The value of headway for a scheduled service

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The marginal social cost of headway for a scheduled service*

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Abstract

This brief paper derives the marginal social cost of headway for a scheduled service, i.e. the cost for users of marginal increases to the time interval between departures. In brief we may call it the value of headway in analogy with the value of travel time and the value of reliability. Users have waiting time costs as well as schedule delay costs measured relative to their desired time of arrival at the destination. They may either arrive at the station to choose just the next departure or they may plan for a specific departure in which case they incur also a planning cost. Then planning for a specific departure is costly but becomes more attractive at longer headways. Simple expressions for the user cost result. In particular, the marginal cost of headway is large at short headways and smaller at long headways. The difference in marginal costs is the value of time multiplied by half the headway.

1 Introduction

This paper proposes the concept of the value of headway for a scheduled service such as a bus route, a train or an air connection. The headway is the time interval between departures and the value of headway is the social cost of marginal increases in the headway. The problem derives from the (rather obvious) observation that users of scheduled services cannot choose their departure time freely, they are constrained to the departure times of the service. Furthermore, users must choose whether to plan for a specific departure or, alternatively, to choose just the time at which to arrive at the station in order to catch the next departure. It is important to know how

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the size of the headway affects user costs. Insights into this may inform analyses of demand as well as welfare economic analyses of changes in the supply of scheduled services.

For a frequent service we may imagine users do not plan to use a specific departure, they just arrive at their chosen time in order to catch the next departure and they might arrive at any time between two departures. Their waiting time at the station can be zero if they are lucky enough to arrive just in time for a departure. At worst, they arrive immediately after a departure and have to wait for a length of time equal to the headway. In general, we expect the waiting time at the station to follow a uniform distribution ranging from zero to the headway. This observation has led some to include just the average waiting time, half the headway, into the user cost. In the present case that accounts for scheduling considerations, users are seen to incur costs of waiting as well as costs due to the uncertainty about the time at which they will arrive at their destination.

For a less frequent service we may imagine that users plan which departure they want to use. In this case, the choice of service is based on scheduling considerations. The choice between planning and not planning we can imagine is governed by a cost of planning, which may include the effort involved in consulting the time table, the timing of the trip to the station as well as a planned wait at the station. We may suppose that the planning cost for a specific trip depends on the frequency of use of the service. Frequent users are able to distribute their planning cost over several trips. They therefore have a small planning cost for a specific trip and will tend more often to plan for a specific departure. Conversely, occasional travellers have a comparatively large planning cost and will tend less often to plan for a specific departure. In general we suppose the planning cost to have a distribution in the population of users such that no user will plan for a specific departure at very short headways, while all users will plan for a specific departure at very long headways. The distribution of planning costs in the population of users implies a smooth transition between the two cases as headway increases.

We consider a scheduled service that runs with a fixed headway of h minutes. The present analysis is based on scheduling preferences, where the user cost is described in terms of waiting time at the station and a schedule delay cost given as a function of the time of arrival at the destination relative to a preferred arrival time. We are not concerned with the travel time on the service and take this to be simply zero in order to simplify the analysis. Hence a user who boards the service at time t also arrives at his final destination at time t. If the travel time is known and not random, the cost of travel time may just be added to the cost expression obtained in this paper. Scheduling preferences are generally used to analyse situations with congestion where users trade travel time against deviations from the desired timing of a trip (see, e.g., Vickrey, 1969; Small, 1982; Arnott et al., 1993).

Here they are used to analyse a situation where users trade waiting time at the station against the desired timing of a trip with a scheduled service.

We consider a continuum of users, each of whom has a preferred arrival time (PAT), such that the PATs are uniformly distributed over time. This is a common idealisation that reflects the assumption that the distribution of PATs does not change much over a span of a few headways. The assumption ties in naturally with the interest in the headway and not the timetable for the scheduled service over, e.g., a day.

Consider a user with a preferred arrival time t^*, who arrives at the station at time t_1 and boards a departure at time t_2. He then has a waiting time at the station of $t_2 - t_1$ and arrives at his final destination at time t_2, since the travel time on the service has been normalised to 0. He has schedule delay cost given by the function D such that the scheduling cost associated with departing at time t_2 is $D(t_2 - t^*)$. The simplest case of D arises when $D(t_2 - t^*) = \gamma(t_2 - t^*)^+ + \beta(t_2 - t^*)^-$. In general we assume that D is convex with minimum at $D(0) = 0$. We also allow for a convex or linear cost term $A(\cdot)$ with $A(0) = 0$ and $A' > 0$ that is a function of waiting time such that the total time cost becomes $A + D$. In the linear case we define $A(t_2 - t_1) = \alpha(t_2 - t_1)$ and obtain the (α, β, γ)−framework that has been used in many papers. A user who arrives at the station at time t_1 and catches a service at time t_2 then incurs a total scheduling cost of $A(t_2 - t_1) + D(t_2 - t^*)$.

A number of previous contributions have focused on the socially optimal joint choice of fare and service frequency but have not presented an analysis of the value of headway. Mohring (1972) investigates the consequences of scale economies for optimal fares and frequency for urban bus services with unplanning users, paying particular attention to the effect of the number of users on travel time. He assumes that the waiting time is proportional to the headway and does not include schedule delay costs in his analysis. The case of planning users was dealt with by Panzar (1979), who considered optimal airline frequency and ticket price for users having schedule delay costs. The cases of unplanning and planning users were integrated by Jansson (1993) who also analysed the socially optimal choice of public transport price and service frequency. He includes a planning cost to select between the two cases, but it is the same for all users such that there is a distinct headway at which all users switch from not planning to planning.

The paper that the present paper resembles most is probably Tisato (1991), who presents an analysis that is similar to the one presented here. Broadly speaking, all the elements of the present analysis also appear in Tisato's. The present paper improves on Tisato's analysis in a number of ways. Most importantly, Tisato considers schedule delay as a function of the difference between the scheduled departure time and the preferred departure time, where the latter is the time which the user would like to be the scheduled departure time. Tisato does not consider schedule delay to be a function of the actual time the user arrives at the station nor of the actual departure time. This means that the waiting time at the station does not affect schedule delay. This seems unreasonable for the case of an unplanning user whose waiting time at the station is random. The formulation of schedule delay seems particularly unreasonable since Tisato allows for randomness of the actual departure times relative to the scheduled departure time. In Tisato's model a planning user will arrive early in order to take random departure time deviations into account. This affects his user cost only through the expected waiting time. It does not affect his schedule delay cost. The user can only affect his schedule delay cost by choosing between scheduled departures. In contrast, the present paper considers schedule delay to occur relative to the arrival time at the destination. Furthermore, Tisato allows only a linear scheduling cost where the marginal cost of schedule delay is the same for early and late delays, i.e. $D(t-t^*) = \beta(t_1-t^*)^+ + \beta(t_1-t^*)^-$. The present analysis allows for a general convex scheduling cost function D.

Tisato's analysis allows actual departure times to deviate randomly from scheduled departure times, whereas the present paper assumes that departures conform to the schedule. The literature has established a number of times (references given by Tisato) that the expected waiting time for an unplanning user is then expanded by a factor depending on the variance of actual random headways.[1] So for the calculation of the expected waiting time it is sufficient to know the variance of headways, one does not need to know the distribution of headways. This simple result does not carry over to the schedule delay cost which is a nonlinear function of the actual departure time. Tisato does not carry out an analysis of the effect of random headways in the case of a planning user. Instead he merely imports some empirical estimates of the effect of average headway and the variance of headway on the average waiting time at the station. This average waiting time is then just plugged into the user cost. Tisato presents no theory to support this.

de Palma and Lindsey (2001) consider the optimal time table under fixed demand, and a fixed number of departures over a period of fixed length. Users have linear, possibly heterogeneous, scheduling cost. They plan for a specific departure and differ with respect to their PAT. As part of their analysis, de Palma and Lindsey obtain the average schedule delay cost as also found in this paper for a planning user with linear schedule delay cost.

The layout of the paper is the following. Section 2 treats the case of long headways where the users plan which departure to use. Section 3 treats the case of a frequent service where users do not bother to consult the time table but merely choose when to appear at the station to catch the next departure. Section 4 integrates these two cases through the concept of a planning cost, such that users will plan for a specific departure if the benefit

[1] For example, in the case of departure times following a Poisson distribution, the expected waiting time is not half the headway but the expected headway.

of planning exceeds the cost. Section 5 discusses how the model may be applied in practice, while section 6 presents some concluding remarks.

2 A service with long headway

We first consider the case of a service with long headway, where a user plans which departure he wants to use. He does not wait and so his cost is given only in terms of the schedule delay cost function D. The waiting time cost A is not relevant. Instead of thinking of users being uniformly distributed over time, we may take the perspective of a single user and consider arrivals to be uniformly distributed over time. We may take a user with PAT=0 as representative of all users.

As figure 1 illustrates, the representative user will choose the departure in the interval $[t-h;t]$ where the time t is defined by the equation

$$D(t-h) = D(t). \tag{1}$$

The equation states that the user is indifferent between arriving t minutes late and $-(t-h)$ minutes early. Equation (1) defines a unique t with $t > 0$ since D is convex and has minimum at 0. Any arrival time inside the interval is preferred to t and $t-h$ and any arrival time outside the interval is strictly worse than any arrival time inside the interval. Since the travel time on the service is normalised to zero, the user will choose a departure in the interval $[t-h, t]$ defined by (1) and be at most t minutes late. In the case of a linear D, we have $t = h\beta/(\gamma+\beta)$. With departures considered to be uniformly distributed over time, the expected scheduling cost is then

$$C_p(h) = \frac{1}{h}\int_{t-h}^{t} D(s)\, ds, \tag{2}$$

where we use the subscript p to denote that this cost applies to a planning user. In the case of a linear schedule delay cost D, we find that $C_p(h) = h\frac{\gamma\beta}{2(\gamma+\beta)}$.[2] Since the user with PAT=0 is representative, we have that $C_p(h)$ is the average scheduling cost for all users.

We may use the convexity of D to find a bound for $C_p(h)$, namely

$$C_p(h) \leq \frac{1}{h}\left(t\frac{D(t)}{2} - (t-h)\frac{D(t-h)}{2}\right) = \frac{D(t)}{2}, \tag{3}$$

with equality when D is linear. This bound will be useful below.

The marginal cost of headway can be found by differentiating the cost with respect to h. Note first that the maximal time a planning user will be

[2] This result may be found in Proposition 2 of de Palma and Lindsey (2001).

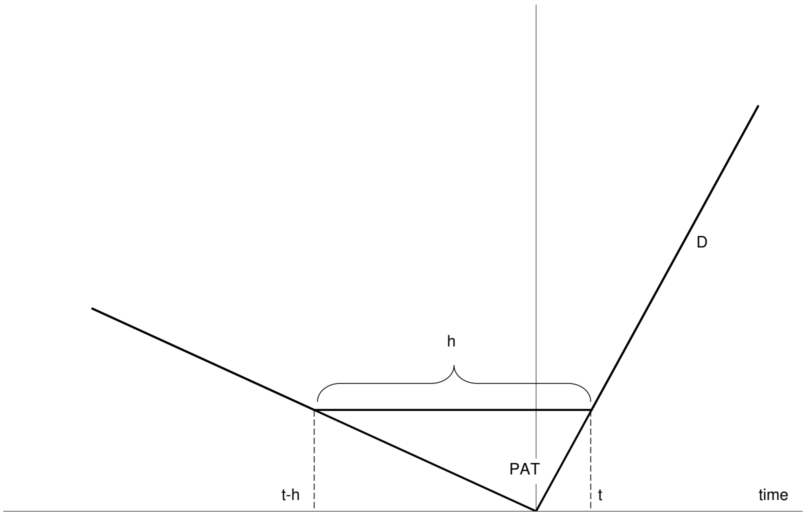

Figure 1: Optimal interval around the PAT

late, t, is a function of h. We may then find how t changes by differentiating (1) with respect to h,

$$D'(t)\,t' = D'(t-h)\left(t'-1\right),$$

such that

$$t' = \frac{-D'(t-h)}{D'(t) - D'(t-h)}.$$

This is positive by the assumptions on D, such that increasing the headway will increase the maximal lateness of a planning user. In the case of a linear D this becomes $t' = \beta/(\gamma+\beta)$.

Using Leibnitz' integral rule to differentiate $C_p(h)$ in (2) we find that

$$\begin{aligned} C_p'(h) &= \frac{D(t)}{h}t' - \frac{D(t-h)}{h}(t'-1) - \frac{C_p(h)}{h} \\ &= \frac{D(t) - C_p(h)}{h}. \end{aligned} \tag{4}$$

In the case of a linear D we have $C_p'(h) = \frac{\gamma\beta}{2(\gamma+\beta)}$, which is constant as a function of h.

Apply the bound on $C_p(h)$ in (3) to the expression for $C'_p(h)$ in (4) to find that

$$C'_p(h) \geq \frac{D(t)}{2h} > 0,$$

such that the marginal cost of headway is strictly positive for general convex D.

It may be of interest how the marginal cost of headway depends on h. Differentiate again to find that

$$C''_p(h) \quad = \quad \frac{D'(t)t' - 2C'_p(h)}{h}.$$

The remainder of this section shows that the second derivative of C_p is in fact positive such that the average scheduling cost of planning users is convex in headway and the marginal cost of headway is increasing.[3] The following lemma uses the scheduling cost function to define a function $K(h) = D(t(h))$, and shows that K is convex. This will be useful in proving the desired result.

Lemma 1 *Let $D : \mathbb{R} \to \mathbb{R}^+$ be a convex function with minimum at $D(0) = 0$. Define the function $K : \mathbb{R}^+ \to \mathbb{R}^+$ by $K(h) = D(t)$ where t is the unique solution to the equation $D(t) = D(t - h)$. Then K is convex.*

Proof. Observe that D is decreasing on $\mathbb{R}^-$ and increasing on $\mathbb{R}^+$. Then D has two inverse functions, denoted $D_+^{-1} : \mathbb{R}^+ \to \mathbb{R}^+$ and $D_-^{-1} : \mathbb{R}^+ \to \mathbb{R}^-$. Note that D_+^{-1} is concave and increasing and that D_+^{-1} is convex and decreasing. For $d > 0$ define $K^{-1}(d) = D_+^{-1}(d) - D_-^{-1}(d)$ and note that K^{-1} is concave and increasing. Then the inverse of K^{-1}, namely K, is convex and increasing. Consider now $K(h)$ and let $t = D_+^{-1}(K(h))$. Then $D(t) = K(h)$ and $D(t - h) = K(h)$ as required. ■

Figure 2 shows the convex and increasing function K. Define now the function $M(h) = hK(h) - \int_0^h K(s)ds$. This function corresponds to the area $M(h^*)$ above the function K, also indicated on figure 2. The derivative of M is $M'(h) = hK'(h)$. Note now that convexity of K implies that $M(h^*)$ is smaller than the triangle bounded by the vertical axis, the horizontal line at $K(h^*)$ and the line tangent to K through the point $(h^*, K(h^*))$. This is clear from inspection of figure 2. The triangle has area $\frac{(h^*)^2}{2}K'(h^*)$, so we note that

$$\frac{(h^*)^2}{2}K'(h^*) \geq M(h^*).$$

The last ingredient needed to prove that $C_p(h)$ is convex is to establish that $M(h) = hD(t) - \int_{t-h}^t D(s)ds$. To see this, first note that both definitions of M have $M(0) = 0$, next use $K(h) = D(t) = D(t - h)$ and note

[3] Robin Lindsey gave me a very helpful suggestion on how to prove this assertion.

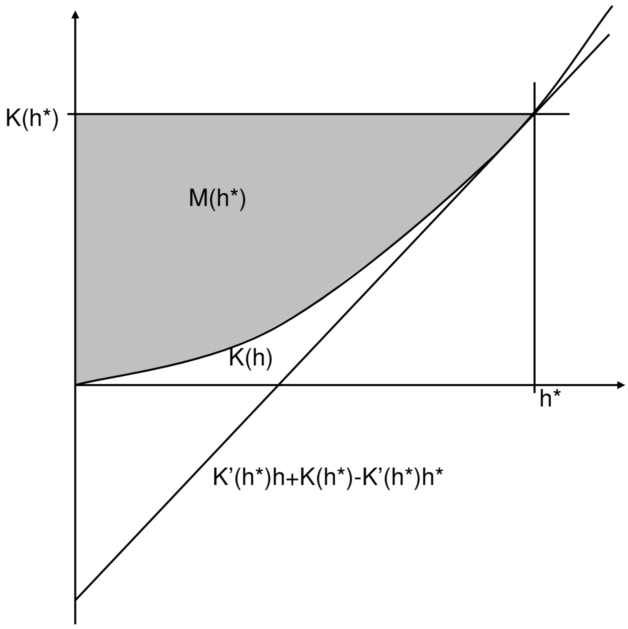

Figure 2: Illustration of proof that $C_p(h)$ is convex

that

$$\begin{aligned}\frac{\partial\left(hK(h)-\int_{t-h}^{t}D(s)ds\right)}{\partial h} &= hK'(h)+K(h)-D(t)t'+D(t-h)(t'-1)\\ &= hK'(h)=M'(h).\end{aligned}$$

We have now established all the necessary ingredients to show that $C_p(h)$ is convex. We need to show that $C_p''(h)\geq 0$. But

$$\begin{aligned}C''p(h) &= \frac{D'(t)t'-2C_p'(h)}{h}\\ &= \frac{K'(h)-2\frac{D(t)-C_p(h)}{h}}{h}\\ &= \frac{K'(h)-\frac{2}{h^2}M(h)}{h},\end{aligned}$$

which may be seen to be positive using the preceding findings.

3 A service with short headway

We now suppose that service is so frequent that a user will not consult the time table but appear at the station in order to catch the next departure, not knowing exactly when that will be. We may again take a user with PAT=0 as representative. If he arrives at the station at, say, time $t-h$, where t will be found below, then the next departure time is uniformly distributed over the interval $[t-h, t]$. His cost if the next departure occurs at time s is

$$A(s-t+h)+D(s)$$

and the expected cost is

$$C_u(t)=\frac{1}{h}\int_{t-h}^{t} A(s-t+h)+D(s)\,ds,$$

where the subscript u denotes that the cost applies to an unplanning user. The optimal arrival time t is found by setting the marginal expected cost equal to zero, using again Leibnitz' integral rule.

$$\frac{\partial C_u(t)}{\partial t}=\frac{1}{h}(A(h)+D(t))-\frac{1}{h}(A(0)+D(t-h))-\frac{1}{h}\int_{t-h}^{t} A'(s-t+h)\,ds=0.$$

Assume that A is linear, $A(s)=\alpha s$. Then the equation reduces to

$$\frac{1}{h}(\alpha h+D(t))-\frac{1}{h}(D(t-h))-\alpha\frac{1}{h}\int_{t-h}^{t} ds=0,$$

such that the first order condition becomes

$$D(t-h)=D(t),$$

which is exactly the same as in the planned arrival case. Taking t to be optimally chosen, we may derive the expected cost for an unplanning user C_u using the expression for the average cost C_p for a planning user in (2). The expected cost then becomes

$$\begin{aligned} C_u(h) &= \frac{\alpha}{h}\left[s^2/2-(t-h)s\right]_{t-h}^{t}+C_p(h) && (5)\\ &= \frac{\alpha h}{2}+C_p(h). && (6)\end{aligned}$$

This shows that the average schedule delay cost is the same for planning and unplanning users, which is quite remarkable. This finding is based on the assumption that the PATs are uniformly distributed over time. It is easy to construct examples of time tables and nonuniform distributions of PATs, where the equality of the average schedule delay cost for planning and unplanning users does not hold.

We may differentiate the expression (5) with respect to h to find the marginal cost of headway for an unplanning user as

$$C_u'(h)=\frac{\alpha}{2}+C_p'(h). \qquad (7)$$

This is exactly $\alpha/2$ larger in the unplanned case than in the planned case. The term $\alpha/2$ corresponds to the average waiting time of an unplanning user. From (7) we see that the marginal cost comprises an additional term, namely the marginal average schedule delay cost of a planning user.

4 To plan or not to plan

We have established the scheduling cost as a function of the headway for two situations. In one the users are assumed to plan for a specific departure while they are not planning in the other. Otherwise the situations are completely identical. To complete the story, we therefore need to explain why some users plan and others do not. At the same time we want the model to have the property that planning is more worthwhile at longer headways.

Assume that a user has a planning cost of $\zeta > 0$. If he plans for a specific departure, he will incur a total cost of $C_p(h) + \zeta$. If he does not plan, then his cost is $\alpha h/2 + C_p(h)$. Choosing the minimum cost option, he will then plan if $\zeta < \alpha h/2$. Assume that planning costs are distributed in the population with some cumulative distribution function Φ and density φ with bounded support. This will result in an interval of headways such that more and more users will decide to plan as the headway increases. The average user cost at headway h is

$$C(h) = C_p(h) + \left(1 - \Phi\left(\frac{\alpha h}{2}\right)\right)\frac{\alpha h}{2} + \int_0^{\frac{\alpha h}{2}} \zeta\varphi(\zeta)\, d\zeta. \tag{8}$$

The average marginal cost of headway then becomes

$$\begin{aligned} C'(h) &= C_p'(h) + \left(1 - \Phi\left(\frac{\alpha h}{2}\right)\right)\frac{\alpha}{2} - \varphi\left(\frac{\alpha h}{2}\right)\frac{\alpha^2 h}{4} + \frac{\alpha^2 h}{4}\varphi\left(\frac{\alpha h}{2}\right) \\ &= C_p'(h) + \left(1 - \Phi\left(\frac{\alpha h}{2}\right)\right)\frac{\alpha}{2}, \end{aligned}$$

which is always positive. Differentiate again to find that

$$C''(h) = C_p''(h) - \varphi\left(\frac{\alpha h}{2}\right)\frac{\alpha^2}{4}$$

such that the cost is concave in headway when scheduling costs are linear. We may suppose that φ has support on some finite interval I with $0 < \min I$, such that nobody will plan for very short headways while everybody will plan for very long headways. Then for very short headways we have $C'(h) = C_p'(h) + \alpha/2$ while for very long headways we have a lower marginal cost of $C'(h) = C_p'(h)$.

5 Application

For the application of the model we need to know first the scheduling preferences. We assume linear scheduling costs expressed by (α, β, γ). A number of studies have estimated these parameters, e.g. Bates et al. (2001) and Small (1982), see the review in Fosgerau et al. (2008), where the parameter

α is the value of travel time. In the present paper, α represents the value of waiting time, which is generally thought to be higher than the value of travel time. Therefore the value of α from the references cited here may be increased correspondingly, relative to β and γ.

It is harder to presume that we know the distribution of planning costs in the population. The planning cost makes no difference if we desire to compare two services where either all users plan or all users do not plan. It is however necessary to know the cost distribution in order to account for the planning cost in the case when two service schedules are compared where at least one of them involve both planning and unplanning users. The following argument is intended to show that it is possible to find an empirical basis for forming an opinion about the share of users who will choose a specific departure at different headways and hence that we may presume that Φ is known.

One possibility is simply to conduct a survey, asking users about their behaviour. Another possibility is to observe the relationship between departure times and the rate at which users arrive at the platform. One would need observations from a range of places to cover a range of headways. At places where users arrive at a constant rate we may conclude that they are not planning. We expect to observe this at places where the service has a small headway. If, in some other place, most users arrive close to the next departure, then we may think they are mostly planning. We expect to observe this at places where the service has a larger headway. In between we will have some users arriving at random and some users arriving close to the next departure. If we can estimate the share of users who plan as a function of headway, and if we know α, then we can identify $\Phi\left(\frac{\alpha h}{2}\right)$. Bowman and Turnquist (1981) undertake such a study.

We hence assume that Φ is known. In particular we know the support of Φ, that is, we know the maximum headway at which no users plan $h_{\min}$ and also the minimum headway at which all users plan $h_{\max}$. This defines the interval over which the planning costs are distributed. Let $\Delta = h_{\max} - h_{\min}$ denote the length of this interval.

Introduce for brevity of notation a function to censor the headway at $h_{\min}$ and $h_{\max}$ by

$$\Lambda(h) = \begin{cases} h_{\min} & \text{if } h < h_{\min} \\ h & \text{if } h_{\min} \leq h < h_{\max} \\ h_{\max} & \text{if } h_{\max} \leq h. \end{cases}$$

We could assume for simplicity that the distribution of planning costs in (8) is uniform, such that

$$\Phi\left(\frac{\alpha h}{2}\right) = \frac{\Lambda(h) - h_{\min}}{\Delta}$$

or

$$\Phi(\zeta) = \frac{2\zeta/\alpha - h_{min}}{\Delta}$$

$$\varphi(\zeta) = \frac{2}{\alpha\Delta}$$

for values of ζ in the interval $[\frac{\alpha}{2}h_{min}; \frac{\alpha}{2}h_{max}]$.

Recall that with linear scheduling cost we have $C_p(h) = C_p'(h)h$ and $C_p'(h) = \frac{\gamma\beta}{\gamma+\beta}$. We can then write the user cost function as

$$\begin{aligned} C(h) &= C_p(h) + \left(1 - \Phi\left(\frac{\alpha h}{2}\right)\right)\frac{\alpha h}{2} + \int_{\frac{\alpha h_{min}}{2}}^{\frac{\alpha\Lambda(h)}{2}} \zeta\varphi(\zeta)\,d\zeta \\ &= C_p'(h)h + \left(\frac{h_{max} - \Lambda(h)}{\Delta}\right)\frac{\alpha h}{2} + \frac{1}{\alpha\Delta}\left(\frac{\alpha^2\Lambda^2(h)}{4} - \frac{\alpha^2 h_{min}^2}{4}\right). \end{aligned}$$

This expression simplifies when h is outside the interval where users change from not planning to planning.

$$\begin{aligned} h &< h_{min} : C(h) = C_p'(h)h + \frac{\alpha h}{2}, C'(h) = C_p'(h) + \frac{\alpha}{2} \\ h &> h_{max} : C(h) = C_p'(h)h + \frac{\alpha}{4}(h_{max} + h_{min}), C'(h) = C_p'(h) \end{aligned}$$

Note here that the last term in $C(h)$ when $h > h_{max}$ is $E(\zeta) = \alpha(h_{max} + h_{min})/4$.

Substituting numerical values for the scheduling parameters yields an expression for the cost associated with headway. Figure 3 uses $(\alpha, \beta, \gamma) = (2, 0.5, 2)$, $h_{min} = 5$ and $h_{max} = 15$. We see that the cost curve is steep with slope $C_p'(h) + \alpha/2$ up to the point h_{min} where some users begin to plan. The curve is dashed in the interval $[h_{min}, h_{max}]$ where more and more users switch to planning. It is drawn here as a straight line but the shape depends on the distribution of planning costs in the population. Thereafter the curve becomes again linear with the smaller slope of $C_p'(h)$. The first and last line segments have been extended with light dashed segments to indicate that the cost curve is bounded above by these lines. The intersection of the extension of the last line segment with the y-axis corresponds to the average planning cost when all users plan.

6 Concluding remarks

This paper provides a model in which the headway of a scheduled service affects user costs in a simple way. It is then straight-forward to work the effect of headway into a user cost expression alongside the effect of various travel time components and monetary cost.

In some situations, one may think that schedule delay is related to the time of departure rather than to the time of arrival at the destination. For

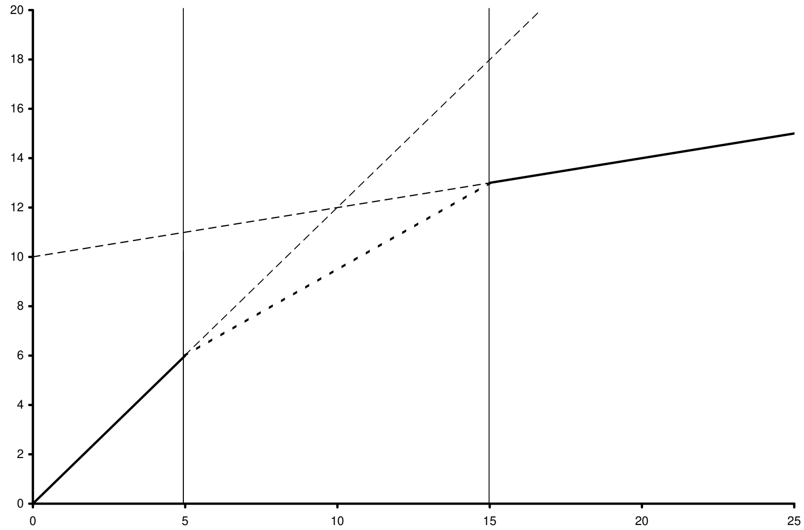

Figure 3: Illustration of the cost function as a function of headway

a planning user, there is no difference, since travel time is fixed and waiting time is zero. Then the choice of a departure simultaneously chooses the departure time and the arrival time. For an unplanning user we have to imagine that the random waiting time does not make him care about being late at his destination. He will just arrive at the station at his preferred time and will incur only waiting cost. So to cover the general case of a user who cares about his departure time, we need only remove the schedule delay term for the unplanning case. Everything else stays the same.

The paper leads up to some issues that may be considered in future research. A main aspect that is missing in the present analysis is the analysis of travel time on the service. While I have argued that the cost of travel time can just be added when travel time is fixed and independent of the departure time, it is relevant to investigate what happens when this is not the case. Incorporating that travel time may be random seems to be an interesting extension to consider. Fosgerau and Karlstrom (2007) use scheduling considerations to obtain a simple expression for the value of reliability, the value of marginal changes to the standard deviation of travel time, for the case of car drivers who can choose their departure time freely. They also show that their result does not carry over to the case of a scheduled service. There is however still the possibility that the results of this paper regarding the value of headway may carry over in some form to the case of travel time risk.

As discussed in the introduction, Tisato (1991) goes some way in allowing for service unreliability in the form of random deviations of departures from schedule. This remains an unresolved issue for the case when users care about being early or late at their destination.

The present paper views the planning cost as heterogenous with a continuous distribution in the population of travellers, where Jansson (1993) views it as being the same for all users. This gives him some problems since his welfare optimisation problem then has potentially more than one solution. It is conceivable that adopting the present paper's assumption about a continuous distribution could be used to streamline the analysis of Jansson. Where adding heterogeneity in many cases makes analysis more complicated, here it could perhaps make analysis both more tractable and more realistic.

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