalities can lead to coordination failures, with no guarantee that the world will end up with a socially efficient monetary system.

By formalizing the role of currency in payments, the model provides a channel through which monetary policy can affect prices, trade, and exchange rates. For example, currency substitution occurs as an endogenous response to local inflation: as it becomes more costly to hold local money, agents start substituting with foreign currency such as dollars. This captures the phenomenon of dollarization common in many Latin American and Eastern European economies. The theory also emphasizes an important influence on the choice of money as an international medium of exchange. Fundamentals, as well as expectations regarding other agents' behavior, jointly determine

[3]History is rife with instances where means of payment have been subject to deceitful intent. The clipping of gold and silver coins in medieval Europe and rampant production of fake banknotes in the 19th century U.S. are notable examples, as documented in Mihm (2007) and discussed in Li, Rocheteau, and Weill (2012).

[4]The idea that what government accepts in payment affects what private agents do dates back to Smith (1963) and Lerner (1947). For example, Lerner (1947) argued that "the modern state can make anything it chooses generally acceptable as money... It is true that a simple declaration that such and such is money will not do... But if the state is willing to accept the proposed money in payment of taxes and other obligations to itself the trick is done." Aiyagari and Wallace (1997) and Li and Wright (1998) provide the first formalization of this insight.

[5]A large literature that explores the link between recognizability, information, and liquidity include Brunner and Meltzer (1971), Alchian (1977), Williamson and Wright (1994), Banerjee and Maskin (1996), Berensten and Rocheteau (2004), Rocheteau (2011), Li, Rocheteau, and Weill (2012), and Lester, Postlewaite, and Wright (2012).

this decision and thereby determine the circulation patterns that arise. Due to inertia, it is difficult to dislodge an incumbent currency from its international role, whose use is associated with low information costs. At the same time, a temporary disruption— such as a change in inflation— can permanently shift payment patterns. International currency use therefore reflects both fundamentals and history, consistent with what we observe in practice.[6]

This paper also explicitly models the strategic interaction among monetary authorities to obtain insights on optimal monetary policy in interdependent economies. The dynamic policy game captures the tradeoffs faced by policymakers and generates an inflation Laffer curve. While some inflation can benefit the issuing country through increased seigniorage from foreigners, too much inflation lowers the purchasing power of money hence trade between countries. At the same time, the threat of losing international status puts an inflation discipline on the issuing country. When monetary authorities interact in a simple policy game, the issuing country must therefore trade off the temptation to inflate and the threat of losing international status to set an optimal inflation rate that will generally deviate from the Friedman rule.

To illustrate these theoretical findings and quantify the welfare effects across countries, the model is calibrated to match international trade data. The regions of interest consist of three trading blocs: the United States, the Eurozone, and China. According to the theory, a country's welfare can be decomposed into seigniorage transfers across countries and the surplus due to liquidity provision to citizens net of any information costs incurred. In turn, the model implies that for the U.S., the welfare benefit of having the dollar as the sole international currency ranges from 1.3% to 2.1% of GDP per year. For the Eurozone, the gain from having the euro as an international currency is 1.4% to 1.9% of GDP, compared with 0.4% of GDP from Portes and Rey (1998) which only include seigniorage gains and the savings due to reduced transaction costs. This suggests that alternative studies may be underestimating the benefit of international liquidity provision since these studies do not take into account the general equilibrium effects an international currency has at expanding trade opportunities abroad.

The study of international currencies with search theory follows a rich tradition. Earlier two-country, two-currency search models with indivisible money include Matsuyama, Kiyotaki, and Matsui (1993), Zhou (1997), Wright and Trejos (2001), Camera and Winkler (2003), and Li and Matsui (2009). However these models cannot address money growth, inflation, or currency substitution due to the assumption of indivisible money holdings. Two-country, two-currency search

[6] This paper provides microfoundations for insights first articulated by Menger (1892), Kindleberger (1967), Swoboda (1969), and Krugman (1980). In more recent work, Rey (2001), Devereux and Shi (2005), and Lyons and Moore (2009) provide theories of vehicle currency use in foreign exchange markets, while Goldberg and Tille (2007) develop a model of invoice currencies. In contrast, this paper emphasizes the medium-of-exchange role for international currencies in cross-border goods transactions and derives this role using search theory.

models following the Shi (1997) framework include Head and Shi (2003) and Liu and Shi (2010).[7]

In contrast with earlier dual-currency search models, this paper features divisible assets by generalizing Lester, Postlewaite, and Wright (2012) to an open-economy setting. Although dollarization and exchange rates are discussed, the closed-economy setting prevents considerations of international currencies and optimal monetary policy in an open economy. Another related work is Geromichalos and Simonovska (2010), which attempts to reconcile the asset home bias puzzle. However, fiat money is not modeled explicitly and the analysis abstracts from policy considerations.

This paper proceeds as follows. Section 2 describes the environment, and Section 3 defines equilibrium. Currency regimes are characterized in Section 4, which also discusses how monetary policy and government transaction policies affect prices, allocations, and welfare. Section 5 considers a simple monetary policy game to determine optimal monetary policy in an open economy. Section 6 calibrates the generalized model using international trade data, and Section 7 calculates the welfare benefits of an international currency. Finally Section 8 concludes.

2 Environment

Time is discrete and continues forever. There are two countries, 1 and 2, populated with a continuum of 2 and $2n$ agents, respectively, where $n \in (0,1)$ denotes relative country size. Each period consists of two sub-periods where economic activity will differ. In the first, agents meet pairwise and at random in decentralized markets (DM) of each country. Here, agents are evenly divided between *buyers* and *sellers*: sellers from $s = \{1,2\}$ can produce output q_s but do not want to consume, while buyers want to consume but cannot produce. In the second sub-period, all trade occurs in a frictionless competitive market (CM). All agents can consume a numéraire good which is produced according to a linear production function in labor. The supply of hours in the CM is h which implies the real wage rate is equal to 1. Figure 1 summarizes the timing of events.

For tractability, instantaneous utilities are additively separable and quasi-linear in hours:

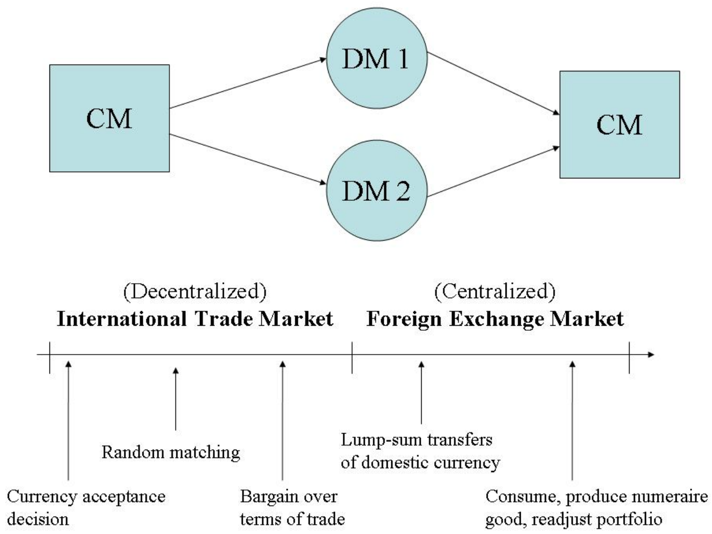
$$\mathcal{U}^B = u(q_s) + U(x) - h,$$

$$\mathcal{U}^S = -c(q_s) + U(x) - h.$$

Functional forms for utilities and cost functions, $u(q)$ and $c(q)$, are assumed to be C^2 with $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$. Also, let $q^* \equiv \{q : u'(q^*) = c'(q^*)\}$ and $x^* \in (0,\infty)$ solve $U'(x^*) = 1$. All goods are perishable, and agents discount the future

[7] There are also one-country models that study currency substitution and dollarization, such as Engineer (2000), Peterson (2001), Ravikumar and Wallace (2002), Curtis and Waller (2003), and Camera, Craig, and Waller (2004).

Figure 1: **Timing of Representative Period**



between periods with a discount factor $\beta \in (0, 1)$. Since individual histories are private information in the DM, credit cannot be used, and a medium of exchange is essential for trade.

Each country issues its own fiat currency, $c = \{1, 2\}$, both perfectly divisible and storable. Currency $m_c \in \mathbb{R}_+$ is valued at ϕ_c, the price of money in terms of the numéraire. The nominal exchange rate is defined here to be the price of currency 2 in terms of currency 1: $e \equiv \frac{\phi_2}{\phi_1}$. Money supplies, M_c, grow or shrink each period at a constant rate $(\gamma_c - 1)$, where $\gamma_c \equiv \frac{M_c'}{M_c}$. I focus on a stationary equilibrium where aggregate real balances in each country are constant. Therefore, the rate of return of currency c in each country is constant and will equal $\gamma_c = \frac{\phi}{\phi'}$. Changes in the money supply are implemented through lump-sum monetary transfers or taxes of domestic currency in the CM to that country's buyers. Since market clearing in the CM implies that the law of one price holds, agents can trade currencies at the market clearing rate. Hence, the CM also functions as a foreign exchange market.

Agents meet pairwise and at random in the DM. Buyers are mobile while sellers are immobile. With probability $\alpha \in [\frac{1}{2}, 1]$, a buyer stays in his country of origin and with probability $1 - \alpha$, visits the foreign country. The number of matches in the DM of country j is given by the matching function $\mathcal{M}_j \equiv \mathcal{M}(\mathcal{B}_j, \mathcal{S}_j) = \frac{\mathcal{B}_j \mathcal{S}_j}{\mathcal{B}_j + \mathcal{S}_j}$, where $\mathcal{B}_j$ and $\mathcal{S}_j$ denotes the measures of buyers and sellers in the DM of country j.[8] In country 1, $\mathcal{B}_1 = \alpha + n(1 - \alpha)$, $\mathcal{S}_1 = 1$, and a buyer meets a seller

[8]Berentsen, Rocheteau, and Shi (2007) specify a general version of this matching function (called the "additive-matching-rate-technology") that nests most matching technologies used in monetary search models.

with probability $a_1 = \frac{1}{1+\alpha+n(1-\alpha)}$ while in country 2, $\mathcal{B}_2 = \alpha n + 1 - \alpha$, $\mathcal{S}_2 = n$, and a buyer meets a seller with probability $a_2 = \frac{1}{1+\alpha+\frac{1-\alpha}{n}}$. Table 1 presents buyers' meeting probabilities across all meeting types.

Table 1: Buyer's Meeting Probabilities

	Seller from 1	Seller from 2
Buyer from 1	αa_1	$(1-\alpha)a_2$
Buyer from 2	$(1-\alpha)a_1$	αa_2

National and international trade frictions are controlled by the parameters n and α. Since $n \in (0,1)$, international meeting probabilities differ across countries. α can be interpreted as the degree of economic integration: as $\alpha \to \frac{1}{2}$, countries become more integrated and meeting a foreigner is more likely, while $\alpha \to 1$ corresponds to a closed economy where only locals trade.

Sellers in the model are further split between *private sellers* and *government sellers*, whose roles will differ in the following way. While domestic currency is perfectly recognizable, it is difficult for private sellers to verify the quality of foreign currency. In particular, they must incur a fixed flow cost, $\psi_s \geq 0$, in order to recognize and accept payment in foreign money. For example, firms must invest in a verification device in order to authenticate genuine foreign notes from counterfeits. The fixed cost is homogenous across sellers within a country but can differ across countries. It is common knowledge in a match whether the seller has invested, and sellers do not accept currencies they do not recognize.[9] Hence trade occurs under full information, and both currencies are accepted if and only if ψ_s is incurred.[10]

Government sellers consume and produce just like private sellers and are subject to the same constraints and matching technology, but have exogenous policies regarding what they accept as payment. Governments that only accept its domestic currency is considered the baseline policy, τ.[11] Government behavior is modeled this way since the purpose is to make precise how the size and influence of government affects realms of circulation and the set of equilibria. Given τ, terms

[9]For example, sellers will reject payment if it is costless to produce worthless counterfeits: if sellers accepted unrecognizable currencies, buyers would just hand over counterfeits in each exchange. This assumption simplifies the pricing mechanism, as discussed by Rocheteau (2008). In related work, Li, Rocheteau, and Weill (2012) allows the production of fraudulent assets at some cost, which generates an endogenous liquidity constraint.

[10]Without this decision, the model would predict that the two currencies are perfect substitutes that circulate at the same rate of return. Consequently, the composition of portfolios and the nominal exchange rate, $e = \frac{\phi_2}{\phi_1}$, is indeterminate, as in Kareken and Wallace (1981). This indeterminacy however is sensitive to the pricing mechanism. An alternative approach such as the Zhu and Wallace (2007) solution that is efficient but treats domestic and foreign currencies asymmetrically can determine the exchange rate, as demonstrated by Nosal and Rocheteau (2011).

[11]Most governments only accept their own local currency— for example, for payment of taxes— but there are a few exceptions where governments specifically demand some payments in foreign currency when their own money has not been well accepted in international markets (see e.g. McBride and Schuler (2012)).

of trade are determined through the same pricing mechanism as with interactions with private sellers.[12]

Table 2: Measures of Agents in Economy

	Country 1	Country 2
Buyer	1	n
Private Seller	$1 - g_1$	$n(1 - g_2)$
Government	g_1	ng_2
Total	2	$2n$

Table 2 summarizes the sizes and composition of the world economy. Agents are equally split between buyers and sellers, each of size 1 and n in country 1 and 2, respectively. The fraction of government sellers in each country, $g_1 \in [0, 1]$ and $g_2 \in [0, 1]$, can be interpreted as government size or the degree of centralized control.[13]

3 Equilibrium

This section describes the equilibrium of the two-country, two-currency model.

3.1 Centralized Market (CM) Value Functions

In the centralized markets, a representative buyer of each country chooses consumption of the numéraire good x, labor h, and real balances to bring forward next period. Portfolios are expressed in real terms: let $\mathbf{z} = (z_i, z_j) \equiv (\phi_i m_i, \phi_j m_j) \in \mathbb{R}^2_+$ represent a buyer from i's portfolio of domestic currency $i = \{1, 2\}$ and foreign currency $j = \{1, 2\} \neq i$. Variables with a prime denote next period's choices. Also let $W_i^B(\mathbf{z})$ and $V_i^B(\mathbf{z})$ denote buyers' value functions in the CM and DM, respectively.

In the beginning of the CM, a representative buyer from country $i = \{1, 2\}$ faces the following maximization problem:

$$W_i^B(\mathbf{z}) = \max_{x,h,\mathbf{z}' \in \mathbb{R}^2_+} \{U(x) - h + \beta V_i^B(\mathbf{z}')\} \tag{1}$$

$$\text{s.t. } x + \phi_i m_i' + \phi_j m_j' = h + z_i + z_j + T_i \tag{2}$$

[12] An alternative is to use a mechanism-design approach similar to Calvalcanti and Wallace (1999) where in a fraction of trades, the government chooses a trading mechanism to maximize the incentives to accumulate the domestic currency, while the remaining trades with private sellers are determined using a pricing mechanism such as bargaining, price posting, or competitive pricing.

[13] Another interpretation is to assume that governments in each country declares which currencies are legal tender. As in Lotz and Rocheteau (2002), the coercive power of the state arises from its ability to monitor transactions. The monitoring technology however is imperfect and governments can only observe a fraction $g_1, g_2 \in [0, 1]$ of trades.

$$T_i \equiv (\gamma_i - 1)\phi_i M_i. \tag{3}$$

The portfolio taken into the next DM is $\mathbf{z}' = (z_i', z_j') = (\phi_i' m_i', \phi_j' m_j')$, while T_i is the lump-sum transfer of domestic currency from the government (expressed in numéraire goods). Notice that because of quasi-linear preferences, T_i does not affect the buyer's problem in the CM. Substituting $m_c' = \frac{z_c'}{\phi_c'}$ for currency $c = \{1, 2\}$ into the budget constraint and then eliminating h yields[14]

$$W_i^B(\mathbf{z}) = U(x^*) - x^* + z_i + z_j + T_i + \max_{\mathbf{z}' \in \mathbb{R}_+^2} \{-\gamma_i z_i' - \gamma_j z_j' + \beta V_i^B(\mathbf{z}')\}. \tag{4}$$

A buyer's lifetime utility at the beginning of the CM is thus the sum of his net consumption in the CM, real balances in domestic and foreign currency, the lump-sum transfer from the local government, and the continuation value at the beginning of the next DM minus the investment in real balances.

A few results from the CM value function are worth highlighting. First, $W_i^B(\mathbf{z})$ is linear in total wealth $\overline{z} = z_i + z_j$: $W_i'(\overline{z}) = 1$. Second, there are no wealth effects since $\mathbf{z}'$ is independent of $\mathbf{z}$, which follows from the quasi-linearity of the utility function. Taking first-order conditions, optimal money holdings must satisfy for each currency $c = \{1, 2\}$:

$$-\gamma_c + \beta \frac{\partial V_i^B(\mathbf{z}')}{\partial z_c'} \leq 0,$$

and with equality if $z_c' > 0$. Provided that DM value functions $V_i^B(\mathbf{z})$ are strictly concave, there will generally be a unique portfolio that satisfies market clearing where all buyers in a country demand the same real balances. A caveat is when the two currencies are perfect substitutes; in that case, buyers can hold different portfolios but they will have the same total value.

I next specify the pricing mechanism in the DM, which will show that the terms of trade do not depend on the seller's portfolio. Consequently, sellers will choose to spend all real balances accumulated in the previous DM. Since sellers have no strict incentive to carry real balances in the DM, their CM value function can be written as $W_i^S(\mathbf{z}) = U(x^*) - x^* + z_i + z_j + \beta V_i^S(0, 0)$, which is also linear in total wealth.

3.2 Terms of Trade

Terms of trade in the DM are determined according to Kalai (1977)'s proportional bargaining rule. This pricing mechanism permits sellers to extract a constant fraction of the match surplus in order

[14] In order to ensure that $h \geq 0$ in equilibrium, it just needs to be checked that $x^* \geq \phi_1(m_1 - m_1') + \phi_2(m_2 - m_2') + T_i$. Alternatively, there may be an additional constraint $h \in [0, \overline{h}]$ but so long as it is not binding, h can be eliminated to obtain (4).

to recover some of their ex-ante investment.[15] Under proportional bargaining, a buyer acquires output in exchange for payment to the seller and receives a constant share, $\frac{\theta}{1-\theta}$, of the seller's surplus, where $\theta \in (0,1)$ measures the buyer's bargaining power, and threat points are given by continuation values. Let q,d (Q, D) denote output and payment in private (government) meetings.

Given the model specification, terms of trade will depend on buyers' portfolios, private sellers' acceptance strategy, and governments' transaction policy. To apply the pricing mechanism, notice that the surplus of a buyer who gets q_s for payment d_s to a private seller is $u(q_s) + W_i^B(\overline{z} - d_s) - W_i^B(\overline{z}) = u(q_s) - d_s$, by the linearity of W_i^B. Similarly, the seller's surplus is $d_s - c(q_s)$.

Consider first a meeting between a buyer and a seller from s that only accepts domestic currency. Under proportional bargaining, quantity traded q_s and payment d_s solves

$$max_{q,d}[u(q_s) - d_s] \tag{5}$$

$$\text{s.t. } u(q_s) - d_s = \frac{\theta}{1-\theta}[d_s - c(q_s)] \tag{6}$$

$$d_s \leq z_s. \tag{7}$$

The bargaining problem maximizes the buyer's surplus, subject to each party receiving a constant share of the match surplus, and a feasibility constraint (7) that says the buyer cannot transfer more money than he has, which is just real balances in the seller's domestic currency z_s. Consequently, the bargaining problem must satisfy

$$q_s \in \text{argmax } \theta[u(q_s) - c(q_s)]$$

$$\text{s.t. } (1-\theta)u(q_s) + \theta c(q_s) \leq z_s.$$

Payment to the sellers is thus $z(q_s) \equiv \theta c(q_s) + (1-\theta)u(q_s)$. The function $z(q_s)$ is continuous, which will guarantee that there exists a solution to the buyer's choice of real balances. As a result, output q_s solves

$$z(q_s) = \min\{z(q^*), z_s\} \tag{8}$$

where

$$z(q_s) \equiv \theta c(q_s) + (1-\theta)u(q_s). \tag{9}$$

[15] Other pricing mechanisms can be used, such as the generalized Nash (1950) solution. Note however that if the buyer makes a take-it-or-leave-it offer, sellers will have no incentive to incur the fixed cost to accept currencies since they do not receive any surplus from trade. As discussed in Arouba, Rocheteau, and Waller (2007), proportional bargaining does not suffer from a shortcoming of Nash bargaining that an agent can end up with a lower individual surplus even if the size of the total surplus increases.

The bargaining solution simply says that when $z_s \geq z(q^*)$, the buyer has enough wealth to finance purchase of the first-best q^*, and payment to the seller will be $z(q^*) = \theta c(q^*) + (1-\theta)u(q^*)$. When $z_s < z(q^*)$, the buyer just gives the seller what he has, z_s, and gets in return $q_s < q^*$.

When instead the seller accepts both currencies by incurring the fixed cost to recognize foreign money, terms of trade will satisfy a similar problem as (5) – (6), but with the feasibility constraint $d_s^b \leq z_1 + z_2$ since the buyer can now pay with both currencies. In what follows, the superscript b is used to distinguish variables when sellers accept both currencies. Consequently, payment to the seller z_s^b and output q_s^b solves

$$z(q_s^b) = \min\{z(q^*), z_1 + z_2\} \tag{10}$$

where

$$z(q_s^b) \equiv \theta c(q_s^b) + (1-\theta)u(q_s^b). \tag{11}$$

Since terms of trade with government sellers follow the same bargaining protocol as with private sellers, output in government meetings, $Q_s(\tau)$, will satisfy similar expressions:

$$z(Q_s(\tau)) = \min\{z(Q^*), z_s\} \tag{12}$$

where

$$z(Q_s(\tau)) \equiv \theta c(Q_s(\tau)) + (1-\theta)u(Q_s(\tau)). \tag{13}$$

3.3 Foreign Currency Acceptance Decision

Before matches are formed in the DM, private sellers can acquire at some cost the information, or technology, in order to accept payment in foreign currency. This decision determines which payment instruments are accepted: only domestic currency or both currencies.[16] Seller's strategies are given by $\sigma_s \in [0,1]$, where $\sigma_s = 0$ if a seller from $s = \{1,2\}$ rejects payment in foreign currency and $\sigma_s = 1$ if foreign currency is accepted. When $\sigma_s \in (0,1)$, both currencies are accepted in a fraction of trades.

Given the bargaining solution, the seller's expected payoff if he rejects payment in foreign currency is

$$\Pi_s \equiv (1-\theta)\{\lambda_{1s}[u(q_s) - c(q_s)] + \lambda_{2s}[u(\widehat{q}_s) - c(\widehat{q}_s)]\}$$

where λ_{is} denotes the probability that a private seller from s meets a buyer from $i = \{1,2\}$ and

[16] Due to the presence of government sellers that always accept local currency, there will be residual demand for both currencies. Without government sellers, there may also be a strategy where private sellers only accept foreign currency or reject payment altogether.

$(1-\theta)$ is the seller's share in the trade surplus. In what follows, hatted variables will refer to a buyer from country 2.

If instead the seller chooses to incur the fixed cost to accept foreign currency, his expected payoff is

$$\Pi_s^b \equiv -\psi_s + (1-\theta)\{\lambda_{1s}[u(q_s^b) - c(q_s^b)] + \lambda_{2s}[u(\widehat{q}_s^b) - c(\widehat{q}_s^b)]\}.$$

The seller's expected gain from accepting both currencies is therefore

$$\begin{aligned}\Delta_s &\equiv \Pi_s^b - \Pi_s \\ &= -\psi_s + (1-\theta)\left\{\lambda_{1s}[S(q_s^b) - S(q_s)] + \lambda_{2s}[S(\widehat{q}_s^b) - S(\widehat{q}_s)]\right\},\end{aligned}$$

where $S(q_s) \equiv u(q_s) - c(q_s)$ is the total trade surplus.

Consequently, the seller will choose to invest if $\Delta_s > 0$ and not invest if $\Delta_s < 0$. When $\Delta_s = 0$, sellers are indifferent and invest with an arbitrary probability. Optimal strategies $\sigma = (\sigma_1, \sigma_2)$ must therefore satisfy

$$\sigma_s = \begin{cases} 1 \\ \in (0,1) \\ 0 \end{cases} \text{if } \Delta_s \begin{cases} > \\ = \\ < \end{cases} 0. \tag{14}$$

3.4 Decentralized Markets (DM) Value Function

Given the bargaining solution, the DM value functions simplify greatly. Since the terms of trade do not depend on sellers' portfolios, the DM value function can be written solely in terms of the buyer's problem.

Consider a representative buyer from country i. Using the linearity of $W_i^B(\mathbf{z})$ and the bargaining solution, the DM value function simplifies to:

$$\begin{aligned}V_i^B(\mathbf{z}) = \alpha a_i &\underbrace{\left\{(1-g_i)\theta[\sigma_i S(q_i^b) + (1-\sigma_i)S(q_i)] + g_i\theta S(Q_i(\tau))\right\}}_{\text{buyer's surplus in domestic meetings}} \\ &+ (1-\alpha)a_j \underbrace{\left\{(1-g_j)\theta[\sigma_j S(q_j^b) + (1-\sigma_j)S(q_j)] + g_j\theta S(Q_j(\tau))\right\}}_{\text{buyer's surplus in foreign meetings}} \\ &+ z_i + z_j + W_i^B(0,0).\end{aligned}$$

The last three terms result from the linearity of $W_i^B(\mathbf{z})$ and is the value of proceeding to the CM with one's portfolio intact. The value function with no government results when $g_1 = g_2 = 0$.

Next, lead the DM value function forward by one period and substitute into the CM value function to yield the buyer's objective function. Letting the interest rate on an illiquid nominal bond denominated in currency c be $1 + i_c = \frac{\phi_c}{\phi'_c \beta}$, the buyer's optimal choice of real balances solve

$$\max_{\mathbf{z} \in \mathbb{R}^2_+} \{ \underbrace{-i_1 z_1 - i_2 z_2}_{\text{cost of holding money}} + \alpha a_i \underbrace{\left\{ (1 - g_i)\theta[\sigma_i S(q_i^b) + (1 - \sigma_i) S(q_i)] + g_i \theta S(Q_i(\tau)) \right\}}_{\text{buyer's surplus in domestic meetings}} \tag{15}$$

$$+ (1 - \alpha) a_j \underbrace{\left\{ (1 - g_j)\theta[\sigma_j S(q_j^b) + (1 - \sigma_j) S(q_j)] + (1 - g_j)\theta S(Q_j(\tau)) \right\}}_{\text{buyer's surplus in foreign meetings}}.$$

Here $i_c = \frac{\phi_c}{\phi'_c \beta} - 1$ is the cost of holding currency c. The objective function simply says that a buyer chooses a portfolio to maximize his expected surplus in domestic and foreign meetings, net of the cost of holding currency. Since (15) is continuous and maximizes over a compact set, there is a solution to the buyer's problem. In Appendix A, I show that this maximization problem is strictly jointly concave so long as $i_1 > 0$ and $i_2 > 0$. When $c \in \{1, 2\}$ and $j \in \{1, 2\} \neq c$, country i's first-order conditions with respect to real balances in currency z_c are:

$$-i_c + \alpha a_c (1 - g_c)\theta \left[\sigma_c S'(q_c^b) \frac{dq_c^b}{dz_c} + (1 - \sigma_c) S'(q_c) \frac{dq_i}{dz_c} \right] + (1 - \alpha) a_j (1 - g_j)\theta \left[\sigma_j S'(q_j^b) \frac{dq_j^b}{dz_c} \right] \tag{16}$$

$$+ \alpha a_c g_f \theta S'(Q_i(\tau)) \frac{dQ_i(\tau)}{dz_c} \leq 0$$

where

$$\frac{dq_c}{dz_c} = \begin{cases} \frac{1}{z'(q_c)} = \frac{1}{\theta c'(q_c) + (1-\theta) u'(q_c)} & : z_c < z(q_c), \\ 0 & : \text{otherwise}, \end{cases}$$

$$\frac{dq_c^b}{dz_c} = \begin{cases} \frac{1}{z'(q_c^b)} = \frac{1}{\theta c'(q_c^b) + (1-\theta) u'(q_c^b)} & : z_1 + z_2 < z(q_c^b), \\ 0 & : \text{otherwise}, \end{cases}$$

$$\frac{dQ_c}{dz_c} = \begin{cases} \frac{1}{p'(Q_c(\tau))} = \frac{1}{\theta c'(Q_c(\tau)) + (1-\theta) u'(Q_c(\tau))} & : z_c < z(Q_c(\tau)) \\ 0 & : \text{otherwise}. \end{cases}$$

Condition (16) is satisfied with equality if $z_c > 0$.

Definition 1. *Given τ, a stationary monetary equilibrium is a list of quantities traded $\{q_s, q_s^b, Q_s(\tau)\}$, sellers' strategies σ_s, and real balances $\boldsymbol{z} \equiv (z_i, z_j)$ $\forall$ $i, j, s = \{1, 2\}$, $i \neq j$ such that*

1. *$\{q_s, q_s^b, Q_z(\tau)\} \in \mathbb{R}^3_+$ solves the bargaining problem 8 – 13;*

2. $\sigma_s \in [0,1]$ *solves sellers' foreign currency acceptance decision 14;*

3. $\boldsymbol{z} \equiv (z_i, z_j) \in \mathbb{R}^2_+$ *solves buyers' portfolio problem 15;*

4. *Money markets clear.*

In a monetary equilibrium where both currencies are valued, $z_1 > 0$ and $z_2 > 0$, DM output across all meetings must satisfy

$$i_1 = \alpha a_1\{(1-g_1)[\sigma_1 L(q_1^b) + (1-\sigma_1)L(q_1)] + g_1 L(Q_1(\tau))\} + (1-\alpha)a_2(1-g_2)[\sigma_2 L(q_2^b)] \quad (17)$$

$$i_2 = \alpha a_1(1-g_1)[\sigma_1 L(q_1^b)] + \{(1-\alpha)a_2(1-g_2)[\sigma_2 L(q_2^b) + (1-\sigma_2)L(q_2)] + g_2 L(Q_2(\tau))\} \quad (18)$$

where

$$L(q) \equiv \frac{\theta[u'(q) - c'(q)]}{\theta c'(q) + (1-\theta)u'(q)}.$$

Buyers wish to bring currencies into the DM since these objects facilitate trade across different meeting types, but doing so is costly as captured by the terms i_1 and i_2 on the left sides of (17) and (18). The function $L(q)$ is the *liquidity premium* and represents the marginal payoff an agent gets from his liquid wealth that can be used to acquire more output in the DM instead of carrying it over to the subsequent CM. Intuitively, the equilibrium conditions equate the marginal benefit of liquidity to its cost. As a result, a currency demands a liquidity premium only if it is accepted in trade, as determined by σ and τ. When no sellers accept a currency, it will not be valued. Also notice that $L(q)$ is strictly decreasing over the relevant range: that is, $L'(q) < 0$ for $q^k \in [0, q^*]$.[17] In what follows, the focus is on equilibria where $\gamma_c \geq \beta$, since there is no solution otherwise.

The following lemma summarizes some basic properties of optimal portfolio holdings.

Lemma 1. *Consider any stationary monetary equilibrium where $i_1 \neq i_2$ (currencies are not perfect substitutes).*

1. *All buyers from the same country hold the same portfolios.*

2. *Buyers from different countries will generally hold different portfolios.*

3. *When there are no asymmetries in meeting arrangements— i.e., when the economy is perfectly integrated ($\alpha = \frac{1}{2}$) and countries and governments are of equal sizes ($n = 1$, $g_1 = g_2$)— then all buyers, irrespective of country origin, will hold the same portfolios.*

[17] Differentiating $L(q) = \frac{\theta[u'(q)-c'(q)]}{\theta c'(q)+(1-\theta)u'(q)}$ yields $L'(q) = \frac{\theta[u''(q)c'(q)-u'(q)c''(q)]}{[\theta c'(q)+(1-\theta)u'(q)]^3} < 0$, given the assumptions on the concavity of $u(q)$ and convexity of $c(q)$. Since the same conditions that make $L'(q) < 0$ make $V_i^B(\mathbf{z})$ strictly concave, this ensures a unique stationary monetary equilibrium that solves the first order conditions.

Table 3: **Equilibrium Currency Regimes**

Regime	σ	Circulation Pattern
N	$(0,0)$	Two National Currencies
I_1	$(0,\sigma_2)$	Currency 1 is International and Currency 2 is National
I_2	$(\sigma_1,0)$	Currency 2 is International and Currency 1 is National
U	$(1,1)$	Two International Currencies

Proof. Part *(1)* of Lemma 1 can be verified by examining the buyer's maximization problem. Due to the strict concavity of the objective, each buyer from a particular country has a degenerate demand for both currencies. Parts *(2)* and *(3)* follow directly from inspection of equations (17) and (18) and the bargaining solution. □

The intuition of Lemma 1 is that buyers from different countries hold different portfolios due to the asymmetry in the matching process: since the probability of meeting a foreigner generally depends on one's nationality, buyers allocate portfolio weights accordingly. Without any asymmetry in meeting arrangements, then buyers' nationalities cease to matter and will all hold symmetric portfolios. Given the model specification, this requires that $n = 1$, $g_1 = g_2$, and $\alpha = \frac{1}{2}$, which implies that it is equally likely to meet compatriots as foreigners.

Finally to close the model, market clearing implies that for each currency, aggregate supply must equal aggregate demand. By Lemma 1, all buyers from the same country hold the same portfolio when currencies are not perfect substitutes. Total demand for money 1 is $2m_1 + 2n\widehat{m}_1$ and for money 2 is $2m_2 + 2n\widehat{m}_2$. Market clearing then implies

$$2m_1 + 2n\widehat{m}_1 = M_1, \tag{19}$$

$$2m_2 + 2n\widehat{m}_2 = M_2. \tag{20}$$

4 Currency Regimes

Having defined monetary equilibrium, I now examine the types of currency regimes that arise in the dual-currency economy. Given government transaction policies under τ, a currency regime is defined as a pair of strategies for private sellers, $\sigma \equiv (\sigma_1, \sigma_2)$, that satisfies their currency acceptance decision. The focus is on the most representative monetary regimes: local circulation of currencies and international circulation of one or both currencies. Table 3 summarizes the currency regimes discussed in the text. In the following, the implications and existence of these types of equilibria are discussed.

4.1 Regime N: Two National Currencies

Consider first a regime where sellers only accept their domestic currency; that is, $\sigma = (0, 0)$. Suppose that country 1 is the U.S. and country 2 is Mexico. Given the government policy τ, this gives rise to the emergence of two national currencies: dollars are only accepted in the United States and pesos only accepted in Mexico.[18] This coincides with a common assumption in many international macroeconomic models, though arises as an equilibrium outcome in this model.

Under τ, output in country 1 must satisfy

$$i_1 = \alpha a_1 L(q_1)$$

$$i_2 = (1 - \alpha) a_2 L(q_2)$$

since output in government meetings will be the same as in private meetings (similar equations can be derived for country 2). The equations above relate the demand by agents from different countries for the two currencies to the cost of holding it. Since DM quantities in each meeting type can be obtained independently from these equations, monetary policies are independent across countries. Real balances can then be obtained from the bargaining solution: $z(q_1) = \phi_1 m_1 > 0$ and $z(q_2) = \phi_2 m_2 > 0$.

Necessary conditions for the two currencies to admit interior solutions are $i_1 < \bar{i}_1 \equiv \alpha a_1 \frac{\theta}{1-\theta}$ and $i_2 < \bar{i}_2 \equiv (1 - \alpha) a_2 \frac{\theta}{1-\theta}$. Buyers must have enough bargaining power in order for currencies to be valued. Hence even with the presence of government sellers that always accept local currency, there may be a non-monetary equilibrium where neither currencies are valued if $i_1 > \bar{i}_1$ or $i_2 > \bar{i}_2$.

So long as $i_1 < \bar{i}_1$ and $i_2 < \bar{i}_2$, buyers hold positive balances of both monies, since each have exchange roles in the issuing country.[19] Neither currency is fundamentally priced since each is essential for some meetings, even if one is being issued at a higher rate and thus has a higher inflation rate. Hence low-return currencies can circulate in equilibrium despite the existence of a

[18] National-currency equilibria are difficult to obtain in earlier dual-currency search models with indivisible money and divisible goods, as pointed out by Yiting Li. To obtain such an equilibrium, Wright and Trejos (2001) must assume that sellers cannot recognize the nationality of the buyer, but only the nationality of the currency the buyer carries. Without this assumption, there are no frictions for sellers to reject foreign money and hence national currencies cannot exist.

[19] A buyer from the U.S. holds both monies since they may meet a Mexican that only accepts pesos, which generates a precautionary demand for foreign currency. Since this allows trade to occur between agents from different countries— though only the seller's domestic currency changes hands— this differs from the "autarky" regime discussed in Matsuyama, Kiyotaki, and Matsui (1993). In the present model, only in a closed economy ($\alpha = 1$) will there be no international trade and hence no precautionary demand for foreign currency. In Appendix B, I consider an alternative formalization of the model where buyers know with whom they will be matched with before making portfolio decisions. In that case, there is no longer a precautionary demand for foreign currency. Moreover, the set of equilibria from the baseline model are preserved in this alternative specification.

competing, higher-return currency. Since sellers only accept their domestic money, only national currencies change hands. Equilibrium money holdings can then be obtained through the market-clearing conditions, $2m_1 + 2n\widehat{m}_1 = M_1$ and $2m_2 + 2n\widehat{m}_2 = M_2$.

Since both currencies can be valued at potentially different rates of return, the nominal exchange rate, $e \equiv \frac{\phi_2}{\phi_1}$, is determinate and will equal

$$e = \frac{z(\widehat{q}_1) + nz(\widehat{q}_2)}{z(q_1) + nz(q_2)} \frac{M_1}{M_2}.$$

As expected, the exchange rate depends on fundamentals and monetary factors in the two countries. For example, if the U.S. increases its money supply then dollars depreciate relative to pesos. The exchange rate is also affected by search frictions through dependence on q.

Turning to existence, this regime will constitute an equilibrium so long as private sellers have no incentive to incur the cost to recognize the foreign currency. This is true if $\Pi_s > \Pi_s^b$, or $\Delta_s < 0$ $\forall s = \{1, 2\}$, which is satisfied when ψ_1 and ψ_2 are sufficiently large:

$$\psi_1 > \overline{\psi}_1 \equiv (1-\theta)\{\lambda_{11}[S(q_1^b) - S(q_1)] + \lambda_{21}[S(\widehat{q}_1^b) - S(\widehat{q}_1)]\},$$

$$\psi_2 > \overline{\psi}_2 \equiv (1-\theta)\{\lambda_{12}[S(q_2^b) - S(q_2)] + \lambda_{22}[S(\widehat{q}_2^b) - S(\widehat{q}_2)]\},$$

where q_s^b is a solution to 17 and 18.

Government transaction policies can also guarantee this outcome. Using the identity $\mu_{ij}\mathcal{B}_i = \lambda_{ij}\mathcal{S}_j$ to derive meeting probabilities for private sellers yields $\lambda_{11} = \alpha\widetilde{a}_1(1-g_1)$, $\lambda_{21} = (1-\alpha)\widetilde{a}_1(1-g_1)$, $\lambda_{12} = (1-\alpha)\widetilde{a}_2(1-g_2)$, and $\lambda_{22} = \alpha\widetilde{a}_2(1-g_2)$,where $\widetilde{a}_1 = [1 + \frac{1}{\alpha + n(1-\alpha)}]^{-1}$ and $\widetilde{a}_2 = [1 + \frac{n}{\alpha n + 1 - \alpha}]^{-1}$. Inserting sellers' meeting probabilities and rearranging equilibrium conditions $\Delta_1 < 0$ and $\Delta_2 < 0$ yields

$$g_1 > \overline{g}_1 \equiv 1 - \frac{\psi_1}{(1-\theta)\widetilde{a}_1\{\alpha[S(q_1^b) - S(q_1)] + (1-\alpha)[S(\widehat{q}_1^b) - S(\widehat{q}_1)]\}},$$

$$g_2 > \overline{g}_2 \equiv 1 - \frac{\psi_2}{(1-\theta)\widetilde{a}_2\{\alpha[S(\widehat{q}_2^b) - S(\widehat{q}_2)] + (1-\alpha)[S(\widehat{q}_2^b) - S(\widehat{q}_2)]\}}.$$

As long as both currencies are valued, $g_1 > \overline{g}_1$ and $g_2 > \overline{g}_2$ ensures that an equilibrium with national currencies will exist. The presence of large national governments that only accept local money can therefore induce private citizens to do the same.

4.2 Regime I_1: Currency 1 is International and Currency 2 is National

This class of equilibria corresponds to the emergence of an international currency that circulates both locally and abroad. Within this class, there will be an equilibrium in pure strategies where all U.S. sellers reject pesos while all Mexican sellers accept both dollars and pesos: $(\sigma_1, \sigma_2) = (0, 1)$. There can also be a mixed equilibrium where Mexican sellers randomize: $(\sigma_1, \sigma_2) = (0, \Phi)$, where $\Phi \in (0, 1)$. As a result, the dollar becomes an international currency while pesos only circulate locally.[20]

Under τ, output in country 1 must satisfy

$$i_1 = \alpha a_1 L(q_1) + (1-\alpha) a_2 (1-g_2)\sigma_2 L(q_2^b),$$

$$i_2 = (1-\alpha) a_2 \{(1-g_2)[\sigma_2 L(q_2^b) + (1-\sigma_2) L(q_2)] + g_2 L(q_2)\}.$$

In this case, buyers hold positive balances of the two monies: $z(q_1) = \phi_1 m_1$, $z(q_2^b) = \phi_1 m_1 + \phi_2 m_2$, $z(q_2) = \phi_2 m_2$. The following table summarizes the effects of inflation and monetary policy in the two countries.[21]

$\frac{\partial \phi_1}{\partial i_1} < 0$	$\frac{\partial \phi_2}{\partial i_1} > 0$	$\frac{\partial e}{\partial i_1} > 0$
$\frac{\partial \phi_1}{\partial i_2} > 0$	$\frac{\partial \phi_2}{\partial i_2} < 0$	$\frac{\partial e}{\partial i_2} < 0$

If γ_2 increases, the peso inflates, which decreases its value, ϕ_2. This also raises the value of dollars, ϕ_1. The exchange rate $e = \frac{\phi_2}{\phi_1}$ falls and dollars appreciate due to increased foreign demand. Intuitively, as domestic inflation increases, locals in Mexico economize on peso holdings, which reduces its price ϕ_2. Since there's less demand for pesos, agents substitute into dollars, which raises its price ϕ_1. Now that the dollar is more valuable, sellers have more incentive to accept it. As a result, the economy dollarizes. This is due to the model's general equilibrium effects that makes currency substitution an endogenous response to local inflation. This situation arises precisely in dollarized economies where high inflation makes transacting in the local currency more costly so that citizens instead adopt the U.S. dollar.[22]

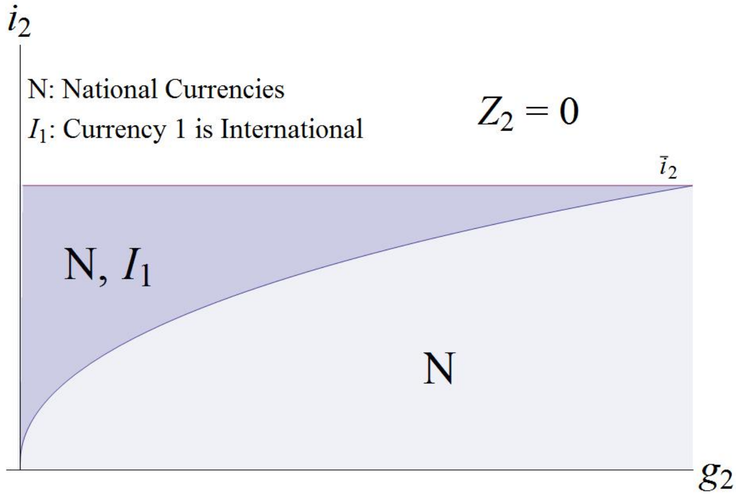
Figures 2 illustrates the effect of inflation on uniqueness or multiplicity. Consider a Mexican seller's decision to accept both currencies rather than just pesos. When peso inflation is low, the benefit of adopting an additional medium of exchange is also low. As monetary policy approaches the Friedman rule $i_2 \to 0$, output approaches q^*, the expected benefit of acquiring information

[20] Another equilibrium, $(\sigma_1, \sigma_2) = \{(1, 0), (\Phi, 0)\}$ has symmetric properties as this regime, so discussion is omitted.

[21] To obtain these comparative statics, I first differentiate the Euler equations with respect to i_1 or i_2 then differentiate the bargaining solution for the final expression. These calculations are provided in Appendix A.

[22] This result is also discussed in Lester, Postlewaite, and Wright (2012) with a fixed one-time cost in a closed-economy setting.

Figure 2: **Monetary Policy and Inflation**



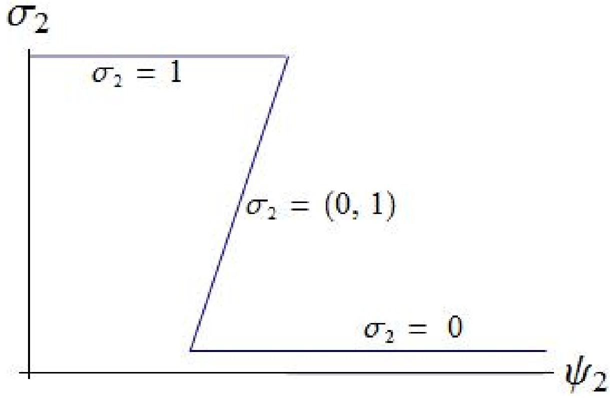
Δ_2 gets small, and there is an equilibrium where sellers do not accept foreign money so that economy ends up in regime N. As inflation increases however, it becomes more costly to use the national money, which decreases ϕ_2 and increases the value of the alternative asset, ϕ_1. This raises the incentive to acquire information and can generate multiple circulation patterns. As a result, currency substitution may be a purely expectational phenomenon: an international money may emerge even if the fundamentals of the economy are consistent with an equilibrium with national currency use. Historical episodes of dollarization in response to high inflation support this idea.[23]

Figure 2 also shows how the government policy variable, g_2, affects circulation patterns. Only in the limiting case where $g_2 = 1$ does the equilibrium where currency 1 is international cease to exist. For $g_2 < 1$, legal tender laws are therefore insufficient to rule out circulation of foreign currency. The non-monetary equilibrium when $i_2 > \bar{i}_2$ therefore exists even when enforcement is at its maximum, $g_2 = 1$. Figure 2 also illustrates the possibility that a local currency may survive and coexist with an international medium of exchange even without government restrictions ($g_2 = 0$), as in Matsuyama, Kiyotaki, and Matsui (1993). More generally, changes in government size will have direct effects on currency values, thereby influencing the monetary equilibrium attained.

In this model, an international currency emerges due to sellers' acceptance decisions and be-

[23] Currency substitution typically arises under high inflation, where the use of foreign currency persists after it has been accepted. For example, dollarization in many Latin American and Asian countries continue even after inflation stabilization, consistent with the model's predictions. Oomes (2001) documents that dollarization in Russia increased from almost zero to over 70% in the 1990's and failed to decrease despite stabilization. Similarly, Guidotti and Rodriguez (1992) report that dollarization in Bolivia went from close to zero in 1985 to nearly 50% in 1987.

Figure 3: **Sellers' Strategies and Multiplicity**



comes valued when a subset of sellers get informed about both currencies. An equilibrium where currency 1 becomes international requires $\Pi_1 > \Pi_1^b$ and $\Pi_2 \leq \Pi_2^b$, which implies that the fixed cost is sufficiently large in country 1 while sufficiently low in country 2:

$$\psi_1 > \overline{\psi}_1 \equiv (1-\theta)\{\alpha\widetilde{a}_1[S(q_1^b) - S(q_1)] + (1-\alpha)\widetilde{a}_1[S(\widehat{q}_1^b) - S(\widehat{q}_1)]\},$$

$$\psi_2 \leq \widetilde{\psi}_2 \equiv (1-\theta)\{(1-\alpha)\widetilde{a}_2[S(q_2^b) - S(q_2)] + \alpha\widetilde{a}_2[S(\widehat{q}_2^b) - S(\widehat{q}_2)]\}.$$

For both currencies to be accepted, the flow cost for country 2 sellers must be less than the increase in their expected surplus associated with accepting both currencies. Figure 3 depicts the strategy of a country 2 seller as a function of the fixed cost, ψ_2, and the measure of country 2 sellers that accept both currencies, σ_2. Since the two horizontal lines overlap for intermediate values of ψ_2, there can be multiple equilibria where regimes N and I_1 coexist.[24]

At the same time, an equilibrium where the dollar is international will exist if government in the issuing country is sufficiently large while sufficiently small in the other country:

$$g_1 > \overline{g}_1 \equiv 1 - \frac{\psi_1}{(1-\theta)\widetilde{a}_1\{\alpha[S(q_1^b) - S(q_1)] + (1-\alpha)[S(\widehat{q}_1^b) - S(\widehat{q}_1)]\}},$$

[24]Figure 3 also shows that the mixed strategy equilibrium with $\sigma_2 = \Phi \in (0,1)$ can deliver a counter-intuitive result that an increase in the cost of accepting dollars can increase the fraction of sellers that accept it. The mixed strategy equilibrium is also unstable in the following sense. When an equilibrium with $\sigma_2 = \Phi$ is changed to $\sigma_2' = \Phi + \epsilon$, where $\epsilon > 0$ is arbitrarily small, it is a best response for all sellers from country 2 to invest, leading to an equilibrium with $\sigma_2 = 1$. This instability however is not robust to a more general formalization of the seller's information cost. For example, if costs are heterogenous across sellers from a given country, then mixed strategy equilibria need not be unstable in the sense described above.

$$g_2 \leq \widetilde{g}_2 \equiv 1 - \frac{\psi_2}{(1-\theta)\widetilde{a}_2\{\alpha[S(\widehat{q}_2^b) - S(\widehat{q}_2)] + (1-\alpha)[S(q_2^b) - S(q_2)]\}}.$$

Since dollars circulates abroad, the Mexican government may want to consider using its transaction policy to drive dollars out of circulation in its country. For this to occur, it must be that Mexican sellers no longer find it optimal to accept dollars. This requires that $g_2 > \widetilde{g}_2$, in which case a sufficiently large local government can de-dollarize the economy by inducing a switch to national currency use. If instead $g_2 \leq \widetilde{g}_2$, then the government can only encourage and promote its currency, but cannot guarantee that the other stops circulating.

4.3 Regime U: Two International Currencies

Now consider an equilibrium where all sellers accept both currencies, $\sigma = (1,1)$, leading to the emergence of two international currencies.[25]

Since the bargaining solution implies that $q_1^b = q_2^b \equiv q^b$, equilibrium conditions simplify to

$$i_1 = [\alpha a_1(1-g_1) + (1-\alpha)a_2(1-g_2)]L(q^b) + \alpha a_1 g_1 L(q_1)$$

$$i_2 = [\alpha a_1(1-g_1) + (1-\alpha)a_2(1-g_2)]L(q^b) + (1-\alpha)a_2 g_2 L(q_2).$$

First notice that when there is no government ($g_1 = g_2 = 0$), for both monies to be valued, it must be that

$$i_1 = i_2.$$

Hence the two currencies are equally liquid and are valued only if they have the same rate of return. As in Kareken and Wallace (1981), citizens are indifferent between currencies with equal returns, so that the two are perfect substitutes. When monies circulate at par, agents may hold different portfolios, but they will have the same total value: $z(q^b) = \phi_1 m_1 + \phi_2 m_2$, where real balances must satisfy the market-clearing conditions $2m_1 + 2n\widehat{m}_1 = M_1$ and $2m_2 + 2n\widehat{m}_2 = M_2$.

Further, the exchange rate is indeterminate since q^b is uniquely determined while M_1 and M_2 must satisfy a single condition.[26] To see this when $\gamma_1 = \gamma_2 = \gamma$, let the world money supply in ϕ_1 units be $M = M_1 + eM_2$, growing at constant rate γ, where $e = \frac{\phi_2}{\phi_1} > 0$. A representative citizen's currency portfolio is constant: $\widetilde{z}(q^b) = \phi_1 M_1 + \phi_2 M_2$. Since this is constant, ϕ_1 and ϕ_2 must be

[25] There is also an equilibrium where sellers in both countries randomize, $\sigma = (\Phi_1, \Phi_2)$, where Φ_s denotes the fraction of sellers in country $s = \{1,2\}$ that accept both currencies. In this case, equilibrium conditions will be given by (17) and (18), with $\sigma_1 = \Phi_1$ and $\sigma_2 = \Phi_2$. In general, the two monies need not be perfect substitutes.

[26] There will be exchange rate indeterminacy when there is a subset of agents who view the two monies as perfect substitutes. While this subset pertains to the fraction of private sellers in this model, King, Wallace, and Weber (1992) provide an early analog of this idea by introducing a group of international currency traders that interact with agents that must satisfy cash-in-advance constraints.

decreasing at rate γ. As both e and ϕ_1 must satisfy a single condition $\widetilde{z}(q^b) = \phi_1[M_1 + eM_2]$, there will be indeterminacy: there can exist equilibria where only one currency is valued and equilibria where both currencies are valued.

While in Kareken and Wallace (1981) the nominal exchange rate is everywhere indeterminate, this will not be the case in this model. In particular, the model's fixed costs for recognizing foreign currency constrains the indeterminacy. If the costs, ψ_1 and ψ_2, are large enough so that no sellers accept both, there is no longer an equilibrium where the two currencies circulate at par.[27] Conversely, an equilibrium where all private sellers accept both currencies exists if $\Pi_s^b > \Pi_s$ $\forall s = \{1, 2\}$, which implies that the fixed cost in both countries must be sufficiently low:

$$\psi_1 < \widetilde{\psi}_1 \equiv (1-\theta)(1-g_1)\{\alpha\widetilde{a}_1[S(q_1^b) - S(q_1)] + (1-\alpha)\widetilde{a}_1[S(\widehat{q}_1^b) - S(\widehat{q}_1)]\},$$

$$\psi_2 < \widetilde{\psi}_2 \equiv (1-\theta)(1-g_1)\{(1-\alpha)\widetilde{a}_2[S(q_2^b) - S(q_2)] + \alpha\widetilde{a}_2[S(\widehat{q}_2^b) - S(\widehat{q}_2)]\}.$$

While the first-best level of output $q^b = q^*$ is achieved under the Friedman Rule, this is not socially efficient since all sellers must incur a real cost $\psi_1 > 0$ and $\psi_2 > 0$.

Alternatively, the existence of national governments can also rule out indeterminacy. An equilibrium where both currencies are accepted by all sellers exists if national governments are sufficiently small:

$$g_1 < \widetilde{g}_1 \equiv 1 - \frac{\psi_1}{(1-\theta)\widetilde{a}_1\{\alpha[S(q_1^b) - S(q_1)] + (1-\alpha)[S(\widehat{q}_1^b) - S(\widehat{q}_1)]\}},$$

$$g_2 < \widetilde{g}_2 \equiv 1 - \frac{\psi_2}{(1-\theta)\widetilde{a}_2\{\alpha[S(\widehat{q}_2^b) - S(\widehat{q}_2)] + (1-\alpha)[S(q_2^b) - S(q_2)]\}}.$$

Consequently, another way to eliminate indeterminacy in this model is by having large enough governments that adopt transaction policies so that currencies no longer circulate at par.

4.4 Multiple Equilibria

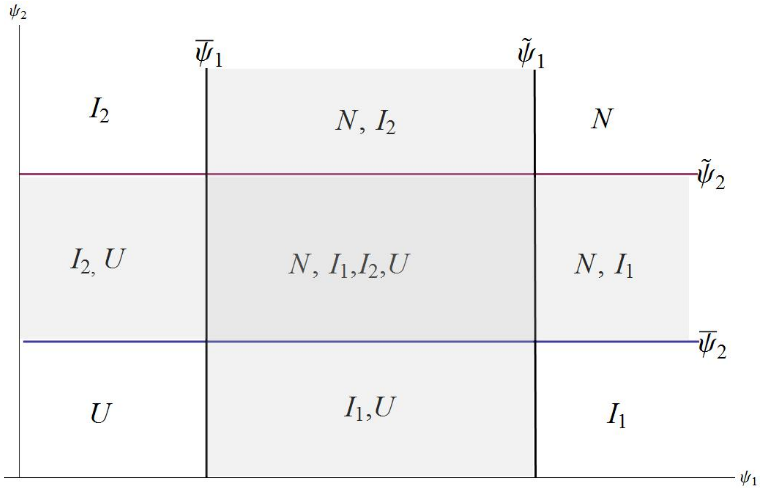
As shown in the previous subsections, the model implies the emergence of distinct currency regimes characterized by different payment patterns and realms of circulation. There can be multiple equilibria, where the share of transactions requiring different currencies is not uniquely determined by fundamentals. Proposition 1 summarizes how the information cost parameter affects multiplicity.

Proposition 1. *Consider a stationary monetary equilibrium where both currencies are valued.*

1. *If $\overline{\psi}_2 < \widetilde{\psi}_2$, then there will be an equilibrium where sellers from 2 only accept local currency*

[27] In an overlapping generations monetary model, Martin (2006) derives an analogous result and rules out indeterminacy of the exchange rate by similarly assuming a fixed cost for sellers to accept two currencies.

Figure 4: **Existence of Equilibria in (ψ_1, ψ_2)-Space**



and an equilibrium where they accept both currencies for any $\psi_2 \in [\overline{\psi}_2, \widetilde{\psi}_2]$ *since conditions* $\Delta_2 < 0$ *and* $\Delta_2 > 0$ *can be simultaneously satisfied.*

2. *If* $\overline{\psi}_1 < \widetilde{\psi}_1$*, then there will be an equilibrium where sellers from* 1 *only accept local currency and an equilibrium where they accept both currencies for any* $\psi_1 \in [\overline{\psi}_1, \widetilde{\psi}_1]$ *since existence conditions* $\Delta_1 < 0$ *and* $\Delta_1 > 0$ *can be simultaneously satisfied.*

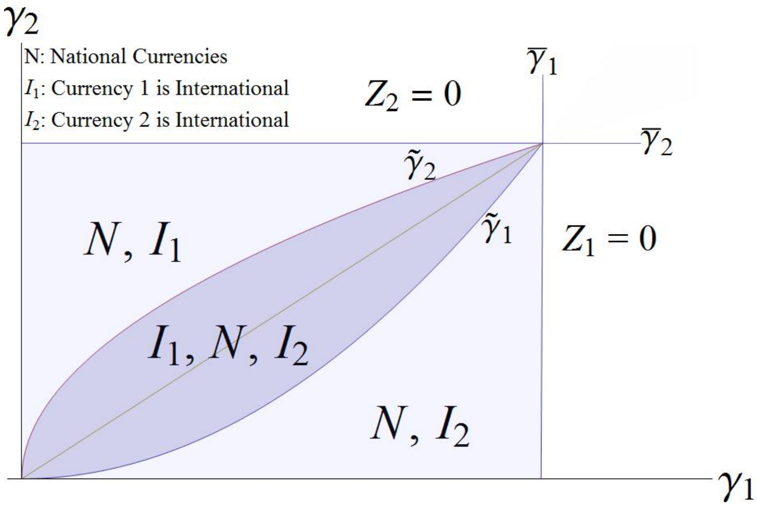
Proposition 1 is illustrated in Figure 4, which depicts the typology of equilibria in the information space, (ψ_1, ψ_2), assuming the same money growth rate in the two countries. Figure 4 shows that there are regions in the parameter space where regimes exist uniquely, if information costs are sufficiently low or high, and regions where regimes coexist.

The intuition for multiplicity operates through the general equilibrium interaction between buyers and sellers: what sellers accept depend on what buyers carry, and what buyers carry depend on what sellers accept. When more sellers accept a currency, it becomes more liquid and thus more valuable in exchange. Since buyers now want to hold more of this currency, its price increases, which increases the incentives for sellers to accept it. Due to this complementarity, multiple equilibria can arise. Consequently, the regime that the economy ends up in will depend on both fundamentals and expectations.

This multiplicity is also present in Figure 5, which depicts the existence of equilibria as a function of the two country's money growth rates, (γ_1, γ_2)-space, assuming that information costs are neither sufficiently high nor low so that Regimes N, I_1, I_2, and U are all possible.

An equilibrium with national currencies (Regime N) exists so long as neither currency is too

Figure 5: **Existence of Equilibria in** (γ_1, γ_2)**-Space**



costly to hold. Both currencies will be valued and sellers only accept domestic currency so long as $\gamma_1 < \overline{\gamma}_1$ and $\gamma_2 < \overline{\gamma}_2$. An equilibrium where currency 1 is international while currency 2 is only locally accepted (Regime I_1) exists so long as $\gamma_1 < \tilde{\gamma}_1$— the rate of return on currency 1 is high enough in order to give sellers from 2 enough incentive to accept currency 1. Symmetrically, an equilibrium where currency 2 is international while currency 1 is only locally accepted (Regime I_2) exists so long as $\gamma_2 < \tilde{\gamma}_2$. An equilibrium where sellers accept both currencies (Regime U) exists as a knife-edge case on the 45-degree line where there is rate of return equality. In that case, the currencies become perfect substitutes and the exchange rate is indeterminate. Finally, there can be a unique non-monetary equilibrium where either one or both currencies are not valued if monies are too costly to hold, which occurs when $\gamma_1 > \overline{\gamma}_1$ or $\gamma_2 > \overline{\gamma}_2$.

4.5 Welfare

This section concludes with a discussion of the model's normative implications. Due to the existence of multiple equilibria, countries may prefer one type of payment regime to another. Welfare in country $i \in \{1, 2\}$ is defined as the steady-state sum of buyers' and sellers' utilities in country i, weighted by their respective measures in the DM, $\mathcal{B}_i$ and $\mathcal{S}_i$:

$$\mathcal{W}_i = \mathcal{B}_i(1-\beta)V_i^B(z_1, z_2) + \mathcal{S}_i(1-\beta)V_i^S(0, 0).$$

Net consumption in the CM, $U(x^*) - x^*$, is normalized to zero with no loss in generality. In Appendix A, I show that welfare can be written as

$$\begin{aligned}\mathcal{W}_i = \mathcal{B}_i\mathcal{T}_i + \alpha\mathcal{M}_i & \underbrace{\{(1-g_i)[\sigma_i S(q_i^b) + (1-\sigma_i)S(q_i)] + g_i S(Q_i(\tau))\}}_{\text{total surplus in domestic meetings}} \\ & + (1-\alpha)\mathcal{M}_j \underbrace{\theta\{(1-g_j)[\sigma_j S(q_j^b) + (1-\sigma_j)S(q_j)] + g_j S(Q_j(\tau))\}}_{\text{buyer } i\text{'s surplus in foreign meetings}} \\ & + (1-\alpha)\mathcal{M}_i \underbrace{(1-\theta)\{(1-g_i)[\sigma_i S(q_i^b) + (1-\sigma_i)S(q_i)] + g_i S(Q_i(\tau))\}}_{\text{seller } i\text{'s surplus in foreign meetings}} - \mathcal{S}_i\sigma_i\psi_i,\end{aligned}$$

where

$$\mathcal{T}_1 \equiv \underbrace{n\phi_1(\widehat{m}_1' - \widehat{m}_1)}_{\text{country 1's seigniorage revenue from country 2}} - \underbrace{\phi_2(m_2' - m_2)}_{\text{country 1's seigniorage transfer to country 2}}, \tag{21}$$

$$\mathcal{T}_2 \equiv \underbrace{\phi_2(m_2' - m_2)}_{\text{country 2's seigniorage revenue from country 1}} - \underbrace{n\phi_1(\widehat{m}_1' - \widehat{m}_1)}_{\text{country 2's seigniorage transfer to country 1}}. \tag{22}$$

As a result, welfare can be decomposed into two components: (i) net seigniorage revenues or transfers given by $\mathcal{T}_i$, and (ii) surplus in DM trades net of information costs. In all equilibria where both currencies are valued, $\mathcal{T}_1 \neq 0$ and $\mathcal{T}_2 \neq 0$, with $\mathcal{T}_1 + \mathcal{T}_2 = 0$ due to market-clearing. Hence each monetary authority is also subject to its budget constraint $T_i = \phi_i(M_i' - M_i)$, which says that it finances lump-sum transfer to buyers through increases in the money supply.

According to the theory, there are two distinct sources of the welfare benefits of having an international currency. First is the increase in welfare due to increased seigniorage that arises from increased demand for real balances by foreigners. Second is the change in welfare due to increased trade. When a currency becomes international, it is more widely used in facilitating transactions which expands international trade.

Consequently, the model implies that welfare is unambiguously higher for a country that successfully has its currency accepted abroad than under a national currency regime. This gain comes from two sources: seigniorage gains from foreigners and an expansion of trade opportunities. However whether the other country also benefits from foreign currency circulation is ambiguous and depends on the benefit from increased international trade and the cost of lost seigniorage and the cost of accepting foreign money. These tradeoffs will be especially important in the next section which considers a simple monetary policy game to determine the optimal choice of inflation in the dual-currency economy.

5 A Simple Monetary Policy Game

In this section, I analyze the strategic choices of monetary authorities by modeling their objective functions and specifying the rules of their strategic interaction.[28] In the baseline analysis, monetary authorities behave non-cooperatively and determine the optimal monetary policy for their country by choosing a money growth rate to maximize the welfare of its citizens, taking as given the other country's money growth rate. In turn, each monetary authority is able to affect the rate of return of its currency and hence impact welfare both at home and abroad. Since the economy is open and policymakers behave strategically, optimal monetary policy in one country depends not only on domestic transaction patterns, but also on choices made by foreign citizens and the foreign policymaker. As a result, the policymaker may generate an externality for the other country that leads optimal policy to deviate from the Friedman rule.

5.1 Non-Cooperative Policy

The analysis begins by representing the strategic choices of monetary authorities as a one-shot non-cooperative game with perfect information. The analysis abstracts from repeated interactions among the monetary authorities since allowing for trigger strategies would substantially enlarge the set of equilibria.

There are three sets of players: citizens and the monetary authorities in countries 1 and 2. The game is divided into two stages. In the first stage, the monetary authority from each country $i = \{1, 2\}$ chooses a money growth rate $\gamma_i \in [\beta, \infty)$ to maximize the welfare of its citizens, taking as given the money growth rate chosen by the other country, $j = \{1, 2\} \neq i$, and optimal choices by citizens. Monetary authorities commit to their policy and choose their policies simultaneously and once-and-for-all.[29] In the second stage, citizens observe the actions of monetary authorities and make their currency acceptance decision, settle terms of trade, and select portfolio holdings. The focus is on finding subgame perfect equilibria of the policy game.

Definition 2. *A subgame perfect equilibrium consists of money growth rates for monetary authorities, (γ_1^*, γ_2^*), and best response functions for agents, $\Theta^*(\gamma_1, \gamma_2)$ such that*

1. For any given action taken by monetary authorities, agents' optimal choices, $\Theta^*(\gamma_1, \gamma_2) \equiv$

[28] Policy games in two-country, two-currency search models with indivisible money include Trejos and Wright (1996), Trejos (2003), and Li and Matsui (2009). However due to restrictions on portfolio holdings, these frameworks cannot be used to to analyze policymakers' choice of inflation. Liu and Shi (2010) consider optimal monetary policy with two symmetric currency areas but focuses on the deviations from the law of one price.

[29] Here, commitment is defined as the ability of an authority to bind future policy choices. Although this assumption can be restrictive, the purpose of this analysis is to establish as a benchmark the choice of optimal policies in the limiting case where policymakers can commit.

$\{(q_s, q_s^b, Q_s(\tau)), (z_i, z_j), \sigma_s\} \in \mathbb{R}_+^3 \times \mathbb{R}_+^2 \times [0,1]$, *satisfy Definition 1 for all* $i, j, s \in \{1,2\}$, $i \neq j$.

2. *Monetary authority* $i \in \{1,2\}$ *chooses a money growth rate,* $\gamma_i^* \in [\beta, \infty)$, *that maximizes welfare for its citizens,* $\mathcal{W}_i$, *taking as given* γ_j^* *for* $j \in \{1,2\} \neq i$ *and* $\Theta^*(\gamma_1, \gamma_2)$*:*

$$\gamma_1^* \in \mathit{argmax}\ \mathcal{W}_1(\gamma_1, \gamma_2^*, \Theta^*(\gamma_1, \gamma_2^*)),$$

$$\gamma_2^* \in \mathit{argmax}\ \mathcal{W}_2(\gamma_1^*, \gamma_2, \Theta^*(\gamma_1^*, \gamma_2)).$$

The game is solved using backwards induction, starting with the choices made by citizens. Section 4 solved for these optimal choices by characterizing the currency regimes that emerge, which forms the Nash equilibria of the final subgame. Throughout this section, it is assumed that government sellers only accept domestic currency.

Monetary Authorities' Choice of (γ_1^*, γ_2^*)

In the first stage, monetary authorities select optimal policies to maximize welfare for its country, anticipating that citizens respond optimally with $\Theta^*(\gamma_1, \gamma_2)$.

First, I show that monetary authorities can increase welfare by deviating from the Friedman rule, which is the typical optimal policy in single-currency economies without entry externalities. This implies that the Friedman rule need not be an optimal policy in the current set-up. Next I establish existence of subgame perfect money growth rates for a given equilibrium selection mechanism that places some continuity on agents' beliefs in parameter regions with multiplicity. Finally, I construct numerical examples to illustrate the main tradeoffs at hand.

Proposition 2. *Suppose country* j *fixes its money growth rate at the Friedman rule,* $\gamma_j = \beta$. *Country* $i \neq j$ *can benefit by setting its money growth rate above the Friedman rule,* $\gamma_i > \beta$, *so long as the economy is open* ($\alpha < 1$) *and there is foreign demand for currency* i. *That is,* $\frac{d\mathcal{W}_i}{d\gamma_i}\Big|_{\gamma_i=\beta} > 0$.

In an open economy, both countries have an incentive to inflate above the Friedman rule if the other country follows the Friedman rule. Since a country can export inflation abroad when foreigners hold its currency, seigniorage becomes a motive for money issue. This temptation to inflate is all the more striking since the monetary authority can resort to lump-sum taxes and does not have any expenditures of its own. As in Schmitt-Grohe and Uribe (2012), the incentive to deviate from the Friedman rule is not simply to finance its budget with seigniorage revenue extracted from foreigners. Rather, the monetary authority imposes an inflation tax on foreigners to increase the total amount of resources available to domestic residents for private consumption.

Further, the Friedman rule can only be the optimal policy if $\mathcal{T}_i = 0$. This would be the case in a closed-economy with $\alpha = 1$ and all sellers only accept domestic currency. Since there is only demand for local currency, buyers no longer hold foreign money and hence neither country receives seigniorage payments from the other country. This results in $\mathcal{T}_i = 0$ and $\frac{d\mathcal{W}_i}{d\gamma_i}\big|_{\gamma_i=\beta} = 0$. In that case, inflating will just reduce the purchasing power of currency for its residents, which is the typical distortion in single-currency economies.

Proposition 2 implies that when monetary authorities cannot cooperate and governments always accept domestic currency, the Friedman rule is not the optimal policy. However, determining equilibrium money growth rates requires examining the best response of one country's money growth rate to any arbitrarily given growth rate of the other country, not just the best response to the Friedman rule. When the other country does not follow the Friedman rule, the policymaker must trade off the positive effect of inflation with the negative effect that inflation has on reducing the purchasing power of currency.

Moreover, establishing the existence of subgame perfect money growth rates requires taking a stand on which equilibrium the economy converges to in regions where multiple regimes exist. For that purpose, I introduce an equilibrium selection mechanism that places some continuity on agents' beliefs. I then show existence for the given equilibrium selection mechanism described.

Definition. *Let $\widetilde{\gamma} \equiv \zeta\widetilde{\gamma}_1 + (1-\zeta)\widetilde{\gamma}_2$, where $\zeta \in [0,1]$ is exogenous. The equilibrium selection mechanism, $\mathcal{G}$, says if $\gamma_1 < \widetilde{\gamma}$, regime $\Omega_1 = \{N, I_1\}$ prevails, and if $\gamma_1 > \widetilde{\gamma}$, then regime $\Omega_2 = \{N, I_2\}$ prevails.*

Figure 6: Equilibrium Selection: $\zeta \in [0, 0.5)$

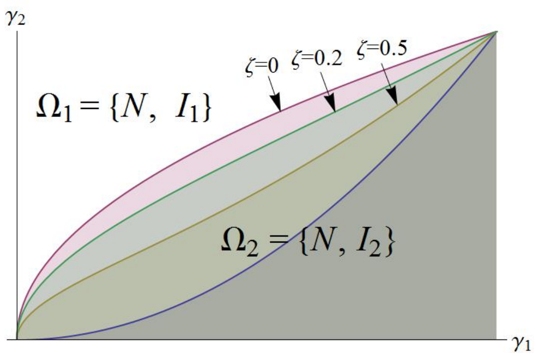
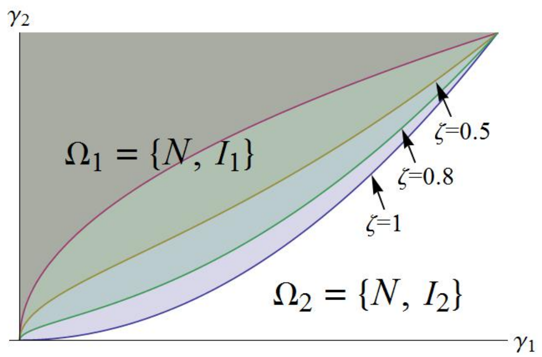


Figure 7: Equilibrium Selection: $\zeta \in (0.5, 1]$



The mechanism $\mathcal{G}$ simply introduces an arbitrary rule, $\widetilde{\gamma}$, that is a weighted average of the two curves in Figure 5, $\widetilde{\gamma}_1$ and $\widetilde{\gamma}_2$. Figures 6 and 7 plots the rule $\widetilde{\gamma}$ for different values of ζ. For example, one specification is that agents believe foreign currency is never accepted for all regions of the parameter space, in which case $\Omega_1 = \{N\}$ and $\Omega_2 = \{N\}$. Another specification is that agents believe currency 1 is international for all $\gamma_1 < \widetilde{\gamma} = \widetilde{\gamma}_1$ while they believe foreign currency is never accepted when $\gamma_1 > \widetilde{\gamma}_1$, in which case $\zeta = 1$, $\Omega_1 = \{I_1\}$, and $\Omega_2 = \{N\}$. The purpose of introducing this mechanism is to place some discipline and monotonicity on agents' beliefs and is not meant to provide ad-hoc microfoundations for how beliefs are actually formed.

Proposition 3. *Given $\mathcal{G}$, there exists subgame perfect money growth rates and it is such that $\gamma_1 > \beta$ and $\gamma_2 > \beta$.*

I now turn to numerical examples to illustrate the motives of monetary authorities under different scenarios in the last subgame.

Case 1: Foreign Currency is Never Accepted

Consider first the case where agents reject payment in foreign currency for all values of (γ_1, γ_2). That is, $\Omega_1 = \{N\}$ and $\Omega_2 = \{N\}$. Since agents always adopt the trading strategy $(\sigma_1, \sigma_2) = (0, 0)$, the welfare function in each country is well-behaved, continuous, and concave. As an example, Figure 8 plots welfare in country 1 as a function of its policy instrument γ_1, for a given γ_2. In what follows, it is implicitly understood that all endogenous variables are indexed with the regime under consideration. The two country's welfare functions are symmetric in this equilibrium and can be written as

$$\mathcal{W}_i|N = \mathcal{B}_i\mathcal{T}_i + \alpha\mathcal{M}_i S(q_i) + (1-\alpha)\mathcal{M}_j\theta S(q_j) + (1-\alpha)\mathcal{M}_i(1-\theta)S(\widehat{q}_i),$$

where net seigniorage for the two countries are $\mathcal{T}_1 = n\phi_1(\widehat{m}_1' - \widehat{m}_1) - \phi_2(m_2' - m_2)$ and $\mathcal{T}_2 = \phi_2(m_2' - m_2) - n\phi_1(\widehat{m}_1' - \widehat{m}_1)$. Policymakers then choose money growth rates, (γ_1, γ_2), that solves the two country's first-order conditions, $\frac{\partial \mathcal{W}_1|N}{\partial \gamma_1} = 0$ and $\frac{\partial \mathcal{W}_2|N}{\partial \gamma_2} = 0$.

Country 1's best response function can be obtained by solving $\frac{\partial \mathcal{W}_1|N}{\partial \gamma_1} = 0$ for their policy instrument, γ_1:

$$\gamma_1 = BR_1(\gamma_1),$$

where $BR_1(\gamma_1) \equiv 1 - \frac{\widehat{z}_1}{z'(\widehat{q}_1)}$ implicitly depends on its own money growth rate, γ_1, but is independent of country 2's policy instrument, γ_2. This is because when only local currency is accepted, the amount of output traded in one country is determined independently of the amount of traded in the other country. Similarly, country 2's best response function is

$$\gamma_2 = BR_2(\gamma_2),$$

where $BR_2(\gamma_2) \equiv 1 - \frac{z_2}{z'(q_2)}$ is also independent of γ_1.

Figure 8: Case 1: Welfare in Country 1

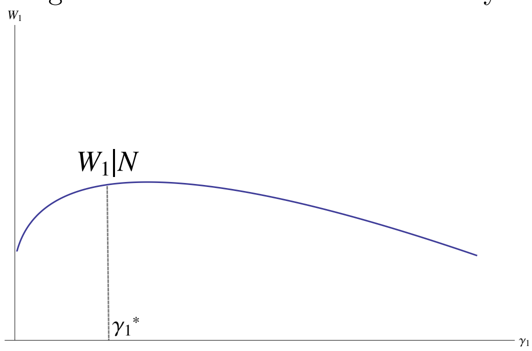


Figure 9: Case 1: Optimal Policies

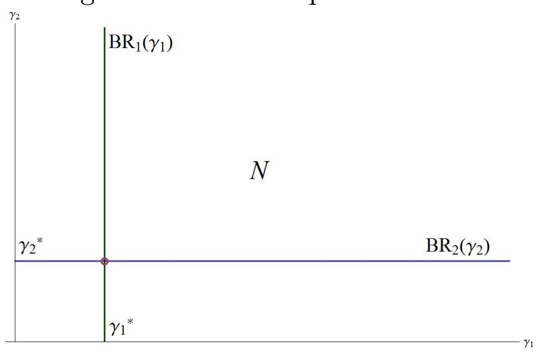


Figure 9 depicts the two country's best response functions when only local currencies are accepted. In this case, monetary policies are independent and there is a dichotomy between the two currencies. In Figure 9, $BR_1(\gamma_1)$ is given by the vertical line and $BR_2(\gamma_2)$ is given by the horizontal line. Optimal policies are given by the intersection of $BR_1(\gamma_1)$ and $BR_2(\gamma_2)$ at (γ_1^*, γ_2^*), which are both strictly above the Friedman rule, (β, β). Since buyers hold both home and foreign currency (due to their precautionary demand for the latter), the policymaker in each country has a temptation to inflate in order to extract seigniorage payments from foreigners. This generates an externality for the other country that neither policymaker takes into account. In equilibrium, monetary authorities trade off the gain from inflating with the cost of distorting allocations for its citizens to set an optimal money growth rate that can deviate from the Friedman rule.

Case 2: Currency 1 is International

Next consider the equilibrium where sellers from country 2 accept currency 1 when $\gamma_1 < \widetilde{\gamma}_1$ and never accept foreign currency otherwise. That is, $\zeta = 1$, $\Omega_1 = \{I_1\}$ and $\Omega_2 = \{N\}$. In this case, both regimes N and I_1 are possible. Although each country's welfare function is continuous within a regime, it is discontinuous at the transition from one regime to another. This is illustrated in Figures 10 and 12, which plots country 1's welfare as a function of its policy instrument, γ_1, for a given γ_2. Country 1's welfare function jumps down as the economy transitions from regime I_1 to regime N since country 1 enjoys higher welfare from issuing an international currency.

Consider the choice of policies when regime I_1 exists. In that case, currency 1 is the sole international currency and welfare in the two countries can be written

$$\mathcal{W}_1|I_1 = \mathcal{B}_1\mathcal{T}_1 + \alpha\mathcal{M}_1 S(q_1) + (1-\alpha)\mathcal{M}_2\theta S(q_2^b) + (1-\alpha)\mathcal{M}_1(1-\theta)S(q_1),$$

$$\mathcal{W}_2|I_1 = \mathcal{B}_2\mathcal{T}_2 + \alpha\mathcal{M}_2 S(q_2^b) + (1-\alpha)\mathcal{M}_1\theta S(q_1) + (1-\alpha)\mathcal{M}_2(1-\theta)S(q_2^b) - \mathcal{S}_2\psi_2.$$

Within Regime I_1, the policymakers' best response functions are obtained by solving the first-order conditions $\frac{\partial \mathcal{W}_1|I_1}{\partial \gamma_1} = 0$ and $\frac{\partial \mathcal{W}_2|I_1}{\partial \gamma_2} = 0$, subject to the constraint that $\gamma_1 \in (\beta, \widetilde{\gamma}_1)$. Country 1's best-response function is

$$\gamma_1 = BR_1(\gamma_1, \gamma_2),$$

where $BR_1(\gamma_1, \gamma_2) \equiv 1 - \frac{\widehat{z}_1}{z'(\widehat{q}_1)} + \frac{(\gamma_2 - 1)z'(q_2) + z_2}{nz'(\widehat{q}_1)}$ depends on its own policy instrument γ_1 implicitly as well as the other country's, γ_2. Consequently, there is no longer a dichotomy between monetary policies when foreigners accept currency 1 for trade. Similarly, country 2's best-response function is

$$\gamma_2 = BR_2(\gamma_1, \gamma_2),$$

where $BR_2(\gamma_1, \gamma_2) \equiv 1 - \frac{z_2}{z'(q_2)} + \frac{n(\gamma_1 - 1)z'(\widehat{q}_1) + \widehat{z}_1}{z'(q_2)}$ depends on both γ_1 and γ_2.

Figure 10: Case 2a: Welfare in Country 1

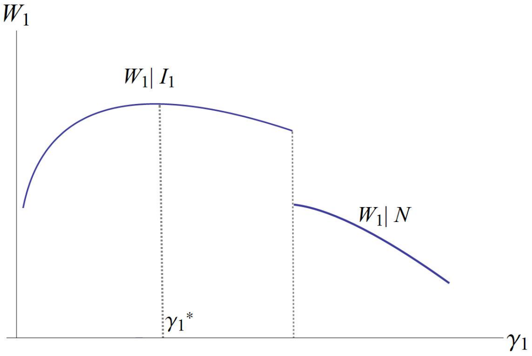


Figure 11: Case 2a: Optimal Policies ($\alpha = 0.8$)

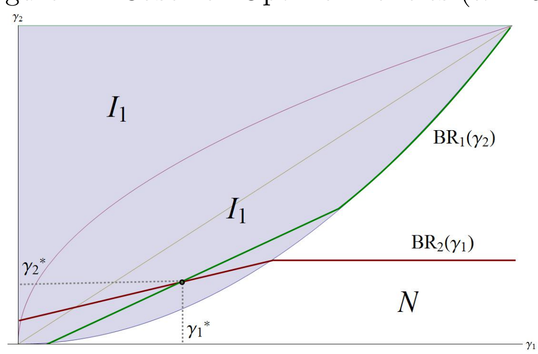


Figure 11 shows an example of optimal policies when currency 1 is internationally accepted in region I_1 and locally accepted otherwise under the assumption that it is much more likely to meet locals than foreigners ($\alpha = 0.8$). Country 1's best response function is given by $BR_1(\gamma_1, \gamma_2)$ and is always in region I_1 since country 1 has strictly higher welfare by having an international currency than by having local currencies. Moreover, $BR_1(\gamma_1, \gamma_2)$ is increasing in γ_2 since a higher γ_2 implies a higher demand for currency 1, which increases country 1's seigniorage revenue and hence incentive to inflate. Foreign demand leads to seigniorage from abroad, which becomes a motive for money issue. Figure 10 plots country 1's welfare as a function of γ_1 given γ_2^* and shows a Laffer Curve effect: as inflation rises beyond γ_1^*, the quantity of money demanded falls and the tax base reduced. As a result, there will be an interior money growth rate that maximizes the gain from inflating with the cost of distorting allocations.

Country 2 on the other hand, does not have its currency accepted abroad but also inflates. When

γ_2 is in Regime N, its best response function is horizontal since monetary policies are independent when only local currencies are accepted. In Regime I_1 however, $BR_2(\gamma_1, \gamma_2)$ depends positively on γ_1 since a higher γ_1 reduces demand for currency 1 while increasing demand for currency 2, which increases country 2's incentive to inflate. Equilibrium money growth rates are at the intersection of the two country's best response functions at (γ_1^*, γ_2^*), which are both above the Friedman rule, and lie within the shaded region where currency 1 is international.

Figure 12: Case 2b: Welfare in Country 1

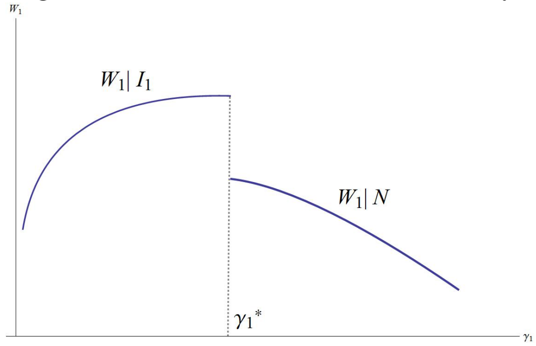


Figure 13: Case 2b: Optimal Policies ($\alpha = 0.5$)

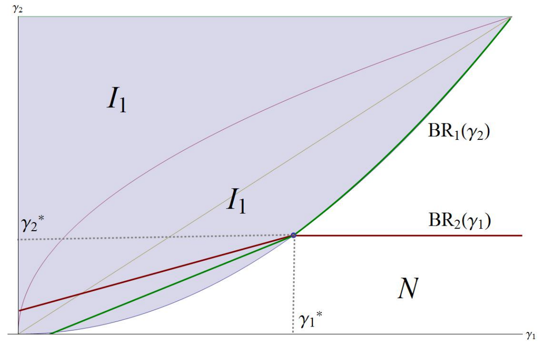


Figure 13 shows how optimal policies change when countries become perfectly integrated ($\alpha = \frac{1}{2}$). In that case, the probability of trading with a foreigner is at the maximum, which increases the amount of seigniorage generated abroad and maximizes the incentive to inflate, all else equal. Figure 12 plots country 1's welfare as a function of γ_1 given γ_2^* and shows that $\mathcal{W}_1$ reaches its maximum value at the transition from regime I_1 to regime N. In this case, the threat of losing international status places an inflation discipline on country 1. At γ_1^*, foreigners are just indifferent between accepting and rejecting payment in currency 1. Consequently, country 1's best response to any γ_2 for $\gamma_1 > \gamma_1^*$ is to inflate on the frontier, $\widetilde{\gamma}_1$, so that its currency remains international.

It is also possible to compare what happens to the optimal choice of inflation for the issuing country if its currency loses international status.[30]

Proposition 4. *A country chooses a higher inflation rate if it is the issuer of international currency than if it loses international status, so long as the other country also inflates. That is,* $\gamma_1^* | I_1 > \gamma_1^* | N$ *if* $\gamma_2^* > 1$.

When $\gamma_2^* > 1$, the positive effect of extracting seigniorage from abroad outweighs the cost of distorting allocations for its citizens. Since this inflationary tendency is curbed by the threat of losing international status, country 1 does not inflate beyond the threshold that would induce

[30] I thank Emmanuel Farhi for this suggestion.

foreigners to stop accepting its currency.[31] When $\gamma_2^* < 1$, both countries are deflating which implies a welfare-decreasing transfer of real resources by the domestic economy to foreigners. To minimize this transfer abroad, the issuing country chooses a lower inflation rate when its currency circulates abroad than when it does not: $\gamma_1^*|I_1 < \gamma_1^*|N$. When $\gamma_1^* = 1$ and $\gamma_2^* = 1$, money supplies are constant. Since neither country receives seigniorage revenues nor makes seigniorage transfers to the other country, $\gamma_1^*|I_1 = \gamma_1^*|N$.

5.2 Cooperative Policy

I now consider the case where the two monetary authorities cooperate by jointly choosing (γ_1, γ_2) to maximize total welfare for the world. Joint welfare is simply measured as the sum of the two countries' welfare functions: $\mathcal{W} = \mathcal{W}_1 + \mathcal{W}_2$.

Proposition 5. *When monetary authorities cooperate at jointly choosing (γ_1^*, γ_2^*) to maximize total welfare for the world, $\mathcal{W}$, and countries are perfectly integrated ($\alpha = \frac{1}{2}$), the unique optimal policy is the Friedman rule, $\gamma_1^* = \gamma_2^* = \beta$. As a result, agents never accept foreign currency and hold perfectly diversified portfolios of the two currencies. Equilibrium is socially efficient since no resources are spent on information costs.*

There can be gains from cooperating that are not realized when each country is pursuing its own best interest. When policymakers can coordinate, there are no longer gains from redistributive policies and hence no more temptation to inflate. In this case, the unique equilibrium is the Friedman rule for both countries. Consequently, private citizens only accept their local currency and society saves on information costs.

6 Quantitative Analysis

The preceding sections presented a simple two-country, two-currency search model that is amenable to policy analysis. To quantify the welfare cost of losing international status (or the gain of achieving it), the framework is generalized to an arbitrary number of countries and currencies and calibrated to match international trade data. Since much of the set-up and analysis carries over from the baseline model, the N-country, N-currency model is in Appendix B.

The regions of interest in the baseline analysis consist of three trading blocs: the United States, the Eurozone, and China. The declining dollar, the advent of the euro and the recent rise of

[31] This desire by the issuing country to extract seigniorage from foreigners is also empirically relevant. Circumstantial evidence from Rogoff (1998) suggests that the European Central Bank's decision to issue large-denomination bills can be interpreted as the desire to extract seigniorage revenue from foreign holders of euros and the underground economy.

China has renewed considerable interest in determining whether the dollar is at risk of losing its international role. This paper provides a new theoretical framework to evaluate this issue. The strategy taken here is to let the data and calibration procedure narrow down the set of equilibria to ones that are empirically plausible. The model is then used to calculate the welfare benefits of having an international currency for the issuing country and the rest of the world.

6.1 Calibration

To calibrate the model, the global economy is split into three trading blocs, or regions: the United States, the Eurozone, and China.[32] After discussing parameters that can be easily estimated or fixed independently to their empirical counterparts, I describe the calibration procedure for the remaining parameters. This procedure uses the model's equations and the parameters calibrated independently in order to find parameter values that match moments in the data. All data used are in annual terms from 1999 to 2005 unless otherwise specified.

Following Lagos and Wright (2005), functional forms for utility and cost functions are $U(x) = Alnx$, $u(q) = ln(q + b) - ln(b)$, and $c(q) = q$. The parameter b is set to $b = 0.0001$ which ensures a solution to the bargaining problem. The discount factor is set to $\beta = 0.966$, consistent with an annual real interest rate of 3.5%. Since the model implies that gross money growth rates are also gross inflation rates in a steady state equilibrium, γ_{us}, γ_{eu}, γ_{ch} are set to average annual inflation rates for the period 1999 to 2005, which is about 2.92% for the U.S., 1.97% for the Eurozone, and 5.04% for China, using data from the World Bank. Different inflation scenarios are also considered in the quantitative exercise. The bargaining power parameter is set to $\theta = 0.5$, consistent with an egalitarian bargaining rule.[33]

The government size parameters are set to match the fraction of state-owned enterprises in a particular country. The share of state-owned enterprises in the U.S. averaged to less less than 5%, with 22% averaged across Europe, and 37% for China. These data are obtained from Szamosszegi and Kyle (2011) and China's Second National Economic Census (2009). The utility parameter A and relative country sizes are jointly calibrated to match the ratio of each country's GDP over world GDP from Source OECD, which results in $A = 2.03$, $n_{us} = 0.36$, $n_{eu} = 0.37$, and $n_{ch} = 0.27$.

The next set of parameters are the model's meeting parameters for each country pair, μ_{is}. The

[32] The Eurozone, or the Euro Area, consists of the 17 European Union member states that have adopted the euro as their common currency and sole legal tender (the euro was officially launched on January 1, 1999). These countries include Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, Netherlands, Portugal, Slovakia, and Spain. Data from each of the 17 member countries are averaged into an aggregate measure for the Eurozone, with country weights determined by GDP.

[33] The bargaining power parameter θ can be calibrated to generate a mark-up consistent with the data, as in Craig and Rocheteau (2008) and Lagos and Wright (2005). However, due to a lack of reliable data on mark-ups for countries other than the United States, I simply set it equal for all countries at the egalitarian solution $\theta = 0.5$.

six international meeting parameters μ_{is} are calibrated with bilateral trade data for the period 1999 to 2005 obtained from the European Commission Bilateral Affairs. Due to an accounting constraint that the total measure of meetings between agents from country i with agents from country s have to be the same as the total measure of meetings between agents from country s with agents from country i, three of the meeting probabilities will not precisely match its targeted value. These values are then backed out using calibrated values for n_i, subject to the accounting constraint.

The final set of parameters are the costs of accepting different currencies. To discipline parameter values for information costs, I use data on the extensive margin of foreign currency holdings—whether or not a country holds a particular foreign currency— and how much of a country's trade is denominated in a particular currency. Information on the extensive margin corresponds to private sellers' acceptance decision $\sigma_s = (\sigma_s^{us}, \sigma_s^{eu}, \sigma_s^{ch}) \in \{0,1\}^3$, while international trade invoicing data will partially determine a country's trade composition in different currencies.[34] This approach is consistent with empirical evidence from Friberg and Wilander (2007) that the currency used in trade invoicing is also the one used in actual payment.

In the U.S., only dollars circulate, so that $\sigma_{us} = (1, 0, 0)$. In the Eurozone and China, dollars are used in international trade invoicing, as reported in Goldberg and Tille (2007). In addition, the Bank for International Settlements reports that U.S. dollars represent most of China's settlement of international trade while the use of euros in China comprise a much smaller share. This results in $\sigma_{eu} = (1, 1, 0)$ and $\sigma_{ch} = (1, 1, 1)$. Next I use data on international trade invoicing to pin down the costs for accepting dollars in Europe and China. Goldberg and Tille (2007) report that the share of dollar-denominated trade in Europe for 2002 ranges from 20.5% in Italy to 71.0% in Greece. I use the reported European average of 32.4% of dollar-invoiced trade to generate the fraction of trades using dollars in the Eurozone. Similarly in Asia, estimates of dollar-denominated trade range from 52.4% to 84.9%. I use the lower bound of 52.4% to determine the fraction of trades using dollars in China. Friberg and Wilander (2007) report an Asia-wide average of 8% of trade denominated in euros. Also as in the model, the cost of accepting one's domestic currency is assumed to be zero.

Another way to think about the model's cost to accept foreign money is a transaction cost or participation fee in foreign exchange markets. As a robustness check and in order to facilitate comparison with previous studies, I also present results for an alternative calibration in Appendix C that uses transaction costs in foreign exchange markets as measured by bid-ask spreads for various currency pairs.[35] In Section 7, welfare calculations from both calibrations have similar magnitudes.

[34]Freeman and Kydland (2000) undertake a similar calibration procedure to determine values for transaction costs by using data on currency-deposit ratios.

[35]The pattern of bid-ask spreads in the data provides a qualitative ranking of costs for the model that then drives the decision to accept a particular currency. Appendix C describes the data and calibration in more detail and also discusses a few shortcomings of this approach.

Table 4: **Parameter Values for 3-Region Model: U.S., Eurozone, China**

	Interpretation	Value	Target
b	–	0.0001	Ensures solution to bargaining problem
A	Output in CM	2.03	World GDP
β	Discount factor	0.966	Annual real interest rate = 3.5%
γ_{us}	Gross inflation (U.S.)	1.03	Average inflation rate (U.S.) = 3%
γ_{eu}	Gross inflation (Eurozone)	1.02	Average inflation rate (Eurozone) = 2%
γ_{ch}	Gross inflation (China)	1.05	Average inflation rate (China) = 5%
θ	Bargaining power	0.5	Egalitarian bargaining solution
g_{us}	Government size (U.S.)	0.05	Fraction state-owned enterprises (U.S.) = 0.05
g_{eu}	Government size (Eurozone)	0.22	Fraction state-owned enterprises (Eurozone) = 0.22
g_{ch}	Government size (China)	0.37	Fraction state-owned enterprises (China) = 0.37
$\psi^{\$}_{eu}$	Cost of accepting dollars in E.U.	0.024	E.U trades invoiced in dollars = 32%
$\psi^{\$}_{ch}$	Cost of accepting dollars in China	0.011	China trades invoiced in dollars = 52%
ψ^{e}_{ch}	Cost of accepting euros in China	0.079	China trades invoiced in euros = 8%

6.2 Parameter Estimates

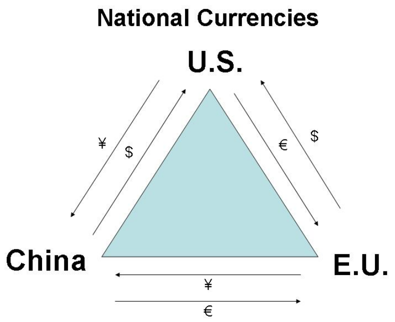
Tables 4 and 5 summarize the baseline calibration results for the three region model. With this calibration, the model can endogenously generate currency portfolios and quantities traded for a given circulation pattern. The next sub-section discusses the payment arrangements that emerge.

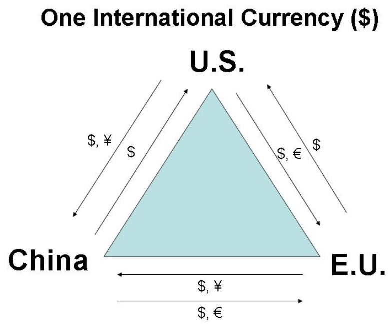
Table 5: **Calibrated Meeting Parameters: U.S., Eurozone, China**

Parameter	Estimate	Target Moment	Model	Data
n_{us}	0.36	GDP share (U.S.)	19.3%	19.1%
n_{eu}	0.37	GDP share (Eurozone)	19.7%	19.8%
n_{ch}	0.27	GDP share (China)	13.9%	14.3%
$\mu_{us,eu}$	0.238	Share of U.S. trade with E.U	11.7%	17.8%
$\mu_{us,ch}$	0.108	Share of U.S. trade with China	10.3%	14.9%
$\mu_{eu,us}$	0.186	Share of E.U. trade with U.S.	14.0%	13.8%
$\mu_{eu,ch}$	0.241	Share of E.U. trade with China	15.4%	13.3%
$\mu_{ch,us}$	0.227	Share of China trade with U.S.	13.8%	13.6%
$\mu_{ch,eu}$	0.268	Share of China trade with E.U.	17.0%	17.0%

6.3 System of World Payments

For the quantitative exercise, it is assumed that government sellers from each country only accept its domestic currency.[36] Note that even with legal restrictions on payments in government transactions, all private citizens are free to use any asset in their portfolio, which does not rule out an international role for any currency.



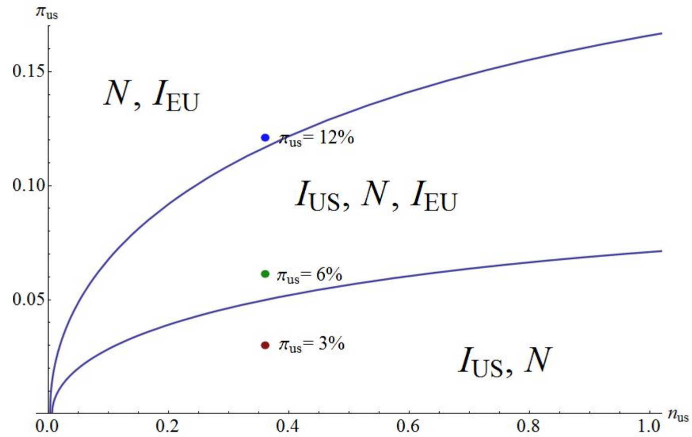


The baseline calibration yields two types of payment patterns: *(i)* national currency circulation, and *(ii)* one international currency (the dollar). Given that this transaction pattern reflects the current state of affairs, what factors can cause hegemony to shift?

The theory implies that changes in inflation is a channel through which monetary policy can influence macroeconomic outcomes. Figure 14 illustrates inflation's effect on international currency use. When U.S. inflation is low, there will be an equilibrium where only the dollar is internationally accepted. As U.S. inflation rises, it becomes costlier to hold dollars which then increases the expected benefit of accepting euros. Citizens in China substitute away from dollars into euros, leading the euro to be accepted alongside the dollar in transactions abroad. If U.S. inflation increases further, it may be possible that sellers from Europe no longer hold dollars and switch to solely accepting euros. In this case, euros circulate abroad while the dollar loses its international status and only circulates at home. Consequently, the extent to which the euro– or more generally, any competing currency– assumes the international role of the dollar will depend on fundamentals, such as monetary policy and inflation, as well as beliefs of market participants.

[36] To deal with China's fixed exchange rate, I assume that its government transaction policy is used to affect the DM market exchange rate with respect to the dollar, $e = q_{us}/q_{ch}$. Suppose that the initial, unresticted equilibrium is $(q_{us}, q_{ch}) = (q^0_{us}, q^0_{ch})$, but the Chinese government announces the official value the yuan to be $q_{ch} = q^0_{ch} - \epsilon$ while the dollar remains at its market value q_{us}. The equilibrium value of dollars stays the same while q_{ch} falls, moving the exchange rate in favor of dollars. The yuan is therefore undervalued and would appreciate if exchange rates were unrestricted. As in Li and Wright (1998), this results in a difference between official and market exchange rates in the DM, as equilibrium q_{ch} differs from the official value $q^0_{ch} - \epsilon$.

Figure 14: **Circulation Patterns and Inflation**



7 Welfare Benefits of International Currency

There are normative consequences of a switch from one type of monetary regime to another, as first discussed in Section 4.5, and these changes will generate real gains and losses for all countries in terms of economic welfare. To study the welfare effects of potential shifts in payment patterns, I follow the approach of Lucas (1987) and ask how much consumption agents demand, or are willing to give up, as compensation to move from regime $\Omega \in \{N, I_{US}, I_{EU}\}$ to another regime $\Omega' \neq \Omega$, where regime N denotes national currency use, regime I_{US} is one where the dollar is international, and regime I_{EU} is one where the dollar and euro share the international role.

Under a given regime Ω, steady-state welfare in each region $i \in \{us, eu, ch\}$ is measured as the steady-state sum of buyers' and sellers' surplus, weighted by their respective sizes: $\mathcal{W}_i = \mathcal{B}_i(1-\beta)V_i^B(z_1, z_2) + \mathcal{S}_i(1-\beta)V_i^S(0,0)$:

$$\begin{aligned}
\mathcal{W}_i(\Omega) = U(x^*\Omega) - x^* + \mathcal{B}_i\mathcal{T}_i + \alpha_i\mathcal{M}_i \sum_k \sigma_i^k S(q_{ii}^k(\Omega)) + \alpha_{ij}\mathcal{M}_j\theta \sum_k \sigma_j^k S(q_{ij}^k(\Omega)) \\
+ \alpha_{ji}\mathcal{M}_i(1-\theta) \sum_k \sigma_j^k S(q_{ji}^k(\Omega)) + \alpha_{i\ell}\mathcal{M}_\ell\theta \sum_k \sigma_\ell^k S(q_{i\ell}^k(\Omega)) \\
+ \alpha_{\ell i}\mathcal{M}_i(1-\theta) \sum_k \sigma_i^k S(q_{\ell i}^k(\Omega)) - \mathcal{S}_i\sigma_i\psi_i,
\end{aligned}$$

where $S(q(\Omega))$ is the equilibrium value for $S(q) \equiv u(q) - c(q)$ given Ω. Welfare depends on three components: (i) net consumption in the CM, (ii) net seigniorage revenue given by $\mathcal{B}_i\mathcal{T}_i$, and (iii) surplus from trading in the decentralized markets net of any information costs.

Suppose the economy moves from Ω to a different equilibrium Ω', but also adjusts consumption of all goods x and q by a common factor Δ. The amount $1-\Delta$ then measures the percentage gain, or loss if $1-\Delta < 0$, of consumption faced by agents per year. Adjusted or compensated welfare then becomes

$$\begin{aligned}\mathcal{W}_i(\Omega) = U(x^*\Omega\Delta) - x^* + \mathcal{B}_i\mathcal{T}_i + \alpha_i\mathcal{M}_i\{\sum_k \sigma_i^k[S(q_{ii}^k(\Omega)\Delta)]\} \\ + \alpha_{ij}\mathcal{M}_j\theta\sum_k \sigma_j^k[S(q_{ij}^k(\Omega)\Delta) + \alpha_{ji}\mathcal{M}_i(1-\theta)\sum_k \sigma_j^k[S(q_{ji}^k(\Omega)\Delta)] \\ + \alpha_{i\ell}\mathcal{M}_\ell\theta\sum_k \sigma_\ell^k[S(q_{i\ell}^k(\Omega)\Delta)] + \alpha_{\ell i}\mathcal{M}_i(1-\theta)\sum_k \sigma_i^k[S(q_{\ell i}^k(\Omega)\Delta)] - \mathcal{S}_i\sigma_i\psi_i.\end{aligned}$$

The compensating variation value $1-\Delta$ that solves $W_i(\Omega') = W_i(\Omega)$ is then the welfare benefit or cost of moving from regime Ω to Ω'. If $1-\Delta > 0$, agents are indifferent between being in Ω and being in Ω' with consumption reduced by $1-\Delta$ percent. Equivalently, agents are willing to give up $1-\Delta$ percent of consumption per year to be in regime Ω' rather than regime Ω.

Table 6: **Welfare Changes in Consumption Equivalent Terms (% of GDP per Year)**

	$\gamma_{US} = 1.03$ N to I_{US}	$\gamma_{US} = 1.05$ N to I_{US}	$\gamma_{US} = 1.06$ I_{US} to I_{EU}	$\gamma_{US} = 1.10$ I_{US} to I_{EU}
$1-\Delta_{US}$	1.41	2.07	-1.25	-1.56
$1-\Delta_{EU}$	0.33	0.24	1.60	1.93
$1-\Delta_{CH}$	0.39	0.32	0.28	0.25

Table 6 summarizes annual consumption equivalent welfare changes for transitions across various steady-state equilibria. For the U.S., the welfare benefit of having the dollar as the sole international currency ranges from 1.41% to 2.07% of consumption per year.[37]

This gain derives from two sources. The first source is from increased seigniorage revenues from foreigners. In reality, foreigners hold large quantities of U.S. dollars: Porter and Judson (1996) report that approximately 60% of dollar banknotes are held abroad. The source of seigniorage for the issuing country is therefore the ability to obtain real resources in exchange for virtually costless notes. In turn, the model implies that the flow of this international seigniorage to the U.S. is approximately 0.11% of GDP, consistent with Alogoskoufis and Portes (1991), Rogoff (1997), and Portes and Rey (1998). The second source however comes from the model's general equilibrium effects of increased international exchange: due to the increased acceptability of the dollar, there is

[37] Under the alternative calibration strategy with bid-ask spread data reported in Appendix C, the welfare gain to the U.S. is 1.33% to 1.98% of annual consumption.

now more surplus from international transactions. As more people use the dollar, its value goes up, which increases the amount of goods that can be purchased for a given unit. Finally, the impact of changes in information costs across different equilibria are quantitatively negligible in terms of annual GDP. While the absolute magnitudes of these costs are small, the model illustrates how their qualitative rankings have significant consequences that drive the choice of an international currency.

Both the Eurozone and China also benefit from using the dollar, owing to increased trade. The gain is larger for China due to a lower inflation tax (3%) than its domestic level (5%). Although Europe now must pay a higher inflation tax in a fraction of its transactions, this does not outweigh the increased surplus from an expansion in trade opportunities with the United States. Column 3 shows that while welfare in the U.S. increases to 2.07% of consumption per year if inflation rises from 3% to 5% due to more seigniorage revenue abroad, welfare in the Eurozone and China falls due to the higher inflation tax. The analysis also shows that for the U.S., the welfare cost of losing international status to the euro is 1.25% of annual consumption at a domestic inflation rate of 6%, which comes at a gain of 1.60% per year for the Eurozone.

The results in Table 6 also highlight the distributional effects of inflation across countries. For the issuing country, some inflation can be beneficial due to increased seigniorage revenue from abroad. However this comes at the cost of harming foreigners who have to pay a higher inflation tax. Monetary policy can therefore have distributional effects across countries by redistributing wealth from foreigners to domestic agents.

7.1 Discussion

The welfare gains of international currency use in this paper are larger than previous estimates. In particular, Portes and Rey (1998) report that the net gains from increased international use of the euro is about 0.4% of GDP for Europe: this comes from a direct effect of increased seigniorage (0.1% of annual consumption) plus gains from a liquidity discount (another 0.1% of GDP) plus welfare gains due to reduced transaction costs on euro financial markets (0.2% of GDP).

The discrepancies between the results presented here and the estimates in Portes and Rey (1998) come from the approach taken in their welfare analysis. The analysis in Portes and Rey (1998) is based on Rey (2001)'s model of vehicle currencies in foreign exchange markets. The model implies an inverse relationship between transaction costs and volumes, and they obtain a measure of the efficiency gains or losses associated with potential shifts in world payment patterns by multiplying transaction costs by volumes exchanged in each market and then summing across all markets. However since transaction costs in Rey (2001) are equilibrium objects, this approach neglects to capture the model's full effects since a reduction in transaction costs will increase volumes, which will in turn reduce transaction costs.

When instead welfare changes are measured using a model-based compensated welfare approach as in this paper, gains or losses are larger due to general equilibrium effects in international trade. While the exact quantitative results presented here will be sensitive to alternative modeling assumptions, such as the assumed pricing mechanism, or the calibration strategy for quantifying the costs of accepting foreign currency, the main message from the analysis is clear: having a model with microfoundations for international payments has quantitatively important implications for welfare that should not be ignored.

8 Conclusion

This paper provides an information-based theory of international currency by generalizing the recent model of asset liquidity by Lester, Postlewaite, and Wright (2012) to an open-economy setting. I investigate some classic issues in international monetary economics, such as the emergence of an international currency, the optimal choice of inflation in an open economy, and the welfare benefits of international currency use. Instead of assuming the payments used in each country, citizens' acceptance decision is made endogenous by letting private citizens choose which currencies to accept. Further, government transaction policies are introduced to examine how certain policies— namely ones which favor the use of a country's national money— affect private agents' acceptance decisions and hence the set of equilibria. Fairly innocuous policies of the kind considered ended up implying the connections observed in practice between currencies and countries.

This paper also explicitly modeled the strategic interaction among money issuers in a dynamic policy game. An inflation Laffer curve emerges and captures the main tradeoffs faced by the country issuing international currency. On the one hand, some inflation can be welfare-improving since this increases the amount of seigniorage extracted from foreigners; but of course too much inflation lowers the purchasing power of money. Since sovereign policymakers are only responsible for welfare for their own citizens, they are not penalized for any negative effects that their policies may have abroad. Non-cooperative behavior can therefore lead to optimal inflations rates above the Friedman rule and hence inefficient outcomes. If instead externalities can be internalized in a cooperative agreement, then all countries may benefit.

Quantitatively, the welfare cost of losing international status is not inconsequential for the issuing country. For the United States, this amounts to about 1.3% to 2.1% of consumption each year. This paper thus provides a first step in examining the effects of transitioning to different types of payment regimes using a microfounded model where credit is imperfect and accepting foreign currency comes at a cost. A useful direction for future work is to examine how these results are affected by extensions to the baseline model, such as incorporating a distribution of costs across countries instead of assuming a fixed information cost or introducing entry decisions by domestic or foreign firms. Using an alternative pricing mechanism, such as a mechanism design approach, is another fruitful direction for future work.

Consistent with historical evidence from Eichengreen (2011), this paper questions the conventional wisdom that competition for international currency status is a winner-take-all game. Just as history shows that several international currencies have often shared this role in the past, the theory implies that a likely situation for the future monetary system is one where several international currencies compete and coexist.

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Appendix B

Model Without Precautionary Demand for Foreign Currency

In the baseline model, there is a precautionary for foreign currency since it allows buyers to self-insure against the risk of not being able to use their domestic currency in some transactions. Here I consider an alternative specification of the model where buyers know the nationality of the seller they are matched with before making portfolio decisions. In particular, while agents are still matched pairwise in the DM, buyers receive a preference shock at the beginning of each CM that specifies which country's goods they prefer and hence which country's seller they match with in the subsequent DM. The instantaneous utility function of a buyer in country 1 is given by

$$\mathcal{U}_1^B = \alpha u(q_1) + (1-\alpha)u(q_2) + U(x) - h,$$

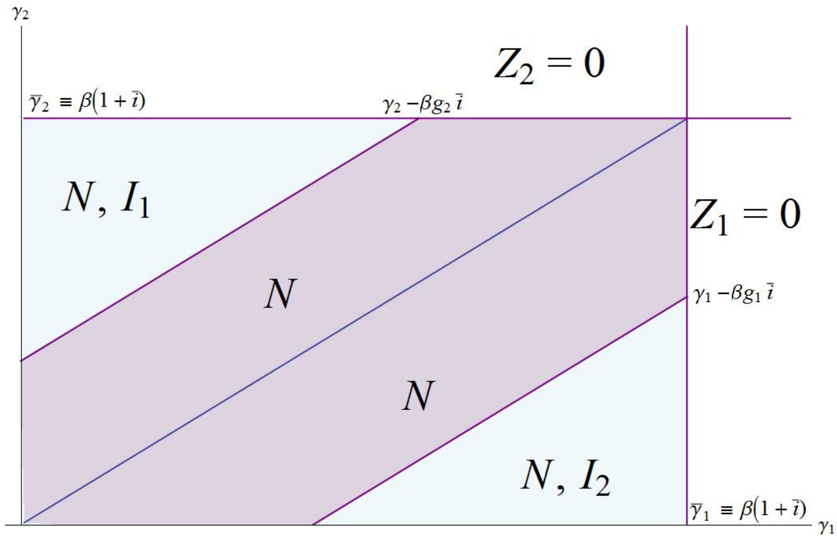
where $\alpha \in \{0,1\}$ is a random preference shock buyers receive in the CM that specifies which country's goods they prefer. When $\alpha = 1$, buyers prefer domestic goods (and hence a buyer from 1 will be matched with a country 1 seller); when $\alpha = 0$, the buyer prefers foreign goods (a buyer from 1 will be matched with a country 2 seller).

The following table summarizes the different types of monetary equilibria that can arise, assuming that all buyers receive an $\alpha = 1$ preference shock. In that case, buyers know they will always match with a domestic seller.

Existence of Equilibria			
Regime	Buyer Conditions	Seller Conditions	Portfolios
N	$i_1 < \frac{\theta}{1-\theta}$ $i_2 < \frac{\theta}{1-\theta}$	$\psi_1 > \overline{\psi}_1$ $\psi_2 > \overline{\psi}_2$	$z_1 > 0, z_2 = 0$ $\widehat{z}_1 = 0, \widehat{z}_2 > 0$
I_1	$i_1 < i_2 - g_2\frac{\theta}{1-\theta}$ $i_2 < \frac{\theta}{1-\theta}$	$\psi_1 > \overline{\psi}_1$ $\psi_2 < \widetilde{\psi}_2$	$z_1 > 0, z_2 = 0$ $\widehat{z}_1 > 0, \widehat{z}_2 > 0$
I_2	$i_1 < \frac{\theta}{1-\theta}$ $i_2 < i_1 - g_1\frac{\theta}{1-\theta}$	$\psi_1 < \widetilde{\psi}_1$ $\psi_2 > \overline{\psi}_2$	$z_1 > 0, z_2 > 0$ $\widehat{z}_1 = 0, \widehat{z}_2 > 0$
U	$i_1 = i_2 \equiv i$ $i < \frac{\theta}{1-\theta}$	$\psi_1 < \overline{\psi}_1$ $\psi_2 < \overline{\psi}_2$	$z_1 + z_2 > 0$ $\widehat{z}_1 + \widehat{z}_2 > 0$

Figure 15 represents the existence of different types of equilibria as a function of the money growth rates or inflation rates in the two countries. As in the baseline model, the presence of government agents affects the prevalence of international currency equilibria. When $g_1 > 0$ and $g_2 > 0$, buyers may still meet government agents that only accept domestic currency even if private

Figure 15: Existence of Equilibria in (γ_1, γ_2)-Space



sellers accept both. As a result, regimes I_1 and I_2 still feature a subset of buyers that hold both currencies. As before, the fact that multiple currencies circulate at potentially different rates of return for some fraction of internal trade means that currency substitution, or dollarization, is occurring.

When government size is at its maximum ($g_1 = 1$ and $g_2 = 1$), the only equilibrium where both currencies are valued is the national currencies equilibrium. In this case, the enforcement power of the state drives foreign currency out of circulation of its country so that only national monies are used. With no government ($g_1 = 0$ and $g_2 = 0$), regimes N, I_1, I_2, and U are all possible and there is a coordination problem for a large region of the parameter space.

The non-monetary equilibria where only one currency is valued while the other is not (corresponding to the regions labeled $Z_1 = 0$ and $Z_2 = 0$ in Figure 15) is a very robust feature of the model and still survives in this alternative formalization. This type of equilibrium is consistent with a fully dollarized world where a country has completely switched to a foreign currency such as the dollar for internal trades while the highly inflationary domestic currency has been driven out of circulation. Finally, an equilibrium where the two currencies become unified and are perfect substitutes is another robust outcome of the model. This case arises when sellers from both countries accept both currencies. As in Kareken and Wallace (1981), if both currencies are acceptable in trade, then the two must have the same rate of return. Consequently, there must be some kind of market incompleteness or friction for two currencies to coexist with different rates of return.

Appendix C

Alternative Calibration with Bid-Ask Spreads

Here I present quantitative results for an alternative calibration for three regions– the U.S., Eurozone, and China– using historical data for bid-ask spreads. As before, the three region world model is used as a framework to analyze alternative steady-state regimes and calculate the welfare benefits of international currency use.

Parameter values for the model's cost of accepting foreign currency are calibrated using bid-ask spreads for currency pairs involving the U.S. dollar, euro, and the Chinese yuan. In general, bid-ask spreads reflect the out-of-pocket costs incurred by investors and are an important factor in determining the trading behavior of market participants. The pattern of bid-ask spreads in the data then provides a qualitative ranking of costs for the model that then drives the decision to accept a particular currency. I use historical bid-ask spread data for the three currency pairs over the sample period 1999 to 2005 from Bloomberg. Table 7 reports the normalized bid-ask spreads in the data, defined as the ask minus bid price divided by the bid-ask midpoint. Magnitudes for these spreads are then mapped into the model's costs by converting to utility units.

Table 7: **Calibrated Information Costs Using Bid-Ask Spreads**

	Interpretation	Value	Target
$\psi_{eu}^{\$}$	Cost of accepting dollars in E.U.	0.0003	Mean euro/$ bid-ask spread = 0.07
$\psi_{ch}^{\$}$	Cost of accepting dollars in China	0.061	Mean $/yuan bid-ask spread = 3.18
ψ_{ch}^{e}	Cost of accepting euros in China	0.068	Mean euro/yuan bid-ask spread = 3.32

Table 8: **Welfare Changes in Consumption Equivalent Terms (% of GDP per Year)**

	$\gamma_{US} = 1.03$ N to I_{US}	$\gamma_{US} = 1.05$ N to I_{US}	$\gamma_{US} = 1.06$ I_{US} to I_{EU}	$\gamma_{US} = 1.10$ I_{US} to I_{EU}
$1 - \Delta_{US}$	1.33	1.98	-1.67	-1.79
$1 - \Delta_{EU}$	-0.20	-0.27	1.41	1.57
$1 - \Delta_{CH}$	-0.36	-0.38	0.07	0.04

As in the baseline analysis, the results in Table 8 reveal quantitatively significant welfare gains for the country issuing an international currency. For the U.S., the welfare gain of having the dollar as the sole international currency ranges from 1.33% to 1.98% of GDP per year and are of a similar magnitude as the baseline calibration. For the Eurozone, the welfare gain of a switch from a regime where the dollar is international to a regime where the euro is international ranges from 1.42% to

1.67% of annual consumption. Gains are slightly larger for the Eurozone than for the U.S. owing to higher GDP and lower average inflation rate in the former.

There are however some discrepancies between the costs in the model and bid-ask spreads in the data. First, bid-ask spreads are variable costs that typically increase with the size of the trade, whereas the model features a fixed cost incurred ex-ante. However bid-ask spreads can also be interpreted as a participation fee that market participants have to incur to trade on foreign asset markets. Second, transaction costs on foreign exchange markets may themselves be endogenous and depend on market factors. It is important for the theory that the cost is incurred ex-ante, before trades occur. While endogenizing this cost is beyond the scope of the present paper, constructing a search-based theory of bid-ask spreads on foreign exchange markets can be done along the lines of Lagos and Rocheteau (2009), where bid-ask spreads per unit of asset traded increase with the size of the trade and decrease with the extent of search frictions.