The value of travel time variance

Fosgerau, Mogens and Engelson, Leonid

Technical University of Denmark

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Mogens Fosgerau*
Technical University of Denmark,
Centre for Transport Studies, Sweden &
Ecole Nationale Supérieure De Cachan, France

Leonid Engelson
Centre for Traffic Research, The Royal Institute of Technology, Sweden

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Abstract

This paper considers the value of travel time variability under scheduling preferences that are defined in terms of linearly time-varying utility rates associated with being at the origin and at the destination. The main result is a simple expression for the value of travel time variability that does not depend on the shape of the travel time distribution. The related measure of travel time variability is the variance of travel time. These conclusions apply equally to travellers who can freely choose departure time and to travellers who use a scheduled service with fixed headway. Depending on parameters, travellers may be risk averse or risk seeking and the value of travel time may increase or decrease in the mean travel time.

1 Introduction

Congestion is widespread in road, rail and air networks and causes delay, entailing significant costs for societies. This cost is generally valued by the value of travel time, a concept with a long and distinguished history starting

*mf@transport.dtu.dk.

from Becker (1965) and DeSerpa (1971). An associated effect of congestion is that travel times become variable and random from the perspective of travellers deciding whether and when to travel. The cost of this travel time variability is thought to be of the same order of magnitude as the cost of delays when congestion is strong. Random travel time variability affects not only transport by car but also users of scheduled services such as buses, trains and airplanes.

The concept of travel time is quite straightforward. The concept of the value of travel time is then just as straightforward, at least in principle: it is the value of marginal changes in travel time. The concept of travel time variability, however, is less straightforward. In general it takes a (countable) infinite number of parameters to characterise a random travel time distribution and so a choice must be made concerning which aspects of the travel time distribution to vary. The implications of this choice are tightly connected to the assumptions made concerning traveller preferences.

Recently, Fosgerau and Karlstrom (2010) presented a derivation of the value of travel time variability based on scheduling preferences adapted from Vickrey (1969) and Small (1982).[1] They derived the time cost for a trip of random duration for a traveller who could freely choose his departure time, with these scheduling preferences and optimal choice of departure time. The time cost is the value of travel time multiplied by the mean travel time plus a constant, the value of travel time variability, times the standard deviation of travel time. As will be discussed below, this result is appealing in some ways but not in other ways. The present paper uses a different formulation of scheduling preferences to derive an alternative expression for the traveller's time cost of a trip of uncertain duration. The alternative result has some advantages over the Fosgerau and Karlstrom (2010) result.

The scheduling preferences used by Fosgerau and Karlstrom (2010) are often referred to as $\alpha - \beta - \gamma$ preferences. A traveller is assumed to have a preferred arrival time, which can be normalised to be time 0. He dislikes being early or late at the destination and he also dislikes travel time. His scheduling utilility υ associated with departing at time t and arriving at time

[1] Following Vickrey (1969), this type of scheduling preferences is often used in bottleneck models of congestion (e.g. Arnott et al., 1993). Noland and Small (1995) and Bates et al. (2001) preceded Fosgerau and Karlstrom (2010) with similar results for some specific travel time distributions.

a is written as

$$-v(t,a) = \alpha(a-t) + \beta \max(0,-a) + \gamma \max(0,a), \tag{1}$$

where $\alpha > 0$ is the marginal value of travel time $(a-t)$, $\beta > 0$ is the marginal cost of earliness $\max(0,-a)$, and $\gamma > 0$ is the marginal cost of lateness $\max(0,a)$. If travel time is random $(a-t) = \mu + \sigma X$, where X is random with CDF Φ, $EX = 0$ and the distribution of X is independent of t, and if the traveller chooses departure time to maximise scheduling utility, then the maximum expected utility is

$$-Ev^* = \alpha\mu + (\beta+\gamma)\sigma \int_{\frac{\gamma}{\beta+\gamma}}^{1} \Phi^{-1}(s)\,ds. \tag{2}$$

The scale of the travel time distribution σ can be taken as a measure of the degree of travel time variability. It can be any measure of scale such as the standard deviation or measures based on quantiles (e.g. Small et al., 2005). The term $\int_{\frac{\gamma}{\beta+\gamma}}^{1} \Phi^{-1}(s)\,ds$ is always positive, since $EX = 0$.[2] Hence travellers are always risk averse. The optimal departure time is $b_* = -\mu - \sigma\Phi^{-1}\left(\frac{\gamma}{\beta+\gamma}\right)$ and the mean arrival time is $b_* + \mu = -\sigma\Phi^{-1}\left(\frac{\gamma}{\beta+\gamma}\right)$.

There are a number of advantages associated with the result (2). First, maximum expected utility is just a linear combination of the mean travel time μ and the travel time variability σ. Second, the result holds for essentially any distribution of travel times Φ. Third, the preferred arrival time does not appear in the expression. It is then not necessary to know the preferred arrival times of travellers in order to apply scheduling preferences. Previously, this was thought to be an obstacle as such information is hard to find. Fourth, the result provides a basis for including a measure of scale of the distribution of travel times directly in the specification of preferences. This has been done in a range of papers, but lacked the justification that is obtained from defining preferences in terms of travel times outcomes rather than the travel time distribution. Fifth, Fosgerau & Karlstrom show that the expression (2) remains a good approximation when μ and σ are allowed to depend (in a limited way) on the departure time t.

[2] The function $H(x) = \int_x^1 \Phi^{-1}(s)\,ds$ satisfies $H(0) = H(1) = 0$. It is increasing for $x < \Phi(0)$ and decreasing for $x > \Phi(0)$. Hence it is always positive.

There are however also disadvantages associated with $\alpha-\beta-\gamma$ preferences (1) and the result (2). First, the value of travel time variability depends on the shape of the travel time distribution through the term $\int_{\frac{\gamma}{\beta+\gamma}}^{1} \Phi^{-1}(s)\,ds$. Second, the expression (2) is not additive over parts of a trip. Additivity would have been a desirable property of a measure of the value of travel time variability, since then the time cost could have been computed separately for different parts and then added. This would have made easier the application of (2) to links in a network. Third, and perhaps most importantly, it is not given that (1) is the best representation of the scheduling preferences of travellers. Finally, the traveller must be able to freely choose his departure time, which is not true for a scheduled service.

Just as many travellers may care about not being late for some activity, they might also care about not leaving some other activity too early. The $\alpha-\beta-\gamma$ preferences treat departure time differently from arrival time. There is a special time for arrivals but no special time for departures. A priori it is not clear why this should be so.

Consider travellers who differ in one respect only, the duration of the trip. The $\alpha-\beta-\gamma$ preferences imply that the traveller with the longer duration would depart earlier but arrive at the same time as the traveller with the shorter duration. This is an empirically testable proposition which may be used to refute (in an appropriately loose sense) $\alpha-\beta-\gamma$ preferences. This is considered in section 5 below.

Vickrey (1973) considered another type of scheduling preferences, recently reused by Tseng and Verhoef (2008).[3] They are introduced in section 2 below. This type of scheduling preferences associates a time varying utility rate with time spent at the origin and a similar time varying utility rate with time spent at the destination. The scheduling utility associated with a trip departing at time t and arriving at time a is the utility gained from being at the origin until time t and at the destination after time a. This is appealing since it connects scheduling preferences with the activities before and after the trip in a symmetric way.[4] The main purpose of this paper

[3] Tseng and Verhoef do not cite Vickrey (1973).

[4] Based on stated-preference data, Ettema et al. (2004) estimated parameters of the time varying utility rates for the departure time choice of complete tours while Tseng and Verhoef (2008) applied non-parametric techniques to estimation of such rates for trips from home to work. Wang (1996) estimate a schedule delay cost function for the morning commute. Zhang et al. (2005) uses utility rates in conjunction with the bottleneck model to carry out an analysis of trip timing.

is to show that such scheduling preferences, under appropriate simplifying assumptions, lead to a value of travel time variability that is as simple and applicable as the Fosgerau & Karlstrom result (2), while not sharing some of its disadvantages. First, it does not depend on the shape of the travel time distribution. Second, the associated measure of travel time variability is the variance of the travel time, which is additive over parts of a journey if the travel time parts are independent. Third, the result applies also to scheduled services. Fourth, under natural assumptions on the time varying utility rates, the value of travel time is increasing with expected trip duration, which is in compliance with empirical evidence (Gunn, 2001). Finally, it may turn out that these scheduling preferences provide a better explanation of observed scheduling behaviour.

The layout of the paper is as follows. Section 2 introduces the alternative formulation of scheduling preferences. Section 3 considers the case of random travel time for a traveller who can choose his departure time freely. Section 4 considers scheduled services. Section 5 discusses empirical implications of the two models of scheduling preferences and evaluate these against empirical data. Section 6 concludes. Table 1 in the appendix provides a list of the notation used.

2 Scheduling preferences

Consider a traveller who departs from the origin of a trip at time t and arrives at the destination at time a, where $t < a$ and who has the following scheduling utility.

$$u(t,a) = \int_0^t (\beta_0 + \beta_1 s)\,ds + \int_a^0 (\gamma_0 + \gamma_1 s)\,ds, t \leq a. \tag{3}$$

The greek letters β and γ are chosen such that β is associated with time spent at the origin of the trip and γ is associated with time spent at the destination. The traveller derives utility at the rate $\beta_0 + \beta_1 s$ from being at the origin at clock time s. He derives utility at the rate $\gamma_0 + \gamma_1 s$ from being at the destination at clock time s. Both rates are assumed to be linear in clock time. They may be interpreted as differences in the utility rate between time spent in the activities at the origin and at the destination from time spent during travel. This scheduling utility is illustrated in Figure 1. The zeros in the integration limits in (3) are chosen for convenience and can be replaced

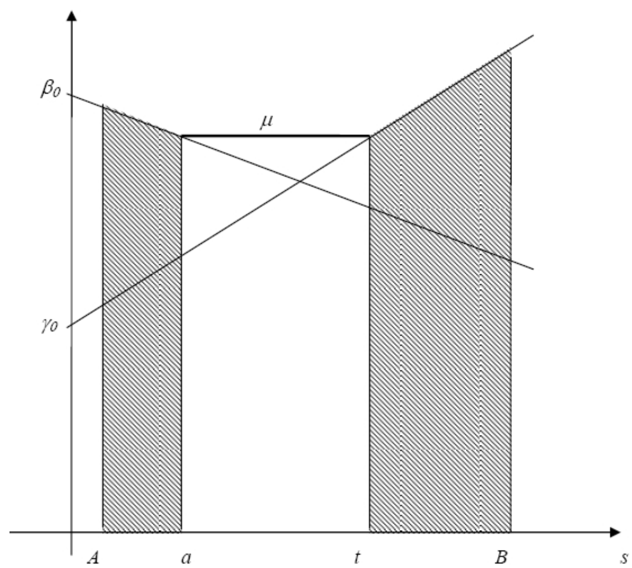

Figure 1: The utility rate functions and the optimal departure and arrival times given travel time μ. Except for a constant, the corresponding scheduling utility (3) is given by the shaded area.

by arbitrary constants, such as A and B in the figure, since utility can be shifted by a constant and represent the same preferences. The requirement that $\beta_1 < \gamma_1$ ensures that there is a time when it becomes preferable to be at the destination rather than at the origin. Note that this requirement does not rule out that β_1 and γ_1 have the same sign.[5] The normalisation $\beta_1 = \gamma_1 - 1$ is imposed for mathematical convenience at no loss of generality. Note that the formulation of utility does not include a term for monetary trip cost.

Consider a trip that lasts μ time units with certainty. Then the optimal departure time t is given by $t_* = \arg\max_t u(t, t+\mu)$. The first-order condition is $\beta_0 + \beta_1 t_* = \gamma_0 + \gamma_1 (t_* + \mu)$, such that the optimal departure time is given as a linear function of μ by $t_*(\mu) = (\beta_0 - \gamma_0) - \gamma_1\mu$. The corresponding arrival time is $a_*(\mu) = (\beta_0 - \gamma_0) - (\gamma_1 - 1)\mu$. If travel time is zero,

[5] The formulation of scheduling preferences does not allow for a discrete lateness penalty such as found by Small (1982).

then the traveller will optimally transfer from origin to destination at time $t_*(0) = \beta_0 - \gamma_0$. This time is normalised at no loss of generality such that $\beta_0 = \gamma_0$ and $t_*(0) = 0$. The second-order condition is always satisfied.[6]

The optimal utility is

$$\begin{aligned} u(t_*, t_* + \mu) &= \int_0^{t_*} (\gamma_0 + \beta_1 s)\, ds + \int_{t_*+\mu}^{0} (\gamma_0 + \gamma_1 s)\, ds \\ &= \gamma_0 t_* + \frac{\beta_1}{2} t_*^2 - \left(\gamma_0 (t_* + \mu) + \frac{\gamma_1}{2} (t_* + \mu)^2 \right) \\ &= -\gamma_0 \mu + \frac{\beta_1 \gamma_1}{2} \mu^2, \end{aligned}$$

such that the value of travel time is $\gamma_0 - \beta_1 \gamma_1 \mu$, which may be increasing or decreasing with the duration of the trip μ.

The typical case would have $\beta_1 < 0 < \gamma_1$, such that the marginal utility of being at the origin is decreasing in clock time and the marginal utility of being at the destination is increasing in clock time. In this case, the value of travel time is increasing with the duration of the trip. As mentioned above, the cases where β_1 and γ_1 have the same sign are not ruled out. In these cases, the value of travel time decreases in the duration of the trip.

3 Random travel time

Consider now the situation where travel time $T = a - t$ is random, but independent of t. Denote the mean travel time by $\mu = ET$ and the variance of travel time by $\sigma^2 = E[T^2] - \mu^2$. The traveller chooses t_* to maximise $E(u|t)$. The first-order condition is

$$\begin{aligned} 0 &= \frac{\partial}{\partial t} E\left[\int_0^t (\gamma_0 + \beta_1 s)\, ds + \int_{t+T}^{0} (\gamma_0 + \gamma_1 s)\, ds \right] \\ &= E[\gamma_0 + (\gamma_1 - 1)t - (\gamma_0 + \gamma_1 t + \gamma_1 T)] \\ &= -E[t + \gamma_1 T] \\ &= -t - \gamma_1 \mu, \end{aligned}$$

[6]With the linear formulation of the utility rates, they become very small or very large far away from the point where they intersect. This is just a mathematical convenience. It does not matter for results what the utility rates are at points in time that are farther away from $t_*(0)$ than the duration of any trip.

such that $t_* = -\gamma_1 \mu$. This is exactly the same as when travel time is certain and does not depend on the distribution of travel time except for the mean. The second-order condition is always satified.

The corresponding optimal expected utility is given by the following expression.

$$
\begin{aligned}
E\left(u|t_*\right) &= \int_0^{t_*} \left(\beta_0 + \beta_1 s\right) ds + E\left[\int_{t_*+T}^{0} \left(\beta_0 + \gamma_1 s\right) ds\right] \\
&= \beta_0 t_* + \frac{\beta_1}{2} t_*^2 - E\left[\beta_0 \left(t_* + T\right) + \frac{\gamma_1}{2} \left(t_* + T\right)^2\right] \\
&= \frac{\beta_1}{2} t_*^2 - \beta_0 \mu - \frac{\gamma_1}{2} \left(t_*^2 + 2 t_* \mu + \sigma^2 + \mu^2\right) \\
&= \frac{\beta_1}{2} \gamma_1^2 \mu^2 - \beta_0 \mu - \frac{\gamma_1}{2} \left(\gamma_1^2 \mu^2 - 2\gamma_1 \mu^2 + \sigma^2 + \mu^2\right) \\
&= -\beta_0 \mu + \frac{\beta_1 \gamma_1}{2} \mu^2 - \frac{\gamma_1}{2} \sigma^2.
\end{aligned}
$$

This shows that the optimal expected utility depends only on the mean and the variance of travel time. The natural measure of travel time variability corresponding to these scheduling preferences is the variance of travel time. The associated value is $\frac{\gamma_1}{2}$. The value of mean travel time $\gamma_0 - \beta_1 \gamma_1 \mu$ is the same as in the deterministic case.

The result based on time varying utility rates (3) has some advantages. A first advantage over $\alpha - \beta - \gamma$ scheduling preferences is that the shape of the travel time distribution plays no role. This is a significant advantage since it implies that a value of travel time variance can be transferred from one situation to another without a need to consider the difference in travel time distributions. A second advantage is that the travel time variability is measured by the variance of travel time. The variance is additive across parts of a trip if the parts of random travel time are independent.

Travellers may be risk seeking or risk averse, depending on the sign of γ_1. If $\gamma_1 < 0$, then travellers are risk seeking. By assumption $\beta_1 < \gamma_1$. Thus travellers are risk-seeking if they are travelling near a time where the marginal utilities of being at the origin or at the destination both are decreasing.

4 Scheduled services

This section extends the previous analysis to the case of scheduled services. The traveller is now seen to use a scheduled service with travel time $\mu + \sigma X$ with the same assumptions as before. The service departs with a fixed headway of h minutes.[7] Consider again a traveller with scheduling preferences (3). These do not comprise an impact of the service schedule. The underlying assumption is still that travellers care only about the time spent at the origin and at the destination of the trip. Hence preferences for waiting time and travel time are the same. Travellers are assumed to know the travel time distribution and with this information the scheduled times make no difference.

The analysis follows that of Fosgerau (2009). Travellers may be planning or unplanning. Unplanning travellers choose a departure time from the trip origin knowing only the headway but not the schedule of the service. They therefore wait at the station until the next scheduled departure. Planning travellers incur a planning cost $\zeta > 0$ in exchange for knowing the schedule and do not wait at the station.

Consider first the case of a planning traveller. His expected scheduling utility associated with choosing a departure at time t is

$$\begin{aligned} E_p(u|t) &= E\left(\gamma_0 t - \gamma_0 (t + \mu + \sigma X) + \beta_1 \frac{t^2}{2} - \gamma_1 \frac{(t + \mu + \sigma X)^2}{2}\right) - \zeta \\ &= -\gamma_0 \mu - \frac{t^2}{2} - \frac{\gamma_1}{2}\left(\mu^2 + \sigma^2 + 2t\mu\right) - \zeta, \end{aligned}$$

where the subscript p denotes that this relates to a planning traveller. The expected utility is concave in t. Therefore the planning traveller will choose uniquely the departure in the interval $[t - h/2, t + h/2]$ defined by $E_p(u|t - h/2) = E_p(u|t + h/2)$. Then t is given uniquely by the equation

$$(t - h/2)^2 + 2\gamma_1 (t - h/2)\mu = (t + h/2)^2 + 2\gamma_1 (t + h/2)\mu,$$

which has solution $t = -\gamma_1 \mu$.

[7] It would be relevant to consider extensions of the present model that allow for non-constant or random headways, as well as random variability of access time. These issues are left for future research.

As previously discussed, the linear specification of the utility rates is convenient but only appropriate in an interval where they do not become very large or small. The requirement that the utility rate at the destination is positive at the time $t+\mu\pm h/2$ is equivalent to $\gamma_0-\gamma_1(\gamma_1-1)\mu > |\gamma_1| h/2$. Similarly, the utility rate at the origin at time $t \pm h/2$ is positive when $\gamma_0 - \gamma_1(\gamma_1 - 1)\mu > |\gamma_1 - 1| h/2$. Together these inequalities imply that

$$\gamma_0 - \gamma_1(\gamma_1 - 1)\mu > \left(\left|\gamma_1 - \frac{1}{2}\right| + \frac{1}{2}\right)\frac{h}{2}. \tag{4}$$

The population of travellers is considered heterogenous in the preferred time of travel but still homogenous in γ_1. It is convenient to instead take the perspective of a single random traveller and consider the departure times of the scheduled service to be uniformly distributed over $[t - h/2, t + h/2]$. The average utility of a planning traveller is then

$$\begin{aligned} E_p u &= -\frac{1}{h}\int_{-\gamma_1\mu-\frac{h}{2}}^{-\gamma_1\mu+\frac{h}{2}} \left(\gamma_0\mu + \frac{t^2}{2} + \frac{\gamma_1}{2}\left(\mu^2+\sigma^2+2t\mu\right)\right) dt - \zeta \\ &= -\gamma_0\mu - \frac{\gamma_1}{2}\left(\mu^2+\sigma^2\right) - \frac{1}{h}\left[\frac{t^3}{6} + \frac{\gamma_1}{2}\mu t^2\right]_{-\gamma_1\mu-\frac{h}{2}}^{-\gamma_1\mu+\frac{h}{2}} - \zeta \\ &= E(u|t_*) - \frac{1}{24}h^2 - \zeta. \end{aligned}$$

This is exactly the same as in the previous section except for the last two terms. The influence of travel time variability is exactly as in the unscheduled case. Hence the value and the measure of travel time variability are unaffected by the service schedule. The term $\frac{1}{24}h^2$ indicates the cost for a planning traveller of being restricted to a schedule. It is zero if headway h is zero. The marginal cost of headway for a planning traveller is $h/12$.

An unplanning traveller chooses his departure time from his origin not knowing the schedule of the service. In addition to the random travel time he also incurs a random waiting time for the next departure of the service. The waiting time is random with a uniform distribution over $[0, h]$, which has mean $h/2$ and variance $h^2/12$. Travel time on the service is independent of his departure time from home and hence the traveller considers travel time on the journey including waiting time to have mean $\mu + h/2$ and variance $\sigma^2 + h^2/12$. Using the result from the previous section, his optimal expected utility is

$$\begin{aligned} E_n u &= -\gamma_0 (\mu + h/2) + \frac{\beta_1 \gamma_1}{2} \left(\mu + \frac{h}{2} \right)^2 - \frac{\gamma_1}{2} \left(\sigma^2 + \frac{h^2}{12} \right) \\ &= E(u|t_*) - \gamma_0 h/2 + \frac{\beta_1 \gamma_1}{2} \frac{h^2}{4} + \frac{\beta_1 \gamma_1}{2} \mu h - \frac{\gamma_1}{2} \frac{h^2}{12} \\ &= E(u|t_*) - \gamma_0 h/2 + \frac{\beta_1 \gamma_1}{2} \mu h + \left(\gamma_1 - \frac{4}{3} \right) \gamma_1 \frac{h^2}{8}, \end{aligned}$$

where subscript n indicates an unplanning traveller. The first term is the optimal expected utility without scheduling constraints. The remaining terms measure the cost associated with being restricted to a schedule. Again, these terms do not depend on travel time variability and so the value and the measure of travel time variability are unaffected by the service schedule. The marginal cost of headway for an unplanning traveller is $\gamma_0/2 - \frac{\beta_1 \gamma_1}{2} \mu - \left(\gamma_1 - \frac{4}{3} \right) \gamma_1 \frac{h}{4}$, which is positive when $\gamma_0 > 0$ and $\gamma_1 - 1 = \beta_1 < 0 < \gamma_1$.

Define now the gain from plannning $\tau(h)$ by $\tau(h) - \zeta = E_p u - E_n u$. A traveller with planning cost ζ chooses to plan when $\tau(h) > \zeta$, where

$$\begin{aligned} \tau(h) &= \gamma_0 h/2 - \frac{\beta_1 \gamma_1}{2} h\mu - \left(\gamma_1 - \frac{4}{3} \right) \gamma_1 \frac{h^2}{8} - \frac{1}{24} h^2 \\ &= \gamma_0 h/2 - \frac{\beta_1 \gamma_1}{2} h\mu - \beta_1 (\gamma_1 - 1/3) \frac{h^2}{8}. \end{aligned}$$

If the planning cost ζ has CDF Ψ in the population with density ψ, then the optimal expected utility for an average traveller is[8]

$$E(u|t_*) + (1 - \Psi(\tau(h))) \left(-\gamma_0 h/2 + \frac{\beta_1 \gamma_1}{2} h\mu + \left(\gamma_1 - \frac{4}{3} \right) \gamma_1 \frac{h^2}{8} \right)$$
$$-\frac{1}{24} h^2 \Psi(\tau(h)) - \int_0^{\tau(h)} \zeta \psi(\zeta) \, d\zeta.$$

Thus, similarly to the case of individual travel where the departure time choice is unrestricted, the only term in the overall travel cost related to travel time variability is proportional to the variance of travel time and does not depend on the shape of travel time distribution.

[8]The formula is valid also when $\tau(h) < 0$, since the planning cost is assumed to be strictly positive such that $\Psi(\tau) = \psi(\tau) = 0$ when $\tau < 0$.

Depending on the parameters and the headway, the gain from planning may be negative, in which case nobody will plan. If the utility rates are always positive in the interval where planning travellers choose departure such that (4) holds, then it may be verified that there is a positive gain from planning for all $\gamma_1 \in \left[-\frac{5}{3}, \frac{\sqrt{22}+5}{3}\right]$.

It is natural to assume that the distribution of the planning cost in the population of travellers is such that the minimum planning cost is positive and the maximum is less than infinity. If furthermore $1/3 < \gamma_1 < 1$ then the gain from planning increases without bound in h, since the coefficient to h^2 is then positive. Then no traveller plans at short headways, all travellers plan at long headways, and there is a transition range of headways in which some travellers plan and some do not.

5 Empirically testable implications

The departure time choices of travellers are observable as are the arrival times. The two alternative models of scheduling preferences have different implications for these aspects of observable behaviour. This gives a possibility for discriminating between the models.

It has been noted above how the optimal departure time depends on mean travel time μ and the variance σ^2. In the case of an $\alpha - \beta - \gamma$ traveller, the optimal departure time is $b_* = -\mu - \sigma\Phi^{-1}\left(\frac{\gamma}{\beta+\gamma}\right)$ and the mean arrival time is $b_* + \mu = -\sigma\Phi^{-1}\left(\frac{\gamma}{\beta+\gamma}\right)$. An isolated increase in μ of Δ minutes will then lead to departure Δ minutes earlier. The mean arrival time is not affected. An isolated increase in σ will lead to earlier departures and to earlier arrivals on average when $\Phi^{-1}\left(\frac{\gamma}{\beta+\gamma}\right) > 0$.

In contrast, travellers with scheduling preferences (3) would optimally depart at time $t_* = -\gamma_1\mu$ with corresponding mean arrival time $-\beta_1\mu$. Assume that $\gamma_1 - 1 = \beta_1 < 0 < \gamma_1$, which would be the typical case. Then $0 < \gamma_1 < 1$. An isolated increase in μ of Δ would lead to departure $\gamma_1\Delta$ minutes earlier, which is less than in the $\alpha - \beta - \gamma$ case. Similarly, the mean arrival time would be $-\beta_1\Delta$ minutes later, whereas the mean arrival time in the $\alpha - \beta - \gamma$ case would be unaffected. An isolated increase in σ^2 would not affect the departure time or the mean arrival time, whereas both will change in the $\alpha - \beta - \gamma$ case. A change in the shape of the standardised travel time distribution Φ would affect the departure time in the $\alpha-\beta-\gamma$ case but not in the case (3).

These observations provide means of distinguishing empirically between travellers with scheduling preferences of the two types considered. We provide a small illustratíon of how this may be done. Consider now identical travellers going to a common destination but located at different distances. They face deterministic travel time. Then $\alpha-\beta-\gamma$ travellers who live further away will depart earlier and arrive at their preferred arrival time. In contrast, travellers with scheduling preferences (3) who live further away will depart earlier but arrive later.

A dataset has been extracted from the Danish national travel survey for the years 2006-2008. The chosen observations are commuting trips by car to the central municipalities of Copenhagen. There are 175 trips that go directly from home to work and end in Copenhagen between 7 and 10 AM. The data record departure and arrival times and the trip distance. Regressing the departure time against the distance from home to work yields that these commuters depart on average 0.73 minutes earlier per km (t-stat 4.2). Regressing the arrival time against the distance from home to work yields that they arrive on average 0.31 minutes later per km (t-stat 1.8). The latter is significantly greater than zero in a one-sided test. Moreover, constant terms in the regressions both indicate that commuters would depart and arrive very close to 8 AM, if the distance was zero. Regressing trip duration against distance shows that the average speed in the data is 57.6 km/h and that there is an additional startup time of 7.3 minutes per trip. The variance of travel time in this sample increases with distance, but it is not clear how much of this is day to day variability since observed trips do not have the same origin and destination.

This small empirical exercise shows a pattern that is consistent with the present model of scheduling preferences (3) but not with $\alpha-\beta-\gamma$ scheduling preferences, when travel time is considered deterministic from the point of view of travellers. It should however be noted that we have not used any controls with our small dataset. Distance could be correlated with other variables that affect trip-timing preferences. For example, highly-paid professionals who have the flexibility to arrive at work when they want may live in affluent suburbs far from where they work. Evidence that professionals do arrive late at work is reported in studies by Ott et al. (1980), Abkowitz (1981) and Moore et al. (1984) . Evidence on the effect of travel distance on trip timing is mixed. Ott et al. (1980) find that individuals with longer commutes tend to arrive later, but Neveu and Koeppel (1980) and Moore et al. (1984) find that they arrive earlier.

6 Concluding remarks

This paper has shown that a certain model of scheduling preferences, based on Vickrey (1973), leads to the variance of random travel time as the relevant measure of travel time variability. The associated value of travel time variability does not depend on the shape of the travel time distribution. The same result applies equally to travellers who can choose departure time freely and to travellers using a scheduled service.

The variance of travel time is an attractive measure of travel time variability since it only requires random travel times on parts of the trips to be independent in order to be additive over parts. The model implies, however, that the cost related to mean travel time is not additive over parts, unless the utility rate at the origin is constant ($\beta_1 = 0$). In this case, the utility rate at the destination must be increasing. A small empirical exercise indicates that actual departure and arrival times are more consistent with this model of scheduling preferences than with $\alpha - \beta - \gamma$ scheduling preferences.

The universe of possible formulations of scheduling preferences contains many more possibilities than the scheduling preferences (3) used in this paper and the $\alpha - \beta - \gamma$ scheduling preferences (1). Both are special cases of general scheduling preferences $U(t, a)$ that are concave, increasing in t and decreasing in a. The main advantages of the two simple types of scheduling preferences are simplicity and convenience. Ultimately, the choice between formulations of scheduling preferences and the associated measures and value of travel time variability should not be based on convenience but on conformity with observable behaviour.

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Table 1: List of notation

Variable	Definition
t	Departure time
t_*	Optimal departure time
a	Arrival time
β_0, β_1	Intercept and slope of utility rate at origin
γ_0, γ_1	Intercept and slope of utility rate at destination
u	Scheduling utility
T	Random travel time
μ, σ^2	Mean and variance of travel time
X	Standardised travel time
$E_p u$, $E_n u$	Expected utility for planning traveller and for nonplanning traveller
h	Headway
ζ	Planning cost
τ	Gain from planning
Ψ, ψ	Cumulative distribution and density of planning cost
Φ	Cumulative distribution of standardised travel time
α, β, γ	Vickrey/Small scheduling parameters