Endogenously Proportional Bargaining Solutions

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Abstract. This paper introduces a class of endogenously proportional bargaining solutions. These solutions are independent of the class of Directional solutions, which Chun and Thomson (1990a) proposed to generalize (exogenously) proportional solutions of Kalai (1977). Endogenously proportional solutions relative to individual i are characterized by *weak Pareto optimality* and *continuity* together with two new axioms that depend on the pairwise total payoff asymmetry of the bargaining problem with respect to each pair involving individual i. Each of these solutions satisfies the basic *symmetry* axiom and also a stronger axiom called *total payoff symmetry*.

Keywords: Cooperative bargaining; proportional solutions; symmetry

JEL Classification Numbers: C71; C78

1 Introduction

In this paper, we introduce a class of endogenously proportional solutions to Nash's (1950) bargaining problem, which is a subset of the n-dimensional euclidean space representing the utility alternatives available to a society involving n individuals. Each

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endogenously proportional solution relative to individual i associates a vector of proportionality to the given bargaining problem. This vector is identical for any two distinct problems that have the same pairwise total payoff asymmetry with respect to each pair involving individual i.

While the proposed class of endogenous solutions is new, exogenously proportional solutions are already known. The first - and most well - known member of this class is the Egalitarian solution, recommended by Rawls (1971). This solution chooses in each bargaining problem the highest utility point with equal coordinates. Characterization of the Egalitarian solution was offered by Kalai (1977), who generalized this solution to a class of exogenously proportional solutions.[1] In this class, given a positive n-tuple p, the corresponding solution selects in each bargaining problem the highest utility point proportional to p. Although the Egalitarian solution has been well studied, other exogenously proportional solutions have received less attention. Among a few studies, Roth (1979) extended Kalai's (1977) generalization to bargaining problems where utilities are not restricted to be freely disposable. Peters (1986) offered alternative characterizations of exogenously proportional solutions, focusing on 'simultaneity of issues and additivity' in bargaining games. Chun and Thomson (1990a) further generalized exogenously proportional solutions to the Directional solutions, focusing on 'uncertain disagreement points' in bargaining games. Characterizations of the Directional solutions and, in particular, exogenously proportional solutions were proposed by Chun and Thomson (1990a, 1990b). Recently, Hougard and Tvede (2010) extended proportional solutions to bargaining games with nonconvex problems.

We show that the class of endogenously proportional solutions are independent of the class of Directional solutions. The proposed solutions relative to individual i

[1] Kalai (1977) simply calls these solutions *proportional solutions*, whereas we call them *exogenously proportional solutions* to highlight the distinction between Kalai's solutions and ours.

are characterized by weak Pareto optimality, continuity, and two new conditions that depend on the pairwise total payoff asymmetry of a given bargaining problem with respect to each pair involving individual i. Moreover, these solutions satisfy a stronger form of the basic *symmetry* axiom that we call *total payoff symmetry.*

The paper is organized as follows: Section 2 introduces the basic structures and Section 3 presents the results. Finally, Section 4 concludes.

2 Basic Structures

A 0-normalized $n-$person bargaining problem for a society of individuals $N = \{1, 2, \ldots, n\}$, where $n \geq 2$, is denoted by S, a non-empty subset of $\mathbb{R}^n_+$, representing von Neumann-Morgenstern utilities attainable through the cooperative actions of the individuals in N.[2] If the individuals fail to agree on any point in S, then each of them receives zero utility (for notational simplicity). Hence, the bargaining problems are 0-normalized. The bargaining problem (simply, problem) S satisfies the following two conditions:

(a) S is convex and compact, and there exists $x \in S$ such that $x > 0$.[3]

(b) S is comprehensive; i.e., if $x \in S$, $y \in \mathbb{R}^n_+$, and $x \geq y$ then $y \in S$ (implying that utility is freely disposable).

Let Σ^n_0 denote the set of all bargaining problems.

A problem S is said to be ***symmetric*** if for all one-to-one functions $\gamma : N \to N$, $S = \{y \in \mathbb{R}^n_+ \,|\, \exists\, x \in S \text{ such that } \; y^{\gamma(i)} = x^i \; \text{ for all } i\}$.

For a problem S, a point $x \in S$ is said to be ***weakly Pareto optimal*** if there exists no $y \in S$ such that $y > x$. Let $WPO(S)$ denote the set of weakly Pareto optimal

[2] $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n \,|\, x^i \geq 0 \text{ for all } \; i\}$ and $\mathbb{R}^n_{++} = \{x \in \mathbb{R}^n \,|\, x^i > 0 \text{ for all } \; i\}$.

[3] Given x and y in $\mathbb{R}^n_+$, $x \geq y$ means $x^i \geq y^i$ for all i and $x > y$ means $x^i > y^i$ for all i.

points in S.

We denote the ***total payoff*** of each $X \subset \mathbb{R}^n_+$ as $TP(X) = \int_{x \in X} dx$. Note that $TP(\lambda X) = \lambda^n TP(X)$ for all $\lambda > 0$.

For each problem S and distinct individuals i and j, define the sets $S^{i,j}_{L,\beta} = \{y \in S \,|\, \beta y^i < y^j\}$ and $S^{i,j}_{R,\beta} = \{y \in S \,|\, \beta y^i > y^j\}$ for each $\beta > 0$.

For each problem S and distinct individuals i and j, also define $\alpha^{i,j}(S)$ such that

$$TP(S^{i,j}_{R,\alpha^{i,j}(S)}) = TP(S^{i,j}_{L,1}) \quad \text{if} \;\; TP(S^{i,j}_{L,1}) \leq TP(S^{i,j}_{R,1})$$

and

$$TP(S^{i,j}_{L,\alpha^{i,j}(S)}) = TP(S^{i,j}_{R,1}) \quad \text{if} \;\; TP(S^{i,j}_{L,1}) > TP(S^{i,j}_{R,1}).$$

Clearly, $\alpha^{i,j}(S)$ always exists and it is unique. In addition, $\alpha^{i,j}(S) = 1/\alpha^{j,i}(S)$. We will call $\alpha^{i,j}(S)$ (a measure of) ***pairwise total payoff asymmetry*** of S with respect to individuals i and j.

A problem S is said to satisfy ***pairwise total payoff symmetry*** with respect to individuals i and j if $\alpha^{i,j}(S) = 1$. Furthermore, S is said to satisfy ***total payoff symmetry*** if it satisfies pairwise total payoff symmetry with respect to each pair involving individual 1. Clearly, if S satisfies total payoff symmetry, then for each i it is true that S satisfies pairwise total payoff symmetry with respect to each pair involving individual i.

For each problem S and distinct individuals i and j, define the set

$$B^{i,j}(S) = \begin{cases} S^{i,j}_{R,1} \backslash S^{i,j}_{R,\alpha^{i,j}(S)} & \text{if} \;\; \alpha^{i,j}(S) \in (0,1), \\ S \backslash (S^{i,j}_{L,1} \cup S^{i,j}_{R,1}) & \text{if} \;\; \alpha^{i,j}(S) = 1, \\ S^{i,j}_{L,1} \backslash S^{i,j}_{L,\alpha^{i,j}(S)} & \text{if} \;\; \alpha^{i,j}(S) > 1. \end{cases}$$

Note that $B^{i,j}(S)$ is always nonempty. We call $B^{i,j}(S)$ the ***pairwise balancing subset*** of S with respect to individuals i and j, given the fact that it balances (the total

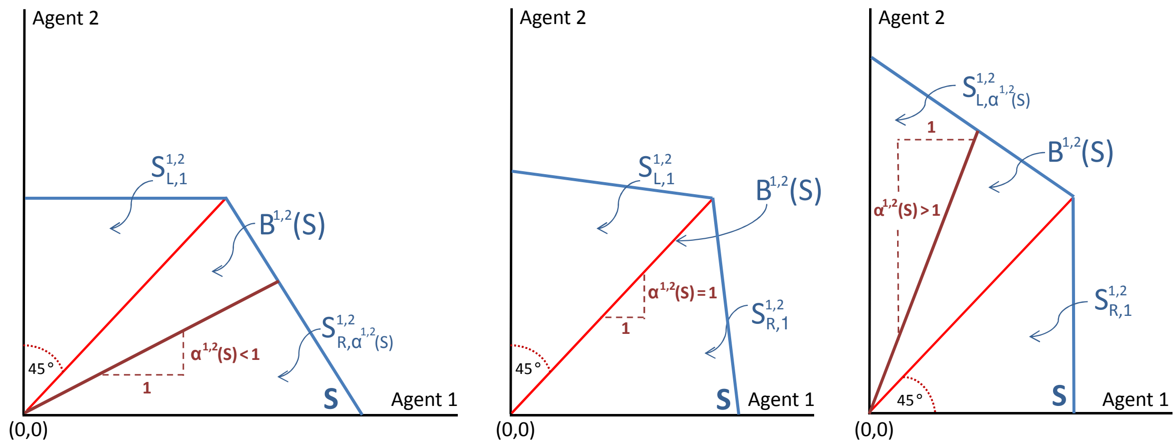

Figure 1. Basic Sets for $n = 2$.

(a) $\alpha^{1,2}(S) < 1$. (b) $\alpha^{1,2}(S) = 1$. (c) $\alpha^{1,2}(S) > 1$.

payoffs of) the sets $S^{i,j}_{R,\alpha^{i,j}(S)}$ and $S^{i,j}_{L,1}$ if $\alpha^{i,j}(S) \in (0,1]$ and the sets $S^{i,j}_{L,\alpha^{i,j}(S)}$ and $S^{i,j}_{R,1}$ if $\alpha^{i,j}(S) > 1$.

Finally, a solution is a function $\mu : \Sigma^n_0 \to R^n_+$ such that $\mu(S) \in S$ for each $S \in \Sigma^n_0$.

3 Results

For each problem S and individual i, define $\boldsymbol{\alpha}^i(S) = (\alpha^{i,j}(S))_{j\neq i}$. We say that a solution μ is ***endogenously proportional*** relative to individual i if there exists a continuous function $r^{i,j} : \mathbb{R}^{n-1}_{++} \to (0,1]$ for all $j \neq i$ such that $\mu(S) = \lambda(S)p(S)$ for all $S \in \Sigma^n_0$, where $p(S) \in \mathbb{R}^n_{++}$ is such that $p^j(S)/p^i(S) = 1 - r^{i,j}(\boldsymbol{\alpha}^i(S)) + r^{i,j}(\boldsymbol{\alpha}^i(S))\alpha^{i,j}(S)$ for all $j \neq i$ and $\lambda(S) = \max\{t \,|\, tp(S) \in B^{i,j}(S) \text{ for all } j \neq i\}$. We will denote by EP_i the class of solutions that are endogenously proportional relative to individual i.

Obviously, the class EP_i is not independent of i since the (weight) functions $r^{i,j}$ and $r^{j,i}$ are independent for all i, j such that $i \neq j$. Any solution μ in the class EP_i is called proportional since for each pair of problems S and T, we have $\mu(S)/\lambda(S) =$

$p(S) = p(T) = \mu(T)/\lambda(T)$ if there exists no individual j that the pairwise total payoff asymmetry with respect to individuals i and j is different for S and T. On the other hand, the proportionality of any solution μ in EP_i is endogenous since the vector of proportionality p is not invariant to changes in the problem S that affect $\boldsymbol{\alpha}^i(S)$, the profile involving pairwise total payoff asymmetries relative to individual i. Note also that the range of the functions $r^{i,j}$ in the definition of EP_i excludes the point 0, which would yield a vector of proportionality corresponding to the Egalitarian solution, an exogenously proportional solution.

The distinction between our solutions and Kalai's (1977) exogenously proportional solutions should be apparent, given the definition that a solution μ over Σ_0^n is ***exogenously proportional*** if there exists $p \in \mathbb{R}_{++}^n$ such that $\mu(S) = \lambda(S)p$ for each $S \in \Sigma_0^n$, where $\lambda(S) = \max\{t \,|\, tp \in S\}$.

Moreover, our solutions are also independent of the class of Directional solutions, to which Chun and Thomson (1990) further generalized exogenously proportional solutions. To see this, consider a class of n-person problems Σ^n, where each problem involves a bargaining set $S \subset \mathbb{R}^n$ satisfying the usual feasibility assumptions and a disagreement point d in S, where the individuals end up if they fail to agree on a point in S. If for a given solution μ there exists a continuous function p from the set of feasible bargaining sets to the n-dimensional simplex Δ^n such that for all $(S,d) \in \Sigma^n$, $\mu(S,d) = d + \lambda(S)p(S)$, where $\lambda(S) = \max\{t \,|\, d + tp(S) \in S\}$, then μ is called ***the Directional solution*** relative to p.

To make a comparison with the Directional solutions, we can simply extend the class of solutions EP_i for any i from Σ_0^n to Σ^n, by setting $\mu(S,d) = d + \lambda(S)p(S)$ for all μ in EP_i and replacing $\alpha^{i,j}(S)$ by $\alpha^{i,j}(IR(S-d,0))$ for all $j \neq i$, where $IR(S-d,0) = \{x \in S-d \,|\, x \geq 0\}$. Obviously, we can, without loss of generality, restrict $p^i(S)$, which is kept free for simplicity in our definition, such that $p(S) \in \Delta^n$ holds as in the definition of the

Directional solutions. But, there remains a significant difference between this class of solutions and ours: For any solution belonging to our class, the vector of proportionality, $p(S)$, on a given problem S is not independent of d over Σ^n. The reason is that over Σ^n, any solution μ in EP_i would require on any problem S a vector of proportionality $p(s)$ satisfying $p^j(S)/p^i(S) = 1 - r^{i,j}(\boldsymbol{\alpha}^i(IR(S-d,0))) + r^{i,j}(\boldsymbol{\alpha}^i(IR(S-d,0)))\alpha^{i,j}(IR(S-d,0))$ for all $j \neq i$, where $\boldsymbol{\alpha}^i(IR(S-d,0)) = (\alpha^{i,j}(IR(S-d,0)))_{j\neq i}$. Therefore, no member of the class of endogenously proportional solutions is a Directional solution.

Below, we present four axioms to characterize our solutions. The first two axioms are well known. The third and fourth axioms are stated for each individual i.

Weak Pareto Optimality (WPO): $\mu(S) \in WPO(S)$.

Continuity (CON): If $\{S_k\}$ converges in the Hausdorff topology to S, then $\{\mu(S_k)\}$ converges to $\mu(S)$.

Balancedness Relative to Individual i (BAL-i): $\mu(S) \in B^{i,j}(S)$ for all $j \neq i$.

Invariance of Payoffs Relative to Individual i under Constant Pairwise Total Payoff Asymmetry (IPRI-i): If S, T are such that $\alpha^{i,j}(S) = \alpha^{i,j}(T)$ for some i and $j \neq i$, then $\mu^j(S)/\mu^i(S) = \mu^j(T)/\mu^i(T)$.

The axiom $BAL{-}i$ requires that the vector of proportionality corresponding to any solution depends on the pairwise balancing subset of the bargaining problem with respect to each pair involving individual i. Finally, $IPRI{-}i$ requires that if the pairwise total payoff asymmetry with respect to individual i and another individual j is the same in two distinct problems, then the utility of individual j relative to individual i must also be the same in these problems.

Below, we will show that *WPO* and *IPRI*$-i$ together imply the well known *homogeneity* axiom. We will use this result in proving our characterization theorem.

Homogeneity (HOM). $\mu(cS) = c\mu(S)$ for all $c > 0$.

Lemma 1. *A solution satisfies* HOM *if it satisfies* WPO *and* IPRI$-i$ *for some* i*.*

Proof. Let a solution μ satisfy *WPO* and *IPRI*$-i$ for some individual i. Fix i. Pick any S and $c > 0$. By *WPO*, $\mu(S) \in WPO(S)$ and $\mu(cS) \in WPO(cS)$. It follows that $c\mu(S) \in WPO(cS)$ since $cWPO(S) = WPO(cS)$. Clearly, $\alpha^{i,j}(cS) = \alpha^{i,j}(S)$ for all $j \neq i$. Then, by *IPRI*$-i$, $\mu^j(cS)/\mu^i(cS) = \mu^j(S)/\mu^i(S)$ for all $j \neq i$. Suppose $\mu^i(cS) > c\mu^i(S)$; then $\mu(cS) > c\mu(S)$, contradicting $c\mu(S) \in WPO(cS)$. Similarly, $\mu^i(cS) < c\mu^i(S)$ would imply $\mu(cS) < c\mu(S)$, contradicting $\mu(cS) \in WPO(cS)$. So, we must have $\mu^i(cS) = c\mu^i(S)$, implying $\mu(cS) = c\mu(S)$. $\square$

Theorem 1. *A solution on* Σ_0^n *satisfies WPO, CON, BAL-i, and IPRI*$-i$ *if and only if it is endogenously proportional relative to individual* i*.*

Proof. Obviously, any solution in the class EP_i satisfies all four axioms. Conversely, let μ be a solution satisfying *WPO, CON, BAL-i*, and *IPRI*$-i$. Pick $\phi^j \in \mathbb{R}_{++}$ for all $j \neq i$. Let $\boldsymbol{\phi} = (\phi^j)_{j\neq i}$. Consider the problem

$$D(i, \boldsymbol{\phi}) = \{y \in \mathbb{R}^n_+ \,|\, y^i \leq 1 \text{ and } y^j \leq \sqrt{\phi^j} \ \text{ for all } \ j\}.$$

Pick $k \neq i$. We have $\alpha^{i,k}(D(i,\boldsymbol{\phi})) = \phi^k$, since

$$TP(D^{i,k}_{L,1}(i,\boldsymbol{\phi})) = \frac{\phi^k}{2}\prod_{\substack{j\neq k\\ j\neq i}}\sqrt{\phi^j} = TP(D^{i,k}_{R,\phi^k}(i,\boldsymbol{\phi})) \quad \text{if} \quad \phi^k \in (0,1], \quad \text{and}$$

$$TP(D^{i,k}_{L,\phi^k}(i,\boldsymbol{\phi})) = \frac{1}{2}\prod_{\substack{j\neq k\\ j\neq i}}\sqrt{\phi^j} = TP(D^{i,k}_{R,1}(i,\boldsymbol{\phi})) \qquad \text{if} \quad \phi^k > 1.$$

By *BAL-i*, we have $\mu(D(i,\boldsymbol{\phi})) \in B^{i,k}(D(i,\boldsymbol{\phi}))$, implying

$$\frac{\mu^k(D(i,\boldsymbol{\phi}))}{\mu^i(D(i,\boldsymbol{\phi}))} \in \begin{cases} [\phi^k, 1) & \text{if} \quad \phi^k \in (0,1), \\ \{1\} & \text{if} \quad \phi^k = 1, \\ (1, \phi^k] & \text{if} \quad \phi^k > 1. \end{cases}$$

Let

$$r^{i,k}(\boldsymbol{\phi}) = \frac{1}{\phi^k - 1}\left(\frac{\mu^k(D(i,\boldsymbol{\phi}))}{\mu^i(D(i,\boldsymbol{\phi}))} - 1\right)$$

if $\phi^k \neq 1$. Clearly, $r^{i,k}$ is defined at all $n-1$ tuples $(\phi^j)_{j\neq i} \in \Re^{n-1}_{++}$ such that $\phi^k \neq 1$. Let $r^{i,k}((\boldsymbol{\phi}^{-k},1)) = \lim_{\phi^k\to 1} r^{i,k}((\boldsymbol{\phi}^{-k},\phi^k))$, where $\boldsymbol{\phi}^{-k} = (\phi^j)_{\substack{j\neq i\\ j\neq k}} \in \mathbb{R}^{n-2}_{++}$ and $\phi^k \in \mathbb{R}_{++}\backslash\{1\}$. (Note that $\mu^k(D(i,(\boldsymbol{\phi}^{-k},\phi^k)))/\mu^i(D(i,(\boldsymbol{\phi}^{-k},\phi^k)))$ is continuous in $(\boldsymbol{\phi}^{-k},\phi^k)$, since μ satisfies CON; hence the above limit exists.) Note that $r^{i,k}(\boldsymbol{\phi}) \in (0,1]$ for all $\boldsymbol{\phi} \in \mathbb{R}^{n-1}_{++}$. Since k was arbitrary, we have constructed a continuous function $r^{i,j} : \mathbb{R}^{n-1}_{++} \to (0,1]$ for each $j \neq i$.

Now pick a problem S. Given $\boldsymbol{\alpha}^i(S) = (\alpha^{i,j}(S))_{j\neq i}$, let $p(S) = \mu(D(i,\boldsymbol{\alpha}^i(S)))$. By construction, $p^j(S)/p^i(S) = 1 - r^{i,j}(\boldsymbol{\alpha}^i(S)) + r^{i,j}(\boldsymbol{\alpha}^i(S))\alpha^{i,j}(S)$ for all $j \neq i$. Let $\lambda(S) = \max\{t \,|\, tp(S) \in B^{i,j}(S) \text{ for all } j \neq i\}$. Clearly, $\lambda(S)p(S) \in WPO(S)$ since μ satisfies *WPO*.

Consider the problem $V(i,S) = \lambda(S)D(i,\boldsymbol{\alpha}^i(S))$. Pick any $j \neq i$. If $\alpha^{i,j}(S) \in (0,1]$, then we have

$$\begin{aligned} TP(V^{i,j}_{L,1}(i,S)) &= [\lambda(S)]^n TP(D^{i,j}_{L,1}(i,\boldsymbol{\alpha}^i(S))) \\ &= [\lambda(S)]^n TP(D^{i,j}_{R,\alpha^{i,j}(S)}(i,\boldsymbol{\alpha}^i(S))) \\ &= TP(V^{i,j}_{R,\alpha^{i,j}(S)}(i,S)). \end{aligned}$$

On the other hand, if $\alpha^{i,j}(S) > 1$, then we have

$$\begin{aligned} TP(V^{i,j}_{R,1}(i,S)) &= [\lambda(S)]^n TP(D^{i,j}_{R,1}(i,\boldsymbol{\alpha}^i(S))) \\ &= [\lambda(S)]^n TP(D^{i,j}_{L,\alpha^{i,j}(S)}(i,\boldsymbol{\alpha}^i(S))) \\ &= TP(V^{i,j}_{L,\alpha^{i,j}(S)}(i,S)). \end{aligned}$$

Since j was arbitrary, it follows that $\alpha^{i,j}(V(i,S)) = \alpha^{i,j}(D(i,\boldsymbol{\alpha}^i(S))) = \alpha^{i,j}(S)$ for all $j \neq i$.

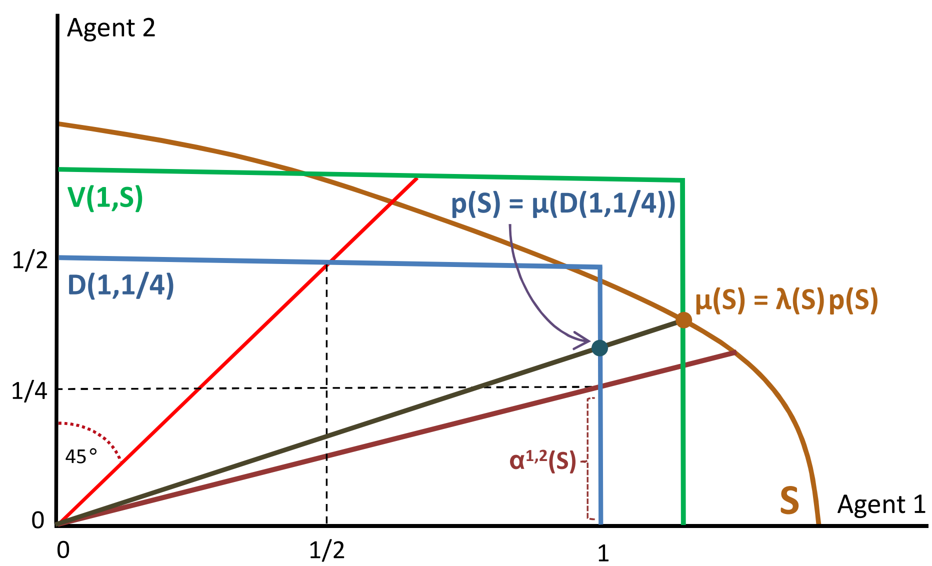

Figure 2. Sketch of the Proof for $n = 2$ and $\alpha^{1,2}(S) = 1/4$.

Also, $\mu(V(i,S)) = \lambda(S)\mu(D(i,\boldsymbol{\alpha}^i(S))) = \lambda(S)p(S)$, since μ satisfies *HOM* by Lemma 1. By *IPRI*–i, $\mu^j(S)/\mu^i(S) = \mu^j(V(i,S))/\mu^i(V(i,S))$ for all $j \neq i$, since

$\alpha^{i,j}(V(i,S)) = \alpha^{i,j}(S)$. Moreover, $\mu(V(i,S)) \in WPO(V(i,S))$, since μ satisfies *WPO*. Then, $\mu^i(V(i,S)) = \mu^i(S)$ for all $j \neq i$, for otherwise we would have either $\mu(V(i,S)) > \mu(S)$ contradicting $\mu(S) \in WPO(S)$ or $\mu(V(i,S)) < \mu(S)$ contradicting $\mu(V(i,S)) = \lambda(S)p(S) \in WPO(S)$. Therefore, $\mu(S) = \mu(V(i,S)) = \lambda(S)p(S)$. $\square$

The axioms *WPO* and *CON* are also satisfied by exogenously proportional solutions, as already shown by Kalai (1977). Besides, these solutions satisfy *IPRI–i* as well, since by definition the vector of proportionality of any exogenously proportional solution is invariant to changes in the bargaining problem. Thus, endogenously and exogenously proportional solutions are only distinguished, in our characterization, by the *balancedness* axiom. It should be evident from the definition of endogenously proportional solutions that any possible alternative characterization of the class EP_i may constantly depend on the axiom *BAL-i*. This dependence is similar to the constant appearance of the *strong individual rationality* (*SIR*) axiom in three alternative characterizations of exogenously proportional solutions offered by Kalai (1977).[4] The axiom *SIR* requires that for each problem the solution should assign a positive utility to each individual.[5] The need for *SIR* by the class of exogenously proportional solutions is obvious as these solutions restrict the vector of proportionality to strictly positive n-tuples. On the other hand, *SIR* is not strong enough to account for the demanding restrictions our solutions put on the vector of proportionality corresponding to each problem. The restrictions put by any solution in EP_i require, for each problem, the exact knowledge of the pairwise total payoff asymmetry with respect to each pair involving individual i,

[4]Kalai (1977) shows that exogenously proportional solutions are characterized by *WPO*, *HOM*, *SIR* together with *monotonicity* or *step-by-step negotiations* or a collection of three axioms involving *independence of irrelevant alternatives*, *individual monotonicity* and *continuity*.

[5]Note that *WPO* and *BAL-i* together imply *SIR*. Therefore any solution in the class EP_i trivially satisfies *SIR*.

hence the direct reflection of these restrictions onto an axiom like *BAL*-i seems to be inevitable.

We should also note that the separation of endogenously and exogenously proportional solutions with regard to the *balancedness* axiom implies that the two classes of solutions also differ with respect to their relation to a basic axiom in the bargaining literature, called *symmetry*.

Symmetry. If S is symmetric, then $\mu^i(S) = \mu^j(S)$ for all i and j.

While *symmetry* is satisfied by no exogenously proportional solution except for the Egalitarian solution, it is satisfied by every endogenously proportional solution. The reason is that given any i, this axiom is implied by $BAL{-}i$, because if S is symmetric, then $\alpha^{i,j}(S) = 1$ and $B^{i,j}(S) = \{y \in S \,|\, y_i = y_j\}$ for all $j \neq i$. In fact, endogenously proportional solutions satisfy a stronger form of symmetry, as well.

Total Payoff Symmetry. If S is total payoff symmetric, then $\mu^i(S) = \mu^j(S)$ for all i and j.

It is clear that *total payoff symmetry* implies *symmetry*, since every bargaining problem is total payoff symmetric if it is symmetric. But, the converse is not true since there are total payoff symmetric problems that are not symmetric. For example, consider $n = 2$ and $S =$ convex hull $(\{(0,0),(0,3/2),(1/2,3/2),(1,1),(7/4,0)\})$. Clearly, S is not symmetric, but it is total payoff symmetric since $TP(S^{1,2}_{L,1}) = TP(S^{1,2}_{R,1}) = 7/8$ and $\alpha^{1,2}(S) = 1$.

Finally, when we eliminate the axiom $IPRI{-}i$ from our list of characterizing axioms, we can further generalize our solutions to a family that we call ***total payoff balancing***

class of solutions. Obviously, each bargaining solution already satisfying *WPO* and *CON* can be simply extended to be a member of this general class of solutions, by picking some individual i and restricting the solution outcome on each problem S to the set $\cap_{j \neq i} B^{i,j}(S)$.

4 Conclusion

In this paper, we have introduced a class of endogenously proportional solutions. The solutions relative to individual i are characterized by *weak Pareto optimality*, *continuity* and two new axioms that depend on the pairwise total payoff asymmetry of a given problem with respect to each pair involving individual i.

Interestingly, endogenously proportional solutions satisfy a stronger form of the *symmetry* axiom, while exogenously proportional solutions, except for the Egalitarian solution, fail to satisfy *symmetry*. Definitely, for non-Egalitarian members of the Kalai's (1977) class of solutions this is not a deficiency per se, since in environments where the players may not have the same bargaining power, asking for *symmetry* would be unreasonable. On the other hand, in environments where the bargaining problem is known to intrinsically contain the bargaining power of the players, it would be natural to focus on solutions that choose symmetric outcomes in symmetric problems. The solutions we propose may enable players in such environments to use proportional solutions without dispensing with *symmetry*. However, one difficult problem that was already addressed by Kalai (1977) for exogenously proportional solutions is what the vector of proportionality should be. For the case of each endogenously proportional solution relative to individual i, this problem boils down to how the weight functions $\{r^{i,j}\}_{j \neq i}$, which determine the direction of the solution inside the set $\cap_{j \neq i} B^{i,j}(S)$ for any problem S, should be constructed.

Finally, new bargaining solutions can be derived from the already known solutions in the literature, restricting the outcome chosen by any proposed solution to lie in the intersection of the pairwise balancing subsets of the bargaining problem relative to a given individual. This procedure can be especially useful for finding symmetric extensions of solutions that fail to satisfy *symmetry*.

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