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October 2012

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Abstract
In this paper we set up an oligopolistic market model, where firms invest in pollution abatement in order to increase the whole market size via an increase in the consumers' reservation price. Moreover, we suppose that the demand function is not a linear one and the resulting game is not a usual linear quadratic one. In the considered model we investigate the open loop, the memory less closed-loop and the collusive patterns equilibrium. Additionally, we examine the social planning perspective. In the case of a convex demand we found the surprising result that the control and state variables have higher values in the open-loop steady state equilibrium than in the closed loop, while in a linear demand case the equilibrium is undetermined. In all cases we find that only if the market demand has concave curvature are the conclusions clear. A number of propositions and remarks are provided.

Keywords: Oligopoly Game; non-linear demand; pollution abatement; reservation price.

JEL Classifications: Q52; Q58; D43;C61; C62.

1. Introduction

Broadly speaking, the main difference between the open-loop equilibrium, on the one hand, and the feedback and closed-loop equilibrium, on the other hand, is that the former does not take into account strategic interaction between players through the evolution of state variables over time and the associated adjustment in controls. According to the open-loop rule, players choose their respective plans at the initial date and commit to them forever. Therefore, in general, open-loop equilibrium is not sub-game perfect, in that it is only weakly time consistent, since players make their action 'by the clock' only. However, there are classes of games where open-loop equilibrium is sub-game perfect (Clemhout and Wan, 1974; Dockner, Feichtinger and Jorgensen, 1985; Reinganum, 1982; Mehlmann, 1988).

A further distinction can be made between the closed-loop equilibrium and the feedback equilibrium, which are both strongly time consistent, therefore sub-game perfect since, at any date τ, players decide 'by the stock' of all state variables. While the closed-loop memory less takes into account the initial and current levels of all the states, the feedback equilibrium takes into account the accumulated stock of each state variable at the current date. If one player decides according to the feedback rule, then it is optimal for the others to do so as well. Hence, the feedback equilibrium is closed-loop equilibrium, while the opposite is not generally true.

The aim of this paper is to assess comparatively the properties of open–loop and closed-loop memory less equilibrium in a dynamic oligopolistic pollution model for which, firstly, the oligopolists face a non-linear market demand and, secondly, they attempt to increase the whole market size via an increase in the consumers' reservation price investing in pollution abatement.

The existing literature in the field of dynamic games devotes a considerable amount of attention to identifying classes of games where the feedback equilibrium degenerates into the open-loop equilibrium. Degeneration means that the Nash equilibrium time path of the control variables coincides under the different strategy concepts. In particular, it entails that the resulting open-loop solution is independent of the vector of the states.

The interest in the coincidence between the equilibrium paths under the different solution concepts is motivated by the following reason: Whenever open-loop equilibrium is degenerate feedback equilibrium, then the former is strongly time consistent. Therefore, one can rely upon the open-loop equilibrium which is, in general, much easier to derive than the closed-loop or feedback ones.

Classes of games where the coincidence arises are illustrated by a number of authors (Fershtman, 1987; Seierstad and Sydsaeter, 1977; Reinganum, 1982). As a whole, the games where open-loop equilibriums are strongly time consistent are known as ‘perfect or state redundant’, precisely because optimal controls derived from open-loop first order conditions depend only upon time and not upon states. A class of games where this clearly applies is that of linear state games, where the Hamiltonian function is linear in the state variables.

From the pollution point of view, it is well known that production process accumulates pollution as a by product, and this external economy must be corrected. Nowadays, the meaning of external diseconomies is essentially synonymous with externality or external effects in the sphere of production and consumption. External effects in production are unpaid side effects of one producer's output or input on other producers. At an earlier stage, external diseconomies in the meaning now given were called technological external diseconomies, reflecting the fact that the effects were transmitted outside the market mechanism and altered the technological relationship between the recipient firms output and the inputs under its control.

The most commonly used methods of correction found in economic literature are pollution permits and Pigouvian taxes on polluting firms. Specifically, the standard economic approach to externalities is typically ascribed to Pigou (1920), who devised a system of taxes and subsidies to allow for the social costs which are not included in private decision-making. In simple terms, a tax is placed on the polluter to bring his cost function into line with the true social cost of production. This approach to externalities has been challenged by pro-market theorists such as Coase (1960). The free-market approach identifies the problem of externalities as the absence of markets and the associated property rights. The assignment of property rights to non-marketed goods, such as air and water, provides a framework in which the parties are encouraged to resolve the problem of negative externalities rather than do nothing (Halkos, 1993, 1994, 1996).

In this paper we consider the basic implications of the pollution in the entire market where firms compete under the assumption that they understand and attempt to endogenize the externalities of the pollution occurrences. As pollution is very important for environmental policy modeling and analysis, an emerging research takes place in the last decades. In our study we adopt a number of sometimes strict assumptions made in other research areas.

First of all, we make the assumption that firms are oligopolistic rather than perfect competitors. The first justification is that perfect competition is not applicable to major polluting generating firms which are oligopolistic in their output markets. Second, we make the assumption that firms undertake the load of abatement, in contrast to the usual assumption where the abatement is undertaken by the government. Forster (1980) makes clear that abatement is a government policy or task.

There are a lot of endogenized variables that firms could take into account. For example, in the recent literature an endogenized variable could be the capital accumulation as an investment decision. Here the proposed model doesn't take the position of the capital accumulation as an investment decision, but rather takes the perspective that pollution abatement is an investment decision variable (Halkos and Papageorgiou, 2012).

The third assumption is that demand in the entire oligopolistic market is not a usual linear demand function but a more flexible one. Specifically, we accept a more general demand function affecting the consumers' reservation price, for which the curvature is determined by a factor α, to allow varying forms of concavity, linearity and convexity. As it is shown in the rest of the paper, the demand function employed affects demand elasticity, consumer surplus and also social welfare.

A last assumption for the proposed model is the dynamic environment within which the game evolves. Former models of Cournot oligopolistic markets are static and with or without linear demand functions. But dynamic modeling is the more general modern perspective that extracts several results depending on the informational structure employed by the game. For this reason we study the two main structures, open and closed loop, and draw the final conclusions.

The structure of the paper is as follows: Section 2 reviews the existing relative literature. In section 3 the basic model with the system dynamics is presented. Sections 4 and 5 describe the Open and Feedback Nash Equilibria. In section 6 we investigate the closed-loop memory less Nash equilibrium, while in section 7 the steady states of open and closed-loop memory less types of equilibriums are compared. Section 8 studies the resulting equilibrium in a firm's collusion proposing the social planning (under the collusion assumption). The last section concludes the paper.

2. Literature review

Nagurney and Dhanda (2000) investigate the modeling analysis and computation of solutions to models of multiproduct, multi-pollutant non compliant oligopolists who are engaged in a market of pollution permits. In order to compute the profit maximized quantities of emissions, they propose and apply an algorithm, along with the equilibrium allocation of licenses and their prices.

Taxing polluting firms or subsiding firms may be considered as an incentive to engage in pollution abatement. The general conclusion produced from a taxing policy, found in literature, is that an optimum tax rule must send to firms the message that the more they pollute now, the higher their future liability will be. The literature on taxing firms in a dynamic context arises from the static oligopoly framework used in Katsoulacos and Xepapadeas (1992) and Kennedy (1994).

The taxing literature consists mainly of two streams. The first one focuses on informational issues without paying much attention to the problem of stock accumulation, while the second one concerns the stock dynamics and assumes perfect information. In the second stream of taxing, studying the two types of equilibrium (open and closed loop) in a dynamic context Benchekroun and Long (1998) found that the optimal tax rules announced from the outset takes similar forms, but with different parameters.

This mode of regulation is exogenous to the market system and needs both social planning and optimal tax rules imposed by the government. One important reason for the firms to endogenize the pollution externality is that firms not engaged in abatement activities may see their profits and possibly the market size shrink due to pollution as well as due to the fall in the consumers' reservation price.

In this paper we consider a dynamic oligopolistic model where demand is not a usual linear function and firms undertake antipollution activities in order to increase the market size. The choice of the non linear demand function is tractable because it implicitly affects consumer surplus and also gives the possibility to increase the market size.

The recent literature in the case where firms compete under the demand constraint and or about demand linearity found is easy to classify in categories. In the first one, the non linear, Naimzada and Sbragia (2005) make the assumption of demand and quadratic costs in the form of $p = a - bQ^{1/2}$ and $C_i = c_{i0} + c_{i1}q_i + c_{i2}q_i^2$, $c_{ik} \geq 0, k = 0,1,2$ respectively. Using the "Gradient Dynamics" (GD) adjustment process, they adjust the firms' production in the direction indicated by their (correct) estimate of the marginal profit. Second, Leonard and Nishimura (1999) employ an arbitrary non linear demand without full information. Firms in a Cournot model do not observe their rival's actions, making mistaken beliefs. The above assumptions destroy the stability of equilibrium and create cycles, so the dynamics of the Cournot model are also affected.

In a different approach given by Bylka et al. (2000) the situation where oligopolistic firms compete with a global demand constraint is explored. The evolution of firms' market demand is determined by all firms' price decisions. Their interest focuses on the analysis of some simple classes of strategies and on finding the best responses to them. To that end, Huck *et al.* (2002) report results of the Cournot best reply process. In a Cournot oligopoly with four firms, linear demand and linear cost functions, the best reply process explode. They also investigate the power of several learning dynamics to explain the unpredicted stability.

To that end, the concept of consumers' valuation of a product or reservation price is used by Giridharan in a Bertrand like model for a new product introduction. In his model, each consumer makes his choice from among the available brands based both on the reservation price and actual prices. He shows, in equilibrium, that the price of the new product is equal to its mean(average) reservation price and those of the old ones are strictly greater than the respective mean reservation prices, denoting the importance of the consumers' valuation (reservation price) for a product market.

Finally, in the proposed model, the induced dynamic game for an oligopolistic market with nonlinear demand function and pollution abatement undertaken by firms is a non quadratic differential game for which the several informational equilibrium structures will be discussed.

3. Open-loop Nash Equilibrium (OLNE)

Let us assume that there are n firms in an oligopoly market that produce and sell a homogenous product. The market demand function is a non linear one, as in Anderson and Engers (1992, 1994)[1] which is used in the static Stackelberg hierarchical model in order to compare the results with the Cournot corresponding static model and is expressed as:

$$q(t) = r(t) - \left(p(t)\right)^{a} \qquad a > 0 \tag{1}$$

[1] To be precise, the market demand used by Anderson and Engers is in the form $Q = 1 - p^{a}$, $a > 0$, where parameter a stands as the curvature of the market demand, and no reference is made about consumers' reservation prices.

with $q(t)$ denoting the aggregate demand $\sum_{i=1}^{n} q_i$ and q_i the individual firm's demand. The positive parameter a shows the curvature of the demand function, so it is convex if $a \in (0,1)$, it is concave if $a > 1$ and it is linear if $a = 1$. The r parameter also measures the reservation price[2], r_j, of each j consumer, aggregating across all consumers. The r_j variable is implicitly an index to the consumer surplus and, consequently, to social welfare; that is, the higher the reservation price, the higher the consumer surplus. With the last observation, the reservation price r clearly affects aggregate demand in the sense that aggregate demand is the number of the consumers whose reservation price is at least r for one unit of the homogenous product.

The inverse demand function of the industry, then, is easily obtained as

$$p(t) = \left(r(t) - q(t)\right)^{1/a} \tag{2}$$

and the price p is always positive, i.e. $p(t) > 0$

The price elasticity of demand, in absolute terms, equals to

$$\left|\varepsilon_{q,p}\right| = \frac{\frac{\partial q}{\partial p}}{\frac{q}{p}} = \frac{p^a a}{r - p^a} = \frac{(r-q)a}{q}$$

and the consumer surplus (CS) is evaluated at the reservation price $q = r$, which also measures the social welfare, that is,

$$\mathrm{CS} = \int_0^r (r-q)^{1/a} dr = \frac{ar^{(a+1)/a}}{a+1}$$

with the above relations verifying that elasticity, consumer surplus and reservation price depend on the curvature of demand a.

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(2) For more details on consumers' reservation price, see Varian (1992).

Firms in the industry seek to increase the aggregate reservation price $r(t)$ and consequently the affected market factors, through investment in pollution abatement (e.g. recycling). Every abatement undertaken at time t by firm i is formally denoted by $A_i(t)$. This implies that firm i is able to increase the market size as a result of the firm's overall abatement. The reservation price is also subject to a constant depreciation rate δ.

Then the following differential equation could describe the dynamics of the system

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = \dot{r}(t) = \sum_{i=1}^{n} A_i(t) - \delta r(t) \tag{3}$$

where the differential $\mathrm{d}r/\mathrm{d}t = \dot{r}(t)$ represents the evolution of the reservation price (aggregating across consumers) in the industry and with $\sum_{i=1}^{n} A_i$ we denote the sum of the pollution abatement undertaken by all involved firms.

Every firm engaged in abatement faces a quadratic cost of the form

$$c_i\left(A_i(t)\right) = b\left(A_i(t)\right)^2, \quad b > 0 \tag{4}$$

In the dynamic problem, then, firms seek to maximize the present value of the future profits, i.e. total revenues minus total costs incurred from the investment in abatement. So, firm's i profits are:

$$\pi_i = \left(r(t) - q_i - q_{-i}\right)^{1/a} q_i - b\left(A_i(t)\right)^2$$

and firm's i problem can be expressed as the maximization of the present value function:

$$\max J_i = \int_0^{\infty} e^{-\rho t} \left[\left[r(t) - q_i(t) - q_{-i}(t)\right]^{1/a} q_i - b\left(A_i(t)\right)^2\right] dt \tag{5}$$

subject to $$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = \dot{r}(t) = A_i(t) + A_{-i}(t) - \delta r(t)$$

where we have set q_{-i} as the quantity produced by all other firms (players) except i firm (player) and $A_{-i}(t)$ the abatement of all other players except i. In this setting, the decision variables are the quantities q_i and the investment decisions A_i, while the state variable is the reservation price, $r(t)$.

If we decide to work with the current value Hamiltonian function denoting by $\mu_i(t)$ the co–state variable, then player's i Hamiltonian is the following function:

$$H_i = \left[\left(r(t) - q_i(t) - q_{-i}(t)\right)^{1/a} q_i - b\left(A_i(t)\right)^2 + \lambda_i\left(A_i(t) + A_{-i}(t) - \delta r(t)\right)\right] e^{-\rho t} \tag{6}$$

where $\lambda_i(t) = \mu_i e^{\rho t}$ is the current value of the co–state variable.

Let us next consider the Open-loopNash Equilibrium, that is, the Nash equilibrium in which every firm draws the time paths of the control variables independently of the states of the game. The only knowledge of the state is its value at the initial time $t = 0$, that is, the informational structure of this type of equilibrium is the singleton of the initial value of the state. The Closed-loop Nash Equilibrium is by definition the concept of equilibrium in which the choice of player's i current action is conditioned on current time t and on the state vector, too. Imposing this assumption on the informational structure of the game, the history of the game clearly becomes important and is reflected in the current value of the state vector.

Consequently, player $i's$ optimal time paths take into account the control variables of the other players at any point of time. This type of equilibrium affects the state variables, requiring a revision of the player's i controls at any time instant. Here we apply the so–called "memory less"[3] closed-loop equilibrium, according to which every player needs to know only the current value instead of the whole history of the state variable. The basic difference between the two type of equilibriums is that the first one, the open loop, is weakly time consistent, while the closed-loop is a strongly time consistent one. Here the time consistent property is in the sense of sub–game perfectness[4].

4. Open-loop Nash Equilibrium (OLNE)

The OLNE is obtained by the following system of FOC equations

$$\begin{aligned}
&\frac{\partial H_i(t)}{\partial q_i(t)} = 0 \\
&\frac{\partial H_i(t)}{\partial A_i(t)} = 0 \\
&-\frac{\partial H_i(t)}{\partial r(t)} = \frac{\mathrm{d}\lambda_i(t)}{\mathrm{d}t} - \rho\lambda_i(t)
\end{aligned} \tag{7}$$

and the transversality condition $\lim_{t\to\infty} r(t)\lambda_i(t)e^{-\rho t} = 0$. The first equation of system (7) can be expressed as

$$\frac{\partial H_i(t)}{\partial q_i} = -\frac{\left(r(t) - q_i(t) - q_{-i}(t)\right)^{(1/a)-1} q_i}{a} + \left(r(t) - q_i(t) - q_{-i}(t)\right)^{1/a} = 0$$

Making use of the symmetry of firms, by setting $q_i(t) = Q(t)$ and $q_{-i} = (n-1)Q$, the above first order condition (FOC) is simplified to

$$\frac{\partial H_i(t)}{\partial q_i} = -\frac{\left(r(t) - Q(t) - (n-1)Q(t)\right)^{(1/a)-1} nQ(t)}{a} + \left(r(t) - Q(t) - (n-1)Q(t)\right)^{1/a} = 0$$

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[3] The memoryless perfect state information pattern is defined by Olsder and Basar (1999) as the informational set whicht has two elements; one is the initial value of the state and the other element is the current value of the state variable.

[4] See Dockner *et al.* (2000) for more details about the sub–game perfectness and the time consistency equilibrium strategies.

and the solution with respect to $Q(t)$ is

$$Q=\frac{ar(t)}{1+an} \tag{8}$$

The second equation of (7) after the appropriate algebraic manipulations gives

$$\frac{\partial H_i(t)}{\partial A_i(t)}=-2bA_i(t)+\lambda_i(t)=0$$

or

$$\lambda_i(t)=2bA_i(t)$$

Differentiation of the last with respect to time t yields

$$\frac{\mathrm{d}A_i(t)}{\mathrm{d}t}=\frac{1}{2b}\frac{\mathrm{d}\lambda_i(t)}{\mathrm{d}t} \tag{9}$$

The third equation of system (7) gives

$$-\frac{q_i(t)}{a}\left(r(t)-q_i(t)-q_{-i}(t)\right)^{(1/a)-1}+\delta\lambda_i(t)=\frac{\mathrm{d}\lambda_i(t)}{\mathrm{d}t}-\rho\lambda_i(t)$$

or

$$\frac{\mathrm{d}\lambda_i(t)}{\mathrm{d}t}=-\frac{q_i(t)}{a}\left(r(t)-q_i(t)-q_{-i}(t)\right)^{(1/a)-1}+(\delta+\rho)\lambda_i(t) \tag{10}$$

Combining equations (9)-(10) we have

$$\frac{\mathrm{d}A_i(t)}{\mathrm{d}t}=-\frac{q_i(t)}{a}\left(r(t)-q_i(t)-q_{-i}(t)\right)^{(1/a)-1}+(\delta+\rho)\lambda_i(t) \tag{11}$$

Having the system of the two differential equations described by

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t}=\dot{r}(t)=A_i(t)+A_{-i}(t)-\delta r(t)$$

$$\frac{\mathrm{d}A_i(t)}{\mathrm{d}t}=-\frac{q_i(t)}{a}\left(r(t)-q_i(t)-q_{-i}(t)\right)^{(1/a)-1}+(\delta+\rho)\lambda_i(t)$$

and making use of the symmetry of firms that is: $q_i(t)=Q(t)$, $A_i(t)=A(t)$ and the relation (8), the system finally turns out to be

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = nA(t) - \delta r(t)$$
$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = (\delta+\rho)A(t) - \frac{1}{2b}\left(\frac{r(t)}{1+an}\right)^{1/a} \tag{12}$$

System (12) in the steady states is defined as $\frac{\mathrm{d}r}{\mathrm{d}t}=0$ and $\frac{\mathrm{d}A}{\mathrm{d}t}=0$. The steady state solution of system (12) is then given by:

$$A^* = \frac{\delta r}{n}$$
$$r^* = \left(\frac{n}{2b\delta(\delta+\rho)}\right)^{a/(a-1)} \frac{1}{(an+1)^{1/(a-1)}} \tag{13}$$

and the production level at the steady state is given by expression (8). Now we are able to analyze the stability properties of equilibrium.

4.1 Stability properties of equilibrium

In order to determine the stability of the dynamic system (12) we linearize this around the steady states, i.e.

$$\begin{bmatrix} \frac{\mathrm{d}r}{\mathrm{d}t} \\ \frac{\mathrm{d}A}{\mathrm{d}t} \end{bmatrix} = \Phi \begin{bmatrix} r(t) \\ A(t) \end{bmatrix}$$

where we have set

$$\Phi = \begin{bmatrix} \frac{\partial f}{\partial r} & \frac{\partial f}{\partial A} \\ \frac{\partial g}{\partial r} & \frac{\partial g}{\partial A} \end{bmatrix} = \begin{bmatrix} -\delta & n \\ -\frac{r^{(1-a)/a}}{2ab(an+1)^{-1/a}} & \delta+\rho \end{bmatrix}$$

and

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = f(A,r) = nA(t) - \delta r(t)$$

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = g(A,r) = (\delta+\rho)A(t) - \frac{1}{2b}\left(\frac{r(t)}{1+an}\right)^{1/a}$$

The trace and the determinant of the Jacobian matrix Φ, evaluated at the steady state level, have the following values

$$\mathrm{tr}(\Phi) = \frac{\partial f}{\partial r} + \frac{\partial g}{\partial A} = -\delta + \delta + \rho = \rho$$

In order to compute the determinant of the Jacobian matrix Φ we make use of the value of r^* previously found from system (12).

So the determinant is:

$$\det(\Phi) = \frac{\partial f}{\partial r}\frac{\partial g}{\partial A} - \frac{\partial g}{\partial r}\frac{\partial f}{\partial A} = -\delta(\delta+\rho) + n\frac{r^{(1-a)/a}}{2ab(an+1)^{-1/a}}$$

Substituting back the value of r into the latter and making the rest of the algebraic manipulations, the final result of the determinant is:

$$\det(\Phi) = (\rho+\delta)\frac{\delta(1-a)}{a}$$

For the saddle point path existence, it suffices the determinant of Jacobian matrix to be less than zero, that is, the following inequality to hold:

$$(\rho+\delta)\frac{\delta(1-a)}{a} < 0$$

This is clearly verified when the market demand is concave, that is, when $a > 1$ as already mentioned. From the previous reasoning the following proposition is obvious.

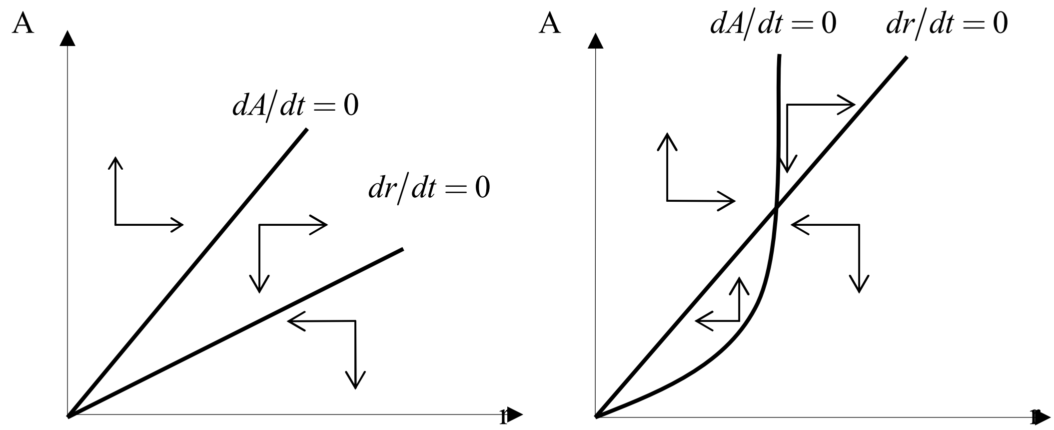
Proposition 1.

The steady state equilibrium of the game described by the solutions of the control variables $A^* = \frac{\delta r}{n}$ *and* $Q^* = \frac{ar^*}{(1+an)}$ *and the state variable* $r^* = \left(\frac{n}{2b\delta(\delta+\rho)}\right)^{a/(a-1)} \frac{1}{(an+1)^{1/(a-1)}}$ *is a saddle point path if and only if the market demand is concave.*

Remark 1.

In the other two cases where the market demand is convex or linear the equilibrium is unstable and degenerated respectively. In the first case of convex demand, both the trace and the determinant of the Jacobian matrix are positive, so equilibrium is unstable. Similarly, in the second case where the value of the state variable is an undetermined magnitude or the determinant of the Jacobian matrix is zero, we have a degenerated equilibrium. In other words, the solution of the dynamical system (12) is a collection of parallel lines.

The phase diagrams for the three curvatures of the demand function are shown in the following panels of the figure 1 at steady state.



(a) (b)

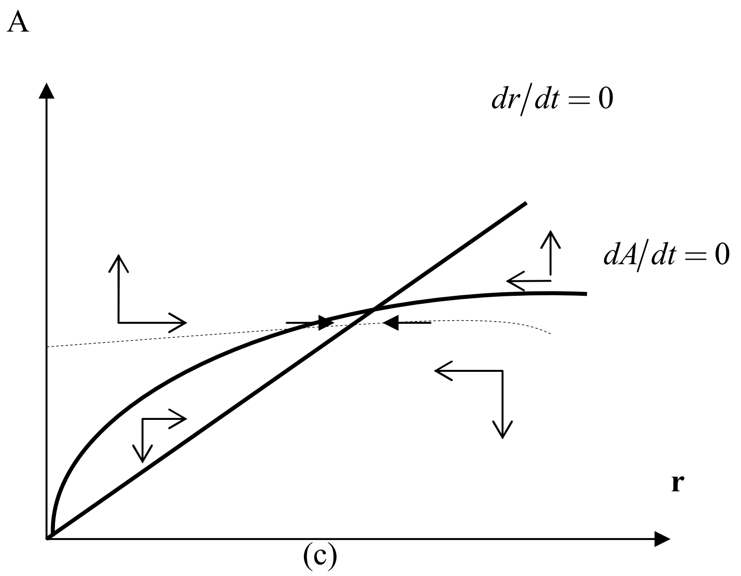


Figure 1: Phase diagrams for the three curvatures of the demand function.

In panel (c) the dotted line identifies the saddle path.

5. The Collusive problem

In this section we study equilibrium under the hypothesis that firms collude in order to maximize the present value of the joint profits. Moreover, we assume symmetry of firms. Then the dynamic problem is defined as follows.

$$\max J = \int_0^\infty e^{-\rho t}\left[n\left(r(t)-nq(t)\right)^{1/a} q(t)-nb\left(A(t)\right)^2\right]dt$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = nA(t)-\delta r(t) \qquad (24)$$

This is a standard optimal control problem and it can be solved using control theoretic methods.

Working with the current value Hamiltonian, this function is expressed as

$$H^c = \left[\left[r(t)-nq(t)\right]^{1/a} nq(t)-nb\left(A(t)\right)^2+\lambda\left[nA(t)-\delta r(t)\right]\right]e^{-\rho t} \qquad (25)$$

Where H^c represents the collusive Hamiltonian function. The first order conditions of (25) according to Pontryagin maximum principle give the following conditions

$$\begin{aligned}
&\frac{\partial H^c(t)}{\partial q(t)} = 0 \Rightarrow -\frac{n^2 q(t)}{a}\left[r(t) - nq(t)\right]^{\frac{1}{a}-1} + n\left[r(t) - nq(t)\right]^{\frac{1}{a}} = 0 \\
&\frac{\partial H^c(t)}{\partial A(t)} = 0 \Rightarrow \lambda(t) = 2bA(t) \qquad\qquad (26) \\
&-\frac{\partial H^c(t)}{\partial r(t)} = \frac{\mathrm{d}\lambda(t)}{\mathrm{d}t} - \rho\lambda(t)
\end{aligned}$$

and the transversality condition $\lim_{t\to\infty} r(t)\lambda(t)e^{-\rho t} = 0$

Solving the first condition of system (26) with respect to $q(t)$, the solution is

$$q(t) = \frac{ar(t)}{n(a+1)} \qquad (27)$$

which, compared with the open-loop solution (8) , is smaller.

From the two remaining conditions of system (26) we obtain the evolution equation of the reservation price as

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = (\rho + \delta)A(t) - \frac{1}{2b}\left(\frac{1}{1+a}\right)^{1/a}\left(r(t)\right)^{1/a} \qquad (28)$$

The dynamic system of the collusive firms is characterized, in the steady states, by the following equations

$$\frac{\mathrm{d}r(t)}{\mathrm{d}t} = 0$$

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = 0$$

where the solution of the first is

$$A = \frac{\delta r}{n}$$

and the solution of the second is

$$A = \frac{r^{(1/a)}}{2b(\rho+\delta)(a+1)^{(1/a)}}$$

The optimal investment in abatement effort corresponds to a curve, which may be convex $(a \in (0,1))$, concave $(a \in (1,+\infty))$ or linear $(a = 1)$ depending again on the curvature parameter a. As a consequence, the collusive solution exists at the steady states and its dynamic properties are the same as in the Cournot cases, i.e. this steady state is a saddle point in the case of $a \in (1,+\infty)$, i.e. the demand function is concave. From the previous reasoning the following result is easily obtained.

Proposition 2.

In the collusive case of firms, the steady states equilibrium of the original model is again a saddle point path if $a \in (1,+\infty)$ and ,in this case, the steady states levels of both r and A are larger than the steady states levels of open and closed-loop equilibrium.

At this point we can compare the collusive solutions with the decentralized ones. In the steady state equilibrium, the reservation price in both open and closed-loop is lower than in the collusive steady state equilibrium. This result is subject to the free riding problem. The economic interpretation of this result is straightforward: Since firms enjoy utility from the higher market reservation price $r(t)$, and -as already mentioned- affects the basic market factors i.e. market size, they have no incentive to invest in abatement but to benefit from the investment of the other firms.

5.1. The social optimum

We adopt a similar procedure to find the present value of the social welfare flows. The social welfare at time t is defined as the sum of the consumer surplus plus the firm's net profits. Taking into account the symmetry hypothesis, the consumer surplus and total profits at time t are respectively:

$$\mathrm{CS}(t)=\int_0^{Q(t)}\left[r(t)-s(t)\right]^{1/a}ds(t)=\frac{a}{1-a}\left[\left(r(t)\right)^{(a+1)/a}-\left(r(t)-Q(t)\right)^{(a+1)/a}\right] \quad (29)$$

$$n\pi(t)=nq(t)\left[r(t)-Q(t)\right]^{1/a}-nb\left(A(t)\right)^2 \quad (30)$$

Supposing that the social planner uses the same discount factor ρ for the social welfare (SW) flows, the dynamic problem is then expressed as:

$$\max \mathrm{SW}=\int_0^{\infty}e^{-\rho t}\left[\frac{a}{1-a}\left[\left(r(t)\right)^{(a+1)/a}-\left(r(t)-Q(t)\right)^{(a+1)/a}\right]+nq(t)\left[r(t)-Q(t)\right]^{1/a}-nb\left(A(t)\right)^2\right]dt$$

subject to $\dfrac{\mathrm{d}r(t)}{\mathrm{d}t}=nA(t)-\delta r(t)$ $\quad (31)$

The solution of problem (26) at the steady states, $\dfrac{\mathrm{d}A(t)}{\mathrm{d}t}=0$, is

$$A=\frac{r^{1/a}}{2b(\rho+\delta)} \quad (32)$$

This is the curve that lays above all its counterparts in the previous regimes for a concave curvature of the demand function. Then the following result is obvious:

Proposition 3

The socially optimal investment in pollution abatement, in the steady states, is higher than any other alternative informational structure, but only for a concave demand function, i.e. for all $a>1$.

Remark 2.

In the case of a convex demand function $a\in(0,1)$, the steady states values of the reservation price, r, and abatement, A, are both smaller than the respective variables of the Cournot game and the steady state of a linear demand, $a=1$, does not exist.

6. Concluding remarks

In this paper we set up an oligopolistic model where firms are engaged in abatement in order to increase the whole market size via an increase in the consumers' reservation price. We additionally assumed that the demand function is not a linear one and the resulting game is not a usual linear quadratic one. In the considered model we investigated the open-loop, the memory less closed-loop and the collusive patterns equilibrium, while in the feedback case only the linear demand is investigated making use of the dynamic programming Hamilton – Jacobi – Bellman equation. Moreover, we examined the social planning perspective.

In the model formulation and in the case of a convex demand we found as result that the control and state variables have higher values in the open-loop steady state equilibrium compared to the memory less closed-loop steady states, while in a linear demand case the equilibrium is undetermined. In all cases, we found that only if the market demand has a concave curvature, the conclusions are clear. Specifically, we found that the social planning case has all the values of the control variables higher than in all the other cases.

In the model presented we considered the reservation price as a kind of a public good. This means that every firm inside the market has access to this price and benefits from its higher value. As it is shown, comparing the collusive steady state equilibrium values of the reservation price $r(t)$ to the open and closed-loop values, the outcome is that an individual firm has no incentive to invest in antipollution effort, but benefits from the investment of the other firms. That is, an implication among the players of the game is the well known free-riding problem.

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