Timing of adoption of clean technologies by regulated monopolies

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Abstract: We consider a monopolistic firm producing a good while polluting and using a fossil energy. This firm can adopt a clean technology by incurring an investment cost decreasing exponentially with the adoption date. This clean technology does not pollute and has a lower production cost because it uses a renewable energy. We determine the optimal adoption date for the firm in the case where it is not regulated at all, and in the case where it is regulated at each period of time i.e. the regulator looks for static social optimality. Interestingly, the regulated firm adopts the clean technology earlier than what is socially-optimal. However, the non-regulated firm adopts later than what is socially-optimal. The regulator can induce the firm to adopt at the socially-optimal date by a postpone adoption subsidy. Nevertheless, the regulator may be interested in the earlier adoption of the firm to encourage the diffusion of the use of clean technologies in other industries.

Keywords: static regulation, clean technology, renewable energy, adoption date.

JEL classification: D62, H57, Q42, Q55.

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1. Introduction

We consider a monopolistic firm producing a good using a polluting technology. This can be the case of a producer of electricity like société tunisienne d’élécricité et du gaz (STEG) which has the monopoly power of producing and distributing electricity in Tunisia. This polluting production uses fossil energy. The firm can adopt a clean technology within a finite time by incurring an investment cost decreasing exponentially as the adoption is delayed. The clean production technology is characterized by no pollution emission and a lower production cost because it uses a renewable energy (e.g. solar energy). We consider the case where the firm is not regulated. We also consider the case where the firm is regulated at each period of time by an emission-tax when it uses the polluting technology; when the firm uses the clean production technology, it receives a per-unit production subsidy that can be considered as a fiscal incentive. In the latter case, the regulator looks for static social optimality.

Interestingly, the regulated firm adopts the clean technology in a finite time and earlier than what is socially-optimal. Therefore, in a dynamic setting, instantaneous regulation, which is socially-optimal, may not be dynamically optimal with respect to the adoption of clean technologies. The regulator can compensate the firm for the losses incurred when it accepts to delay its adoption to the socially-optimal adoption date. We also show that the non-regulated firm adopts the clean technology in a finite time but later than what is socially-optimal.

Indeed, when the regulated firm switches to the clean technology, it no longer pays a pollution tax, receives a production subsidy and its production cost decreases. Consequently, its instantaneous net profit increases importantly and that’s why it adopts the clean technology very soon. In the same time, the instantaneous social welfare level increases because there are no environmental damages and production costs are lower. However, this last increase is less important than that of the instantaneous net profit of the regulated firm. For this reason the regulated firm adopts the clean technology earlier than what is socially-optimal. When the non-regulated firm adopts the clean technology, its production cost decreases.
Consequently, its instantaneous net profit increases, but not importantly, and less than the increase of the social welfare. For this reason, the adoption date of the non-regulated firm is later than what is socially-optimal.

This work can be placed in the international context where encouraging the use of renewable energies, such as solar energy or wind energy, in place of fossil energy is one of the most stimulating debates of the recent years. Indeed, countries are more conscious that fossil energy is becoming scarce and they are now experiencing the harmful effects of climate change. Moreover, petrol multinationals have gained too much money in the last decade and are now ready to invest in the promotion of renewable energies.

Dosi and Moretto (1997) studied the regulation of a firm which can switch to a clean technology by incurring an irreversible investment cost. This technological switch is expected to provide benefits surrounded, however, by a certain degree of uncertainty. To bridge the gap between the private and the policy-maker’s desired timing of innovation, they recommended that the regulator stimulate the innovation by subsidies and by reducing the uncertainty surrounding the profitability of the clean technology through appropriate announcements. Dosi and Moretto (2010) extended the previous study to an oligopolistic industry and studied the incentives of not being the first firm adopting the clean technology.

Soest (2005) analyzed the impact of environmental taxes and quotas on the timing of adoption when the date at which improved energy-efficient technologies become available is uncertain, and when the investment decision is irreversible. He found that neither policy instrument is unambiguously preferred to the other. Nasiri and Zaccour (2009) proposed a game-theoretic approach to model and analyze the process of utilizing biomass for power generation. They considered three players: distributor, facility developer, and participating farmer. They characterized the subgame-perfect Nash equilibrium and discussed its features. Ben Youssef (2010) considered a monopolistic firm that can adopt a cleaner technology within a finite time by incurring an investment cost. It has been shown that the socially-optimal adoption date of incomplete information is delayed compared to that of complete information.
Wirl and Withagen (2000) considered a model where a clean technology is available and requires costly investments but is characterized by low variable costs (e.g., solar energy or wind power). They showed that, in a competitive equilibrium, pollution-control policy is not necessarily optimal in the sense of leading to the social optimum. Fischer, Withagen and Toman (2004) developed a model of a uniform good that can be produced by either a polluting or a clean technology. This latter is more expensive and requires investment in capacity. They showed that the optimal transition path is quite different with a clean or dirty initial environment.

Some empirical studies have been interested in clean technologies. Whitehead and Cherry (2007) estimated the annual benefits of the regional amenities associated with a green energy program in North Carolina. Varun, Prakash and Bhat (2009) found that wind and small hydro are the most sustainable sources for the electricity generation. Li et al. (2009) estimated how much US households would be willing to pay annually to support increased energy R&D activities designed to replace fossil fuels. Caspary (2009) assessed the likely competitiveness of different forms of renewable energy in Colombia over the next 25 years. Pillai and Banerjee (2009) reviewed the status and potential of different renewable energies (except biomass) in India and established a diffusion model as a basis for setting targets.

The most important feature of the present work is that the clean technology has a lower production cost than the polluting technology. Moreover, we compare the socially-optimal adoption date and the optimal adoption date of the instantaneous regulated and non-regulated firm. Theses comparisons have not been reported by previous studies. One important question to which we try to respond is whether the regulator should intervene in the adoption of clean technologies by firms, and how?

Our main results are in contrast with the findings of Ben Youssef (2010) who showed that, because of the positive marginal social cost of public funds, the instantaneous net profit of the regulated firm is nil and, consequently, the firm never adopts the cleaner technology unless it receives an innovation subsidy. Also, in Dosi and Moretto (1997) study, the regulator objective is the abandonment of the polluting technology and the adoption of the green one before a “critical” date, whereas in the present paper the regulator maximizes his intertemporal social welfare function for
the determination of the socially-optimal adoption date. Moreover, these authors have not considered the case where the firm is instantaneously regulated.

The paper is structured as follows. Section 2 introduces the model. Section 3 studies the non-regulated firm case. Section 4 studies the instantaneous regulated firm case. In Section 5, we derive the optimal adoption dates and we compare them. Section 6 concludes and an Appendix contains some proofs.

2. The model

We consider a monopolistic firm producing a good in quantity \( q \) sold on the market at price \( p(q) = a - bq \), \( a, b > 0 \).

The consumption of this good gives a consumers’ surplus equal to

\[
CS(q) = \int_0^q p(z) dz - p(q)q = \frac{b}{2} q^2.
\]

At the beginning i.e. at date 0, the firm uses a polluting production technology using fossil fuels and characterized by a positive emission/output ratio \( e > 0 \).

Therefore, the pollution emitted by the firm is \( E = eq \), which causes environmental damages equal to \( D = \alpha E \), where \( \alpha > 0 \) is the marginal disutility of pollution. Let us point out here that we suppose that damages caused to the environment are due to the flow of emissions and not to the stock of pollution.

With the polluting technology, the unit production cost is \( d > 0 \) and the profit of the firm\(^1\) is \( \Pi_d = p(q)q - dq \).

The firm behaves for an infinite horizon of time and can adopt a clean production technology within a period of time \( \tau \). This clean technology does not pollute at all, uses a renewable energy (solar energy for instance) and therefore has a lower unit production cost \( c \) verifying \( 0 < c < d \). Thus, the profit of the firm is \( \Pi_c = p(q)q - cq \).

An investment cost is necessary to get the clean technology. This investment cost could comprise the R&D cost or the cost of acquisition and installation of the clean technology.

\(^1\) In what follows, the subscripts \( d \) and \( c \) refer to the polluting and clean technologies, respectively.
The investment cost of adopting the clean technology at date $\tau$ actualized at date $0$ is:

$$V(\tau) = \theta e^{-m \tau},$$

(1)

where $\theta > 0$ is the cost of immediate adoption of the clean technology, $r > 0$ is the discount rate, and the parameter $m$ denotes that the investment cost of adoption decreases more rapidly when it is greater. We suppose that $m > 1$.\(^2\)

Function $V$ is decreasing because of the existence of freely-available scientific research enabling the firm to reduce the investment cost of adopting the clean technology when it delays its adoption, and is convex because the adoption cost increases more rapidly when the firm tries to accelerate the adoption date. Let’s remark that $\tau = +\infty$ means that the firm never adopts the clean technology.

3. Non-regulated firm

In this section, we will study the case where, at each period of time, the monopoly is not regulated even when it uses the polluting technology.

When it uses the polluting technology, the firm maximizes its profit $\Pi_d$ to get the optimal level of production:\(^3\)

$$q_d^n = \frac{a - d}{2b}$$

(2)

When it uses the clean technology, the firm maximizes its profit $\Pi_c$ to get the optimal level of production:

$$q_c^n = \frac{a - c}{2b}$$

(3)

Because of condition (6), $q_d^n > 0$ and $q_c^n > 0$.

It is easy to verify that the firm produces more with the clean technology because of its lower production cost ($q_c^n > q_d^n$).

\(^2\) The restriction $m > 1$ is necessary for the optimal adoption dates to be positive. Also, it guarantees the second-order condition when determining the optimal adoption dates (see the Appendix).

\(^3\) The superscript $n$ refers to the non-regulation case.
4. Regulated firm

In this section, we study the case where the firm is regulated at each period of time. Rather than directly looking to the socially-optimal regulatory instruments, we will determine the socially-optimal production quantities. Next, we determine the regulatory instruments. Thus, we have a leader-follower relationship where the regulator is a leader and the monopoly is a follower.

When the firm uses the polluting technology, the instantaneous social welfare is equal to the consumers’ surplus, minus damages plus the profit of the firm:

\[ S_d = CS(q) - D(q) + \Pi_d(q) \]  \hspace{1cm} (4)

Maximizing the expression given by (4) with respect to \( q \) gives the socially-optimal production level with the polluting technology:

\[ \hat{q}_d = \frac{a - d - \alpha e}{b} \]  \hspace{1cm} (5)

We assume the following condition so that production quantities are positive:

\[ a > d + \alpha e \]  \hspace{1cm} (6)

Therefore, the maximum willingness to pay for the good must be higher than the marginal cost of production plus the marginal damage of production.

Since the firm is a polluting monopoly, it is regulated. An emission-tax per-unit of pollution \( \lambda \) is sufficient to induce the socially-optimal level of production.

Indeed, the instantaneous net profit of the firm is:

\[ U_d = \Pi_d(q) - \lambda E(q) \]  \hspace{1cm} (7)

The socially-optimal per-unit emission-tax that induces the firm to produce \( \hat{q}_d \) is:

\[ \lambda = \frac{a - d - 2b\hat{q}_d}{e} \]  \hspace{1cm} (8)

Using the expression of \( \hat{q}_d \) given by (5), we can show that:

\[ \lambda > 0 \iff a - d < 2\alpha e \]  \hspace{1cm} (9)

Therefore, the emission-tax is positive when the marginal damage of production is high enough. Otherwise, it is negative meaning that the regulator subsidizes production to deal with the monopoly distortion.
When the firm uses the clean technology, the instantaneous social welfare is equal to the consumers’ surplus plus the profit of the firm:

\[ S_c = CS(q) + \Pi_c(q) \]  \hspace{1cm} (10)

Maximizing the expression given by (10) with respect to \( q \) gives the socially-optimal production level with the clean technology:

\[ \hat{q}_c = \frac{a - c}{b} > 0 \]  \hspace{1cm} (11)

It is easy to verify that \( \hat{q}_c > \hat{q}_d \). Therefore, the clean technology enables to produce more and without pollution.

We can establish that:

\[ \hat{q}_d > q^*_d \iff a - d > 2ae \]  \hspace{1cm} (12)

Indeed, with the polluting technology, the socially-optimal production takes into account both environmental damages and monopoly distortion. That’s why it is higher than the optimal level of production for the non-regulated firm only when the marginal damage of production is low enough. However, with the clean technology, there is no pollution, and we always have \( \hat{q}_c > q^*_c \) as it is commonly known.

Since the production process is clean, the regulator gives the firm a subsidy \( s \) for each unit produced, which can be considered as a fiscal incentive. One may think about production of electricity. A per-unit production subsidy can be given by the regulator when the production process is clean (using solar energy, for instance). This per-unit subsidy is chosen so that it induces the socially-optimal level of production.\(^4\)

Indeed, the instantaneous net profit of the firm is:

\[ U_c = \Pi_c(q) + sq \]  \hspace{1cm} (13)

The socially-optimal per-unit subsidy that induces the firm to produce \( \hat{q}_c \) is:

\[^4\text{Note that, in expressions (4) and (10), taxes and subsidies don’t appear because they are pure transfers from the firm to the regulator. Indeed, we suppose that there is no marginal social-cost of public funds and no transaction costs: the tax diminished from the firm’s profit is added to the consumers’ welfare, and the subsidy added to the firm’s profit is diminished from the consumers’ welfare.}\]
Using the expression of $\hat{q}_c$ given by (11), we can show that $s > 0$.

In the Appendix, we show that:

$$0 < \Pi_c(q^n_c) - \Pi_d(q^n_d) < S_c(\hat{q}_c) - S_d(\hat{q}_d) < U_c(\hat{q}_c) - U_d(\hat{q}_d)$$  \hspace{1cm} (15)

The above inequalities enable us to establish the following proposition:

**Proposition 1.** The instantaneous gain from using the clean technology is greater for the regulated firm than for the regulator. This latter instantaneously benefits more from the clean technology than the non-regulated firm.

Indeed, when the regulated firm adopts the clean technology, it no longer pays a pollution tax, receives a production subsidy and its unit production cost decreases. This increases its instantaneous net profit significantly. The instantaneous social welfare level increases because of the absence of environmental damages and the lower production costs. However, this last increase is less important than that of the regulated firm. The unique benefit of the non-regulated firm from adopting the clean technology is the reduction of its unit production cost. Consequently, its instantaneous net profit increase is less important than that of the instantaneous social welfare.

5. Optimal adoption dates

The intertemporal payoffs, of the regulator or the firm, are equal to the instantaneous payoffs actualized at date zero minus the investment cost of adopting the clean technology at date $\tau$. Therefore, the intertemporal social welfare, intertemporal net profits of the regulated firm and non-regulated firm are, respectively:

$$IS(\tau) = \int_0^\tau S_d(\hat{q}_d)e^{-\tau} dt + \int_\tau^{\infty} S_c(\hat{q}_c)e^{-\tau} dt - \theta e^{-m\tau}$$  \hspace{1cm} (16)

$$IU(\tau) = \int_0^\tau U_d(\hat{q}_d)e^{-\tau} dt + \int_\tau^{\infty} U_c(\hat{q}_c)e^{-\tau} dt - \theta e^{-m\tau}$$  \hspace{1cm} (17)
In order to have positive adoption dates, we need the following condition, which can be always verified by choosing the parameters $\theta$ and/or $m$ high enough:\footnote{Notice that the left expression of (19) is independent of parameters $\theta$, $m$ and $r$.}

\[ U_c(\hat{q}_c) - U_d(\hat{q}_d) < \theta m s \]

(19)

The regulator or the firm maximizes its intertemporal payoff function with respect to $\tau$ to get the optimal adoption date. In the Appendix, we determine the socially-optimal adoption date, the optimal adoption dates for the regulated and nonregulated firm, which are respectively:

\[
\hat{\tau} = \frac{1}{(1-m)r} \ln \left( \frac{S_c(\hat{q}_c) - S_d(\hat{q}_d)}{\theta m s} \right) > 0
\]

(20)

\[
\tau^* = \frac{1}{(1-m)r} \ln \left( \frac{U_c(\hat{q}_c) - U_d(\hat{q}_d)}{\theta m s} \right) > 0
\]

(21)

\[
\tau^n* = \frac{1}{(1-m)r} \ln \left( \frac{\Pi_c(q^n_c) - \Pi_d(q^n_d)}{\theta m s} \right) > 0
\]

(22)

**Proposition 2.** We have the following ranking for the optimal adoption dates:

\[ 0 < \tau^* < \hat{\tau} < \tau^n* \]

Therefore, the optimal adoption date for the regulated firm is earlier than the socially-optimal adoption date, which is earlier than the optimal adoption date for the non-regulated firm.

*Proof.* See the Appendix.

The above results show that socially-optimal instantaneous regulation may not be dynamically optimal with respect to the adoption of clean technologies. They are due to the fact that the incentives to adopt are, in order, greater for the regulated firm, the regulator and the non-regulated firm. This is clearly established by the inequalities in (15).

Paradoxically, if the regulator desires that the regulated firm delays its adoption to the socially-optimal adoption date, he must compensate the firm for the losses it incurs by this adoption delay. If we consider our previous example where the firm is
given a subsidy for each unit of electricity produced with the clean technology using solar energy; one important reason for the very early adoption by the regulated firm is the per-unit production subsidy received from the regulator; this engenders important investment cost of adoption for the intertemporal social welfare; to overcome this, we propose that the regulator gives a postpone adoption subsidy to the firm.

If the intertemporal net profits of the regulated firm are $IU(\tau^*)$ and $IU(\hat{\tau})$ when the adoption dates are $\tau^*$ and $\hat{\tau}$, respectively, then the postpone adoption subsidy (compensation) is:

$$g = IU(\tau^*) - IU(\hat{\tau}) > 0$$

**Proposition 3.** The regulator can push the regulated firm to delay its adoption of the clean technology by giving it a postpone adoption subsidy that compensates the firm for the losses it incurs when the latter delays its optimal adoption date to the socially-optimal adoption date.

6. Conclusion

In this paper, we consider a monopolistic firm producing a good using a polluting technology. However, this firm can adopt a clean technology within a finite time by incurring an investment cost decreasing exponentially as the adoption is delayed. The clean production technology is characterized by no pollution emission and by a lower production cost because it uses a renewable energy. We consider the case where the firm is not regulated. We also consider the case where the firm is regulated at each period of time by an emission-tax when it uses the polluting technology, and by a production subsidy when it uses the clean one. The regulator maximizes the instantaneous social welfare.

When the regulated firm switches to the clean technology, it no longer pays a pollution tax, receives a production subsidy and its unit production cost decreases. Consequently, its instantaneous net profit increases significantly. The instantaneous social welfare level increases because of the absence of environmental damages and the lower production costs. However, this instantaneous benefit of social welfare
from the clean technology is less important than that of the firm. When the non-
regulated firm adopts the clean technology, its unit production cost decreases.
Consequently, its instantaneous net profit increases, but not importantly, and less
than the increase of the instantaneous social welfare. These results induce the
following.

The non-regulated firm adopts the clean technology in a finite time but later than
what is socially-optimal. Interestingly, the regulated firm adopts the clean
technology in a finite time and earlier than what is socially-optimal. The regulator
can compensate the firm for the losses incurred if he desires that the firm delays its
adoption to the socially-optimal adoption date. However, the regulator may be
interested by allowing the firm to adopt earlier to encourage the diffusion of the use
of clean technologies in other industries.

Appendix

A) Instantaneous gains from the clean technology

*From expressions (4) and (10), we have:

\[ S_c(\hat{q}_c) - S_d(\hat{q}_d) = \left[ a - \frac{b}{2}(\hat{q}_d + \hat{q}_c) - c \right] (\hat{q}_c - \hat{q}_d) + (d - c)\hat{q}_d + \alpha e \hat{q}_d \]

By using the expressions of \( \hat{q}_d \) and \( \hat{q}_c \) given by (5) and (11), the above bracketed
expression is equal to \( \frac{d - c + \alpha e}{2} \). Therefore, we have:

\[ S_c(\hat{q}_c) - S_d(\hat{q}_d) = \frac{d - c + \alpha e}{2} (\hat{q}_c + \hat{q}_d) > 0 \]

(25)

*From expressions (7) and (13), we have:

\[ U_c(\hat{q}_c) - U_d(\hat{q}_d) = \left[ a - b(\hat{q}_c + \hat{q}_d) \right] (\hat{q}_c - \hat{q}_d) + (s - c)\hat{q}_c + d\hat{q}_d + \lambda e \hat{q}_d \]

By changing the emission tax \( \lambda \) and the production subsidy \( s \) by their expressions in
function of \( \hat{q}_d \) and \( \hat{q}_c \) given by (8) and (14), we obtain:

\[ U_c(\hat{q}_c) - U_d(\hat{q}_d) = b(\hat{q}_c^2 - \hat{q}_d^2) > 0 \]

(26)

*We can easily show that:

\[ \Pi_c(q_c^n) - \Pi_d(q_d^n) = \left[ a - b(q_c^n + q_d^n) \right] (q_c^n - q_d^n) + dq_d^n - cq_c^n \]
By replacing \( q^n_c \) and \( q^n_d \) between the above brackets by their values given by (2) and (3), we get:

\[
\Pi_c(q^n_c) - \Pi_d(q^n_d) = \frac{d - c}{2}(q^n_c + q^n_d) > 0
\]  

(27)

Therefore, the clean technology improves the instantaneous social welfare when production levels are socially-optimal. It also increases the instantaneous net profit of both regulated and non-regulated firm.

**B) Comparison of the instantaneous gains**

*By using expressions (25) and (26), we have:

\[
(U_c(\hat{q}_c) - U_d(\hat{q}_d)) - (S_c(\hat{q}_c) - S_d(\hat{q}_d)) = \left[b(\hat{q}_c - \hat{q}_d) - \frac{d - c + \alpha e}{2}\right](\hat{q}_c + \hat{q}_d)
\]

By using the expressions of \( \hat{q}_d \) and \( \hat{q}_c \), given by (5) and (11), in the above bracketed expression, we obtain:

\[
(U_c(\hat{q}_c) - U_d(\hat{q}_d)) - (S_c(\hat{q}_c) - S_d(\hat{q}_d)) = \frac{d - c + \alpha e}{2}(\hat{q}_c + \hat{q}_d) > 0
\]  

(28)

*By using expressions (25) and (27), we get:

\[
(S_c(\hat{q}_c) - S_d(\hat{q}_d)) - \left[\Pi_c(q^n_c) - \Pi_d(q^n_d)\right] = \frac{d - c + \alpha e}{2}(\hat{q}_c + \hat{q}_d) - \frac{d - c}{2}(q^n_c + q^n_d)
\]

\[
= \frac{d - c}{2}[\hat{q}_c + \hat{q}_d - q^n_c - q^n_d] + \frac{\alpha e}{2}(\hat{q}_c + \hat{q}_d)
\]

By replacing \( q^n_d \), \( q^n_c \), \( \hat{q}_d \) and \( \hat{q}_c \), in the above brackets by their values, given by (2), (3), (5) and (11), we obtain:

\[
(S_c(\hat{q}_c) - S_d(\hat{q}_d)) - \left[\Pi_c(q^n_c) - \Pi_d(q^n_d)\right] = \frac{d - c}{2}\left[\frac{2a - c - d - 2\alpha e}{2b}\right] + \frac{\alpha e}{2}(\hat{q}_c + \hat{q}_d)
\]

Using condition (6) for the above bracketed term gives:

\[
(S_c(\hat{q}_c) - S_d(\hat{q}_d)) - \left[\Pi_c(q^n_c) - \Pi_d(q^n_d)\right] > \frac{(d - c)^2}{4b} + \frac{\alpha e}{2}(\hat{q}_c + \hat{q}_d) > 0
\]  

(29)

Thus, we have the following ranking:

\[
0 < \Pi_c(q^n_c) - \Pi_d(q^n_d) < S_c(\hat{q}_c) - S_d(\hat{q}_d) < U_c(\hat{q}_c) - U_d(\hat{q}_d)
\]  

(30)
The instantaneous gain from using the clean technology is greater for the regulated firm than for the regulator, which benefits from the clean technology more than the non-regulated firm.

C) Optimal adoption dates

*To get the socially-optimal adoption date, the regulator maximizes his intertemporal social welfare function given by (16) with respect to \( \tau \):

\[
\frac{\partial IS}{\partial \tau} = (S_d(\hat{q}_d) - S_c(\hat{q}_c))e^{-r\tau} + \theta mre^{-mr\tau} = 0
\]  

Equation (31) is equivalent to:

\[
S_d(\hat{q}_d) - S_c(\hat{q}_c) + \theta mre^{(1-m)r\tau} = 0 \iff \hat{\tau} = \frac{1}{(1-m)r}\ln\left(\frac{S_c(\hat{q}_c) - S_d(\hat{q}_d)}{\theta m}\right)  
\]  

Because of \( m>1 \), condition (19) and inequality (30), \( \hat{\tau} > 0 \).

We have: \( \frac{\partial^2 IS}{\partial \tau^2} = r(S_c(\hat{q}_c) - S_d(\hat{q}_d))e^{-r\tau} - \theta (mr)^2 e^{-mr\tau} \).

Using the first-order condition given by (31), we get:

\[
\frac{\partial^2 IS(\hat{\tau})}{\partial \tau^2} = (1-m)\theta r^2 e^{-mr\hat{\tau}} < 0
\]

The second-order condition of optimality is verified.

*The regulated firm maximizes its intertemporal net profit given by (17) with respect to \( \tau \):

\[
\frac{\partial IU}{\partial \tau} = (U_d(\hat{q}_d) - U_c(\hat{q}_c))e^{-r\tau} + \theta mre^{-mr\tau} = 0
\]  

Equation (33) is equivalent to:

\[
U_d(\hat{q}_d) - U_c(\hat{q}_c) + \theta mre^{(1-m)r\tau} = 0 \iff \tau^* = \frac{1}{(1-m)r}\ln\left(\frac{U_c(\hat{q}_c) - U_d(\hat{q}_d)}{\theta m}\right)
\]  

Because of \( m>1 \) and inequality (19), \( \tau^* > 0 \).

We have: \( \frac{\partial^2 IU}{\partial \tau^2} = r(U_c(\hat{q}_c) - U_d(\hat{q}_d))e^{-r\tau} - \theta (mr)^2 e^{-mr\tau} \).

Using the first-order condition given by (33), we obtain:

\[
\frac{\partial^2 IU(\tau^*)}{\partial \tau^2} = (1-m)\theta r^2 e^{-mr\tau^*} < 0
\]
Therefore, the second-order condition of optimality is verified.

*The non-regulated firm maximizes its intertemporal net profit given by (18) with respect to $\tau$:

$$\frac{\partial U^n}{\partial \tau} = (\Pi_d(q^n_d) - \Pi_c(q^n_c))e^{-r\tau} + \theta mr e^{-mr\tau} = 0$$  \hspace{1cm} (35)

The above equality implies:

$$\Pi_d(q^n_d) - \Pi_c(q^n_c) + \theta mr (1-m)r\tau = 0 \iff \tau^n = \frac{1}{(1-m)r} \ln \left( \frac{\Pi_c(q^n_c) - \Pi_d(q^n_d)}{\theta mr} \right)$$  \hspace{1cm} (36)

Because of $m>1$, inequalities (19) and (30), $\tau^n > 0$.

We have:

$$\frac{\partial^2 U^n}{\partial \tau^2} = r(\Pi_c(q^n_c) - \Pi_d(q^n_d))e^{-r\tau} - \theta (mr)^2 e^{-mr\tau}.$$

Using the first-order condition given by (35), we obtain:

$$\frac{\partial^2 U^n(\tau^n)}{\partial \tau^2} = (1-m)m^2 e^{-mr\tau^n} < 0$$

The second-order condition of optimality is verified.

**D) Comparison of the optimal adoption dates**

Inequalities (30) and the assumption $m>1$, enable us to make the following ranking:

$$0 < \tau^* < \hat{\tau} < \tau^n$$

The regulated firm adopts sooner than what is socially-optimal, whereas the non-regulated firm adopts later.

References


