Liquidity Traps and the Price (In)Determinacy of Monetary Rules

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Abstract:

This paper proposes a new methodology for assessing price indeterminacy to supplant the discredited nonexplosive criterion. Using this methodology, we find that nominal GDP targeting and price-level targeting do determine prices when the central bank follows a sufficiently strong feedback rule for setting the nominal interest rate. However, inflation targeting leads to price indeterminacy, a result consistent with the principles of calculus. This price indeterminacy of inflation targeting could manifest itself in a liquidity trap or zero bound for nominal interest rates rendering central banks impotent. Nominal GDP targeting could overcome this liquidity-trap effect.

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A predominate view in monetary economics today is that a central bank should be transparent concerning its goals and objectives rather than following a cloudy discretionary policy. That transparency can be reflected in an explicit monetary target. Prior to the 2008 Financial Crisis, the fad monetary target in central banking was inflation. Recently, economists and central banks have been considering alternative monetary targets such as the price level and nominal GDP (NGDP).

A very important issue facing the choice of monetary target is whether the price level of an economy can be determined by a central bank pegging the interest rate according to a monetary rule that pursues that monetary target. This issue is very relevant since most central banks set a short-term interest rate rather than a monetary aggregate. If the central bank cannot determine the price level, then the price level will be free to take multiple or infinite time paths. As recognized by some researchers such as Atkeson et al (2009), one extreme price time path would be hyperinflation. Another more immediate, but less discussed extreme of price indeterminacy would be a liquidity trap where the central bank runs into a zero bound for the nominal interest rate and becomes impotent.

Economists have long debated the issue of whether an interest-setting rule can determine prices since Sargent and Wallace's (1975) argument that doing so cannot determine prices. While many economists have argued that inflation targeting (IT) or price-level targeting (PLT) can determine prices when the central bank follows a Taylor-like monetary rule for setting the interest rate, no literature has addressed whether nominal GDP targeting[1] (NGT) determines prices. Demonstrating that NGT does determine prices is the first task this paper accomplishes.

Even with regard to IT and PLT, the price determinacy issue is unresolved. While many economists, such as Woodford (2003) and Clarida et al (1999), have previously argued that IT does determine the price level, most of that literature has made errors by applying a nonexplosive criterion without having any economic basis for doing so. More recent literature recognizes these errors. Cochrane (2010) notes that while transversality conditions can rule out real explosive behavior, nothing in economics can rule out nominal explosions. Also, while many economists assumed that those using the nonexplosive criterion could justify it with transversality conditions, McCallum (2009, p. 1103) admits that there is no such transversality condition that justifies the use of the nonexplosive criterion in the case of fiat money. Some economists have argued that in order for IT to determine price levels, we have to look for fiscal transversality conditions, but the economic profession has not endorsed the idea of using a government transversaility condition to justify the price determinacy of monetary policy.

Thus, even with IT and PLT, price determinacy has become an open question with some research saying that IT leads to price indeterminacy: See Cochrane (2010), Eagle (2007), and Nakajima and Polemarchakis (2005).

The earlier literature on price determinacy used a nonexplosive criterion, which now McCallum recognizes is not justified by any transversality condition. This leaves the state of the price determinacy literature in limbo. While McCallum (2009) has tried to use learnability as a means to address price determinacy, Cochrane (2010) has argued against learnability as a sufficient way to address price determinacy.

[1] Another term for nominal GDP targeting is nominal income targeting.

Since the methodology used in the past for addressing price determinacy is now discredited, this paper takes a different approach to the issue of price determinacy. The issue of price determinacy is hampered by the infinite time we use in our economic models. If the time horizon was finite, we would just apply Bellman's principle, work backwards in time with expectations, and there would be no question about whether monetary policy under a given monetary target would determine the price level. However, to apply Bellman's principle we need some future time to start our backwards expectations work. When the economy lasts forever, then there is no future time to begin our backwards expectations calculations work.

The approach taken in this paper is to assume an arbitrarily large, but finite horizon in the economy, then apply Bellman's principle to apply expectations backwards in time. We add to this the issue of the public's confidence concerning what the central bank will do in the last period of the economy. To assess price determinacy, we propose two criteria. The first criterion is that the price level should be determined in a finite economy under a particular monetary targeting rule. The second criteria is that the public's confidence in expected price levels should be maintained as the economic horizon increases to infinity. The methodology proposed in this paper is that price determinacy should require both criteria to be met. This sense of price determinacy carries important economic understanding, not just blindly applying rules that economists have invented (and that conflict with basic principles of calculus).

The next section sketches an economic model and quickly derives the Fisher Euler equation that is the basis in the price-determinacy literature for how expectations about future price levels is translated into the current price level. Section III then discusses the price determinacy of NGT and introduces a feedback rule for the central bank to use to set the nominal interest rate as the central bank pursues its nominal GDP target. This section also discusses the issue of the public's deteriorating confidence of what the central bank will do in the final period of the economy as that finite economic horizon increases. This section then shows that that NGT does determine the price level as long as that feedback rule is sufficiently strong to compensate for public's deteriorating confidence in the central bank's final-period action as the time horizon increases.

Section IV discusses the price determinacy of PLT and finds that PLT does determine the price level as long as the central bank follows a sufficiently strong McCallum feedback rule. Section V discusses the price determinacy of IT and finds that IT does not determine the price level because even in a finite economy, there are fewer equations than unknowns. Section VI discusses how IT may have appeared to work because the public confused IT with PLT; this section also discusses the base-drift difference between IT and PLT.

Section VII discusses how the price determinacy of IT may have manifested itself in the liquidity traps or zero-bound nominal interest rates interfering with the ability of central banks to stimulate nominal spending in the economy. Section VIII concludes and discusses how central banks could help avoid liquidity traps by following NGT rather than IT.

II. The Fisher-Euler Equation:

Assume a representative consumer with an expected utility function of the form $\sum_{t=1}^{T}\left(\beta^{t} E_{0}\left[U(c_{t})\right]\right)$ where β is the time discount factor and T is the arbitrarily large, but finite economic horizon. When the consumer maximizes the utility function subject to typical budget constraints involving money and nominal bonds, the previous literature[2] has shown that these assumptions lead to following Euler equation:

$$\frac{U'(c_t)}{P_t} = \beta R_t E_t\left[\frac{U'(c_{t+1})}{P_{t+1}}\right] \tag{1}$$

where R_t is the gross nominal interest rate which equals one plus the regular nominal interest rate. Equation (2) states that marginal utility of consumption per "buck" today equals the gross nominal interest rate times the expected marginal utility of consumption per "buck" next year multiplied by the time discount factor. Here "buck" refers to the particular monetary unit of the particular economic system we are looking at.

We also assume a pure exchange economy so that aggregate consumption equals aggregate output, $c_t = Y_t$. Then this Euler equation becomes:

$$\frac{U'(Y_t)}{P_t} = \beta R_t E_t\left[\frac{U'(Y_{t+1})}{P_{t+1}}\right] \tag{2}$$

Equation (2) is often referred to as the Fisher-Euler equation because it relates the gross nominal interest rate to expected inflation and the determinants of the real interest rate.

Equation (2) is standard in the price determinacy literature except often that literature linearizes around a steady state. In this paper, we will be able to speak more generally than a linearization around a steady state and therefore we will continue to work with equation (2).

For the model of this paper, assume a logarithmic utility function so $U'(Y_t) = 1/Y_t$. We assume this utility function for two reasons. First, we can easy model NGT with this utility function without any further assumptions. Second, the logarithmic utility function has a constant relative risk coefficient, which if applied to diverse consumers with different borrowing and

[2] See for example Woodford (2003) and Carlstrum and Fruest (2001).

lending needs would require perfectly successful NGT in order for borrowers and lenders to efficiently share real aggregate supply risks (See Eagle and Domian, 2005; Koenig, 2011; and Eagle and Christensen, 2012).

With the logarithmic utility function, (2) becomes

$$\frac{1}{P_t Y_t} = \beta R_t E_t \left[\frac{1}{P_{t+1} Y_{t+1}} \right] \tag{3}$$

By the equation of exchange, $M_t V_t = N_t = P_t Y_t$, (3) can be expressed as:

$$\frac{1}{N_t} = \beta R_t E_t \left[\frac{1}{N_{t+1}} \right] \tag{4}$$

where N_t is the level of nominal GDP at time t.

III. The Price Determinacy of Nominal GDP Targeting (NGT)

This section accomplishes two tasks. First, it shows that NGT does determine the price level when the central bank follows a sufficiently strong feedback rule for setting the nominal interest rate. Second, this section introduces this paper's methodology for assessing price determinacy.

Assume the central bank targets NGDP where its NGDP target at time t is given by N_t^*. To do so, assume the central bank uses the following feedback rule for setting the nominal interest rate:

$$\hat{R}_t = \frac{N_{t+1}^*}{\beta N_t^*} \left(\frac{N_t}{N_t^*} \right)^{\tau} \tag{5}$$

Substitute (5) for R_t in (4) to get:

$$\frac{1}{N_t} = \beta\left(\frac{N_{t+1}^*}{\beta N_t^*}\left(\frac{N_t}{N_t^*}\right)^{\tau}\right)E_t\left[\frac{1}{N_{t+1}}\right]$$

which can be simplified to:

$$\frac{N_t^*}{N_t} = E_t\left[\frac{N_{t+1}^*}{N_{t+1}}\right]^{\frac{1}{1+\tau}} \tag{6}$$

Remember we are now assuming an arbitrarily large, but finite horizon for the economy. We will apply Bellman's principle and work backwards in time. For time T, the Fisher-Euler equation (2) does not apply; at time T there will be no tomorrow; there will be no loans from time T to time T+1 and hence no interest rate at time T. As a result, at time T the central bank will unable to peg the interest rate; hence, the central bank will have to set the money supply. To model the economic response to the central bank setting the money supply in the last period of the economy, assume a link between money and the economy such as a cash-in-advance constraint. That link between money and the economy may not be a perfect one, so the central bank may be unable to control N_T perfectly by setting M_T. As a result, we will assume the central bank sets M_T so that:

$$E_T\left[\frac{1}{N_T}\right] = \frac{1}{N_T^*}$$

which implies that:

$$E_T\left[\frac{N_T^*}{N_T}\right] = 1 \tag{7}$$

Apply Bellman's principle, we move backwards in time to time t=T-1. Taking the expectations of (7) based on the information set at time t=T-1, we get:

$$E_{T-1}\left[\frac{N_T^*}{N_T}\right]=1 \qquad (8)$$

At time t=T-1, the Fisher-Euler equation (4) applies, and the central bank follows (5); therefore, (6) applies. Applying (6) to time t=T-1 gives:

$$\frac{N_{T-1}^*}{N_{T-1}}=\left(E_{T-1}\left[\frac{N_T^*}{N_T}\right]\right)^{\frac{1}{1+\tau}}$$

Substituting (8) into the above gives $N_{T-1}^*/N_{T-1}=1$. Since we are assuming that the central bank is transparent in its nominal GDP target, the public's expectation of the above at time t=T-2 is $E_{T-2}\left[N_{T-1}^*/N_{T-1}\right]=1$. Substituting the above in (6) at time t=T-2 gives:

$$\frac{N_{T-2}^*}{N_{T-2}}=E_{T-2}\left[\frac{N_{T-1}^*}{N_{t-1}}\right]^{\frac{1}{1+\tau}}$$

$$\frac{N_{T-2}^*}{N_{T-2}}=1$$

As we work backwards through time applying Bellman's principle, we find that, for any t in [0,T-1], $E_t\left[\frac{N_{t+1}^*}{N_{t+1}}\right]=1$. Substituting this into (6) applied at time t gives $\frac{N_t^*}{N_t}=1$, which by backwards recursion shows that $\frac{N_t^*}{N_t}=1$ for t=0,1,…,T-1. This also means that $N_t=N_t^*$ for t=0,1,…,T-1. In other words, N_t is determined for t=0,1,…,T-1 in this model. Since $P_t=N_t/Y_t$ and Y_t is exogenous, then P_t is determined for t=0,1,…,T-1. In other words, the price level is determined under NGT in this model with a finite horizon.

We should note that while it is exogenous, Y_t can be stochastic so that P_t may be stochastic. Nevertheless, P_t is still determined in the sense that given the exogenous Y_t, P_t is determined.

This paper proposes the following methodology to assess the price determinacy of a monetary target/rule: This methodology requires that two criteria be met in order for us to declare prices to be determined in the model. The first criterion we call the "finite criterion," which requires the price level be determined for a finite horizon. The second criterion we call the "public's confidence criterion," which requires that the public's confidence interval in the expected value determining the price level be finite as the economic horizon approaches infinity.

For NGT the preceding discussion shows that the first criterion is met. To assess the second criterion, note that by backwards recursion, (6) and (8) imply that

$$\frac{N_0^*}{N_0} = \left(E_0 \left[\frac{N_T^*}{N_T} \right] \right)^{\frac{1}{(1+\tau)^T}} \tag{9}$$

The above equation shows the expectations that determines the value of N_0 and hence the value of $P_0=N_0/Y_0$ where Y_0 is exogenous. Hence the issue about whether the public's confidence in the expectations determining N_0 and P_0 is whether the public's confidence in the right-hand side of (9) is maintained when T approaches infinity.

Let $\Omega_{0,\alpha}\left[N_T^*/N_T\right]$ be the public's α% confidence interval at time 0 for N_T^*/N_T given the public's information set at time 0. It may very well be the case that the $\lim_{T\to\infty}\Omega_{0,\alpha}\left[N_T^*/N_T\right]$ is infinite, which means that the public's confidence that central bank will be close to its NGDP target at time T will decrease (confidence interval increase), the further in the future is the last period.

Let us first look at the case where $\tau = 0$ in the central bank's feedback rule (5) for setting the interest rate. Then by (9), the public's confidence in the expectations that determines N_0^*/N_0 would be the same as $\Omega_{0,\alpha}\left[N_T^*/N_T\right]$. Hence, if $\lim_{T\to\infty} \Omega_{0,\alpha}\left[N_T^*/N_T\right] = \infty$, then the public's confidence interval in the expectations determining N_0^*/N_0 would also be infinite meaning the public would have no confidence in N_0^*/N_0. This in turn implies that the public would have no confidence in N_0. Since $P_0=N_0/Y_0$, this would also imply that the public would have no confidence in P_0. It is in this sense that Sargent and Wallace (1975) argued that the price level would not be determined when the central bank pegged the interest rate rather than setting the money supply. Hence, for $\tau = 0$, a central bank following the feedback rule (5) while pursuing a NGDP target would not lead to price determinacy as defined by the methodology proposed in this paper. While it would meet the first criterion of determining the price level in a finite economy, NGT with $\tau = 0$ would not meet the second criterion; the public's confidence in the expectations determining that price level would disappear as the economy's horizon approaches infinity.

However, if τ is sufficiently positive, meaning a bigger central bank's interest rate response when NGDP deviates from is targeted path, then the damping effect of τ in (9) may offset the increase in $\Omega_{0,\alpha}\left[N_T^*/N_T\right]$ as T approaches infinity. For this sufficiently positive τ, if the public's confidence in their expectations determining N_0^*/N_0 is maintained as T approaches infinity, the public confidence in N_0 and P_0 will be maintained as well.

Hence, it may be the case that for τ sufficiently positive; the public's confidence in their expectations determining the expected value for N_0^*/N_0 as reflected in (9) will be maintained because the dampening effect of the positive τ offsets the decreasing public's confidence is the central bank's action at time T as T increases to infinity. In conclusion, a central bank following the monetary rule (5) for a sufficiently positive τ will meet both criteria for price determinacy. First, for a finite economy, NGT does determine prices, and second, for a sufficiently positive τ, the public's confidence in their expectations of the determinant of the price level may be maintained.

Because we have not made specific assumptions about how the public's confidence interval changes as T increases, we cannot say for sure that there exists a sufficiently positive value of τ. However, if such a sufficiently positive τ does exist, then this explains how this strong reaction of the interest rate to deviations of NGDP from its target helps maintain the public's confidence in the price level and in monetary policy. This explanation is in sharp contrast to Cochrane (2010), where he argued that the use of the nonexplosive criterion as used in the previous price-determinacy literature was that the central bank was threatening the public with explosive monetary policy if the public's expectations were not stable. Cochrane's view is one that looks forward in time rather than backwards in time. Expectations works backwards in time. When we realize that expectations works backwards in time, then we recognize that the positive τ has a dampening rather than explosive effect on expectations.

While this section discussed the methodology in terms of NGT, the previous literature has discussed usually PLT and IT. How the second criterion is related to the previous literature is made clearer in the next sections that assess the price determinacy of PLT and IT.

IV. The Price Determinacy of Price-Level Targeting (PLT)

Let us now assume that instead of targeting NGDP, the central bank targets the price level, where its price level target at time t is given by P_t^*. To try to meet this target, the central bank sets the interest rate according to the following feedback rule:

$$\hat{R}_t = \frac{1}{\beta P_t^* Y_t E_t\left[\frac{1}{P_{t+1}Y_{t+1}}\right]} E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right]\left(\frac{P_t}{P_t^*}\right)^{\tau} \tag{10}$$

If substitute the right side of (10) for R_t in (3), we get:

$$\frac{1}{P_t Y_t} = \beta\left(\frac{1}{\beta P_t^* Y_t E_t\left[\frac{1}{P_{t+1}Y_{t+1}}\right]} E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right]\left(\frac{P_t}{P_t^*}\right)^{\tau}\right) E_t\left[\frac{1}{P_{t+1}Y_{t+1}}\right]$$

which can be simplified to

$$\frac{P_t^*}{P_t} = \left(E_t\left[\frac{P_{t+1}^*}{P_{t+1}}\right]\right)^{\frac{1}{1+\tau}} \tag{11}$$

Let us first check whether the price level is determined for a finite horizon. Again, at time T, the Fisher-Euler equation (2) does not apply, there is no interest rate that applies to a loan from time T to the nonexistent time T+1. Hence, the central bank cannot peg the interest rate at time T. Assume the central bank sets the money supply at time T so that $E_T[1/P_T] = 1/P_T^*$, which implies that:

$$E_T\left[\frac{P_T^*}{P_T}\right] = 1 \tag{12}$$

Applying Bellman's principle, we move backwards in time to time t=T-1. Taking expectations of (12) based on the information set at time T-1 gives $E_{T-1}\left[P_T^*/P_T\right] = 1$. At time t=T-1, the Fisher-Euler equation (3) applies, and the central banks follows (10); therefore, (11) applies. Substituting in $E_{T-1}\left[P_T^*/P_T\right] = 1$ gives:

$$\frac{P_{T-1}^*}{P_{T-1}} = \left(E_{T-1}\left[\frac{P_T^*}{P_1} \right] \right)^{\frac{1}{1+\tau}} = 1$$

Hence, $P_{T-1} = P_{T-1}^*$. By backwards recursion, we conclude that $P_t = P_t^*$ for t=0,1,…,T-1. This then shows that PLT satisfies this paper's first criterion of price determinacy: PLT does determine prices in a finite economy.

For the second criterion, we can solve (11) backwards to get:

$$\frac{P_0^*}{P_0} = \left(E_0\left[\frac{P_T^*}{P_T} \right] \right)^{\frac{1}{(1+\tau)^t}} \tag{13}$$

Let $\Omega_{0,\alpha}\left[P_T^* / P_T\right]$ be the public's α% confidence interval at time 0 for P_T^* / P_T. Again, it may very well be the case that $\lim_{T\to\infty} \Omega_{0,\alpha}\left[P_T^* / P_T\right]$ is infinite. If the τ in the central bank's feedback rule (10) is 0, then the public's confidence in the expectations leading to P_0 would disappear when T approaches infinity. This is in essence the point Sargent and Wallace (1975) made where they in essence assumed the central bank targeted the price level. They said that the current price level depended too much on what the central bank did in the far distant future.

However, for a sufficiently positive τ, the public's confidence may be maintained even though $\lim_{T\to\infty} \Omega_{0,\alpha}\left[P_T^* / P_T\right]$ is infinite. This is the point of McCallum's (1981) argument that a central bank could determine prices when setting the interest rate if the reaction of the central bank to disparities of the price level from its target is strong enough. When we look backwards in time, we find that positive τ decreases the degree to which current expectations depend on what the central bank will do in the far distant future. Again, this interpretation differs from Cochrane (2010) where he argues that the central bank is threatening the public with explosive prices if the public does not constrain their expectations.

V. The Price Indeterminacy of Inflation Targeting (IT)

In this section, we assume the central bank targets the inflation rate, where its inflation target at time t is given by π_t^* and we define $\pi_t \equiv P_t / P_{t-1}$. To try to meet this target, the central bank sets the nominal interest rate by following the following feedback rule:

$$\hat{R}_t = \frac{\pi_{t+1}^* E_t\left[\frac{1}{\pi_{t+1}}\right]}{\beta Y_t E_t\left[\frac{1}{\pi_{t+1} Y_{t+1}}\right]} \left(\frac{\pi_t}{\pi_t^*}\right)^{1+\tau} \tag{14}$$

Substituting the right side of (14) for R_t in (3) gives:

$$\frac{1}{P_t Y_t} = \beta \left(\frac{\pi_{t+1}^* E_t\left[\frac{1}{\pi_{t+1}}\right]}{\beta Y_t E_t\left[\frac{1}{\pi_{t+1} Y_{t+1}}\right]} \left(\frac{\pi_t}{\pi_t^*}\right)^{1+\tau} \right) E_t\left[\frac{1}{P_{t+1} Y_{t+1}}\right]$$

which simplifies to:

$$\left(\frac{\pi_t^*}{\pi_t}\right) = E_t\left[\frac{\pi_{t+1}^*}{\pi_{t+1}}\right]^{\frac{1}{1+\tau}} \tag{15}$$

First, we need to investigate whether IT determines the price level when the economy has a finite horizon. Applying Bellman's principle, we start with time t=T. At time the Fisher-Euler equation does not apply since there are no loans and hence no interest rate for time T to the nonexistent time T+1. Hence, the central bank must set the money supply at time T. Assume the central bank targets the money supply so that $E_T\left[1/\pi_T\right] = 1/\pi_T^*$, which implies that

$$E_T\left[\frac{\pi_T^*}{\pi_T}\right] = 1 \tag{16}$$

Continuing to apply Bellman's principle, we move backwards in time to time t=T-1. Applying expectations based on the information set at time t=T-1 to both sides of (16), we get that $E_{T-1}\left[\pi_T^*/\pi_T\right] = 1$. The Fisher-Euler equation (3) does apply at time t=T-1 and the central bank follows the feedback rule (14) so that (15) applies. Applying (15) at time t=T-1, we conclude that:

$$\left(\frac{\pi_{T-1}^*}{\pi_{T-1}}\right) = \left(E_{T-1}\left[\frac{\pi_T^*}{\pi_T}\right]\right)^{\frac{1}{1+\tau}} = 1$$

Which implies that $\pi_{T-1} = \pi_{T-1}^*$. As we continue to move backwards in time, applying Bellman's principle, we find that $\pi_{t+1} = \pi_{t+1}^*$, which also implies that $E_t\left[\pi_{t+1}^* / \pi_t\right] = 1$. When we substitute this result in (15), we then conclude that $\pi_t = \pi_t^*$. Therefore, by backwards recursion, we conclude that $\pi_t = \pi_t^*$ for t=1,2,…,T-1.

In summary, by applying Bellman's principle, we have concluded the following:

$\pi_t = \pi_t^*$ for t=1,2,…,T-1.

$E_{T-1}\left[\pi_T^*/\pi_T\right] = 1$

This is a total of T equations. By targeting inflation, which is the "speed" or derivative of the price level, the central bank will be unable to determine the price level in an economy with a finite horizon. To see this, first note that there are only T equations above. However, we need to determine either the actual price level or an expectation involving the price level for t=0,1,…,T;

which means there are T+1 unknowns. Since there are more unknowns than equations, the price level is not determined.

To a mathematician, it should be of no surprise that the price level cannot be determined by targeting the derivative of the price level. In calculus, it is well known that one cannot obtain the original function by merely integrating the original function; one needs an additional condition to do so. In the case of inflation targeting, we do not have that additional condition.

Examples are the clearest proof that inflation targeting does not determine the price level. For this example, let us assume that a cash-in-advance constraint exists at time t=T so that $M_T = P_T Y_T$. Assuming also that the central bank has perfect control over M_T, then for a give Y_T, the central bank would be able to determine a value of P_T to get $\pi_T = \pi_T^*$. With this stronger assumption, we now have $\pi_t = \pi_t^*$ for t=1,2,…,T. Let us also assume in this example that $\pi_t^* = 1.02$. Then, we have $\pi_t = \pi_t^* = 1.02$. In other words, the inflation rate in our model is always 2%. Any price path that meets the condition, $P_t = P_0(1.02)^t$, will have this 2% inflation rate. Clearly, P_0 is not determined. Again, this should be obvious from an understanding of calculus. If P_0 was 100 or 40 or 9878, as long as $P_t = P_0(1.02)^t$, the inflation will be 2%.

We are talking about a flexible price model where P_{-1} is given. Some might wonder about the inflation rate at time 0, $\pi_0 = P_0 / P_{-1}$. However, take the Fisher-Euler equation (3), multiply both sides by P_t and remember that $\pi_t \equiv P_t / P_{t-1}$. We get

$$\frac{1}{Y_t} = \beta R_t E_t \left[\frac{1}{\pi_{t+1} Y_{t+1}} \right]$$

The first period that this applies is t=0. This means the first inflation rate that we get the inflation rate π_1. The Fisher-Euler equation does not apply to π_0; hence π_0 is not determined in this model.

We could modify the model from a flexible price model to a model with price rigidities, and assume P_0 is fixed. However, by doing so, the Fisher-Euler equation must be changed to deal with these price rigidities. Once we do that in a consistent fashion, we will have one less Fisher-Euler equation. With one less equation and one less unknown, we still have price indeterminacy under IT.

VI. Why IT Temporary May Have Determined the Price Level

Despite the theoretical argument that IT leads to price indeterminacy, some may wonder why IT worked, at least before the 2008 crisis. My answer is that if the public was confused and thought the central bank was actually targeting the price level, then IT would have determined prices until the public learned differently.

The difference between IT and PLT is what Taylor (2006) calls "let bygones be bygones" and what McCallum (2011) calls "base drift." For example, assume the inflation target is 2% per annum. Assume the current price is P_0=100. Then if the central bank were to forever meet its inflation target, then $P_t = 100(1.02)^t$ for all future t. Assume that the public is thinking the central bank is actually targeting the price level at $P_t^* = 100(1.02)^t$. Since the public is more familiar with the term inflation than the price level, even if it were targeting the price level, the central bank would probably still communicate its intentions to the public in terms of the inflation rate. As a result, it is very possible the public could be confused. As long as the inflation rate is as expected, then the project path for the price level would be the same for both IT and PLT: $P_t = 100(1.02)^t$.

However, suppose at time t=1, the price level was 104 instead of 102, meaning that the central bank missed its target. Under PLT, the central bank would try to get the price level back to its targeted path of $P_t^* = 100(1.02)^t$. However, with IT the central bank would "let bygones be bygones" and merely try to get future inflation to equal its target. Therefore, the central bank's new intended price path would be $P_t = 104(1.02)^{t-1}$, which is greater than its original intended price path of $P_t = 100(1.02)^t$. In essence, with IT the central bank "forgives" its past mistakes.

On the other hand if the price level was 101 instead of 102 at time t=1, again the central bank missed its target. Under PLT, the central bank would try to increase the price level back to its targeted path of $P_t^* = 100(1.02)^t$. However, with IT the central bank would "let bygones be bygones and merely try to get future inflation to meet its inflation target; the central bank would not try to make up for the lower inflation that occurred in period 1. Under IT, the central banks new intended price path would be $P_t^* = 101(1.02)^{t-1}$, which is less than its original intended price path of $P_t^* = 100(1.02)^t$. McCallum (2011) calls this "base drift" since if the price level strays from course, the central bank will allow this drift rather than steering the price level back to its original levels.

As Eagle (2012) shows, the price base drift inherent in IT will manifest itself in NGDP base drift. For example, assume N_0=1000 , if the long-run growth rate in the economy is 3% and the central bank wants a long-run inflation rate of 2%, then under NGDP targeting (NGT), the central bank would target the path of NGDP at $N_t^* = 1000\left((1.02)(1.03)\right)^t$. Under IT, the initial intended path for NGDP would also be $N_t = 1000\left((1.02)(1.03)\right)^t$ as long as real GDP (RGDP) increased at its 3% long-run growth rate. Thus, initially both NGT and IT have the same intended path for NGDP. However, if N_1 turns out being 1030 instead 1050 because π_1 turned out being 0% instead of 2%, then under NGT, the central bank would try to return to its targeted NGDP path. However, under IT, the central bank would "let bygones be bygones" and change its intended NGDP path to $N_t = 1030((1.02)(1.03))^{t-1}$, which is less than its initial intend NGDP path. We call this drift of NGDP "NGDP base drift."

Eagle (2012) documents the statistical significant of NGDP base drift for the three U.S. recessions since 1990, which is within the time frame many economists have viewed the Federal Reserve as a "closet inflation targeter."

VII. How the Price Indeterminacy of IT can Manifest Itself in a Liquidity Trap

This section argues the price indeterminacy of IT can manifest itself in a liquidity trap or a zero-bound for the nominal interest rate that would cause the central bank to be impotent, which is a similar situation facing many central banks today. My argument is in a much broader context than the simple flexible-price model for which I discussed the price determinacy of NT, PLT, and IT. In particular, this section assumes the economy does have nominal rigidities especially with respect to wages. As a result of these nominal rigidities, the immediate impact of a drop in NGDP is a drop in RGDP rather than in the price level. Because the price level in the short-run may be unresponsive to drops in NGDP, I talk about the NGDP base drift caused by IT rather than the price-level base drift.

After the Financial Crisis of 2007-2008, NGDP dropped for most economies in the world. These NGDP drops exerted downward pressure on prices and wages, but because of the nominal rigidities, these prices and wages did not immediately drop. In fact, because of delayed cost-of-living adjustments (COLAs), many prices and wages continued to increase. Since NGDP fell relative to its prerecession trend and prices and wages stagnated or increased, RGDP fell relative to its prerecession trend. If the central bank totally focused on prices, the central bank would not even notice that NGDP dropped. However, in the last recession, all central banks were aware that NGDP was dropping below its prerecession trend. Most of these central banks took action to try to boost nominal aggregate spending. In particular, they lowered the nominal interest rates they set.

However, NGDP continued to fall. If investors put their money into real capital, that capital likely would have a negative return in the economy since the demand for the products produced by that real capital had decreased because of the drop in NGDP. Even if the investor could find some non-capital assets that experienced little or no depreciation, one possible future outcome would be that those assets would depreciate in nominal terms in the future. If investors expect that nominal depreciation, then these investors would be better off holding cash rather than the asset. This is especially true since we know that the drop in NGDP will be putting downward pressure on all prices. Even if those prices are slow to adjust, if the investor realizes than that downward pressure exists, then the investor should recognize that this is not the time to buy those assets.

Another possibility is that NGDP will rebound. However, given the experience in the U.S. since 1990 as documented by Eagle (2012), that rebound in NGDP is very unlikely. That empirical evidence gives investors knowledge about how the central bank will react when the central bank is targeting inflation. They know that the central bank will "let bygones be bygones" and let NGDP remain at its lower level. So the investors do not invest in real capital or other assets. Instead, they hold cash, which is Keynes' liquidity trap. Since they do not invest in real capital or other assets, NGDP falls. The holding of cash by these investors rather than spending this cash contributes to keeping NGDP from increasing in the future. The investors holding this cash rather than spending the cash becomes a self-fulfilling prophecy – NGDP does not increase and the central banks "let bygones be bygones" deciding to let the NGDP base drift downward and just try to get NGDP to increase in the future in a manner consistent with their inflation target. By focusing on inflation, not nominal aggregate spending, the central banks does not try to return NGDP to its prerecession trend.

Instead of targeting inflation, suppose a central bank targets NGDP. Assume that this central bank is both transparent and credible in its pursuit of the NGDP target. When NGDP falls, investors know that sooner or later the central bank will reverse the drop of NGDP back to its prerecession trend, which we assume is also the targeted level of NGDP. Also, recognize that since NGDP fell, prices and wages did not increase as much as the inflation targeted by the central bank. The central bank's intended increase of NGDP back to its prerecession trend/targeted path will therefore mean inflation will at some point have to be higher than the long-run inflation rate desired by the central bank. If investors believe the central bank will pursue this NGDP target and will not "let bygones be bygones," then the investors should know that the question is not **if** the higher inflation will occur but **when** will it occur. They also will recognize that the more NGDP falls below its target, the higher inflation will have to be as a result of the return of NGDP to its target; investors know that sometime in the future if they hold cash instead of investing that money in assets, they will lose value because of the inflation. This will discourage investors from holding cash and encourage them to spend their cash on real capital or other assets.

In summary, if the central bank targets NGDP rather than allowing permanent NGDP base drift, the expectation of the higher inflation will discourage investors leaving their money in cash and will reduce the likelihood of a liquidity trap.

VIII. Conclusions and a Caveat

This paper proposes a new methodology for assessing price determinacy of monetary targeting regimes when the central bank pegs the interest rate instead of setting the money supply. This methodology involved two criteria. The first criterion is whether the price level is determined in an economy with a finite horizon. The second criterion concerned whether the public's confidence in monetary policy is maintained as the finite horizon increases to infinity. Using that methodology, we find that NGDP targeting (NGT) and price-level targeting (PLT) do lead to price determinacy when the central bank follows a sufficiently strong feedback rule for setting the interest rate. On the other hand, we find that inflation targeting (IT) does not determine prices even when following a Taylor-like rule; IT fails the first criterion; IT cannot determine prices in an economy with a finite horizon. The conclusion that IT does not lead to price determinacy is consistent with the principle from calculus which states that one cannot recover the original function by merely integrating the derivative.

We also argue that the price indeterminacy of inflation targeting can manifest itself into a liquidity trap or zero bound for nominal interest rates as currently being experienced in many economies of the world.

In our investigation of price determinacy, we were able to discuss global price determinacy rather than just local determinacy because we did not linearize the model around a steady state. Also, we used central-bank feedback rules to result in difference equations involving deviations from the targets that were the same as appeared in most of the price determinacy literature.

However, at this point we add a caveat. We have found an inconsistency concerning "current IT," "current PLT," and "current NGT." The reason is because the previous literature on "current IT" and "current PLT" have not used consistent and rigorous central bank feedback rules. For the final contribution of this paper, we now discuss the logical inconsistency of these central bank feedback rules.

Our formulation of the feedback rules is similar to those by Carlstrum and Fruest (2001), but which is in essence consistent with other researchers as well. For PLT, our feedback rule was equation (10), where the central bank looked at how the current price level differed from the targeted price level when it pegged the nominal interest rate. However, in a flexible price model, the current price level is not yet determined. Thus, the logic of the central bank observing the current price level, then setting the interest rate, which then determines the current price level is really circular logic.

Similarly for IT, the feedback rule (14) had the central bank observing how the current inflation rate compared to the targeted inflation rate as the central bank pegged the interest rate, which then (according to previous literature) would determine the current price level.

Also for NGT, the feedback rule (5) had the central bank observing the current level of NGDP to peg the interest rate, but that current level of NGDP was being determined by the Fisher-Euler equation and the central bank's pegging of that interest rate. A more logically consistent approach would be to look at past prices, inflation, or NGDP instead of current prices, inflation, or NGDP. For PLT and IT, Eagle(2007) does use such feedback rules; while the resulting formulas are more complex, Eagle still reaches the conclusions that PLT leads to price determinacy but IT does not.

Hence, we can look at our basing these feedback rules on current prices, inflation, and NGDP as something that simplifies our results without having an impact on our basic conclusions.

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