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Abstract: This paper studies the consequences of labor-market frictions for the real effects of steady inflation when cash is required for households' consumption purchases and firms' wage payments. Money growth may generate a positive real effect by encouraging vacancy creation and raising job matches. This may result in a positive optimal rate of inflation, particularly in an economy with moderate money injections to firms and with nonnegligible labor-market frictions in which wage bargains are not efficient. This main finding holds for a wide range of money injection schemes, with alternative cash constraints, and in a second-best world with pre-existing distortionary taxes.

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1 Introduction

The real effect of money growth and the welfare cost of steady inflation have been two central issues in monetary economics since mid-1960s. Due to their simplicity, dynamic general equilibrium models with cash-in-advance constraints have been the most prototypical setups to address these issues.[1] Even when cash is required only for consumption purchases, Wang and Yip (1992) establish a negative real effect of steady inflation in the presence of labor-leisure trade-off when capital and labor are Pareto complements in production. Their conclusion remains valid under the endogenous growth framework, as shown by Gomme (1993), Jones and Manuelli (1995), and many others. In our paper, we revisit this issue by examining whether steady inflation may foster real activity and whether the optimal inflation rate may depart from the Friedman rule if the underlying labor market is subject to search and entry frictions. The incorporation of labor-market frictions is particularly relevant because the central force for generating a negative real effect of money growth is the labor-leisure trade-off, which depends naturally on labor-market conditions.

It is noted that the empirical evidence fails to lend consistent support to a negative relationship between steady inflation and real economic activity. For example, while conventional studies by Fischer(1983) and Cooley and Hansen (1989) document such a negative relationship in cross countries, recent works by Bullard and Keating (1995) and Ahmed and Rogers (2000) find zero or even positive correlation between money and output growth in low-inflation industrialized economies. Moreover, empirical evidence seems to suggest that the real effect of anticipated inflation is largely insignificant when the level of inflation is low.[2]

In our paper, we provide a plausible theory to explain the empirical findings: a positive real effect of money growth might be present in low-inflation regimes when the economy lacks a central planner (or Walrasian auctioneer) and exhibits labor-market frictions. More precisely, by allowing labor-market frictions and several important dimensions of labor-related trade-offs, our paper identifies a new channel that characterizes the long-run effects of inflation on employment and capital accumulation. This new channel may account for different consequences of money growth on the macroeconomic variables as well as for normative prescriptions that depart from the Friedman rule,

[1] Lucas (1980) shows money superneutrality in a one-sector neoclassical growth framework when only consumption purchases require cash in advance. By requiring investment purchases to use cash, Stockman (1981) identifies a negative real effect of money growth via discouragement to capital accumulation. By allowing inflation uncertainty in a Lucasian monetary economy, Dotsey and Sarte (2000) find insignificant real effects of money growth.

[2] For example, Khan and Senhadji (2000) find that inflation significantly reduces growth only when the level of inflation is above 1-3% for developed countries and above 7-11% for developing countries.

yielding important monetary policy implications.

Specifically, we follow the idea developed by Diamond (1982), Mortensen (1982), Pissaridis (1984) and Laing, Palivos and Wang (1995) by postulating that both vacancy creation and job search are costly and that vacancies and job seekers are brought together by a matching technology exhibiting constant returns to scale. We depart from this prototypical random search and matching setup to consider competitive search that was developed by Peters (1991) and extended by Andolfatto (1996) and Merz (1995). This framework allows us to model capital accumulation under a dynamic general equilibrium setting in a tractable manner while allowing for market frictions highlighted in search and matching models.

Since the purpose of our paper is to examine the consequence of labor market frictions for inflation and growth in a monetary economy, our model also depart from the Merz-Andolfatto framework in two aspects.[3] On the one hand, we follow the neoclassical transactions-time monetary model by assuming that costly activities such as vacancy creation and job search intensity are all in terms of labor and time allocation. This feature enables us to assess thoroughly the real effects of steady inflation via the labor-leisure-search trade-off in the presence of labor market frictions. On the other hand, rather than focusing exclusively on Hosios' (1990) rule of efficient matching, we also consider the possibility of an inefficient bargain whereupon a firm, facing an outside option normalized to zero, decides whether to accept a take-it-or-leave-it offer of an additional employee by a "large" household. In so doing, we obtain an equilibrium income distribution resembling the neoclassical benchmark, thus enabling better understanding about the role of labor-market frictions (in lieu of strategic bargaining) played in the long-run relationship between steady inflation, employment and economic welfare and providing more clear contrast with findings in frictionless neoclassical models. In short, our model structure permits various crucial decisions: firm's vacancy creation (intensive margin) and household's labor participation (extensive margin) as well as search intensity (intensive margin). The interaction between extensive margin and intensive margin is magnified by the matching technology where vacancies and searching workers are complementary to lead to thick matching, which may be referred to as the *matching externality effect*.

The punch-line finding is that in addition to the conventional negative effect of steady inflation on real activity via labor-leisure trade-off, money growth may generate positive real effects on employment and output. Specifically, higher money growth raises the cost of holding money, thus

[3] It should be noted that while Andolfatto and Merz study how labor market frictions influence the effect of technology shocks on the business cycle in a real production economy, our paper examines the long-run real effect of steady inflation under monetary exchanges.

reducing real money balances held by households and firms and restricting purchases that require cash in advance (household consumption and firm wage payment). A direct consequence is to encourage firms to shift from "production" to "non-production" activities, devoting more manpower to vacancy creation and maintenance. As a result, the job finding rate facing each searching worker is higher. While the shift from production to non-production activities lowers firm's labor demand, a higher job finding rate raises job matches and the steady-state level of employment. When there are at least some moderate amount of money being injected to firms and when agents' responses to labor-market frictions are sufficiently strong, the matching externality effect dominates the conventional labor demand effect and hence equilibrium employment rises. This therefore creates a channel for higher money growth to induce possibly higher welfare: the betterment in job matching generates a large time saving effect in job search by the households, resulting in sufficiently higher leisure to outweigh the modest drop in consumption. The welfare-maximizing inflation rate could be positive, departing from the Friedman rule. A positive optimal rate of money growth can never arise in a standard cash-in-advance model with labor-leisure choice in the absence of labor market frictions. The results of positive real effects of money growth and positive rate of optimal inflation may be established when firms and households are responsive to labor market frictions, which is more likely in relatively low inflation regimes, with low job finding rates, and under inefficient wage bargains. These findings remain robust, over a wide range of money injection schemes, under alternative cash constraint setups, and with pre-existing distortionary taxes in a second-best world.

2 The Model

Time is discrete. The basic economy features three theatres of economic activities: a continuum of identical infinitely lived competitive firms (of measure one), a continuum of identical infinitely lived households (of measure one) and a (passive) monetary authority. All individual agents have perfect foresights. There are two productive factors: capital and labor, both owned by households. Firms and households exchange in both goods and factor markets. The goods market is Walrasian and the capital market is perfect, but the labor market exhibits search/entry frictions. While each firm can create multiple vacancies and each household can choose search intensity endogenously, both vacancy creation and search intensity are costly.

To avoid complexity involved in managing the distribution of the employed, the unemployed and their respective cash holdings, we adopt the "large households" framework proposed by Lucas (1990). Specifically, each household can be thought of containing a continuum of "members" who are employed (workers) and unemployed (job seekers or searching workers), with the sum of their mass normalized to unity. All members pool their income as well as their enjoyment of the fruit of employment (consumption) and unemployment (leisure). Vacancies and job seekers are brought together through a Diamond (1982) type matching technology, where the flow matches depends on the masses of both matching parties. Each vacancy can be filled by exactly one searching workers. At an exogenous rate, filled vacancies and workers are separated every period and separated workers immediately become job seekers.

Finally, the monetary authority is passive, determining nothing but the issuance of (fiat) money and the rule of money injections. In order to offer a direct comparison with Lucas (1980) and Wang and Yip (1992), we consider a benchmark with cash required only for firm's wage payments and household's consumption purchases, where firm's spending in capital rental and household's spending in capital investment are not subject to cash-in-advance (CIA) constraints. In Section 4.1, we will discuss the implications of relaxing these assumptions.

2.1 Firms

At period t, a representative firm rents capital k_t (beginning-of-period measure) from households at a gross rental rate r_t and employs labor n_t at a real market wage rate w_t to produce a single final good y_t under a constant-returns Cobb-Douglas technology. Not all employed workers at the representative firm are devoted to production. A mass of workers of measure Φ are employed solely to maintain the vacancies v_t, which can be thought of covering the costs of posting vacancies, managing personnel-related documentations, as well as providing and maintaining the office space. Since such costs are likely to exhibit scale economies, we postulate: $\Phi(v_t) = \phi v_t^{\varepsilon}$, where $\varepsilon \in (0,1)$ reflects the scale economies and $\phi > 0$ captures any exogenous shift in the cost of vacancy management. Accordingly, the measure of workers used for manufacturing is $n_t - \Phi(v_t)$ and the output of the representative firm can now be specified as: $y_t = Ak_t^{\alpha}\left(n_t - \Phi(v_t)\right)^{1-\alpha}$, with $\alpha \in (0,1)$ and $A > 0$.

Let ψ be the (exogenous) job separation rate and η_t be the (endogenous) employee recruitment rate. Since each vacancy can be filled by only one worker, the inflow of workers to employment is $\eta_t v_t$ and the outflow is ψn_t. Employment within the representative firm thus evolves according to the following birth-death process: $n_{t+1} - n_t = \eta_t v_t - \psi n_t$, or, by rearranging terms:

$$n_{t+1} = (1-\psi)n_t + \eta_t v_t. \tag{1}$$

Denote the nominal money stock held by the representative firm at the beginning of period t as M_t^F, the aggregate price level prevailed in period t as P_t, and the rate of inflation from period

$t-1$ to period t as π_t. Further denote by m_t^F the real money balances held by the representative firm at the beginning of period t: $m_t^F = \frac{M_t^F}{P_{t-1}}$. Define the incremental holding of nominal money balances as $Z_t^F = M_{t+1}^F - M_t^F$. Then, the incremental holding of real money balances is given by: $z_t^F = \frac{M_{t+1}^F}{P_t} - \frac{M_t^F}{P_t} = m_{t+1}^F - \frac{m_t^F}{1+\pi_t}$, or, rewriting in forms of the evolution of real money balances by the representative firm:

$$m_{t+1}^F = \frac{m_t^F}{1+\pi_t} + z_t^F. \tag{2}$$

We assume that firms must hold money to finance their wage payments, that is, the following CIA constraint must hold true: $W_t n_t \leq M_t^F$. Since vacancy creation and maintenance require labor inputs, the above specification implies that the firm's expenses on labor in *both* production $(W_t(n_t - \Phi(v_t)))$ and vacancy activities $(W_t\Phi(v_t))$ are cash constrained. By rewriting, the firm's CIA constraint becomes $P_t w_t n_t \leq m_t^F P_{t-1}$, or,

$$w_t n_t \leq \frac{m_t^F}{1+\pi_t}. \tag{3}$$

Notably, since matching is not instantaneous in this frictional labor market, employment becomes a state (rather than control) variable. Therefore, given real money injection x_t^F defined as $\frac{X_t^F}{P_t}$ and under constraints (1), (2) and (3), the representative firm will, facing the state (n_t, m_t^F), choose vacancies, capital demand and incremental real money balances (v_t, k_t, z_t^F) to maximize its value, $\Gamma(n_t, m_t^F)$, which is the sum of the discounted real profit flows $y_t - w_t n_t - r_t k_t - z_t^F + x_t^F$ (inclusive of net benefits related to money holding). Applying the standard dynamic programming techniques, we can express the representative firm's optimization problem in Bellman equation form as:

$$\Gamma(n_t, m_t^F) = \max_{v_t,\, k_t,\, z_t^F} \left[Ak_t^{\alpha} \left(n_t - \Phi(v_t)\right)^{1-\alpha} - w_t n_t - r_t k_t - z_t^F + x_t^F \right] + \frac{1}{1+r_t}\Gamma(n_{t+1}, m_{t+1}^F), \tag{4}$$

subject to constraints (1), (2) and (3).

2.2 Households

Facing a pooled resource, the representative "large" household has a unified preference capturing enjoyment of all its members: the employed, whose fraction is n_t, and the unemployed, whose fraction is $1-n_t$. The employed is assumed to work full time (normalized to one), while the unemployed are divided into job searchers, whose fraction is s_t and leisure takers, whose fraction is $1-s_t$. Thus, the overall fractions of workers, job seekers and leisure takers are n_t, $s_t(1-n_t)$ and $(1-s_t)(1-n_t)$, respectively.

The representative household value both consumption, c_t, and leisure, $(1-s_t)(1-n_t)$. Under this framework, it is obvious that not all unemployed time can be regarded as leisure, because search intensity takes away such an enjoyment. Accordingly, the representative household's preference can be written in a standard time-additive form as:

$$\Omega = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t U\left(c_t, (1-s_t)(1-n_t)\right),$$

where $\rho > 0$ and $U\left(c_t, (1-s_t)(1-n_t)\right) = c_t^{1-\sigma} \left[(1-s_t)(1-n_t)\right]^{\sigma}$, $\sigma \in (0,1)$.

Since the household owns capital k_t and holds real money balances m_t^H, its budget constraint must include both capital investment and incremental holding of real money balances z_t^H: $m_t^H = \frac{M_t^H}{P_{t-1}}$. The incremental holding of real money balances is: $z_t^H = \frac{M_{t+1}^H}{P_t} - \frac{M_t^H}{P_t} = m_{t+1}^H - \frac{m_t^H}{1+\pi_t}$, or,

$$m_{t+1}^H = \frac{m_t^H}{1+\pi_t} + z_t^H. \tag{5}$$

Let $\delta \in (0,1)$ denote the constant rate of capital depreciation. Given the market rental and wage rates (r_t, w_t), consumption, gross capital investment $(k_{t+1} - k_t + \delta k_t)$, and incremental real money balances can be supported by wage and rental income, plus real money injection x_t^H:

$$k_{t+1} = (1 - \delta + r_t)k_t + w_t n_t - c_t - z_t^H + x_t^H. \tag{6}$$

Denote by μ_t the (endogenous) job finding rate. Recall that each vacancy can be filled by only one worker, so the inflow of workers to employment $\mu_t s_t(1-n_t)$ net of the outflow ψn_t must be equal to incremental employment. Thus, similar to those specified in the firms decision above, employment in the household perspective evolves according to:

$$n_{t+1} = (1-\psi)n_t + \mu_t s_t (1-n_t). \tag{7}$$

With only consumption requiring cash, the CIA constraint facing the household becomes:

$$c_t \leq \frac{m_t^H}{1+\pi_t}. \tag{8}$$

The representative household's optimization problem in Bellman equation form is then:

$$\Omega(k_t, n_t, m_t^H) = \max_{c_t,\, s_t,\, z_t^H} U\left(c_t, (1-s_t)(1-n_t)\right) + \frac{1}{1+\rho}\Omega(k_{t+1}, n_{t+1}, m_{t+1}^H), \tag{9}$$

subject to constraints (5), (6), (7) and (8).

2.3 The Aggregate Economy

A main departure from the real-business cycle search framework is that we do not impose Hosios' (1990) rule of efficient matching for firms and workers to share the surplus. Instead, we follow closely the neoclassical distribution of income. Specifically, we consider each firm's outside option as Π_0. Under competitive search, Π_0 must be treated as given by all individuals. We then consider that a large household makes a take-it-or-leave-it-offer of a potential employee to the firm with wage demand w. Normalizing $\Pi_0 = 0$ yields a "zero profit" solution, which is a special benchmark bargaining outcome. This enables us to focus exclusively on the consequence of labor market frictions (rather than strategic bargaining) for inflation and growth to contrast with findings in frictionless neoclassical models. In Section 4.2, we shall return to the case of efficient matching under Hosios' rule to further elaborate the role of efficient bargain played in the welfare cost of inflation.

Since the goods market is perfectly competitive, all firms must reach zero profit in equilibrium:

$$Ak_t^\alpha \left(n_t - \Phi\left(v_t\right)\right)^{1-\alpha} - w_t n_t - r_t k_t - z_t^F + x_t^F = 0. \tag{10}$$

Because there is only a single good in the economy, the resource constraint requires that aggregate goods supply must be equal to aggregate goods demand, which is the sum of consumption and gross investment:

$$c_t + [k_{t+1} - (1-\delta)k_t] = Ak_t^\alpha \left(n_t - \Phi\left(v_t\right)\right)^{1-\alpha}. \tag{11}$$

While the capital market is perfect as in the conventional Walrasian models, the labor market exhibit search frictions. Given the job separation rate ψ and the aggregate flow matches E_t, aggregate employment evolves according to the following birth-death process:

$$N_{t+1} = (1-\psi)N_t + E_t. \tag{12}$$

Similar to Diamond (1982), the aggregate flow matches depend on the masses of both matching parties, namely, search intensity augmented job seekers, $s_t(1-N_t)$, and vacancies, V_t. Assume the matching technology exhibits constant-returns-to-scale property, as suggested by the empirical evidence in Blanchard and Diamond (1990) using the U.S. data. We can specify:

$$E_t = B\left[s_t(1-N_t)\right]^\beta V_t^{1-\beta}, \tag{13}$$

where $B > 0$ measures the degree of matching efficacy and $\beta \in (0,1)$.

Since each vacancy can be filled by exactly one searching workers, the tightness of the labor market to firms, denoted θ_t, can be measured by the ratio of vacancies to searching workers:

$$\theta_t \equiv \frac{V_t}{1-N_t}. \tag{14}$$

We can then rewrite the aggregate flow matches in terms the current employment, the search intensity and the tightness of labor market to firms:

$$E_t = B(1-N_t)s_t^\beta \theta_t^{1-\beta}. \tag{15}$$

Using (15) and (14), we obtain the employee recruitment rate (η_t) and the job finding rate (μ_t):

$$\eta_t = \frac{E_t}{V_t} = B\left(\frac{\theta_t}{s_t}\right)^{-\beta}, \tag{16}$$

$$\mu_t = \frac{E_t}{s_t(1-N_t)} = B\left(\frac{\theta_t}{s_t}\right)^{1-\beta}. \tag{17}$$

Thus, the tighter the labor market is to firms, the lower the employee recruitment rate and the higher the job finding rate are. An increase in job seekers' search intensity crowds out job seeking opportunities and hence reduces the average job finding rate, although it eases firms' recruiting process by raising their employee recruitment rate.

Finally, we specify the monetary authority's money injection rules. While we follow the convention assuming that money is injected in a lump-sum fashion, we permit a general money injection rule that nests the proportional injection rule commonly used in the literature (cf. Lucas 1972 and many subsequent studies) as a special benchmark. Specifically, let $g \equiv \frac{X_t}{M_t} \equiv \frac{M_{t+1}-M_t}{M_t}$ denote the (constant) growth rate of aggregate money supply. At equilibrium, money is injected to each firm and household according to: $X_t^F = \bar{\zeta}_t^F X_t$ and $X_t^H = \bar{\zeta}_t^H X_t$, with $\bar{\zeta}_t^F + \bar{\zeta}_t^H = 1$. Rewriting in real terms and defining relative injections as ratios to agents' respective holdings, $\zeta_t^F = \frac{\bar{\zeta}_t^F}{m_t^F/m_t}$ and $\zeta_t^H = \frac{\bar{\zeta}_t^H}{m_t^H/m_t}$, we then have:

$$x_t^F = \frac{g}{1+\pi_t}\zeta_t^F m_t^F, \tag{18}$$

$$x_t^H = \frac{g}{1+\pi_t}\zeta_t^H m_t^H. \tag{19}$$

There are three special cases of interest: (i) benchmark proportional injection: $\left(\bar{\zeta}_t^F, \bar{\zeta}_t^H\right) = (m_t^F/m_t, m_t^H/m_t)$ and $\left(\zeta_t^F, \zeta_t^H\right) = (1,1)$; (ii) injections to firms only: $\left(\bar{\zeta}_t^F, \bar{\zeta}_t^H\right) = (1,0)$ and $\left(\zeta_t^F, \zeta_t^H\right) = \left(m_t/m_t^F, 0\right)$; and (iii) injections to households only: $\left(\bar{\zeta}_t^F, \bar{\zeta}_t^H\right) = (0,1)$ and $\left(\zeta_t^F, \zeta_t^H\right) = \left(0, m_t/m_t^H\right)$. Combining zero profit condition with resource constraint, we can easily see that aggregate incremental money holdings must be equal to aggregate money injections, yielding the money market equilibrium condition:

$$x_t^H + x_t^F = z_t^H + z_t^F. \tag{20}$$

3 Equilibrium

In this section, we focus on a steady-state analysis, examining how an increase in money growth would affect, in the long run, the key economic variables such as employment, output, capital-labor ratio as well as the variables related to search such as probabilities of job matching, search intensity and vacancy rate. We will also discuss the optimal money growth rate in terms of output and welfare considerations.

The constraints in the representative firm's optimization problem consist of two evolution equations on the state variables, n and m^F, and a CIA constraint on wage payments. The constraints in the representative household's optimization problem consist of the laws of motion for three state variables, k, n and m^H, and a CIA constraint on consumption. Both optimization problems can be solved using standard dynamic programming techniques. For brevity, we relegate the mathematical details to the Appendix. Instead, we will proceed with solving and characterizing the steady-state analysis in our model. It will become clear that more assumptions are necessary in order for us to secure the uniqueness of the steady state.

3.1 Steady State

To economize on notations, we will, in the rest of the paper, use small-case letters without time subscript to denote the equilibrium aggregate variables in a steady state. Since we are interested in the long-run perspective and will not look into the transitional dynamics, this abuse of notations should not cause any confusion.

Labor-market matching in the steady state implies:

$$\psi n = \mu s(1-n) = \eta v. \tag{21}$$

which can be used with other labor-market relationships, (15), (14), (16) and (17), to obtain the steady-state Beveridge curve,

$$\eta = B^{1/(1-\beta)} \mu^{-\beta/(1-\beta)}, \tag{22}$$

as well as the steady-state number of vacancies, search intensity and market tightness measure:

$$v = \frac{\psi n}{\eta}, \tag{23}$$

$$s = \frac{\psi n}{\mu(1-n)}, \tag{24}$$

$$\theta \equiv \frac{\psi n}{(1-n)\,\eta}. \tag{25}$$

It is clear that in the steady state, the inflation rate equals the money growth rate: $\pi = g$. From the definition of incremental real money holdings and under the general money injection rule, we have:

$$x^i = \frac{g}{1+g} \zeta^i m^i = \zeta^i z^i \quad (i = F, H). \tag{26}$$

Combining the budget constraint (6) and the zero profit condition (10) yields,

$$c + \delta k = rk + wn + x - z = rk + wn + \left(\zeta^H - 1\right) z^H = rk + wn + \left(1 - \zeta^F\right) z^F, \tag{27}$$

where it can be easily verified that $\left(\zeta^H - 1\right) z^H = \left(1 - \zeta^F\right) z^F$. Under perfect foresight, the two CIA constraints both hold with equality in the steady state. Thus,

$$m^F = (1+g)wn, \tag{28}$$

$$m^H = (1+g)c. \tag{29}$$

In the steady state, the real interest rate is pinned down by,

$$r = \rho + \delta, \tag{30}$$

which, together with the capital demand condition, yields the familiar modified golden rule:

$$\alpha A k^{\alpha-1} \left(n - \Phi\left(v\right)\right)^{1-\alpha} = \rho + \delta. \tag{31}$$

Using (27), (31) and (37), we can express the real wage as:

$$w = \frac{(\rho+\delta)\left(\frac{1-\alpha}{\alpha}\right)}{1 + \left(1 - \zeta^F(g)\right) g} \left(\frac{k}{n}\right) \tag{32}$$

We then write aggregate consumption as:

$$c = \left(\frac{\rho+\delta}{\alpha} - \delta\right) k, \tag{33}$$

Substituting these two expressions above into (28) and (29) yields the money holding ratio of households to firms:

$$\frac{m^H}{m^F} = \frac{c}{wn} = \frac{[\rho + (1-\alpha)\delta]\left[1 + \left(1 - \zeta^F(g)\right) g\right]}{(1-\alpha)(\rho+\delta)}, \tag{34}$$

We turn next to three important trade-off and equilibrium conditions. As shown in the Appendix, the *vacancy creation-production trade-off* is captured by,

$$\left[1 - \frac{\Phi'(v)\left(\psi + r\right)}{\eta}\right] (1-\alpha) A k^{\alpha} \left(n - \Phi\left(v\right)\right)^{-\alpha} = (1+r)(1+g)w, \tag{35}$$

whereas the relative money injection to firm and the *zero profit condition* become,

$$\zeta^F(g) = \bar{\zeta}^F \left\{1 + \frac{[\rho + (1-\alpha)\delta]\left[1 + \left(1 - \bar{\zeta}^F\right) g\right]}{(1-\alpha)(\rho+\delta) + [\rho + (1-\alpha)\delta]\,\bar{\zeta}^F g}\right\} \tag{36}$$

$$A k^{\alpha} \left(n - \Phi\left(v\right)\right)^{1-\alpha} = rk + \left[1 + \left(1 - \zeta^F(g)\right) g\right] wn. \tag{37}$$

We then arrive at the following condition governing the *labor-leisure trade-off* (see the Appendix),

$$(\rho+\psi+\mu)\, U_2\left(c,(1-s)(1-n)\right)=\frac{\mu w U_1\left(c,(1-s)(1-n)\right)}{(1+g)\,(1+\rho)}. \tag{38}$$

To understand the channels through which economic aggregates are affected by the rate of money growth in the steady state requires thorough investigation of the role played by labor market frictions. As a part of such frictions, vacancy creation and maintenance are costly. Thus, the equilibrium wage determined by the zero profit condition (37) must be lower than the marginal product of labor (MPN), given by,

$$w=(1-D)\cdot MPN, \tag{39}$$

with D measuring the *wage discount*. However, various money injection schemes may lead to redistribution between firms and households. As a result, the wage discount D can be larger or smaller than the vacancy cost ratio $\frac{\Phi(v)}{n}$. From (31), (37), and (39), these two measures are related as follows:

$$\frac{\Phi\left(v\right)}{n}=1-\left[1+\left(1-\zeta^{F}\left(g\right)\right)g\right](1-D), \tag{40}$$

from which we can see that $D=\frac{\Phi(v)}{n}$ under the proportional money injection rule ($\zeta^F=1$).

Another equation that relates w and D comes from equation (35) that captures the vacancy creation-production trade-off:

$$\left[1-\frac{\varepsilon\left(\psi+\rho+\delta\right)}{\psi}\left\{1-\left[1+\left(1-\zeta^{F}\left(g\right)\right)g\right](1-D)\right\}\right]MPN=(1+\rho+\delta)(1+g)w \tag{41}$$

The impact of a higher money growth rate, g, on equilibrium wage-marginal product ratio, $\frac{w}{MPN}$, and the wage discount, D, can be seen clearly from Figure 1, which depicts two straight lines given by equations (39) and (41). The two lines represent two different interpretations of D, wage discount (WD) in equation (39) and vacancy creation-employment ratio (VCER) in equation (41), respectively. Since MPN is determined by $k/(n-\Phi\left(v\right))$ and is, according to (31), independent of g, so is line WD. On the other hand, the line VCER will be affected by g depending on the money injection rule (to be further elaborated in the next subsection). Thus, we can solve explicitly from WD and VCER for the wage discount as a function of the money growth rate (see the Appendix):

$$D(g)=\frac{\psi\left[(1+\rho+\delta)(1+g)-1\right]-\varepsilon\left(\psi+\rho+\delta\right)\left(1-\zeta^{F}\left(g\right)\right)g}{\psi(1+\rho+\delta)(1+g)-\varepsilon\left(\psi+\rho+\delta\right)\left[1+\left(1-\zeta^{F}\left(g\right)\right)g\right]}. \tag{42}$$

It can be readily seen that, if the scale economies of vacancy creation is sufficiently large such that $\frac{\psi}{\psi+\rho+\delta}>\varepsilon$, then $D(g)$ is always bounded above by one.

The equations determining the steady state can be re-arranged in a recursive fashion that is conducive to perform comparative statics. Essentially, we could obtain two equations determining μ and n. The rest of endogenous variables can then be derived easily. The first of the two equations has to do with the vacancy creation-production trade-off (35), and is henceforth named the VP locus:

$$n = \left[\frac{\phi}{\tilde{D}(g)}\right]^{1/(1-\varepsilon)} \left(\psi B^{-\frac{1}{1-\beta}} \mu^{\frac{\beta}{1-\beta}}\right)^{\varepsilon/(1-\varepsilon)}, \tag{43}$$

where

$$\tilde{D}(g) = \frac{\psi \left\{(1+\rho+\delta)(1+g) - \left[1+(1-\zeta^F(g))g\right]\right\}}{\psi(1+\rho+\delta)(1+g) - \varepsilon\left(\psi+\rho+\delta\right)\left[1+(1-\zeta^F(g))g\right]},$$

which summarizes the channel through which money growth affects the vacancy creation-production trade-off. Notice that other than $\tilde{D}(g)$, the VP locus takes the same form regardless of the money injection rule. The second of the two equations comes from the labor-leisure decision (38), and is henceforth named the LL locus:

$$n = \left\{1 + \frac{\psi}{\mu} + \frac{\sigma}{1-\sigma}\frac{1}{1-\alpha}\frac{\rho+(1-\alpha)\delta}{\rho+\delta}\left[1+\left(1-\zeta^F\left(g\right)\right)g\right]\left(1+g\right)\left(1+\rho\right)\left(\frac{\rho+\psi+\mu}{\mu}\right)\right\}^{-1}. \tag{44}$$

Detailed derivations for the VP locus and the LL locus can be found in the Appendix.

It is easy to see that the LL locus is upward sloping in (μ, n)-space, with n starting from 0 when $\mu = 0$ and approaching a finite upper bound as μ approaches infinity. The LL locus is concave. To rule out possibility of non-degenerate multiple equilibria, we assume that $\varepsilon > 1-\beta$ so that the VP locus is upward sloping and convex, with n starting from 0 when $\mu = 0$ and approaching infinity as μ approaches infinity. The interior intersection of VP and LL loci defines the steady-state values of μ and n (see Figure 2). With the steady-state values of μ and n solved, the rest of endogenous variables can be derived in a recursive manner.

Finally, applying the modified golden rule, the steady-state capital-labor ratio becomes:

$$\frac{k}{n} = \left(\frac{\alpha A}{\rho+\delta}\right)^{1/(1-\alpha)}\left[1+\left(1-\zeta^F\left(g\right)\right)g\right]\left(1-D\left(g\right)\right), \tag{45}$$

which is negatively related to the wage discount $D\left(g\right)$. The aggregate real output is:

$$y = A^{1/(1-\alpha)}\left(\frac{\rho+\delta}{\alpha}\right)^{-\alpha/(1-\alpha)}\left[1+\left(1-\zeta^F\left(g\right)\right)g\right]\left(1-D\left(g\right)\right)n, \tag{46}$$

which also depends on the wage discount negatively. Additionally, we can derive economic welfare measured by steady-state lifetime utility facing the representative large household:

$$U(c,(1-s)(1-n)) = \left(\frac{\rho+\delta}{\alpha} - \delta\right)^{1-\sigma} n^{1-\sigma}\left(\frac{k}{n}\right)^{1-\sigma}\left(1-n-\frac{\psi n}{\mu}\right)^{\sigma}. \tag{47}$$

3.2 Comparative Statics

We are now prepared for a complete examination of the real effect of money. To begin, let us focus on the benchmark case with proportional money injections. This is an important benchmark not only because it has been commonly used in the literature but also because it removes any arbitrary redistribution between firms and household as a result of money injections – in this case, the seigniorage tax collected by the monetary authority is fully refunded to money holders in a neutral, nondistortionary manner.[4] Thus, under this benchmark money injection rule, we can study the real effect of money purely through the neoclassical channel via capital accumulation and the channel via costly job search and vacancy creation highlighted in this paper.

Recall that under the proportional money injection rule, $\zeta^F(g) = 1$ and hence,

$$\frac{\Phi(v)}{n} = \tilde{D}(g) = D(g) = \frac{\psi\left[(1+\rho+\delta)(1+g)-1\right]}{\psi(1+\rho+\delta)(1+g)-\varepsilon\left(\psi+\rho+\delta\right)}, \tag{48}$$

which is strictly increasing in g as long as $\frac{\psi}{\psi+\rho+\delta} > \varepsilon$. Diagrammatically, the line VCER in this benchmark case rotates down as g increases. The intuition for the rotation is as follows. As money growth rate increases, the shadow cost of employment tends to increase (see the coefficient of w on the RHS of 41) because the wage payment is subject to the CIA constraint. For any fixed $D < \frac{\psi}{\varepsilon(\psi+\rho+\delta)}$, the wage rate, w, needs to drop to compensate the tendency for the increase in the shadow cost. Under the condition $\psi > \varepsilon(\psi+\rho+\delta)$ (easily satisfied when $\varepsilon < 1$ and $\rho+\delta$ is small relative to ψ as in our calibration), the two lines intersect with VCER flatter than WD.

Intuitively, since in our benchmark model money is injected in a lump-sum fashion that is ex post proportional to the beginning-of-period money holdings by firms and consumers, the seigniorage tax imposed on firms and consumers are fully refunded ex post. However, when firms are making decision, the injections are taken as given. When money growth increases, the cost of holding money facing firms (and consumers) rises. Other things being equal, the profit is lower. Since employment is a state variable in a labor search model (and cannot be adjusted within a period), the wage will be renegotiated. Given the outside option of zero profit for the firm, the firm must bargain down the real wage, leading to higher wage discount. Since the wage discount is equivalent to the fraction of employment used to maintain and create vacancy, this employment fraction would be

[4] In a typical monetary model with firms, money is often injected to the firms, through an intermediary/financial institution (e.g., see Carlstrom and Fuerst 1995 and Christiano and Eichenbaum 1995). It may be noted as an exception that in the money-in-the-production-function framework developed by Fischer (1974), firms rent capital and real money balances from the representative household. Since the household is the owner of real money balances, it is natural that money is injected to the household alone.

higher, which in turn improves job matching and raises future employment. Mathematically, $D(g)$, which has the alternative interpretation as the vacancy creation-employment ratio, is an increasing function of money growth rate, g. In other words, higher money growth shifts employment from production to non-production activities and raises steady state employment through enhanced job matching. This establishes the new channel that an increase in money growth encourages vacancy creation and raises job matches.

Also recall that the VP locus varies with the money injection rule only through $\tilde{D}(g)$, which in the benchmark case becomes:

$$\tilde{D}(g) = \frac{\psi\left[(1+\rho+\delta)(1+g)-1\right]}{\psi(1+\rho+\delta)(1+g)-\varepsilon\left(\psi+\rho+\delta\right)}.$$

The LL locus is now,

$$n = \left\{1+\frac{\psi}{\mu}+\frac{\sigma}{1-\sigma}\frac{1}{1-\alpha}\frac{\rho+(1-\alpha)\delta}{\rho+\delta}\left(1+g\right)\left(1+\rho\right)\left(\frac{\rho+\psi+\mu}{\mu}\right)\right\}^{-1}.$$

It is easily seen that more rapid money supply growth (higher g) will cause both VP and LL to shift downward, hence n could either increase or decrease (Figure 2 depicts the situation when n increases as g rises). Intuitively, a higher money growth rate increases the cost of holding money, thereby reducing real money balances held by households and firms and restricting purchases that require cash in advance. On the one hand, households are forced to lower their consumption c. On the other hand, firms are forced to shift from production to non-production activities, by devoting more manpower to vacancy creation and maintenance (which can be seen from (48) that higher g increases $\frac{\Phi(v)}{n}$, thus raising v for a given level of employment n). As a result, the job finding rate facing each searching worker is higher (referred to as the matching externality effect). While the shift from production to non-production activities lowers firm's labor demand, a higher job finding rate raises job matches and the steady-state level of employment. When agents' responses to labor-market frictions are sufficiently strong, the matching externality effect dominates the conventional labor demand effect and, in this case, equilibrium employment rises.

Thus, our model provide a channel through which higher money growth may induce higher output. Particularly from (46), if an increase in steady inflation raises n more significantly than raising $D\left(g\right)$, it could lead to a higher real output. This is more likely to arise if labor-market frictions are severe and wage bargains are not efficient. Moreover, a higher job finding rate also reduces job search time. From (47), should the reduction in total job search time, measured by $(1-n)S = \frac{\psi n}{\mu}$, outweigh the rise in work time n, leisure would increase. Thus, despite the loss in consumption, a representative household's welfare may be higher as a result of higher leisure.

While the presence of search friction may induce a positive real effect of steady inflation similar to the findings in the asset substitution setup of Mundell (1963) and Tobin (1965), the channels are very different.[5] Indeed, both the matching-externality-induced positive effect of money growth on employment and the search-time-induced positive effect of money growth on welfare highlighted in our model are absent in a conventional frictionless Walrasian economy.

Notably, when bargaining is inefficient, a more likely scenario in developing countries, firms expand their activity in creating and maintaining vacancies as money growth increases in the benchmark case when the firms' outside option is zero profit and when money injections are proportional to agents' respective holdings. While our result depends on firms' outside option, it remains valid, by continuity argument, even if we use a small positive profit as the outside option for the firms. Thus, the remaining issue left to be addressed is how much our result would depends on the money injection rule, which we now study.

From (36), the relative money injection to firms is captured by,

$$\zeta^F(g) = \begin{cases} 1 + \frac{\rho+(1-\alpha)\delta}{(1-\alpha)(\rho+\delta)+[\rho+(1-\alpha)\delta]g} & \text{firms only} \\ 0 & \text{households only} \end{cases}$$

In the former case with injections to firms only, we arrive at:

$$\begin{aligned} \tilde{D}(g) &= \frac{\psi\left\{(1+\rho+\delta)(1+g)\left[1+\frac{\rho+(1-\alpha)\delta}{(1-\alpha)(\rho+\delta)}g\right]-1\right\}}{\psi(1+\rho+\delta)(1+g)\left[1+\frac{\rho+(1-\alpha)\delta}{(1-\alpha)(\rho+\delta)}g\right]-\varepsilon(\psi+\rho+\delta)} \\ n &= \left\{1+\frac{\psi}{\mu}+\frac{\sigma}{1-\sigma}\frac{[\rho+(1-\alpha)\delta](1+\rho)(1+g)}{(1-\alpha)(\rho+\delta)+[\rho+(1-\alpha)\delta]g}\left(\frac{\rho+\psi+\mu}{\mu}\right)\right\}^{-1} \end{aligned}$$

It is clear that $\tilde{D}(g)$ is increasing in g, so the VP shifts downward unambiguously as g increases; moreover, the LL locus shifts upward (see the Appendix for a graphical illustration). Therefore the effect of money growth on employment is unambiguously positive when money injections are distributed to firms. In the latter case with injections to households only, we have:

$$\begin{aligned} \tilde{D}(g) &= \frac{\psi(\rho+\delta)}{\psi(1+\rho+\delta)-\varepsilon(\psi+\rho+\delta)} \\ n &= \left\{1+\frac{\psi}{\mu}+\frac{\sigma}{1-\sigma}\frac{1+\rho}{1-\alpha}\frac{\rho+(1-\alpha)\delta}{\rho+\delta}(1+g)^2\left(\frac{\rho+\psi+\mu}{\mu}\right)\right\}^{-1} \end{aligned}$$

Thus, the VP locus is independent of the money growth rate, whereas the LL locus shifts downward. As a result, higher money growth reduces employment unambiguously. Notably, because the new

[5] Under the Lucasian CIA constraint, Chang and Tsai (2003) show the presence of positive effects of money growth when individuals value wealth as social status.

channel emphasized in this paper must be through the vacancy creation and production trade-off, such a trade-off requires firms to receive money injections – otherwise, monetary expansion can never be beneficial for firms. This is why monetary expansion always hurt employment when money is only injected to households. Of course, just how much such injections to firms are needed for monetary expansion to be potentially welfare-enhancing is a quantitative issue, to which we now turn.

3.3 Calibration and Welfare Analysis

We calibrate the benchmark economy with parameter values matching the U.S. annual data over the post-WWII period. We set the subjective rate of time preference to $\rho = 2\%$, the rate of capital depreciation to $\delta = 3.5\%$ and the capital share to $\alpha = 0.38$. As it can be seen below, these parameter values will yield reasonable consumption-output ratio and real rental rate. Over the period mentioned above, the money growth rate is averaged about $g = 6.5\%$. Defining the search intensity augmented unemployment measure as $u = s(1-n))$, we can calibrate $n+u$ to match the labor force participation rate of 61.5%. Based on Shimer (2005), the monthly separation rate is 3.4%, the monthly job finding rate is 45%, and the matching elasticity is $\beta = 0.72$. These give the annual separation rate and annual job finding rate as $\psi = 1-(1-0.034)^{12} = 0.339724$ and $\mu = 1-(1-0.45)^{12} = 0.999234$, respectively. Using (21), we have: $n+u = n(1+\frac{\psi}{\mu}) = n(1+\frac{0.339724}{0.999234}) = 0.615$, which gives the calibrated employment rate, n, at 0.458961. We then follow Shimer (2005) to normalize the vacancy-unemployed searching worker ratio ($\frac{v}{u}$) as one, from which we can utilize (21) to calibrate $v = 0.156039$, $\theta = S = 0.288407$, and $B = \eta = \mu = 0.999234$. Further normalizing $A = 1$ and choosing a reasonable wage discount at $D = 0.2$, we can now use (44) to calibrate $\sigma = .317088$ and use (42) and (43) jointly to calibrate $\phi = 0.206034$ and $\varepsilon = .435237$. From equations (45)–(34), we then obtain: $\frac{k}{n} = 18.071520$, $m^H/m^F = 1.222874$, $c = 0.910170$ and $y = 1.200464$. Based on the flow of funds data computed by Cole and Ohanian (1998, Figure 3), the household-to-firm money demand ratio in the U.S. from 1952 to 1997, excluding the volatile high inflation period during the oil crises, is mostly between 1 and 2 (32 out of 35 years). Our calibrated household-to-firm money demand ratio falls right in the range. This calibrated ratio corresponds to a firm money injection ratio of $\bar{\zeta}^F = 44.9868\%$ in the benchmark and we will check how our quantitative results may change in response to different firm injection ratios. While the capital-labor ratio yields a realistic real rental rate of 5.5%, the consumption-output ratio of approximately 75% is also reasonable.

We can now revisit the issue of optimum quantity of money. Given the calibration above

($\mu = \eta = 0.999234$) following Shimer's normalization, the Cobb-Douglas matching function implies that as the money growth rate varies slightly from benchmark of 6.5%, either μ or η will hit the upper bound of unity. In particular, we find that the only range of money growth rates relevant for our consideration is $[0.06386, 0.065113]$, within which the welfare declines as the money growth rate rises. It is thus safe to say that the benchmark money growth rate is close to the optimum.

The question on the optimal inflation rate would become much more interesting if an economy features a less efficient labor market. Consider for illustrative purposes a *modified benchmark* that is identical to the U.S. economy except a lower job finding rate at $\mu = 0.8 \cdot 0.999234$ (which corresponds to a monthly job finding rate of 12.5% or an average unemployment spell of 8 months). With more labor-market frictions, the optimal inflation rate turns out to be $g^* = 7.5592\%$, which is significantly above the benchmark rate of 6.5% (see Figure 3). This suggests that developing countries may have an optimal inflation rate that is higher than in developed countries if the former has a labor market with greater frictions. Notably, Khan and Senhadji (2000) reach a similar conclusion for inflation in an empirical investigation in terms of economic growth. In Table 1, we compare key variables under the benchmark money growth rate and the optimal money growth rate.

Table 1. Comparison between the modified benchmark and the optimal case

Endogenous Variables	$g = 0.065$	$g^* = 0.075592$
n	0.431585	0.438905
μ	0.799387	0.857199
η	0.799387	0.668006
k/n	18.071520	17.750489
v	0.183415	0.223212
S	0.322678	0.310013
θ	0.322678	0.397815
D	0.200000	0.214212
m^H/m^F	1.222874	1.222874
y	1.128860	1.127614
c	0.855881	0.854936
U	0.664649	0.665318

As discussed in the analytical results above, higher money growth induces the firms to put more effort in vacancy creation, thereby raising the job finding rate. Under our benchmark parametrization, the matching externality effect is dominant and thus higher money growth leads to higher employment. The trade-off is to reduce consumption and output, though the magnitude is negligible due to the opposing effects on the employment rate (positive) and the capital-labor ratio (negative). Since the betterment in job matching as a result of a suitable increase in money growth generates a large time saving effect in job search by the households, it results in sufficiently higher leisure to outweigh the modest drop in consumption. As a consequence, an increase in money growth from the benchmark rate of 6.5% to g^* enhances economic welfare.

We now return to the issue concerning how the welfare result may change under different money injection rules. It is best illustrated by solving quantitatively the optimal inflation rate g^* when the firm injection ratio $\bar{\zeta}^F$ rises from 0 to 1. As depicted in Figure 4, we can see that the optimal inflation rate is positive for all $\bar{\zeta}^F > 2.2602\%$. Recall from the previous subsection that when money is only injected to households ($\bar{\zeta}^F = 0$), the VP locus is independent of the money growth rate and the matching externality effect through vacancy creation and production is fully shut down. Thus, monetary expansion always hurt employment and positive inflation can never be welfare-enhancing. When a small amount of money is injected to firms, there is a positive effect of money growth in conjunction with a negative effect via labor-leisure trade-off. As $\bar{\zeta}^F$ exceeds the above-mentioned threshold, the optimal money growth rate becomes positive. Our numerical analysis shows that even when $\bar{\zeta}^F$ is at an extremely low value of 3%, the optimal inflation is about 5.4%, far above zero. When $\bar{\zeta}^F$ rises to about 9%, the optimal inflation rate increases to 16.9%. Afterward, the optimal inflation starts declining, falling to 3.6% as $\bar{\zeta}^F = 1$ where money is injected to firms only. This hump-shaped result can be understood as follows. As $\bar{\zeta}^F$ continues to rise, the rebate of the seigniorage tax to households declines and the detrimental labor-leisure trade-off effect increases rapidly, eventually dominating the matching externality effect through vacancy creation and production. Thus, the optimal money growth rate starts to decrease when $\bar{\zeta}^F$ becomes too large. Overall, not only is the optimal inflation rate positive for most injection schemes, but over a wide range of $\bar{\zeta}^F \in (3.2\%, 53\%)$ the optimal inflation rate turns out to exceed the benchmark value of 6.5%. Thus, while we need "some" money injection to firms to ensure that the vacancy creation-production channel can work, we do not need "much" of such injections quantitatively. Notably, while too much of such injections would lower the optimal rate of inflation, we find that when money is injected to the firms only, the resulting welfare with the optimal inflation rate of 3.6% is higher than under other injection rules (see Figure 4). This confirms that, in our calibrated economy, the channel through firms' vacancy creation and production is the ultimate force driving the welfare outcomes.

In summary, the Friedman rule does not hold in our economy. Note that in a standard CIA model with labor-leisure choice in the absence of labor market frictions, the optimal rate of money growth can never be positive (as documented in Wang and Yip 1992, Gomme 1993, and many others). Moreover, even in the case with a positive real effect of steady inflation under our setting, there is a crucial difference between ours and the Mundell-Tobin model. In their framework, higher steady inflation causes a substitution from real balances to capital, implying a higher capital-labor ratio in the steady state. In ours, higher steady inflation induces the firms to put more manpower in managing vacancy, which leads to more job matches and increase in employment, thus lowering the capital-labor ratio in the steady state. Our result is found very robust to a wide range of money injection rules.

4 Further Discussion

In this section, we check the robustness of our main findings to alternative CIA constraints or alternative distribution of matching surplus, under the benchmark proportional money injection rule. We also check the validity of our main findings remain in a second-best world with pre-existing distortionary taxes.

4.1 Alternative Cash-in-Advance Constraints

Consider generalized CIA constraints, where a fraction q^F of firm's spending in capital rental and a fraction q^H of household's spending in capital investment require cash:

$$w_t n_t + q^F r_t k_t \leq \frac{m_t^F}{1+\pi_t} \tag{49}$$

$$c_t + q^H \left(k_{t+1} - k_t + \delta k_t\right) \leq \frac{m_t^H}{1+\pi_t}. \tag{50}$$

In the steady state, firm's capital demand becomes:

$$\alpha A k^{\alpha-1} \left(n - \Phi(v)\right)^{1-\alpha} = Q_0 r, \tag{51}$$

where $Q_0 \equiv \left[\left(1-q^F\right) + q^F (1+r)(1+g)\right]$ $(= 1$ if $q^F = 0)$. Moreover, household's intertemporal trade-off and labor-leisure trade-off are given by,

$$r = \frac{\rho+\delta}{Q_1} \tag{52}$$

$$(\rho+\psi+\mu) U_2 (c, (1-s)(1-n)) = \frac{Q_2 \mu w U_1 (c, (1-s)(1-n))}{(1+g)(1+\rho)}, \tag{53}$$

where

$$Q_1 \equiv \frac{1-\rho q^H(1+g)\left[1-\frac{1}{(1+\rho)(1+g)}\right]}{1+q^H(1+g)\left[1-\frac{1}{(1+\rho)(1+g)}\right]} \leq 1 \quad (=1 \text{ if } q^H=0)$$

$$Q_2 \equiv \frac{(1+\rho)(1+g)\left\{1-\rho q^H(1+g)\left[1-\frac{1}{(1+\rho)(1+g)}\right]\right\}}{(1+\rho)(1+g)-\rho q^H(1+g)\left[1-\frac{1}{(1+\rho)(1+g)}\right]} \leq 1 \quad (=1 \text{ if } q^H=0).$$

On the firm side, (51) implies that money growth generates an additional negative effect on capital demand as a result of a higher user cost of capital. On the household side, the steady-state real interest rate under generalized CIA constraints is higher than the benchmark case (see (52)), which discourages capital accumulation; the marginal benefit of labor is lower than the benchmark case (see (53)), which lowers employment. Not surprisingly, both favor a reversed Tobin effect as in the canonical Walrasian setups in Stockman (1981) and Wang and Yip (1992). However, one may inquire how large these negative output effects of money growth will be. We argue that they are indeed modest. On the one hand, capital rental is much smaller than the wage payment while capital investment is much smaller than consumption. On the other, the fractions $\left(q^F, q^H\right)$ are in the order of 0.2 or lower, based on Chang and Li (2004) and Liu, Tsou and Wang (2008). Using our benchmark parametrization with the capital income share of $\alpha = 0.38$ and $q^F = q^H = 0.2$, we can recalibrate this modified economy to compute $Q_0 = 1.02472$, $Q_1 = 0.98303$ and $Q_2 = 0.99997$, which are all very close to one. That is, the key relationships are not much different from the benchmark CIA setup. Thus, the magnitude of the additional negative output effects of money growth induced by generalized CIA constraints is not large enough to upset our main findings, particularly concerning the departure from the Friedman rule.

4.2 Alternative Distribution of Matching Surplus

We turn now to examining whether applying Hosios' rule of efficient matching would change our main findings. Under Hosios' rule, the equilibrium sharing of the matching surplus will be tied to the matching elasticities. In the benchmark case, the zero profit condition (37) together with the modified golden rule (31) yields the equilibrium wage as a fraction of the marginal product. Under Hosios' rule, worker's wage (denoted w^*) is lower than that in the benchmark case (the proof of all the results in this subsection is relegated to the Appendix):

$$w^* = \beta\left(1+\frac{1-\psi}{\psi}\frac{\varepsilon\Phi}{n-\Phi}\right)w + (1-\beta)\left[\frac{(1+\rho)(1+g)\frac{\sigma n}{1-\sigma}\frac{\rho k}{n}}{(1-n)-(1+\rho)(1+g)\frac{\sigma n}{1-\sigma}}\right], \tag{54}$$

which is a weighted average of the competitive wage rate and a lower value, analogous to the findings in Andolfatto (1996). We can rewrite (54) as: $w^* = \Delta \cdot w$, where $w = \left(1 - \frac{\Phi}{n}\right) MPN$ and

$$\Delta = \beta \left(1 + \frac{1-\psi}{\psi} \frac{\varepsilon \Phi}{n - \Phi}\right) + (1-\beta) \left[\frac{\frac{\alpha\rho}{(1-\alpha)(\rho+\delta)}}{\frac{(1-n)(1-\sigma)}{n(1+\rho)(1+g)\sigma} - 1}\right]. \tag{55}$$

Due to the double holdup problems, this wage ratio generally differs from one. In the absence of the double holdup problems, the bargained wage rate equals the competitive wage and thus $\Delta = 1$.

Under Hosios' rule, the labor-leisure trade-off (LL) locus need be modified as:

$$\frac{\frac{\alpha\rho}{(1-\alpha)(\rho+\delta)}}{\frac{\frac{1-\sigma}{\sigma}\left(\frac{1-n}{n} - \frac{\psi}{\mu}\right)}{(1+\rho)(1+g)\left(1+\frac{\rho+\psi}{\mu}\right)} - 1} = \beta \left(1 + \frac{1-\psi}{\psi} \frac{\varepsilon}{\frac{1}{\Lambda(\mu,n)} - 1}\right) + (1-\beta) \left[\frac{\frac{\alpha\rho}{(1-\alpha)(\rho+\delta)}}{\frac{(1-n)(1-\sigma)}{n(1+\rho)(1+g)\sigma} - 1}\right], \tag{56}$$

where Λ measures the fraction of labor devoted to vacancy creation and maintenance and is a function of (μ, n),

$$\Lambda(\mu, n) \equiv \frac{\phi}{n^{1-\varepsilon}} \left[\psi \left(\frac{\mu^\beta}{B}\right)^{\frac{1}{1-\beta}}\right]^\varepsilon.$$

While the LHS of (56) is decreasing in μ and increasing in n, the RHS is increasing in μ and may increase or decrease in n. Thus, we cannot pin down the slope of the LL locus. Should the double holdup problems be absent, the RHS reduces to one and the LL locus is upward-sloping. When the household's bargaining power is weaker (smaller β), the LL locus is steeper.

Under efficient bargain, the VP locus becomes,

$$\frac{1\text{-}\frac{\varepsilon(\psi+\rho+\delta)\Lambda(\mu,n)}{\psi}}{(1+\rho+\delta)(1+g)\left[1\text{-}\Lambda(\mu,n)\right]} = \beta \left(1 + \frac{1\text{-}\psi}{\psi} \frac{\varepsilon}{\frac{1}{\Lambda(\mu,n)}\text{-}1}\right) + (1\text{-}\beta) \left[\frac{\frac{\alpha\rho(1+\rho)(1+g)\sigma n}{(1\text{-}\alpha)(\rho+\delta)(1\text{-}\sigma)}}{1\text{-}n\text{-}\frac{(1+\rho)(1+g)\sigma n}{1\text{-}\sigma}}\right]. \tag{57}$$

This relationship captures the production-vacancy creation trade-off under efficient bargain. When $\varepsilon(\psi + \rho + \delta) < \psi$, both the LHS and the RHS are increasing in μ and decreasing in n. As a result, we cannot determine the slope of the modified VP locus. However, if the double holdup problems are absent, then the RHS reduces to one and the modified VP locus is still upward-sloping. When the household's bargaining power is weaker (smaller β), the VP locus becomes flatter.

Under the calibrated value of bargaining parameter $\beta = 0.72$, the double holdup problem is not negligible. In the calibrated equilibrium mimicking the post-WWII U.S. economy with a lower job finding rate $\mu = 0.8 \cdot 0.999234$, the LL locus is still upward sloping but the VP locus becomes downward sloping. As money growth increases, the VP locus shifts downward and the LL locus also shifts downward but barely. As a result, μ and n both decrease in response to higher money growth. Hence, the optimal inflation rate is now much lower than the benchmark case ($g^* = 0.002149$) when μ^* hits the upper bound of unity.

4.3 Second-Best Tax Incidence Analysis

One may now inquire whether our main findings remain valid in a second-best world with an array of nonzero pre-existing taxes where a full access to the lump-sum tax is unavailable.[6] We are particularly interested in comparing the inflation tax with either a consumption tax or a general income tax.

Denote the consumption tax and general income tax rates as τ_c and τ_y, respectively, and the exogenous government spending as G_t (which is for simplicity assumed to be nonproductive and to yield no utility). Then household's budget constraint (6) is modified as:

$$k_{t+1} = \left[1 - \delta + (1 - \tau_y)\, r_t\right] k_t + (1 - \tau_y)\, w_t n_t - (1 + \tau_c)\, c_t - z_t^H,$$

where the direct money injection is now removed to be consistent with the assumption in Chari, Christiano and Kehoe (1996). The CIA constraint (8) becomes:

$$(1 + \tau_c)\, c_t \leq \frac{m_t^H}{1 + \pi_t},$$

where it is reasonable to assume the sales taxes are paid in cash when consumption purchases are made. The resource constraint (11) is now,

$$c_t + [k_{t+1} - (1 - \delta)k_t] + G_t = A k_t^{\alpha} \left(n_t - \Phi\left(v_t\right)\right)^{1-\alpha},$$

where, in the absence of a lump-sum tax and under the assumption of proportional money injections, the following government budget constraint is met:

$$G_t = \tau_y \left(r_t k_t + w_t n_t\right) + \tau_c c_t + g\left[w_t n_t + (1 + \tau_c) c_t\right],$$

where the last term captures the (real) inflation tax on firm's wage spending and household's consumption purchase.

While firm's optimizing conditions are all unchanged (as the removal of the direct money injection from firm's flow profit would not alter any decisions), some of household's optimizing conditions and equilibrium conditions need be modified. As a consequence, several steady-state equilibrium relationships are different from their benchmark counterparts. For brevity, we only highlight a few key relationships and relegate all mathematical details to the Appendix. While the LL locus is unchanged (all tax effects cancelled out on the margin because τ_y applies on both capital income and wage income), the VP locus is now modified as:

$$n = \left(\frac{\phi}{D}\right)^{1/(1-\varepsilon)} \left(\psi B^{\frac{-1}{1-\beta}} \mu^{\frac{\beta}{1-\beta}}\right)^{\varepsilon/(1-\varepsilon)}.$$

[6] We are grateful to an anonymous referee for alerting us to this useful exercise.

where the wage discount D is now a function of (g, τ_y),

$$D = \frac{\psi\left[\left(1+\frac{\rho+\delta}{1-\tau_y}\right)(1+g)-1\right]}{\psi\left[\left(1+\frac{\rho+\delta}{1-\tau_y}\right)(1+g)-1\right]+\psi(1-\varepsilon)-\varepsilon\frac{\rho+\delta}{1-\tau_y}}.$$

Note that the consumption tax τ_c does not appear in the steady state relationships (VP and LL loci) or in the wage discount expression. An increase in the general income tax τ_y has two effects on the firms' vacancy creation-production trade-off, both through the wage discount. First, it raises the real interest rate ($r = \frac{\rho+\delta}{1-\tau_y}$), which reduces firm's incentive to create vacancy because the benefit from higher matching and higher employment next period would worth less in present value (represented by the term $-\varepsilon\frac{\rho+\delta}{1-\tau_y}$ above). The reduction in the firm's marginal benefit from vacancy creation requires a larger wage discount in order to maintain zero profit. Second, an increase in the real interest rate raises the nominal interest rate, $(1+\frac{\rho+\delta}{1-\tau_y})(1+g)-1$, thereby raising the opportunity cost of financing wage payments due to the presence of the firm' CIA constraint and leading to a higher wage discount. The resulting increase in the wage discount through both channels in turn leads to a lower level of employment (VP locus shifts down). Notably, while the inflation tax also affect the level of employment via the nominal interest rate channel, it does not have the additional effect via the real interest rate as does the general income tax.

To evaluate the welfare effects of money growth and taxes, we express the capital-labor and consumption-capital ratios as follows:

$$\begin{aligned} \frac{k}{n} &= \left[\frac{\alpha A}{(\rho+\delta)/(1-\tau_y)}\right]^{1/(1-\alpha)}(1-D), \\ \frac{c}{k} &= \frac{[(1-\tau_y)-(1-\alpha)g]\frac{\rho+\delta}{\alpha(1-\tau_y)}-\delta}{(1+\tau_c)(1+g)}. \end{aligned}$$

Thus, the welfare measured by steady-state lifetime utility is decomposed into four components,

$$U(c,(1-s)(1-n)) = \left(\frac{c}{k}\right)^{1-\sigma} n^{1-\sigma}\left(\frac{k}{n}\right)^{1-\sigma}\left(1-n-\frac{\psi n}{\mu}\right)^{\sigma}.$$

Through the wage discount channel, an increase in τ_y lowers the capital-labor ratio. Additionally, τ_y also has a direct negative effect on the capital-labor ratio via the real interest rate channel ($(\rho+\delta)/(1-\tau_y)$). By contrast, neither the wage discount nor the capital-labor ratio is affected by the consumption tax τ_c – the only effect of τ_c is on the consumption-capital ratio: by taxing consumption purchases, this ratio is unambiguously lower. Turning now to the effects on leisure $(1-n-\frac{\psi n}{\mu})$, we note that there is an extensive margin (via $1-n$) and an intensive margin (by economizing job search, captured by $-\frac{\psi n}{\mu}$). As discussed above, both g and τ_y have positive impacts on the level of employment, thus lowing leisure on the extensive margin. Similar to the analysis using Figure 2, however, both tax instruments induce large downward shifts in the VP locus accompanied by small shifts in the LL locus, leading to lower employment-job finding rate ratio (n/μ). Thus, an increase in g or τ_y economizes job search and raises leisure on the intensive margin. In our calibrated economy, we always find the intensive margin to dominate the extensive margin. We thus expect that g and τ_y will generate positive leisure effects in the steady state.

In summary, an increase in the consumption tax does not affect any other economic aggregate but the consumption-output ratio negatively, thereby reducing welfare unambiguously. A higher general income tax or a higher inflation tax, on the one hand, raises employment and (most plausibly) leisure, and reduces the capital-labor ratio, on the other. Thus, both affects welfare ambiguously even when their effects on leisure are dominated by the intensive margin.

We recalibrate the modified benchmark economy (with $\mu = 0.8 \cdot 0.999234$) under the zero profit setup and obtain $\sigma = 0.316530$, $\phi = 0.162182$ and $\varepsilon = 0.371870$. While σ is very close to the modified benchmark figure, ϕ and ε are both lower than their counterparts. Concerning the second-best tax incidence analysis, we set the pre-existing inflation tax rate the same as before ($g = 6.5\%$) and choose the consumption and general income tax rates as $\tau_c = 5\%$ and $\tau_y = 20\%$ (which are commonly chosen in the dynamic tax incidence literature calibrating the U.S. economy). These yield a government spending $G = 0.295544$.

We conduct three government revenue-neutral tax incidence exercises. In the first, we fix the consumption tax rate at $\tau_c = 5\%$ and $G = 0.295544$ to maintain revenue-neutral. We find that the optimal tax mix is: $(g^*, \tau_y^*) = (10.57\%, 14.20\%)$. In the second exercise, we fix the general income tax rate at $\tau_y = 20\%$ and again $G = 0.295544$ to obtain the optimal tax mix $(g^*, \tau_c^*) = (8.64\%, 0.36\%)$. These two pair-wise exercises suggest a shift toward the inflation tax. Nevertheless it is never optimal to fully replace consumption or general income taxes by the inflation tax. Finally, in the last exercise, we calculate the global optimal tax mix given $G = 0.295544$ to obtain $(g^*, \tau_y^*, \tau_c^*) = (10.48\%, 0\%, 26.77\%)$. Notice that under the pre-existing tax rates given above, inflation, general income and consumption taxes account for about 24.9%, 66.7% and 8.4%, respectively, of the total government revenue. In the optimal tax mix scheme, their revenue shares become 49.7%, 0% and 50.3%, respectively. To understand the global optimal tax mix result, we note that the direct negative effect of the general income tax is quantitatively too large to be compensated by the positive employment and (most plausibly) leisure effects. As a consequence, it is optimal for income to be tax-exempted. Thus, when it is optimal to fully eliminate the distortionary general income tax, tax burdens must fall on the inflation and the consumption taxes. However, the welfare-cost trade-off between the inflation tax and the consumption tax is never strong enough to lead to an optimal tax scheme with one tax being fully replaced by another.

To the end, we compare our findings with those established by Chari, Christiano and Kehoe (1996) and Chari and Kehoe (1999). Their papers consider both cash and credit consumption goods, where money can be regarded as an intermediate good that produces the final consumption good. By the intermediate good principle (cf. Diamond and Mirrlees 1971), the inflation tax is distortionary. Moreover, the uniform taxation principle (cf. Atkinson and Stiglitz 1972) requires that both cash and credit goods be taxed equally. Thus, an inflation tax levied only on the cash good is distortionary. To avoid both distortionary margins, the Friedman rule is optimal in a class of homothetic preferences that are separable in consumption and leisure. In our paper, money can also be regarded as an intermediate good serving to facilitate transactions in firm's wage payments and household's consumption purchases. However, in the presence of labor-market frictions, our model exhibits trade-offs in vacancy creation and production use of labor, as well as trade-offs in labor, job search and leisure. Such trade-offs yield a new channel through which steady inflation raises employment and leisure in the long run, thus invalidating the Friedman rule in our calibrated economy.

5 Concluding Remarks

By constructing a monetary growth model where cash is required for wage payments and consumption purchases, we have shown that labor market frictions play an important role in creating new channels through which steady inflation influences the real activity in the long run. The key elements of labor market frictions considered in our benchmark framework include costly vacancy creation and job search as well as imperfect job matches. While there is a prototypical detrimental effect of money growth via labor-leisure trade-off, we have identified, with at least some moderate amount of money being injected to firms, there exists a positive real effect due to the encouragement of steady inflation to create new vacancies and to raise job matches.

Some valuable lessons from our calibration exercises are summarized as follows.

- When the economy exhibits relatively low inflation, positive real effects of money growth may arise in which higher steady inflation, via more vacancy creation and better job matches, raises employment and saves job search time. In this case, the optimal rate of inflation is positive, departing from the Friedman rule.

- When wages are determined by competitive profit conditions, the greater labor-market frictions, the larger the positive real effect of money growth and the optimal inflation rate.

- When bargaining inefficiency is removed under Hosios' rule in an economy with significant labor-market frictions, the benefit of money growth is significantly reduced. Under our calibrated economy with a job finding rate 20% lower than one in the U.S., the real effect of money growth is almost absent and the optimal inflation rate is close to zero.

- Even in a second-best world with pre-existing distortionary consumption and general income taxes, the Friedman rule still fails to hold under our benchmark parametrization with wages being determined by competitive profit conditions. Under our calibrated economy with a job finding rate 20% lower than one in the U.S., it is optimal to fully eliminate general income tax and to have positive inflation and consumption tax.

Our quantitative results are generally consistent with recent empirical studies investigating the long-run money-output relationship.

We have focused on providing a thorough characterization of the real effects of steady inflation via the labor-leisure-search trade-off in the presence of labor market frictions and have been abstracting any pecuniary costs associated with job search and vacancy creation. Our model may be generalized to include such costs. In doing so, one may mimic better the real world and conduct calibration analysis matching better with the observed rates of job turnover and unemployment. Another simplifying assumption is the utility function specification. Should the utility function be nonhomothetic (in consumption and search-intensity augmented effective leisure), the employment rate may affect the marginal rate of substitution and the marginal product of labor differently and hence steady inflation may affect the ratio of household to firm money holding, depending on the severity of labor market frictions. This extension may thus provide a plausible explanation for the sharp movements in the ratio of money holding over the past few decades in the U.S. Finally, our framework is ready for a comprehensive study of monetary transmission over the business cycle. In particular, one may set up the stochastic processes for the technological factor (A) and the monetary growth rate (g) and then characterize monetary transmission by log-linearizing the system governing the equilibrium dynamics. Search frictions may be viewed as "real rigidities" that may permit a better fit with the data in impulse responses of output, employment and factor returns with respect to monetary shocks.

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Figure 1: Impact of Money Growth on Wage and Wage Discount

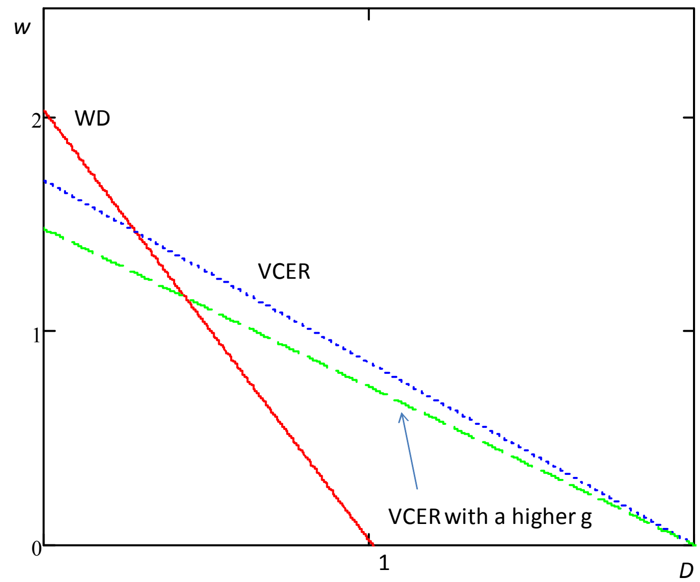


Figure 2: An Increase in the Growth Rate of Money

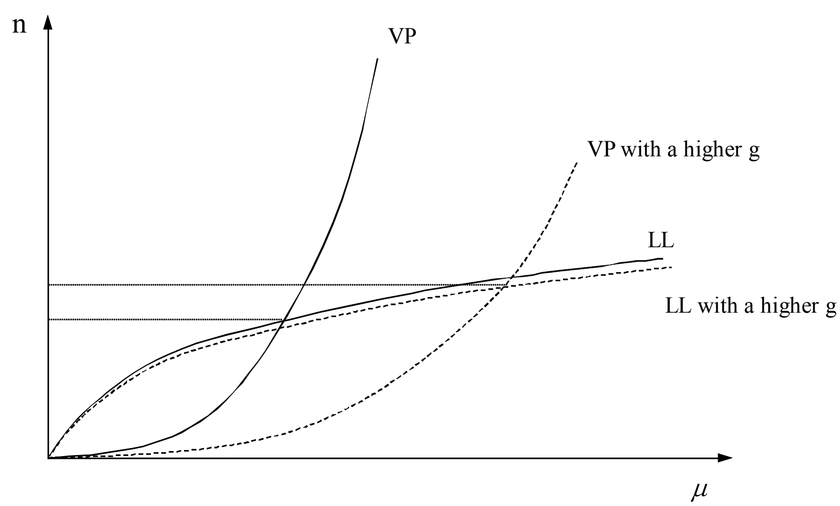


Figure 3: Optimal Growth Rate of Money

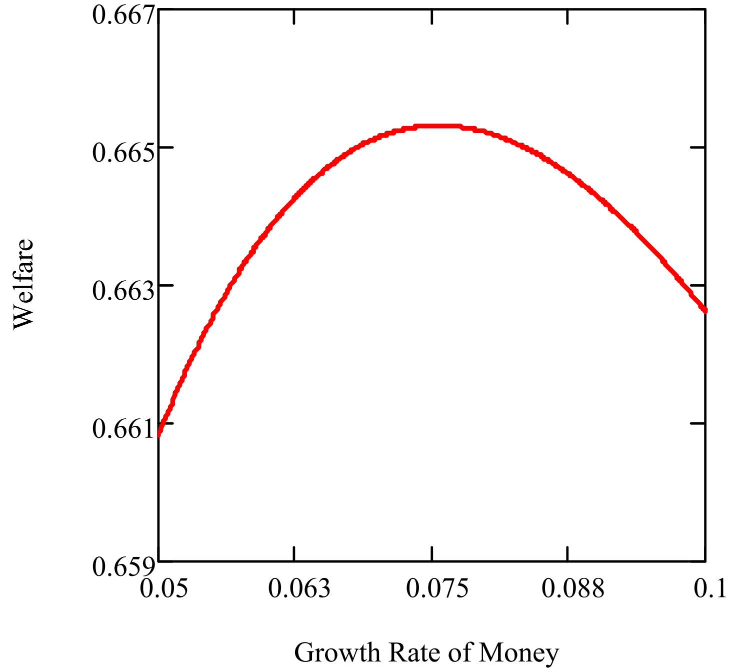
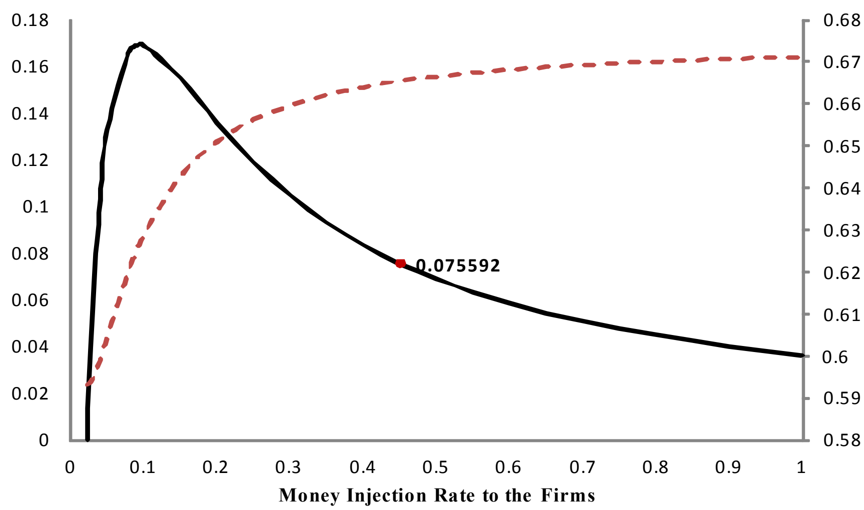


Figure 4. Optimal Money Growth (solid, left scale)
and the Resulting Maximum Welfare (dash, right scale)
under Different Money Injection Rules



Appendix

(Not Intended for Publication)

In this appendix, we derive the optimizing conditions for the representative firm and the representative household, the second-order condition for vacancy creation, three important relationships presented in the main text, the wage and the equilibrium solution under Hosios' rule, as well as the conditions governing the second-best tax incidence analysis.

1. *Optimizing Conditions for a Representative Firm*

Denote the Lagrangian multiplier associated with the firm's CIA constraint as λ^F. By substituting the evolution equations into the household's next-period value function, the representative firm's optimization problem can be written in Bellman equation form as follows:

$$\begin{aligned}\Gamma(n, m^F) \quad = \quad & \max_{v,\, k,\, z^F} \left[Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{1-\alpha} - wn - rk - z^F + x^F\right] \\ & + \frac{1}{1+r}\Gamma\left((1-\psi)n + \eta v, \frac{m^F}{1+\pi} + z^F\right) + \lambda^F\left(\frac{m^F}{1+\pi} - wn\right).\end{aligned}$$

Note that the subscript t is no longer necessary and is dropped. The firm decides on (i) how many vacancies to create (v); (ii) how much capital to rent (k), and (iii) how much real money balance to acquire (z^F). Let $\mathcal{F}$ denote the firm's vector of state variables this period, namely, $\mathcal{F} = (n, m^F)$. Let $\mathcal{F}'$ denote the same vector next period. The first-order conditions are:

$$\frac{\eta}{1+r}\Gamma_1\left(\mathcal{F}'\right) = (1-\alpha)\Phi'(v)Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{-\alpha}, \tag{58}$$

$$\alpha Ak^{\alpha-1}\left(n-\Phi\left(v\right)\right)^{1-\alpha} = r, \tag{59}$$

$$\Gamma_2(\mathcal{F}') = 1+r. \tag{60}$$

Equation (58) states that for optimality, the benefit from creating additional vacancy in order to make new hires should equal the cost in terms of staff-time necessary to make these vacancies available to the searching workers. In Section 3 of this Appendix, we derive the second-order condition for vacancy creation, which is then verified in the numerical exercises. Equation (59) is the standard capital demand equation. Equation (60) indicates that the return from acquiring additional real money balance this period, which comes in the form of a more relaxed CIA constraint next period, should equal the opportunity cost of holding money.

In addition, the optimality also requires the Benveniste-Scheinkman conditions, which can be simplified as follows by making use of the first order conditions above:

$$\Gamma_1(n, m^F) = \left[1 + \frac{\Phi'(v)\left(1-\psi\right)}{\eta}\right](1-\alpha)Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{-\alpha} - (1+\lambda^F)w, \tag{61}$$

$$\Gamma_2(n, m^F) = \left(\frac{1}{1+r}\right)\left(\frac{1}{1+\pi}\right)\Gamma_2\left((1-\psi)n + \eta v, \frac{m^F}{1+\pi} + z^F\right) + \frac{\lambda^F}{1+\pi},$$

which, when using (60), gives rise to:

$$\Gamma_2(n, m^F) = \frac{1+\lambda^F}{1+\pi}. \tag{62}$$

2. *Optimizing Conditions for a Representative Household*

Denote the Lagrangian multiplier associated with the household's CIA constraint λ^H. By substituting the evolution equations into the household's next-period value function, the representative household's optimization problem can be written in Bellman equation form as follows:

$$\Omega(k, n, m^H) = \max_{c,\, s,\, z^H} U\left(c, (1-s)(1-n)\right)$$
$$+\frac{1}{1+\rho}\Omega\left((1\text{-}\delta\text{+}r)k + wn - c - z^H + x^H, (1\text{-}\psi)n + \mu s(1\text{-}n), \frac{m^H}{1+\pi}+z^H\right) + \lambda^H\left(\frac{m^H}{1+\pi} - c\right).$$

Let $\mathcal{H}$ denote the vector of state variables this period, namely, $\mathcal{H} = (k, n, m^H)$. Let $\mathcal{H}'$ denote the triplets next period. The first-order conditions are:

$$U_1\left(c, (1-s)(1-n)\right) = \frac{1}{1+\rho}\Omega_1(\mathcal{H}') + \lambda^H, \tag{63}$$

$$\frac{\mu}{1+\rho}\Omega_2(\mathcal{H}') = U_2\left(c, (1-s)(1-n)\right), \tag{64}$$

$$\Omega_3(\mathcal{H}') = \Omega_1(\mathcal{H}'). \tag{65}$$

In equation (63), the appearance of λ^H captures the additional shadow cost of increasing consumption due to the presence of the CIA constraint. Equation (64) states that the employment gain next period from a marginal increase in search intensity this period equals the disutility from the corresponding reduction in leisure. Equation (65) equates the benefit of acquiring an additional real money balance that relaxes the CIA constraint for the next period, to the opportunity cost of foregone capital accumulation.

The Benveniste-Scheinkman conditions are given as follows:

$$\Omega_1(\mathcal{H}) = \frac{(1-\delta+r)}{1+\rho}\Omega_1(\mathcal{H}'), \tag{66}$$

$$\Omega_2(\mathcal{H}) = -(1-s)U_2\left(c, (1-s)(1-n)\right) + \frac{w}{1+\rho}\Omega_1(\mathcal{H}') + \frac{(1-\psi-\mu s)}{1+\rho}\Omega_2(\mathcal{H}'), \tag{67}$$

$$\Omega_3(\mathcal{H}) = \frac{1}{(1+\rho)\,(1+\pi)}\Omega_3(\mathcal{H}') + \frac{\lambda^H}{1+\pi}. \tag{68}$$

3. *The Second-order condition for vacancy creation:*

From the optimization problem is given by,

$$\begin{aligned}\Gamma(n,m^F) &= \max_{v,\,k,\,z^F}\left[Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{1-\alpha}-wn-rk-z^F+x^F\right]\\&\quad+\frac{1}{1+r}\Gamma\left((1-\psi)n+\eta v,\frac{m^F}{1+\pi}+z^F\right)\\&\quad+\lambda^F\left(\frac{m^F}{1+\pi}-wn\right).\end{aligned}$$

The first-order condition with respect to v is:

$$-(1-\alpha)Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{-\alpha}\Phi_v+\frac{\eta}{1+r}\Gamma_1\left(\mathcal{F}'\right)=0,$$

while the second-order condition requires:

$$\begin{aligned}&-\alpha(1-\alpha)Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{-\alpha-1}\left(\Phi_v\right)^2-(1-\alpha)Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{-\alpha}\Phi_{vv}+\frac{\eta^2}{1+r}\Gamma_{11}\left(\mathcal{F}'\right)\\=\;&-\frac{\eta}{1+r}\Gamma_1\left(\mathcal{F}'\right)\frac{\alpha\varepsilon}{v}\left[\frac{\Phi\left(v\right)}{n-\Phi\left(v\right)}-\frac{(1-\varepsilon)}{\alpha\varepsilon}\right]+\frac{\eta^2}{1+r}\Gamma_{11}\left(\mathcal{F}'\right)<0,\end{aligned}$$

where $\Phi_{vv}=-(1-\varepsilon)\frac{\Phi_v}{v}$.

Note that defining the vacancy cost ratio as $VCR\equiv\frac{\Phi(v)}{n}$, we then have:

$$\begin{aligned}\Gamma_1(n,m^F) &= (1-\alpha)Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{-\alpha}-w+\frac{1-\psi}{1+r}\Gamma_1(\mathcal{F}')-\lambda^F w\\&=\left\{\frac{1+\frac{(1-\psi)}{\psi}\varepsilon\cdot VCR}{1-VCR}-(1+g)(1+\rho+\delta)\right\}\frac{1-\alpha}{\alpha}(\rho+\delta)\frac{k}{n},\end{aligned}$$

$$\begin{aligned}\Gamma_{11}(n,m^F) &= -\alpha(1-\alpha)Ak^{\alpha}\left(n-\Phi\left(v\right)\right)^{-\alpha-1}\\&=-(1-\alpha)\frac{(\rho+\delta)}{\left(n-\Phi\left(v\right)\right)(1-VCR)}\frac{k}{n}.\end{aligned}$$

Thus, the second-order condition requires:

$$\begin{aligned}&-\frac{\eta}{1+r}\Gamma_1\left(\mathcal{F}'\right)\frac{\alpha\varepsilon}{v}\left[\frac{\Phi\left(v\right)}{n-\Phi\left(v\right)}-\frac{(1-\varepsilon)}{\alpha\varepsilon}\right]+\frac{\eta^2}{1+r}\Gamma_{11}\left(\mathcal{F}'\right)\\=\;&-\frac{\eta}{1+r}\left\{\frac{1+\frac{(1-\psi)}{\psi}\varepsilon\cdot VCR}{1-VCR}-(1+g)(1+\rho+\delta)\right\}\frac{(1-\alpha)(\rho+\delta)}{\alpha}\frac{k}{n}\frac{\alpha\varepsilon}{v}\left[\frac{\Phi\left(v\right)}{n-\Phi\left(v\right)}-\frac{1-\varepsilon}{\alpha\varepsilon}\right]\\&-\frac{\eta^2}{1+r}\frac{(1-\alpha)(\rho+\delta)}{\left(n-\Phi\left(v\right)\right)(1-VCR)}\frac{k}{n}<0,\end{aligned}$$

which we verify in our numerical exercises.

4. *The Vacancy-Production Trade-off, the Relative Money Injection to Firms, the Zero Profit Condition and the Labor-leisure Trade-off Equations:*

Substituting (60) and $\pi = g$ into (62), one obtains: $\lambda^F = r + (1+r)g$. This expression can then be used in (61) and the resulting equation can be combined with (58) to yield:

$$\begin{aligned} & \frac{\eta}{1+r}\left(\left[1+\frac{\Phi'(v)\,(1-\psi)}{\eta}\right](1-\alpha)Ak^{\alpha}\,(n-\Phi\,(v))^{-\alpha}-(1+r)(1+g)w\right) \\ = \;& (1-\alpha)\Phi'(v)Ak^{\alpha}\,(n-\Phi\,(v))^{-\alpha}\,, \end{aligned}$$

which can be simplified to arrive at the vacancy creation-production trade-off relationship (35) after replacing η by $\psi n/v$.

From (26), (28) and (29), we obtain:

$$\begin{aligned} \left(1-\zeta^F\right)z^F &= \left(1-\zeta^F\right)\frac{g}{1+g}m^F = \left(1-\zeta^F\right)gwn, \\ \zeta^F &= \bar{\zeta}^F\frac{m}{m^F} = \bar{\zeta}^F\left(1+\frac{c}{wn}\right). \end{aligned}$$

Substituting (34) into the latter expression, we can derive (36). With this, (37) follows immediately.

Next, from (65) and (68), we have:

$$\lambda^H = (1+g)\left[1-\frac{1}{(1+\rho)(1+g)}\right]\Omega_1(\mathcal{H}),$$

which can be substituted into equation (63) to arrive at:

$$U_1 = (1+g)\Omega_1(\mathcal{H}).$$

Combining this latter equation with (64) and (67), one obtains:

$$\left(1-\frac{1-\psi-\mu s}{1+\rho}\right)\frac{1+\rho}{\mu}U_2 = -(1-s)U_2 + \frac{w}{1+\rho}\frac{U_1}{1+g}$$

which can be simplified to produce the labor-leisure trade-off equation (38).

5. *The Wage Discount Function:*

Equations (39) and (41) can be combined to yield,

$$1-\frac{\varepsilon\,(\psi+\rho+\delta)}{\psi}\left\{1-\left[1+\left(1-\zeta^F\,(g)\right)g\right](1-D)\right\} = (1+\rho+\delta)(1+g)(1-D)$$

or, manipulating,

$$D = 1-\frac{1-\frac{\varepsilon(\psi+\rho+\delta)}{\psi}}{(1+\rho+\delta)(1+g)-\frac{\varepsilon(\psi+\rho+\delta)}{\psi}\left[1+\left(1-\zeta^F\,(g)\right)g\right]},$$

which can be simplified to generate the wage discount function (42).

6. *The VP Locus:*

From (35) and (39), we have:

$$\left[1 - \frac{\Phi'(v)\,(\psi+\rho+\delta)}{\eta}\right] MPN = (1+\rho+\delta)(1+g)(1-D)MPN,$$

or, by eliminating MPN and using (21), (22) and (40),

$$\left[1 - \frac{\varepsilon\,(\psi+\rho+\delta)\,\Phi(v)}{\eta v}\right] = (1+\rho+\delta)(1+g)\frac{1-\frac{\psi\Phi(v)}{\eta v}}{\left[1+\left(1-\zeta^F(g)\right)g\right]}$$

or,

$$\frac{(1+\rho+\delta)(1+g) - \left[1+\left(1-\zeta^F(g)\right)g\right]}{\psi(1+\rho+\delta)(1+g) - \varepsilon\,(\psi+\rho+\delta)\left[1+\left(1-\zeta^F(g)\right)g\right]} = \phi\,(\psi n)^{\varepsilon-1}\left(B^{-\frac{1}{1-\beta}}\mu^{\frac{\beta}{1-\beta}}\right)^{\varepsilon}$$

which can be simplified to yield the VP locus (43).

Non-negativity of employment requires that

$$\psi + \frac{\psi - \varepsilon\,(\psi+\rho+\delta)}{(\rho+\delta)+(1+\rho+\delta)g} > 0, \text{ namely, } \frac{\psi(1+\rho+\delta)(1+g)}{(\psi+\rho+\delta)} > \varepsilon.$$

7. *The LL Locus:*

From labor-leisure decision (38),

$$\begin{aligned} w &= (1+g)\,(1+\rho)\frac{(\rho+\psi+\mu)}{\mu}\frac{U_2}{U_1} \\ &= (1+g)\,(1+\rho)\frac{(\rho+\psi+\mu)}{\mu}\left(\frac{\sigma}{1-\sigma}\right)\frac{c}{(1-s)(1-n)} \end{aligned}$$

From (37), (31), (26), (28),

$$\frac{1-\alpha}{\alpha}(\rho+\delta)k = wn + \left(1-\zeta^F(g)\right)z^F = \left[1+\left(1-\zeta^F(g)\right)g\right]wn$$

or,

$$w = \frac{(\rho+\delta)\left(\frac{1-\alpha}{\alpha}\right)}{1+\left(1-\zeta^F(g)\right)g}\left(\frac{k}{n}\right)$$

Equating the two expressions to eliminate w yields,

$$\begin{aligned} \frac{(\rho+\delta)\left(\frac{1-\alpha}{\alpha}\right)}{1+\left(1-\zeta^F(g)\right)g}\left(\frac{k}{n}\right) &= \left(\frac{\sigma}{1-\sigma}\right)(1+g)\,(1+\rho)\left(\frac{\rho+\psi+\mu}{\mu}\right)\frac{c}{(1-s)(1-n)} \\ &= \left(\frac{\sigma}{1-\sigma}\right)(1+g)\,(1+\rho)\left(\frac{\rho+\psi+\mu}{\mu}\right)\frac{\rho+(1-\alpha)\delta}{\alpha(1/n-(1+\frac{\psi}{\mu}))}\left(\frac{k}{n}\right) \end{aligned}$$

or,

$$\frac{1}{n} = 1 + \frac{\psi}{\mu} + \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{1}{1-\alpha}\right)\frac{\rho+(1-\alpha)\delta}{\rho+\delta}\left[1+\left(1-\zeta^F(g)\right)g\right](1+g)\,(1+\rho)\left(\frac{\rho+\psi+\mu}{\mu}\right)$$

which generates the LL locus (44).

8. *Intertemporal and Labor-Leisure Trade-offs under Generalized CIA Constraints:*

Under generalized CIA constraints, some of household's optimizing conditions need be modified as follows:

$$U_1\left(c,(1-s)(1-n)\right) = \frac{1}{1+\rho}\Omega_1(\mathcal{H}') + \lambda^H\left(1-\theta^H\right)$$

$$\Omega_3(\mathcal{H}') + \lambda^H\theta^H = \Omega_1(\mathcal{H}')$$

$$\Omega_1(\mathcal{H}) = \frac{(1-\delta+r)}{1+\rho}\Omega_1(\mathcal{H}') - \lambda^H\theta^H r$$

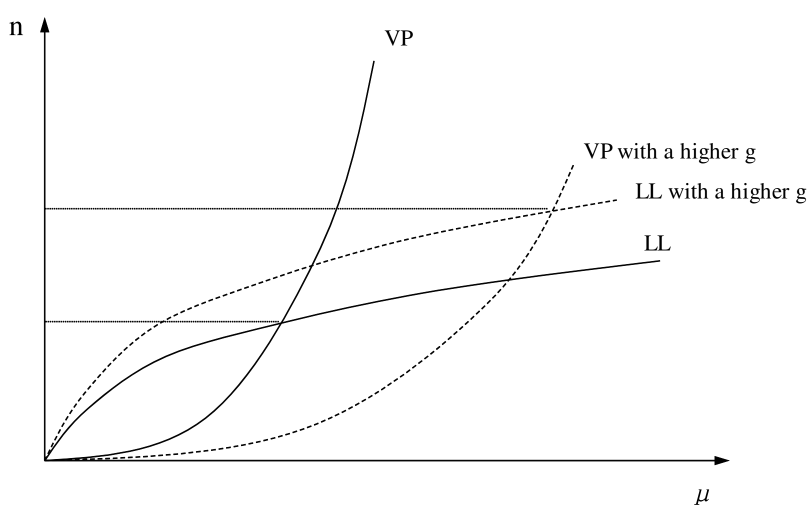
$$\Omega_2(\mathcal{H}) = -(1-s)U_2\left(c,(1-s)(1-n)\right) + \frac{w}{1+\rho}\Omega_1(\mathcal{H}') + \frac{(1-\psi-\mu s)}{1+\rho}\Omega_2(\mathcal{H}') - \lambda^H\theta^H w$$

Using the first three equalities together with (64) to express λ^H, $\Omega_1(\mathcal{H}')$, $\Omega_2(\mathcal{H}')$, and $\Omega_3(\mathcal{H}')$ in terms of U_1 and U_2, and substituting these results into the last two equations, we obtain modified intertemporal and labor-leisure trade-offs, (52) and (53).

9. *Diagrammatic analysis when money injections are distributed to firms:*

When money injections are distributed to firms only, the VP and LL loci are depicted in the following.

Figure 5: An Increase in the Growth Rate of Money
When Money Injections are Distributed to Firms



10. *Hosios' rule:*

Recall that the competitive wage rate under zero profit is given by $w = \left(1-\frac{\Phi}{n}\right)MPN$, where $MPN = (1-\alpha)A\left(\frac{k}{n-\Phi(v)}\right)^{\alpha}$. Denote the efficient matching wage rate under Hosios' rule as w^*. Since the flow cost of vacancy creation is $-dy/dv = \Phi_v \cdot MPN$, we can compute firm's flow profit per match as follows:

$$\begin{aligned}R &= \max_{k/n}\left\{\frac{y}{n}-r\frac{k}{n}-w^*\right\}\\ &= \left(1-\frac{\Phi}{n}\right)\cdot MPN-w^*\\ &= w-w^*.\end{aligned}$$

Firms' unmatched value (Π^U) and matched value (Π^M) accrued from a successful bargain with their employees can be specified as:

$$\Pi^U=-\Phi_v\cdot MPN+\frac{1}{1+r}\left[\eta\Pi^{M'}+(1-\eta)\Pi^{U'}\right], \tag{69}$$

$$\Pi^M=\left(1-\frac{\Phi}{n}\right)\cdot MPN-w^*+\frac{1}{1+r}\left[(1-\psi)\Pi^{M'}+\psi\Pi^{U'}\right]. \tag{70}$$

In the absence of firm's entry cost, we have: $\Pi^{U'}=\Pi^U=0$. Thus, applying the functional form of $\Phi(v)$ and (21), we use (69) and (70) to derive:

$$\Pi^{M'}=\frac{1+r}{\eta}\Phi_v\cdot MPN, \tag{71}$$

$$\begin{aligned}S^F &= \Pi^M-\Pi^U\\ &= \left[\left(1-\frac{\Phi}{n}\right)+\frac{1-\psi}{\psi}\varepsilon\frac{\Phi}{n}\right]\cdot MPN-w^*.\end{aligned} \tag{72}$$

Combining (63) and (68) and making use of (65), we can eliminate λ^H to obtain $\Omega_1(\mathcal{H}')=\Omega_3(\mathcal{H}')=\frac{1}{1+\pi}U_1$. In the steady state, we can substitute the previous expression into (67) to derive:

$$\Omega_2=\frac{1}{(\rho+\psi+\mu s)}\left[-(1+\rho)(1-s)U_2+\frac{w^*}{1+\pi}U_1\right].$$

Equations (27) and (30) yield:

$$c=\rho k+w^*n. \tag{73}$$

From the competitive wage rate equation mentioned above and (31), we have:

$$\frac{k}{n}=\frac{\alpha}{(1-\alpha)(\rho+\delta)}w. \tag{74}$$

Now, by applying (21), (27) and $\pi=g$, and making use of the two equations above, the matching surplus accrued to a household from a successful match (in unit of goods) can be derived as:

$$\begin{aligned}S^H &= \frac{\Omega_2}{U_1}\\ &= \frac{1-n}{\psi+\rho(1-n)}\left[-(1+\rho)\frac{\sigma}{1-\sigma}\frac{c}{1-n}+\frac{w^*}{1+g}\right]\\ &= \frac{1}{\psi+\rho(1-n)}\left\{-(1+\rho)\frac{\sigma}{1-\sigma}\rho k+\left[\frac{(1-n)-(1+\rho)(1+g)\frac{\sigma n}{1-\sigma}}{1+g}\right]w^*\right\}.\end{aligned} \tag{75}$$

In a Nash bargain, the wage is solved by maximizing the joint surplus, taking as given the competitive rental rate, matching probabilities and state variables. Hosios' rule implies that to reach the bargaining frontier, the bargaining shares have to be the same as the powers in the matching function. Thus, the bargaining problem is given by,

$$\max_{w^*} \left(S^H\right)^{\beta} \left(S^F\right)^{1-\beta}.$$

Noting that $v = \frac{\psi n}{\eta}$ is taken as given (as do Φ, k, MPN and $\overline{w}$) and thus both S^F and S^H are linear in w^*. As a consequence, the first-order condition to the Nash bargain problem exhibits the conventional form:

$$\beta S^F = (1-\beta) \frac{(1+g)\left[\psi + \rho(1-n)\right]}{(1-n) - (1+\rho)(1+g)\frac{\sigma n}{1-\sigma}} S^H, \tag{76}$$

which, together with (72) and (75), implies:

$$\beta \left\{ \left(1 + \frac{1-\psi}{\psi} \frac{\varepsilon \Phi}{n - \Phi}\right) w - w^* \right\} = (1-\beta) \left\{ -\frac{(1+\rho)(1+g)\frac{\sigma n}{1-\sigma}\frac{\rho k}{n}}{(1-n) - (1+\rho)(1+g)\frac{\sigma n}{1-\sigma}} + w^* \right\}.$$

Thus, the wage can be solved as in (54).

Next, we will reduce all the steady state conditions into a 2×2 system of equations in (μ, n). From (38), (21) and (73), we have:

$$\begin{aligned} w^* &= (1+\rho)(1+g)\frac{\rho + \psi + \mu}{\mu} \frac{U_2}{U_1} \\ &= (1+\rho)(1+g)\left(1 + \frac{\rho+\psi}{\mu}\right) \frac{\sigma}{1-\sigma} \frac{\rho \frac{k}{n} + w^*}{\frac{1-n}{n} - \frac{\psi}{\mu}}, \end{aligned}$$

or, solving w^* leads to:

$$\Delta = \frac{\frac{\alpha\rho}{(1-\alpha)(\rho+\delta)}}{\frac{1-\sigma}{\sigma} \frac{\left(\frac{1-n}{n} - \frac{\psi}{\mu}\right)}{(1+\rho)(1+g)\left(1 + \frac{\rho+\psi}{\mu}\right)} - 1}. \tag{77}$$

From (22) and (23), $\eta = B^{1/(1-\beta)} \mu^{-\beta/(1-\beta)}$ and $v = \frac{\psi n}{\eta}$, so we have:

$$\Phi = \phi \left[\psi n \left(\frac{\mu^{\beta}}{B}\right)^{\frac{1}{1-\beta}} \right]^{\varepsilon},$$

which is an increasing function of n and μ. Also, one can get the shares of employees in creating/maintaining vacancies and in production:

$$\frac{\Phi}{n} = \frac{\phi}{n^{1-\varepsilon}} \left[\psi \left(\frac{\mu^{\beta}}{B}\right)^{\frac{1}{1-\beta}} \right]^{\varepsilon} \equiv \Lambda(\mu, n); \quad \frac{n - \Phi}{n} = 1 - \Lambda(\mu, n),$$

where Λ is increasing in μ and decreasing in n. Substituting these expressions into (55) yields:

$$\Delta = \beta \left(1 + \frac{1-\psi}{\psi} \frac{\varepsilon}{\frac{1}{\Lambda(\mu,n)} - 1}\right) + (1-\beta) \left\{ \frac{\frac{\alpha\rho}{(1-\alpha)(\rho+\delta)}}{\frac{(1-n)(1-\sigma)}{n(1+\rho)(1+g)\sigma} - 1} \right\}. \tag{78}$$

Comparing (78) with (77) , we obtain (56).

We now turn to another fundamental relationship, using the vacancy creation condition (35), which can be rewritten as:

$$\Delta = \frac{1}{(1+\rho+\delta)(1+g)} \frac{1 - \frac{\varepsilon(\psi+\rho+\delta)}{\psi}\Lambda(\mu, n)}{1 - \Lambda(\mu, n)}, \tag{79}$$

which is increasing in n and μ. This expression can be compared with (78) to yield (57).

11. *Sensitivity Analysis:*

While our pre-set parameters in the calibration exercises are all justified, some of the calibration criteria and some of the calibrated parameter values may be argued questionable. We therefore perform a sensitivity analysis to check the robustness of our results. In particular, we consider the following alternatives:

- Since our calibration of the labor market follows the recent work by Shimer (2005), we for consistency set, in the benchmark case, the matching technology parameter $\beta = 0.72$ as in Shimer, which is arguably high as compared to Blanchard and Diamond (1990) with $\beta = 0.40$ and Hall (2008) with $\beta = 0.54$. In the first sensitivity analysis, we recalibrate β to the lowest value 0.40.

- We calibrated B based on an assumed vacancy-unemployed search worker ratio of one as in Shimer (2005). We now explore the alternative by allowing B to be 10% below or above its benchmark value.

- We also allow four other crucial parameters to be 10% below or above its benchmark value to check how robust our main findings are.

The sensitivity analysis results are reported in Table 2 below. Our results deliver two important messages. First, under the zero profit setup with an inefficient wage bargain, the Friedman rule is never valid in a wide set of parametrization. The more frictional the labor market is (lower matching efficacy B, higher job separation ψ, or more costly vacancy creation ϕ), the greater the optimal inflation rate will be. Second, the optimal inflation rate is always lower when the wage bargain is efficient, satisfying Hosios' rule. In some cases, the Friedman rule may hold with an efficient wage bargain – it is the case when the matching elasticity of unemployed workers is low (low β), the matching efficacy is low (low B), the job separation rate is low (low ψ), the vacancy creation is more costly (high ϕ), the scale economies of vacancy creation is low (high ε), and when welfare is more sensitive to leisure (high σ). It is of particular interest to note that when $\beta = 0.40$, there are in fact two nondegenerate equilibria. The one that is not reported in Table 2 involves an unrealistically low employment rate (about 17%) and implausibly high unemployment duration (about 7.5 years).[7]

Table 2. Sensitivity Analysis on Optimal Inflation			
		g^* under zero profit	g^* under Hosios' rule
Benchmark		0.075592	0.002149
$\beta = 0.40$		0.069569	0
B	up 10%	0.066394	0.039491
	down 10%	0.086607	0
ψ	up 10%	0.087162	0.039022
	down 10%	0.063729	0
ϕ	up 10%	0.081890	0
	down 10%	0.068979	0.024613
ε	up 10%	0.053374	0
	down 10%	0.104442	0.046782
σ	up 10%	0.077166	0
	down 10%	0.074117	0.010785

12. *Second-Best Tax Incidence Analysis:*

In the second-best tax incidence model, three household's optimizing conditions, namely, (63), (66) and (67), must be modified:

$$U_1\left(c,(1-s)(1-n)\right) = (1+\tau_c)\left[\frac{1}{1+\rho}\Omega_1(\mathcal{H}') + \lambda^H\right],$$

$$\Omega_1(\mathcal{H}) = \frac{1-\delta+(1-\tau_y)\,r}{1+\rho}\Omega_1(\mathcal{H}'),$$

$$\Omega_2(\mathcal{H}) = -(1-s)U_2\left(c,(1-s)(1-n)\right) + \frac{(1-\tau_y)\,w}{1+\rho}\Omega_1(\mathcal{H}') + \frac{(1-\psi-\mu s)}{1+\rho}\Omega_2(\mathcal{H}').$$

In the steady state, (27), (29), (30), (31), and (38) become:

$$\begin{aligned}
(1+\tau_c)\,(1+g)c+\delta k &= (1-\tau_y)\,(rk+wn)-gwn,\\
m^H &= (1+\tau_c)\,(1+g)c,\\
r &= \frac{\rho+\delta}{1-\tau_y},\\
\alpha A k^{\alpha-1}\left(n-\Phi\left(v\right)\right)^{1-\alpha} &= \frac{\rho+\delta}{1-\tau_y},\\
(\rho+\psi+\mu)\,U_2\left(c,(1-s)(1-n)\right) &= \left(\frac{1-\tau_y}{1+\tau_c}\right)\frac{\mu w U_1\left(c,(1-s)(1-n)\right)}{(1+g)\,(1+\rho)}.
\end{aligned}$$

[7] When $\beta \to 0$, the VP locus becomes concave and stays above the LL locus so that a nondegenerate steady state no longer exists.

Repeating the same procedures as in appendix sections 3 and 4 above, we can derive the VP and LL loci. In addition to those steady-state expressions reported in the main text, the following steady-state relationships also differ from the benchmark ones:

$$
\begin{aligned}
\frac{c}{n} &= \frac{\left[(1-\tau_y) - (1-\alpha)\, g\right] \frac{\rho+\delta}{\alpha(1-\tau_y)} - \delta}{(1+\tau_c)\,(1+g)} \frac{k}{n}, \\
w &= \frac{1-\alpha}{\alpha} \frac{\rho+\delta}{1-\tau_y} \frac{k}{n}, \\
\frac{m}{n} &= (1+\pi) \left[(1+\tau_c)\, \frac{c}{n} + w\right] = \left[\frac{(1+\frac{1-\alpha}{1-\tau_y})(\rho+\delta)}{\alpha} - \delta\right] \frac{k}{n}, \\
\frac{y}{n} &= \frac{\rho+\delta}{\alpha\,(1-\tau_y)} \frac{k}{n}, \\
y &= A^{1/(1-\alpha)} \left[\frac{\rho+\delta}{\alpha\,(1-\tau_y)}\right]^{-\alpha/(1-\alpha)} (1-D)\, n.
\end{aligned}
$$