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June 2003

Online at https://mpra.ub.uni-muenchen.de/42286/
MPRA Paper No. 42286, posted 31 Oct 2012 12:11 UTC
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Abstract

We consider the allocation of corporate control in a company with two large and a continuum of small shareholders. Control is determined in a shareholders’ meeting, where the large shareholders submit competing proposals in order to attract the vote of small shareholders. The presence of multiple shareholders reduces private benefits through competition for control. In the optimal ownership structure, the more efficient blockholder will hold just enough shares to gain control, but a large fraction of shares is allocated to the less efficient shareholder in order to reduce rents. We investigate when the large shareholders would want to trade parts or all of their share blocks among them, and show that the concern about retrading will lead to a larger than optimal stake of the controlling shareholder.

Keywords: Multiple shareholders, corporate control, contestability, block trading.

JEL Classification: G32, G34.

∗ The authors would like to thank Marco Becht, Neil Brisley, Mike Burkart, Bernard Caillaud, Thomas Gehrig, Florian Heider, Lisa Meulbroek, Piet Moerland, Giovanna Nicodano, Darius Palia, Peter Roosenboom, Paolo Volpin and Jürgen Weigand for helpful conversations and suggestions. We are grateful to conference participants at WFA (Los Cabos), EFA (Barcelona), AFFI (Namur), DGF (Vienna) as well as faculty seminars at Freiburg, Leuven, Namur, CERGE Prague, Rotterdam, Tilburg, Toulouse, Warwick and Koblenz for stimulating discussions.

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1 Introduction

Since Berle and Means (1932), the central conflict of interest in the modern corporation was seen as opposing employed managers and dispersed shareholders, and this view was tacitly extended to the world outside the United States. But the idea of shareholder dispersion was increasingly at odds with more accurate documentation of corporate ownership patterns, which showed that in virtually every country other than the US and the UK, the vast majority of large publicly traded companies have large shareholders, often with controlling share blocks. Consequently, the research agenda has recently broadened to address the conflict between large shareholders and minority shareholders. Symptomatic for this trend, Johnson et al. (2000) suggest that problems of “tunneling”, the term they propose to generally describe “the transfer of resources out of a company to its controlling shareholder”, are endemic in civil law countries.

But for all the new emphasis on the role of large shareholders, another misconception may yet have taken hold of the discussion, namely the idea that the typical ownership structure pits a single large (and presumably controlling) shareholder against a sea of dispersed shareholders, each endowed with too small an equity stake to wield significant influence. This picture has dominated the theoretical and empirical literature on the role of large shareholders since the seminal contributions of Shleifer and Vishny (1986) and Demsetz and Lehn (1985).

Recent empirical literature on ownership structure, however, shows that many large companies have several shareholders with significant blockholdings. In eight out of nine of the largest stock markets in the European Union, the median size of the second largest voting block in large publicly listed companies exceeds five percent, according to results of the European Corporate Governance Network; and in Germany, the only exception on this list, between 25 and 40 percent of listed firms have two or more large shareholders (Becht and Boehmer (2000), Lehmann and Weigand (2000)). Perhaps most surprisingly, in the United Kingdom, long seen as the country with the least shareholder concentration, the size of the second and third largest blocks appears to be larger than in the European average. By contrast, for NYSE and NASDAQ-listed corporations in the United States, significant voting blocks apart from the largest share stake are apparently much rarer (Becht and Mayer (2000)).

The objective of this paper is to investigate the structure of corporate governance and the allocation of corporate control in the presence of multiple large shareholders. We see three principal...
motivations behind this research.

First, it pertains to the understanding of corporate governance mechanisms outside the United States. Confronted with this reality, empirical researchers have used various measures of ownership concentration, but, in the words of LaPorta et al. (1999, p. 476), “a theoretically appropriate measure requires a model of the interactions between large shareholders, which we do not have.” Our paper aims to contribute to the filling of this gap.

Second, such a model is also interesting because the coexistence of several block holdings frequently occurs by design rather than historical accident. After their IPO, many young firms have several large owners, for example founders, venture capitalists, or corporate allies. And in privatizations, governments often bring in a number of strategic investors, along with the wide dissemination of the stock among the general public and employees. As an illustration, the Spanish government first sold substantial equity blocks of Iberia, the national air carrier, to two categories of strategic shareholders, to domestic banks representing local roots and to airline alliance partners British Airways and American Airlines to bring in industry clout, two years before the privatization was completed with an IPO in April 2001.

Third, as institutional investors try to become active shareholders in the hope of stimulating firm performance, the question is how successfully they can challenge the control of large incumbent shareholders.

We analyze the strategic interplay between the various blockholders on the one hand, and between any of the blockholders and small shareholders on the other hand, and the consequences for corporate performance and shareholder value. We limit the analysis in this paper to the case where two blockholders compete for effective control in a company. In our model, the allocation for control is decided by a vote, which is interpreted as the vote for the composition of the board of directors in a shareholders’ meeting. The two large shareholders submit competing proposals to the vote, and small shareholders will only vote if they are sufficiently relevant, in view of the cost of participation. In order to lure small shareholders and assemble a majority of votes, the two large shareholders can pledge to limit the private benefits of control they will take. The relative merit of the proposals also depends on the shareholders’ specific competence to develop the company’s strategy.

We investigate this triangle of strategic interactions in order to find answers to the following questions: What is the relationship between small shareholders and multiple blockholders who are competing for control, and who will seize corporate control? Is the competition between multiple

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5 Interestingly, another block was set aside for employees, an aspect which we do not address in the current paper.

6 For the frontrunner of this strategy, the Californian public sector pension fund CalPERS, the influence on corporate control seems to be well-documented. The findings on performance improvement are mitigated at best (Smith (1996), Romano (2000)), in contrast to evidence that ownership concentration and the involvement of large shareholders improve performance, for example by Ang et al. (2000) for smaller US companies, and by Claessens et al. (2000) for Asian economies.
share blocks successful in limiting “tunneling”? What is the optimal ownership structure of a company in an initial share sell-off, say in an IPO or a privatization? Is the design of multiple share blocks a stable structure, or would subsequent trading of blocks or on the stock exchange be likely to alter the ownership structure? Should we expect large shareholders to really compete for control, or are they rather likely to collude against small shareholders?

Our key results are as follows. The relevant concept of control power is the *contestability* of the incumbent shareholder’s position, and not just ownership concentration. In analogy to the contestable markets concept\(^7\), we say that corporate control is *contestable* if the incumbent cannot increase the level of control rents without losing control in a control contest. We suggest that contestability is a useful concept to predict the identity and allocation of control and the capacity to extract benefits depend in fact on contestability. The maximum rent extraction under contestable control is determined by two factors, the shareholders’ ability to create value when in control on the one hand and the relative size of their blocks on the other hand. Both factors are substitutes, so rents decrease as the controlling shareholder loses appeal as a wealth creator or as her block decreases in size relative to the competitor’s. The design of the ownership structure can exploit this in order to minimize control rents, and allocate a relatively larger block to the less efficient shareholder in order to make the competition more equal. But if the more competent shareholder can provide monitoring services at sufficiently lower costs, then the shareholder in control should also be the largest shareholder.

In an extension, we consider the possibility that large shareholders trade parts or their entire blockholdings between them. By merging the blocks in a block trade, competition can be mitigated, so at first glance this appears to be attractive for the large shareholders. But block mergers may in turn imply that the company value is reduced, because the blockholders complement each other in creating value. We show that multiple blocks will only coexist if ownership is substantially concentrated in the hands of the largest shareholder, and more concentrated than appears desirable in our analysis of the optimal share allocation.

Our model leads to additional empirical implications. Based on contestability, we propose a simple measure of the concentration of voting power as the difference in size between the leading blocks, normalized by the free float. Multiple blockholders are more likely if they belong to different categories of shareholders. Contestability of control, and not ownership concentration per se, should determine firm performance. The optimal size of the largest block is increasing in the free float, but the size of the controlling block is only increasing in the free float if heterogeneity about monitoring skills is important. Our analysis shows that the conventional idea of multiple shareholders acting as substitutes for poor legal protection of dispersed shareholders is incomplete. When the legal protection is weak, then the deadweight losses that shareholder competition help

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\(^7\)In essence, a market in industrial economics is (perfectly) contestable if no entrant can cut the price and make a profit when supplying the equilibrium quantity (Baumol et al. (1982)).
to avoid are usually less important, and multiple shareholders may be less beneficial than with strong protection. Our analysis suggest that block premia can be used to identify whether control is associated with a block.

Two other recent papers have recently explicitly addressed the issue of competing blocks. In Gomes and Novaes (2000), there are two blockholders sharing control. The consequence is less tunneling, but also a risk to miss out on valuable projects due to their internal disagreement. Bennedsen and Wolfenzon (2000) take for granted that a controlling coalition of shareholders will be formed. The winning coalition is the minimal coalition garnering a majority of votes, and the larger it is, the less private benefits will it extract at the expense of minority shareholders.

In these two papers, multiple shareholders exercise control jointly, whereas our model is centered around the contestability of a single controlling shareholder. All three papers have in common the idea that the presence of multiple shareholders imposes limits on the extraction of private benefits. Our contribution to this literature is to consider simultaneously a strategic role of blockholders and of dispersed shareholders, within an explicit voting contest. Moreover, our paper is innovative in introducing shareholder heterogeneity and showing that the competence of large shareholders is a second independent dimension, besides voting power, determining the allocation of control. We also suggest new approaches to the analysis of block trading.

Related themes of multiple large shareholders have also been visited in other work. Pagano and Roell (1998) suggest that multiple blocks commit the firm to protect minority investors. A number of papers argue that multiple blockholders are unlikely to emerge. In Zwiebel's (1995) general equilibrium model, investors are sorting such that only one of them holds a block in any given firm, precisely because they want to eschew the sort of competition over benefits that we model. Winton (1993) emphasizes the free-rider problem in monitoring efforts among multiple large shareholders. Similarly, in Bolton and von Thadden's (1998) liquidity-control trade-off, multiple large shareholders would increase the liquidity costs without offering compensating advantages in monitoring.

The paper is organized as follows. In Section 2, the model is laid out. Section 3 contains the basic analysis of the bidding strategies and the voting outcome. The determinants and the comparative statics of the optimal ownership structure are investigated in Section 4. In Section 5, we analyze the conditions of retrading-proofness vis-a-vis block trades. Empirical implications are discussed in Section 6. Section 7 concludes.

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8By contrast, in Gomes and Novaes (1999) there is no role for minority shareholders whereas Bennedsen and Wolfenzon (2000) do not consider a non-cooperative interaction of the large shareholders.
2 The Model

We analyze a control contest in a company with two large blockholders, $i = 1, 2$, and a continuum of small shareholders, $s \in [0, 1]$. The company could be either privately held or publicly listed. Prior to the contest, the ownership structure of the firm is chosen by the initial owner of the firm. We suppose that the initial seller of the firm (a founder-entrepreneur, a venture capitalist, a company spinning off assets, or a government privatizing state enterprises) seeks to partition the equity blocks in order to maximize proceeds. We denote by $\alpha_1$ and $\alpha_2$ the fractions of shares owned by the two large shareholders, and by $\bar{\alpha}$ the total amount of shares sold as blocks to the large shareholders, $\bar{\alpha} = \alpha_1 + \alpha_2$. The remainder of the shares, $1 - \bar{\alpha}$, are distributed uniformly among the small shareholders.

A shareholders’ meeting is convened in order to allocate control power. At the meeting, each of the two large shareholders proposes a plan to run the company. The plans describe measures that the controlling shareholder will implement in order to limit her control power and to protect the interests of minority shareholders (large and small). We suppose that the large shareholders’ plans can be summarized by the maximal amount of private benefits that they can capture if they control the firm, denoted $B_1$ and $B_2$. The proposals limiting the control benefits to $B_1$ and $B_2$ are binding commitments that will be enshrined in the company charter and cannot be revoked by the board. There is an upper limit $\bar{B}$ that $B_1$ and $B_2$ cannot exceed.

The extraction of private benefits $B_i$ results in a linear value loss to the company of $\gamma B_i$. The parameter $\gamma$ indicates how easy it is for a controlling shareholder to convert company resources into private resources. Hence, the parameter $\gamma$ can be interpreted as a measure of the legal protection of minority shareholders, with larger values of $\gamma$ corresponding to a better protection of minority interests, as in Shleifer and Wolfenzon (2000) and Burkart and Panunzi (2000) for example. We assume $\gamma > 1$ to capture the idea that benefit extraction is costly for the shareholders as a whole.

The allocation of control power to one of the two large blockholders results from a vote of all the shareholders of the company, small and large. The allocation of control power should be understood as the vote for the composition of the board of directors. Assuming that the number of seats in the board is uneven, the outcome of the shareholders’ vote will be an unequivocal allocation of control power to one of the two large shareholders. Each share carries one vote, and the controlling shareholder is elected by simple majority of the votes effectively cast.\(^9\) While the two large shareholders always participate in the meeting, the attendance of small shareholders is not guaranteed. Specifically, we assume that small shareholders incur a cost to participate in the contest.

\(^9\)In order to break ties, we assume that when the two plans receive the same number of votes, the winning shareholder is the one who receives the largest number of votes of small shareholders. If each receives the same number of votes of small shareholders, the efficient shareholder wins the contest.
meeting. This cost represents not only the transportation and opportunity cost of attending the meeting (which could be alleviated by proxy voting), but also the cost of gathering and processing information about the two large shareholders’ types and proposals at the meeting. Different small shareholders have different voting costs, and we assume that the distribution of voting costs of small shareholders is uniform on \([0, 1]\).\(^{10}\)

The winning shareholder’s use of her control powers is an important determinant of firm value. The two large blockholders differ in their ability to add value to the company when in control. We represent this ability by a parameter \(\theta_i\), called competence, which captures all of a shareholder’s attributes that translate control into value, for example her capacity to define and implement the company’s strategy. We let \(\theta_1\) and \(\theta_2\) denote the competence of the two shareholders when they control the firm and assume, without loss of generality, that Shareholder 1 is more competent than Shareholder 2, \(\theta_1 \geq \theta_2\). The difference in competence is denoted \(\Delta \theta = \theta_1 - \theta_2\).

Once the vote has taken place, and the winner of the control contest is chosen, the two large shareholders provide monitoring efforts \(e_1\) and \(e_2\). They incur convex monitoring costs given by \(c_1(e_1) = e_1^2\) and \(c_2(e_2) = e_2^2\). Finally, the value of the firm is realized. The firm value is a simple additively separable function of the shareholders’ monitoring efforts, the controlling shareholder’s competence level, and the private benefits extracted by the controlling shareholder,

\[
v = \theta_i + e_1 + e_2 - \gamma B_i .
\]

Our assumption that both shareholders can add value through monitoring services is motivated by the idea that shareholders of different types and expertise may share in the ownership of a company, for example families, banks/financials or institutional shareholders, alongside corporate owners, which again are different if they are unrelated or horizontally or vertically related. There is evidence in support of this idea. Boehmer (2000) finds that multiple shareholders improve the performance of takeover decisions of German listed companies, but only if one shareholder is a bank and the other shareholder a family or a corporate owner, not if both are of the same type. Also, corporate shareholders with close industry knowledge may play a particular value-enhancing role, as we hinted in our example of the Spanish air carrier Iberia. Allen and Phillips (2000) show that over half of all corporate block acquisitions are made by firms in related industries, and that the entry of corporate blockholders leads to large and significant value gains, especially if the blockholder has also direct industrial ties.

We assume throughout that the total amount of shares sold as blocks to the two large shareholders, \(\bar{\alpha}\), is exogenously given. This will arise whenever the initial seller of the firm first chooses the amount of shares floated to the general public, and then decides how to partition equity blocks among the two large shareholders. The initial seller’s decision on the number shares sold to the

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\(^{10}\)This is the aggregate voting cost of a mass of 1 of small shareholders, i.e. a mass just sufficient to hold the entire company. If there are \(N\) small shareholders each holding one share, each would bear \(1/N\) of this cost, where we consider \(N \to \infty\) in our continuous limit.
general public is not explicitly modeled here. It could result, for example, from a motivation to sell
shares to the small shareholders in order to increase the market liquidity for the stock. A more
liquid stock gives direct benefits to the company like a higher transparency and informational
accuracy, and is likely to be reflected in a liquidity premium in the stock valuation. Alternatively,
the initial seller may pursue a “Machiavellian objective” as in Biais and Perotti (2001),
and choose to sell shares to the public in order to influence their future decisions. While we
account for the possibility of block trades between the large shareholders (Section 5), we assume
that $\alpha$ remains fixed throughout, so the large shareholders do not sell or buy stock on the open
market. In support of this assumption, we refer to the argument that large shareholders should
normally find it in their interest to stick to the existing ownership structure, since changes in their
holdings will lead them to endogenously adjust their monitoring efforts, an effect which should be
rationally anticipated in the stock valuation and forestall trading (Burkart et al. (1997)). Also,
large shareholders should take the trade-off between liquidity and control into account, since the
large shareholders themselves will value the exit option that the liquidity of the stock grants to
them (Bolton and von Thadden (1998)).

In order to make the analysis interesting, we suppose that neither of the two large shareholders
is initially offered a majority stake in the company, $\alpha_1 < \frac{1}{2}$ and $\alpha_2 < \frac{1}{2}$ and that the controlling
shareholder always has an incentive to extract private benefits, for any split of the ownership
structure, $1 - \gamma\alpha > 0$.

Our model thus incorporates two different sources of heterogeneity among the two large block-
holders: they may differ in their competence to run the company $\theta_1$ and $\theta_2$, and in their monitoring
costs, $c_1$ and $c_2$. Another (endogenous) source of heterogeneity stems from the distribution of
shares, $\alpha_1$ and $\alpha_2$. As we will see in the next sections, these asymmetries play a fundamental role
in the determination of the firms’ value and the optimal ownership structure.

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| ownership structure chosen | blockholders bid $B_1, B_2$ | voting decides on control $e_1, e_2$ | efforts chosen | value $v$ realized |

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*Figure 1: Time Line*

Figure 1 illustrates the time line of the model. We proceed to solve the model by backward
induction. Section 3 analyzes the outcome of the control contest and the determination of the
private benefits and the company’s value, for a fixed ownership structure $(\alpha_1, \alpha_2)$. Section 4
considers the optimal ownership structure chosen by the initial seller seeking to maximize the
firm’s value.

7
3 Control Contests

3.1 Optimal Levels of Monitoring

At the last stage of the model, the two large blockholders choose their monitoring effort levels in order to maximize their utility levels. We adopt the convention to denote the controlling shareholder as blockholder $i$, and the non-controlling shareholder as shareholder $j$. The utilities are given by

$$U_i = \alpha_i(\theta_i + e_i + e_j) + (1 - \gamma \alpha_i)B_i - c_i \frac{e_i^2}{2}$$

$$U_j = \alpha_j(\theta_i + e_i + e_j) - \gamma \alpha_j B_i - c_j \frac{e_j^2}{2}.$$

As the firm’s value is additively separable in all its terms, the optimal monitoring efforts are independent of the identity of the controlling shareholder and the private benefits. They are given by the first-order conditions of the utility functions as

$$e_i^* = \frac{\alpha_i}{c_i}, \quad e_j^* = \frac{\alpha_j}{c_j}.$$

Substituting back these optimal effort levels, the utilities of the two large shareholders and of the small shareholders can be rewritten directly as a function of the ownership structure $(\alpha_1, \alpha_2)$ as

$$U_i = \alpha_i \left( \theta_i + \frac{\alpha_1}{c_1} + \frac{\alpha_2}{c_2} \right) + (1 - \gamma \alpha_i)B_i - \frac{\alpha_i^2}{2c_i}$$

$$U_j = \alpha_j \left( \theta_i + \frac{\alpha_1}{c_1} + \frac{\alpha_2}{c_2} \right) - \gamma \alpha_j B_i - \frac{\alpha_j^2}{2c_j}$$

$$U_s = (1 - \bar{\alpha}) \left( \theta_i + \frac{\alpha_1}{c_1} + \frac{\alpha_2}{c_2} \right) - \gamma (1 - \bar{\alpha})B_i.$$

3.2 Voting Equilibrium

At the voting stage of the model, all shareholders evaluate the proposals $B_1$ and $B_2$. A rapid inspection of the payoffs of the three types of shareholders shows that the efficient shareholder (Shareholder 1) always prefers her plan, $B_1$, to the plan of the other shareholder. Shareholder 2 prefers her plan if and only if $(1 - \gamma \alpha_2)B_2 + \gamma \alpha_2 B_1 \geq \alpha_2 \Delta \theta$. Hence, if the difference in competence is high enough, the inefficient blockholder prefers to give control of the company to the other blockholder. Small shareholders favor the plan of the efficient shareholder if and only if $\gamma (B_1 - B_2) \leq \Delta \theta$. Figure 2 graphs the preferences of the second large shareholder and the small shareholders in the plane $(B_1, B_2)$. Notice in particular that when Shareholder 2 prefers the plan $B_1$, the small shareholders also prefer the plan of the efficient blockholder.
The equilibrium strategies of the two large blockholders are easily characterized. As there are only two alternatives, it is a weakly dominant strategy for each blockholder to vote for her preferred plan. On the other hand, small shareholders face a coordination problem, as their participation to the meeting is costly, and their ability to influence the outcome of the vote depends on the participation decisions of other small shareholders. To solve this coordination problem, we restrict our attention to strong equilibria of the voting game, i.e. equilibria such that no group of agents with positive measure has an incentive to deviate.\footnote{See Aumann (1959) for this concept. We emphasize that we adopt a non-cooperative approach. The use of strong equilibria is merely an equilibrium refinement in order to reduce the number of equilibria in the small shareholders’ coordination problem, but does not indicate the use of cooperative game theory concepts. It indicates that when the two large shareholders propose plans they anticipate the least favorable outcome (for the largest shareholder) or the most favorable outcome (for the smallest shareholder).}

![Figure 2: Preferences](image)

Since voting is costly, it is a dominant strategy for small shareholders not to participate in the meeting when their preferences agree with those of the largest blockholder. Furthermore, a situation where both large blockholders prefer the plan $B_1$ but small shareholders prefer the plan $B_2$ can never arise. Hence, the only case where the vote of small shareholders matters is when they favor the plan of the smallest blockholder. We are thus left with two cases to consider: (i) one where $\alpha_2 \leq \alpha_1$ and small shareholders prefer $B_2$ to $B_1$ and (ii) one where $\alpha_1 \leq \alpha_2$ and small shareholders prefer $B_1$ to $B_2$.

The first case ($\alpha_2 \leq \alpha_1$) is illustrated in Figure 3A.\footnote{The formal proposition and proofs are given in the Appendix.} Shareholder 2 wins the contest if and only if she attracts the votes of a fraction $\frac{\alpha_1 - \alpha_2}{1 - \alpha}$ of the small shareholders. This implies that the plan $B_2$ is adopted if and only if a fraction $\frac{\alpha_1 - \alpha_2}{1 - \alpha}$ of the small shareholders has a voting cost $\kappa$. 
satisfying\textsuperscript{13} \( \theta_2 - \gamma B_2 - \kappa \geq \theta_1 - \gamma B_1 \), or

\[ \kappa \leq \gamma(B_1 - B_2) - \Delta \theta \]

As voting costs are uniformly distributed, this occurs if and only if \( \frac{\alpha_1 - \alpha_2}{1 - \alpha} \leq \gamma(B_1 - B_2) - \Delta \theta \), or

\[ \frac{\Delta \theta}{\gamma} + \frac{\alpha_1 - \alpha_2}{\gamma(1 - \alpha)} \leq B_1 - B_2 . \]

Comparing Figure 2 with Figure 3A, we thus see that the region of plans \((B_1, B_2)\) where Shareholder 2 wins the contest can be obtained by a parallel shift of the line representing the preferences of small shareholders. The inefficient shareholder wins the contest only if the difference in plans is high enough to overcome the cost of voting of a large enough fraction of the small shareholders.

Consider now the second case \((\alpha_1 \leq \alpha_2)\). By a similar reasoning, Shareholder 1 wins the contest if and only if she attracts the votes of a fraction \( \frac{\alpha_2 - \alpha_1}{1 - \alpha} \) of the small shareholders. This will arise whenever \( \frac{\alpha_2 - \alpha_1}{1 - \alpha} \leq \gamma(B_2 - B_1) + \Delta \theta \), or

\[ \frac{\alpha_2 - \alpha_1}{\gamma(1 - \alpha)} - \frac{\Delta \theta}{\gamma} \leq B_2 - B_1 . \]

Again, the voting costs of small shareholders induce a parallel shift of the line representing the preferences of small shareholders. However, when the inefficient shareholder has a larger fraction of shares, this parallel shift may result in two configurations, depending on the exact position of the crossing point of the new line representing small shareholders’ preferences and the line

\textsuperscript{13}Recall that the voting cost \( \kappa \) is normalized to a unit mass of shareholders.
representing the preferences of the second large shareholder. Figures 3B and 3C illustrate the two possible subcases.

In Figure 3B, the two lines cross in the interior of the positive orthant. This defines two connected regions where Shareholders 1 and 2 win the contest. In Figure 3C, the two lines cross outside the positive orthant, and the regions where Shareholder 1 wins the contest are disconnected. Shareholder 1 either wins (for low values of $B_2$) because all shareholders unanimously agree on the plan $B_1$ or (for high values of $B_2$) because she manages to attract enough votes of small shareholders to defeat the plan of Shareholder 2. However, there is an intermediate range of plans $B_2$ for which Shareholder 2 always wins the contest, as she prefers to vote against the plan $B_1$ and Shareholder 1 cannot attract enough votes to defeat the plan of the inefficient shareholder.

![Figure 3B: Voting Outcome if $\alpha_1 < \alpha_2$ and Shareholder 1 Wins Control](image)

**3.3 Control Contests**

We now turn to the stage where the two large shareholders simultaneously choose the plans $B_1$ and $B_2$. The competition between the two large shareholders is reminiscent of a model of Bertrand competition between firms with asymmetric costs (see Shy (1995) p.109), and, with the help of Figures 3A, 3B and 3C, we can characterize the unique equilibrium values of the plans proposed by the two large blockholders.

In Figures 3A and 3C, one of the two shareholders has a clear advantage over the other (Shareholder 1 in Figure 3A and Shareholder 2 in Figure 3C). Each large blockholder seeks to undercut her competitor, and the equilibrium is obtained when the disadvantaged shareholder offers a plan with zero private benefits (points $E$ in the two figures). At this equilibrium, the advantaged shareholder is able to extract strictly positive private benefits.\(^{14}\) The situation of

\(^{14}\)Formally, as in a Bertrand model with asymmetric firms, this equilibrium exists only if there is a discrete money
Figure 3B is more complex to analyze. As we show in the Appendix, the unique equilibrium is obtained at point $E$, which is the maximal point at which Shareholder 1 wins the contest irrespective of the choice $B_2$. We summarize our findings in the following Proposition.

![Figure 3C: Voting Outcome if $\alpha_1 < \alpha_2$ and Shareholder 2 Wins Control](image)

**Proposition 1** (i) Suppose $\alpha_1 \geq \alpha_2$ (Figure 3A). In the unique equilibrium of the control contest, Shareholder 1 wins and extracts private benefits $B_1^* = \frac{\Delta \theta}{\gamma} + \frac{\alpha_1 - \alpha_2}{\gamma(1 - \alpha)}$. Shareholder 2 proposes $B_2^* = 0$.

(ii) Suppose $\alpha_2 \geq \alpha_1$ and $\frac{\alpha_2 - \alpha_1}{1 - \alpha} \leq \frac{\Delta \theta}{1 - \gamma \alpha_2}$ (Figure 3B). In the unique equilibrium of the control contest, Shareholder 1 wins and extracts private benefits $B_1^* = \frac{\Delta \theta}{\gamma} - \frac{(1 - \gamma \alpha_2)(\alpha_2 - \alpha_1)}{\gamma(1 - \alpha)}$. Shareholder 2 proposes $B_2^* = \frac{\alpha_2(\alpha_2 - \alpha_1)}{1 - \alpha}$.

(iii) Suppose $\alpha_2 \geq \alpha_1$ and $\frac{\alpha_2 - \alpha_1}{1 - \alpha} > \frac{\Delta \theta}{1 - \gamma \alpha_2}$ (Figure 3C). In the unique equilibrium of the control contest, Shareholder 2 wins and extracts private benefits $B_2^* = \frac{\alpha_2 - \alpha_1}{\gamma(1 - \alpha)} - \frac{\Delta \theta}{\gamma}$. Shareholder 1 proposes $B_1^* = 0$.

*Proof:* See the Appendix.

4 Optimal Ownership Structure
We now turn to the computation of the optimal ownership structure chosen by the initial seller of the firm. We rewrite the value created by monitoring efforts and the equilibrium cost of monitoring as

\[ V(\alpha_1, \alpha_2) = \frac{\alpha_1}{c_1} + \frac{\alpha_2}{c_2}, \quad C_1(\alpha_1) = \frac{\alpha_1^2}{2c_1}, \quad C_2(\alpha_2) = \frac{\alpha_2^2}{2c_2}. \]

We suppose that the objective of the initial owner is to maximize the total value of the firm for all shareholders:

\[ W = V(\alpha_1, \alpha_2) - C_1(\alpha_1, \alpha_2) - C_2(\alpha_1, \alpha_2) + \theta_i - (\gamma - 1)B_i(\alpha_1, \alpha_2), \]

where \( i \) denotes the identity of the controlling shareholder.

Notice that the choice of the shares \( \alpha_1 \) and \( \alpha_2 \) affects the total value of the firms through two different channels. On the one hand, the ownership structure determines the value created by the monitoring efforts of the two large shareholders, \( V(\alpha_1, \alpha_2) - C_1(\alpha_1, \alpha_2) - C_2(\alpha_1, \alpha_2) \); on the other hand, the choice of \( \alpha_1 \) and \( \alpha_2 \) affects the amount of private benefits, \( B_i(\alpha_1, \alpha_2) \), extracted by the controlling shareholder. In general, the interplay between these two effects is very complex, and the optimal ownership structure is difficult to characterize.\(^\text{15}\) Suppose for instance that Shareholder 1 has a lower monitoring cost than Shareholder 2. The maximization of the value created by monitoring efforts would then prescribe to increase the share of the first shareholder. If, however, Shareholder 1 is the controlling shareholder of the firm, this increase results in an increase in private benefits and may reduce the total value of the firm. The global effect of an increase in the share of the first shareholder thus depends on the relative magnitude of the effects on the monitoring value and the private benefits.

Recall that the total amount of shares sold as blocks to the two large shareholders, \( \bar{\alpha} \), is exogenously given.\(^\text{16}\) We can thus simply replace \( \alpha_2 = \bar{\alpha} - \alpha_1 \). We recall that the monitoring value of the firm is given by

\[ V(\alpha_1, \alpha_2) - C_1(\alpha_1, \alpha_2) - C_2(\alpha_1, \alpha_2) = \frac{\alpha_1}{c_1} + \frac{(\bar{\alpha} - \alpha_1)}{c_2} - \frac{\alpha_2^2}{2c_1} - \frac{(\bar{\alpha} - \alpha_1)^2}{2c_2}. \]

The private benefits can be computed as

\[^{15}\text{For arbitrary values of } \alpha_1 \text{ and } \alpha_2, \text{ the total value of the firm is neither a convex nor a concave function of the ownership structure. Hence, we cannot use first order conditions to determine the optimal distribution of shares.}\]

\[^{16}\text{Recall also the assumption that } 1 - \gamma \alpha_i > 0 \text{ is satisfied. Otherwise, the initial seller could decide to sell a very large fraction of shares to the controlling shareholder, who would then optimally decide not to extract any private benefits.}\]
Hence, the optimal distribution of shares necessarily lies in the interval 
without loss of generality restrict our attention to the case 

It is immediate to check that, when \( c_1 \leq c_2 \) (larger control competence is aligned with lower 
monitor costs), \( \bar{\alpha}_1 \leq \frac{\bar{\alpha}}{2} \leq \alpha_1 \). Furthermore, offering more than \( \alpha_1 \) shares to Shareholder 1 is 
inefficient, as a decrease in the shares results both in an increase in the monitoring value and a 
reduction in private benefits. Similarly, offering less than \( \bar{\alpha}_1 \) shares is a dominated choice, since 
an increase in the number of shares increases the monitoring value and decreases private benefits.

Hence, the optimal distribution of shares necessarily lies in the interval \([\bar{\alpha}_1, \alpha_1]\) and Shareholder 1 ultimately gets control of the firm. However, as noted above, the computation of the optimal 
distribution of shares \( \alpha^*_1 \) in the interval \([\bar{\alpha}_1, \alpha_1]\) is a difficult exercise, as the objective function is 
a highly irregular function of the variable \( \alpha_1 \) and the parameters \( c_i, \Delta \theta, \bar{\alpha} \) and \( \gamma \). In order to gain 
some understanding of the optimal distribution of shares and its relation to the parameters, we 
distinguish between two polar cases: (i) one where the two firms are identical in their competence 
\( (\Delta \theta = 0) \) but differ in their monitoring costs \( (c_1 \leq c_2) \), and (ii) one where the two firms have 
identical monitoring costs \( (c_1 = c_2 = c) \) but differ in their competence \( (\Delta \theta \geq 0) \).

### 4.1 Heterogeneous Monitoring Costs

When the two shareholders have equal competence to control \( \Delta \theta = 0 \), then \( \bar{\alpha}_1 = \frac{\bar{\alpha}}{2} \), so we can 
without loss of generality restrict our attention to the case \( \frac{\bar{\alpha}}{2} \leq \alpha_1 \leq \bar{\alpha}_1 \). The total value of the 
firm is then given by the expression:

\[
B_i(\alpha_1, \alpha_2) = \begin{cases} 
\frac{\Delta \theta}{\gamma} + \frac{2\alpha_1 - \bar{\alpha}}{\gamma(1-\bar{\alpha})} & \text{if } 2\alpha_1 \geq \bar{\alpha} \\
\frac{\Delta \theta}{\gamma} - \frac{(1-\gamma(\bar{\alpha}-\alpha_1))(\bar{\alpha}-2\alpha_1)}{\gamma(1-\bar{\alpha})} & \text{if } 2\alpha_1 \leq \bar{\alpha} \text{ and } \frac{(\bar{\alpha}-2\alpha_1)}{1-\bar{\alpha}} \leq \frac{\Delta \theta}{1-\gamma(\bar{\alpha}-\alpha_1)} \\
\frac{(\bar{\alpha}-2\alpha_1)}{\gamma(1-\bar{\alpha})} - \frac{\Delta \theta}{\gamma} & \text{if } \frac{(\bar{\alpha}-2\alpha_1)}{1-\bar{\alpha}} > \frac{\Delta \theta}{1-\gamma(\bar{\alpha}-\alpha_1)} 
\end{cases}
\]

Figure 4 graphs the monitoring value of the firm and private benefits as a function of the 
share of the efficient shareholder. We let \( \hat{\alpha}_1 \) and \( \tilde{\alpha}_1 \) denote the ownership structure maximizing 
the monitoring value and minimizing private benefits respectively. Straightforward computations 
show that, in the relevant range of parameters\(^{17}\),

\[
\hat{\alpha}_1 = \frac{c_2 - c_1 + c_1 \bar{\alpha}}{c_1 + c_2} \\
\tilde{\alpha}_1 = \frac{3\bar{\alpha}}{4} - \frac{1}{2\gamma} + \frac{1}{4\gamma} \left[ (3\gamma \bar{\alpha} - 2)^2 + 8\gamma(\bar{\alpha} - \gamma \tilde{\alpha} - \Delta \theta(1 - \tilde{\alpha})) \right]^{rac{1}{2}}. 
\]

\(^{17}\)In order to construct Figure 4 and to compute the values maximizing monitoring and minimizing private 
benefits, we restrict parameters so that \( \hat{\alpha}_1 \) and \( \tilde{\alpha}_1 \) assume interior values in the interval \([0, \bar{\alpha}]\).
Figure 4: Monitoring Value $V - C_1 - C_2$ and Private Benefits $B_i(\alpha_1, \alpha_2)$ as a Function of $\alpha_1$

$$W = \frac{\alpha_1}{c_1} + \frac{\bar{\alpha} - \alpha_1}{c_2} - \frac{\alpha_1^2}{2c_1} - \frac{(\bar{\alpha} - \alpha_1)^2}{2c_2} + \theta_1 - \frac{(\gamma - 1)(2\alpha_1 - \bar{\alpha})}{\gamma(1 - \bar{\alpha})}.$$

The objective of the initial seller is to select $\alpha_1 \in [\frac{\bar{\alpha}}{2}, \hat{\alpha}_1]$ in order to maximize the total value $W$ of the firm, which is a strictly concave function of $\alpha_1$. Depending on the value of the parameters, the optimal choice can either be an interior solution,

$$\alpha_1^* = \frac{c_2 - c_1 + \bar{\alpha}c_1}{c_1 + c_2} - \frac{(\gamma - 1)2c_1c_2}{\gamma(c_1 + c_2)(1 - \bar{\alpha})},$$

or a corner solution, $\alpha_1^* = \frac{\bar{\alpha}}{2}$ or $\alpha_1^* = \hat{\alpha}_1 = \bar{\alpha}$.\(^\text{18}\)

To understand the nature of the optimal ownership distribution, we analyze the comparative statistics with respect to the key parameters $c_i$, $\Delta \theta$, $\bar{\alpha}$ and $\gamma$. From the perspective of comparative corporate governance, $\bar{\alpha}$ (capturing ownership dispersion) and $\gamma$ (measuring the difficulty to transform company resources into private resources, or the level of institutional protection of small shareholders), appear to be particularly interesting.

The comparative statics of the parameters $c_1$ and $c_2$ can easily be obtained, as they only affect the interior solution. A reduction of the monitoring cost of Shareholder 1 increases the amount of shares of the efficient shareholder in the optimal monitoring outcome and yields an increase in the amount of shares sold to her. Finally, in the relevant range (that is taking $\alpha_1^* \geq \frac{\bar{\alpha}}{2}$), an increase

\(^{18}\)As the interior solution always satisfies: $\alpha_1^* \leq \hat{\alpha}_1$, the only case where the upper bound is reached is when $\alpha_1^* = \hat{\alpha}_1 = \bar{\alpha}$. 

in the monitoring cost of Shareholder 2 also increases the shares of the efficient shareholder in the optimal monitoring outcome and results in an increase in the number of shares sold to Shareholder 1.

The analysis of the parameter $\gamma$ is straightforward for the same reason. An increase in $\gamma$, the cost of extracting benefits, makes the equilibrium level of benefit extraction more costly for the initial seller, who loses $\gamma - 1$ times the benefit. Thus, the seller becomes less tolerant to control rent taking, and this reduces the optimal amount of shares sold to Shareholder 1, in order to increase competitive pressure on benefits.

The effect of changes in $\bar{\alpha}$, the total volume of blockholdings, can be characterized as well, but is a bit more intricate. Starting at $\bar{\alpha} = 0$, there must be an interval where any difference in the size between the two large shareholders is so small in absolute terms that it can be easily neutralized by the small shareholders, and hence $B_1 \approx 0$. Thus, there must a region for small $\bar{\alpha}$ where only the monitoring value effect counts and where the optimal structure is to sell all the shares to the most efficient shareholder, $\alpha_1 = \bar{\alpha}$. Next, for the curve of interior solutions $\alpha^*_1(\bar{\alpha})$, we observe that

$$\frac{d\alpha^*_1}{d\bar{\alpha}} = \frac{c_1}{c_1 + c_2} - \frac{(\gamma - 1)^2 c_1 c_2}{\gamma (c_1 + c_2) (1 - \bar{\alpha})^2} < \frac{1}{2}.$$
there are always three regions for the comparative statics of \( \alpha \), and as \( \bar{\alpha} \) increases, the size of \( \alpha_1 \) relative to \( \alpha_2 \) is (weakly) decreasing: it is constant in the lower and upper region, where corner solutions obtain, and strictly falling in the intermediate region. The reason for the fall is that a decrease in \( \bar{\alpha} \) means less free float, so the difference between the large shareholders must decrease in order to maintain competitive pressure on benefits.

Figure 5 plots the optimal distribution of shares, \( \alpha^*_1 \) against \( \bar{\alpha} \) for typical parameter values \( (\gamma = 1.5, c_1 = 0.1, c_2 = 0.2) \), showing an example where in a parameter range of the intermediate region, the absolute number of shares sold to the largest shareholder is decreasing with \( \bar{\alpha} \). This is certainly a surprising result, but based on a clear intuition: the need to give more power to Shareholder 2 becomes more and more important, and so much so that at some point Shareholder 1’s absolute holdings must decline, until the lower bound \( \bar{\alpha}/2 \) is hit. This model also presents a very natural explanation (in the upper region of values of \( \bar{\alpha} \)) of why we may observe that blockholders hold blocks of equal size, even if their monitoring skills differ.

### 4.2 Heterogeneous Competence

We now turn to the second polar case, where the two large shareholders have the same monitoring costs, \( c_1 = c_2 = c \), but differ in their competence \( \theta_i \). In that case, \( \bar{\alpha}_1 = \frac{\bar{\alpha}}{2} \) and we can restrict our attention to values \( \bar{\alpha}_1 \leq \alpha_1 \leq \frac{\bar{\alpha}}{2} \). The total value of the firm is then given by

\[
W = \frac{\bar{\alpha}}{c} - \frac{\alpha_1^2}{2c} - \frac{(\bar{\alpha} - \alpha_1)^2}{2c} + \theta_1 + \frac{(\gamma - 1)(1 - \gamma(\bar{\alpha} - \alpha_1))(\bar{\alpha} - 2\alpha_1)}{\gamma(1 - \bar{\alpha})} - \frac{(\gamma - 1)\Delta \theta}{\gamma}.
\]

The objective function is strictly concave in \( \alpha_1 \). Hence the optimal ownership structure is either given by the interior solution

\[
\alpha^*_1 = \frac{\bar{\alpha}\gamma(1 - \bar{\alpha}) + 3c\gamma(\gamma - 1)\bar{\alpha} - 2c(\gamma - 1)}{2\gamma[c(\gamma - 1) + (1 - \bar{\alpha})]}
\]

or by a corner solution, \( \alpha^*_1 = \bar{\alpha}_1 \) or \( \alpha^*_1 = \frac{\bar{\alpha}}{2} \).

Comparative statics on the effects of the parameters \( c \) and \( \Delta \theta \) are easily obtained, as in the previous subsection, because the parameter \( c \) only affects the interior solution, and \( \Delta \theta \) the boundary value \( \bar{\alpha}_1 \). An increase in the monitoring cost \( c \) lowers the share of the controlling shareholder, as it increases the effect of private benefits extraction. An increase in the asymmetry between the two shareholders’ competence level reduces the lower boundary \( \bar{\alpha}_1 \). For low values of \( \Delta \theta \), this boundary will typically be binding. The effect of private benefits extraction is predominant. Hence, as private benefits are increasing in the difference in competence, the initial seller should reduce Shareholder 1’s block in order to reduce private benefits, so \( \alpha_1 \) is decreasing in \( \Delta \theta \). As \( \Delta \theta \)
increases, the lower boundary ceases to be binding. Reducing the share of the efficient shareholder to decrease private benefits now becomes too costly, and it is not optimal any longer to squeeze equilibrium benefits $B_1$ to zero. The optimal ownership structure then becomes independent of $\Delta \theta$.

The effect of an increase in the controlling shareholder’s ability to extract private benefits, $\gamma$, is more complex, since $\gamma$ affects both the lower boundary $\tilde{\alpha}_1$ and the interior solution $\alpha_1^\ast$. The relationship is not always monotonic, as it was the case of heterogeneous monitoring costs. At $\gamma = 1$, the lowest level of protection from expropriation, the optimal ownership structure is $\alpha_1^\ast = \bar{\alpha}/2$, the maximum among all candidate values. In fact, Shareholder 1’s fraction of shares reaches its global maximum at $\gamma = 1$, since $\alpha_1^\ast < \bar{\alpha}/2$ for all $\gamma > 1$. We also verify that $\alpha_1^\ast$ is strictly decreasing in $\gamma$ around the lower bound $\gamma = 1$. But this relationship is non-monotonic: At the upper bound $\gamma = \gamma_{\text{max}} = \frac{1}{\bar{\alpha}}$, the internal solution $\alpha_1^\ast$ is increasing in $\gamma$. This does not mean, however, that Shareholder 1’s fraction of equity is necessarily increasing for large $\gamma$, since the lower corner solution $\alpha_1^\ast = \tilde{\alpha}_1$ may already have been reached earlier, especially if $\Delta \theta$ is small. Note that this corner boundary solution $\tilde{\alpha}_1$ is always strictly decreasing in $\gamma$ as long as $\tilde{\alpha}_1 > \frac{3}{4} \bar{\alpha} - \frac{1}{2\bar{\alpha}}$, for which sufficient conditions are $\bar{\alpha}_1 < \frac{\bar{\alpha}}{4}$ or $\alpha < \frac{2}{3}$. Overall, our results are that $\alpha_1^\ast$ is decreasing in $\gamma$, as long as $\gamma$ and $\Delta \theta$ are not large. As the protection of small shareholders increases, the initial seller should decrease the share of the controlling shareholder in order to reduce the loss from private benefits extraction. For large $\gamma$ and $\Delta \theta$, $\alpha_1^\ast$ may be increasing in $\gamma$. In this case, the controlling shareholder internalizes the deadweight loss sufficiently and the

\[ \gamma_{\text{max}} = \frac{1}{\bar{\alpha}} \] is the maximum value consistent with the assumption that the controlling shareholder always wants to take private benefits.
Figure 7: Heterogeneous Skills: Optimal $\alpha_1$ as a Function of $\bar{\alpha}$

competence effect becomes dominant. Figure 6 plots the optimal ownership structure against $\gamma$ for a set of parameter values ($c = 0.1$, $\Delta \theta = 0.1$, $\bar{\alpha} = 0.5$) where $\Delta \theta$ is small enough for $\bar{\alpha}_1$ to become binding, and the relationship is therefore monotonically decreasing throughout.

The comparative statics of $\bar{\alpha}$ are simpler to characterize. The optimal number of shares of the controlling shareholder increases with $\bar{\alpha}$, the total number of shares sold to the two large blockholders. When this number is very low, it is optimal to assign zero shares to the controlling shareholder, as the small shareholders will nevertheless put her in control of the firm. When the fraction of shares sold to the two large blockholders becomes large enough, the initial planner should sell exactly $\bar{\alpha}_1$ shares to the efficient shareholder, or choose the ownership structure is chosen in order to eliminate the extraction of private benefits. We observe that the upper boundary $\bar{\alpha}_2$ is never reached, i.e. it is never optimal to split the shares equally if the two shareholders have identical monitoring costs. On the other hand, there is a range of parameters for which the lower boundary $\bar{\alpha}_1$ is reached: the initial seller chooses the ownership structure in such a way that private benefits are minimized. Overall, the block size of Shareholder 1 relative to Shareholder 2’s block increases with $\bar{\alpha}$. Figure 7 shows the optimal ownership structure against $\bar{\alpha}$ for parameter values comparable to those used before ($c = 0.1$, $\Delta \theta = 0.1$, $\gamma = 1.5$).

4.3 Synopsis

We have, in each round of the preceding analysis, eliminated one of the sources of blockholder heterogeneity that drive the solution in the overall trade-off as depicted in Figure 4. The most conspicuous difference is as to the block size of the controlling shareholder, who holds the larger block when monitoring costs are important and the second block when control competence is important. This difference is not really surprising: The reason for the controlling shareholder to
be set on the minority block is to compensate for her edge in competence, and absent that reason, she should be majority owner, provided she has also the lower monitoring costs.

It is then easy to see that in any intermediate scenario between the two extreme cases of heterogeneous monitoring costs and heterogeneous control competence, the attribution of the majority block simply depends on the comparative difference between the two blockholders, along the two dimensions of heterogeneity.

The comparative statics show interesting differences: First, while the block size of Shareholder 1 is mostly decreasing in $\gamma$, which we can interpret to measure differences in the degree of legal protection of small shareholders, an important exception emerges for large $\gamma$ and $\Delta \theta$. A positive correlation between the controlling shareholder’s block and legal protection indicates then that heterogeneity in control competence is important. Second, the relative size of the controlling shareholder’s block decreases in $\bar{\alpha}$ if shareholder heterogeneity concerns mostly the monitoring efficiency, and it increases in $\bar{\alpha}$ if shareholder heterogeneity is mostly about control-specific attributes. Since the shareholder in control has the smaller block in the latter case, we predict that the relative size of the largest block always increases in the free float $1 - \bar{\alpha}$.

5 Block Trading

We have so far assumed that the initially fixed ownership structure will not be altered. In this Section, we consider when a given ownership structure would be immune to retrading opportunities. This question is important for the seller, for if retrades are possible and the seller’s target ownership structure is not viable, then the shareholders are initially willing to pay only prices that reflect their value under the sustainable ownership structure. Recall that $\bar{\alpha}$ is exogenous in this paper, so any share trades can only occur between the two large shareholders. Such trades are commonly referred to as block trades, frequently negotiated separately from the market floor, or “upstairs”. We consider such upstairs retrading, i.e. the price of the block transfer could be freely negotiated between the large shareholders.

In order to economize on notation, we go back to the heterogeneous competence model, i.e. $c_1 = c_2 = c$, and $\Delta \theta > 0$. We begin with the case where Shareholder 1 is initially endowed with more shares than Shareholder 2, $\alpha_1 > \alpha_2$. The sum of utilities for the two large shareholders is initially given by

$$U_1(\alpha_1, \alpha_2) + U_2(\alpha_1, \alpha_2) = \bar{\alpha} \left( \theta_1 + \frac{\bar{\alpha}}{c} \right) - \frac{\alpha_1^2}{2c} - \frac{(\bar{\alpha} - \alpha_1)^2}{2c} + (1 - \gamma \bar{\alpha}) \left( \frac{2\alpha_1 - \bar{\alpha}}{\gamma(1 - \bar{\alpha})} + \frac{\Delta \theta}{\gamma} \right).$$

We consider first the case where one of the two large blockholders, say Shareholder 1, considers a complete purchase of all shares from Shareholder 2. We assume that after selling out, Shareholder
can still launch a bid (say from holding on to a single share, like a small shareholder), but her position is then seriously undermined by the lack of voting power. Shareholder 1 faces no serious competition in the control contest and is thus able to extract a benefit of $B_1(\bar{\alpha}, 0)$.\textsuperscript{20} The sum of utilities obtained by the two large shareholders after the sale is thus given by

$$U_1(\bar{\alpha}, 0) + U_2(\bar{\alpha}, 0) = \bar{\alpha} \left( \theta + \frac{\bar{\alpha}}{c} \right) - \frac{\bar{\alpha}^2}{2c} + (1 - \gamma \bar{\alpha}) B_1(\bar{\alpha}, 0).$$

By comparing the sum of utilities obtained by the two large blockholders before and after the sale, we obtain a simple condition when Shareholder 1 is not interested in buying Shareholder 2’s entire block. This is the case if the combined utility would be reduced from the transaction, or if

$$(1 - \gamma \bar{\alpha}) \left( B_1(\bar{\alpha}, 0) - \frac{2\alpha_1 - \bar{\alpha}}{\gamma (1 - \bar{\alpha})} - \frac{\Delta \theta}{\gamma} \right) < \frac{\alpha_1(\bar{\alpha} - \alpha_1)}{c} = \frac{\bar{\alpha}^2}{2c} - \frac{(\bar{\alpha} - \alpha_1)^2}{2c} - \frac{\Delta \theta^2}{2c}.
$$

Notice that the left hand term in the above inequality represents the increase in private benefits due to the absence of competition in the control contest. The right hand term corresponds to the loss in value due to the increase in monitoring costs. The condition is obviously satisfied if $1 - \gamma \bar{\alpha} > 0$, that is if the free float is so unimportant that small shareholders would be automatically protected by the remaining shareholder’s internalizing of the social cost of benefit-taking after the sale. As the total amount of private benefits that can be extracted by the controlling shareholder, $B_1$, is reduced, the left hand term goes down, and the condition is more likely to be satisfied. Similarly, if the difference in competence increases, then the private benefits initially extracted by the controlling shareholder go up, and the inequality is more likely to be satisfied. As the monitoring cost $c$ goes down, the value loss due to the formation of a single block increases, and the condition is more likely to be satisfied. Clearly, because of the competence difference, it is even less attractive for Shareholder 2 to buy Shareholder 1’s entire block.

We also need to consider whether the two blockholders could benefit from a partial retrade of their shares. To investigate this question, suppose there is any ownership structure $(\alpha'_1, \alpha'_2)$ different from the initial distribution $(\alpha_1, \alpha_2)$, which could be the result of such a partial share trade among the two large shareholders. Again, this can only be the case if the combined utility is larger after the trade than before, or if

$$U_1(\alpha'_1, \alpha'_2) + U_2(\alpha'_1, \alpha'_2) \geq U_1(\alpha_1, \alpha_2) + U_2(\alpha_1, \alpha_2).$$

Now for any such post-trade ownership structure $(\alpha'_1, \alpha'_2)$, we need to take into account that the benefits $B_1$ will adjust to the change in ownership. We find the following conditions for an allocation $(\alpha_1, \alpha_2)$ to be immune against any re trading attempt on the upstairs market:

\textsuperscript{20}$B_1(\bar{\alpha}, 0) = \min \{ \bar{B}, \frac{\bar{\alpha}}{\gamma (1 - \alpha_2)} + \frac{\Delta \theta}{\gamma} \}$ is determined by either the maximal amount $\bar{B}$ or the highest benefits that allow Shareholder 1 to win against the counter-bid of $B_2 = 0$ of Shareholder 2 (who is now stripped of any significant shareholding).
Proposition 2 (Block Trading) If $c_1 = c_2 = c$ and $\Delta \theta > 0$, then there is a unique allocation $(\alpha_1, \alpha_2)$ where there will be no partial or complete retrade of shares between the two blockholders, given by

$$
\alpha_1 = \min \left\{ \frac{\bar{\alpha}}{2} + \frac{c(1 - \gamma \bar{\alpha})}{\gamma(1 - \bar{\alpha})}, \frac{\bar{\alpha}}{2} + \frac{1 - \bar{\alpha}}{2} (\gamma \bar{B} - \Delta \theta), \bar{\alpha} \right\},
$$

$$
\alpha_2 = \max \left\{ \frac{\bar{\alpha}}{2} - \frac{c(1 - \gamma \bar{\alpha})}{\gamma(1 - \bar{\alpha})}, \frac{\bar{\alpha}}{2} - \frac{1 - \bar{\alpha}}{2} (\gamma \bar{B} - \Delta \theta), 0 \right\}.
$$

Proof: See Appendix.

The finding in Proposition 2 can be intuitively explained as follows. For any reshuffling of the ownership distribution, there are two effects, an increase in the benefits $B_1$ as well as a repartition of the monitoring efforts. As long as both effects will be positive, a share sale will occur. Therefore, a necessary condition for a retraining-proof ownership structure is that the further repartition of the monitoring effort has a negative effect, or that $\alpha_1 > \bar{\alpha}$ (because of the assumption of equal costs, $c_1 = c_2 = c$). The more we move away from an equal distribution of shares $\alpha_1 = \alpha_2$, the stronger this negative effect. On the other hand, the effect of Shareholder 1’s share purchase on benefits $B_1$ remains constant, as $\frac{\partial B_1}{\partial \alpha_1} = \frac{2}{\gamma(1-\bar{\alpha})}$. The retraining-proof ownership structure is attained where the effort repartition effect and the benefit effect are exactly offsetting. Now there are two cases: either this point is reached before the upper limit on benefits $\bar{B}$ is reached; then $\alpha_1 = \frac{\bar{\alpha}}{2} + \frac{c(1 - \gamma \bar{\alpha})}{\gamma(1 - \bar{\alpha})}$. Or the benefit effect stills outweighs the repartition effect when $\bar{B}$ is reached; in this case if $\alpha_1 = \frac{\bar{\alpha}}{2} + \frac{1-\bar{\alpha}}{2} (\gamma \bar{B} - \Delta \theta)$. In the Appendix, we show that this retraining-proof ownership structure is unique and always exists within the bound $\alpha_1 \leq \bar{\alpha}$.

From a more empirical point of view, the most interesting finding is that a retraining-proof ownership distribution always requires that $\alpha_1 > \frac{\bar{\alpha}}{2}$. This is in marked contrast to our analysis of the optimal ownership structure, which for the case of identical monitoring costs ($c_1 = c_2$) requires that Shareholder 2 should be majority owner, $\bar{\alpha}_1 \leq \alpha_1^* \leq \frac{\bar{\alpha}}{2}$, in order to offset Shareholder 1’s advantage of better control competence.

Thus, our analysis hints that the empirical finding that majority owners are typically in possession of corporate control is a second best: It is not the optimal ownership structure, but the only one robust to share-trading, implying that equilibrium benefit taking or tunneling is considerable larger than under the optimal ownership structure.

6 Empirical Implications

We summarize the most important empirical implications that arise from our paper. When formulating hypotheses for empirical work, we also need to take into account that retraining is a
likely determinant of the ownership structures that are observed in reality. For a number of the
implications listed below, one needs to find proxies in order to put them to tests. For example,
heterogeneity in competence among shareholders could be proxied by subdividing the shareholder
sample into different groups: corporate shareholders (which might be further distinguished ac-
cording to distance in terms of industry classification, or geographic distance / country of origin),
financial intermediaries and institutional investors, individuals and holdings, families.

• A satisfactory measurement of ownership concentration should take the contestability of
control into account.

Contestable control is the contribution of our model to the debate about the right measure
of shareholder concentration. From the viewpoint of contestability, the identity of the controlling
shareholder and the maximum control are derived from two independent dimensions, competence
and voting power. Competence is hard to capture empirically. But based on voting power
alone, the measure of shareholder power that comes out of our contestability model would be the
difference between the leading blocks, normalized by the free float, or $\alpha_1 - \alpha_2$. The same measure
can actually be in the presence of $N > 2$ blocks, as it is straightforward to extend our model. Even
if more than two blockholders can submit proposals, only the two strongest blockholders determine
the outcome (like in a second-bid auction). The equilibrium rent of the winning shareholder is
then determined by the value if everyone rallies against her. Therefore, we suggest to use as
the logical extension of our concentration measure $\max\left\{\frac{\alpha_1 - \sum_{i=2}^{N} \alpha_i}{1 - \bar{\alpha}}, 0\right\}$. We emphasize that a
satisfactory analysis of contestability should include measures of competence as well.

• Control benefits are smaller on average if multiple blockholders are present. Control benefits
are increasing on average in the difference in block size between the leading shareholders.

This implication follows easily since a single large shareholder holding $\alpha_1 < 1/\gamma$ would always
take maximum benefits $B$. While control benefits are not directly observable, they might be
proxied by measures of corporate performance, as for example argued by Ang et al. (2000). In
our model, benefits are determined by the size difference $\alpha_1 - \alpha_2$ as well as by the competence
difference $\Delta \theta$. Thus, to test this hypothesis, one needs again to control for competence differences,
for example by classifying shareholders into various groups.

Consistent with this hypothesis, Lehmann and Weigand (2000) report that, in a regression
of ownership variables on return on assets for German stock-listed companies, “the presence of
a strong second or third large shareholder enhances profitability.” Volpin (2001) investigates
the impact of ownership structure on top executive turnover in Italy; while the presence of a
strong minority shareholder does not increase the performance-sensitivity of executive turnover,
the presence of an explicit shareholder’s agreement among leading shareholders does.
• If the leading shareholder is a particular good monitor, then it is more likely that there is only a single share block. If monitoring skills are equally distributed, then multiple blocks are likely. Multiple blocks are more frequent if shareholders are likely to perform independent or complementary monitoring functions.

This result is supported by our analysis of the optimal ownership structure, and corroborated by our investigation of upstairs re trading. As pointed out, blockholders will only abstain from re trading shares if both perform valuable functions for the company. A good proxy for capturing complementarities in monitoring could be heterogeneity among shareholders, again proxied by their group adherence. Conversely, if the existence of multiple blocks is observed and blockholders seem to be fairly similar, this should indicate that equilibrium private benefits are small. If shareholders are proxied to be more heterogeneous, equilibrium private benefits should be more substantial.

Evidence consistent with this prediction is provided by Boehmer (2000) in a study of acquisitions by German listed firms. While the presence of a second large shareholder does not generally improve the quality of the acquisitions (interpreted as a proxy for good investment choices), there is a beneficial influence if the second blockholder complements a bank as shareholder, or if a bank complements a family or corporate controlling shareholder.

• A strictly positive fraction of firms should have leading shareholders with equal block size, in particular if the free float is small.

The equal size of blocks is optimal when there is no difference in control competence ($\Delta \theta = 0$) and the difference in monitoring skills is not too big, in particular if there are few small shareholders around that might help keep the power of the largest block in check.

• The relative size of the largest block increases in the free float. If the relative size of the controlling block decreases in the free float, then shareholder heterogeneity is more determined by control-specific competence than by control-independent monitoring skills, and vice versa.

These results have been established in Section 4. The simple observation that the largest shareholder is in control is less useful as an indicator that monitoring skills are the main determinant of shareholder heterogeneity, since in the re trading-proof ownership structure, the controlling shareholder always holds the largest block.

• For a high level of investor protection, multiple shareholders should be observed more frequently. The size of the controlling block is decreasing in improvements in investor protection if the level of investor protection is low, but may be increasing if investor protection and differences in control competence are large.
Note that a system with a poor state of investor protection is a system where the costs of transform company resources into private resources are small, i.e. $\gamma$ is small, as in Shleifer and Wolfenzon (2000) and Burkart and Panunzi (2000).

This insight debunks the popular idea that the presence of multiple shareholders may act as a substitute for poor investor protection (see e.g. LaPorta et al. (1999), p. 502). The flaw in this idea, from the viewpoint of our analysis, is that it confounds legal protection with the equilibrium level of deadweight costs. If there is no legal protection ($\gamma = 1$), controlling shareholders will steal, but for every dollar stolen, the company loses only one dollar. As we consider an increase in $\gamma$, this will reduces the equilibrium level of benefits $B$, but the total waste from benefit extraction is benefits times $\gamma - 1$, a product which increases with protection. So the better is protection, the more is to be gained from competition among shareholders. This is the reason why we find robustly an inverse relationship between the size of $\alpha_1$ and $\gamma$, whether we look at optimal or retrading-proof ownership structures.

While the empirical support for this hypothesis seems to be weak, it may at least help to understand some puzzling evidence as to the presence of second shareholders. LaPorta et al. (1999) find that the presence of a single large shareholder is strongly more likely with poor investor protection, but they also find that the probability for the presence of a second large shareholder is the same in countries with high and with low anti-director rights. And in Europe, there is no evidence that second or third shareholders are particularly prevalent in countries with poor investor protection (See Becht and Mayer (2000)).

- **Controlling blocks trade at a premium, non-controlling blocks trade at a discount.**

The studies of Barclay and Holderness (1991), Crespi-Cladera and Renneboog (2000) and Nico-dano and Sembenelli (2000) show that while blocks typically trade at a premium, block transfers at a discount are frequent. Our analysis suggests that discounts are naturally explained since minority blockholders, unlike dispersed shareholders, spend on monitoring e ff ort, but are unlikely to reap control benefit. The premium/discount can thus be viewed as a convenient variable whether effective control was transferred with a share block or not. Similarly, our model contributes to the prediction of announcement effects: Positive block premia coincide with negative stock price reactions if the sale is to a less efficient shareholder or the sale is a block merger. Positive block premia coincide with positive stock price reactions if the sale is to a more efficient shareholder.

## 7 Conclusion

In this paper, we investigate when a seller of a corporation, like an owner bringing a company public or a government privatizing a state-run firm, would find it desirable to sell share blocks not just to one, but to several strategic investors. The benefit of having several blockholders is
seen as limiting rent extraction, in an effort to secure the vote of minority shareholders. If private benefits are value-destroying, this device to protect minority shareholders is in the interest of the initial seller.

This model is a preliminary attempt to model corporate governance by explicitly referring to voting games played in shareholder meetings. We emphasize that the strength in the control contest is derived from two sources, the competence to run the firm and the voting power. Our results strongly indicate that empirical research on ownership structure is incomplete if it does not try to proxy for contestability. To eliminate control rents, the seller should give the smaller share block to the controlling shareholder, but this may not be the optimal ownership structure from the point of view of the seller when taking account of monitoring incentives. If only the second largest block is allocated to the controlling shareholder, then the ownership structure is likely to be undone through subsequent block trades, and the anticipation of this unraveling would harm the initial valuation.

We have included only one reason why multiple share blocks may not be mutually sold out in block mergers, namely complementarity in monitoring services. But there are, in our opinion, a couple of other mechanisms leading to retrading-proofness, and our model can easily be accommodated to include those, thus giving richer testable hypotheses: (i) Multiple blocks allow a better risk sharing between large shareholders;\(^{22}\) (ii) the implicit commitment to protect minority shareholders is attractive for firms that expect to go back to the capital market (firms with high growth opportunities) in the future; (iii) a second blockholder allows for a contingent shift in control; (iv) asymmetric information about the firm’s prospects or about private benefits.

Two possible extensions are worth to be explored in future work. First, it is highly desirable to endogenize the size of the free float, \(1 - \bar{\alpha}\). We mentioned the idea that stock prices include a liquidity premium, and it is reasonable to assume that it is an increasing function of the free float. Such an endogeneization would also allow to explicitly address possible stock purchases or sales of large stockholders on the downstairs market.

Second, we have assumed that large shareholders remain locked into a contest for control. But collusion among large shareholders, and even explicit voting pacts and shareholder agreements, are indeed ubiquitous. While the formal analysis of what makes shareholders compete or collude is in its infancy, there seems to be some empirical support for the idea that shareholder agreements are arrangements that may break under stress, to the benefit of shareholder value since competition arises when it is most needed (see Volpin (2001)). It would be interesting to explore these ideas in the framework of our contestability model.

\(^{22}\)Risk aversion is known to be a major obstacle to the sustainability of large share blocks (Admati, Pfleiderer and Zechner (1993)).
Appendix A: Strong Equilibria of the Voting Game

We will establish the following proposition.

Proposition 3  
(i) There exists a unique strong equilibrium of the voting game.  
(ii) Suppose that \( \alpha_1 \geq \alpha_2 \). If \( \frac{\Delta \theta}{\gamma} + \frac{\alpha_1 - \alpha_2}{\gamma(1 - \alpha)} > B_1 - B_2 \), then no small shareholder participates in the vote, and Shareholder 1 wins the control contest. If \( \frac{\Delta \theta}{\gamma} + \frac{\alpha_1 - \alpha_2}{\gamma(1 - \alpha)} \leq B_1 - B_2 \), then a fraction \( \frac{\alpha_2}{\delta} \) of the small shareholders vote, and Shareholder 2 wins the contest. 
(iii) Suppose that \( \alpha_2 \geq \alpha_1 \) and \( (1 - \gamma \alpha_2) B_2 + \gamma \alpha_2 B_1 \geq \alpha_2 \Delta \theta \). If \( \frac{\alpha_2 - \alpha_1}{\gamma(1 - \alpha)} - \frac{\Delta \theta}{\gamma} > B_2 - B_1 \), then no small shareholder participates in the vote, and Shareholder 2 wins the contest. If \( \frac{\alpha_2 - \alpha_1}{\gamma(1 - \alpha)} - \frac{\Delta \theta}{\gamma} \leq B_2 - B_1 \), then a fraction \( \frac{\alpha_2 - \alpha_1}{\delta} \) of the small shareholders vote, and Shareholder 1 wins the contest.  
(iv) If \( \alpha_2 \geq \alpha_1 \) and \( (1 - \gamma \alpha_2) B_2 + \gamma \alpha_2 B_1 < \alpha_2 \Delta \theta \), no small shareholder participates in the vote, and Shareholder 1 wins the control contest.

Proof of Proposition 3: Suppose \( \alpha_1 \geq \alpha_2 \) and \( \frac{\Delta \theta}{\gamma} + \frac{\alpha_1 - \alpha_2}{\gamma(1 - \alpha)} > B_1 - B_2 \). We show that the only strong equilibrium is the equilibrium where nobody votes. If a fraction \( \varepsilon < \frac{\alpha_1 - \alpha_2}{\delta} \) of the small shareholders vote, their votes do not change the outcome of the control contest, and hence it is a dominant strategy for them not to vote. If a fraction \( \varepsilon \geq \frac{\alpha_1 - \alpha_2}{\delta} \) of the small shareholders vote, as \( \gamma(B_1 - B_2) - \Delta \theta < \frac{\alpha_1 - \alpha_2}{\delta} \), there must exist a positive measure \( \delta \) of small shareholders for whom \( \kappa > \gamma(B_1 - B_2) - \Delta \theta \), and who prefer not to vote. Suppose now that \( \frac{\Delta \theta}{\gamma} + \frac{\alpha_1 - \alpha_2}{\gamma(1 - \alpha)} \leq B_1 - B_2 \). We show that there is a strong equilibrium where a fraction \( \frac{\alpha_2}{\delta} \) of small shareholders with voting cost \( \kappa \leq \gamma(B_1 - B_2) - \Delta \theta \) participates in the vote. Small shareholders who do not vote have no incentive to deviate since they already obtain their preferred outcome, and do not incur the voting cost. If now a measure \( \varepsilon \) of voting small shareholders chooses to deviate, the outcome of the vote will be Shareholder 1’s plan, and as \( \kappa \leq \gamma(B_1 - B_2) - \Delta \theta \) for all voting small shareholders, this will induce a lower payoff. It is clear that there cannot be a strong equilibrium where a larger fraction of small shareholders chooses to vote, since the non-pivotal voters have an incentive to deviate. There does not exist a strong equilibrium where no small shareholder votes either, since a positive measure of shareholders has an incentive to vote, in order to ensure that Shareholder 2 wins the control contest.

Similar arguments show that, when \( \alpha_2 \geq \alpha_1 \) and \( (1 - \gamma \alpha_2) B_2 + \gamma \alpha_2 B_1 \geq \alpha_2 \Delta \theta \), the prescribed strategies form a strong equilibrium of the voting game. Finally, if \( (1 - \gamma \alpha_2) B_2 + \gamma \alpha_2 B_1 < \alpha_2 \Delta \theta \), all shareholders unanimously prefer Shareholder 1’s plan, and the small shareholders do not vote.  
QED.
Appendix B: Proofs

Proof of Proposition 1: The competition between the two large shareholders is reminiscent of a model of Bertrand competition between two firms with different marginal costs (see e.g. Shy (1995), p. 109). Hence, in order to be able solve for an equilibrium in pure strategies, we use the standard technique of introducing a smallest money unit \( \eta \). We say that money is continuous if \( \eta = 0 \) and discrete if \( \eta > 0 \). When money is discrete, the choices of private benefits are \( B_1 = b_1 \eta \) and \( B_2 = b_2 \eta \) for integer values \( b_1 \) and \( b_2 \).

(i) We start with the case \( \alpha_1 \geq \alpha_2 \) (Figure 3A). Suppose that there exists a discrete money unit \( \eta \) and that plans are labeled in terms of the money unit, \( B_1 = \eta b_1 \) and \( B_2 = \eta b_2 \) for integer values \( b_1 \) and \( b_2 \). From Figure 3A, we can divide the plane \((B_1, B_2)\) into two regions, region \( A \), where Shareholder 1 wins the control contest, and region \( B \) where Shareholder 2 wins the control contest. We first claim that there cannot be an equilibrium where \((B_1, B_2)\) belong to region \( B \). To see this note that, whenever \( B_1 = 0 \), Shareholder 1 wins the control contest, so that for any point \((B_1, B_2)\) in region \( B \), Shareholder 1 has a profitable deviation, \( B_1 = 0 \). Now consider a point \((B_1, B_2)\) in region \( A \), with \( B_1 \geq B_1^* \). As \( B_1 \geq B_1^* \) and \((B_1, B_2)\) belongs to region \( A \), we must have \( B_2 > 0 \). By choosing \( B_2 = 0 \), player 2 wins the control contest. Furthermore, in the region where \( B_1 \geq B_1^* \), Shareholder 2 prefers to win the control contest, irrespective of the values of \( B_1 \) and \( B_2 \). Hence, Shareholder 2 has a profitable deviation by choosing \( B_2 = 0 \). Next, consider a point \((B_1, B_2)\) in region \( A \), with \( B_2 > 0 \) and \( B_1 < B_1^* \). As Shareholder 1’s utility is increasing in \( B_1 \) in region \( A \), \( B_1 \) must be the maximal private benefit that player 1 can extract while winning the control contest. In other words, we must have

\[
(b_1 + 1) \eta \geq b_2 \eta + \frac{(\alpha_1 - \alpha_2)}{\gamma(1 - \bar{\alpha})} + \frac{1}{\gamma} \Delta \theta.
\]

However, as \( B_1 < B_1^* \), we have

\[
b_1 \eta < \frac{(\alpha_1 - \alpha_2)}{\gamma(1 - \bar{\alpha})} + \frac{1}{\gamma} \Delta \theta.
\]

Clearly, the two inequalities 1 and 2 are inconsistent for any value \( b_2 \geq 1 \). Hence, the only possible equilibrium candidate is given by \( B_2 = 0 \) and \( B_1 = \max \{b_1 \eta | b_1 \eta < \frac{(\alpha_1 - \alpha_2)}{\gamma(1 - \bar{\alpha})} + \frac{1}{\gamma} \Delta \theta \} \). These strategies form an equilibrium, as Shareholder 1 has no incentive to deviate, and Shareholder 2 always loses the control contest and is indifferent among all possible values of \( B_2 \). As the discrete money unit converges to zero, the equilibrium converges to \((B_1^*, 0)\).

(ii) Suppose now that \( \alpha_2 \geq \alpha_1 \) and \( \frac{\alpha_2 - \alpha_1}{1 - \bar{\alpha}} \leq \frac{1}{1 - \gamma \alpha_2} \Delta \theta \) (Figure 3B) and suppose that money is continuous. Consider a point \((B_1, B_2)\) in the region where Shareholder 2 wins the contest. As Shareholder 1 always wins the contest by offering \( B_1 = 0 \), she has a profitable deviation by choosing \( B_1 = 0 \). Consider now a point \((B_1, B_2)\) in the region where Shareholder 1 wins with
$B_1 > B_1^*$. As $B_1 > B_1^*$, there exists a value $B_2$ such that \( \frac{\alpha_1 \Delta \theta}{1 - \alpha_2} - \frac{\gamma \alpha_2 B_1}{\gamma \alpha_2} < B_2 < \frac{\alpha_2 - \alpha_1}{\gamma (1 - \bar{\alpha})} - \frac{\Delta \theta}{\gamma} + B_1. \)

Hence, shareholder can win the contest by proposing $B_2$ when Shareholder 1 proposes $B_1$. As $B_1 > B_1^*$, Shareholder 2 strictly prefers to win the contest, and this deviation is profitable. Note now that, for any value $B_1 \leq B_1^*$, Shareholder 1 wins the contest. Hence any offer $B_1 < B_1^*$ is dominated for Shareholder 1 by the offer $B_1^*$. To finish the proof, note that $(B_1^*, B_2^*)$ does indeed for an equilibrium, as Shareholder 2 has no incentive to deviate (he loses the contest for any value of $B_2$ when Shareholder 1 proposes $B_1^*$), and Shareholder 1 has no incentive to deviate, as any value $B_1 > B_1^*$ would make Shareholder 2 win the contest.

(iii) Finally suppose that $\alpha_2 \geq \alpha_1$ and $\frac{\alpha_2 - \alpha_1}{\alpha_2 - \bar{\alpha}} > \frac{1}{\gamma \alpha_2 - 1} \Delta \theta$ (Figure 3C) and let money be labeled in discrete units. This case turns out to be very similar to the case of Figure 3a. We can again divide the plane $(B_1, B_2)$ into two regions, region $A$, where Shareholder 1 wins the control contest, and region $B$ where Shareholder 2 wins the control contest. Let the smallest money unit $\eta$ be small enough so that $\eta < \frac{\alpha_2 - \alpha_1}{\alpha_2 - \bar{\alpha}} - \frac{1}{\gamma \alpha_2} \Delta \theta$. Then, for any possible point $(B_1, B_2)$ in region $A$ there exists a deviation for Shareholder 2 which makes her win the control contest, and such that $(1 - \gamma \alpha_2) B_2 + \gamma \alpha_2 B_1 > \alpha_2 \Delta \theta$, i.e. Shareholder 2 strictly prefers to win the contest. Hence, Shareholder 2 has a profitable deviation and $(B_1, B_2)$ cannot be an equilibrium. For any point $(B_1, B_2)$ in region $B$ with $B_2 \geq B_2^*$, Shareholder 1 has a profitable deviation, by offering $B_1 = 0$ and winning the contest. Finally, by an argument similar to the argument of the case $\alpha_1 \geq \alpha_2$, a point $(B_1, B_2)$ in region $B$ with $B_2 < B_2^*$ and $B_1 > 0$ cannot be an equilibrium. Hence the only possible equilibrium is given by $B_1 = 0$ and $B_2 = \max \{ b_2 \eta | b_2 \eta < \frac{\alpha_2 - \alpha_1}{\gamma (1 - \bar{\alpha})} - \frac{1}{\gamma} \Delta \theta \}$ and it is easy to see that these strategies indeed form a equilibrium, which converges to $(0, B_2^*)$ as the smallest money unit $\eta$ goes to zero. \textbf{QED.}

\textbf{Proof of Proposition 2:} We will show that there is a unique point where the joint surplus $U_1(\alpha_1, \alpha_2) + U_2(\alpha_1, \alpha_2)$ is maximized. Under our assumption $\alpha_1 > \alpha_2$, this joint surplus is given by

\[ U_1(\alpha_1, \alpha_2) + U_2(\alpha_1, \alpha_2) = \bar{a} \left( \theta_1 + \frac{\bar{\alpha}}{c} \right) - \frac{\alpha_1^2}{2c} - \frac{(\bar{\alpha} - \alpha_1)^2}{2c} + (1 - \gamma \bar{\alpha}) \min \left\{ \bar{B}, \frac{2\alpha_1 - \bar{\alpha}}{\gamma (1 - \bar{\alpha})} + \frac{\Delta \theta}{\gamma} \right\}. \]

(i) We assume first that $\bar{B} > \frac{\bar{\alpha}}{\gamma (1 - \bar{\alpha})} + \frac{\Delta \theta}{\gamma}$ (maximum benefit is never attained). Then the objective function becomes

\[ U_1(\alpha_1, \alpha_2) + U_2(\alpha_1, \alpha_2) = \bar{a} \left( \theta_1 + \frac{\bar{\alpha}}{c} \right) - \frac{\alpha_1^2}{2c} - \frac{(\bar{\alpha} - \alpha_1)^2}{2c} + (1 - \gamma \bar{\alpha}) \left( \frac{2\alpha_1 - \bar{\alpha}}{\gamma (1 - \bar{\alpha})} + \frac{\Delta \theta}{\gamma} \right), \]

and the first-order condition gives:

\[ \frac{2\alpha_1 - \bar{\alpha}}{c} + \frac{2(1 - \gamma \bar{\alpha})}{\gamma (1 - \bar{\alpha})} = 0, \]

or a maximizer of

\[ \alpha_1 = \frac{\bar{\alpha}}{2} + \frac{c(1 - \gamma \bar{\alpha})}{\gamma (1 - \bar{\alpha})} \equiv \alpha_1^R, \]

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Next, notice that $U_1(\alpha_1, \alpha_2) + U_2(\alpha_1, \alpha_2)$ is strictly concave, hence the maximum $\alpha_1^R$ is unique. For all $\alpha_1 < \alpha_1^R$, the value of the objective function is strictly increasing in $\alpha_1$, so the corner solution $\bar{\alpha}$ is the unique optimum if $\bar{\alpha} < \alpha_1^R$.

(ii) Second, we consider the case of $B < \frac{\alpha}{\gamma(1-\alpha)} + \Delta \theta$, then the maximum benefit $\bar{B}$ may be attained for $\alpha_1 < \bar{\alpha}$. Define the threshold where this upper limit is reached as

$$\alpha_1^+ = \frac{\bar{\alpha}}{2} + \frac{1 - \bar{\alpha}}{2} (\gamma \bar{B} - \Delta \theta)$$

Consider $\alpha_1^R < \alpha_1^+$. Then for all $\alpha_1 > \alpha_1^R$, the value of the objective function is strictly falling in $\alpha_1$ and strictly below the value in case (i), hence $\min\{\alpha_1^R, \bar{\alpha}\}$ is the unique maximizer. Consider $\alpha_1^R > \alpha_1^+$. Since $\alpha_1^+ > \frac{\bar{\alpha}}{2}$, it follows that for all $\alpha_1 > \alpha_1^+$, the value of the objective function is falling in $\alpha_1$ and strictly below the value in case (i), hence $\min\{\alpha_1^+, \bar{\alpha}\}$ must be the unique maximizer. QED.
References


