A Monetary Union Model with Cash-in-Advance Constraints

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Abstract

We characterize the monetary competitive equilibrium in a two-country monetary union model involving cash-in-advance constraints both in the factor markets and in the good markets. Simulations show that common money inflation in the union have asymmetric effects on the welfare of workers in the two countries which are technologically differentiated. We also find that the distribution of the money stock within the union may affect labor flow across the countries.

Keywords: Monetary union, cash-in-advance, monetary policy.

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1 Introduction

This paper studies the effects of monetary policy on the flow and welfare of labor within an open-economy monetary union facing liquidity constraints. In closed economy models, implications of cash-in-advance constraints in production and/or investment expenses on equilibrium were successfully studied, for example, by Stockman (1980), Grossman and Weiss (1983) and Rotemberg (1984). Their applications to the business cycles literature have also been considerably successful (Fuerst, 1992, Christiano, Eichenbaum and Evans, 1997, 1998). However, the literature that incorporates liquidity constraints into open economy models is quite thin. Amongst the notable studies, the one by Carlstrom and Fuerst (1999) considers a small open economy operating with fiat money to deal with the issues of optimal money inflation and the indeterminacy of equilibria. Arseneau (2004) investigates the optimal inflation in a two-country open economy with imperfect competition.

Unlike the previous literature, we integrate in this paper the open economy framework with cash-in-advance models allowing for factor trade between countries. Our model that partially borrows from the closed economy models of Basci and Saglam (1999, 2003, 2005) considers a two-country monetary union with a common central bank. We assume that there are infinitely-lived representative workers and producers in each country. Workers own labor endowments which are perfectly mobile between countries whereas producers have constant returns to scale technologies in each time period. Countries are asymmetric with respect to their production technologies. Using labor as the single factor of production, each country produces a single good, which is sold (at different prices) and consumed in both countries. We assume that in each country the labor market opens before the good market and all transactions occur by the common currency of the union. The common central bank determines the uniform money growth rate, while the implied money supply in each country is controlled through money transfers to - or taxes on - producers. In each period, producers derive utility from the consumption of the two goods whereas workers receive utility from both consumption and leisure.

We characterize the monetary competitive equilibrium in the described monetary union model and run some computer simulations to analyze the effects of the distribution and the rate of growth of the aggregate money stock in the monetary union on the welfare and flow of labor.

The paper proceeds as follows. Section 2 presents the model. Section 3 contains the characterization of the monetary competitive equilibrium and simulation results. Finally, Section 4 concludes.

2 Model

2.1 Basic Structures

2.1.1 Monetary Union

We consider a monetary union involving two countries, denoted by h and f. Countries are indexed by j. The unionized economy operates with fiat money. By m_t^j we denote the aggregate money stock in country j at the end of period t. We assume that m_0^j is given, and money stock evolves over time according to $m_{t+1}^j = (1+\alpha)m_t^j$, where α is the common growth rate of money in the union.

2.1.2 Workers and Producers

Each country is populated with two types of agents, workers, w, and producers, p. Agents are indexed by i. All agents live for infinite periods. There is no population growth. In every time period, the numbers of workers and producers in country j are denoted by n^{wj} and n^{pj}, respectively.

Workers in country j have constant labor endowments, $\bar{l}^{wj}$, in each period of their lives. They value leisure in units of the consumption goods through the function $v^{wj}(x,y) = \ln(x+\delta^j) - \rho^j y$, which is increasing in its first argument, the leisure, and decreasing in its second argument, the labor supplied in the neighboring country. We assume that $\rho^f > \rho^h$ and $\delta^h, \delta^f > 1$ to ensure that labor asymmetrically flows from country h to country f in equilibrium.

Producers in country j have no labor endowments; instead they own a constant returns to scale production technology, $f^{pj}(x) = \eta^j x$ with $\eta^j > 0$ to convert any labor, $x \geq 0$, into a country-specific good, called good j, which is consumed in the whole union. We assume that the technology in country f is more productive, i.e. $\eta^f > \eta^h$. (For notational convenience, we also introduce $\bar{l}^{pj} = 0$, $f^{wj}(x) = 0$, and $v^{p,j}(x,y) = 0$ for all j and for all nonnegative x and y.)

Each agent i in country j has the life-time utility

$$W^{ij} = \sum_{t=0}^{\infty} (\beta^{ij})^t [\gamma^{ij} \ln(c_t^{ijj}) + (1-\gamma^{ij}) \ln(c_t^{ijk}) + v^{ij}(\bar{l}^{ij} + l_t^{ijj} + l_t^{ijk}, l_t^{ijk})]$$

for $k = \{h,f\} \setminus \{j\}$ and $\beta^{ij}, \gamma^{ij} \in (0,1)$. Here, c_t^{ijr} denotes consumption of each agent $i \in \{w,p\}$ in country $j \in \{h,f\}$ of good $r \in \{h,f\}$ and l_t^{ijr} denotes labor demand (supply if negative) of each agent $i \in \{w,p\}$ from country $j \in \{h,f\}$ in country $r \in \{h,f\}$.

2.1.3 Money Transfers

The common central bank changes the money stock in the union through lump-sum transfers/taxes at the beginning of each period. Each agent i in country j enters period t with the end of period $t-1$ cash balance m_{t-1}^{ij}. While no worker gets money transfer during her lifetime ($x_t^{wj} = 0$, for all t), each producer receives $x_t^{pj} = (1+\alpha)m_{t-1}^j / n^{pj}$ units of money transfer at the beginning of period t.

2.2 Transactions

The timing of transactions is as follows: We assume that the labor market opens before the good market in each country. Each type i agent in country j starts period t with the money balance $m_{t-1}^{ij}+x_t^{ij}$ that is equal to the sum of the balance carried from the end of period $t-1$ and money transfers/taxes. Then labor markets open simultaneously. Factor trades take place at the nominal wage rates w_t^h and w_t^f in countries h and f, respectively, and all wage bills are paid in advance of production. Production of country-j good occurs with the labor l_t^{pjh} and l_t^{pjf} that producers in country j employ from countries h and f, respectively. Good markets open simultaneously. Transactions are made by cash at the price p_t^{jk} for $j,k \in \{h,f\}$. The end-of-period cash balances realize as $m_t^{ij} = m_{t-1}^{ij}+x_t^{ij} - w_t^j l_t^{ijj} - w_t^k l_t^{ijk} - p_t^{jj} q_t^{ijj} - p_t^{jk} q_t^{ijk}$, where $k = \{h,f\} \setminus \{j\}$, and q_t^{ijr} denotes commodity demand (supply if negative) of each agent $i \in \{w,p\}$ from country $j \in \{h,f\}$ for (of) good $r \in \{h,f\}$.

2.3 Agents' Problems

The representative agent of type i in country j faces the following maximization problem given her life time endowment structure, $\bar{l}^{ij}$, and strictly positive prices $\{w_t^h, w_t^f, p_t^{hh}, p_t^{hf}, p_t^{fh}, p_t^{ff}\}$:

$$\max \sum_{t=0}^{\infty} (\beta^{ij})^t [\gamma^{ij} \ln(c_t^{ijj}) + (1-\gamma^{ij}) \ln(c_t^{ijk}) + v^{ij}(\bar{l}^{ij} + l_t^{ijj} + l_t^{ijk}, l_t^{ijk})], \qquad k \neq j$$

subject to:

$$m_t^{ij} = m_{t-1}^{ij} + x_t^{ij} - w_t^h l_t^{ijh} - w_t^f l_t^{ijf} - p_t^{jh} q_t^{ijh} - p_t^{jf} q_t^{ijf} \tag{1}$$

$$-\bar{l}^{ij} \leq l_t^{ijh} + l_t^{ijf} \leq (m_{t-1}^{ij} + x_t^{ij})/w_t^j \tag{2}$$

$$c_t^{ijj} = q_t^{ijj} + f^{ij}(\bar{l}^{ij} + l_t^{ijh} + l_t^{ijf}) \tag{3}$$

$$c_t^{ijk} = q_t^{ijk}, \qquad k \neq j \tag{4}$$

$$-p_t^{ij} f^{ij}(\bar{l}^{ij} + l_t^{ij}) \leq p_t^{jh} q_t^{ijh} + p_t^{jf} q_t^{ijf} \leq m_{t-1}^{ij} + x_t^{ij} - w_t^h l_t^{ijh} - w_t^f l_t^{ijf} \tag{5}$$

Equation (1) describes end-of-period cash holdings. Constraint (2) must be read as that in each period supply of labor is bounded from above by labor endowments of workers whereas demand for labor is constrained by the amount of cash with which producers enter the factor market. Constraints (3) and (4) state that consumption of each agent equals the sum of her production (if any) net of sales. Constraint (5) imposes technology and liquidity constraints in the goods market.

2.4 Monetary Competitive Equilibrium

We are now ready to define our equilibrium concept.

Definition 1. The list $\langle w_t^j, p_t^{jk}, c_t^{ijk}, l_t^{ijk}, q_t^{ijk}, m_t^{ij} | t \in \{0,1,...\infty\}$, $i \in \{w,p\}$ and $j,k \in \{h,f\}\rangle$ is a *Monetary Competitive Equilibrium* (MCE) for the described economy if $w_t^j, p_t^{jh}, p_t^{jf} > 0$ for all j, and

(i) $\langle c_t^{ijk}, l_t^{ijk}, q_t^{ijk}, m_t^{ij} | k \in \{h,f\}\rangle$ solves the maximization problem of each agent i in country

j under the sequence $\{w_t^h, w_t^f, p_t^{jh}, p_t^{jf}\}$,

(ii) $n^{wh} l_t^{whj} + n^{wf} l_t^{wfj} + n^{ph} l_t^{phj} + n^{pf} l_t^{pfj} = 0$,

(iii) $n^{wh} q_t^{whj} + n^{wf} q_t^{wfj} + n^{ph} q_t^{phj} + n^{pf} q_t^{pfj} = 0$,

(iv) $n^{wj} m_t^{wj} + n^{pj} m_t^{pj} = m_t^j$.

Condition (i) states that representative agents make optimal choices under perfect foresight of future prices and price taking behaviour. Conditions (ii)-(vi) ensure the clearing of the three markets.

3 Results

Below, we characterize the monetary competitive equilibrium of the described economy.

Proposition 1. *Monetary Competitive Equilibrium* $\langle w_t^j, p_t^{jk}, c_t^{ijk}, l_t^{ijk}, q_t^{ijk}, m_t^{ij} |$ *for* $t \in \{0, 1, \ldots, \infty\}$, $i \in \{w, p\}$ *and* $j, k \in \{h, f\}\rangle$ *exists if* $(v_1^{wf} + v_2^{wf}) - (v_1^{wf} w_t^h)/w_t^f < 0$ *and is uniquely characterized by (6)-(20):*

$$\frac{w_t^j}{p_t^{jj}} = \eta^j \tag{6}$$

$$\frac{w_t^h}{w_t^f} = \frac{l_t^{whf}}{l_t^{whf} - \rho^h(\bar{l}^{wh} + l_t^{whh} + l_t^{whf} + \delta^h)} \tag{7}$$

$$\frac{w_t^j}{p_t^{jj}} = \frac{q_t^{wjj}}{\gamma^{wj}(\bar{l}^{wj} + l_t^{wjh} + l_t^{wjf} + \delta^j)} \tag{8}$$

$$\frac{p_t^{jj}}{p_t^{jk}} = \frac{\gamma^{ij}}{1-\gamma^{ij}} \frac{q_t^{pjk}}{q_t^{pjj} + \eta^j(l_t^{pjh} + l_t^{pjf})}, \quad k \neq j \tag{9}$$

$$\frac{p_t^{jj}}{p_t^{jk}} = \frac{\gamma^{ij}}{1-\gamma^{ij}} \frac{q_t^{wjk}}{q_t^{wjj}}, \quad k \neq j \tag{10}$$

$$q_t^{wjj} = -\frac{\gamma^{wj}}{p_t^{jj}}(w_t^h l_t^{wjh} + w_t^f l_t^{wjf}) \tag{11}$$

$$q_t^{wjk} = \frac{\gamma^{wj} - 1}{p_t^{jk}}(w_t^h l_t^{wjh} + w_t^f l_t^{wjf}), \quad k \neq j \tag{12}$$

$$q_t^{pjj} = -\frac{\gamma^{pj}}{p_t^{jj}} m_t^{pj} + (1-\gamma^{pj})\eta^j \tag{13}$$

$$q_t^{pjk} = \frac{p_t^{jj}}{p_t^{jk}} \frac{1-\gamma^{pj}}{\gamma^{pj}} (q_t^{pjj} + \eta^j), \quad k \neq j \tag{14}$$

$$l_t^{pjj} = \frac{m_{t-1}^{pj} + x_t^{pj}}{w_t^j} \tag{15}$$

$$l_t^{wfh} = 0 \tag{16}$$

$$m_t^{wj} = 0 \tag{17}$$

$$m_t^{pj} = \frac{m_t^j}{n^{pj}} \tag{18}$$

$$c_t^{ijj} = q_t^{ijj} + f^{ij}(\bar{l}^{ij} + l_t^{ijh} + l_t^{ijf}) \tag{19}$$

$$c_t^{ijk} = q_t^{ijk}, \quad k \neq j \tag{20}$$

Proof. See Appendix.

An immediate remark about the characterized equilibrium is that the rate of money growth has no effect on the real wage rate in any of the two countries, as observed from equation (6). However, a further analytical (in)validation of a superneutrality result is not possible, since there exists no closed-form solution for the other model variables. Thus, we run simulations to uncover the real effects of money inflation in an artificial economy. The values for the model parameters are listed in Table 1.

TABLE 1

Parameter Values

Utility from Consumption	$\gamma^{wh} = \gamma^{wf} = \gamma^{ph} = \gamma^{pf} = 0.5$
Utility from Leisure	$\delta^h = \delta^f = 1.2,\ \rho^h = 0.001,\ \rho^f = 0.002$
Time Preference	$\beta^{wh} = \beta^{wf} = \beta^{ph} = \beta^{pf} = 0.05$
Labor Endowment	$\bar{l}^{wh} = \bar{l}^{wf} = 5$
Productivity	$\eta^h = 1.5,\ \eta^f = 2$
Population	$n^{wh} = n^{wf} = n^{ph} = n^{pf} = 1$
Policy Variable	$\alpha \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$

For a symmetric distribution of the money stock in the union, with $m_0^h = m_0^f = 50$, we observe no labor flow between the two countries. Each worker in country h supplies 88.4 percent of her labor endowment while the supply rate is only 62 percent for each worker in country f, where the production technology is more efficient. We find that the labor in both countries is supplied independently from the growth rate of money stock. However, money inflation clearly affects the (relative) prices of the goods, hence the trade and consumption of the goods within the union. Figure 1 shows that the prices of good h in the two countries are increasing with the money growth rate, albeit at a much faster rate in contry h. (In the below figures, superscripts of the relevant variables appear inside parantheses, while *alpha* denotes α).

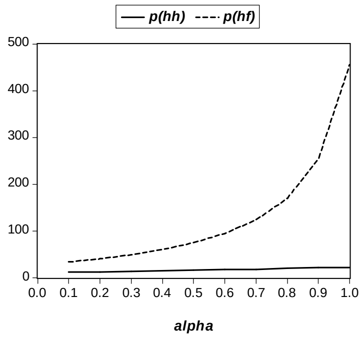

Figure 1

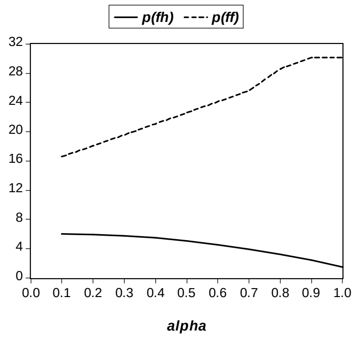

Figure 2

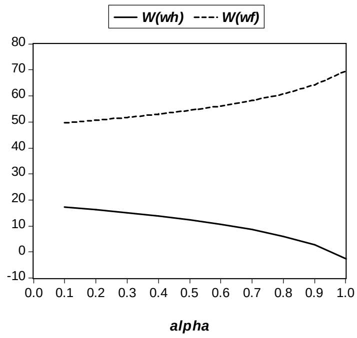

Figure 3

Similarly, Figure 2 illustrates a positive relation between money inflation and the price of good f in its country of production, p^{ff}. But the graph of price p^{fh} of good f in country h, as shown in the same figure, is clearly decreasing in the money growth rate. The positive relation between money inflation and the relative competitiveness of country-f good in country h significantly affects the welfare of workers in the two countries through the channel of good trades. Figure 3 establishes that with higher rates of money growth, the lifetime utility of each worker becomes lower in country h, whereas higher in country f, where the production technology is more efficient.

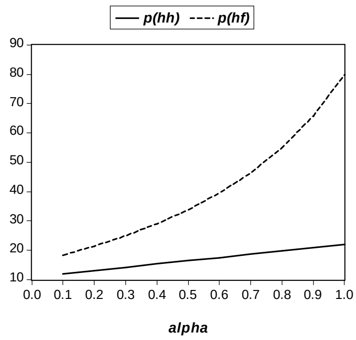

Figure 4

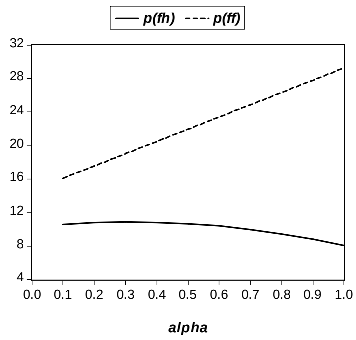

Figure 5

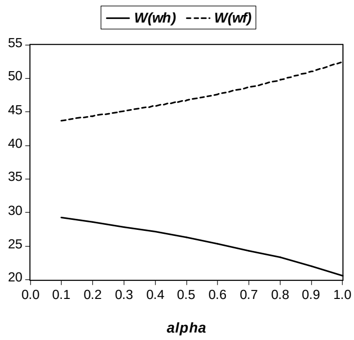

Figure 6

When the distribution of the money stock within the union changes in favor of country f, with $m_0^h = 40$ and $m_0^f = 60$, we have similar nominal effects as well as welfare effects of money inflation, as exhibited in Figures 4-6. However, in the resulting equilibria we observe factor trades, as well. While each worker in country f remains to supply (still 62 percent of her endowed) labor only to her domestic country, workers in country h now sell 7.4 percent of their labor endowment to producers in country f, while 73 percent to producers in their home country.

4 Conclusion

In this paper, we characterize the monetary competitive equilibrium in a two-country monetary union where agents face liquidity constraints both in the factor markets and in the good markets. Our simulations show that the expansion of the money stock in the union yields some real effects through the channel of the relative prices of the goods traded in the two countries. In response to a rise in the common money growth rate within the union, the welfare of each worker increases in the country where production is technologically superior while it decreases in the rest of the union. We also establish that the distribution of the total money stock may affect the factor trade within the union. A reduction in the share of the money stock held by the technologically inferior country may lead to a partial flow of its labor into the technologically superior country, while money inflation seems to have no impacts on the labor flow and the domestic labor supply.

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Appendix

Proof of Proposition 1. We will show that every MCE satisfies (6)-(20). Given the problems of workers and producers in the two countries, we have the following first order conditions:

First Order Conditions for Workers in Country j:

$$(\beta^{wj})^t \gamma U'(q_t^{wjj}) - \lambda_t^{wj} p_t^{jj} = 0 \tag{21}$$

$$(\beta^{wj})^t (1-\gamma) U'(q_t^{wjk}) - \lambda_t^{wj} p_t^{jk} = 0, \quad k \neq j \tag{22}$$

$$(\beta^{wj})^t v_1^j - \lambda_t^{wj} w_t^j = 0 \tag{23}$$

$$(\beta^{wj})^t (v_1^j + v_2^j) - \lambda_t^{wj} w_t^k \begin{cases} = 0 & \text{if } j = h \text{ and } k = f \\ < 0 & \text{if } j = f \text{ and } k = h \end{cases} \tag{24}$$

$$m_t^{wj} + p_t^{jh} q_t^{wjh} + p_t^{jf} q_t^{wjf} = -w_t^h l_t^{wjh} - w_t^f l_t^{wjf} + m_{t-1}^{wj} \tag{25}$$

First Order Conditions for Producers in Country j:

$$(\beta^{pj})^t \gamma U'(q_t^{pjj} + f(\bar{l}^{pj} + l_t^{pjh} + l_t^{pjf})) - \lambda_t^{pj} p_t^{jj} = 0 \tag{26}$$

$$(\beta^{pj})^t (1-\gamma) U'(q_t^{pjk}) - \lambda_t^{pj} p_t^{jk} = 0, \quad k \neq j \tag{27}$$

$$(\beta^{pj})^t \gamma U'(q_t^{pjj} + f(\bar{l}^{pj} + l_t^{pjh} + l_t^{pjf})) f'(\bar{l}^{pj} + l_t^{pjh} + l_t^{pjf}) - \lambda_t^{pj} w_t^j = 0 \tag{28}$$

$$m_t^{pj} + p_t^{jh} q_t^{pjh} + p_t^{jf} q_t^{pjf} = -w_t^h l_t^{pjh} - w_t^f l_t^{pjf} + m_{t-1}^{pj} + x_t^{pj} \tag{29}$$

In addition, we have the following market clearing conditions:

Labor Market Clearing Condition in Country j:

$$n^{wj}(l_t^{wjj}) + n^{wk}(l_t^{wkj}) + n^{pj}(l_t^{pjj}) = 0, \quad k \neq j \tag{30}$$

Good Market Clearing Condition in Country j:

$$n^{wh}(q_t^{whj}) + n^{wf}(q_t^{wfj}) + n^{ph}(q_t^{phj}) + n^{pf}(q_t^{pfj}) = 0 \tag{31}$$

Money Market Clearing Condition in Country j:

$$n^{wj}m_t^{wj} + n^{pj}m_t^{pj} = m_t^j \tag{32}$$

First note that the plans (6)-(20) satisfy aggregate feasibility (market clearing) conditions (30)-(32). Moreover, they satisfy all individual feasibility conditions. On the workers' side constraint (2) is satisfied at the interior while constraint (5) is satisfied at the upper bound. On the producers' side constraint (2) is satisfied at the upper bound, while constraint (5) is satisfied in the interior. Equality constraints (1), (3) and (4) clearly hold.

Equations (11)-(14) follow from (1) at the optimal choices of money holding while (19) and (20) are restatements of (3) and (4) in equilibrium. If $w_t^j/p_t^{jj} \leqslant f'(\bar{l}^{pj} + l_t^{pjj} + l_t^{pjk})$ then cash-in-advance constraints are binding for producers in the labor market; hence $l_t^{phh} = (m_{t-1}^{ph} + x_t^{ph})/w_t^h$ and $l_t^{pff} = (\, m_{t-1}^{pf} + x_t^{pf})/w_t^f$ as in (15). Equality (6) is obtained from the first order conditions (26)-(29) for producers. (9) and (10) are marginal rate of substitution conditions for producers. Condition $(v_1^{wf} + v_2^{wf}) - (v_1^{wf}w_t^h)/w_t^f < 0$ ensures that $l_t^{wfh} = 0$ by the first-order condition (24) for each worker's problem in country f, as in (16). Finally (7) and (8) derive from the first-order conditions (21)-(25) for workers in the two countries.

(17) and (18) follow from the optimality conditions along with the money market clearing condition (32). To see this, differentiate twice the lifetime utility of each agent i in country j with respect to her money demand m_t^{ij}. We have

$$\partial W^{wj}/\partial m_t^{wj} = \gamma^{wj}\frac{1}{q_t^{wjj}}\left(-\frac{1}{p_t^{jj}}\right) + (1-\gamma^{wj})\frac{1}{q_t^{wjk}}\left(-\frac{1}{p_t^{jk}}\right) < 0$$

and

$$\partial^2 W^{wj}/\partial (m_t^{wj})^2 = -\gamma^{wj}\left(\frac{1}{q_t^{wjj}}\right)^2\left(\frac{1}{p_t^{jj}}\right)^2 - (1-\gamma^{wj})\left(\frac{1}{q_t^{wjk}}\right)^2\left(\frac{1}{p_t^{jk}}\right)^2 < 0$$

for $k \neq j$. So, we must have $m_t^{wh} = m_t^{wf} = 0$ as in (17). For producers, we have

$$\partial W^{pj}/\partial m_t^{pj} = \gamma^{pj}\frac{\eta^j}{q_t^{pjj} + \eta^h(m_t^{pj}/w_t^j)}\frac{1}{w_t^j} > 0$$

and

$$\partial^2 W^{pj}/\partial (m_t^{pj})^2 = -\gamma^{pj}\left(\frac{\eta^j}{q_t^{pjj} + \eta^h(m_t^{pj}/w_t^j)}\right)^2\frac{1}{w_t^j} < 0.$$

So, we must have $m_t^{pj} > 0$. Thus, (18) is the unique money holding plan consistent with a symmetric equilibrium. ∎