Duration dependence test for rational speculative bubble: the strength and weakness

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Duration Dependence Test for Rational Speculative Bubble:
The Strength and Weakness


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Abstract

This review highlights the strength and weakness of duration dependence test used by Mokhtar, Nassir and Hassan (2006) to detect the rational speculative bubble in the Malaysian stock market. It is found that despite the test’s strength over the other tests, it is however sensitive to different specifications and therefore may produce contrasting results.

Keyword: Rational Speculative Bubble, Duration Dependence Test, Stock market, Asian Financial Crisis.

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1. Introduction

The study by Mokhtar, Nassir, and Hassan in 2006 (henceforth MNH) “Detecting Rational Speculative Bubbles in the Malaysian Stock Market” investigates the presence of rational speculative bubble in Malaysian stock market by employing the duration dependence test (DDT) developed by McQueen and Thorley (1994) using the Malaysian stock indices’ monthly abnormal real returns for a period from 1994 to 2003. The findings suggest the presence of bubble in Malaysian stock market before and after the Asian Financial Crisis (AFC) 1997.

MNH revisits a situation when stock market, representing the well being of the economy, falls into troubles caused by the speculative trading which eventually plunges the stock market into crash as shown in the boom-bust cycles in Malaysian stock market in the 1990s, most noticeably the Asian Financial Crisis in 1997. The main motivation of their study is to ascertain the presence of bubble in Malaysian stock market which would provide an essential implication for both investors and policy makers.

This paper is a modest attempt to review MNH study by looking into the DDT and other econometric tests for bubble detection in the literature, to evaluate the tests’ strengths and weaknesses by examining the findings of other studies using DDT. On overall, it can be concluded that the various econometric tests for bubble detection including DDT cannot be achieved with a satisfactory degree of certainty and each method has its fair share of criticism. Despite its strength over cointegration tests, DDT is however sensitive over specification decisions and therefore its efficacy may be called into question.
This paper is organised as follows: this introduction being Section 1. Subsequently, Section 2 discusses several theoretical definitions of rational speculative bubble, followed by Section 3 that explains various econometric tests of bubble detection including DDT. Section 4 outlines several problems with MNH (2006) study and Section 5 concludes.

2. Definition of rational speculative bubble

MNH (2006) begin their paper with several theoretical definitions of asset price bubble as suggested by Kindleberger (1978), Garber (1990) and rational price bubble by DeLong et al. (1990), Chan et al. (1998), but they do not discuss long on it. They however mention an important feature of rational speculative bubbles i.e. the investors realized share prices exceed their fundamental value but they believe there is high probability that the bubble will continue to expand and lead to a high return, which compensates them for the probability of crash.

In general, the fundamental solution of an asset price ($p_t^*$) in the rational expectation approach is given as the sum of all future discounted dividends. The model explicitly assumes that there are no informational asymmetries, no time-varying risk premia which make consumer risk-neutral, that discount rate is constant which make the sum of the discounted dividend stream to be finite, and its generating process is not expected to change, and that transversality condition which make the sum of future resale price of the asset equals to zero must hold (Gurkaynak, 2005).
The presence of bubble, however, suggests there is other possible solution to the asset pricing model, noted by Shiller (1978), Blanchard and Watson (1982) and West (1987) as the following\textsuperscript{1}:

\begin{equation}
 p_t = p_t^* + b_t
\end{equation}

where $p_t$ is actual price and $b_t$ is the bubble component. Equation (1) assumes the transversality assumption is relaxed and therefore the asset pricing model consists of not only dividend component (in $p_t^*$), but also the bubble component. The bubble component must satisfy the following condition:

\begin{equation}
 E_t(b_{t+1}) = (1 + r)b_t
\end{equation}

where the bubble factor must at least grow progressively at the rate of $1 + r$, hence it is a sub-martingale. The growth rate of the bubble factor increases each period the bubble survives to compensate investors for the potential crash of a progressively larger bubble. When it survives, the explosive bubble becomes a more dominant component in the model, causing higher and higher observed (abnormal) returns\textsuperscript{2} leading up to crash. In other words, the rational speculative bubble is present when an asset price expands continuously to a level beyond its fundamental value. The persistent overvaluation of the asset is often followed by a market collapse or a burst in the asset price bubble.

\textsuperscript{1} For a review of the theoretical and empirical literature and a more formal derivation of the rational speculative bubble model, refer Camerer (1989). Blanchard and Watson (1982) suggest that bubble can grow and burst (and re-grow) with probability assigned to each case. Diba and Grossman (1987, 1988) observe that if there is a rational bubble, it must have existed from day one of trading, and rule out negative bubble. Chan et al. (1998) also highlight that bubble cannot exist when investors have infinite investment horizon, in assets with terminal values and when the supply of stock is not constant.

\textsuperscript{2} Abnormal returns normally defined as the excess difference between the actual returns and the expected returns.
As mentioned by MNH, an important feature in the rational speculative bubbles is that the investors are assumed to be rational, i.e. they realize that prices exceed the fundamental values, but they still believe that, with high probability, the bubble will continue to expand and yield high returns which compensate them for the probability of crash. Thus, the investors are considered rational despite their overstaying in the market after the sharp rise in asset price, justified by the perceived continuous expansion of the bubble with high probability of getting high returns to compensate them for the probability of the bubble bursts.

3. Econometric tests for bubble detection

MNH discusses several econometric tests of bubble detection in their study including tests for bubble premiums, tests for excess volatility, tests for non-stationarity and cointegration of asset prices and dividends, and DDT\(^3\). They conclude that the first three econometric tests of bubble detection have received their fair share of critiques and comments from various scholars because of their lack of robustness in testing the existence of bubbles. They also highlight DDT is a more widely accepted technique in detecting rational speculative bubbles in stock price.

A bubble premium is the excess returns the investors demand above the fundamental return in the presence of speculative bubble. It incorporates the actual excess return of the stock over the risk-free rate and has explosive nature over time. Examples of studies employing these tests are by Hardouvelis (1988), DeLong et al. (1990), Rappoport and White (1993), and Liu et al. (1995). The tests for the presence of

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\(^3\) In Gurkaynak (2005) test for excess volatility is called variance bound test. He reviews a number econometric test for bubble detection including the variance bound tests, West’s two-step tests, cointegration tests, and intrinsic bubbles. He concludes that the available econometric tests for detection of asset price bubble cannot be achieved with satisfactory degree of certainty, and the bubbles from time-varying or regime-switching fundamentals are still indistinguishable.
bubble premium, however, face serious problem as they are not able to prove or adequately disprove the existence of rational speculative bubble.

Tests for excess volatility examine the stock markets’ variance, and if the speculative bubble is presence, the variance of a stock price will be greater than the variance of its fundamental price. Among the researchers who used this technique are Friedman (1953), Baumol (1957), Kohn (1978), Flood and Garber (1980), LeRoy and Porter (1981), Shiller (1981, 1997), Hart and Kreps (1986), Mash and Merton (1986), Kleidon (1986), West (1987), and Dezhbakhsh and Demirguc-Kunt (1990). These tests have problem with the implementation of a specific bound for the variance that makes them unsuitable for bubble detection, but MNH fail to mention this problem.

The tests for cointegration of asset prices and dividends as proposed by Diba and Grossman (1988) are the widely used method to test for rational bubbles. It involves determining non-stationarity and cointegration between the stock prices and the fundamental variables (including dividends) determining the prices. A cointegration relation between prices and dividends implies price convergence to fundamental value to satisfy the long run equilibrium condition. Conversely, the lack of cointegration indicates the presence of bubble as price will diverge from the long run fundamental value.


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4 For more discussion on variance bound problem, please refer Gurkaynak (2005).

Criticism for the cointegration tests comes from those arguing that unit-root based tests have difficulties of detecting bubbles; either they are periodically collapsing bubbles (Evans, 1991) or bubbles existing from a stochastic explosive root (Charemza and Deadman, 1995). Taylor and Peel (1998) point out that although rational speculative bubble implies non-cointegration between asset prices and dividends, the traditional cointegration tests are subject to size distortion or specification error especially in small samples.
Specifically, the cointegration test depends mainly on the correct identification of the fundamental variables. The lack of cointegration could therefore be interpreted as an evidence of a bubble or the result of missing fundamental factors (Brooks and Katsaris, 2003; Anderson et al., 2003). Lim (2005) and Jiang and Lee (2005) show that, beside dividends, accounting data such as earnings provide useful information about stock price movements. Jirasakuldech et al. (2008) report no evidence of cointegration relationship among prices, dividends and earnings indicating the presence of bubbles in Thailand equity market for pre-Asian financial crisis 1997. For post-crisis, however, prices appear to be in line with fundamentals. Conventional cointegration tests have low power when limited data span are used, and furthermore the cointegration results are sensitive to data that are subject to regime shifts or structural changes (Shiller and Perron, 1985; Brooks and Katsaris, 2003). A few, but large highly persistent shocks in the systems or a change in the economic regime will bias the cointegration test in favour of no cointegration relationship (Chow, 1998). The above concerns with the cointegration methodology call to question the definitiveness of the conclusions in the previously mentioned studies.

Duration dependence technique or DDT introduced by McQueen and Thorley (1994) is employed by MNH for the detection of rational speculative bubble in the Malaysian stock market. As discussed in Section 2 earlier, the rational speculative bubble leads to explosive price changes, i.e. the bubble grows each period it survives and eventually it dominates the asset price model. If the bubble continues, its innovation is positive and small relative to an infrequent but large negative innovation. In other words, negative abnormal returns become less likely and

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5Besides MNH, DDT is also employed by Chan et al. (1998), Lavin and Zorn (2001), Harman and Zuehlke (2004), Hassan and Suk-Yu (2007), Jirasakuldech et al. (2008), and Abdul-Haque et al. (2008), to name a few.
generally occur when the bubble bursts. Thus, if a stock price is considered to have a rational speculative bubble, a run of positive abnormal returns will exhibit negative duration dependence, i.e. the conditional probability of a run ending, given its duration, is a decreasing function of the duration of the run. In other words, with the assumption of rational speculative bubble presence, if currently a stock is experiencing a run of positive abnormal returns, the probability of obtaining negative abnormal returns will decline the longer the run established. Because the speculative bubble cannot be negative, the similar situation however cannot be inferred to a run of negative abnormal returns.

The duration dependence tests are conducted by analyzing the hazard rate \( h_i \) for positive and negative runs. The hazard rate is the probability of obtaining a negative return \( (\varepsilon_t < 0) \) given a sequence of \( i \) prior positive returns \( (\varepsilon_{t-1} > 0) \). In the presence of rational expectation bubbles, the hazard rate, \( h_i = \text{Prob}(\varepsilon_t < 0 \text{ given } \varepsilon_{t-1} > 0, \varepsilon_{t-2} > 0, ..., \varepsilon_{t-i} > 0, \varepsilon_{t-i-1} < 0) \) decreases with \( i \) or \( h_{i+1} < h_i \) for all \( i \).

As presented in their study, MNH explain that in order to apply the duration dependence test, firstly real returns are transformed into a series of run lengths of positive and negative observed abnormal returns. For example, a return series of five positive abnormal returns followed by four negative, three positive and finally four negative abnormal returns is converted in two data sets: a set of runs of positive abnormal returns with values 5 and 3 and a set of runs of negative abnormal returns with values 4 and 4. The separation into the two groups follows the same way as

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adopted by Blanchard and Watson (1982), Evans (1986), and McQueen and Thorley (1994).

Formally, the data consists of a set of $S_T$ of $T$ observations on the random length, $I$. A run is defined as a sequence of abnormal returns of the same signs. Thus, $I$ is a positive valued discrete random variable generated by some discrete density function $f_i \equiv \text{Prob} (I = i)$, and corresponding cumulative density function $F_i \equiv \text{Prob} (I < i)$. Define $N_i$ as the count of completed runs of length $i$ in the sample, the density function likelihood is:

$$L (\theta | S_T) = \sum_{i=1}^{\infty} N_i \ln f_i + P_i \ln (1 - F_i)$$

where $\theta$ is a vector parameters, the hazard function $h_i \equiv \text{Prob} (I = i \mid I \geq i)$ represents the probability that a run ends at $i$ given that it lasts at least until $i$.

A hazard function specification describes data in terms of conditional probabilities in contrast to the density function specification which focuses on unconditional probabilities. The choice between a hazard and density specification depends on the economic concept in question of interest. The study of bubble detection using DDT usually questions whether the probability that a return run continues depend on the length of the run, thus hazard specification is appropriate. The hazard function is related to the density function by:

$$h_i = \frac{f_i}{1-f_i} \quad \text{and} \quad f_i = h_i \prod_{j=1}^{i-1} (1 - h_j)$$

Using the relationship above, the hazard function version of the log likelihood is:

$$L (\theta | S_T) = \sum_{i=1}^{\infty} N_i \ln h_i + M_i \ln (1 - h_i) + Q_i \ln (1 - h_i)$$
where $N_i$ is the number of completed runs of length $i$ in the sample, and $M_i$ and $Q_i$ are the numbers of completed and partial runs with length greater than $i$, respectively. To test the null hypothesis of no rational expectations bubble test, MNH used two functional forms of the hazard function, first is Log-logistic hazard model specified by McQueen and Thorley (1994), and secondly Weibull hazard model as specified by Harman and Zuehlke (2004). Log-logistic hazard function is:

$$h_i = \frac{1}{1 + e^{-(\alpha + \beta \ln i)}}$$

(6)

The duration dependence test for Log-logistic hazard function is performed by substituting Equation (6) into Equation (5) and maximizing the log likelihood function with respect to $\alpha$ and $\beta$.

The Weibull hazard model is as the following:

$$S(t) = \exp(-\alpha t^{\beta t + 1})$$

(7)

Where $S(t)$ is the probability of survival in a state to at least time ($t$), and the corresponding hazard function is:

$$H(t) = \alpha(\beta + 1)t^\beta$$

(8)

where $\alpha$ is the shape parameter of the Weibull distribution, and $\beta$ is the duration elasticity of the hazard function. The fundamental assumption of the Weibull hazard model is a linear relationship between the log of the hazard function and the log of duration, where:
\[
\ln [h(t)] = \ln \left[ \alpha (\beta + 1) \right] + \beta \ln (t)
\]  

(9)

The duration dependence test for Wiebull hazard function is performed by substituting Equation (9) into Equation (5) and maximizing the log likelihood function with respect to \( \alpha \) and \( \beta \). The null hypothesis of no rational expectation bubbles implies that the probability of a positive run ending is unrelated to prior runs, in other words the hazard rate should be constant (\( H_0 : \beta = 0 \)). The alternative bubble hypothesis suggests that the probability of a positive run ending should decrease with the length run, i.e. decreasing hazard rate. Under the null hypothesis of no bubble, the likelihood ratio test (LRT) is asymptotically distributed \( \chi^2 \) with 1 degree of freedom.

MNH point out that Log-logistic hazard model and Weibull hazard model can be applied to detecting rational speculative bubble in Malaysian stock market since the results are reliable and error of bubble size between these models is small. The duration dependence tests use all returns without presupposing periods of bubbles and crashes, and allow for the nonlinearity inherent in returns if bubble presence. Additionally, duration dependence is more unique to bubbles than attributes such as autocorrelation, skewness and kurtosis. Positive autocorrelation, negative skewness and leptokurtosis may reflect the possibility of bubble presence but they also can be caused by changes in fundamental values (Chan et al. 1998).\(^7\)

The strength of DDT over cointegration tests is that it is more flexible and does not require the correct identification of the fundamental variables. It is therefore able to

\(^7\) Refer Chan et al. (1998) for more on factors that may affect fundamental values resulting in positive autocorrelation, negative skewness and leptokurtosis.
overcome the most serious criticism levelled against the cointegration tests that it is a joint test of the null hypothesis of “no bubble” and “no model misspecification.” The additional benefit of DDT is that it does not require the time series under investigation i.e. returns to be normally distributed (Jirasakuldech et al. 2008).

4. Criticism against MNH (2006) study

This review finds that MNH however do not to elaborate on the general characteristics of bubble except that they mention asymmetric and leptokurtic innovations may be indicative of bubble presence not fundamental alone. Furthermore, they fail to discuss the strength of DDT over cointegration test in a more detailed account.

In their testing, MNH split the data they used into three subperiods i.e. before (1994-1996), during (1997-1998) and after Asian financial crisis (1999-2003) but fail to mention the purpose of doing so. Jirasakuldech et al. (2008), in their study for detection of bubble in Thailand stock market, similarly divide the data into two subperiods i.e. pre- and post- AFC (1975-1997 and 1998-2006, respectively) and by doing so they aim to test for sensitivity of the results in the presence of an extraordinary event; in this case AFC 1997. Such subperiod analyses may be useful because in general the use of long time series decreases the probability of detecting bubble as the long time series interval smooth out the effect of many small bubble episodes. Jirasakuldech et al. however warn that testing for a bubble during a period of market crash (as in 1997) increase the probability of detecting a bubble as a result of “data snooping,” hence they report results for both the entire period and the subperiods to compare these two effects. MNH however do not report any result for the full period in their study.
Mixed results from the studies detecting speculative bubble using cointegration tests, and the advantage of DDT over cointegration tests apparently do not make DDT yielding entirely robust results. Like the studies using cointegration tests, the studies using DDT also show mixed findings.

In their study, MNH report that rational speculative bubbles are presence in Malaysian stock market for the period before and after AFC, but not during the crisis period. They also discover the average size of bubbles during post-crisis period is smaller than those of pre-crisis period. Meanwhile, Jirasakuldech et al. (2008), in their study using monthly abnormal returns of Thailand stock market, report evidence of the presence of rational speculative bubbles during the full period (1975-2006) and the period before crisis, but not after the crisis. Abdul-Haque et al. (2008) test for the presence of bubble in China stock markets using weekly abnormal returns of Shanghai and Shenzen stock markets for the period 1990-2007 and 1991-2007, respectively, and detect the presence of rational speculative bubbles. Chan et al. (1998) use both monthly and weekly abnormal returns in six Asian stock markets (Hong Kong, Japan, Korea, Malaysia, Thailand and Taiwan) and the U.S. stock market for the period 1975-1994 and report none of the seven markets have return characteristics that completely conform to the predictions of the rational speculative bubbles. Only in Thailand’s weekly returns does it conform to the duration dependence. Hasan and Suk-Yu (2007) report that despite the extreme fluctuations in the Middle East and North African stock markets, they do not find strong evidence of rational speculative bubbles in the perspective of both domestic and U.S.-based investors. Their study uses monthly price indices of various time periods of eight
stock markets in Middle East and North African countries to compute monthly returns and employs DDT to test for the presence of rational speculative bubbles.

If cointegration tests are criticized for its requirement for correct identification of the fundamental variables, Harman and Zuehlke (2004) highlight the weakness of DDT that it is sensitive to several specification decisions. They report contrasting conclusions to their tests of bubble detection with various specification choices including hazard models of Continuous Weibull, Interval Weibull, Discrete Weibull, and Discrete Log-logistic for monthly and weekly returns of equally-weighted and value-weighted portfolios of New York Stock Exchange (NYSE) securities and New York Stock Exchange-American Stock Exchange (NYSE-AMEX) securities. They find that DDT is sensitive to choice of sample period, the method of controlling for discrete observation of continuous duration, the use of equally weighted versus value-weighted portfolios, and the use of weekly versus monthly returns. They conclude that the sensitivity of DDT tests to so many specification choices calls into question the efficacy of using hazard models to test for speculative bubbles.

5. Concluding remarks

The paper by Mokhtar, Nassir and Hassan (2006), “Detecting Rational Speculative Bubbles in the Malaysian Stock Market,” is among the many studies in the literature employing duration dependence tests for rational speculative bubbles detection. Using the Malaysian stock indices’ monthly abnormal returns for a period from 1994 to 2003, they report evidence of bubble in Malaysian stock market before and after the Asian Financial Crisis (AFC) 1997. This review paper is a modest analysis of MNH study, and it briefly discusses the fundamental asset price solution and rational speculative bubble before moving on to investigating the DDT and other econometric
tests for bubble detection available in the literature. In general, despite the recent advances in econometric methodologies, the various econometric tests for bubble detection including DDT cannot be achieved with a satisfactory degree of certainty and have their fair share of criticism. Despite its strength over cointegration tests, DDT is found to be sensitive over specification decisions and its uses in studies for rational speculative bubbles detection are therefore still disputable.
References


