Information provision by regulated public transport companies

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*Highlights

Information provision by regulated bus companies (*)

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Research highlights

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Abstract

We study the interaction between pricing, frequency of service and information provision by public transport firms offering scheduled services, and we do so under various regulatory regimes. The model assumes that users can come to the bus stop or rail station at random or they can plan their trips; the fraction of users who plan their trips is endogenous and depends on the frequency of service and on the quality of information provided. Four institutional regimes are considered, reflecting various degrees of government regulation. A numerical example illustrates the theoretical results. Findings include the following. First, fare regulation induces the firm to provide less frequency and less information than is socially optimal. Second, if information and frequency did not affect the number of planning users a higher fare always induces the firm to raise both frequency and the quality of information. With endogenous planning, however, this need not be the case, as the effect of higher fares strongly depends on how frequency and information quality affect the number of planners. Third, a profit-maximizing firm offers more information than a fare-regulated firm. Fourth, if the agency regulates both the fare and the quality of information then more stringent information requirements induce the firm to reduce frequency; this strongly limits the welfare improvement of information regulation. Finally, of all institutional structures considered, socially optimal fares, frequency and quality of information stimulate passengers least to plan their trips, because the high frequency offered reduces the benefits of trip planning.

JEL: L15,L32,L92,L98
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1. Introduction

Investment in the provision of accurate and easily accessible information to potential customers has become an important policy instrument for firms in almost all industries. Public transport firms -- typically offering scheduled services -- are no exception, because better information reduces the generalized cost of public transport trips for consumers. This generalized cost includes the fare, the time costs of traveling and of waiting at the bus stop or at the rail station (Mohring (1972)), and the schedule delay costs associated with arriving earlier or later than desired (Small (1982), Arnott, de Palma and Lindsey (1993)). Moreover, it is now generally recognized that it also includes the costs of planning the trip (see, e.g., Jansson (1993), De Palma and Lindsey (2001) and Fosgerau (2009)). The level of the information provided to passengers when planning their trips then affects the generalized cost in two ways. It directly reduces the costs associated with planning these trips; indirectly, as it leads more travelers to plan their trips, it also saves on passengers' overall waiting time costs.

Although a large literature exists on fares and frequency decisions by public transport firms, the role of information provision has not been studied before[1]. In this paper, we study the interaction between pricing, frequency of service and information provision by public transport firms offering scheduled services, and we do so under various regulatory regimes[2]. Four regimes are considered, reflecting different degrees of government regulation. First, in line with regulatory policies in some European countries, we look at a setting where a price-regulated profit maximizing public transit firm is responsible for determining frequency of service and the provision of information, conditional on a regulated price imposed by a government agency. The interaction between the firm and the government agency is modeled as a leader-follower game with the agency acting as the leader. The government sets the fare so as to maximize social welfare, taking into account the benefits to users as well as the firm's profit. Next we

[1] Of course, there is a large literature on the economics of information in industrial organization. This literature studies, among others, the advantages of private information (see, e.g., Einy, Moreno, Shitovitz (2002) and Chokler, Hon-Snir, Kim and Shitovitz (2006)), the relation between information acquisition and market structure (Dimitrova and Schlee (2003) and Iossa and Staffolini (2002)), the welfare effects of ignoring private information (Vives (2002)), and the optimal disclosure of privately held information (see Milgrom (2008) for a recent survey)).

[2] With minor qualifications, see section 3.2 below, the model equally applies to bus and railroad companies offering scheduled passenger services.

study the interaction between pricing, frequency and the provision of information assuming that the government agency regulates both fare and the quality of information provided to passengers. This is a policy-relevant exercise: although in most countries the quality of information is not explicitly regulated, there are notable exceptions. For example, in the UK, contracts between the regulatory agency (The Office of Rail Regulation) and private railroad operators explicitly impose restrictions on the information provided to passengers[3]. Finally, we compare the outcomes under regulation with the social optimum and with the outcomes under pure profit maximizing behavior. Numerical analysis illustrates the theoretical results.

Public transport firms typically offer various types of information (e.g., about time tables, changes in schedules, or expected delays) through several different channels, including websites, information boards on platforms, etc. The main purpose of providing high quality information is that this facilitates trip planning by passengers. Of course, the cost of providing information can be very substantial, and it increases with the quality of information offered. It may involve, for example, designing and maintaining websites, providing real-time information about route and schedule changes, etc. The model we study below applies to any type of information that is made available by the firm, but where passengers incur a cost of learning or extracting the exact information they need. It is assumed that this cost is lower for higher quality information. Moreover, the cost to the firm of providing the information is assumed to be independent of the number of travelers. As an example, think of the information about the time schedules and expected delays public transport firms offer on their website. Extracting the required information is costly, but the firm can reduce this cost by offering high quality search procedures and investing in a user-friendly website; moreover, the cost of providing the information does not depend on the number of travelers[4].

Our findings may be summarized as follows. First, we show that a fare-regulated public transport firm will (conditional on a given fare) provide less frequency and invest less in information provision to passengers than is socially optimal. If information and

[3] See the operator licensing information on www.rail-reg.gov.uk.

[4] As noted by a referee, the assumption that the cost to the firm is independent of the number of passengers implies that the model does not apply to all types of information. For example, information provided via (printing and distributing) physical booklets is not captured by the model.

frequency do not affect the number of planning users, then a higher regulated fare always induces the firm to raise both frequency and the quality of information provision. However, this is no longer necessarily the case once one accounts for the endogeneity of the number of people planning their trips. If providing more information is very effective in reducing planning cost, increasing the regulated fare may in fact reduce frequency. Similarly, a higher regulated fare may reduce information provision if people attach a high value to waiting time, so that offering more frequency is more effective at reducing the generalized cost of trips. In general, the firm puts more emphasis on providing more information relative to raising frequency if more people plan their trips. Second, if the government agency not only regulates the fare but also the quality of information provided to passengers, this induces the firm to offer very low frequency. Because of this, the welfare improvement due to information regulation is limited. Third, a profit-maximizing firm offers more information than a fare-regulated firm. Fourth, the numerical illustration suggests that delegating all decisions to a profit maximizing firm yields frequencies and provision of information that are close to the social optimum, but large welfare losses still result due to the high fare. Finally, of all institutional structures considered, socially optimal fares and service qualities stimulate passengers least to plan their trips: the high frequency offered under the socially optimal policy reduces the benefits of trip planning.

The structure of the paper is as follows. In the next section, we briefly review earlier literature on public transport policy-making and the role of information provision. In Section 3, we develop the structure of the model, focusing on the introduction of information and the cost of planning. Section 4 briefly considers two benchmark cases, viz. the social optimum and the maximum profit solution. Fare regulation is dealt with in Section 5. We study the behavior of a price-regulated public transport firm that is responsible for determining its headway and the level of information provided to users, conditional on the price set by government. The fare is determined by the government agency taking into account the firm's responses to price adjustments. In Section 6 we study the case where the government agency not only controls the fare but also imposes information requirements on the firm. A comparison of outcomes for price, frequency and information provided by the firm is given in Section 7 under the four different scenarios studied: the social optimum, the two regulatory regimes and the profit maximum. Numerical analysis illustrates the theoretical outcomes in Section 8. A final section summarizes the main findings.

2. Previous literature

This paper builds upon several strands of literature. First, as providing information can be seen as a quality indicator of public transport supply, our model relates to the literature on quality and quality regulation originating with the seminal paper by Spence (1975). He shows that a profit maximizing monopolist may offer lower or higher quality than is socially optimal, depending on the relative quality valuation of marginal and infra-marginal consumers (also see Sheshinski (1976)). Moreover, fare regulation may provide such firms with incentives to reduce quality (see, for example, Sappington and Weisman (1996), and Brueckner (2004)); a variety of policies have been designed to mitigate this problem, including revenue-sharing and profit-sharing penalty schemes (for further discussion, see Weisman (2005)).

Second, there is an extensive literature dealing with frequency and pricing decisions in the transport sector which has, with very few exceptions, ignored the role of information provision[5]. In a seminal paper, Mohring (1972) investigates the consequences of scale economies for optimal fares and frequency for urban bus services, paying particular attention to the effect of the number of users on waiting time. He assumes that the waiting time is proportional to the headway (the time between two departures); neither schedule delay costs (costs associated with arriving earlier or later than desired) nor planning costs are included in his analysis. His work was extended in various directions to capture, among others, optimal vehicle size (Jansson (1980)), the role of accident risks (Evans and Morrisson (1997)), and optimal fleet size under capacity constraints (Jara-Diaz and Gschwender (2003)). Moreover, Frankena (1981, 1983) studied the role of different objective functions of public transit operators and different

[5] De Palma and Lindsey (2001) consider the optimal time table of a firm offering scheduled services under fixed demand, and assuming a fixed number of departures over a period of fixed length. Users have linear, possibly heterogeneous, scheduling costs. They plan for a specific departure and differ with respect to their preferred arrival time. However, the authors do not study the interaction between price, frequency and information provision.

government subsidy formulas for optimal pricing and frequency decisions. Finally, a number of papers have explicitly taken into account competition from other transport modes. For example, Viton (1983) determines optimal modal composition in the peak period, using car and bus prices, the supply of highway lanes, and service characteristics of public transport (routes, frequency per route per hour) as policy variables. Similarly, Kraus (1989) developed a simulation model to study optimal pricing of car and bus use together with optimal frequency provision by the public bus mode, focusing on the relative efficiency of different pricing instruments in the presence of un-priced road congestion. De Borger and Wouters (1998) introduce optimal frequency and fleet size into a model of optimal pricing of transport services in the presence of congestion. Most recently, Bilotkach, Fageda and Flores-Fillol (2010) consider the differential behavior of scheduled service providers in terms of pricing and frequency decisions, depending on the distance of the service route and whether they face a competing mode.

Third, the literature on pricing and frequency decisions surveyed above typically ignored schedule delay costs due to undesirable arrival times; moreover, it implicitly focused on users who are not planning their trips, in the sense that they randomly arrive at the place of departure. This may be realistic for very frequent services. If people do not plan their trips then, in the worst case, if they arrive immediately after a departure they have to wait for a length of time equal to the headway, the time between two departures. However, for less frequent services (air service, many rail and bus services) or for frequent users of a particular service, it is more plausible that people do not arrive randomly but plan their arrivals at the place of departure (by consulting time tables before heading for the station, etc.). This paper also extends a small literature initiated by Panzar (1979), who introduced schedule delay costs and planning users in a model of optimal airline frequency and ticket price. Combining the cases of planning and non-planning users, Jansson (1993) analyzed the socially optimal choice of fare and service frequency. To distinguish users who plan from those who do not, he included a fixed cost of planning a trip, the same for everyone. Most recently, Fosgerau (2009) allows more flexibility, assuming a distribution for the cost of planning (which may be very different for trips an individual makes very frequently compared to trips that are not frequently made). The model implies a smooth transition between the two cases (planning and non-planning users) as headway increases. The author uses the model to determine the marginal cost of headway, but he does not study the implications of planning for optimal pricing and frequencies, nor does he analyze the effects of providing better information and the optimal investment in information.

3. Structure of the model

In this section, we present the structure of the model. Given the focus on information provision, we consider a single bus or rail line throughout and ignore many of the other complications considered in the literature (optimal vehicle size, optimal fleet size, etc.). A crucial ingredient of the model is that it allows for planned and unplanned trips, endogenously determining the number of planned trips as a function of the frequency and information provided by the public transport firm[6].

3.1. Generalized costs for planned and non-planned trips

We assume that travelers can plan their trips or just go to the bus stop or rail station at random. To make the distinction as transparent as possible, we assume that planning users incur planning costs but do not incur waiting time at the stop. Users who do not plan a particular trip have no planning costs but incur waiting time costs[7]. Moreover, both planning and non-planning travelers pay the fare, and they incur schedule delay costs associated with deviations between the scheduled arrival time and their preferred arrival time. The cost of travel time is ignored, because it is unaffected by the policy variables studied in this paper.

First, consider schedule delay costs. In order to focus on the trade-off between waiting time costs and planning costs, we keep the specification of schedule delay costs

[6] In principle, the firm may direct different types of information towards different types of passengers. For example, users that plan their trips ex ante may benefit substantially from a high-quality website. Passengers not planning their trips ex ante do not benefit from this type of information, but they may benefit from information boards at the bus or rail stop. In the paper, we do not explicitly distinguish different types of information.

[7] This setup implies that the model is best suited to describe the case of ex ante trip planning using, for example, information made available on websites. However, with minor adjustment it can also deal with information provided at stops. What is crucial in our model is the trade-off between planning costs and the costs of waiting. Although improving the quality of information boards or electronic announcements of delays at stops and stations does not reduce waiting time, it facilitates planning the remainder of the trip and reduces the cost of waiting time, allowing passengers to use waiting times more efficiently.

as simple as possible. If the costs of being early or late are linear in time, and assuming the preferred arrival times are uniformly distributed over time, it can be shown that expected schedule delay costs of planning and non-planning users are equal (Fosgerau (2009)). Moreover, under these conditions this expected scheduling cost is linear in headway (de Palma and Lindsey (2001), Fosgerau (2009)). Since the focus of this paper is on information provision we will stick to these assumptions. Denote the headway offered by the firm on the bus or rail line under consideration as *h*. This allows us to assume that, both for planning and non-planning users, the expected schedule delay cost is just given by

$$\delta h,$$

where δ depends on the costs of arriving early and late. Raising headway (reducing frequency) increases the cost of schedule delay.

Next consider the costs of waiting time at the stop and the cost of planning trips. Whether or not people plan a particular trip depends on the relative costs of the two options. We assume that not planning implies that the expected waiting time public transport users incur is half the headway; denoting the waiting cost per time unit as α, the expected value of this cost is therefore $\frac{\alpha h}{2}$. Of course, planning is costly as well. The cost of planning for an individual is specified as

$$\psi(\sigma, I) \qquad \psi_\sigma > 0; \psi_I < 0 \tag{1}$$

The planning cost depends on the quality of information, denoted *I*, provided by the firm. Better information (quality and user-friendliness of timetables, availability of information on different platforms, availability and quality of trip planning websites, how the firm deals with unexpected changes in schedules, with unforeseen delays, etc.) reduces planning costs for all passengers. However, dealing with the information provided by the firm requires time and effort on the part of the passenger. This is captured by an individual-specific parameter σ that reflects the effort it takes the individual to collect the required trip information from the supply of information provided by the firm (going on the web, looking up timetables, checking whether the trip is not subject to unexpected delays, etc). To a large extent, it represents the individual's "efficiency" in getting the information he is interested in. For a particular trip, it may differ substantially across users depending on, among others, hardware and software available, on the person's familiarity with using modern technology, etc. One expects the effort to collect information also to depend on whether the individual makes the trip frequently or not. For example, for trips made very often it is unnecessary to check the timetable but it suffices to check for delays. Looking up information requires relatively more effort for trips not frequently made.

The above discussion then implies that the generalized cost of a trip for an individual depends on whether he is planning the trip or not. If the user does not plan, the generalized cost consists of the fare (denoted as *p*), the schedule delay cost and the waiting time cost at the stop

$$p+\delta h+\frac{\alpha h}{2}. \tag{2}$$

In the case a user does plan a particular trip, the generalized cost of this trip is the fare plus the schedule delay cost plus the individual specific effort cost

$$p+\delta h+\psi(\sigma,I). \tag{3}$$

3.2. The decision whether or not to plan

Consider the decision of a rational individual user who has to decide whether to plan the trip or not. Given his individual 'effort' parameter σ, he will plan if the cost of doing so is less than the cost of not planning. So this person plans as long as[8]

$$\psi(\sigma,I)\leq\frac{\alpha h}{2}. \tag{4}$$

Denote the solution of $\psi(\sigma,I)=\frac{\alpha h}{2}$ for σ by $k(\frac{\alpha h}{2},I)$. It follows that the person will plan as long as

$$\sigma\leq k\left(\frac{\alpha h}{2},I\right). \tag{5}$$

The function $k(.)$ can be interpreted as the maximum effort level the individual is willing to incur to plan his trip. It follows from the definition that

[8] Note that we could use the same model to allow a distribution of time values in a setting without individual-specific planning cost. A cutoff for the time value would then determine whether an individual would be planning or not.

$$\psi(k,I)=\frac{\alpha h}{2}. \tag{6}$$

The implicit function theorem then implies

$$\frac{\partial k}{\partial h}=\frac{\alpha}{2\psi_\sigma}>0; \quad \frac{\partial k}{\partial I}=-\frac{\psi_I}{\psi_\sigma}>0. \tag{7}$$

Raising the headway (reducing frequency) or raising the level of information provided by the firm increases the maximal effort the individual is willing to incur to plan trips.

Following Fosgerau (2009), assume that the distribution of the individual-specific planning cost parameter σ has support on an interval (σ^-,σ^+) such that nobody plans for $k(\frac{\alpha h}{2},I)<\sigma^-$ and everyone plans when $k(\frac{\alpha h}{2},I)>\sigma^+$. The intuition for this assumption is clear. For example, suppose the headway offered is below a particular very low threshold. Then it seems reasonable that nobody will plan, no matter how much information is provided. Indeed, at very high frequency planning is simply unnecessary[9]. Alternatively, suppose frequency is very low; then a user who is not planning his trip will incur extremely long average waiting times, and hence it seems reasonable to assume that everyone will plan.

The fraction of all users who will plan their trips (or the probability that an arbitrary user plans) can be written as

$$\Phi\left[k(\frac{\alpha h}{2},I)\right]=\int_{\sigma^-}^{k(\frac{\alpha h}{2},I)}\phi(\sigma)d\sigma. \tag{8}$$

where ϕ is the density function. We find by differentiating (8) that

$$\frac{\partial \Phi}{\partial h}=\phi\frac{\alpha}{2\psi_\sigma}>0; \quad \frac{\partial \Phi}{\partial I}=-\phi\frac{\psi_I}{\psi_\sigma}>0. \tag{9}$$

The number of planning users rises when headway goes up (higher headway or lower frequency raises the benefits of planning) and when better information is provided. Moreover, (8) also means that more people plan when the waiting time cost α increases: this makes planning more beneficial.

[9] Note that this assumption may be slightly less realistic for rail than for bus service. In large rail stations, even at high frequencies some planning is necessary, as passengers have to find out on what platform the train arrives and, in some cases, which car to board.

3.3. The specification of total trip demand

The specification of the demand side of the model is kept simple. Travelers differ in planning efficiency, captured by the individual-specific parameter σ; otherwise, they are identical. The generalized cost is, conditional on σ, given by (2) or (3) above. Moreover, travel demand is assumed to be linear in the generalized price of trips. Given these assumptions, it is straightforward to show that total expected demand for bus trips becomes linear in the expected generalized cost of trips.[10] Specifically, aggregate demand *N(.)* can be written as (see Appendix 1)

$$N\left(p+f(h,I)\right); \quad N'<0, N''=0 \tag{10}$$

where

$$f(h,I)=\delta h+\left\{1-\Phi\left[k\left(\frac{\alpha h}{2},I\right)\right]\right\}\frac{\alpha h}{2}+\int_{\sigma^-}^{k(\frac{\alpha h}{2},I)}\psi(I,\sigma)\phi(\sigma)d\sigma. \tag{11}$$

Note that $\left(p+f(I,h)\right)$ is the expected generalized cost across all planning and non-planning users. The function *f(h,I)* is the expected non-monetary component of the generalized cost of the trip. It consists of the expected schedule delay plus waiting (for non-planning users) and planning (for people planning their trips) cost. The first term is schedule delay cost, the second term is the waiting time cost for users who do not plan. Indeed, $\frac{\alpha h}{2}$ is the average waiting time cost for an un-planning user, and the number of such users equals $1-\Phi\left[k\left(\frac{\alpha h}{2},I\right)\right]$. The third term on the right hand side is the expected planning cost of planning users.

To analyze the effect of the policy variables on demand, it is instructive to derive the impact of changes in headway and the level of information provision on *f(.)*. This is obtained by differentiating (11). We find after simple algebra that

[10] Linearity will prove to strongly facilitate the derivations; it does not affect the qualitative nature of our findings.

$$\frac{\partial f}{\partial h} = \delta + \left[1 - \Phi(.)\right]\frac{\alpha}{2} > 0,$$
$$\frac{\partial f}{\partial I} = \int_{\sigma^-}^{k(\frac{\alpha h}{2}, I)} \phi(\sigma)\psi_I(I,\sigma)d\sigma < 0. \tag{12}$$

Here, the last inequality follows from $\psi_I(\sigma, I) < 0$ (see (1) above). Expression (12) implies that increasing the headway (lower frequency) increases the overall expected cost of a trip. This effect is decreasing in the number of planning users. Higher frequency reduces the cost of planning, because scheduling costs and waiting time costs of un-planning users decline; this effect is smaller when more people plan because the fraction of un-planning people is lower. Similarly, providing more information reduces the cost of planning, and it does so more when more people plan.

Using (12) and (10), it then follows that the impact of the three policy instruments of interest on demand is given by

$$\frac{\partial N\left(p + f(h,I)\right)}{\partial p} = N' < 0; \quad \frac{\partial N\left(p + f(h,I)\right)}{\partial h} = N'\frac{\partial f}{\partial h} < 0; \quad \frac{\partial N\left(p + f(h,I)\right)}{\partial I} = N'\frac{\partial f}{\partial I} > 0$$

As expected, increasing the fare reduces demand, more frequency and better information both raise demand[11].

3.4. Costs of headway and information

The firm incurs costs of running the scheduled service; moreover, providing information to potential passengers is costly. Fixed costs are ignored; they do not affect the results. It is assumed that the cost of running a bus is constant and equal to *c*, so that total running costs for a given headway *h* amount to

$$\frac{c}{h}.$$

The costs associated with providing a level of information *I* are captured by $g(I) \geq 0$, where $g(0) = 0$, and $g' > 0$, $g'' > 0$. Better information is costly at an increasing rate. Note that the cost is assumed to be independent of the number of users. As noted above, this

[11] Our model implies that the provision of information affects consumer choices. There is substantial recent evidence that this is indeed the case (Farag and Lyons (2010), Kiesel and Villas-Boas (2011)). Specifically for the problem studied in this paper, evidence suggests that users acquire public transport information more intensively for unfamiliar trips, for arrival-time sensitive trips, long distance trips, and for leisure trips (Farag and Lyons (2010)).

implies that the model does not capture information provision (for example, of timetables) via physical booklets.

In view of this discussion, the firm's operational profit can be written as

$$\pi(p,h,I) = pN\left(p + f(h,I)\right) - \frac{c}{h} - g(I). \tag{13}$$

4. Benchmark cases: the social optimum and pure profit maximizing behavior

Before proceeding to the analysis of regulated markets, in this section we first briefly study two special cases. At one extreme, we look at the fare, frequency and quality of information that would be provided by a welfare-maximizing government agency. At the other extreme, we consider the choices of the policy variables that would be made by a profit-maximizing unregulated monopolist. Both cases serve as benchmarks for the regulatory settings considered further in the paper.

4.1. Perfect government control and the social optimum

Consider a government agency that controls all three decision variables. Assume the agency maximizes the following social welfare function

$$W = \left(\int_{p+f(h,I)}^{\infty} N(s)ds \right) + \lambda \left[p\, N\left(p + f(h,I)\right) - \frac{c}{h} - g(I) \right] \tag{14}$$

The first component of the welfare function is net consumer surplus, the second term is the profit of the firm, weighted by a parameter λ reflecting the social value the government assigns to the firm's operating profit or deficit. The first-order conditions are

$$\frac{\partial W}{\partial p} = (\lambda - 1)N + \lambda pN' = 0 \tag{15a}$$

$$\frac{\partial W}{\partial h} = -N\frac{\partial f}{\partial h} + \lambda\left[pN'\frac{\partial f}{\partial h} + \frac{c}{h^2} \right] = 0 \tag{15b}$$

$$\frac{\partial W}{\partial I} = -N\frac{\partial f}{\partial I} + \lambda\left[pN'\frac{\partial f}{\partial I} - g' \right] = 0 \tag{15c}$$

Using (15a) in (15b)-(15c), the system can be rewritten as

$$p = \frac{(1-\lambda)}{\lambda}\frac{N}{N'} \tag{16a}$$

$$N\frac{\partial f}{\partial h}=\frac{c}{h^2} \tag{16b}$$

$$N\frac{\partial f}{\partial I}=-g' \tag{16c}$$

The welfare-optimal price depends on the social value of profits and on the price sensitivity of demand. If $\lambda=1$, we obtain the first-best outcome. Not surprisingly, as the production cost is independent of the number of passengers (it only depends on frequency) and there are neither external costs nor benefits in the model, this implies a zero fare. For higher values of λ, the government charges a positive fare to users. The expressions for optimal headway and information provision simply require equality of the marginal benefits to users (left-hand side) and the marginal cost to the firm (right-hand side).

4.2. Behavior of a profit-maximizing unregulated monopolist

How would a pure profit-maximizing firm set the fare, the headway and the level of information provided to passengers? Denoting profit by π, the firm maximizes

$$\pi = p\,N\big(p+f(h,I)\big)-\frac{c}{h}-g(I) \tag{17}$$

with respect to p,h and I. This implies the first-order conditions

$$\frac{\partial \pi}{\partial p}=\pi_p=pN'+N=0 \tag{18a}$$

$$\frac{\partial \pi}{\partial h}=\pi_h=pN'\frac{\partial f}{\partial h}+\frac{c}{h^2}=0 \tag{18b}$$

$$\frac{\partial \pi}{\partial I}=\pi_I=pN'\frac{\partial f}{\partial I}-g'=0 \tag{18c}$$

Using (18a) in (18b)-(18c) and rearranging, we find

$$p=-\frac{N}{N'} \tag{19a}$$

$$N\frac{\partial f}{\partial h}=\frac{c}{h^2} \tag{19b}$$

$$N\frac{\partial f}{\partial I}=-g' \tag{19c}$$

The monopolist charges a fare consistent with a unitary price elasticity of demand. Moreover, it equates the firm's marginal revenues and marginal costs, both for headway and information provision.

Comparing systems (19) and (16) reproduces two well-known results. First, conditional on an arbitrary fare, optimal frequency and quality of information are identical at the social optimum and at the profit-maximizing outcome[12]. Second, if the weight on profit λ is infinitely large, the social optimum and the profit-maximizing solution coincide.

5. Fare regulation

In this section, we assume that the public transport firm operates in a regulated environment in which the price is imposed by a supervising public agency. Conditional on the price, the firm can freely set the headway and determine the quality of information it provides to facilitate trip planning of passengers so as to maximize its operating profit. Both reduction in headway (increase in frequency) and increased information provision on its schedules are costly to the firm.

Of course, the interaction between the government and the firm can be modeled in different ways. The most plausible setting seems to be a leader-follower framework in which the government agency acts as the leader by making the first move. The agency first sets the fare to be charged by the public transport firm; conditional on the fare, the firm decides on frequency and the quality of information offered to passengers. When fixing the fare, the agency takes into account how the firm will respond to different fare levels[13].

[12] Although he assumes fixed demand rather than a given price, this result is consistent with Spence (1975) who shows that if marginal and infra-marginal consumers value quality equally, the profit-maximizing and socially optimal quality levels are identical. Also see Oum, Zhang and Zhang (2004) and De Borger and Van Dender (2006) for similar findings in different types of models.

[13] In a previous version of the paper, we also considered a Nash game between the government agency and the firm. As noted by a referee, a leader-follower setting seems more realistic than a Nash game: it seems plausible that the fare is known to the firm when it makes its decisions on frequency and information. In a Nash game, we found that, if the firm raises headway (reduces frequency) and, as a consequence, reduces demand, the government reacts by lowering the fare so as to stimulate demand. If the firm provides more

Although the contract specifications governing the relations between public transport firms and the regulating agencies are highly complex in practice, the setting studied in this section does serve as a highly stylized description of the authority division between public transit firms and the government in a number of European countries[14]. In what follows, we solve the leader-follower game backwards: we first look at the decisions of the firm, then consider fare determination by the government agency.

5.1. Frequency and information provision by a price-regulated firm

In this section, we study the implications of fare regulation for the frequency of service and for the quality of information the firm provides to passengers. We also want to find out how the firm reacts to changes in the fare imposed by the government agency.

The firm maximizes profit, given by (17), with respect to h and I, conditional on the regulated fare p. Of course, this just gives the first-order conditions that were reported before, see expressions (18b)-(18c). It then immediately follows that, for a given fare, the firm will provide less frequency and less information than a welfare-maximizing government. To see this, substitute (18b)-(18c) in (15b)-(15c) to find

$$\frac{\partial W}{\partial h} = -N\frac{\partial f}{\partial h} < 0 \tag{20a}$$

$$\frac{\partial W}{\partial I} = -N\frac{\partial f}{\partial I} > 0 \tag{20b}$$

This says that, at the profit-maximizing values for headway and level of information provision by the bus company, a further increase in frequency and information would be welfare-increasing. This gives the following proposition.

information, the government raises the fare. In other words, the government agency 'punishes' the firm for reducing frequency or offering less information by reducing the fare.

[14] There is little doubt that the government agency makes the first move in almost all European countries. Often, but not always, this happens by auctioning the right to provide public transport service under pre-specified regulatory conditions. Moreover, in several countries (including Belgium and Denmark) fare regulation seems to be an acceptable approximation to the complex contracts between government agencies and public transport operators; the contracts either explicitly stipulate fares to be charged or impose strong restrictions on fare adjustments during the term of the contract, but they do allow some flexibility in scheduling and information provision. However, not all regulatory settings fit our model description. For example, although in France more than two thirds of all operators are private firms, most decisions on fares, routes and schedules are taken by local government authorities under fixed price or cost-plus contracts. For a detailed description of the French system, see Gagnepain and Ivaldi (2002).

Proposition 1: *Conditional on the regulated fare imposed, the firm will provide less frequency and invest less in information provision to passengers compared to the welfare optimum.*

This finding is just a minor extension of a well-known result derived by, among others, Brueckner (2004). Within the context of competition between airlines, he shows that price-regulated firms will provide lower service quality[15]. Proposition 1 above shows that this finding holds for both frequency and the quality of information provision. The intuition is clear. The profit-maximizing firm ignores the effect of its frequency and information decisions on net consumer surplus (see the first term of the welfare function in (14) above). The reason it ignores these benefits is that it cannot recoup the costs by raising the fare. Ignoring the extra benefits, it provides both insufficient frequency and information.

Now turn to the question of how the firm reacts to an increase in the regulated fare. We derive the effect of a higher regulated fare on the headway and the information provided by the public transport firm in Appendix 2. Unfortunately, the sign and magnitude of these effects depend in a complex way on the price sensitivity of demand, on the distribution of σ, on the planning cost function $\psi(\sigma, I)$, on the cost of operating buses, and on the shape of the information cost function $g(I)$. It will be instructive, therefore, to start with some special cases.

First, if no one plans, we find (see Appendix 2) that a higher price reduces headway (hence raises frequency) but does not have any effect on information. This is intuitive. The firm raises frequency because this reduces the generalized price of a trip and, hence, it stimulates demand. However, the impact on information provision is zero; information is costly but does not yield benefits when no one plans. Second, if everyone plans, we find (see Appendix 2) that a higher fare reduces headway (raises frequency) and raises information provision. Again, this is plausible. Even though providing more information does not increase the number of people that plan their trips, it does reduce the generalized cost of a trip for all passengers, and it therefore stimulates demand.

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[15] See his Proposition 6. A similar result was also derived in a different context by Bilotkach et al. (2010, p. 65-66).

In between zero and universal planning there is a smooth transition from fewer to more planners. To facilitate the interpretation of the general case, it is instructive to assume a linear individual information cost function $\psi(h,I)$. In Appendix 2, we show that the effect of the fare on headway and information is given by the following expressions

$$\frac{dh}{dp}=\frac{N'}{D}\left\{g''\left(\delta+(1-\Phi)\frac{\alpha}{2}\right)-[Z]\frac{\partial\Phi}{\partial I}\right\} \tag{21}$$

$$\frac{dI}{dp}=\frac{N'}{D}\left\{\frac{2c}{h^3}\psi_I\Phi+[Z]\frac{\partial\Phi}{\partial h}\right\} \tag{22}$$

where

$$[Z]=\left[pN'\psi_I\left(\delta+\frac{\alpha}{2}\right)\right]>0;\quad D=\pi_{hh}\pi_{II}-\pi_{hI}\pi_{Ih} \tag{23}$$

These results can be interpreted as follows. If offering more frequency and better information did not affect the number of planners ($\frac{\partial\Phi}{\partial h}=\frac{\partial\Phi}{\partial I}=0$), then a higher regulated fare would induce the firm to reduce headway and increase information. Indeed, in that case

$$\frac{dh}{dp}=\frac{N'}{D}\left\{g''\left[\delta+(1-\Phi)\frac{\alpha}{2}\right]\right\}<0$$

$$\frac{dI}{dp}=\frac{N'}{D}\left\{\frac{2c}{h^3}\psi_I\Phi\right\}>0.$$

Note that a higher cost c of operating extra buses means that a fare increase will imply more investment in information provision (rather than offering more frequency). Not surprisingly, a larger number of planners Φ means that the firm puts more emphasis on providing more information relative to raising frequency.

If information does raise the number of planners, (21) suggests that a fare increase leads to less extra frequency (note that $N'<0$, $Z>0$). The intuition is that when more passengers are planning their trips, this makes increasing frequency less beneficial to the firm, so it puts less emphasis on raising frequency. If the impact of information on planning is large, the final term between brackets in (21) may dominate so that the firm may even reduce frequency. Similarly, expression (22) implies that if headway raises the number of people that are planning their trips then the firm provides less extra information after a fare increase. Indeed, offering more frequency reduces the number of planners, making information less useful. If planning is strongly affected by headway the firm may actually reduce information provision.

Slightly rewriting (21)-(22) also shows that the effect of the fare on information and headway crucially depends on the relative importance of the value of waiting time (α) and on the impact information has on passengers' costs of planning (as captured by ψ_I). Using (9) we can reformulate (21)-(22) as

$$\frac{dh}{dp}=\frac{N'}{D}\left\{\psi_I\left[G\right]+g''\left[\delta+\left(1-\Phi(.)\right)\frac{\alpha}{2}\right]\right\} \tag{24}$$

$$\frac{dI}{dp}=\frac{N'}{D}\left[\frac{\alpha}{2}\left[G\right]+\psi_I\left(\frac{2c}{h^3}\Phi\right)\right] \tag{25}$$

where

$$\left[G\right]=pN'\left(\delta+\frac{\alpha}{2}\right)\phi\frac{\psi_I}{\psi_\sigma}>0.$$

If the value of waiting time (α) is large relative to the effect of information on planning cost (ψ_I), the firm puts much more emphasis on having more frequency. If, on the contrary, waiting time is not important but the firm's information provision has a large effect on individuals' planning costs, then more attention goes to raising information. It is also clear from (24)-(25) that a higher fare may well reduce frequency or information provision. For example, if time values are very high and information is ineffective in reducing planning costs, then the firm reacts to a higher fare by strongly raising frequency and in fact reducing information provision.

We summarize our findings on the effect of the fare in the following proposition.

Proposition 2: *(a) If the number of people planning their trips is constant, then a fare increase raises both frequency and information provision. (b) The firm puts more emphasis on providing more information relative to raising frequency if more people plan their trips. (c) If the number of planners depends on the frequency offered and the information provided by the firm, higher fares may reduce frequency or information* *provision. Higher fares reduce frequency when providing more information is very effective in reducing planning cost. Similarly, if time values are sufficiently high, a fare increase leads the firm to provide less information.*

Finally, in Appendix 3 we show that a higher cost of operating a public transport trip (a larger c) raises headway (and, hence, reduces frequency); moreover, it increases optimal information provision by the firm. We have

$$\frac{dh}{dc} > 0; \quad \frac{dI}{dc} > 0.$$

Proposition 3: *An increase in the cost of offering public transport service implies that the firm offers more information and less frequency.*

5.2.The pricing decision of the government agency

When deciding on the price to impose on the public transport operator, the government takes account of the firm's response to the fare when it determines information provision and frequency offered. As argued in Section 4, we assume the agency cares about both the net surplus of users and the profit of the firm. It solves

$$\max_{p} \quad W = \left(\int_{p+f(h,I)}^{\infty} N(s)ds \right) + \lambda \left[pN\left(p + f(h,I)\right) - \frac{c}{h} - g(I) \right].$$

Let us denote the responses of the public transport firm to the regulated fare by writing $h(p), I(p)$, where the derivatives $\frac{dh(p)}{dp}, \frac{dI(p)}{dp}$ have been determined in the previous section, see (21) and (22). The fare will therefore affect welfare directly, but also indirectly via the adjustment of headway and information investment.

The first-order condition of the government's problem is

$$\frac{dW}{dp} = \frac{\partial W}{\partial p} + \frac{\partial W}{\partial h}\frac{dh(p)}{dp} + \frac{\partial W}{\partial I}\frac{dI(p)}{dp} = 0. \qquad (26)$$

The first term is the welfare effect of a price increase at given frequency and information levels. The second and third terms capture the induced welfare effect by the firm's reaction to price changes. Expression (26) shows that the government 'corrects' the firm's profit maximizing behavior with respect to headway and information provision by adjusting the fare.

Using (15a) and (20a)-(20b) in (26), straightforward algebra shows:

$$p = \frac{(1-\lambda)}{\lambda}\frac{N}{N'} + \frac{N}{\lambda N'}\left(\frac{\partial f}{\partial h}\frac{dh(p)}{dp} + \frac{\partial f}{\partial I}\frac{dI(p)}{dp}\right). \tag{27}$$

The first term on the right hand side is the expression for the welfare optimal price, provided the government controlled all policy variables (see (16a)). The sign of the 'correction' term

$$\frac{N}{\lambda N'}\left(\frac{\partial f}{\partial h}\frac{dh(p)}{dp} + \frac{\partial f}{\partial I}\frac{dI(p)}{dp}\right)$$

is indeterminate in general and depends on the signs of the reaction functions of the firm. If higher fares raise frequency and information provision then the correction term is positive. This leads to an optimal price that is "structurally" higher than the welfare optimal price. This makes sense: the higher fare induces the firm to offer more frequency and better information, both of which were under-provided. Of course, as argued above, in particular circumstances a higher fare may reduce frequency and/or information quality; in those cases, the last term on the right hand side of (27) may be negative.

6. Fare and information regulation

In this section, we study the case where the government agency regulates both the fare and the quality of information provision. There are good reasons for analyzing this case. First, the theory presented in the previous section suggested that, conditional on the fare, the firm provides insufficient information from a social viewpoint. Although the government agency adjusts the fare to take the firm's responses into account, the result may well be that fare regulation results in low information quality. Imposing restrictions on information provision (for example, requiring time tables in a particular format, requiring specific information on platforms, quality of websites etc.) may then be a useful instrument to mitigate this tendency to provide low-quality information[16]. Not

[16] As an analogy, in several industries regulators have imposed quality restrictions combined with penalty schemes if firms do not comply (Weisman (2005)).

surprisingly, at least some European public transport regulators (for example, the UK) do impose such minimum quality restrictions. Second, we are interested in the implications of regulating information provision for welfare and for the other major decisions variables in the public transport sector, i.e., fares and frequency[17].

6.1. The firm's frequency decision

The first-order condition for profit-maximizing headway was derived before (see (18b); it is reproduced here for convenience

$$\frac{\partial \pi}{\partial h} = \pi_h = pN'\frac{\partial f}{\partial h} + \frac{c}{h^2} = 0 \tag{28}$$

Substituting (28) in (15b), it again immediately follows that, conditional on the regulated fare and information quality, the firm underprovides frequency compared to the socially optimal level.

Differentiating (28), and maintaining our assumption of linear demand, we find

$$\left[pN'\frac{\partial^2 f}{\partial h^2} - \frac{2c}{h^3}\right]dh + \left[N'\frac{\partial f}{\partial h}\right]dp + \left[pN'\frac{\partial^2 f}{\partial h \partial I}\right]dI = 0$$

The first term between square brackets is negative by the second-order condition, so we have[18]

$$\frac{dh}{dp} = -\frac{N'\frac{\partial f}{\partial h}}{pN'\frac{\partial^2 f}{\partial h^2} - \frac{2c}{h^3}} < 0; \quad \frac{dh}{dI} = -\frac{pN'\frac{\partial^2 f}{\partial h \partial I}}{pN'\frac{\partial^2 f}{\partial h^2} - \frac{2c}{h^3}} > 0 \tag{29}$$

A higher regulated fare now unambiguously (i.e., independent of the impact on the number of planners) leads the firm to raise frequency. Not surprisingly, the requirement

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[17] We also briefly considered the case where the government agency regulates the quality of information but leaves decisions on both fare and frequency to the firm. The impact of higher standards for the quality of information has ambiguous effects on fares and frequency: if the value of time is small but information strongly reduces planning costs then requiring higher information quality reduces frequency as well as fares; if time values are high and information is not successful in reducing generalized cost, then better information raises frequency and raises fares. How much information the government agency imposes upon the firm further depends on the relative cost of operating trips and the price sensitivity of demand. It may be above or below the socially optimal level.

[18] The sign of the second expression follows from $\frac{\partial^2 f}{\partial h \partial I}$<0; this is easily shown by differentiating (12). Also see Appendix 2.

to provide better information raises headway and hence reduces frequency: one quality attribute is substituted for another.

6.2. The agency's decision: fare and information provision

The government sets fare and information provision, taking into account the reaction by the firm. It solves

$$\max_{p,I} \quad W=\left(\int_{p+f(h,I)}^{\infty} N(s)ds \right)+\lambda\left[pN\left(p+f(h,I)\right)-\frac{c}{h}-g(I) \right].$$

As shown above, the firm's optimal headway depends on the regulated fare and information quality. We write *h(p,I)* , where the relevant derivatives are given by (29).

The first-order conditions with respect to p and I are given by, respectively

$$\frac{dW}{dp}=\frac{\partial W}{\partial p}+\frac{\partial W}{\partial h}\frac{dh}{dp}=0. \tag{30}$$

$$\frac{dW}{dI}=\frac{\partial W}{\partial I}+\frac{\partial W}{\partial h}\frac{dh}{dI}=0 \tag{31}$$

where we reproduce earlier results for convenience:

$$\frac{\partial W}{\partial p}=(\lambda-1)N+\lambda pN' \tag{32a}$$

$$\frac{\partial W}{\partial h}=-N\frac{\partial f}{\partial h}+\lambda\left[pN'\frac{\partial f}{\partial h}+\frac{c}{h^2} \right]=-N\frac{\partial f}{\partial h}<0 \tag{32b}$$

$$\frac{\partial W}{\partial I}=-N\frac{\partial f}{\partial I}+\lambda\left[pN'\frac{\partial f}{\partial I}-g' \right] \tag{32c}$$

Note that in (32b) we used the first-order condition for profit maximizing choice of headway. Use (32a)-(32b) in (30) to find:

$$p=\frac{(1-\lambda)}{\lambda}\frac{N}{N'}+\frac{N}{\lambda N'}\left(\frac{\partial f}{\partial h}\frac{dh}{dp} \right). \tag{33}$$

Then substitute (32b)-(32c) in (31) and use (33). We obtain:

$$\frac{dW}{dI}=-\lambda\left[N\frac{\partial f}{\partial I}+g' \right]+N\frac{\partial f}{\partial h}\left[\frac{\partial f}{\partial I}\frac{dh}{dp}-\frac{dh}{dI} \right]=0. \tag{34}$$

Expressions (33)-(34) capture the government agency's behavior when setting fares and the quality of information imposed on the firm. If neither the fare nor the quality of information had any impact on the firm's choice of headway ($\frac{dh}{dp} = \frac{dh}{dI} = 0$), then we just reproduce the social optimum (see system (16)). If headway choices do depend on the government's regulated fares and information quality, using our previous results (more precisely $\frac{\partial f}{\partial h} > 0, \frac{dh}{dp} < 0$) in (33) imply that the fare structurally exceeds the socially optimal fare. Moreover, as the sign of the final term between square brackets in (34) is ambiguous, this expression suggests that, conditional on fare and frequency, the government agency's choice of information quality may be higher or lower than socially optimal.

The economic intuition underlying (33)-(34) can best be illustrated by considering some simplifying examples. Suppose that the public transport firm strongly raises frequency after an increase in the regulated fare ($\frac{dh}{dp}$ large in absolute value), but that it does not substantially reduce frequency when more stringent information requirements are imposed ($\frac{dh}{dI}$ small). Then (33)-(34) jointly imply a relatively high fare and high quality of information. The high fare strongly stimulates the firm to offer more frequency but it also raises generalized prices for passengers; offering high quality information dampens this increase in generalized price while – under our assumptions – not affecting frequency much. Alternatively, suppose that exactly the opposite assumptions hold ($\frac{dh}{dp}$ small, $\frac{dh}{dI}$ large). Then (33)-(34) imply low fares and poor information quality. The former yields low generalized prices but it does not stimulate more frequent service provision by the firm; therefore, frequency is stimulated by providing relatively low quality information.

We summarize our findings in this section in the following proposition.

Proposition 4. *Suppose the government agency regulates both the fare and the quality of information the public transport firm has to provide to passengers. Then*

(a) Imposing more stringent restrictions on the quality of information unambiguously reduces the frequency offered by the firm.

(b) The regulated fare structurally exceeds the socially optimal value

(c) The quality of information offered to passengers may be better than socially optimal

7. Comparing different institutional settings

In the previous sections we analyzed fares, frequency and the quality of information provided to passengers under different regulatory regimes. In order of increasingly more stringent regulation we looked at pure profit maximizing behavior, fare regulation, fare plus information regulation, and full government control (the social optimum). For each of the four regulatory structures considered, the outcomes for the policy variables can be described as the solution to a system of three simultaneous equations. In Table 1 we summarize the various equation systems.

Of course, comparing sets of simultaneous equations does not give unambiguous predictions for comparison of numerical outcomes, because demand and several of the derivatives occurring in the different expressions depend on all policy variables. Numerical analysis is needed to get insight into the implications of different regulatory regimes for optimal information provision, frequency and fares. Based on our earlier discussion and on Table 1, there are just a few speculative observations we can make.

First, compare the social optimum with decisions by a fully profit-maximizing firm. As argued before, conditional on the price, the social optimum and the maximum profit solution yield the same headway and information provision. Of course, for reasonable values of the cost of funds, Table 1 suggests a much lower price at the social optimum. Lower fares raise demand, suggesting higher frequency and information provision at the social optimum. Second, by the same arguments we expect higher fares and both lower frequency and information quality under fare regulation as compared to the social optimum. Third, although the comparison is ambiguous theoretically, we expect higher quality information and lower frequency under fare plus information regulation compared to fare regulation only; the effect on fares is unclear a priori. Finally, whether fare plus information regulation will lead to better information than the social optimum is ambiguous, although in the former case we expect much lower frequency.

Table 1: Comparing optimality rules

	No regulation (Profit maximizing outcome)	Fare regulation	Fare and information regulation	Social optimum
Fare	$p=-\frac{N}{N'}$	$p=\left(\frac{1-\lambda}{\lambda}\right)\frac{N}{N'}+\frac{N}{\lambda N'}\{M\}$	$p=\frac{(1-\lambda)}{\lambda}\frac{N}{N'}+\frac{N}{\lambda N'}\{Q\}.$	$p=\left(\frac{1-\lambda}{\lambda}\right)\frac{N}{N'}$
Headway	$N\frac{\partial f}{\partial h}=\frac{c}{h^2}$	$pN'\frac{\partial f}{\partial h}=-\frac{c}{h^2}$	$pN'\frac{\partial f}{\partial h}=-\frac{c}{h^2}$	$N\frac{\partial f}{\partial h}=\frac{c}{h^2}$
Information Quality	$N\frac{\partial f}{\partial I}=-g'$	$pN'\frac{\partial f}{\partial I}=g'$	$-\lambda\left[N\frac{\partial f}{\partial I}+g'\right]+N\frac{\partial f}{\partial h}\left[\frac{\partial f}{\partial I}\frac{dh}{dp}-\frac{dh}{dI}\right]=0$	$N\frac{\partial f}{\partial I}=-g'$

Note: $M=\left(\frac{\partial f}{\partial h}\frac{dh(p)}{dp}+\frac{\partial f}{\partial I}\frac{dI(p)}{dp}\right);\frac{dh(p)}{dp},\frac{dI(p)}{dp}\ \textit{given by}\ (21)-(22).$

$$Q=\frac{\partial f}{\partial h}\frac{dh(p,I)}{dp};\frac{dh(p,I)}{dp}\ \ \textit{given by}\ (29)$$

8. A numerical example

In this section, we provide a simple numerical example to illustrate our findings. We first present the functional forms chosen for the demand function, the planning cost function and the information cost function; we explain our choice of distribution for the individual-specific planning cost parameter σ, and we describe the parameter values used. We then discuss the numerical results and look at some sensitivity results.

8.1. Choice of functional forms and parameters

The demand function is taken to be linear

$$N\left(p+f(h,I)\right)=a-b\left(p+f(h,I)\right).$$

The planning cost function takes the following simple form[19]

$$\psi(\sigma,I)=\frac{\sigma}{I}.$$

The cost is increasing in the individual-specific parameter σ and declining in information. Note that this specification implies $k\left(\frac{\alpha h}{2},I\right)=\frac{\alpha hI}{2}$. Using this information, the scheduling plus planning cost *f(h,I)* can be written as

$$f(h,I)=\delta h+\left\{1-\Phi\left(\frac{\alpha hI}{2}\right)\right\}\frac{\alpha h}{2}+\int_0^{\frac{\alpha hI}{2}}\left(\frac{\sigma}{I}\right)\phi(\sigma)d\sigma.$$

The information cost function was specified as

$$g(I)=g_0e^{g_1 I},\qquad g_0>0,g_1>0.$$

This implies the desirable properties

$$g(I)>0;\quad g'=g_1\left(g(I)\right)>0;\quad g''=\left(g_1\right)^2\left(g(I)\right)>0.$$

Finally, we let the individual cost parameter σ be gamma distributed with parameters s,θ; specifically

$$\phi(\sigma;s,\theta)=\sigma^{s-1}\frac{e^{-\frac{\sigma}{\theta}}}{\theta^s\Gamma(s)}.$$

[19] Note that we avoided a linear specification as this may easily lead to negative overall planning costs. A desirable characteristic of the specification used is that it is the simplest form that avoids negative planning costs.

The gamma distribution was chosen because it is naturally positive. Moreover, $\sigma \cdot \phi(\sigma)$ is easily integrated, which facilitates the numerical implementation.

The parameters chosen for the various functions are summarized in Table 2, including the 'cost of funds' parameter $\lambda = 1.2$ used in the simulations.

Demand function	**Cost of bus service**	**Scheduling cost**	**Information cost function**	**Distribution of σ**	**Cost of funds**
a=2	*c=2*	$\alpha = 0.8$	$g_0 = 0.1$	*s=1*	$\lambda = 1.2$
b=0.1		$\delta = 0.16$	$g_1 = 0.1$	$\theta = 1$	

Table 2. Parameter values for the numerical illustration

8.2. Numerical results

We numerically studied the behavior of the government and the public transport firm under the four regimes analyzed above. The results are summarized in Table 3.

First, consistent with the theory of the previous sections, the social optimum yields somewhat lower headway (more frequency) and better information provision, but a much lower fare as compared to the profit maximizing solution. Second, imposing a regulated fare and making the firm responsible for frequency and information provision only is welfare-improving compared to the maximum profit scenario, but the lower fare does induce the firm to offer less frequency and reduce information provision. Third, combining information requirements with fare regulation strongly increases information provision, but does so at the expense of lower frequency: the government forces the firm to provide more information, but the firm reacts by saving on another quality aspect, viz. frequency. Fourth, a profit maximizing firm controlling all policy variables invests more in information to passengers than a fare regulated firm. Finally, interestingly, the social optimum implies the lowest fraction of planning users; this holds because of the very high frequency, which reduces the benefits of planning trips, and despite more information being provided.

	No regulation: Profit Max	Fare regulation	Fare and information regulation	Social optimum
Fare (p)	9.46	4.31	3.97	2.74
Headway (h)	2.90	4.69	5.50	2.04
Information (I)	1.27	1.09	1.86	1.36
Fraction of people planning (Φ)	0.77	0.87	0.98	0.67
Demand (N)	0.95	1.41	1.46	1.64
Scheduling cost f(h,I)	1.07	1.55	1.41	0.82
Information cost g(I)	0.16	0.12	0.35	0.18
Profit	8.11	5.54	5.09	3.34
Welfare	14.21	16.66	16.80	17.52

Table 3. Results under different institutional settings

The results are obviously sensitive to the parameters of the problem (time values, cost of bus service, price sensitivity of demand, etc.). In Table 4, for example, we consider the sensitivity of the results with respect to the assumed value of time. We compare the results for a low time value (half the base value) with the base value (equal to 0.8, as used in Table 3) and a high value (twice the base value). For each type of market structure, we report the most important outcomes in Table 4. To facilitate comparison, the base results of Table 3 are indicated in bold in Table 4.

We observe that higher value of time induces more people to plan in order to avoid waiting time costs at the bus stop. Not surprisingly, having more planners then makes it worthwhile for the firm to provide more information. It also implies that headway rises and, hence, frequency goes down; given the specification of the model, offering more information reduces the marginal benefit of higher frequency. Observe that demand, profit and welfare are not very sensitive to the assumed time values. These observations hold under all regulatory settings considered. In terms of the comparison between regulatory settings, we see that the difference between fare regulation only and fare plus information regulation becomes quite small at low time values. The latter case yields just a bit more information and lower frequency. Finally, as argued above, the relative rankings of different market structures are not affected by the assumed time values.

	No regulation: Profit Max			Fare regulation			Fare and information regulation			Social optimum		
	L	B	H	L	B	H	L	B	H	L	B	H
Fare (p)	9.57	**9.46**	9.41	4.25	**4.31**	4.26	4.24	**3.97**	3.89	2.76	**2.74**	2.72
Headway (h)	2.59	**2.90**	3.56	4.02	**4.69**	5.36	4.08	**5.50**	5.67	1.93	**2.04**	2.67
Information (I)	0.53	**1.27**	1.65	0.52	**1.09**	1.28	0.60	1**.86**	2.00	0.54	**1.36**	1.96
Fraction of people planning (Φ)	0.24	**0.77**	0.99	0.34	**0.87**	0.99	0.39	**0.98**	0.99	0.18	**0.67**	0.98
Demand (N)	0.96	**0.95**	0.94	1.45	**1.41**	1.41	1.45	**1.46**	1.47	1.66	**1.64**	1.63
Profit	8.35	**8.11**	8.03	5.61	**5.54**	5.48	5.61	5**.09**	4.96	3.52	**3.34**	3.32
Welfare	14.6	**14.2**	14.1	17.1	**16.7**	16.5	17.2	**16.8**	16.7	18.0	**17.5**	17.3

Table 4. Sensitivity with respect to value of time. L: low (0.4), B: base (0.8), H: high (1.6)

As a second illustration, let us look at the implications of changing the cost of operating the bus service[20]. Results are in Table 5. Higher costs of operating buses strongly reduces frequency under all market structures. This induces more people to plan, so that it becomes beneficial for the firm to offer more information. Note that there is almost no effect of higher operating cost on the fare; this is due to the fact that the cost of bus service is independent of the number of passengers. Finally, demand, profit and welfare all slightly go down.

[20] Although we did not do a fully comprehensive sensitivity analysis, the results of varying some other parameters (for example, the price sensitivity of demand) are also available. For the limited set of parameter values that were used, the rankings of the various policy variables for the four cases studied was not affected.

	Profit Max			**Fare regulation**			**Fare and information regulation**			**Social optimum**		
	L	B	H	L	B	H	L	B	H	L	B	H
Fare (p)	9.62	**9.46**	9.30	4.15	**4.31**	4.46	4.14	**3.97**	4.15	2.77	**2.74**	2.70
Headway (h)	1.59	**2.90**	4.90	2.62	**4.69**	7.25	2.75	**5.50**	7.74	1.06	**2.04**	3.61
Information (I)	0.72	**1.27**	1.55	0.71	**1.09**	1.25	0.83	1**.86**	1.96	0.72	**1.36**	1.81
Fraction of people planning (Φ)	0.37	**0.77**	0.95	0.52	**0.87**	0.97	0.60	**0.98**	0.99	0.28	**0.67**	0.93
Demand (N)	0.96	**0.95**	0.93	1.47	**1.41**	1.36	1.47	**1.46**	1.41	1.66	**1.64**	1.62
Profit	8.57	**8.11**	7.59	5.66	**5.54**	5.36	5.65	5.09	4.95	3.70	**3.34**	2.94
Welfare	14.9	**14.2**	13.4	17.5	**16.7**	15.7	17.6	**16.8**	15.9	18.3	**17.5**	16.7

Table 5. Sensitivity to the cost of bus operations. L: low (1), B: base (2), H: high (4)

Jointly with the theoretical findings, the numerical illustration allows some modest policy implications. First, although delegating all decisions to a profit maximizing firm yields frequencies and provision of information that are close to the social optimum, large welfare losses result due to the high fare. Second, government regulation of the fare indeed leads to much lower fares, but it comes at a cost in terms of quality of service: the firm provides lower quality information and lower frequency compared to both the social optimum and the profit maximizing outcome. Fare regulation does perform well in terms of stimulating people to plan their trips. Third, information regulation may resolve the problem of low information provision by fare regulated firms, but it does so at the expense of lower frequency. The welfare improvement of imposing informational requirements on fare regulated firms may therefore be relatively small.

9. Summary and conclusions

We studied the interaction between pricing, frequency and the provision of information by public transport companies offering scheduled services under different regulatory regimes. We allowed for users who plan their trip and others who do not. The fraction of users who plan their trips was treated as endogenous and depended on the frequency of service offered by the firm and on the level of information provided. Several institutional settings were considered. We studied two cases of government regulation where the government agency acts as the leader in a leader-follower game. In the first case, the agency regulated the fare and the firm decided on frequency and information provision. In a second situation we also studied fare plus information regulation by the government agency. Finally, for purposes of comparison we analyzed fares, frequencies and information provision for a profit maximizing firm deciding on all decision variables, and the case of a welfare maximizing government agency controlling all policy variables.

Our theoretical findings include the following. First, conditional on the fare imposed, a fare-regulated firm will provide less frequency and less information compared to the welfare optimum. If information and frequency did not affect the number of planning users, then a higher regulated fare always induces the firm to raise both frequency and information provision. However, this is no longer necessarily the case once one accounts for an endogenous number of people planning their trips. If providing more information is very effective in reducing planning cost, increasing the regulated fare may in fact reduce frequency. Similarly, a higher regulated fare may reduce information provision if people attach a high value to waiting time, so that offering more frequency is much more effective at reducing the generalized cost of trips. Moreover, we show that the firm puts more emphasis on providing more information relative to raising frequency if more people plan their trips. Second, a fare-regulated firm may well provide lower quality information to passengers than a profit maximizing unregulated firm. Third, fare plus information regulation results in provision of high quality information, but it induces the firm to reduce frequency of service. As a consequence, the welfare improvement due to imposing information requirements on fare-regulated firms is limited. Fourth, the numerical illustration suggests that delegating all decisions to a profit maximizing firm yields frequencies and provision of information that are close to the social optimum, but large welfare losses still result due to the high fare. Finally, of all institutional structures considered, socially optimal fares and service qualities stimulate passengers least to plan their trips: the high frequency offered under the socially optimal policy reduces the benefits of trip planning.

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Appendix 1: Derivation of the aggregate demand for trips

We assume travelers differ only in their individual planning efficiency σ; otherwise, they are identical. Moreover, we assume that individual demand is linear in the generalized price P of a trip. This will substantially simplify the aggregate demand function without affecting the qualitative results[21]. Specifically, let individual demand be given by

$$a - bP$$

where

$$P = p + \delta h + \frac{\alpha h}{2} \quad for \quad k(I, \frac{\alpha h}{2}) < \sigma < \sigma^{+}$$

$$P = p + \delta h + \psi(I,\sigma) \quad for \quad \sigma^{-} < \sigma < k(I, \frac{\alpha h}{2})$$

Total expected demand N is then obtained as

$$\int_{k(\frac{\alpha h}{2},I)}^{\sigma^{+}} \left[a - b\cdot(p + \delta h + \frac{\alpha h}{2}) \right] \phi(\sigma) d\sigma + \int_{\sigma^{-}}^{k(\frac{\alpha h}{2},I)} \left[a - b\cdot\left(p + \delta h + \psi(I,\sigma)\right)\right] \phi(\sigma) d\sigma$$

Rearranging, noting that

$$\int_{\sigma^{-}}^{k(\frac{\alpha h}{2},I)} \phi(\sigma) d\sigma = \Phi$$

and simple algebra yields

$$N = a - b * \left\{ p + \delta h + \left[1 - \Phi\left(k(\frac{\alpha h}{2}, I) \right) \right] \frac{\alpha h}{2} + \int_{\sigma^{-}}^{k(\frac{\alpha h}{2},I)} \psi(I,\sigma)\phi(\sigma) d\sigma \right\}$$

This implies demand is linear in $\left(p + f(I,h)\right)$, where

$$f(h,I) = \delta h + \left[1 - \Phi\left(k(\frac{\alpha h}{2}, I) \right) \right] \frac{\alpha h}{2} + \int_{\sigma^{-}}^{k(\frac{\alpha h}{2},I)} \psi(I,\sigma)\phi(\sigma) d\sigma$$

---

[21] In principle, individual demand functions can be allowed to differ between individuals (i.e., that can be made dependent on the individual's type, indexed by σ. More specifically, it is easy to show that the result shown below (viz. that aggregate demand can be written as a linear function of the expected generalized price) still holds when demand functions differ in intercept (a_{σ}) but have a common slope (b). Allowing the slope parameter to be individual-specific complicates the technical analysis dramatically.

The function *f(h,I)* captures the expected non-monetary component of generalized cost; it consists of the schedule delay cost, waiting time costs (for non-planning users) and expected planning (for people planning their trips) cost. In other words, expected demand for trips is linear in the expected generalized price. Reflecting linearity in $\left(p+f(I,h)\right)$, in the main body of the paper we formulate demand as $N\left(p+f(I,h)\right)$ with $N'<0$, $N''=0$.

Appendix 2: Derivation of the impact of the fare on headway and information provision

In this appendix, we derive the effect of the regulated fare on headway and information provided by the firm. We first derive general expressions for these effects, then look at some special cases and, finally, we elaborate and interpret the general case.

Derivation of the general case

We derive the effect of an exogenous increase in the regulated fare on the public transport firm's optimal choice of headway and quality of information provided to potential passengers. To derive these effects, differentiate the first-order conditions with respect to optimal headway and information quality (18b)-(18c) and write the result in matrix notation as:

$$\begin{bmatrix} \pi_{hh} & \pi_{hI} \\ \pi_{Ih} & \pi_{II} \end{bmatrix} \begin{bmatrix} dh \\ dI \end{bmatrix} = \begin{bmatrix} -\pi_{hp}\ dp - \pi_{hc}\ dc \\ -\pi_{Ip}\ dp - \pi_{Ic}\ dc \end{bmatrix}$$

where

$$\begin{aligned} \pi_{hh} &= p\left[N''\left(\frac{\partial f}{\partial h}\right)^2 + N'\frac{\partial^2 f}{\partial h^2}\right] - \frac{2c}{h^3} \\ \pi_{II} &= p\left[N''\left(\frac{\partial f}{\partial I}\right)^2 + N'\frac{\partial^2 f}{\partial I^2}\right] - g'' \\ \pi_{hI} &= \pi_{Ih} = p\left[N''\left(\frac{\partial f}{\partial h}\right)\left(\frac{\partial f}{\partial I}\right) + N'\frac{\partial^2 f}{\partial h \partial I}\right] \end{aligned} \qquad \text{(A2.1)}$$

and

$$
\begin{aligned}
\pi_{hp} &= \frac{\partial f}{\partial h}\left(N' + pN''\right) < 0 \\
\pi_{hc} &= \frac{1}{h^2} > 0 \\
\pi_{Ip} &= \frac{\partial f}{\partial I}\left(N' + pN''\right) > 0 \\
\pi_{Ic} &= 0
\end{aligned}
\tag{A2.2}
$$

Using Cramer's rule, the effect of a price increase on the optimal headway and on optimal information investment is, therefore:

$$\frac{dh}{dp} = \frac{1}{D}\left\{\pi_{Ip}\pi_{hI} - \pi_{hp}\pi_{II}\right\} \tag{A2.3}$$

$$\frac{dI}{dp} = \frac{1}{D}\left\{\pi_{hI}\pi_{hp} - \pi_{hh}\pi_{Ip}\right\} \tag{A2.4}$$

where $D = \pi_{hh}\pi_{II} - \pi_{hI}\pi_{Ih}$. The second order conditions to the firm's optimization problem imply

$$\pi_{hh} < 0, \pi_{II} < 0, \quad D > 0$$

Straightforward algebra, using expressions (A2.1-A2.2) in (A2.3) and (A2.4), then leads to

$$\frac{dh}{dp} = \frac{N'}{D}\left\{\left[pN'\left(\frac{\partial f}{\partial I}\frac{\partial^2 f}{\partial h \partial I} - \frac{\partial f}{\partial h}\frac{\partial^2 f}{\partial I^2}\right) + g''\left(\frac{\partial f}{\partial h}\right)\right]\right\} \tag{A2.5}$$

$$\frac{dI}{dp} = \frac{N'}{D}\left\{\left[pN'\left(\frac{\partial^2 f}{\partial h \partial I}\frac{\partial f}{\partial h} - \frac{\partial^2 f}{\partial h^2}\frac{\partial f}{\partial I}\right) + \frac{2c}{h^3}\left(\frac{\partial f}{\partial I}\right)\right]\right\} \tag{A2.6}$$

The first derivatives of the planning and scheduling cost function *f(h,I)* have been derived above, see (12). Differentiating (12) leads to

$$
\begin{aligned}
&\frac{\partial^2 f}{\partial h^2} = -\left(\frac{\alpha}{2}\right)^2 \frac{\phi}{\psi_\sigma} < 0 \\
&\frac{\partial^2 f}{\partial h \partial I} = \frac{\alpha}{2}\phi\frac{\psi_I}{\psi_\sigma} < 0 \\
&\frac{\partial^2 f}{\partial I^2} = -\phi\frac{(\psi_I)^2}{\psi_\sigma} + \int_{\sigma^{=}}^{k(\frac{\alpha h}{2}, I)} \phi(\sigma)\psi_{I,I}(I,\sigma)d\sigma\,; \qquad \psi_{I,I} = \frac{\partial^2 \psi}{\partial I^2}
\end{aligned}
\tag{A2.7}
$$

The signs of the first two expressions of (A2.7) follow from expression (1) in the main body of the paper. The sign of the final expression is ambiguous in general. The first term on the right hand side is negative but, plausibly assuming $\psi_{I,I}(I,\sigma)$ >0, the second one is positive.

The signs of the impacts of the regulated fare on headway and information (see (A2.5)-(A2.6)) are unambiguous, and depend on the various parameters of the model (time value, demand sensitivity, cost of operating buses, the effect of information on individual planning cost, etc.). It will be instructive, therefore, to start with some special cases.

Some special cases

First, let us consider the case where $k(\frac{\alpha h}{2}, I) < \sigma^{-}$ so that no one plans. In that case the average cost of planning users is zero, and $\Phi = 0$. This implies that (11) reduces to

$$f(h,I) = \delta h + \frac{\alpha h}{2}$$

We then have

$$\frac{\partial f}{\partial h} = \delta + \frac{\alpha}{2}; \quad \frac{\partial f}{\partial I} = 0; \quad \frac{\partial^2 f}{\partial I^2} = \frac{\partial^2 f}{\partial I \partial h} = \frac{\partial^2 f}{\partial h^2} = 0$$

Substituting in (A2.5)-(A2.6) leads to

$$\frac{dh}{dp} = \frac{N'}{D}\left\{\left[g''\left(\delta + \frac{\alpha}{2}\right)\right]\right\} < 0$$

$$\frac{dI}{dp} = 0$$

If no one plans, a higher price reduces headway (hence raises frequency) but does not have any effect on information.

Alternatively, assume $k(\frac{\alpha h}{2}, I) > \sigma^{+}$ so that everyone plans; then waiting time costs of un-planning users are zero and $\Phi = 1$. The planning plus scheduling cost function *f(h,I)* then reduces to

$$f(h,I) = \delta h + \int_{\sigma^{-}}^{\sigma^{+}} \psi(I,\sigma)\phi(\sigma)d\sigma$$

It follows

$$\frac{\partial f}{\partial h}=\delta;\quad \frac{\partial f}{\partial I}=\int_{\sigma^-}^{\sigma^+}\psi_I(I,\sigma)\phi(\sigma)d\sigma<0;\quad \frac{\partial^2 f}{\partial I^2}=\int_{\sigma^-}^{\sigma^+}\psi_{I,I}(I,\sigma)\phi(\sigma)d\sigma>0;\quad \frac{\partial^2 f}{\partial h^2}=\frac{\partial^2 f}{\partial h\partial I}=0$$

Substituting in (A2.5)-(A2.6) then finally yield:

$$\frac{dh}{dp}=\frac{N'}{D}\left\{\left[-\delta pN'\left(\int_{\sigma^-}^{\sigma^+}\psi_{I,I}(I,\sigma)\phi(\sigma)d\sigma\right)+g''(\delta)\right]\right\}<0$$

$$\frac{dI}{dp}=\frac{N'}{D}\left\{\left[\frac{2c}{h^3}\left(\int_{\sigma^-}^{\sigma^+}\psi_I(I,\sigma)\phi(\sigma)d\sigma\right)\right]\right\}>0$$

where the signs follow from $\psi_I<0;\quad \psi_{I,I}>0$. We again find that a higher fare reduces headway (raises frequency). Moreover, it raises information provision.

Interpreting the general case

To facilitate the interpretation of the case with endogenous number of people planning their trips, assume that the individual planning cost function $\psi(\sigma,I)$ is linear, so that ψ_σ,ψ_I are both constant. Note that under these assumptions it follows from (12) that

$$\frac{\partial f}{\partial I}=\int_{\sigma^-}^{k(\frac{\alpha h}{2},I)}\phi(\sigma)\psi_I d\sigma=\psi_I\Phi$$

Using the first (see (12)) and second derivatives (see (A2.7)) of the cost function *f(.)* it is then easily shown that:

$$\left(\frac{\partial f}{\partial I}\frac{\partial^2 f}{\partial h\partial I}-\frac{\partial f}{\partial h}\frac{\partial^2 f}{\partial I^2}\right)=\left(\delta+\frac{\alpha}{2}\right)\phi\frac{(\psi_I)^2}{\psi_\sigma}$$
$$\left(\frac{\partial^2 f}{\partial h\partial I}\frac{\partial f}{\partial h}-\frac{\partial^2 f}{\partial h^2}\frac{\partial f}{\partial I}\right)=\left(\delta+\frac{\alpha}{2}\right)\frac{\alpha}{2}\phi\frac{\psi_I}{\psi_\sigma}$$

Substituting in (A2.5)-(A2.6) and using (9) we find after simple manipulations

$$\frac{dh}{dp}=\frac{N'}{D}\left\{g''\left(\delta+(1-\Phi)\frac{\alpha}{2}\right)-[Z]\frac{\partial\Phi}{\partial I}\right\}$$

$$\frac{dI}{dp}=\frac{N'}{D}\left\{\frac{2c}{h^3}\psi_I\Phi+[Z]\frac{\partial\Phi}{\partial I}\right\}$$

where

$$[Z] = \left[pN'\psi_I \left(\delta + \frac{\alpha}{2} \right) \right] > 0$$

Appendix 3. The effect of the cost of operating buses on headway and information

The impact of increasing the cost of operating a bus on the optimal headway is given by, using (A2.1)-(A2.2)

$$\frac{dh}{dc} = \frac{1}{D}\left\{ -\pi_{hc}\pi_{II} \right\}$$

We know $\pi_{II} < 0$ by the second order condition and $\pi_{hc} > 0$ by (A2.2). Hence

$$\frac{dh}{dc} > 0$$

The effect on information provision is:

$$\frac{dI}{dc} = \frac{1}{D}\left\{ \pi_{hc}\pi_{hI} \right\} = \frac{1}{D}\left\{ \frac{1}{h^2} p \left[N' \frac{\partial^2 f}{\partial h \partial I} \right] \right\}$$

As $\frac{\partial^2 f}{\partial h \partial I}$ <0 (see (A2.7)), for linear demand more costly bus operations raise the level of information provided to passengers.