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Monetary Policy, Informality and Business Cycle Fluctuations in a Developing Economy Vulnerable to External Shocks

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ABSTRACT

This paper develops an open economy dynamic stochastic general equilibrium (DSGE) model based on New-Keynesian micro-foundations. Alongside standard features of emerging economies, such as a combination of producer and local currency pricing for exporters, foreign capital inflow in terms of foreign direct investment and oil imports, this model also incorporates informal labor and production sectors. This customization intensifies the exposure of a developing economy to internal and external shocks in a manner consistent with the stylized facts of Business Cycle Fluctuations. We then focus on optimal monetary policy analysis by evaluating alternative interest rate rules and calibrate the model using data from Pakistan economy. The learning and determinacy analysis suggest monetary authority in developing economies to follow Taylor principle in large and to put some weight on exchange rate fluctuations even if there is relatively less inertia in the setting of policy interest rate.

Key Words: *Monetary Policy, Informal Economy, Business Cycles, DSGE*
JEL Classifications: *E26, E32, E35, E37.*

1.   INTRODUCTION

Modelling the sources of Business Cycle Fluctuations (BCF)[1] in an open economy Dynamic Stochastic General Equilibrium (DSGE) framework is a fascinating area of research. The main advantage of this framework over traditional modeling approach is due to an additional feature of micro-foundations in terms of welfare optimization. This feature allows structural interpretation of deep parameters in a way that is less skeptical to Lucas critique (Lucas, 1976). In DSGE modeling context, the sources of BCF are normally viewed as exogenous shocks, which have potential power to propagate the key endogenous variables within the system. This requires a careful identification, as the transmission of these shocks may emanate from internal side, such as, political instability; weak institutional quality in terms of low governance, or from external side, such as, natural disaster (like, earth quacks and floods); international oil and commodity prices; sudden stops in foreign capital inflows; changes in term of trade and exchange rate, or any combination of shocks from both sides. Also, the nature and magnitude of these shocks may vary, depending upon their variances and persistence levels.

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[1] According to Frisch's view on business cycle, a BCF is the cyclical change in the macroeconomic indicator around its trend. See for instance, Frisch (1933), Lucas (1977), Blanchard and Watson (1986) and Bormotov (2009).

After the seminal work of Kydland and Prescott (1982), a substantial body of research has been conducted to identify key possible sources of BCF and to understand propagation mechanisms of these exogenous shocks. But the earlier attempts have mainly focused on high-income countries, like US and Euro-Zone. But for developing courtiers, like Pakistan, small amount of efforts have been made to understand the dynamics of BCF.[2] Data limitations have often considered as root cause (Batini, *et al.*, 2011a and 2011b). As, there is an inherent lack of microeconomic-based surveys and even high-frequency data on major macroeconomic variables is mostly unavailable (Ahmad, *et al.*, 2012).

Further, the structure of economy in developing countries is partially different as compared with the advanced countries, due to the existence of large informal sector. Structures of goods, labour and credit markets are quite different in formal and informal sector of economy due to variations in endowments and constraints of agents. When relative size of informal sector is small (as in developed economies) then ignoring informal sector may be plausible on the ground that it has very limited impact on aggregates. However, if informal sector represents a non-trivial (Schnieder, *et al.*, 2010) fraction of an economy as observed in many developing economies then neglecting informal sector in some micro-founded DSGE model may not be justified.

Table 1: Stylized Facts about Business Cycles: A comparison of Developed and Developing Countries

Countries	Business Cycle Statistics							
	$\sigma(Y)$	$\sigma(C)/\sigma(Y)$	$\sigma(I)/\sigma(Y)$	$\sigma(NX)/\sigma(Y)$	$\rho(Y_t,Y_{t-1})$	$\rho(C,Y)$	$\rho(I,Y)$	$\rho(NX,Y)$
Pakistan*	4.48	1.20	2.76	4.26	0.60	0.92	0.52	-0.38
Developing Economies**	2.74	1.46	3.91	3.22	0.76	0.72	0.77	-0.51
Argentina	3.68	1.38	2.53	2.56	0.85	0.90	0.96	-0.70
Brazil	1.98	2.01	3.08	2.61	0.65	0.41	0.62	0.01
Ecuador	2.44	2.39	5.56	5.68	0.82	0.73	0.89	-0.79
Israel	1.95	1.60	3.42	2.12	0.50	0.45	0.49	0.12
Korea	2.51	1.23	2.50	2.32	0.78	0.85	0.78	-0.61
Malaysia	3.10	1.70	4.82	5.30	0.85	0.76	0.86	-0.74
Mexico	2.48	1.24	4.05	2.19	0.82	0.92	0.91	-0.74
Peru	3.68	0.92	2.37	1.25	0.64	0.78	0.85	-0.24
Philippines	3.00	0.62	4.66	3.21	0.87	0.59	0.76	-0.41
Slovak Republic	1.24	2.04	7.77	4.29	0.66	0.42	0.46	-0.44
South Africa	1.62	1.61	3.87	2.46	0.88	0.72	0.75	-0.54
Thailand	4.35	1.09	3.49	4.58	0.89	0.92	0.91	-0.83
Turkey	3.57	1.09	2.71	3.23	0.67	0.89	0.83	-0.69
Developed Economies**	1.34	0.94	3.41	1.02	0.75	0.66	0.67	-0.17

*Author's personal estimates based on Pakistan data [1951-2011].
**Based on Aguiar and Gopinath (2007).

However, recent studies[3] come forward with some stylized-facts of BCF in developing countries. Table-1 provides a summary of business cycle statistics of various macroeconomic indicators in both absolute and relative terms. This table shows that: aggregate income is more volatile in developing countries as compared with developed countries, private consumption and investment relative to aggregate income are substantially more volatile, net exports is countercyclical with aggregate income and real interest rates. However, if we focus on Pakistan economy exclusively and compare its business cycle statistics with other developing countries, then we can observe that aggregate income is more volatile. This volatility is mainly triggered from net exports, which is a main component of aggregate demand. The remaining results are in line with business cycle statistics as we observed in rest of selected developing countries.

Based upon these facts, this study carries two dimensional motivation agenda. First, in developing economies, like Pakistan, with complex economic structures, one of the enduring research

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[2] Haider and Khan (2008), Ahmad, *et al.*, (2012) and Choudhri and Malik (2012) are few notable examples.
[3] See for instance, Batini, *et al.*, (2011a, 2011b), Agenor and Montiel (2010), Aguiar and Gopinath (2007), Neumeyer and Perri (2005), Agenor, *et al.*, (2000).

questions is to construct and calibrate a valid micro-founded DSGE model featured with nominal and real rigidities. This issue is really challengeable as such economic model which comprehensively explores the transmission mechanism of economic behaviors in the developing economies is scarcely available due to unavailability of high frequency data and because of a major share of the undocumented economy in the observed economic data. Furthermore, due to nature of small open emerging economy, BCF are mainly prone to external shocks, like international oil and commodity price shocks and sudden stops in capital inflows mainly in terms of foreign direct investment. This requires an intensive customization of readily available DSGE models which are capable to answer these dynamics especially in the context of developing countries. Therefore, this study comes forward to meet these challenges by constructing a small open economy DSGE model feature with informal sectors vis-à-vis various external shocks.

More specifically, we develop a two-bloc DSGE model of a small open economy (SOE) interacting with the rest of the world. Alongside standard features of SOE, such as a combination of producer and local currency pricing for exporters, foreign capital inflow in terms of foreign direct investment and oil imports [see for instance, Batini *et al.,* (2010a), Kolasa (2008), Medina and Soto (2007), Liu (2006), Gali and Monacelli (2005) and Lubik and Schorfheide (2005)], our model also incorporates informal sectors [while considering informal goods production and informal labor supply decisions by households, see for example, Ahmad *et al.*, (2012) and Batini *et al.*, (2011a)]. This intensifies the exposure of a SOE to internal and external shocks in a manner consistent with the stylized facts listed above. We then focus on optimal monetary policy analysis, by calibrating the model using data from Pakistan economy.

The rest of the essay is organized as follows: section two provides a comprehensive literature review, section three discusses some stylized-facts of Pakistan economy, section four layout the structure of the model; section five discusses empirical calibration results; and finally last section concludes.

2. LITERATURE REVIEW

DSGE modeling based on New-Keynesian (NK) framework[4] has emerged as a powerful tool to analyze various macroeconomic policies, which are essentially forward-looking in nature. The term DSGE was originally ascribed by Kydland and Prescott (1982) in their seminal work on Real Business Cycle (RBC) model. This modeling approach is based on classical-axioms of flexible prices and money neutrality. The initial contribution on RBC augments the neo-classical Ramsey–Cass–Koopmans growth model by introducing stochastic technology shocks.[5] Kydland and Prescott (1982), Altug (1989) and many of their followers, empirically show that such a modeling approach is capable of reproducing a number of stylized facts of the business cycle of US economy. Another reason of popularity of these models in the early 1990's that these are featured with solid micro-foundations of economic agents in terms of welfare optimizations, subject to various incentive constraints [for example, budget constraints, technological constraints].[6] The agents-optimization process implicitly considers Rational Expectation Hypothesis (REH).

These innovations give many advantages over the use of traditional Tinbergen (1953) type reduce-form macroeconomic models. Among them, the most important advantage is that the structural interpretations of deep parameters of RBC type models are less vulnerable to Lucas critique (Lucas, 1976). The traditional macro-econometric models contained equations linking variables of interest of explanatory factors such as economic policy variables. One of the uses of these models is therefore to examine how a change in economic policy affects these variables of interest, other things being equal. However, in the RBC approach, since the equilibrium conditions for aggregate variables can be computed from the optimal individual behavior of consumers and firms. Further, the REH enables this optimal

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[4]In macroeconomic literature, the terms "new-Keynesian" or "new-neoclassical synthesis" are being used synonymously; see, Clarida, Gali and Gertler (1999), Gali and Gertler (2007), Goodfriend (2007), Goodfriend and King (1997), Mankiw (2006) and Romer (1993, 2011).

[5] This approach is inspired with Frisch's view of the business cycles (Frisch, 1933).

[6]Specifically, RBC type models deals infinitely-lived representative agents, whose objective is to maximize its utility by choosing an optimal path for consumption, real money balances and leisure, as well as a representative firm whose objective, is to maximize profits.

behavior of private agents to use available information rationally, so in this way they should respond to economic policy announcements by adjusting their supposedly actions. Therefore, results obtain from various policy simulations in those reduce-form models, which do not use REH are highly skeptical to Lucas critique. But, as RBC type models are based on optimizing agents with REH, so structural parameters are invariant to Lucas-skepticism.

Despite these over-riding advantages, the RBC models had criticized in terms of the usefulness for monetary or fiscal policy simulations. This is because of the critical main assumption of classical dichotomy. This assumption assume that fluctuations of real quantities are caused by real shock only; that is, only stochastic technology or government spending shocks play their role. On the one hand, many researchers felt that the non-existence impact of monetary policy on business cycles is likely downplayed the role of market inefficiencies. On the other hand, the way in which the empirical fit of these models was measured came under strong criticism.[7] See for instance, Summers (1986), Cooley (1995), Rebelo (2005) and Romer (2011).

To address inherent weaknesses in RBC models, later research, however included Keynesian short-run macroeconomic features (usually called nominal rigidities or gradual both of price and wage adjustment), such as Calvo (1983) type staggered pricing behavior and Taylor (1980, 1998) type wage contracts. This provides plausible short-run dynamic of macroeconomic fluctuations with fully articulated description of the monetary-cum-fiscal policy transmission mechanisms [see, for instance, Christiano et al., (2005) and Smets and Wouters (2003, 2005)]. Such new modeling framework labeled interchangeably as New-Neoclassical Synthesis (NNS) or New-Keynesian (NK) modeling paradigm.[8] Interestingly, the inclusion of NK ideas, into an otherwise RBC model, proved to be extremely successful in terms of reception by the economic profession as well as in terms of explanatory power of the empirical evidence. In particular, the introduction of nominal rigidities together with market imperfections sufficient to break the neutrality of money typical of RBC models, and hence it opened a new avenue for monetary policy analysis. Due to these reasons, the last decade shows a sharp interest in academics, international policy institutions and central banks in developing small-to-medium, even large-scale NK-DSGE models.[9]

The earlier attempts are based on the construction of closed economy NK-DSGE models [see, Christiano et al., (2005), Smets and Wouters (2003, 2007) and the references within], with three main reduce-form equations: the New Keynesian Investment-Saving (NKIS) equation, the Hybrid New-Keyenesain Phillips Curve (NKPC) and the monetary policy rule [like, Taylor (1993, 1999) or McCullam (1988)]. But the later emphasis changed from NK Closed economy frameworks to NK Open economy Macroeconomics (NOEM). This approach is based on the seminal work of Obstfeld and Rogoff (1995) with open economy foundations as suggested in Mundell-Fleming framework [See for example, Adolfson et al. (2007a, 2007b, 2008), Dib et al. (2008), Justiniano and Preston (2004, 2005), Liu (2006) , Gali and Monacelli (2005) and Lubik and Schorfheide (2005)].

The salient features of NOEM models are: optimization-based dynamic general-equilibrium modeling, sticky prices and/or wages in, at least, some sectors of the economy, incorporating of stochastic shocks, and evaluation of monetary policies based on household welfare [Gali and Monacelli (2005), Monacelli (2005) and Lubik and Schorfheide (2003,2005)], the persistence of real and nominal exchange rates [Chari et al, (2002, 2007)], exchange rate pass-through [Devereux and Engel (2002), Monacelli (2005) and Adolfson, et al., (2008)] and international oil price shocks [An and Kang (2009) and Medina and Soto (2005, 2006, 2007)]. More specifically, the model developed by Lubik and Schorfheide

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[7]The failure of these models to replicate some of the empirical regularities such as liquidity effects, co-movement of productivity and employment or the co-movement of real wages and output (Kremer et al., 2006).

[8]Romer (1993) and Goodfriend and King (1997) are in the view that such combined modeling framework is the result of a synthesis of real business cycle (RBC) theory and New Keynesian theory.

[9] Some well-known NK-DSGE models developed by most of the central banks and international policy institutions as noted by Tovar (2008) are: (a) Bank of Canada (TotEM), (b) Bank of England (BEQM), (c) Central bank of Brazil (SAMBA), (d) Central bank of Chile (MAS), (e) Central bank of Peru (MEGA-D), (f) European Central bank (NAWM), (g) Norges Bank (NEMO), (h) Sveriges Riksbank (RAMSES), (i) US Federal Reserve (SIGMA) and (j) IMF (GEM and GIMF).

(2005) is a simplified and straightforward version of Kollmann (2001) and Galí and Monacelli (2005). Like its closed-economy counterpart, the model consists of an (open economy) forward looking IS curve and a NKPC type relationship which determine output and inflation by allowing for monopolistic competition and staggered re-optimization in the import market as in Calvo-type staggered price setting respectively. The term of trade is introduced via the definition of the consumer price index (CPI) and under the assumption of purchasing-power parity (PPP). The exchange rate is derived from the uncovered interest rate parity condition. Monetary policy is described by a nominal interest rate rule and model is simulated by using Bayesian estimation approach.

Although, modern open economy NK-DSGE models fit the data well empirically and able to explain many policy related questions together with the propagation of exogenous shocks. But these models are essentially constructed for advanced (US or European) countries. But, in context of developing countries; however the objective (for example, empirical fit) cannot be achieved by simply replicating NK-DSGE models build for developed countries As the structure of economy in developing countries is partially different as compared with the advanced countries, due to the existence of large informal sector. Structures of goods, labour and credit markets are quite different in formal and informal sector of economy due to variations in endowments and constraints of agents. When relative size of informal sector is small (as in developed economies) then ignoring informal sector may be plausible on the ground that it has very limited impact on aggregates. However, if informal sector represents a non-trivial (Schnieder, 2010) fraction of an economy as observed in many developing economies then neglecting informal sector in some micro-founded NK-DSGE model may not be justified. Keeping in view of potential implications of informal sectors, especially in the context of emerging market economies, new modeling research tries to extend NK-DSGE models with the dynamics of informality. Batini, *et. al.*, (2010b) provide a comprehensive survey of general equilibrium models with informal sectors. This survey covers issues ranging from definitional problems, attached with underground economy to DSGE models incorporating informality. With a focus on informal labour and credit markets, this survey emphasizes the development of DSGE models for better understanding of costs, benefits and policy implications associated with informal sector. In a subsequent research, Batini, *et. al.* (2011a) analyze costs and benefits of informality using a dynamic NK-DSGE model. In their model, formal sector is taxed, capital intensive, highly productive and has frictions in labour market. Informal sector, on the other hand, is untaxed, labour intensive, less productive and has frictionless labour market. Incidence of tax burdon only on formal sector and fluctuations stemming from tax financing are major costs whereas wage flexibility has been listed as benefit of informality. Policy experiments of tax smoothing lead this study to conclude that costs of informality are greater than its benefits.

Ahmad *et. al.* (2012) develop a closed economy DSGE model for Pakistan economy, where they incorporate informality in labour and product markets. This study finds that transmissions of productivity, fiscal and monetary policy shocks to informal sector are weak. In this way, informal sector damps the impact of shocks to economy. Gabriel *et. al.* (2010) constructs in a similar framework a closed economy NK-DSGE model for Indian economy and estimates it using Bayesian methods. This model has more features of liquidity constrained consumers, financial accelerator and informal sector in product and labour markets. This study step-by-step adds features of financial frictions and informal sector to a canonical dynamic New Keynesian model and concludes that addition of financial frictions and informal sector improves model fit in case of Indian economy.

Aruoba (2010) documents the association of institutions with informality, inflation and taxation using data set of 118 countries. This study finds better institutions are associated with low level of informality, high income taxation and low inflation taxation. On the other hand, poor institutions are associated with high level informality, low income taxation and high inflation taxation. After discussing these stylized facts from data, the study presents a general equilibrium model in which households optimally decides about quantum of informal activity for exogenous condition of institutions. Similarly governments optimally decide about their mix of income tax and inflation tax to finance their expenditures. The study claims this model does a reasonable job in explanation of cross differences in inflation, informal activity and taxation.

Ngalawa and Viegi (2010) develop a DSGE model to study inter-dependence of formal and informal financial sectors and impact of informal financial sector on overall economic activity and monetary policy in context of quasi emerging market economies. The model simulations reveal complementarity between the two financial sectors. Increase in formal credit causes a parallel increase in informal sector credit. In response to monetary policy shock, interest rates in the two sectors move in opposite directions; making the conduct of monetary policy hard in presence of large informal financial sectors.

Zenou (2007) develops two-sector general equilibrium model to study labour mobility between formal and informal labour markets under different labour policies. In the model design, formal labor market has search and matching frictions and informal labour market exhibits perfect competition. The study concludes that reduction in unemployment benefits or formal firms' entry cost causes increase in formal employment and an inverse impact on informal sector employment. Although not directly regulated, yet informal sector labour market is not independent of policies applicable on formal labour market due to interdependence of both markets.

Antunes and Cavalcanti (2007) study the impact of regulation costs and financial contract enforcement on size of informal economy and per capita GDP using a small open economy general equilibrium model. The study concludes that regulation costs are more important in accounting for informality. However, in a country where enforcement is very poor e.g. Peru, regulation costs and enforcement of financial contracts are equally important for explanation of informal sector. Regulation costs and enforcement are not much helpful for explaining income difference across countries.

Koreshkova (2006) studies the consequences of tax-evading informal sector for budget financing that ultimately affects inflation. Using cross country data, the study shows that size of informal sector, financing of government expenditure through seigniorage and inflation are positively associated. After establishment of stylized facts, the study uses a two-sector general equilibrium model to analyze implications of informal sector for inflation. Cross country simulations of the model show that in presence of large informal sectors where taxes are not paid, financing of government expenditures through inflation tax is consistent with solution of Ramsay problem.

Conesa *et. al.* (2002) explains the negative relationship between participation rate and GDP fluctuation observed in cross country data through existence of informal sector. Using a dynamic general equilibrium model incorporating informal sector in labour and product markets, the study shows that agents switch between formal and informal sectors during productivity shocks. This transition enhances the impact of shocks on registered output and culminates into amplification of fluctuations.

3. PAKISTAN ECONOMY: SOME STYLIZED-FACTS

The history of Pakistan economy has showing a high degree of political uncertainty. After every autocratic regime, economy rebounded in the troubling position by posing stagnant economic growth and unstable prices. In most of politically-elected regimes, Pakistan has experienced with high inflation rates, large budget deficits and low growth in private sector credit. These regimes also witnessed wobbly external sector along with large trade deficit and declining trend in capital inflows both in the form of foreign direct investments and portfolio investments, which indicates low level of confidence of foreign investors in the domestic economy. The most recent episode of alternative regimes, in Pakistan, is an obvious illustration of this belief. After having seven successful years (autocratic regime of General Pervaiz Musharaf especially from FY01 to FY08) of high growth, balanced fiscal and external position and high investments, the country suffered historic inflation, faltering economic growth, increase in twin deficit, record low investment and rapid accumulation of public debt that increases inter-temporal debt burden.

Table 2 portrays the performance of selected macroeconomic variables during current episode. Specifically, the consumer price inflation increased to around 19 percent during FY09 and remained persistent afterwards. The economy grew around 3 percent during the last four years as compared with over 7 percent average growth observed during autocratic regime. The interest rate increased in tandem

to the overall inflation that has negative impact on investment. The impact of low investment is also reflected from the contraction of large scale manufacturing during the last few years. On external side, the trade deficit widened to historical high level of over 13 percent of GDP in FY08 and remained unabated afterwards. The current account deficit increased to unsustainable level of over 8 percent in FY08. All these developments are the manifestation of political uncertainty, deteriorating law and order situation, prolong power outages and severe energy shortages. As a consequence foreign investment dried up, which put unprecedented pressure on external accounts. The exchange rate depreciated significantly against US dollar and liquid foreign exchange reserves declined considerably due to high twin deficit. In addition, the role of external shocks in worsening of domestic economic situation cannot be disregarded. A significant rise in international commodity prices in 2008 has not only affected negatively Pakistan's trade balance but also seeped into domestic inflation. Moreover, oil imports contribute around 30 percent in Pakistan's total imports and increase in oil price in international market also put pressure on external account. Therefore, analysis of these shocks in shaping in domestic economic condition is central in DSGE modeling.

Table 2: Key Pakistani Macroeconomic Indicators, FY08 - FY11

	2008				2009				2010				2011			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Inflation[a]	7.43	7.60	8.91	12.00	15.64	18.42	19.44	17.03	13.80	11.01	9.68	10.10	11.26	12.94	13.50	13.66
International Oil Prices[b]	73.57	87.62	95.47	121.11	115.47	56.09	44.21	59.17	68.22	75.51	77.05	78.14	75.50	85.44	99.68	110.12
Growth Rate[c]	3.7				1.7				3.8				2.4			
Growth Rate of IPLSM[d]	7.13	2.33	5.65	1.24	-5.72	-3.48	-12.06	-9.70	-0.80	4.02	8.41	5.39	-1.13	1.51	4.34	-0.45
Interest Rate[e]	9.83 (8.96)	10.00 (9.10)	10.33 (9.37)	11.83 (10.33)	13.00 (12.37)	14.33 (13.36)	15.00 (12.65)	14.00 (12.94)	13.33 (12.10)	12.67 (12.33)	12.50 (12.05)	12.50 (12.04)	13.00 (12.43)	13.83 (12.95)	14.00 (13.46)	14.00 (13.31)
Fiscal Balances[f]	-7.83 (-2.69)				-5.61 (-0.19)				-6.60 (-1.90)				-6.98 (-2.90)			
Domestic Debt[g]	2.70	2.87	3.03 (33.01)	3.27	3.42	3.57	3.75 (31.88)	3.85	4.01	4.30	4.49 (33.09)	4.89	5.19	5.50	5.59 (34.82)	6.23
Current Account Balance[h]	-2225	-3827	-3636 (-8.75)	-4181	-4213	-3625	-545 (-6.01)	-878	-981	-1589	-536 (-2.35)	-840	-597	483	52 (0.27)	604
Trade Balance[i]	-1.25	-1.94	-2.28 (-13.20)	-1.77	-1.90	-1.41	-1.05 (-11.12)	-1.31	-1.12	-1.39	-0.93 (-9.19)	-1.58	-0.82	-0.57	-0.97 (-7.79)	-1.44
External Debt[j]	44.87 (28.32)				51.06 (33.15)				54.78 (32.67)				59.12 (29.56)			
International Reserves[k]	16.24	16.07	14.02 (7.43)	11.77	9.70	8.84	10.81 (7.91)	12.16	14.04	15.09	15.06 (9.74)	16.07	16.65	17.31	18.17 (9.27)	18.31
Real Exchange Rate[l]	97.21 (0.31)	96.32 (-0.49)	94.46 (-2.03)	93.57 (-2.21)	91.02 (-6.37)	94.74 (-1.64)	96.72 (2.38)	95.30 (1.85)	93.25 (2.45)	92.18 (-2.71)	95.46 (-1.29)	100.49 (5.44)	101.41 (8.76)	101.74 (10.37)	102.08 (6.94)	101.34 (0.84)

Source: State Bank of Pakistan

Note: The Annual/Quarterly observations mentioned here correspond to the fiscal years; for example, 2008 is FY08.

[a]Annual average growth rate of consumer price index (CPI).
[b]International Oil Prices (US$ per Barrel)
[c]Annual percentage change in real gross domestic product (GDP).
[d]YoY percentage change in Industrial production of Large Scale Manufacturing.
[e]SBP Discount rate; figures in parenthesis are 6-month T-bill rate.
[f]Budget Balance as percent GDP; figures in parenthesis are primary balance as percent GDP.
[g]Domestic debt in billion of rupees;. figures in parenthesis are public debt as percent of GDP
[h]Current Account Balance in Million of Dollars; figures in parenthesis are current account as percent of GDP.
[i]Trade Balance in Million of Dollars; figures in parenthesis are trade balance as percent of GDP.
[j]External debt billion of dollars; figures in parenthesis are external debt as percent of GDP .
[k]International reserves in billions of dollars; figures in parenthesis are international reserves as percent GDP
[l]Real effective exchange rate (REER; a rise in the index indicates appreciation of rupee); figures in parenthesis are percentage App/Depr.

In this study we investigate the impact of various external shocks - like oil price, commodity price, Calvo shock (sudden stops) - viz-a-viz conventional domestic shocks – productivity shock, government spending shock, monetary policy shock. The significance of external shocks is evaluated in the perspective of the economic structure of Pakistan. Pakistan is a small open economy, where trade to GDP ratio stands at 40 percent – ranked third after Sri-Lanka and India in the region- but much lower than Malaysia and other South Asian countries (see, Figure 1). Importantly, textile and textile products contribute around 50 percent in overall Pakistan's exports. Pakistan is considered in

top 10 textile exporting countries of the world and 4th largest producer of cotton yarn and cloths. In addition, Pakistan is also considered as 3rd largest player in Asia with a spinning capacity of 5 percent of total world production. Like other countries in the region, Pakistan also started to liberalize, though partially, its capital account during early 1990s. In term of financial openness, based on the sum of foreign direct investment (FDI) and portfolio equity investments (PI) as percent of the GDPs, Pakistan is ranked third after Malaysia and India while it is much ahead of Bangladesh and Sri Lanka (see Figure 1).

Figure 1: Average Degree of Openness [FY01 – FY11]
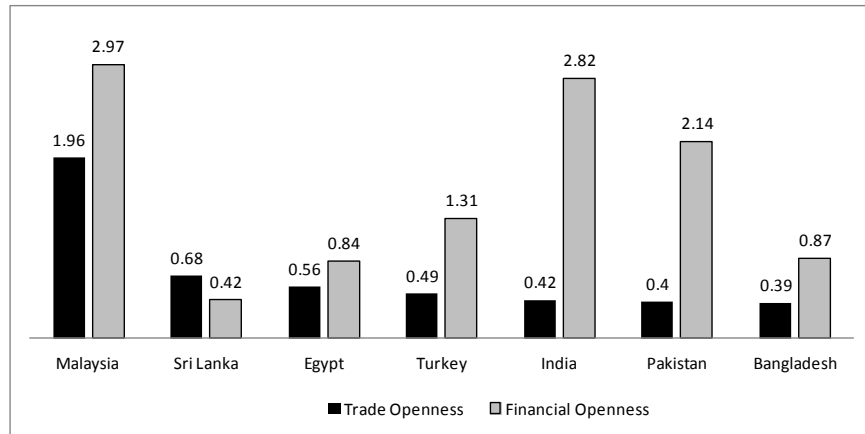
Note: Trade Openness: = (Exports + Imports)/GDP;
Financial Openness: = (Foreign Direct Investment + Portfolio Investment) / GDP
Data Source: International Financial Statistics, IMF Database

However, based on the average of South Asian countries and the world, Pakistan falls short in this respect. On the other hand, Pakistan's imports are largely dominated by petroleum and petroleum products, foods items and Agriculture/chemical products. Therefore, any shocks to commodity and energy price in international market, have a likely impact on domestic imports. Anecdotal evidence suggests that the impact of such shocks is not only observes in overall external account but also emerges in domestic inflation. This evidence confirms in a recent study by Khan and Ahmad, (2011) while analyzing impulse responses in macroeconomic variables by introducing a positive oil price shock. They find that an increase in international oil prices not only put pressure on domestic currency to depreciate but also increases domestic inflation. Similarly, interest rate also tends to rise and domestic economy wanes (see Figure 2).

Figure 2: One standard deviation shock to international oil prices
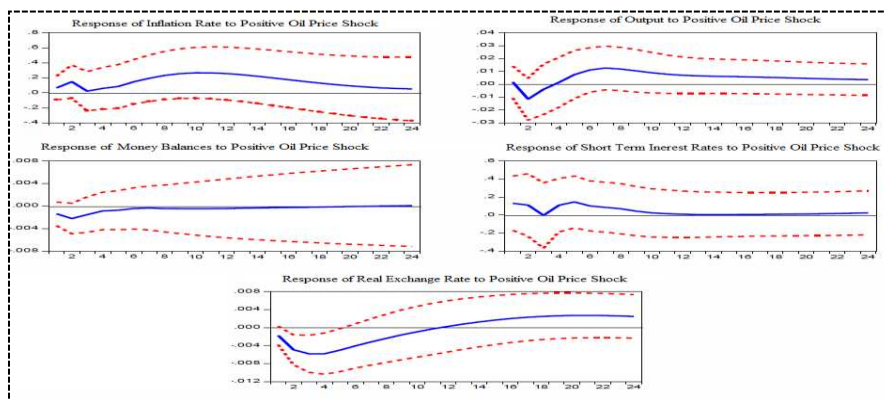
Source: Khan and Ahmed (2011)

Being a small open economy, Pakistan received capital inflows largely in the shape of foreign remittance, foreign debt, portfolio and foreign direct investment. Importantly, Pakistan has received significantly large inflows of foreign capital during 2000s as compared with earlier decades. Until FY04, these inflows limited to unrequited transfers for instance, workers' remittance, grants and logistic supports. But large capital inflows took place during FY05 and onwards, when government adopted pro-cyclical policies by going sovereign, allowing institutions to generate funds from external sources, privatizing more public sector enterprises and financial institutions and provide free access to foreign investors in domestic equity market, thus creating capital inflow bonanza. Importantly, widening current account deficit remained unnoticed during this period as healthy inflows not only financed burgeoning current account deficit, but also resulted in accumulation of foreign exchange reserves. However, this trend seems no longer continue during FY08 and beyond. Calvo and Reinhart (2000) has rightly pointed out that capital flow bonanzas should not be mistaken as blessings and great harm is done when policymakers and investors start treating the bonanza as a permanent phenomenon rather than a temporary shock.

Figure 3: Vulnerability of Sudden Stops
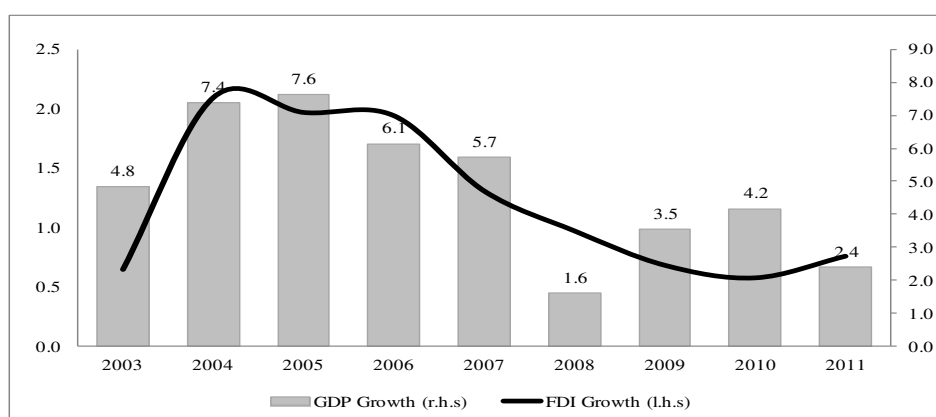
Data Source: Annual Reports, State Bank of Pakistan.

The recent decline in foreign investment in Pakistan and then reversal of portfolio investments is an obvious illustration of Calvo shock (see Figure 3). Importantly, the origin of a capital account shock or sudden stop (SS) may be systemic and exogenous, for instance see Calvo, *et al*., (2004). The systemic sudden stop or 3S, initially triggered by factors, which is exogenous to a country, but then a weak economic fundamentals and financial shallowness exacerbate the situation and ultimately lead to "full-fledge SS" (Calvo, *et al*., 2008). Understanding these differences and carefully modeling the transmission mechanism of internal and external shocks is crucial to the design of stabilization programs and the conduct of economic policies.

Another important characteristic of Pakistan's economy is the existence of large informal sector. Although the size of informal sectors, in term of GDP, trenched during the last decades, nevertheless, it is still high in the region and as compared with other developing countries (see Table 3). As most of the economic activities in Pakistan, particularly agriculture, are undocumented that employed large portion of unskilled or partially skilled labor. Specifically, Pakistan's economy is considered agrarian economy in term of labor as around 70 percent of total work force is informally employed in this sector. In addition, in term of the size of informal sector as a percent of non-agriculture Pakistan stands out, among the competing countries, (see Figure 4). Specifically, this size of 70 percent is slightly lower than HIPIC countries (sub-Sahara region) but much higher compared with emerging Asian economies. Due to its large size, the importance of informal sector in designing of DSGE model cannot be ignored.

Table 3: Size of Informal Economy (as percent of formal GDP)

	Malaysia*	Sri Lanka*	Egypt*	Turkey*	India*	**Pakistan****	Bangladesh*
1999	32.2	45.2	35.5	32.7	23.3	**33.8**	36
2000	31.1	44.6	35.1	32.1	23.1	**40.2**	35.6
2001	31.6	44.6	35.2	32.9	22.9	**39.4**	35.5
2002	31.5	44.1	35.7	32	22.7	**37.6**	35.7
2003	31.2	43.8	35.4	31.2	22.2	**35.5**	35.6
2004	30.7	43.9	35.0	30.4	21.9	**33.6**	35.5
2005	30.4	43.4	34.8	29.6	21.4	**33.2**	35.1
2006	30.0	42.9	34.1	29.5	21	**34.0**	34.5
2007	29.6	42.2	33.1	29.1	20.7	**35.0**	34.1
2008	--	--	--	--	--	**31.3**	--
2009	--	--	--	--	--	**27.4**	--
2010	--	--	--	--	--	**28.7**	--
Average	30.9	43.9	34.9	31.1	22.1	**35.8**	35.3

* Based on Schneider, et al. (2010)

** Based on Gulzar, *et al.* (2010)

Figure 4: Informal Employment (as % of Non-Agricultural Employment)
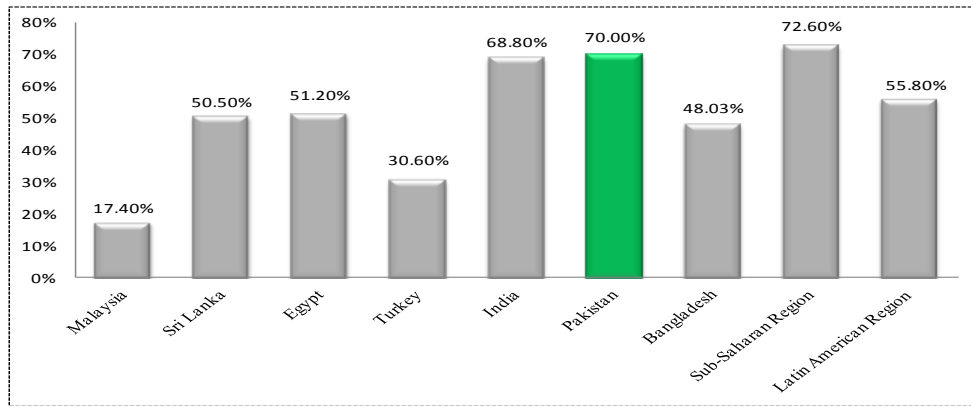
Source: ILO, June 2011

4. DESCRIPTION OF THEORETICAL MODEL

This section presents a multi-sector small open economy DSGE model for Pakistan. Following mainly Gali and Monacelli (2005), Smets and Wouters (2007), Medina and Soto (2007), Haider and Khan (2008), and Batini *et al.*, (2010a) the model structure begins with the world-economy as inhabited by a continuum of infinite-lived households, (indexed by $j \in [0,1]$) who take decisions on the consumption and saving, in a standard rational optimizing manner.[10] They hold real and financial assets and earn income by providing labor to different types of firms working in formal and informal sectors.

There is a set of formal sector firms that produce differentiated varieties of intermediate tradable goods. These firms produce goods using labour, capital and oil as inputs. They have monopoly power over the varieties they produce and set prices in a staggered way. They sell their varieties to assemblers that sale a composite home good in the domestic and foreign markets. A second group of formal sector firms are importers that distribute domestically different varieties of foreign goods. These firms have monopoly power over the varieties they distribute, and also set prices in a Calvo (1983) type staggered

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[10] Each household lives in one of two countries, individual defined on the interval, $j \in [0,n]$ lives in the home-country, and remaining on the interval $j \in [0,n]$ lives in the foreign-country. The value of n measures the relative size of the home-country.

fashion. A third group of firms is producing informal non-tradable intermediate goods. These firms do not pay any tax to government and relatively considered less productive as compared with formal sector firms. These firms produce varieties using labour and oil as inputs and have monopoly power over the goods they distribute, and also set prices in a similar staggered fashion.

Along with manufacturing of goods in both formal and informal sectors, agriculture production is also taking place. For simplicity, it is assumed that commodities produced in this sector are completely exported abroad. The formal sector firms have access to rent capital from capital leasing firms working domestically and abroad. The rest of the model assumes symmetric preferences and technologies, and allowing potentially rich exchange rate and current account dynamics. Government in this model deals with fiscal issues and central bank conducts monetary policy using interest rate as a policy instrument.

4.1 Households

The domestic economy is inhabited by a representative household who derives its utility from consumption, C_t, real money balances, $M_t / P_{C,t}$, and leisure $1 - \ell_t$. Its life time preferences are described by an intertemporal utility function as:

$$U(.) = E \sum_{t=s}^{\infty} \beta^{s-t} u_s \left(C_s, M_s(j) / P_{C,s}, 1 - \ell_s(j) \right) \tag{1}$$

Where $\beta \in (0,1)$ is the intertemporal discount factor which describe rate of time preferences, E is expectation operator and $P_{C,t}$ is aggregate price index for core consumption bundle. In these preferences, we introduce external habit formation in consumption for the optimization household as, $C_t = C_t(j) - hC_{t-1}$ with degree of intensity[11] indexed by $\hat{h}$, where C_{t-1} is the aggregate part of consumption index. The functional specification of utility function u_s is given as:

$$u_s \left(C_s, M_s(j) / P_{C,s}, 1 - \ell_s(j) \right) = \zeta_{C,s} \left\{ \log(C_s(j) - hC_{s-1}) + \frac{\zeta_M}{\mu} \left(\frac{M_s(j)}{P_{C,s}} \right)^{\mu} - \zeta_{L,s} \frac{\ell_s(j)^{1+\sigma_L}}{1+\sigma_L} \right\}$$

In this specification, $\zeta_{C,t}$ is consumption preference shock (a kind of taste shock), μ is the semi-elasticity of money demand to nominal interest rate, ζ_M is relative weight assign to real money balances, σ_L is the inverse of wage elasticity of labour supply and $\zeta_{L,s}$ is a labour supply shock. As usual, it is assumed that, $\mu < 0$ and $\sigma_L > 1$. It is also assume that $u_s \left(C_s, M_s(j) / P_{C,s}, 1 - \ell_s(j) \right)$ is an increasing function with diminishing returns in each of its arguments. The household does want to maximize its utility level subject to the following budget constraints at time t:

$$C_t(j) + \frac{M_t(j)}{P_{C,t}} + \frac{E_t(Q_{t,t+1} D_{t+1}(j))}{P_{C,t}} + \frac{\varepsilon_t B_t^*(j)}{(1+i_t^*)\Theta(\beta_t)P_{C,t}}$$

$$= \frac{W_t(j)\ell_t(j)}{P_{C,t}} + \frac{D_t(j)}{P_{C,t}} + \frac{M_{t-1}(j)}{P_{C,t}} + \frac{\varepsilon_t B_{t-1}^*(j)}{P_{C,t}} + \frac{TR_t}{P_{C,t}} - \frac{\tau_{P,t}}{P_{C,t}} \tag{2}$$

Where $Q_{t,t+1}$ is defined as a stochastic discount factor for assessing consumption streams[12] (or asset price kernel) with the property that the price in period t of any bond portfolio with random values

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[11] It also shows habit persistence parameter to reproduce observed output, rages from $0 \le \hat{h} \le 1$.

[12] In terms of this discount factor, the riskless short term nominal interest rate R_t corresponds to the solution to the equation: $1/(1+i_t) = E_t(Q_{t,t+1})$.

D_t (denotes nominal payoffs from a portfolio of assets at time *t-1*) in the following period is given by: $B_t = E_t[Q_{t,t+1}D_{t+1}]$.[13] W_t is the nominal wage for labour services provided to firms, ε_t is nominal exchange rate and B_t^* is foreign bond holdings with rate of return i_t^*. In this budget constraint, $\Theta(.) = \Theta$ is the premium that domestic household have to pay when they borrow from abroad. The premium is a function of the net foreign asset positions relative to gorss domestic product (GDP) and define as: ß $_t = \varepsilon_t B_t^* / P_{Y,t} Y_t$. The premium function also satisfies two properties: $\Theta(.) = \Theta$ and $(\Theta^{'}/\Theta)$ ß $= \rho$. Finally, *TR* is nominal transfer and $\tau_{P,t}$ is nominal lump-sum tax, which every household have to pay to governmnet. The household optimization process solves the following Langrangian function as:

$$\underset{C_s,M_s,\ell_s,D_{s+1},B_s^*}{\text{Max}} \pounds_t = E_t \sum_{t=s}^{\infty} \beta^{s-t} \left[\begin{array}{l} \zeta_{C,s} \left\{ \log(C_s(j)-hC_{s-1}) + \frac{\zeta_M}{\mu}\left(\frac{M_s(j)}{P_{C,s}}\right)^{\mu} - \zeta_{L,s}\frac{\ell_s(j)^{1+\sigma_L}}{1+\sigma_L} \right\} + \\ \lambda_s(j)\left\{ \begin{array}{l} \left[\frac{W_s(j)\ell_s(j)}{P_{C,s}} + \frac{D_s(j)}{P_{C,s}} + \frac{M_{s-1}(j)}{P_{C,s}} + \frac{\varepsilon_s B_{s-1}^*(j)}{P_{C,s}} + \frac{TR_s}{P_{C,s}} - \frac{\tau_{P,s}}{P_{C,s}}\right] \\ -C_s(j) - \frac{M_s(j)}{P_{C,s}} - \frac{E_s(Q_{s,s+1}D_{s+1}(j))}{P_{C,s}} - \frac{\varepsilon_s B_s^*(j)}{(1+i_s^*)\Theta(ß_s)P_{C,s}} \end{array}\right\} \end{array} \right]$$

The solution to this problem yields the following first order conditons (FOCs):

$$\lambda_t(j) = \zeta_{C,t}\left(C_t(j)-hC_{t-1}\right)^{-1}$$

$$\zeta_{C,t}\zeta_M\left(\frac{M_t(j)}{P_{C,t}}\right)^{\mu-1} = \lambda_t(j)\left\{1 - \beta E \frac{\lambda_{t+1}(j)}{\lambda_t(j)}\frac{P_{C,t}}{P_{C,t+1}}\right\}$$

$$\zeta_{C,s}\zeta_{L,t}(\ell_t(j))^{\sigma_L} = \frac{W_t(j)\lambda_t(j)}{P_{C,t}}$$

$$\beta E \frac{\lambda_{t+1}(j)}{P_{C,t+1}} = Q_{t,t+1}\frac{\lambda_t(j)}{P_{C,t}}$$

$$\beta E \frac{\varepsilon_{t+1}\lambda_{t+1}(j)}{P_{C,t+1}} = \frac{\varepsilon_t\lambda_t(j)}{(1+i_t^*)\Theta(ß_t)P_{C,t}}$$

The above FOCs can further simlify by solving them simultanously, and yield four important results, intertemporal Euler equation of consumption, real money demand equation, labour supply function and uncovered interest parity condition.

The Euler equation of conumption is given as:

$$\beta E_t\left\{(1+i_t)\frac{P_{C,t}}{P_{C,t+1}}\frac{\zeta_{C,t+1}}{\zeta_{C,t}}\left(\frac{C_t(j)-hC_{t-1}}{C_{t+1}(j)-hC_t}\right)\right\} = 1 \tag{3}$$

where, $E_t Q_{t,t+1} = \dfrac{1}{1+i_t}$

The real money demand function is given as:

$$\left(\frac{M_t(j)}{P_{C,t}}\right)^{\mu-1} = \frac{1}{\zeta_M\left(C_t(j)-hC_{t-1}\right)}\left\{\frac{i_t}{(1+i_t)}\right\} \tag{4}$$

The labour supply function is given as:

$$\zeta_{L,t}(\ell_t(j))^{\sigma_L} = \frac{W_t(j)}{P_{C,t}} \frac{1}{\left(C_t(j) - hC_{t-1}\right)} \tag{5}$$

Finally, the uncovered parity condition is given as:

$$\frac{(1+i_t)}{(1+i_t^*)\Theta(\beta_t)} = \frac{E_t\left\{\dfrac{P_{C,t}^*}{P_{C,t+1}^*}\dfrac{\varepsilon_{t+1}}{\varepsilon_t}\dfrac{\zeta_{C,t+1}^*}{\zeta_{C,t}^*}\left(\dfrac{C_t^* - hC_{t-1}^*}{C_{t+1}^* - hC_t^*}\right)\right\}}{E_t\left\{\dfrac{P_{C,t}}{P_{C,t+1}}\dfrac{\zeta_{C,t+1}}{\zeta_{C,t}}\left(\dfrac{C_t - hC_{t-1}}{C_{t+1} - hC_t}\right)\right\}}$$

Using the fact that: $E_t Q_{t,t+1} = E_t Q_{t,t+1}^* \dfrac{\varepsilon_{t+1}}{\varepsilon_t}$, so it simplifies as:

$$\frac{(1+i_t)}{(1+i_t^*)\Theta(\beta_t)} = E_t \frac{\varepsilon_{t+1}}{\varepsilon_t} \tag{6}$$

This equation implies that the interest rate differential is related with expected future exchange rate depreciation and international risk premium, which defined as un-covered interest parity condition.

4.1.1 Household Consumption Decisions

The aggreagate consumption bundle, C_t, for the jth household is a composite of core consumption bundle, $C_{z,t}$, and oil consumption, $C_{O,t}$. Its composition is given by the constant elasticity of substitution (CES) aggregator:

$$C_t(j) = \left[\alpha_c^{1/\omega_c} C_{Z,t}(j)^{\frac{\omega_c-1}{\omega_c}} + (1-\alpha_c)^{1/\omega_c} C_{O,t}(j)^{\frac{\omega_c-1}{\omega_c}}\right]^{\frac{\omega_c}{\omega_c-1}} \tag{7}$$

Where, ω_c is the elasticity of intertemporal substitution between core consumption and oil consumption bundle. Larger the value of ω_c implies that goods are closer substitutes. α_c measures the proportion (persentage share) of core goods in the consumption of households. If its value equals to one then it implies households only consume core goods and there is no oil consumption taking place. The representative household aims at maximizing the utility from consumption of both goods by minimizing the expenditure on these two varieties, while maintaing a certain target level of consumption. Solving this problem of optimal allocation of expenditure on core and oil goods yields the following demand functions for these goods.

$$C_{Z,t}(j) = \alpha_c \left(\frac{P_{Z,t}}{P_{C,t}}\right)^{-\omega_c} C_t(j) \tag{8}$$

$$C_{O,t}(j) = (1-\alpha_c)\left(\frac{P_{O,t}}{P_{C,t}}\right)^{-\omega_c} C_t(j) \tag{9}$$

Where $P_{z,t}$ and $P_{o,t}$ are prices of core and oil consumption bundles. $P_{c,t}$ is the aggregate consumer price index (CPI) and defined as:

$$P_{C,t} = \left[\alpha_c P_{Z,t}^{1-\omega_c} + (1-\alpha_c)P_{O,t}^{1-\omega_c}\right]^{\frac{1}{1-\omega_c}} \tag{10}$$

The core consumption goods are produced either in the formal sector or in undocumented (also known as: informal or hidden) sector. Therefore, the consumption of this bundle is determined by a CES index composed of formal sector goods, $C_{D,t}$, and informal goods, $C_{U,t}$, as follows:

$$C_{Z,t}(j) = \left[\upsilon_c^{1/\varphi_c} C_{D,t}(j)^{\frac{\varphi_c-1}{\varphi_c}} + (1-\upsilon_c)^{1/\varphi_c} C_{U,t}(j)^{\frac{\varphi_c-1}{\varphi_c}} \right]^{\frac{\varphi_c}{\varphi_c-1}} \tag{11}$$

Where, φ_c is the elasticity of intertemporal substitution between formal goods consumption and informal goods consumption bundle. Larger the value of φ_c implies that goods are closer substitutes. υ_c measures the proportion (persentage share) of formal sector goods in the core consumption of households. As with the case of total consumption above, expenditure minimization problem on the core consumption goods yeilds the following demand functions for the formal sector and informal sector goods.

$$C_{D,t}(j) = \upsilon_c \left(\frac{P_{D,t}}{P_{Z,t}} \right)^{-\varphi_c} C_{Z,t}(j) \tag{12}$$

$$C_{U,t}(j) = (1-\upsilon_c) \left(\frac{P_{U,t}}{P_{Z,t}} \right)^{-\varphi_c} C_{Z,t}(j) \tag{13}$$

Where $P_{D,t}$ and $P_{U,t}$ are prices of formal and informal consumption bundles. $P_{z,t}$ is the aggregate price index of core consumption bundle and given as:

$$P_{Z,t} = \left[\upsilon_c P_{D,t}^{1-\varphi_c} + (1-\upsilon_c) P_{U,t}^{1-\varphi_c} \right]^{\frac{1}{1-\varphi_c}} \tag{14}$$

The formal sector consumption bundle is given by the following CES aggregator of home and foreign goods:

$$C_{D,t}(j) = \left[\gamma_c^{1/\eta_c} C_{H,t}(j)^{\frac{\eta_c-1}{\eta_c}} + (1-\gamma_c)^{1/\eta_c} C_{F,t}(j)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}} \tag{15}$$

Where, η_c is the elasticity of intertemporal substitution between home goods consumption and foreign goods consumption bundle. Larger the value of η_c implies that goods are closer substitutes. γ_c measures the proportion (persentage share) of home goods in the formal goods consumption by the households. As with the case of core consumption above, expenditure minimization problem on the core consumption goods yeilds the following demand functions for the home and foreign goods.

$$C_{H,t}(j) = \gamma_c \left(\frac{P_{H,t}}{P_{D,t}} \right)^{-\eta_c} C_{D,t}(j) \tag{16}$$

$$C_{F,t}(j) = (1-\gamma_c) \left(\frac{P_{F,t}}{P_{D,t}} \right)^{-\eta_c} C_{D,t}(j) \tag{17}$$

Where $P_{H,t}$ and $P_{F,t}$ are prices of home and foreign consumption bundles. $P_{D,t}$ is the aggregate price index of formal consumption goods and given as:

$$P_{D,t} = \left[\gamma_c P_{H,t}^{1-\eta_c} + (1-\gamma_c) P_{F,t}^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \tag{18}$$

4.1.2 Household Labour Supply Decisions

In this model, a fraction Λ_L of households provide labor to formal sector firms and rest of them, $(1-\Lambda_L)$, provide labor to informal sector. The aggregate labour is given by the following CES aggregator of formal sector labour, $\ell_{D,t}$ and informal sector labour, $\ell_{U,t}$.

$$\ell_t(j) = \left[\Lambda_L^{-\varphi_L} \ell_{D,t}(j)^{1+\varphi_L} + (1-\Lambda_L)^{-\varphi_L} \ell_{U,t}(j)^{1+\varphi_L} \right]^{\frac{1}{1+\varphi_L}} \tag{19}$$

Where, $\wp_L$ is the inverse elasticity of intertemporal substitution between formal and informal sector labour. The household optimization problem based on wage earnings, yeilds the following supply functions for the formal and informal labour:

$$\ell_{D,t}(j) = \Lambda_L \left(\frac{W_{D,t}(j)/P_{D,t}}{W_t(j)/P_{Z,t}} \right)^{\frac{1}{\wp_L}} \ell_t(j) \tag{20}$$

$$\ell_{U,t}(j) = (1-\Lambda_L) \left(\frac{W_{U,t}(j)/P_{U,t}}{W_t(j)/P_{Z,t}} \right)^{\frac{1}{\wp_L}} \ell_t(j) \tag{21}$$

Where $W_{D,t}$ and $W_{U,t}$ are nominal wages in the formal and informal sector respectively. W_t is the aggregate wage index which is defined as:

$$W_t(j) = \left[\Lambda_L W_{D,t}(j)^{\frac{\wp_L}{1+\wp_L}} + (1-\Lambda_L) W_{U,t}(j)^{\frac{\wp_L}{1+\wp_L}} \right]^{\frac{1+\wp_L}{\wp_L}} \tag{22}$$

The supply functions (20) and (21) show that supply of each type of labor depend on relative wage of respective labor and on aggregate labor supply. The simultaneous solution to above supply functions with aggregate wage index yield following real wage functions:

$$W_{D,t}(j)/P_{D,t} = \Lambda_L \left(\frac{P_{D,t}}{P_{Z,t}} \right)^{\wp_L} W_t(j)/P_{Z,t} \tag{23}$$

$$W_{U,t}(j)/P_{U,t} = (1-\Lambda_L) \left(\frac{P_{U,t}}{P_{Z,t}} \right)^{\wp_L} W_t(j)/P_{Z,t} \tag{24}$$

Following Ahmad *et al.*, (2012), we assume that formal labor is a composite of labor differentiated on basis of different levels of skill represented by *s*. Using this assumption, aggregate formal labor supply can taken as:

$$\ell_{D,t} = \left[\int_0^{\Lambda_L} \ell_{D,t}(s)^{\frac{\epsilon_L-1}{\epsilon_L}} ds \right]^{\frac{\epsilon_L}{\epsilon_L-1}} \tag{25}$$

Where, ϵ_L is the elasticity of substitution between different labor skills in the formal sector. Using this aggregator, it is easy to define aggregate wage in the formal sector as:

$$W_{D,t} = \left[\int_0^{\Lambda_L} W_{D,t}(s)^{\epsilon_L-1} ds \right]^{\frac{1}{\epsilon_L-1}} \tag{26}$$

This condition allows each household to have some market power to set wages on basis of its skill s. Therefore, household optimizes following wage income function as:

$$\max_{W_{D,t}(s)} W_{D,t}(s)\ell_{D,t}(s) - W_{D,t}\ell_{D,t}(s) \tag{27}$$

The solution to this problem yields the following wage markup condition:

$$W_{D,t}(s) = \left(\frac{\epsilon_L}{\epsilon_L-1} \right) W_{D,t} \tag{28}$$

4.2 Capital Leasing Firm and Investment Decisions

There is a representative capital leasing firm that rents capital goods to formal sector firms producing intermediate varieties. Due to formal documentation and legal requirments, this firm does not intract with informal sector firms. It also decides how much capital to accumulate each period in the formal

sector. New capital goods are assembled using a CES technology that combines home and foreign goods as follows:

$$I_{D,t} = \left[\gamma_I^{1/\eta_I} (\zeta_{H,t}^I I_{H,t})^{\frac{\eta_I-1}{\eta_I}} + (1-\gamma_I)^{1/\eta_I} (\zeta_{F,t}^I I_{F,t})^{\frac{\eta_I-1}{\eta_I}} \right]^{\frac{\eta_I}{\eta_I-1}} \tag{29}$$

Where, $I_{D,t}$ is the total private investment in the formal sector. η_I is the elasticity of intertemporal substitution between home and foreign investment goods and γ_I measures the proportion (persentage share) of home goods in total foraml sector investment. The invesetment optimization problem yeilds the following demand functions for the home and foreign goods:

$$\zeta_{H,t}^I I_{H,t} = \gamma_I \left(\frac{P_{H,t}}{P_{I,t}} \right)^{-\eta_I} I_{D,t} \tag{30}$$

$$\zeta_{F,t}^I I_{F,t} = (1-\gamma_I) \left(\frac{P_{F,t}}{P_{I,t}} \right)^{-\eta_I} I_{D,t} \tag{31}$$

Where, $\zeta_{H,t}^I$ and $\zeta_{F,t}^I$ are shocks to the domestic investment and foreign investment respectively, which are assumed to follow first-order autoregressive processes with *IID*-Normal error terms: $\zeta_{H,t}^I = \rho_{\zeta_H^I} \zeta_{H,t}^I + \xi_{H,t}^I$ and $\zeta_{F,t}^I = \rho_{\zeta_F^I} \zeta_{F,t}^I + \xi_{F,t}^I$. $P_{I,t}$ is the aggregate price index of total investment and given as:

$$P_{I,t} = \left[\gamma_I P_{H,t}^{1-\eta_I} + (1-\gamma_I)P_{F,t}^{1-\eta_I} \right]^{\frac{1}{1-\eta_I}} \tag{32}$$

As in Smets and Wouters (2003, 2007) and Christiano *et al.* (2005) we introduce a delayed response of investment observed in the data. Capital leasing firm combine existing capital, $K_{D,t}$, leased from the entrepreneurs to transform an input $I_{D,t}$, gross investment, into new capital according to:

$$K_{D,t+1} = \zeta_t^I I_{D,t} S\left(\frac{I_{D,t}}{I_{D,t-1}} \right) + (1-\delta)K_{D,t} \tag{34}$$

Where $I_{D,t}$ is gross formal sector investment, δ is the depreciation rate and the adjustment cost function $S(\bullet)$ is a positive function of changes in investment. $S(\bullet)$ equals zero in steady state with a constant investment level [$S(1)=0$ and $S(1+g_y)=1$]. In addition, we assume that the first derivative also equals zero around equilibrium, so that the adjustment costs will only depend on the second-order derivative ($S'(.)=0, S''(.)=-\mu_s < 0$) as in Christiano *et al.* (2005). We also introduced a shock to the total investment, which is assumed to follow a first-order autoregressive process with an *IID*-Normal error term: $\zeta_t^I = \rho_{\zeta^I} \zeta_t^I + \xi_t^I$.

Formal sector firms choose the capital stock and investment in order to maximize their profit function. Let Z_t is the rental price of capital. The representative formal sector firm must solve the following optimization problem:

$$\underset{K_{D,t+i}, I_{D,t+i}}{\text{Max}} \quad E_t \left\{ \sum_{i=0}^{\infty} \Xi_{t,t+i} \frac{Z_{t+i}K_{D,t+i} + P_{I,t+i}I_{D,t+i}}{P_{C,t+i}} \right\} \tag{33}$$

The first-order conditions result in the following two equations:

$$\frac{P_{I,t}}{P_{C,t}} = \frac{Q_t}{P_{C,t}}\left\{ S\left(\frac{I_{D,t}}{I_{D,t-1}}\right) + S'\left(\frac{I_{D,t}}{I_{D,t-1}}\right)\frac{I_{D,t}}{I_{D,t-1}}\right\}\zeta_t^I$$
$$-E_t\Xi_{t,t+1}\frac{Q_{t+1}}{P_{C,t+1}}\left\{ S'\left(\frac{I_{D,t+1}}{I_{D,t}}\right)\left(\frac{I_{D,t+1}}{I_{D,t}}\right)^2\right\}\zeta_{t+1}^I$$

(35)

$$\frac{Q_t}{P_{C,t}} = E_t\Xi_{t,t+1}\left\{\frac{Z_{t+1}}{P_{C,t+1}} + (1-\delta)\frac{Q_{t+1}}{P_{C,t+1}}\right\}$$

(36)

These equations samultanously determine the evolution of the shadow price of capital, Q_t and real invesetmtent expenditure.

4.3 Domestic Formal Sector Production

In domestic formal sector, there are three types of firms: domestic formal sector retailers, intermediate goods producing firms and import goods retailers. Domestic formal sector retailers are net buyers of domestic intermediate varieties produced by domestic intermediate goods producing firms and assemble them as final home goods. These firms sell a quantity of formal home goods, in domestic formal goods market and also export remaining quantity abroad. Import goods retailers on the other hand purchase foreign goods at world market prices, and sell to domestic consumers. These firms charge a markup over their cost, which creates a wedge between domestic and import prices of foreign goods, when measure in the same currency.

4.3.1 Formal Sector Retailers

Retailers in the formal sector produce a quantity of home goods, $Y_{H,t}$ sold domestically and $Y_{H,t}^*$ the quantity of goods sold abroad. These quantities of final goods are assembled using CES technology with a continuum of intermediate goods, $Y_{H,t}(z_H)$ and $Y_{H,t}^*(z_H)$ respectively as:

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(z_H)^{\frac{\epsilon_H-1}{\epsilon_H}}dz_H\right]^{\frac{\epsilon_H}{\epsilon_H-1}}$$

(37)

$$Y_{H,t}^* = \left[\int_0^1 Y_{H,t}^*(z_H)^{\frac{\epsilon_H-1}{\epsilon_H}}dz_H\right]^{\frac{\epsilon_H}{\epsilon_H-1}}$$

(38)

Where, ϵ_H, is the elasticity of substitution between differentiated formal intermediate varieties. Under the assumption of perfectly competitive environment, the profit function for both quantities of final goods can be written as:

$$\Pi_{H,t} = (1-\tau_H)P_{H,t}Y_{H,t} - \int_0^1 P_{H,t}(z_H)Y_{H,t}(z_H)dz_H$$

(39)

$$\Pi_{H,t}^* = (1-\tau_H)P_{H,t}^*Y_{H,t}^* - \int_0^1 P_{H,t}^*(z_H)Y_{H,t}^*(z_H)dz_H$$

(40)

Where, τ_H is the flat tax rate on final goods, $P_{H,t}(z_H)$ is the price of variety Z_H, when used to assemble home goods sold in the domestic market, and $P_{H,t}^*(z_H)$ is the foreign currency price of this

variety when used to assemble home good sold abroad. Formal sector retailers try to optimize their profit functions while taking decision on how much intermediate variety Z_H to purchase given its price and demand elasticity. This optimization problem yields the following demand functions for the particular intermediate variety Z_H as:

$$Y_{H,t}(z_H) = \left(\frac{P_{H,t}(z_H)}{(1-\tau_H)P_{H,t}} \right)^{-\in_H} Y_{H,t} \tag{41}$$

$$Y^*_{H,t}(z_H) = \left(\frac{P^*_{H,t}(z_H)}{(1-\tau_H)P^*_{H,t}} \right)^{-\in_H} Y^*_{H,t} \tag{42}$$

4.3.2 Intermediate Goods Production in the Formal Sector

In the formal sector, there is a set of monopolistic compretivie firms, which produce intermediate goods using labour, capital and oil as key inputs. These firms maximize profits by choosing the prices of their differetiated good subject to the corresponding demands and the available CES technology of the following type:

$$\Upsilon_{H,t}(z_H) = A_{H,t} \left[\alpha_H^{1/\omega_H} V_{H,t}(z_H)^{\frac{\omega_H-1}{\omega_H}} + (1-\alpha_H)^{1/\omega_H} O_{H,t}(z_H)^{\frac{\omega_H-1}{\omega_H}} \right]^{\frac{\omega_H}{\omega_H-1}} \tag{43}$$

Where, $A_{H,t}$ is the exogenous level of technology available to all firms, $V_{H,t}(z_H)$ is a composite of labour and capital inputs used in the production process and $O_{H,t}(z_H)$ is the amount of oil input used in production of intermediate vareites. In this technology, ω_H is the elasticity of intertemporal substitution between oil and other factor of inputs. Larger the value of ω_H implies that oil and other factor of imputs are closer substitutes. α_H measures the proportion (persentage share) of non-oil factor inputs in the production of intermediate varieties. The composite of labour and capital is given by the following Cobb-Douglas technology:

$$V_{H,t}(z_H) = [T_{D,t}\ell_{D,t}(z_H)]^{\eta_H} [K_{D,t}(z_H)]^{1-\eta_H} \tag{44}$$

Where, $\ell_{D,t}(z_H)$ is the amount of domestic formal labour being used in the formal sector production and $K_{D,t}(z_H)$ is the amount of physical capital rented from capital leasing firm. In this Cobb-Douglas technology, η_H represents labour share and $T_{D,t}$ represents stochastic trend in the labour productivity and it evolves according to the follwing expression:

$$\frac{T_{D,t}}{T_{D,t-1}} = \varsigma_{DT,t} \tag{45}$$

The aggregate technology shock in CES technology (43) and formal sector labour productivity shock in Codd-Douglas technology (44) are defined in the following manner:

$$A_{H,t} = A_{H,t-1}^{\rho_{a_H}} \exp(\xi_{a_H,t}) \text{ and } \varsigma_{DT,t} = (1+g_y)^{1-\rho_{DT}} \varsigma_{DT,t-1}^{\rho_{DT}} \exp(\xi_{DT,t})$$

Where, $\xi_{a_H,t}$, $\xi_{DT,t}$ are stochastic respective iid innovations and $A_{H,t-1}^{\rho_{a_H}}$, $\varsigma_{DT,t-1}^{\rho_{DT}}$ are respective persistence levels. Following, Calvo (1983) staggered-price setting is assumed. This means that domestic

formal sector differentiating goods are defined subject to Calvo-type price-setting. In this setup, at each period, only $1-\phi_H^i$ fraction of randomly selected domestic firms set prices optimally for domestic market and $1-\phi_{H^*}^i$ fraction of randomly selected domestic firms set prices optimally for foreign market, while remaining firms keep their prices unchanged[14]. As a result the average duration of a price contract is given by $1/(1-\phi_H^i)$ for domestic market and $1/(1-\phi_{H^*}^i)$ for foreign market. This implies that if a firm does not receive a signal, it revises its price following a simple rule that weights past inflation and the inflation target set by the central bank. [15] Therefore, when a firm receives a signal to adjust its price for domestic formal market then it solves the following optimization problem:

$$\underset{P_{H,t}(z_H)}{\text{Max}} E_t \left\{ \sum_{i=0}^{\infty} \Xi_{t,t+i} \phi_H^i \frac{\Gamma_{H,t}^i P_{H,t}(z_H) - MC_{H,t+i}}{P_{C,t+i}} Y_{H,t+i}(z_H) \right\} \tag{46}$$

Subject to demand constraint (41) and updating rule for prices, $\Gamma_{H,t}^i$. Similarly, when a firm receives a signal to adjust its price for foreign market then it solves the following optimization problem:

$$\underset{P_{H,t}^*(z_H)}{\text{Max}} E_t \left\{ \sum_{i=0}^{\infty} \Xi_{t,t+i} \phi_{H^*}^i \frac{\varepsilon_{t+i} \Gamma_{H^*,t}^i P_{H,t}^*(z_H) - MC_{H,t+i}}{P_{C,t+i}} Y_{H,t+i}^*(z_H) \right\} \tag{47}$$

Subject to demand constraint (41) and updating rule for prices, $\Gamma_{H^*,t}^i$. In above expressions (46) and (47), $MC_{H,t+i}$ is the nominal marginal cost which is defined as follow:

$$MC_{H,t} = \frac{W_{D,t} \ell_{D,t}(z_H) + Z_t K_{D,t}(z_H) + P_{O,t} O_{H,t}(z_H)}{\Upsilon_{H,t}(z_H)} \tag{48}$$

Given this price structure, the optimal path for inflation is given by a New-Keynesian Phillips Curve with indexation.

4.3.3 Import Goods Retailers in the Formal Sector

In the formal sector, there is a set of competitive assemblers that use a CES technology to combine a continuum of differentiated imported varieties to produce final foreign good, $Y_{F,t}$.

The CES aggregator is given as:

$$Y_{F,t} = \left[\int_0^1 Y_{F,t}(z_F)^{\frac{\epsilon_F - 1}{\epsilon_F}} dz_F \right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{49}$$

Where, ϵ_F, is the elasticity of substitution between differentiated formal intermediate imported varieties. Under the assumption of perfectly competitive environment, the profit function for imported

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[14] ϕ_H^i firms adjust prices according to steady state inflation rate π. This notion introduces inflation persistence by allowing for price indexation to previous inflation.

[15] The degree of price stickiness is assumed to be same as the fraction of past inflation indexation. The reason of this crude assumption is that it validates a basic rationale of Phillips curve. "In the long-run Phillips Curve is vertical", see for instance, Gali and Gertler (1999).

final goods can be written as:

$$\Pi_{F,t} = (1-\tau_F)P_{F,t}Y_{F,t} - \int_0^1 P_{F,t}(z_F)Y_{F,t}(z_F)dz_F \tag{50}$$

Where, τ_F is the flat tax rate on imported final goods, $P_{F,t}(z_F)$ is the price of variety Z_F, when used to assemble imported goods sold in the domestic market. Formal sector import retailers try to optimize their profit functions while taking decision on how much intermediate imported variety Z_F to purchase given its price and demand elasticity. This optimization problem yields the following demand function for imported variety Z_F as:

$$Y_{F,t}(z_F) = \left(\frac{P_{F,t}(z_F)}{(1-\tau_F)P_{F,t}}\right)^{-\epsilon_F} Y_{F,t} \tag{51}$$

Following Gali and Monacelli (2005) and Monacelli (2005), it is assumed that the law-of-one price (LOP) holds at the wholesale level for imports. But, endogenous fluctuations from purchasing power parity (PPP) in the short run arise due to the existence of monopolistically competitive intermediate variety importers. Since, they keep domestic import prices over and above the marginal cost. As a result, the LOP fails to hold at the retail level for domestic imports. Importers purchase foreign goods at world-market prices $P_{F,t}^*(z_F)$ so that the law of one price holds at the border. These purchased foreign goods are then sell to domestic consumers and a mark-up is charged over their cost, which creates a wedge between domestic and import prices of foreign goods when measured in the same currency. [16] Now following a similar staggered-pricing argument (46) as defined in the case of domestic formal sector producer, the optimal price setting behavior for the domestic monopolistically competitive importer could be defined as:

$$\underset{P_{F,t}(z_H)}{\text{Max}} E_t \left\{\sum_{i=0}^{\infty} \Xi_{t,t+i}\phi_F^i \frac{\Gamma_{F,t}^i P_{F,t}(z_F) - \varepsilon_{t+i}P_{F,t}^*(z_F)}{P_{C,t+i}} Y_{F,t+i}(z_F)\right\} \tag{52}$$

Importing firms try to optimize (52) subject to demand constraint (51) and use updating rule for prices as $\Gamma_{F,t}^i$.

4.4 Domestic Informal Sector Production

In this model setup, it is assume that along with domestic formal sector, informal production is also taking place. There are two types of firms: domestic informal sector retailers and informal intermediate goods producing firms. The informal retailers are working in a symmetric fashion to the formal sector retailers. The only two exceptions distinguish their role from formal to informal that these retailers pay no taxes and can only use intermediate varieties produced in the informal intermediate goods producing sector.

4.4.1 Informal Sector Retailers

Retailers in the informal sector produce a quantity of goods, $Y_{U,t}$ which is completely consumed domestically. This quantity of informal final goods is assembled using CES technology with a continuum of informal intermediate varieties, $Y_{U,t}(z_U)$ as:

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[16] If PPP holds, then l.o.p gap implies that pass-through from exchange rate movements to the domestic currency prices of imports is imperfect as importers adjust their pricing behavior to extract optimal revenue from consumers. See for instance, Monacelli (2005).

$$Y_{U,t} = \left[\int_0^1 Y_{U,t}(z_U)^{\frac{\epsilon_U - 1}{\epsilon_U}} dz_U \right]^{\frac{\epsilon_U}{\epsilon_U - 1}} \tag{53}$$

Where, ϵ_U, is the elasticity of substitution between differentiated informal intermediate varieties. Under the assumption of perfectly competitive environment, the profit function of final informal goods producing retailer can be written as:

$$\Pi_{U,t} = P_{U,t} Y_{U,t} - \int_0^1 P_{U,t}(z_U) Y_{U,t}(z_U) dz_U \tag{54}$$

Where, $P_{U,t}(z_U)$ is the price of variety Z_U, when used to assemble informal goods sold in the domestic informal market. Informal sector retailers try to optimize their profit functions while taking decision on how much intermediate variety Z_U to purchase given its price and demand elasticity. This optimization problem yields the following demand function for the intermediate variety Z_U as:

$$Y_{U,t}(z_U) = \left(\frac{P_{U,t}(z_U)}{P_{U,t}} \right)^{-\epsilon_U} Y_{U,t} \tag{55}$$

4.4.2 Intermediate Goods Production in the Informal Sector

Similar to formal sector, there is a set of monopolistic compretitive informal firms, which produce intermediate goods using labour and oil as key inputs. However, these firms have no access to rent capital as an input of production. These firms maximize profits by choosing the prices of their differetiated good subject to the corresponding demands and the available CES technology of the following type:

$$\Upsilon_{U,t}(z_U) = A_U \left[\alpha_U^{1/\omega_U} V_{U,t}(z_U)^{\frac{\omega_U - 1}{\omega_U}} + (1 - \alpha_U)^{1/\omega_U} O_{U,t}(z_U)^{\frac{\omega_U - 1}{\omega_U}} \right]^{\frac{\omega_U}{\omega_U - 1}} \tag{56}$$

Where, A_U is the constant exogenous level of technology available to all informal sector firms. It is also assumed that these firms are less productive as compared with formal sector firms. In the production technology of these firms, $V_{U,t}(z_U)$ is a composite of labour input only and $O_{U,t}(z_U)$ is the amount of oil input used in production of intermediate informal vareites. Furthermore, ω_U is taken as the elasticity of intertemporal substitution between oil and other factor of inputs. Larger the value of ω_U implies that oil and other factor of imputs are closer substitutes. α_U measures the proportion (persentage share) of non-oil factor inputs in the production of intermediate informal varieties. The labor composit is given by the following form:

$$V_{U,t}(z_U) = [T_{U,t} \ell_{U,t}(z_U)] \tag{57}$$

Where, $\ell_{U,t}(z_U)$ is the amount of domestic informal labour being used in the informal sector production. In this composite technology, $T_{U,t}$ represents stochastic trend in the informal labour productivity and it evolves according to the follwing expression:

$$\frac{T_{U,t}}{T_{U,t-1}} = \varsigma_{UT,t} \tag{58}$$

It is assumed that informal labor is less productive $(T_{U,t} < T_{D,t})$ and labour productivity shock $\varsigma_{UT,t}$ is defined as: $\varsigma_{UT,t} = (1 + g_y)^{1 - \rho_{UT}} \varsigma_{UT,t-1}^{\rho_{UT}} \exp(\xi_{UT,t})$, where, $\xi_{UT,t}$ is stochastic iid innovations and

$\varsigma_{UT,t-1}^{\rho_{UT}}$ is respective persistence levels. Similar to formal sector firms, it is also assumed that informal sector firms set prices according to Calvo (1983) staggered-price setting scheme. This implies that at each period, only $1-\phi_U^i$ fraction of randomly selected domestic informal sector firms set prices optimally for domestic informal market, while remaining informal sector firms keep their prices unchanged. As a result the average duration of a price contract is given by $1/(1-\phi_U^i)$ for domestic informal market. This implies that if any informal sector firm does not receive a signal, it revises its price following a simple rule that weights past inflation and the inflation target set by the central bank. Therefore, when this firm receives a signal to adjust its price for domestic formal market then it solves the following optimization problem as:

$$\underset{P_{U,t}(z_U)}{\text{Max}} E_t\left\{\sum_{i=0}^{\infty} \Xi_{t,t+i}\phi_U^i \frac{\Gamma_{U,t}^i P_{U,t}(z_U)-MC_{U,t+i}}{P_{C,t+i}} Y_{U,t+i}(z_U)\right\} \tag{59}$$

Subject to demand constraint (55) and updating rule for prices, $\Gamma_{U,t}^i$. In this expression, $MC_{U,t+i}$ is the nominal marginal cost which is defined as following:

$$MC_{U,t} = \frac{W_{U,t}\ell_{U,t}(z_U)+P_{O,t}O_{U,t}(z_U)}{\Upsilon_{U,t}(z_U)} \tag{60}$$

Given this price structure for informal firms, the optimal path for inflation is given by a New-Keynesian Phillips Curve with indexation.

4.5 Agriculture Commodity Producing Sector

In this model, aggriculture production is also taking palce along with the manufacturing of formal and inforaml goods. It is assumed that there is sigle firm produces a quantity of homogenous aggriculture commodities that is completely exported abroad. Production technology evolves with the same stochastic trend as other aggregate variables and requires no inputs as:

$$\Upsilon_{S,t} = \left(\frac{T_{U,t}}{T_{U,t-1}}\Upsilon_{S,t-1}\right)^{\rho_{Y_S}}\left(T_{U,t}\Upsilon_{S,0}\right)^{1-\rho_{Y_S}} \exp(\xi_{Y_s,t}) \tag{61}$$

Where, $\Upsilon_{S,t}$ is total aggriculture output, $\xi_{Y_s,t}$ is stochastic *iid* technology and ρ_{Y_S} represents persistence of shock in aggriculture output. As there is no factor input in this production process, so an increase in aggriculture commodity production implies directly an increase in domestic output. This increase in production can be taken as a windfall gain. It also may increase exports, if no counteracting effect on home goods exports dominates. We would expect that, as with any increase of technological frontier of tradable goods, a boom in this sector would induce an exchange rate appreciation. Lastly, it is also assumed that aggriculture production is positively related with informal labour productivity indirectly through prduction pssibility frontier.

4.6 Monetary Policy

It is assumed that monetary authority follows Taylor-type reaction functions. Since the basic objective of the central bank is to stabilize both output and inflation. So in order to specify this reaction function, it needs to adjust nominal interest rate in response to deviations of inflation, a measure of output and exchange rate depreciation from their targets. Following Clarida, Gali and Gertler (2001) and Gali and Monacili (2005), simple open economy version of reaction function is defined as:

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r}\right)^{\psi_i}\left(\frac{Y_t}{Y_{t-1}}\frac{\overline{Y}_{t-1}}{\overline{Y}_t}\right)^{(1-\psi_i)\psi_y}\left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_t}\right)^{(1-\psi_i)\psi_\pi}\left(\frac{RER_t}{\overline{RER}}\right)^{(1-\psi_i)\psi_{rer}} \exp(\xi_{m,t}) \tag{68}$$

Where, $\pi_{C,t} = (P_{C,t} - 1)/P_{C,t-1}$ is total consumer price index and $r_t = (1+i_t)/(1+\pi_{C,t})$. The parameter ψ_i is the degree of interest rate smoothing and $(1-\psi_i)\psi_y$, $(1-\psi_i)\psi_\pi$ and $(1-\psi_i)\psi_{rer}$ are relative weights on output, inflation and real exchange rate respectively. $\xi_{m,t}$ is *iid*-innovation, defined as a monetary policy shock.

4.7 Fiscal Policy

In this model setup, government finances its expenditures by seigniorage (printing money, $M_t - M_{t-1}$) and imposing lump-sum tax $\tau_{P,t}$ on households and flat taxes on final goods produced in the domestic formal sector as, $\tau_H P_{H,t} Y_{H,t}$ and on imported goods from abroad as, $\tau_F P_{F,t} Y_{F,t}$. These expenditures are consisting of spending on goods and services, $P_{G,t} G_t$ and making lump-sum transfers to households, TR_t. The deficit in any case is finance using foreign bonds $\varepsilon_t B_{G,t}^*$ and domestic bonds, $B_{G,t}$, on which it pays interest back as well. Therefore, the government's budget constraint can be written as:

$$\frac{\varepsilon_t B_{G,t}^*}{(1+i_t^*)\Theta(\frac{\varepsilon_t B_t^*}{P_{Y,t} Y_t})} + \frac{B_{G,t}}{(1+i_t)} + P_{G,t} G_t = \varepsilon_t B_{G,t-1}^* + B_{G,t-1}$$

$$+ \tau_H P_{H,t} Y_{H,t} + \tau_F P_{F,t} Y_{F,t} + \tau_{P,t} + (M_t - M_{t-1}) - TR_t \tag{62}$$

We assume that the basket consumed by the government includes both home, $G_{H,t}$ and foreign goods, $G_{F,t}$. Its composition is given by the constant elasticity of substitution (CES) aggregator:

$$G_t = \left[\gamma_G^{1/\eta_G} G_{H,t}^{\frac{\eta_G-1}{\eta_G}} + (1-\gamma_G)^{1/\eta_G} G_{F,t}^{\frac{\eta_G-1}{\eta_G}} \right]^{\frac{\eta_G}{\eta_G-1}} \tag{63}$$

Where, η_G is the elasticity of intertemporal substitution between home and foeign consumption good. Larger the value of η_G implies that goods are closer substitutes. γ_G measures the proportion (persentage share) of home good in government consumption goods. If its value equals to one then it implies government only consume home good and there is no foreign good consumption taking place. The government decides the composition of its consumption basket by minimizing its cost. Solving the cost optimizaiton problem of optimal allocation of government expenditure yields the following demand functions:

$$G_{H,t} = \gamma_G \left(\frac{P_{H,t}}{P_{G,t}} \right)^{-\eta_G} G_t \tag{64}$$

$$G_{F,t} = (1-\gamma_G) \left(\frac{P_{F,t}}{P_{G,t}} \right)^{-\eta_G} G_t \tag{65}$$

Where, the deflator of government expenditure is given by:

$$P_{G,t} = \left[\gamma_G P_{H,t}^{1-\eta_G} + (1-\gamma_G) P_{F,t}^{1-\eta_G} \right]^{\frac{1}{1-\eta_G}} \tag{66}$$

Finally, it is assumed that government follows a simple structural fiscal balance rule according to which government aggregate expenditure as percent of GDP evolves as follows:

$$\frac{P_{G,t}G_t}{P_{Y,t}Y_t} = \left[\frac{\tau_t}{P_{Y,t}Y_t} + \left(1 - \frac{1}{(1+i^*_{t-1})\Theta_{t-1}}\right)\left(\frac{\varepsilon_t}{\varepsilon_{t-1}} \frac{\varepsilon_{t-1}B^*_{G,t-1}}{P_{Y,t-1}Y_{t-1}} \frac{P_{Y,t-1}Y_{t-1}}{P_{Y,t}Y_t}\right)\right]\exp(\xi_{G,t}) \tag{67}$$

Where, $\xi_{G,t}$ is exogenous government spending shock defined as a autoregressive of order one process with *iid* innovation.

4.8 Foreign Sector Economy

Agents in the foreign sector economy demand both the agriculture commodity goods and formal sector home goods. The demand for the agriculture commodity good is assumed to be completely elastic at the international price, $P^*_{S,t}$. If the law of one price holds for this good, then its domestic-currency price can be defined by:

$$P_{S,t} = \varepsilon_t P^*_{S,t} \tag{69}$$

Similar to pricing assumption of agriculture commodities in the international market, we assumed that demand for oil commodity is completely elastic at the international price, $P^*_{O,t}$ and if the law of one price holds then the oil price in domestic currency is given as:

$$P_{O,t} = \varepsilon_t P^*_{O,t} \tag{70}$$

The real exchange rate is defined as the product of nominal exchange rate, ε_t with relative price of a foreign price index, P^*_t, and the price of the consumption bundle in the domestic economy, $P_{C,t}$:

$$RER_t = \frac{\varepsilon_t P^*_t}{P_{C,t}} \tag{71}$$

As usual the foreign price index is not necessarily equal to the price of imported goods. However, under the assumption of long-run relationship, we can write as:

$$P^*_{F,t} = P^*_t \varsigma^*_{F,t} \tag{72}$$

Where, $\varsigma^*_{F,t}$ is a stationary transitory shock to the relative price of imports abroad. This shock may be related to changes in the relative productivity across sector in the foreign economy. Foreign demand for home goods depends on the relative price of this type of goods abroad and the total foreign aggregate demand:

$$Y^*_{H,t} = \varsigma^*(\frac{P^*_{H,t}}{P^*_t})^{-\eta^*}Y^*_t \tag{73}$$

Where, ς^* corresponds to the share of domestic intermediate goods in the consumption basket of foreign agents, η^* is the price elasticity of the demand and Y^*_t the foreign output. This demand can be obtained from a CES utility function with an elasticity of substitution across varieties equal to that parameter.

4.9 Aggregate Equilibrium

Using the above model setup, we can drive general equilibrium dynamics around their steady-state level. The general equilibrium is achieved from goods market equilibrium and labour market equilibrium. The goods market equilibrium derived from aggregate demand side forces and labour market equilibrium

dynamics emerge from aggregate supply side forces. So, the general equilibrium of the whole model is achieved from these market equilibriums and key equilibrium results are given as:

$$\Upsilon_{H,t}(z_H) = \left(\frac{P_{H,t}(z_H)}{(1-\tau_H)P_{H,t}}\right)^{-\epsilon_H} Y_{H,t} + \left(\frac{P_{H,t}^*(z_H)}{(1-\tau_H)P_{H,t}^*}\right)^{-\epsilon_H} Y_{H,t}^* \tag{74}$$

Where the aggregate resource constraint of the formal sector is defined as:

$$Y_{H,t} = C_{H,t} + I_{H,t} + G_{H,t} \tag{75}$$

The above aggregate equilibrium implies that total labor demand by intermediate varieties producers in the formal sector must be equal to labor supply implied in this sector. Since, economy is interacting with the rest of the world, therefore foreign demand for home goods depends on the relative price of these formal sector goods abroad and the total foreign aggregate demand is defined as:

$$Y_{H,t}^* = \varsigma^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta^*} Y_t^* \tag{76}$$

Where ς^* is taken as the share of domestic intermediate formal sector goods, used in the consumption basket of foreign agents, η^* is the price elasticity of the export demand function and Y_t^* is the aggregate foreign output. Under certain assumption, this demand can be derived from a CES utility function with an elasticity of substitution across intermediate arieties equal to that parameter. Similar to formal sector aggregate equilibrium conditions as jointly defined in (77) and (75), the equilibrium informal sector equates informal output to informal consumption.

$$\Upsilon_{U,t}(z_U) = \left(\frac{P_{U,t}(z_U)}{P_{U,t}}\right)^{-\epsilon_U} Y_{U,t} \tag{77}$$

This equilibrium also implies that total labor demand by intermediate varieties producers in the informal sector must be equal to labor supply implied in this sector. Therefore, the aggregate resource constraint of informal sector is given as:

$$Y_{U,t} = C_{U,t} \tag{78}$$

The joint combination of goods market equilibrium conditions, the budget constraint of the government and the aggregate budget constraint of households, it is easy to obtain an expression for the aggregate accumulation of international bonds:

$$\frac{\varepsilon_t B_{G,t}^* / P_{Y,t} Y_t}{(1+i_t^*)\Theta(\frac{\varepsilon_t B_{,t}^*}{P_{Y,t}Y_t})} = \frac{\varepsilon_{t-1}B_{t-1}^*}{P_{Y,t}Y_t} + \frac{P_{X,t}X_t}{P_{Y,t}Y_t} - \frac{P_{M,t}M_t}{P_{Y,t}Y_t} \tag{79}$$

This expression also shows net foreign asset position of the external sector of the economy. Corresponding to this expression with price deflator to each aggregate demand component the total GDP at current prices can be defined as the following relation:

$$P_{Y,t}Y_t = P_{C,t}C_t + P_{G,t}G_t + P_{I,t}I_t + P_{X,t}X_t - P_{M,t}M_t \tag{80}$$

Finally, the total value of nominal exports and the total value of nominal imports are given by:

$$P_{X,t}X_t = \varepsilon_t(P_{H,t}^*Y_{F,t}^* + P_{s,t}^*Y_{s,t}) \tag{81}$$

$$P_{M,t}M_t = \varepsilon_t(P_{F,t}^*Y_{F,t} + P_{O,t}^*(C_{O,t} + O_{H,t} + O_{U,t})) \tag{82}$$

Where, aggregate resource constraint in terms of $Y_{F,t}$ can be defined as:

$$Y_{F,t} = C_{F,t} + I_{F,t} + G_{F,t} \tag{83}$$

4.10 Alternative Monetary Policy Rules and Welfare

We analyze the likely impact of alternative monetary policy rules based on social welfare loss function minimization. Among various monetary policy regimes, the optimal monetary policy or Ramsey policy rule is defined by maximizing the intertemporal household welfare (U_t) subject to a set of non-linear structural constraints of the model. To be more precise, a Ramsey equilibrium is a competitive equilibrium such that:

(i)   *Given a sequence of shocks, prices, policy instrument and quantities* P_t; R_t; $Q_{tt=0}^{\infty}$ *it maximizes the representative agent lifetime utility,* U_t.

(ii) $r_t > 0$.

In order to analyze essentially the macroeconomic stabilization properties of the monetary policy, we assume subsidies on labor and goods markets are offsetting first order distortions. In that case, the flexible price equilibrium is Pareto optimal. The Ramsey policy problem is written using an infinite horizon Lagrangian:

$$\Im = U_t + E_t \lambda_r \sum_{j=0}^{J} \beta^j (r_{t+j} - \overline{r})^2 + \lambda \ (Model \ Constraints) \tag{84}$$

Where,

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r}\right)^{\psi_i} \left(\frac{Y_t}{Y_{t-1}} \frac{\overline{Y}_{t-1}}{\overline{Y}_t}\right)^{(1-\psi_i)\psi_y} \left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_t}\right)^{(1-\psi_i)\psi_\pi} \left(\frac{RER_t}{\overline{RER}}\right)^{(1-\psi_i)\psi_{rer}}$$

and

$$U_t = \zeta_{C,t} \left\{ \log(C_t - hC_{t-1}) + \frac{\zeta_M}{\mu}\left(\frac{M_t}{P_{C,t}}\right)^{\mu} - \zeta_{L,t} \frac{\ell_t^{1+\sigma_L}}{1+\sigma_L} \right\}$$

In above Lagrangian function, λ_r is the weight associated to the cost on interest rate fluctuations. We introduce an interest rate objective in this problem in order to make the Ramsey policy operational. Following, Woodford (2004), Gali (2008) and Walsh (2010) we will take second order approximation of the agents lifetime expected utility function through Taylor's series.[17] In this framework the central bank tries to maximize the social welfare when there is a trade-off between the aggregate consumer price inflation and the changes in output.

First we can write the approximate utility of consumption as:

$$\log(C_t - hC_{t-1}) = (1-h)\overline{C} + (\tilde{C}_t - h\tilde{C}_{t-1}) \tag{85}$$

We can also write the disutility of labor about its flexible price equilibrium as:

$$\frac{\ell_t^{1+\sigma_L}}{1+\sigma_L} = \frac{\overline{\ell}^{1+\sigma_L}}{1+\sigma_L} + \overline{\ell}^{1+\sigma_L}\left(\tilde{n}_t + \frac{1}{2}(1+\sigma_L)\tilde{n}_t^2\right) + o(\|a\|^3) \tag{86}$$

Where, $\ell_{i,t} = \left(\frac{Y_{i,t}}{A_{i,t}}\right)\int_0^1 \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\omega_t} di, \qquad i \in (D,U)$.

Using these results with optimization condition, we can write the second order approximation to the small open economy's utility function as follows:

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[17] A second-order log-linear approximation to the function (U_t) around its steady state ($\overline{U}$) is given by:

$U_t = e^{\log U_t} \approx \overline{U} + \overline{U}[\log U_t - \log \overline{U}] + \frac{1}{2}\overline{U}[\log U_t - \log \overline{U}]^2 + o(\|a\|^3)$. In general, if $\tilde{U}_t = U_t - \overline{U}$ is deviation of (U_t) around its steady state ($\overline{U}$) and $\hat{U}_t = \log(U_t/\overline{U})$ is log-deviation of (U_t) around its steady state ($\overline{U}$), then second order approximation can be obtained as: $\tilde{U}_t = \overline{U}(\hat{U}_t + \frac{1}{2}\hat{U}_t^2) + o(\|a^3\|)$.

$$W \cong -\Omega E_0 \sum_{t=0}^{\infty} \beta_t \left\{ \frac{\omega_C}{\lambda} \hat{\pi}_{C,t}^2 + (1+\sigma_L)(\hat{y}_t - \hat{y}_t^{flexible})^2 \right\} + o(\| a \|^3) \tag{87}$$

Taking unconditional mean and letting $\beta \longrightarrow 1$, the expected welfare loss of any policy that deviated from Ramsey policy can be written in terms of variances of inflation and output gap as:

$$W^{social} \cong -\Omega E_0 \sum_{t=0}^{\infty} \beta_t \left\{ \frac{\omega_C}{\lambda} \text{var}(\hat{\pi}_{C,t}^2) + (1+\sigma_L) \text{var}(\hat{y}_t - \hat{y}_t^{flexible})^2 \right\} \tag{88}$$

Gali and Monacelli (2005) and Alba *et al.*, (2012) studies argued that that using this social welfare maximization criteria along with the assumptions of purchasing power parity and uncovered interest parity, the combined effects of market power and terms of trade distortions could be offset so that under flexible price equilibrium. In this framework, domestic inflation targeting is the optimal monetary policy. However, apart from domestic inflation targeting in a strict sense, central banks in most of emerging market economies, put weight to some other secondary objectives of monetary policy, like economic growth stability and smoothing of exchange rate fluctuations. Therefore, optimal monetary policy in such developing economies is one produce minimum welfare loss of the central bank. Therefore, the optimal monetary policy rule under Ramsey policy framework is characterized as:

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r} \right)^{\psi_i} \left(\frac{Y_t}{Y_{t-1}} \frac{\overline{Y}_{t-1}}{\overline{Y}_t} \right)^{(1-\psi_{i,optimal})\psi_{y,optimal}} \left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_t} \right)^{(1-\psi_{i,optimal})\psi_{\pi,optimal}} \left(\frac{RER_t}{\overline{RER}} \right)^{(1-\psi_{i,optimal})\psi_{rer,optimal}} \tag{89}$$

This framework measures welfare loss as a second order approximation to the utility loss of the domestic consumer resulting from deviations from optimal monetary policy. Following, Lucas (1987) and Woodford (2004), alternative monetary policies regimes are specified in the context of a simple DSGE model and the welfare losses are compared to draw policy implications. In this paper, we have also considered four alternative monetary policy regimes along with optimal monetary policy rule. The first alternative policy regime is considered as less aggressive anti-inflation policy. This regime put less weight to inflation and assigned zero weight to both changes in output and exchange rates fluctuations. The policy rule associated with this regime can be defined as:

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r} \right)^{\psi_i} \left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_t} \right)^{(1-\psi_i)\psi_{\pi,1}} \tag{90}$$

The second alternative monetary policy regime is considered as more aggressive anti-inflation policy. This regime put relatively more weight to inflation and assigned zero weight to both changes in output and exchange rates fluctuations. The policy rule associated with this regime can be defined as:

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r} \right)^{\psi_i} \left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_t} \right)^{(1-\psi_i)\psi_{\pi,2}} \text{, where, } \psi_{\pi,2} > \psi_{\pi,1}. \tag{91}$$

The third alternative monetary policy regime is taken in which central bank put less weight to changes in output together with price stability objective. However, this regime assigns zero weight to exchange rates fluctuations. The policy rule associated with this regime can be defined as:

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r} \right)^{\psi_i} \left(\frac{Y_t}{Y_{t-1}} \frac{\overline{Y}_{t-1}}{\overline{Y}_t} \right)^{(1-\psi_i)\psi_{y,1}} \left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_t} \right)^{(1-\psi_i)\psi_\pi} \tag{92}$$

The forth alternative monetary policy regime is taken in which central bank put more weight to changes in output together with price stability objective. Similar to previous alternative regimes, this policy rule assign zero weight to exchange rates fluctuations. The policy rule associated with this regime can be defined as:

$$\frac{1+r_t}{1+r} = \left(\frac{1+r_{t-1}}{1+r} \right)^{\psi_i} \left(\frac{Y_t}{Y_{t-1}} \frac{\overline{Y}_{t-1}}{\overline{Y}_t} \right)^{(1-\psi_i)\psi_{y,2}} \left(\frac{1+\pi_{C,t}}{1+\overline{\pi}_t} \right)^{(1-\psi_i)\psi_\pi} \text{, where, } \psi_{y,2} > \psi_{y,1}. \tag{93}$$

5. NUMERICAL SOLUTION AND CALIBRATION RESULTS

The model is solved numerically using the general methodology as provided in Uhlig (1999), Klein (2000) and Sims (2002). In order to obtain numerical solutions, it is required first to transform model's complete set of non-linear equilibrium relations to its log-linearized form. This is done by taking first order Taylor approximations to each equilibrium condition around its steady state path. A brief description of this approach along with log-linearized equilibrium conditions are provided in Appendix-A. The numerical solutions are then obtained by employing the method of undetermined coefficients. This step considers the autoregressive shocks as key exogenous processes. Our DSGE model consists of sixteen exogenous shocks, among which nine are domestic and rest are propagated from external sources. Based on the propagation mechanism of these shocks, numerical algorithm computes model empirical moments, impulse responses of endogenous variables to each exogenous process and variance decomposition results. These results allow us to examine the empirical fit of the model and to understand the behavior of economy to various structural shocks.

5.1 Model Parameterization

Model parameterization step requires assigning numbers to structural parameters of the model.
We calibrate the model at quarterly frequency with the choice of parameter values that are approximately consistent with key features of developing economy in general and Pakistan's economy in particular. Almost all parameter values used in this model have initially been calibrated using partial estimation approach. The rest of few parameter values whose data for estimation is unavailable, are then taken from the existing DSGE/RBC literature on emerging market economies. The chosen values of these parameters can be gleaned from personal introspection to reflect strongly held beliefs about the validity of economic theories. Therefore, the selection must reflect researcher confidence about the likely location of structural parameter of the model. In our model, there are forty-three structural and thirty-two shock related parameters. The estimated values of structural parameters are given in Table C1, whereas values to the shock related parameters are given in Table C2 of Appendix C.

The first category of structural parameters is related to household preferences. The parameter value of discount factor (β) is taken as 0.991. This value is consistent with the quarterly estimates of discount factor β for Pakistan economy as given in Ahmed, et $al.,$ (2012). This value is set in order to obtain historical mean of real interest rate in the steady state. Ahmed, et $al.$, (2012) estimates suggest that the long run real interest rate is lower in most of developing countries. Therefore, the selected parameter value of intertemporal discount factor is quite useful for our model calibrations as our prime concern is to replicate business cycle fluctuations of a developing economy like Pakistan. The degree of external habit persistence ($\hat{h}$) in consumption is set as 0.36 (Haider and Khan, 2008). This parameter value implies that degree of habit persistence in consumption is quite low as compared with advanced economies; see for instance, Lubik and Schorfeide (2005). The semi-elasticity of money demand to interest rate (μ) is taken as -0.15 (Haider, et $al.$, 2012). It shows that money demand is less elastic with respect to nominal interest rate. The relative weight in preferences assigned to real money balances (ζ_M) is 0.25 (Ahmad, et $al.$, 2012). The parameter value of wage elasticity of labour supply (σ_L) taken as 1.5. This value is consistent with the estimates reported by Ahmad, et $al.$, (2012) and Fagan and Messina (2009). The share of core goods in the consumption basket of household (α_c) is taken as 0.75. This value is computed from Pakistan's Household Integrated Economic Survey (HIES), 2010-11. A similar estimated value is used by Batini (2010b) for Indian case. This shows that subsistence level of consumption is high in most of developing economies and people spend approximately 75% of their budget on core-consumption related goods. The rest of share is allocated to oil and energy related items. The elasticity of intertemporal substitution between core and oil goods consumption bundle is fixed at 0.35. This value is consistent with posterior estimates given in An and Kang (2009) for the Korean and Medina and Soto (2007) for Chilean economies.

The share of formal sector goods in the core consumption basket (v_c) is set to be 0.55. The estimate is closer to value given in Ahmad, *et al.*, (2012) and Khan and Khan (2011). The elasticity of substitution between formal and informal goods consumption bundle (φ_c) is taken as 0.70. This high value of substitution elasticity shows significant share of informal goods consumption in the core consumption bundle. The share of home goods in the formal consumption basket (γ_c) is fixed at 0.65. The corresponding elasticity of substitution between home and foreign goods consumption bundle (η_c) is taken as 1.12. These parameter values are consistent with the posterior estimates given in An and Kang (2009) and Haider and Khan (2008). The share of formal labor in aggregate labor supply (Λ_L) is taken as 0.29. This value is consistent with estimates used in Choudhri and Malik (2012) and Ahmad *et al.*, (2012).[18] This is due to the fact that in developing countries about 70% of the non-agriculture labor is employed in the informal sector. The corresponding elasticity of substitution between formal and informal labour supply ($\wp_L$) is fixed to be 2.00 and elasticity of substitution between different labor skills in the formal sector ($\in_L$) is taken as 0.80. Ahmad *et al.*, (2012) has estimated these values using labour force survey data from Pakistan.

The second category of parameters is related to aggregate investment and production side of the economy. The share of home investment in aggregate private investment (γ_I) is fixed at 0.52. The corresponding elasticity of substitution between home and foreign private investment (η_I) is taken as 1.20. These parameter values are consistent with Medina and Soto (2007). Labor share in formal sector production (η_H) is fixed at 0.54. This parameter value is taken from Bukhari and Khan (2008). The capital depreciation rate (δ) is taken as 0.03. It implies capital depreciates annually around 12%. Bukhari and Khan (2008), Haider and Khan (2008) and Ahmad et al., (2012) studies used a similar estimates for depreciation rate for Pakistan economy. For simplicity, the elasticity of substitution between differentiated formal intermediate varieties ($\in_H$) is fixed at 1.00. Following, Medina and Soto (2007) flat tax rate on both final home goods (τ_H) and final imported goods (τ_F) are fixed at 0.15. The share of non-oil factor inputs in the production of intermediate formal sector varieties (α_H) is fixed 0.65 and for intermediate informal sector varieties (α_U) at 0.75. The corresponding elasticity of substitution between oil and other factor of inputs in formal production (ω_H) is taken 0.85 and for factor of inputs in informal production (ω_U) at 0.95.

The third category of parameters is related to price setting behavior in both formal and informal sectors. Recent survey studies on the frequency of price change in emerging market economies suggest that prices are more flexible as compared with the developed countries (see for instance, Choudhary *et al.*, 2011). In Calvo (1983) staggered pricing sense, less degree of stickiness provide reasonable notion about frequent price changes in developing economies. This means, probability of not changing price is quite low in a given quarters. Therefore, following survey estimates as given in Choudhary *et al.*, (2011), the parameter values of degree of price stickiness for formal sector home goods sold domestically (ϕ_H^i) is fixed at 0.24 and for formal sector home goods sold abroad ($\phi_{H^*}^i$) is at 0.64 respectively. On the other hand, the degree of price stickiness for formal sector imported goods (ϕ_F^i) is taken as 0.70. Finally, the degree of price indexation to each category is adjusted to replicate flexible nature of prices. These parameter values price stickiness along with the degree of indexation are also consistent with the posterior estimates as given in Haider and Khan (2008) for Pakistan and de-Castro, *et al.*, (2011) for of

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[18] Ahmad *et al.*, (2012) calculate this parameter value by taking average of ratios of number of people employed in the formal sector to total number of people employed in the non-agricultural sector during 1990-1991 to 2008-2009. The labour force data used in the calculation of these ratios is taken from various issues of the Labor Force surveys, Pakistan Bureau of Statistics.

Brazilian case.[19] Choudhary *et al.*, (2011) survey finding also suggest us to set degree of price stickiness for informal sector goods (ϕ_U^i) at 0.21. The degree of price indexation for these goods (χ_U) is fixed at 0.70.

The last category of structural parameters is associated with the central bank reaction function. We have estimated these parameters which satisfied optimal monetary policy criteria in a Ramsey policy sense. We assume optimal monetary policy as a baseline case. The estimated optimal parameter values suggest inflation coefficient (ψ_π) to be fixed at 1.21, which is slightly low as compared with Taylor (1993) suggestions for the US case. The optimal relative weight related to changes in output growth (ψ_y) is taken at 0.60 and to exchange rate fluctuations (ψ_{rer}) at 0.05. These estimated values show that central bank in a developing economy also put significant weights on growth and exchange rate stability objectives along with inflation. Finally, the optimal weight associated with AR(1) term of policy rate shows considerable inertia, which is around 63%. These parameter values for the baseline case are also consistent with an empirical study by Ahmed and Malik (2011) for Pakistan case. We have also used different parameter values of monetary policy reactions function to evaluate alternative monetary policy regimes in the context of developing countries.

The parameter values related to sixteen exogenous shocks are reported in Table C2 of Appendix C. We have computed persistence level and standard deviation corresponding to each shock. For the benchmark developing economy, we have used data from Pakistan to estimate these parameters. The results show that external shocks are more volatile as compare with domestic one. Also, these shocks signify high persistence, which suggests developing economies are more prone to shocks propagate mainly from the external side of the economy. Finally, for model calibrations, steady state values of key endogenous variables are given in Table C3 of Appendix C. These values are calculated by taking long term averages to each variable. For this purpose the data is taken from Pakistan economy. However, estimates related to informal output is taken from Gulzar, *et al.*, (2010).

5.2 Quantitative Assessment and Empirical Fitness of the Model

In this section, we try to assess the quantitative performance of the model by drawing comparisons with quantitative features of the business cycle statistics. The main purpose of this quantitative assessment is to test empirical fitness of the model. It examines, whether a constructed DSGE model is really capable to replicate standard features of business cycles which prevail in the developing economies, like Pakistan. The standard RBC/DSGE literature tries to compare statistical moments of the data from those generated by the model. Therefore, following this approach, we focus on the model's prediction with respect to the volatility of key macroeconomic variables, relative volatility of these selected variables with respect to formal sector output and the contemporaneous correlations of these variables with each other. The results are reported in Table C4, C5 and C6 of Appendix C.

The Table C4 shows the standard deviations for formal consumption, informal consumption, formal sector output, informal sector output, agriculture commodity output, formal sector inflation, informal sector inflation, real exchange rate, aggregate labour, aggregate wages, domestic investment, foreign investment, oil consumption, domestic interest rate, government consumption and current account. The table also provides results of relative standard deviations of these variables with respect to formal sector output. The model matches the observed volatility in formal sector consumption, informal sector consumption, inflation in the formal and informal sector and aggregate output in the Pakistan, which turns out to be high but not very different from the volatility in these two key variables in other developing economies.[20] It over predicts the volatility in few other endogenous variables, like agriculture commodity output, domestic and foreign investments and oil consumption. The predicted volatility of the domestic interest rates and current account are slightly lower than observed in Pakistan but higher

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[19] These low parameter values shows the less proportion of firms that do not re-optimize their prices in a given quarters. Furthermore, these staggered price coefficients also imply that the average duration of price contracts is around one to two quarters for domestic firms. This duration is calculated as: $1/(1-\phi)$.

[20] See for instance, Aguiar and Gopinath (2007) for a comparison with other emerging countries.

than observed in the emerging countries and these are about equal to the mean volatility for the panel of all selected countries as taken in Aguiar and Gopinath (2007). The same is true for volatility of the rest of the variables; the model's predicted value being higher than Pakistan's and lower than in the other developing countries but approximately equal to the average volatility for the complete set of countries.

In terms of the relative standard deviations, the model predicts higher volatility of formal and informal consumption relative to GDP. This is a unique stylized business cycle fact of emerging counties and the model is fairly capable to replicate this fact. However, it predicts a higher volatility of domestic and foreign investment relative to formal sector output than observed in Pakistan and other emerging economies. As far as the relative volatility in rest of the selected endogenous variables with output is concerned, this model underestimates the results as compared to the data.

The Table C5 presents the contemporaneous correlation and Table C6 shows auto-correlations of all these sixteen variables. Among these results, contemporaneous correlations of all endogenous variables with respect to aggregate formal sector output have prime importance due to theoretical moment matching concerns of the model. Auto-correlations results indicate non-stationary behavior of selected variables at level. Our DSGE model does well in matching these correlations, producing results with correct signs that lie between the observed values for Pakistan and other emerging economies. Broadly speaking, the model does quite well quantitatively, producing moments that are roughly consistent with empirically observed counterparts in developing economies in general and Pakistan economy in particular.

5.3 Impulse Responses

The impulse response functions compute dynamic responses of model variables to the fundamental economic disturbances. These are plotted against the number of quarters that have elapsed since the shock occurred. [21] We have computed impulse-response of the key endogenous variables to the sixteen exogenous shocks hitting the domestic economy, under five different monetary policy regimes. These alternative regimes are represented by conventional Taylor (1993) type interest rate rules with various policy assumptions.[22] These assumptions vary with respect to the different responsiveness of the central bank to its various key objectives, like inflation, economic growth, interest rate smoothing and exchange rate stability. However, among all these policy specifications, price stability is taken as a primary objective of the central bank.

The first specification named as baseline policy, which follows optimal Ramsey policy rule defined in terms of welfare optimization criteria. We have calculated optimal reaction parameters using this specification. The values are as follows: $\psi_i = 0.63$, $\psi_\pi = 1.21$, $\psi_y = 0.60$ and $\psi_{rer} = 0.05$. These relative weights associated with the baseline rule are characterized by a moderate reaction to inflation, a stronger response to changes in economic growth, a significant degree of interest rate smoothing and a marginal reaction to exchange rate movements. The second specification assumes considerable inertia in the policy rate and central bank in this case only respond to inflation. However, the responsiveness is less aggressive. The policy reaction parameters used in this specification are: $\psi_i = 0.90$, $\psi_\pi = 1.01$, $\psi_y = 0$ and $\psi_{rer} = 0$. The third specification is similar to second one. However, in this case response of central bank to inflation is more aggressive. The policy reaction parameters used in this specification are: $\psi_i = 0.90$, $\psi_\pi = 1.65$, $\psi_y = 0$ and $\psi_{rer} = 0$. The forth specification assumes considerable inertia in the policy rate and

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[21] The impulse responses to a one unit increase in the various structural shocks are calculated using 10,000 Monte-Carlo simulations. These simulations are performed using MATLAB version 2010b.

[22] The fundamental reason to consider alternative monetary policy regimes based on Taylor type rule is due to the fact that monetary policy in most of the emerging countries switched from the traditional monetary aggregation rule to the interest rate rule in the late 1990s (Alba *et al.,* 2012). The current monetary policy practice in such economies to achieve the objective of price stability no longer involve setting quantitative target for any nominal variable – for example, broad money growth or exchange rate – as an intermediate target (Hussain, 2012).

central bank in this case respond to both inflation and output. However, the output response is less aggressive. The policy reaction parameters used in this specification are: $\psi_i = 0.90$, $\psi_\pi = 1.21$, $\psi_y = 0.53$ and $\psi_{rer} = 0$. Last specification of monetary policy rule is similar to forth with a difference is that here response to output is more aggressive. The policy reaction parameters used in this specification are: $\psi_i = 0.90$, $\psi_\pi = 1.21$, $\psi_y = 0.95$ and $\psi_{rer} = 0$. Based on these alternative monetary policy specifications, we have simulated impulse responses and results are displayed in Figures C1 –to- C16 of Appendix C.

We start by illustrating the dynamic effects of an international oil price shock on a number of endogenous variables. Figure C1 of Appendix C represents the impulse responses to a unit positive innovation in international oil price under the five alternative monetary policy regimes. This shock has a first round impact on marginal costs of formal and informal sector production. Therefore, inflation rises in both these sectors. Due to increase in inflation, output and consumption fall in each sector respectively and then converges to its steady state level. On the household side, oil price increase creates a negative income effect that reduces domestic consumption. As a result, the demand for different types of goods in the consumption basket falls. There is also a substitution effect that tends to increase the demand for both formal and informal goods. However, since the degree of substitution between oil and the other types of goods is low, this effect does not counteract the negative income effect on the demand for core goods. Moreover, this shock also pushes up the cost of formal and informal sector firms producing these types of goods, and their prices relative to the price of foreign goods increases. This shock also has a negative impact on domestic and foreign investment. Exchange rate depreciates in this case and this shock forces a further monetary tightening in the policy interest rate.[23] We have also notice that both monetary policy specifications generate a similar kind of responses, unlike the specification with more aggressive reaction to output, in which more adverse consequences in all endogenous variables are being observed.

Next, we have computed dynamic impulse responses associated to unit negative shock in domestic and foreign investment. On average, the responsiveness of endogenous variables to these shocks under all policy regimes is similar. However, in terms of magnitude, the negative shock to foreign investment has more adverse consequences as compared with domestic one. The results are displayed in Figure C2 and C3 of Appendix C. Following these shocks, output drops and inflation rise up both in formal and informal sectors. Exchange rate depreciates, as investment goods are relatively more import intensive than other final goods. The monetary policy response under most of regime specifications to these shocks lead to a surge in the interest rate. Similar kinds of results are associated with negative adjustment cost shock which is displayed in Figure C4 of Appendix C. These results suggest implications associated with the sudden stops in foreign capital inflows and their likely adverse consequences on the key endogenous variables. Domestic economy ends up with stagflation situation in the form of high inflation and a reduction in output.

Figure C5 of Appendix C show impulse responses to a negative foreign demand shock. Due to this shock, all domestic endogenous variables behave according to the theory. A reduction in the foreign demand leads to an increase in domestic formal sector inflation, a tightening of monetary policy, and a fall in output, consumption and employment. On the other hand a negative commodity price shock generates an output contraction, a reduction in employment, and a surge in inflation. This last effect is explained by the currency depreciation, which increases imported inflation and makes capital goods expensive. This creates a burden on marginal costs and it forces a reduction in real wages. Finally, current account faces deficit position and interest rate rises due to this shock. These results are displayed in Figure C6 of Appendix C.

The next figure plots the impulse responses to a positive import price shock. The impact of this shock on the model endogenous variables is quite similar with international oil price shock. In response to this shock, domestic formal sector inflation increases, as higher import prices pushing up the cost of production causes as a surge in domestic inflation. Aggregate consumption decreases due to a foreign price surge relative to domestic prices. The economic interpretation of this reduction is that domestic agents substitute out of foreign produced goods into home produced goods in response to positive

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[23] These results are similar with a recent empirical study by Khan and Ahmed (2011) for the case of Pakistan.

import price shock, which causes expenditure switching effect and hence leads to a decline in the aggregate consumption. This shock also leads to exchange rate depreciation and a reduction in output and all kind of investments. The results associated with import price shock are displayed in Figure C7 of Appendix C. Next, we have observed response of endogenous variables due to a positive foreign interest rate shock. This shock affects negatively investment decisions and it increases consumption of foreign sector goods and leads to a reduction in aggregate output and in employment. This shock also generates a real appreciation of the currency and a reduction in foreign investment. Optimal monetary policy response to this shock suggests an increase in the policy rate to boost up foreign investment. The results of this shock are displayed in Figure C8 of Appendix C. The next figure plots the impulse responses to a positive foreign inflation shock. Due to this shock, consumption of foreign sector goods decline whereas informal sector goods consumption rises up. This is mainly due to increase in the price of imported items which forces a substitution affect in the formal and informal consumption goods. This shock helps the domestic economy by increasing domestic and foreign investment. Monetary policy reaction is loose to this shock by decreasing policy interest rate. The results are given in Figure C9 of Appendix C.

The impulse responses associated with negative transitory and permanent productivity shocks and negative agriculture commodity production shock are displayed respectively in Figures C10, C11 and C12 of Appendix C. The productivity shocks have a negative impact on formal and informal sector output. These shocks also imply an immediate surge in inflation, as they increase marginal costs of production. However, in response to the permanent productivity shock, inflation rises significantly above its steady state after some periods. The monetary authority tights its policy rate in response to the surge in inflation. For both shocks, employment initially rises because the reduction of aggregate demand associated with the monetary contraction. . The negative transitory technology shock tends to depreciate the real exchange rate, however the negative permanent productivity shock leads to a real appreciation of the currency, explained by the monetary policy tightening that follows some periods after the shock to curb inflation. Similar results have been observed for the case of agriculture commodity production shock. The preference shock on the other hand increases formal and informal sector goods consumption. Due to rise in consumption demand forces inflation to rise up. The optimal monetary policy response to this shock suggests a further tightening of policy interest rates. We have also observed impulse responses to positive domestic labour supply shock. Due to this shock, output initially rises and then after one quarter it declines from its steady state. The later decrease in output shows that agent's substitution between working and leisure dominates the lower cost of production that arises from the increase in labour supply. The results associated with these shocks are displayed respectively in Figures C13 and C14 of Appendix C.

Next figure shows the impulse response to a positive interest rate shock. This shock can be thought of a contractionary monetary policy shock. Following an unanticipated surge in the policy interest rate, a decline in inflation and output is observed both in formal and informal sectors. On the other hand, exchange rate depreciates due to this shock before returning to its equilibrium level. This shock also reduces domestic and foreign investment by increasing cost of business and there is a fall in the aggregate employment. We have also notice that both monetary policy specifications generate a similar kind of responses, unlike the specification with less aggressive reaction to inflation, in which more contraction in most of endogenous variables are being observed. These results are displayed in Figure C15 of Appendix C. The last figure shows impulse responses to positive shock to government spending. This shock forces domestic policy interest rate to rise which creates a burden on formal sector firms to invest in private capital. It results in a crowding-out effect on domestic vis-à-vis foreign investments. This shock also lowers aggregate wage and increase employment at a cost of inflation. This shock produces current account deficit and exchange rate depreciates before returning to its steady state level. We have also observed that baseline monetary policy yields more optimal responses as computed with other alternative policy regimes in terms of contraction in key endogenous variables. These results are displayed in Figure C16 of Appendix C.

5.4 Variance Decompositions

In the previous subsection, we carefully analyzed and understand the transmission mechanisms of exogenous shocks and corresponding responsiveness of key endogenous variables to each shock. We also observed that these shocks propagate from domestic and external sources. Now question arises, how much do these shocks contribute both as a group and individually to economic fluctuations in a representative emerging market economy? It depends not just on the magnitude of the response when a shock of a given size occurs, but also how often and, on average, what size of shocks hit the domestic economy. This problem can be tackled by considering a famous empirical technique known as variance decompositions, which compute the percentage of the forecast error variances at various forecast horizons that are attributable to each of the individual shocks or a group of shocks. We focus here on a medium term horizon which defined three years of time interval. The results are reported in Table C7 of Appendix C.

   We have observed that external shocks explain around 50% of the fluctuations in consumption of formal sector goods. Whereas, consumption of informal sector goods is mainly explained by domestic shocks which uncover 73% of its fluctuations. Similar results can be observed with respect to domestic formal and informal sector output. External shock mostly explain forecast error in formal sector output, which is around 52.3%, whereas fluctuation in informal sector output is mainly explained by domestic shocks. In contrast with these results, however, business cycle fluctuations in both formal and informal sector inflation are manily determined by domestic shocks which cover around 70% of total variations. Fluctuations in real exchange rate and oil consumption are mainly explained by external shocks whereas, the rest of endogenous variables are hit by shocks propogated from domestic sources. Finally, we notice that domestic shocks are relatively more important in explaining movements in variables over longer horizons whereas, short run fluctuations are mainly determined from external shocks.

5.5 Performance of Alternative Monetary Policy Rules under Learning

This section evaluates the performace of baseline optimal monetary policy rule with four alternative rules incorporating different responses to inflation, changes in economic growth, interest rate smoothing and exchange rate fluctuations. We evaluate performace of these rules with two approaches: first conventional welfare loss criterion based on quadratic approximation of household utility, and secondly, through monetary policy learning in terms of analyzing conditions of expectational stability (E-stability) and In-determinacy.[24]

   Table C8 of Appendix C reports the welfare losses associated with the five monetary policy rules analyzed in the previous section: baseline policy, less aggressive anti-inflation policy, more aggressive anti-inflation policy, less aggressive reaction to output and more aggressive reaction to output. There are five panels in this table. The first panel reports welfare losses in the case of our baseline parameterization, while the remaining four panels display the effects of alternative specifications. Correspond to each panel, we have reported volatility associated with each endogenous variables. Along with volatility results, we have also reported welfare loss results associated to formal and informal sectors. Among these calibration results, baseline policy out performs all other regime specifications. It produces less volatility in endogenous variables and yield minimum welfare loss both in formal and informal sectors. Finally, we have also observed that more-aggressive anti-inflation policy yield second best results. The implied welfare losses in this case are quantitatively small as compared with all other policy regimes.

   Next, we follow Bullard and Mitra (2002, 2007) to assess optimal monetary policy through learning in terms of E-stability and In-determinacy conditions. Since it is hard to derive clear analytical results due to complex open economy DSGE model with formal and informal sectors, we present a numerical simulation on a calibrated version of our economy and check the determinacy area. We consider four alternative cases: i) less inertia in monetary policy and no reaction to exchange rate, ii) more inertia in monetary policy but no reaction to exchange rate, iii) less inertia in monetary policy with

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[24] A brief discussion on monetary policy learning is given in Appendix B.

reaction to exchange rate, and iv) more inertia in monetary policy with reaction to exchange rate. For each case, along with all possible values of pair (ψ_π, ψ_y), our numerical routine [25] checks the Eigen-values of complete model solution to determine whether all the eigenvalues have real part less than unity. Regions where the solution is determinate (and thus E-stable) are shown in dark green color format. Regions where at least one eigenvalue have a real part greater than unity are white, i.e. the solution is indeterminate. The resulting graphs are displayed in Figure C17 to C20 of Appendix C.

From these results, we have noted several policy implications. The first implication is associated with Taylor Principle. This means that each case must ensure that model E-stability and equilibrium determinacy are possible only when central bank sets relative weight to inflation, which is greater or equal to one. The likelihood of in-determinacy is maximum in the first case. This means a policy with less inertia in policy rate along with zero-reaction to exchange rate is not optimal. The second case also generates E-instability area, even in a case, where central bank follows Taylor principle. Third and forth policy combinations produce relatively more desirable results. These cases ensure more likelihood of determinacy and E-stability. However, results of first monetary policy evaluation criteria based on society welfare loss meet with the third specification of monetary policy learning in terms of E-stability and determinacy. It indicates that central bank in emerging market economy must follow Taylor Principle and put some with on exchange rate fluctuations even, there is less inertia in the policy interest rate.

6. CONCLUDING REMARKS

In this paper, we develop a two-bloc open economy DSGE model interacting with the rest of the world. Alongside standard features of emerging economies, such as a combination of producer and local currency pricing for exporters, foreign capital inflow in terms of foreign direct investment and oil imports, our model also incorporates informal labor and production sectors. This intensifies the exposure of a developing economy to internal and external shocks in a manner consistent with the stylized facts of business cycle fluctuations. More specifically, we have considered nine domestic and seven external shocks. In the presence of these shocks, our model reasonably captures the likely responses of key endogenous variables, which are consistent with the existing empirical literature available for developing countries. We also evaluate the performance of the model by other conventional measures in terms of theoretical moments matching, like, standard deviations, contemporaneous correlations, auto-correlations etc. Broadly speaking, our model comprehensively matches patterns of business cycle statistics consistent with the empirical facts from emerging market economies. We then focus on optimal monetary policy analysis by evaluating alternative interest rate rules and calibrating the model using data from Pakistan economy as benchmark emerging economy case. The learning and determinacy analysis suggest monetary authority in developing economies to follow Taylor principle and to put some weight on exchange rate fluctuations, even if there is relatively less inertia in the setting of policy interest rate. Finally, for the future research, this model can be extended by incorporating banking and non-banking financial sectors to understand dynamics associated with fiscal borrowing from the banking system, and its likely consequences on monetary expansion and inflation. This helps to explain fiscal dominance issue, which is also an important feature of developing economies in large.

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[25] These numerical routines are implemented by using Global Sensitivity Analysis toolkit, available with Dynare 4.3.

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This section proceeds by a model solution methodology with the log-linearization and canonical representation of the model along with its foreign sector counterpart. In order to solve the model, we first state the first order nonlinear dynamic system that characterizes the competitive equilibrium. In order to calculate the steady state we transform the system equations into their deterministic steady state representation and solve using numerical methods. Then we log-linearize around the deterministic steady state where $x_t = \ln(x_t) - \ln(\bar{x})$. At this stage the system is expressed in terms of relative deviations from the steady state. After solving the model using the method of Klein (2000)[26] we obtain matrices M and H which generate the dynamic solution by iterating on the following two equations:

$$Y_t = HX_t$$
$$X_{t+1} = MX_t + R\eta_{t+1}$$

Where Y_t is a vector composed by control, co-state and flow variables, X_t is a vector of endogenous and exogenous states, H characterizes the policy function and M the state transition matrix. η_{t+1} is an innovation vector and R is a matrix composed of zeros, ones or a parameter instead of a one. This matrix determines which variables are hit by the shock and in what magnitude. Given a set of values of the parameters of the model, this state space representation will help us to compute the relevant statistics of the model such as the spectrum of the data, the likelihood function, among others.

Log-linearized Equilibrium Relations

The small open economy model consists of the following log-linearized equations for endogenous variables and equations for the exogenous processes expressed in terms of AR(1) processes.

➤ Household's aggregate consumption:

$$\tilde{c}_t = \frac{1}{1+h} E_t(\tilde{c}_{t+1}) + \frac{h}{1+h}(\tilde{c}_{t-1}) + \frac{1-h}{1+h} E_t(\tilde{i}_t - \pi_{C,t+1}) + \frac{1-h}{1+h} E_t[\zeta_{C,t} - E_t(\zeta_{C,t+1})]$$

$$- \frac{h}{1+h} E_t[\zeta_{TD,t} - E_t(\zeta_{TD,t+1})] - \frac{h}{1+h} E_t[\zeta_{UT,t} - E_t(\zeta_{UT,t+1})] \qquad \text{(S.01)}$$

➤ Household's real demand for money:

$$m_t = \left(\frac{1-h}{1+h}\right)\left\{\left(\frac{1}{1+h}\right)\tilde{c}_t - \left(\frac{h}{1+h}\right)\tilde{c}_{t-1} + \tilde{i}_t\right\} \qquad \text{(S.02)}$$

➤ Aggregate labour supply:

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[26]Any other method can also be used to solve the log-linear approximation to the rational expectations solution, e.g., Sims (2002).

$$w_t = \sigma_L \tilde{\ell}_t + \left(\frac{1}{1+h}\right)\tilde{c}_t - \left(\frac{h}{1+h}\right)\tilde{c}_{t-1} + \zeta_{L,t} \tag{S.03}$$

➢ Supply of Documented and Undocumented labour:

$$\tilde{\ell}_{D,t} = \frac{1}{\wp_L}\left(w_{D,t} - w_t\right) + \tilde{\ell}_t \tag{S.04}$$

$$\tilde{\ell}_{U,t} = \frac{1}{\wp_L}\left(w_{U,t} - w_t\right) + \tilde{\ell}_t \tag{S.05}$$

➢ Composite wage index:

$$w_t = \frac{1}{\overline{w}^{-\frac{1+\wp_L}{\wp_L}}}\left[\Lambda_L \overline{w}_D^{\frac{1+\wp_L}{\wp_L}} w_{D,t} + (1-\Lambda_L)\overline{w}_U^{\frac{1+\wp_L}{\wp_L}} w_{U,t}\right] \tag{S.06}$$

Where,

$$w_{D,t} = \left(\frac{\varepsilon_L}{\varepsilon_L - 1}\right)w_{U,t} \tag{S.07}$$

$$w_{U,t} = p_{U,t} - p_{Z,t} \tag{S.08}$$

➢ Uncovered interest parity condition:

$$\tilde{i}_t = \tilde{i}_t^* + \mathfrak{A}\mathbf{b}_t^* + E_t(\Delta\tilde{e}_{t+1}) \tag{S.09}$$

➢ Aggregate consumption bundles:

$$\tilde{c}_{Z,t} = \tilde{c}_t - \omega_C\, p_{Z,t} \tag{S.10}$$

$$\tilde{c}_{O,t} = \tilde{c}_t - \omega_C\, p_{O,t} \tag{S.11}$$

$$0 = \alpha_C\, p_{Z,t} + (1-\alpha_C)p_{O,t} \tag{S.12}$$

➢ Core consumption bundles:

$$\tilde{c}_{D,t} = \tilde{c}_{Z,t} - \varphi_C\, p_{D,t} \tag{S.13}$$

$$\tilde{c}_{U,t} = \tilde{c}_{Z,t} - \varphi_C\, p_{U,t} \tag{S.14}$$

$$p_{Z,t} = \upsilon_C\, p_{D,t} + (1-\upsilon_C)p_{U,t} \tag{S.15}$$

➢ Consumption bundles of Documented goods:

$$\tilde{c}_{H,t} = \tilde{c}_{D,t} - \eta_C\, p_{H,t} \tag{S.16}$$

$$\tilde{c}_{F,t} = \tilde{c}_{D,t} - \eta_C\, p_{F,t} \tag{S.17}$$

$$p_{D,t} = \gamma_C\, p_{H,t} + (1-\gamma_C)p_{F,t} \tag{S.18}$$

➢ Equation of motion of capital stock:

$$k_{D,t+1} = \left(1 - \frac{(1-\delta)}{(1+n)(1+g_y)}\right)(inv_{D,t} + \zeta^I_{D,t}) + \frac{(1-\delta)}{(1+n)(1+g_y)}k_{D,t} \tag{S.19}$$

➢ Investment goods bundles of documented sector:

$$(inv_{H,t} + \zeta^I_{H,t}) = inv_{D,t} - \eta_I(p_{H,t} - p_{I,t}) \tag{S.20}$$

$$(inv_{F,t} + \zeta^I_{F,t}) = inv_{D,t} - \eta_I(p_{F,t} - p_{I,t}) \tag{S.21}$$

$$p_{I,t} = \gamma_I p_{H,t} + (1-\gamma_I)p_{F,t} \tag{S.22}$$

➢ Supply and demand for investment goods in documented sector:

$$p_{I,t} = \frac{\overline{Q}}{\overline{P_I}}(q^I_t + \zeta^I_{D,t}) - \frac{\overline{Q}}{\overline{P_I}}(1 + \frac{1}{1+r})\mu_S(1+g_y)^2 inv_{D,t} +$$

$$\frac{\overline{Q}}{\overline{P_I}}\mu_S(1+g_y)^2 inv_{D,t-1} + \frac{\overline{Q}}{\overline{P_I}}\mu_S\left(\frac{1}{1+r}\right)(1+g_y)^2 E_t(inv_{D,t+1}) \tag{S.23}$$

$$q^I_t = \left(\frac{1}{1+r}\right)\left(\frac{\overline{Z}}{\overline{Q}}\right)E_t(\tilde{z}_{t+1}) + \left(\frac{1}{1+r}\right)(1-\delta)E_t(q_{t+1}) - E_t(\tilde{i}_t - \pi_{C,t+1}) \tag{S.24}$$

➢ FOCs for cost minimization and marginal cost (Formal Sector):

$$\left[k_{D,t} - \zeta_{DT,t} - \tilde{\ell}_{D,t}\right] = w_{D,t} - \tilde{z}_t \tag{S.25}$$

$$\frac{1}{\omega_H}o_{H,t} - \left(\left(\frac{1}{\omega_H} + \frac{1}{\alpha_H}\right)\eta_H - \frac{1}{\alpha_H}\right)\tilde{\ell}_{D,t} - \left(\frac{1}{\omega_H} + \frac{1}{\alpha_H}\right)(1-\eta_H)\left(k_{D,t} - \zeta_{DT,t}\right) + p_{O,t} - w_{D,t} = 0 \tag{S.26}$$

$$mc_{H,t} = \frac{\overline{Z}k_D}{MC_H Y_H}(\tilde{z}_t + k_{D,t}) + \frac{W_D \ell_D}{MC_H Y_H}(w_{D,t} + \tilde{\ell}_{D,t}) + \frac{P_O O_H}{MC_H Y_H}(p_{O,t} + o_{H,t}) - y_{H,t} \tag{S.27}$$

➢ FOCs for cost minimization and marginal cost (Informal Sector):

$$-\left[\zeta_{UT,t} + \tilde{\ell}_{U,t}\right] = w_{U,t} \tag{S.28}$$

$$\frac{1}{\omega_U}o_{U,t} - \frac{1}{\omega_U}\tilde{\ell}_{U,t} + p_{O,t} - w_{U,t} = 0 \tag{S.29}$$

$$mc_{U,t} = \frac{W_U \ell_U}{MC_U Y_U}(w_{U,t} + \tilde{\ell}_{U,t}) + \frac{P_O O_U}{MC_U Y_U}(p_{O,t} + o_{U,t}) - y_{U,t} \tag{S.30}$$

➢ New-Keynesian Phillips Curve for domestic formal-sector goods consumed at home:

$$\pi_{H,t} = \frac{\beta}{1+\beta\chi_H}E_t\left[\pi_{H,t+1}\right] + \frac{\chi_H}{1+\beta\chi_H}\left[\pi_{H,t-1}\right] + \frac{\kappa_H}{1+\beta\chi_H}\left[mc_{H,t} - p_{H,t}\right] \tag{S.31}$$

where, $\kappa_H = \dfrac{(1-\beta\phi_H)(1-\phi_H)}{\phi_H}$

- New-Keynesian Phillips Curve for domestically produced formal-sector exported goods consumed at abroad:

$$\pi^*_{H,t} = \frac{\beta}{1+\beta\chi^*_H} E_t\left[\pi^*_{H,t+1}\right] + \frac{\chi^*_H}{1+\beta\chi^*_H}\left[\pi^*_{H,t-1}\right] + \frac{\kappa^*_H}{1+\beta\chi^*_H}\left[mc_{H,t} - rer_t - p^*_{H,t}\right] \qquad (S.32)$$

$$\text{where, } \kappa^*_H = \frac{(1-\beta\phi^*_H)(1-\phi^*_H)}{\phi^*_H}$$

- New-Keynesian Phillips Curve for the imported goods:

$$\pi_{F,t} = \frac{\beta}{1+\beta\chi_F} E_t\left[\pi_{F,t+1}\right] + \frac{\chi_F}{1+\beta\chi_F}\left[\pi_{F,t-1}\right] + \frac{\kappa_F}{1+\beta\chi_F}\left[\zeta^*_{F,t} + rer_t - p_{F,t}\right] \qquad (S.33)$$

$$\text{where, } \kappa_F = \frac{(1-\beta\phi_F)(1-\phi_F)}{\phi_F}$$

- New-Keynesian Phillips Curve for domestic Informal-sector goods consumed at home:

$$\pi_{U,t} = \frac{\beta}{1+\beta\chi_U} E_t\left[\pi_{U,t+1}\right] + \frac{\chi_U}{1+\beta\chi_U}\left[\pi_{U,t-1}\right] + \frac{\kappa_U}{1+\beta\chi_U}\left[mc_{U,t} - p_{U,t}\right] \qquad (S.34)$$

$$\text{where, } \kappa_U = \frac{(1-\beta\phi_U)(1-\phi_U)}{\phi_U}$$

- The foreign demand for domestically produced goods:

$$y^*_{H,t} = y^*_t - \eta^* p^*_{H,t} \qquad (S.35)$$

- Law of one price of commodity-goods:

$$p_{S,t} = rer_t + p^*_{S,t} \qquad (S.36)$$

$$p_{O,t} = rer_t + p^*_{O,t} \qquad (S.37)$$

- Law of motion of relative prices:

$$\pi_{Z,t} = p_{Z,t} - p_{Z,t-1} + \pi_{C,t} \qquad (S.38)$$

$$\pi_{H,t} = p_{H,t} - p_{H,t-1} + \pi_{C,t} \qquad (S.39)$$

$$\pi^*_{H,t} = p^*_{H,t} - p^*_{H,t-1} + \pi^*_t \qquad (S.40)$$

$$\pi_{F,t} = p_{F,t} - p_{F,t-1} + \pi_{C,t} \qquad (S.41)$$

$$\pi_{U,t} = p_{U,t} - p_{U,t-1} + \pi_{C,t} \qquad (S.42)$$

$$\Delta\tilde{e}_t = rer_t - rer_{t-1} + \pi_{C,t} - \pi^*_t \qquad (S.43)$$

- Evaluation of Government Consumption:

$$\frac{P_G G}{P_{DY} Y_D} g_t = \frac{\tau}{P_{DY} Y_D}(\tilde{\tau}_t - y_{D,t}) + \left(\hbar - \frac{\hbar}{\Theta(1+i^*)}\right)\left[\Delta \tilde{e}_t - \pi_{C,t} + \tilde{b}_{G,t-1} - \Delta p_{DY,t} - \Delta y_{D,t} - \varsigma_{T,t}\right]$$

$$+ \frac{\hbar}{\Theta(1+i^*)} \tilde{i}^*_{H,t-1} + (\varsigma_{G,t} + p_{H,t} - p_{DY,t} - y_{D,t})$$

(S.44)

$$\text{Where,} \qquad \hbar = \frac{B_G}{P_Y Y} \frac{1}{(1+\pi^*)(1+g_{DY})(1+n)} \qquad \text{and}$$

$$\tilde{i}^*_{H,t} = \tilde{i}^*_t + \rho \mathbf{b}^*_t.$$

➢ Choice of Fiscal Policy instrument:

$$g_t - p_{H,t} - p_{DY,t} - y_{D,t} = 0$$

(S.45)

➢ Evaluation of Fiscal net asset position:

$$\frac{\varepsilon B^*_G}{P_{DY} Y_D} \frac{1}{\Theta(1+i^*)} \tilde{b}_{G,t} = \frac{\tau}{P_{DY} Y_D} \tilde{\tau}_t - \frac{P_G G}{P_{DY} Y_D} g_t + \frac{B_G}{P_{DY} Y_D} \frac{1}{\Theta(1+i^*)} \tilde{i}^*_{H,t-1}$$

$$+ \hbar \frac{\varepsilon B^*_G}{P_{DY} Y_D}\left[\Delta \tilde{e}_t - \pi_{C,t} + \tilde{b}_{G,t-1} - \Delta p_{DY,t} - \Delta y_{D,t} - \varsigma_{T,t}\right]$$

(S.46)

$$\text{Where, } \hbar = \frac{B_G}{P_Y Y} \frac{1}{(1+\pi^*)(1+g_{DY})(1+n)} \text{ and } \tilde{i}^*_{H,t} = \tilde{i}^*_t + \rho \mathbf{b}^*_t.$$

➢ Monetary policy rule:

$$\tilde{r}_t = \psi_i \tilde{r}_{t-1} + (1-\psi_i)\psi_\pi \pi_{C,t} + (1-\psi_i)\psi_y \Delta y_{D,t}$$

$$+ (1-\psi_i)\psi_{rer} rer_t + \xi_{m,t}$$

(S.47)

$$\text{Where, } \tilde{r}_t = \tilde{i}_t - E_t(\pi_{C,t+1}) \text{ is real rate of interest.}$$

➢ The total aggregate demand for domestically produced goods in the formal-sector is:

$$\frac{P_H Y_H}{P_Y Y} y_{H,t} = \gamma_C \frac{P_H C_H}{P_Y Y} \tilde{c}_{H,t} + \frac{P_G G}{P_Y Y}\left(g_t - p_{H,t} + p_{DY,t} + y_{D,t}\right) + \gamma_I \frac{P_I I_D}{P_Y Y} inv_{H,t} + \frac{P_H Y^*_H}{P_Y Y} y^*_{H,t} \quad \text{(S.48)}$$

➢ The total aggregate demand for domestically produced goods in the Informal-sector is:

$$\frac{P_U Y_U}{P_Y Y} y_{U,t} = (1-\upsilon_C)\frac{P_U C_U}{P_Y Y} \tilde{c}_{U,t}$$

(S.49)

➢ The total supply for domestically produced goods in the formal-sector is:

$$y_{H,t} = a_{H,t} + \alpha_H^{1/\omega_H}\left(A_H \frac{V_H}{Y_H}\right)^{(\omega_H-1)/\omega_H} \eta_H \tilde{\ell}_{D,t} + (1-\alpha_H)^{1/\omega_H}\left(A_H \frac{O_H}{Y_H}\right)^{(\omega_H-1)/\omega_H} o_{H,t}$$

$$+ \alpha_H^{1/\omega_H}\left(A_H \frac{V_H}{Y_H}\right)^{(\omega_H-1)/\omega_H}(1-\eta_H)(k_{D,t-1} - \varsigma_{T,t})$$

(S.50)

> The total supply for domestically produced goods in the Informal-sector is:

$$y_{U,t} = \alpha_U^{1/\omega_U} \left(A_U \frac{V_U}{Y_U} \right)^{(\omega_U-1)/\omega_U} \tilde{\ell}_{U,t} + (1-\alpha_U)^{1/\omega_U} \left(A_U \frac{O_U}{Y_U} \right)^{(\omega_U-1)/\omega_U} o_{U,t} \qquad (S.51)$$

> Real formal-sector GDP:

$$y_{D,t} = \frac{P_H C_H}{P_Y Y} \tilde{c}_{H,t} + \frac{P_G G}{P_Y Y} \left(g_t - p_{H,t} + p_{DY,t} + y_{D,t} \right) + \frac{P_I I_D}{P_Y Y} inv_{H,t} + \frac{P_{EXP} EXP}{P_Y Y} \exp_t - \frac{P_{IMP} IMP}{P_Y Y} imp_t \; (S.52)$$

> Real informal-sector GDP:

$$y_{U,t} = \frac{P_U C_U}{P_Y Y} \tilde{c}_{U,t} \qquad (S.53)$$

> Balance of payments:

$$\frac{(1-\rho)\mathbf{B}^*}{(1+i^*)\Theta(\mathbf{B}^*)} \tilde{b}_t^* = \frac{\mathbf{B}^*}{(1+i^*)\Theta(\mathbf{B}^*)} \tilde{i}_t^* + \frac{\mathbf{B}^*}{(1+\pi^*)(1+n)(1+g_{DY})} (\Delta \tilde{e}_t - \pi_{C,t} - \Delta p_{DY,t} - \Delta y_{D,t} + \tilde{b}_{t-1}^* - \varsigma_{T,t})$$

$$+ \frac{P_{EXP} EXP}{P_{DY} Y_D} (p_{EXP,t} + \exp_t - p_{DY,t} - y_{D,t}) - \frac{P_{IMP} IMP}{P_{DY} Y_D} (p_{IMP,t} + imp_t - p_{DY,t} - y_{D,t}) \qquad (S.54)$$

Where, $\mathbf{B}^* = \dfrac{\varepsilon B_G^*}{P_{DY} Y_D}$.

> Real exports and corresponding price-deflator:

$$\exp_t = \frac{\varepsilon P_S^* Y_S}{P_{EXP} EXP} y_{S,t} + \left(1 - \frac{\varepsilon P_S^* Y_S}{P_{EXP} EXP} \right) \tilde{c}_{H,t}^* \qquad (S.55)$$

$$p_{EXP,t} = \frac{\varepsilon P_S^* Y_S}{P_{EXP} EXP} p_{S,t} + \left(1 - \frac{\varepsilon P_S^* Y_S}{P_{EXP} EXP} \right) \left(p_{H,t}^* - rer_t \right) \qquad (S.56)$$

> Real imports and corresponding price-deflator:

$$imp_t = (1-\gamma_C) \frac{P_C C}{P_{IMP} IMP} \tilde{c}_{F,t} + (1-\gamma_I) \frac{P_I I_D}{P_{IMP} IMP} (inv_{F,t})$$

$$+ \frac{P_O(C_O + O_H + O_U)}{P_{IMP} IMP} \left(\frac{C_O}{C_O + O_H + O_U} \tilde{c}_{O,t} + \frac{O_H}{C_O + O_H + O_U} o_{H,t} + \frac{O_U}{C_O + O_H + O_U} o_{U,t} \right) \qquad (S.57)$$

$$p_{IMP,t} = rer_t + \frac{P_O(C_O + O_H + O_U)}{P_{IMP} IMP} p_{O,t} + \left[1 - \frac{P_O(C_O + O_H + O_U)}{P_{IMP} IMP} \right] \zeta_{F,t}^* \qquad (S.58)$$

> List of Exogenous Shocks:

$$\xi_t = \rho_\xi \xi_{t-1} + \varsigma_{\xi,t} \qquad (S.59)$$

Where, ξ_t is vector of 16 exogenous shocks and $\varsigma_{\xi,t}$ is a vector of *iid* processes.

APPENDIX B: Determinacy and E-Stability Conditions under Monetary Policy Learning

This section provides technical details about determinacy and expectational-stability (E-Stability) conditions under learning of alternative monetary policy rules. A more general discussion can be found in Evans and Honkapohja (2001) and Bullard and Mitra (2002, 2007). The fundamental notion of determinacy encapsulate under a necessary and sufficient condition which ensure equilibrium to exist. This condition for the uniqueness of such a solution in a system with no pre-determined variables is that correct number of eigenvalues lie inside the unit circle. This idea was initially highlighted by Blanchard and Kahn (1980) and later extended by McCallum (1983), Farmer (1992) and Klien (2000) for more general cases. Here we elaborate Blachard and Kahn (1980) method which is more feasible for model determinacy solution.

B1: Conditions of Determinacy/Local-Indeterminacy:

Consider a model given by the general form:
$$\Phi_1 E_t(Y_{t+1}) = \Phi_2 E_t(Y_t) + \Theta_1 \eta_t$$

Where, Y_t is a vector of endogenous variables, Φ_1, Φ_2 and Θ_1 are matrices of coefficients and η_t is a vector of exogenous variables which is assumed to follow a stationary VAR. If Φ_1 is invertible, then we can write the system as:
$$E_t(Y_{t+1}) = \Phi_1^{-1}\Phi_2 E_t(Y_t) + \Phi_1^{-1}\Theta_1 \eta_t$$

Let us assume: $\Phi = \Phi_1^{-1}\Phi_2$ and $\Theta = \Phi_1^{-1}\Theta_1$, then the above system can re-written as:
$$E_t(Y_{t+1}) = \Phi E_t(Y_t) + \Theta \eta_t$$

Using the notion of Jordan-decomposition, we can write matrix Φ as: $\Phi = A\Lambda A^{-1}$, where A is the matrix of eigenvectors of Φ and Λ is the diagonal matrix of eigenvalues. Since vector Y_t may contain backward and forward looking variables, so we can easily make a partition of Y_t into two sub vectors such that YB_t is a vector of backward looking variables and YF_t is a vector of forward looking variables. Therefore, we can write as:
$$Y_t = \begin{bmatrix} YB_t \\ YF_t \end{bmatrix}$$

Under these settings, we can express the whole system into its decomposition form as:
$$\begin{bmatrix} YB_{t+1} \\ E_t(YF_{t+1}) \end{bmatrix} = A\Lambda A^{-1} \cdot \begin{bmatrix} YB_t \\ YF_t \end{bmatrix} + \Theta \eta_t$$

If we pre-multiply both sides by $A^{-1} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}$, then we get the following result as:
$$\begin{bmatrix} KB_{t+1} \\ E_t(KF_{t+1}) \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \cdot \begin{bmatrix} KB_t \\ KF_t \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \eta_t$$

Thus we can easily separate each equation as:
$$KB_{t+1} = \Lambda_1 KB_t + \Gamma_1 \eta_t$$
$$E_t(KF_{t+1}) = \Lambda_2 KF_t + \Gamma_2 \eta_t$$

Based upon above decomposed system into two separate equations, Blachard and Kahn (1980) provide general determinacy conditions as:

Condition (a): if $|diag(\Lambda_1)| \ll 1$ and $|diag(\Lambda_2)| \gg 1$ both condition true, then the system has a unique solution (Unique Equilibria)

Condition (b): if $|diag(\Lambda_1)| \ll 1$ holds but $|diag(\Lambda_2)| \gg 1$ does not hold, then the system has many solutions (Multiple Equilibria)

Condition (c): if $|diag(\Lambda_2)| \gg 1$ holds but $|diag(\Lambda_1)| \ll 1$ does not hold, then the system has no solution. (In-determinacy)

These joint determinacy conditions guide us that, while both eigenvalues of matrix A can be shown to be real and positive, the largest is always greater than one. As a result there exists a continuum of solutions in a neighborhood of (0, 0) that satisfy the equilibrium conditions (local indeterminacy) and one cannot rule out the possibility of equilibria displaying fluctuations driven by self-fulfilling revisions in expectations.[27] Gali and Monacili (2005) have argued that these conditions can help to understand various combinations of alternative monetary policy rules. Their results shown that any kind of indeterminacy problem can be avoided, and the uniqueness of the equilibrium allocation restored, by having the central bank follow a rule which would imply that the interest rate should respond to inflation and/or the output gap, if these variables to deviate from their (zero) target values. It requires a credible threat by the central bank to vary the interest rate sufficiently in response to any deviations of these variables from target; yet, the very existence of that threat makes its effective application unnecessary.

In a more general case with complex model structure, Blanchard and Kahn (1980) conditions guide to categorize determinacy and indeterminacy regions numerically using any weighting scheme of generalized Taylor-type monetary policy rule.[28]

B2: Conditions of E-Stability under learning:

This section briefly describes learning framework of alternative monetary policy rules. Using this framework, we also discuss expectational stability (E-stability) conditions as proposed by Evans and Honkapohja (2001). Under learning, the agents do not have rational expectations, instead they form their expected values with adaptive learning rules which are updated as data is produced by the system. The fundamental idea is that at each period private agents possess the Perceived Law of Motion (PLM) whose form is similar to the Minimum-State Variable (MSV) solutions. Since the agents do not know the parameter values of the system, they use a kind of recursive least square updating rule, which is conditional upon E-stability. According to Evan and Honkapohja (2001) and Bullard and Mitra (2002), this E-stability is a notional time concept correspond to stability under real time adaptive learning under general conditions. According to them, under E-stability, recursive least square learning solution is locally convergent to rational expectation equilibrium. They have also argued that under weak assumptions, if rational expectation equilibrium is not E-stable, then the probability of convergence of the recursive least squares solution to rational expectation equilibrium is zero.

To explain this framework, we consider a model of the form:

$$Y_t = A + BE_t(Y_{t+1}) + CY_{t-1} + D\eta_t$$

$$\eta_t = \rho\eta_{t-1} + \varepsilon_t$$

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[27] This is also known as: stationary sunspot fluctuation.

[28] This is especially under such cases where analytical solution is not possible.

Where, Y_t is a vector of endogenous variables, A, B, C and D are matrices of coefficients and η_t is a vector of exogenous variables which is assumed to follow a stationary VAR. Given this general form with $C \neq 0$, an MSV rational expectational equilibrium takes the following form:[29]

$$Y_t = \bar{a} + \bar{b}Y_{t-1} + \bar{c}\eta_t$$

Where, $\bar{a}$, $\bar{b}$ and $\bar{c}$ are conformable and are to be calculated by the method of undermined coefficients. In order to define E-stability, we consider PLM of the same form of the MSV as:

$$Y_t = a + bY_{t-1} + c\eta_t$$

Evan and Honkapohja (2001) and Bullard and Mitra (2002) analyze different information assumption about how agents update their PLM. The first assumption treats expectations as determined before the current values of endogenous variables are to be realized. Under this assumption, the next period expectation is:

$$E(Y_{t+1}) = a + b(a + bY_{t-1} + c\eta_t) + c\eta_t$$

By substituting it into original model form, we can compute Actual Law of Motion (ALM) as:

$$Y_t = A + (B(I+b))a + (Bb^2 + C)Y_{t-1} + (B(bc + c\rho) + D)\eta_t$$

To analyze the E-stability conditions, we have to check the stability of the mapping T from the PLM to ALM:

$$T(a,b,c) = (A + (B(I+b))a, Bb^2 + C, B(bc + c\rho) + D)$$

Using this mapping we can easily define principle of E-stability, which comes from analyzing the following matrix of differential equation:

$$\frac{d}{d\tau}T(a,b,c) = T(a,b,c) - (a,b,c)$$

Using this differential equation, Evan and Honkapohja (2001) and Bullard and Mitra (2002) have shown three equilibrium conditions, which are:

(a) $DT_a = B(1 + \bar{b})$

(b) $DT_b = \bar{b}' \otimes B + I \otimes B\bar{b}$

(c) $DT_c = \rho' \otimes B + I \otimes B\bar{b}$

The rational expectational equilibrium $(\bar{a}, \bar{b}, \bar{c})$ is E-stable or learnable if all real parts of the eigenvalues of DT_a, DT_b and DT_c are lower than 1. The solution is E-unstable, if any of them have real part higher than 1. Alternatively, E-stability holds, if all eigenvalues of $DT_a - 1$, $DT_b - 1$ and $DT_c - 1$ have negative real parts. Bullard and Mitra (2002) have shown that these E-stability conditions actually govern stability under adaptive learning and therefore, really helpful to understand behavior of alternative monetary policy rules in more complex set of DSGE models.

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[29] See, McCallum (1983) for more details on this solution form.

APPENDEX-C: Model Calibration Results

Table C1: Key Structural Parameter Values for Model Calibrations (on quarterly basis)

Parameters	Description	Value
β	Subjective discount factor	0.99
$\hat{h}$	Degree of habit formation	0.36
ζ_M	Relative weight in preferences assigned to real money balances	0.25
μ	Semi-elasticity of money demand to interest rate	-0.15
σ_L	Inverse of wage elasticity of labor supply	1.50
α_C	Share of core goods in the consumption basket	0.75
ω_C	Elasticity of substitution between core and oil goods consumption bundle	0.35
υ_c	Share of formal sector goods in the core consumption basket	0.55
φ_c	Elasticity of substitution between formal and informal goods consumption bundle	0.70
γ_c	Share of home goods in the formal consumption basket	0.65
η_c	Elasticity of substitution between home and foreign goods consumption bundle	1.12
Λ_L	Share of formal labor in aggregate labor supply	0.29
$\wp_L$	Elasticity of substitution between formal and informal labor	2.00
$\in_L$	Elasticity of substitution between different labor skills in the formal sector	0.80
γ_I	Share of home investment in aggregate private investment	0.52
η_I	Elasticity of substitution between home and foreign private investment	1.02
δ	Capital depreciation rate	0.03
$\in_H$	Elasticity of substitution between differentiated formal intermediate varieties	1.00
τ_H	Flat tax rate on final home goods	0.15
α_H	Share of non-oil factor inputs in the production of intermediate formal sector varieties	0.65
ω_H	Elasticity of substitution between oil and other factor of inputs in formal production	0.85
η_H	Labor share in formal sector production function	0.54
$\in_F$	Elasticity of substitution between differentiated formal intermediate imported varieties	1.25
τ_F	Flat tax rate on final imported goods	0.15
$\in_U$	Elasticity of substitution between differentiated informal intermediate varieties	0.78
α_U	Share of non-oil factor inputs in the production of intermediate informal sector varieties	0.75
ω_U	Elasticity of substitution between oil and other factor of inputs in informal production	0.95
ϕ_H^i	Calvo degree of price rigidity in formal sector home goods	0.24
χ_H	Indexation of price of formal sector home goods	0.65
$\phi_{H^*}^i$	Calvo degree of foreign price rigidity in formal sector home goods	0.64
χ_H^*	Indexation of foreign price of formal sector home goods	0.55
ϕ_F^i	Calvo degree of price rigidity in formal sector imported goods	0.70
χ_F	Indexation of price of formal sector imported goods	0.45
ϕ_U^i	Calvo degree of price rigidity in informal sector home goods	0.21
χ_U	Indexation of price of informal sector home goods	0.70
ψ_i	Relative weight of interest rate inertia in monetary policy Rule	0.63
ψ_π	Relative weight of inflation in monetary policy Rule	1.21
ψ_y	Relative weight of output in monetary policy Rule	0.60
ψ_{rer}	Relative weight of real exchange rate in monetary policy Rule	0.05
γ_G	Share of government consumption of home goods in aggregate government consumption	0.75
η_G	Elasticity of substitution between government consumption of home and foreign goods	1.50
ς^*	share of domestic intermediate goods in the consumption basket of foreign agents	0.04
η^*	price elasticity of the foreign demand of domestic goods	0.78

Table C2: Data for Benchmark Model Calibrations
(Shock Process Paramters)

Exogenous Shocks	Persistence in Shocks (ρ_ξ)	Volatility in Shocks (σ_ξ)
Transitory negative productivity shock in formal sector	0.86	0.05
Negative agriculture commodity production shock	0.75	1.45
Negative foreign commodity price shock	0.89	1.82
Negative foreign demand shock	0.65	3.55
Positive foreign interest rate shock	0.55	0.37
Positive foreign inflation price shock	0.81	0.27
Domestic tight monetary policy shock	0.31	0.03
Domestic labor supply shock	0.85	1.02
Positive preference shock	0.81	2.51
Domestic fiscal policy shock	0.78	0.15
Negative investment adjustment cost shock	0.35	4.02
Negative domestic investment shock	0.65	4.55
Negative foreign investment shock	0.68	4.58
Positive import price shock	0.89	4.16
Positive international oil price shock	0.95	6.25
Permanent negative productivity shock	0.92	0.04

Table C3: Data for Benchmark Model Calibrations
(Annualized Steady State Values)

Variables	Steady State Values
Formal sector output growth	5.0 %
Informal sector output growth	3.5%
Formal sector overall inflation	7%
Informal sector inflation	9%
Current account to GDP ratio	2.5%
Formal sector consumption to Output ratio	70%
Informal sector consumption to informal output ratio	75%
Domestic private investment to output ratio	12%
Foreign private investment to output ratio	9%

Table C4: Standard Deviations and Relative Volatility with Output
(Calibration results from Baseline version of the Model)

Variables	S.D	Relative S.D with Formal Output
Formal Consumption	5.365	1.109
Informal Consumption	9.316	1.926
Formal sector Output	4.837	--
Informal sector Output	4.811	0.995
Agriculture Commodity Output	7.269	1.503
Inflation in Formal Sector	1.376	0.284
Inflation in Informal sector	2.18	0.451
Real Exchange Rate	5.718	1.182
Aggregate Labour	6.071	1.255
Aggregate Wages	4.792	0.991
Domestic Investment	20.219	4.180
Foreign Investment	24.992	5.167
Oil Consumption	17.269	3.570
Domestic Interest Rate	0.554	0.115
Government Consumption	9.414	1.946
Current Account	2.816	0.582

Table C5: Pairwise Correlation Matrix
(Calibration results from Baseline version of the Model)

	Var.01	Var.02	Var.03	Var.04	Var.05	Var.06	Var.07	Var.08	Var.09	Var.10	Var.11	Var.12	Var.13	Var.14	Var.15	Var.16
Var.01	1.00	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
Var.02	0.20	1.00	--	--	--	--	--	--	--	--	--	--	--	--	--	--
Var.03	0.74	0.39	1.00	--	--	--	--	--	--	--	--	--	--	--	--	--
Var.04	0.16	0.13	0.33	1.00	--	--	--	--	--	--	--	--	--	--	--	--
Var.05	0.02	0.02	0.06	0.00	1.00	--	--	--	--	--	--	--	--	--	--	--
Var.06	-0.02	-0.31	-0.23	0.09	0.05	1.00	--	--	--	--	--	--	--	--	--	--
Var.07	0.01	-0.25	-0.07	0.03	0.03	0.85	1.00	--	--	--	--	--	--	--	--	--
Var.08	0.28	0.07	0.42	-0.08	-0.02	-0.39	-0.15	1.00	--	--	--	--	--	--	--	--
Var.09	0.31	0.36	0.49	0.39	0.07	0.39	0.23	-0.05	1.00	--	--	--	--	--	--	--
Var.10	0.30	0.62	0.45	-0.01	-0.01	-0.67	-0.44	0.32	-0.25	1.00	--	--	--	--	--	--
Var.11	0.18	0.55	0.70	0.05	0.03	-0.22	-0.06	0.13	0.54	0.36	1.00	--	--	--	--	--
Var.12	-0.02	0.72	0.47	0.06	0.03	-0.30	-0.14	0.08	0.41	0.45	0.87	1.00	--	--	--	--
Var.13	0.46	0.40	0.36	0.07	0.01	-0.18	-0.13	0.04	0.20	0.48	0.25	0.24	1.00	--	--	--
Var.14	-0.13	0.01	0.03	0.29	0.17	0.10	0.05	-0.12	0.19	-0.12	0.09	0.05	-0.04	1.00	--	--
Var.15	-0.41	-0.37	-0.64	-0.32	0.04	0.22	0.16	-0.25	-0.47	-0.22	-0.66	-0.54	-0.29	-0.14	1.00	--
Var.16	-0.13	-0.38	-0.38	0.31	0.07	-0.11	-0.35	-0.08	-0.32	-0.19	-0.67	-0.68	-0.11	0.07	0.24	1.00

Table Note:

Var.01	Formal Consumption	Var.05	Agriculture Commodity Output	Var.09	Aggregate Labour	Var.13	Oil Consumption
Var.02	Informal Consumption	Var.06	Inflation in Formal Sector	Var.10	Aggregate Wages	Var.14	Domestic Interest Rate
Var.03	Formal sector Output	Var.07	Inflation in Informal sector	Var.11	Domestic Investment	Var.15	Government Consumption
Var.04	Informal sector Output	Var.08	Real Exchange Rate	Var.12	Foreign Investment	Var.16	Current Account

Table C6: Autocorrelations
(Calibration results from Baseline version of the Model)

	Lag Order 1	Lag Order 2	Lag Order 3	Lag Order 4	Lag Order 5
Formal Consumption	0.9428	0.8856	0.8282	0.7708	0.7143
Informal Consumption	0.9613	0.8998	0.8262	0.7480	0.6704
Formal sector Output	0.9007	0.8222	0.7553	0.6948	0.6379
Informal sector Output	0.6700	0.4489	0.3008	0.2015	0.1350
Agriculture Commodity Output	0.7700	0.5929	0.4565	0.3515	0.2707
Inflation in Formal Sector	0.8690	0.7124	0.5828	0.4849	0.4136
Inflation in Informal sector	0.7727	0.5205	0.3270	0.1933	0.1052
Real Exchange Rate	0.8996	0.8120	0.7336	0.6617	0.5951
Aggregate Labour	0.7864	0.6464	0.5500	0.4779	0.4199
Aggregate Wages	0.9893	0.9693	0.9436	0.9149	0.8848
Domestic Investment	0.9071	0.8155	0.7264	0.6395	0.5550
Foreign Investment	0.9401	0.8657	0.7821	0.6937	0.6039
Oil Consumption	0.9687	0.9380	0.9079	0.8786	0.8501
Domestic Interest Rate	0.5795	0.3564	0.2368	0.1702	0.1309
Government Consumption	0.9103	0.8358	0.7701	0.7097	0.6528
Current Account	0.7950	0.6284	0.4894	0.3724	0.2734

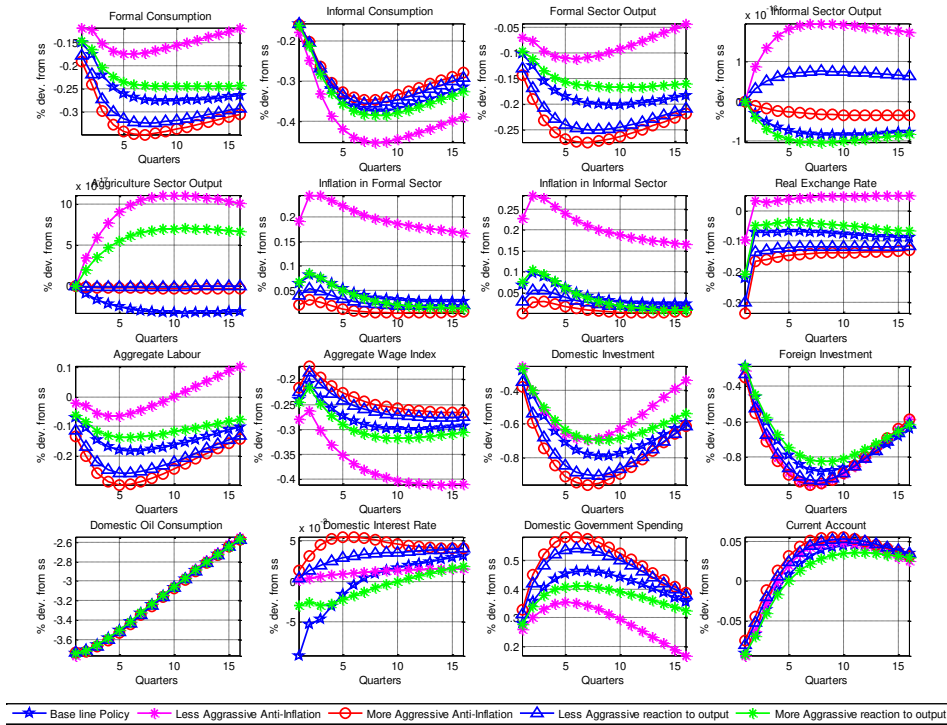# Figure C1: Impulse Response to positive international oil price shock

Legend: Base line Policy — Less Aggrassive Anti-Inflation — More Aggrassive Anti-Inflation — Less Aggrassive reaction to output — More Aggrassive reaction to output

Figure C2: Impulse response to negative domestic investment shock

Legend: Base line Policy — Less Aggrassive Anti-Inflation — More Aggrassive Anti-Inflation — Less Aggrassive reaction to output — More Aggrassive reaction to output
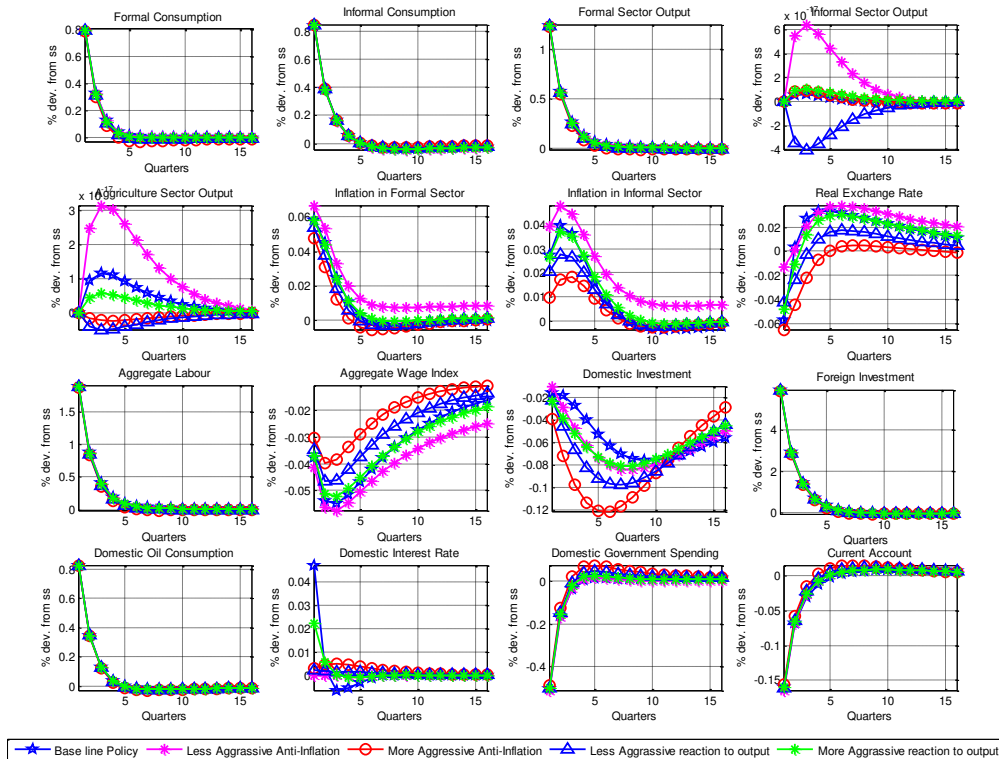
Figure C3: Impulse response to negative foreign investment shock
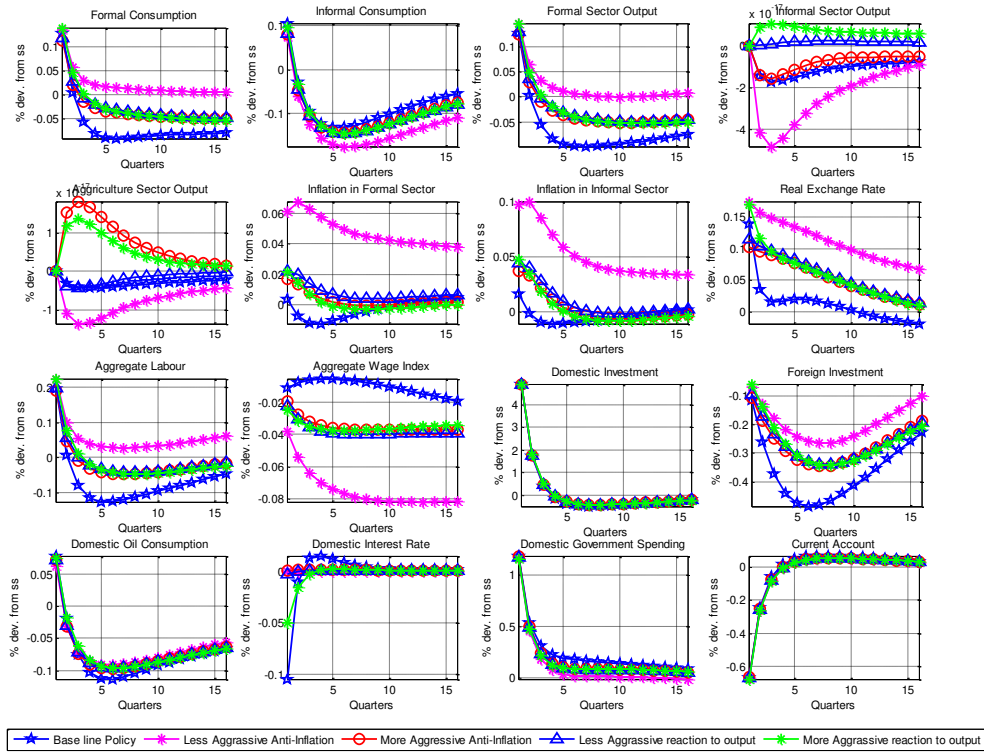
Figure C4: Impulse Response to negative investment adjustment cost shock
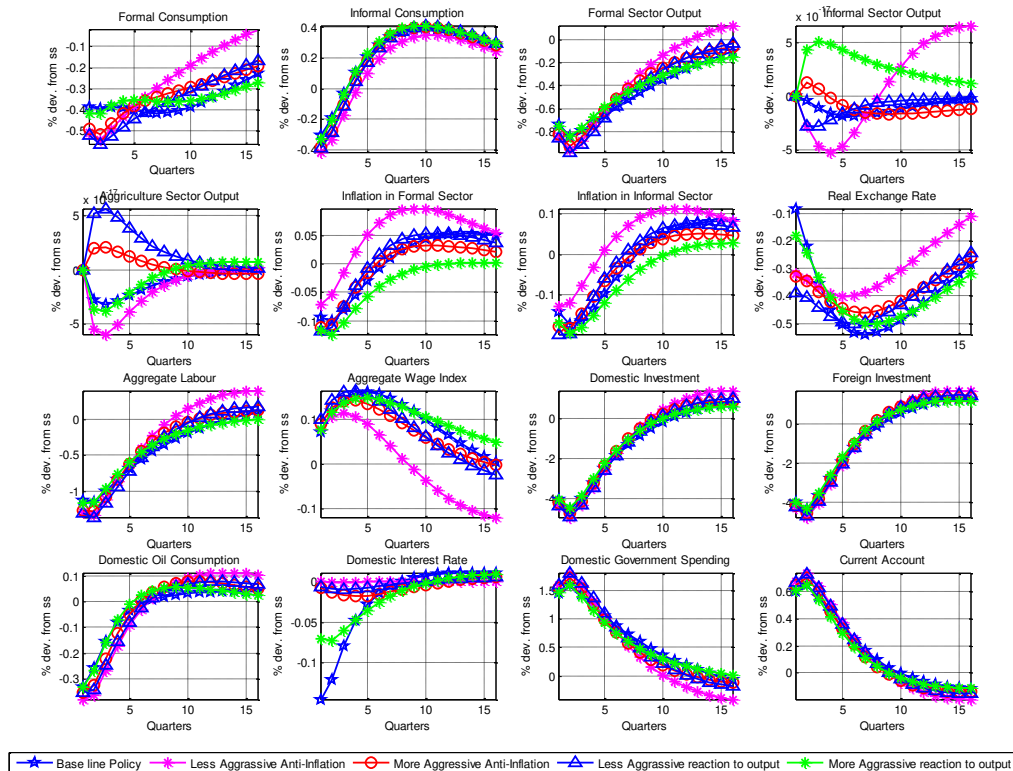
Figure C5: Impulse response to negative foreign demand shock
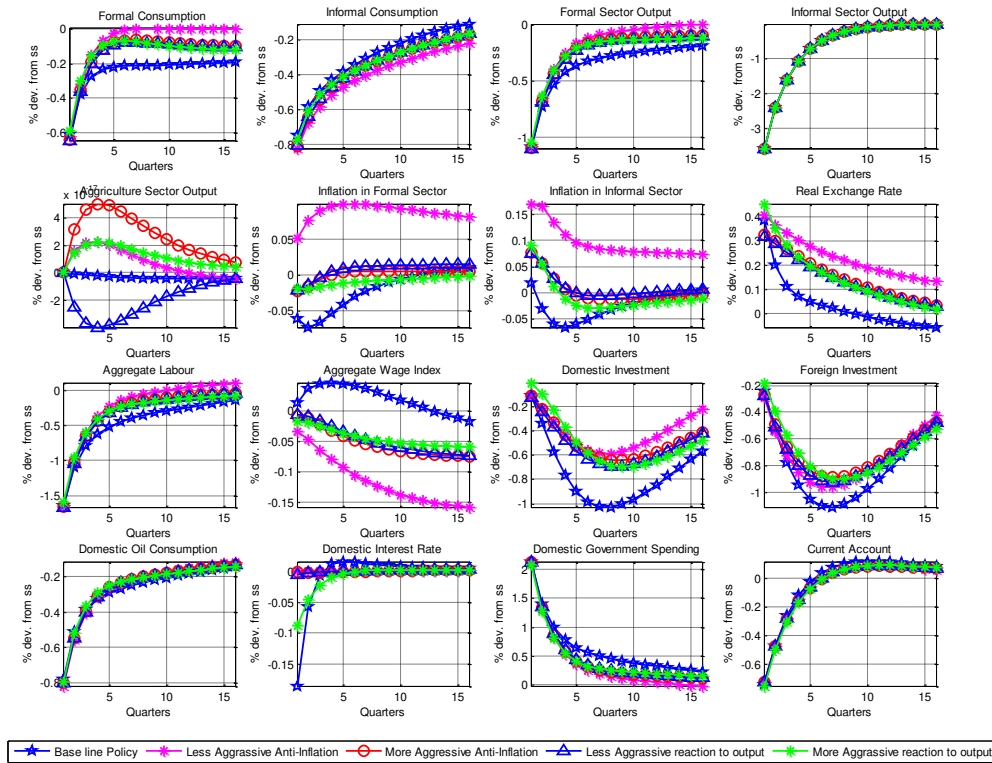
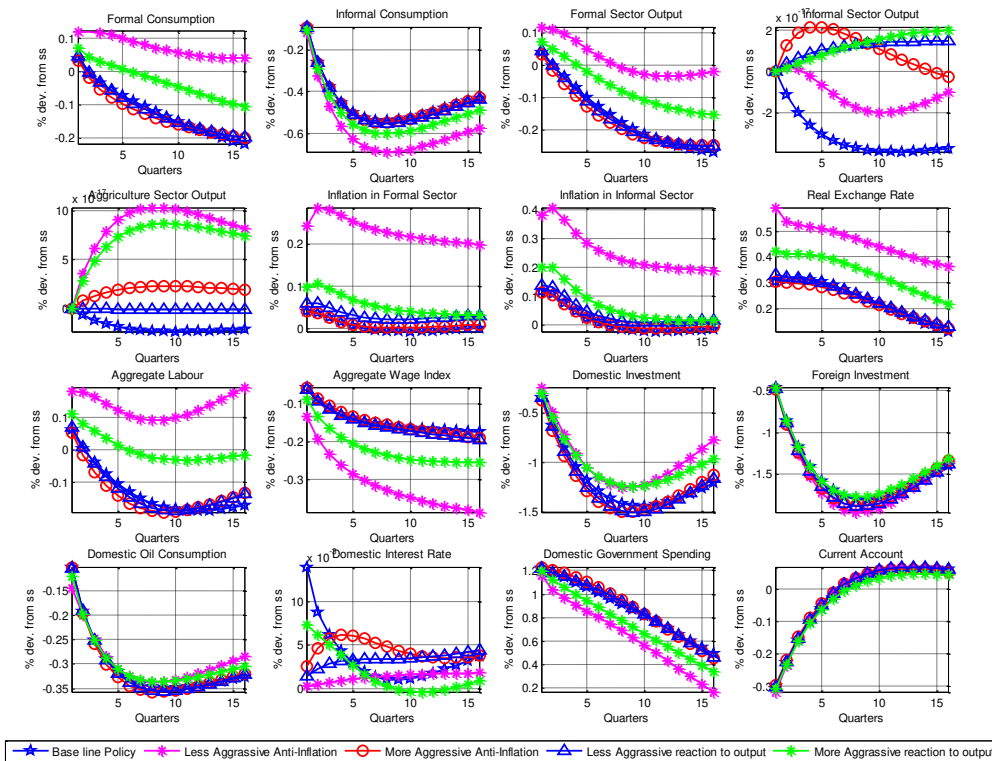## Figure C6: Impulse response to negative foreign commodity price shock

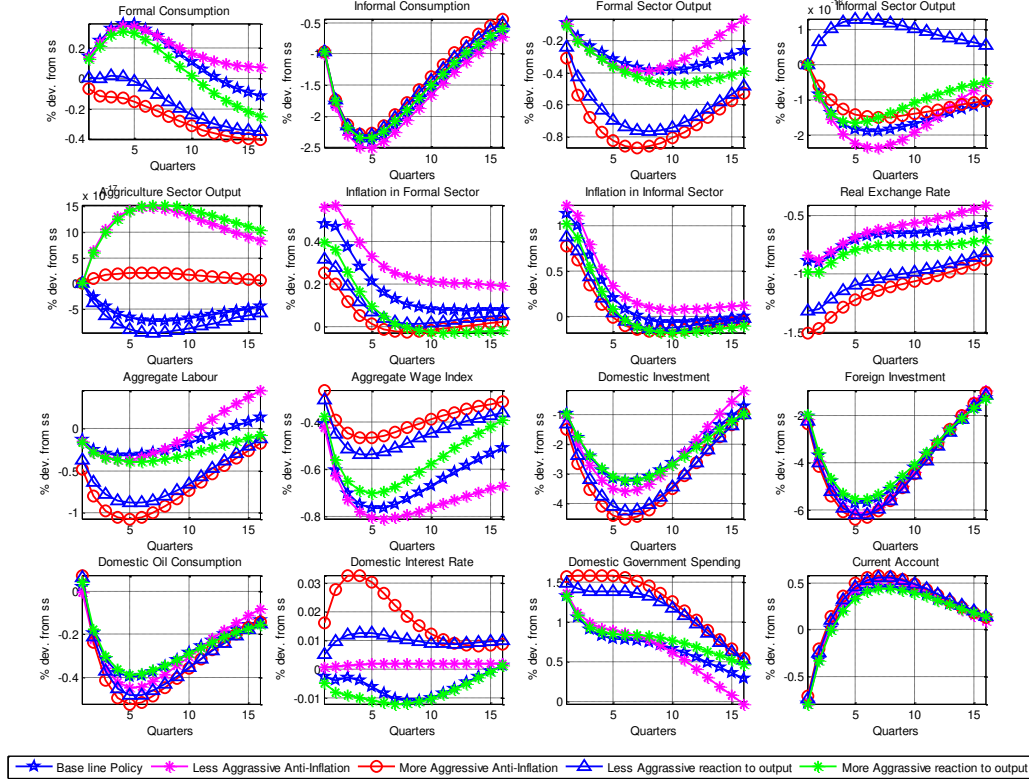## Figure C7: Impulse response to positive import price shock

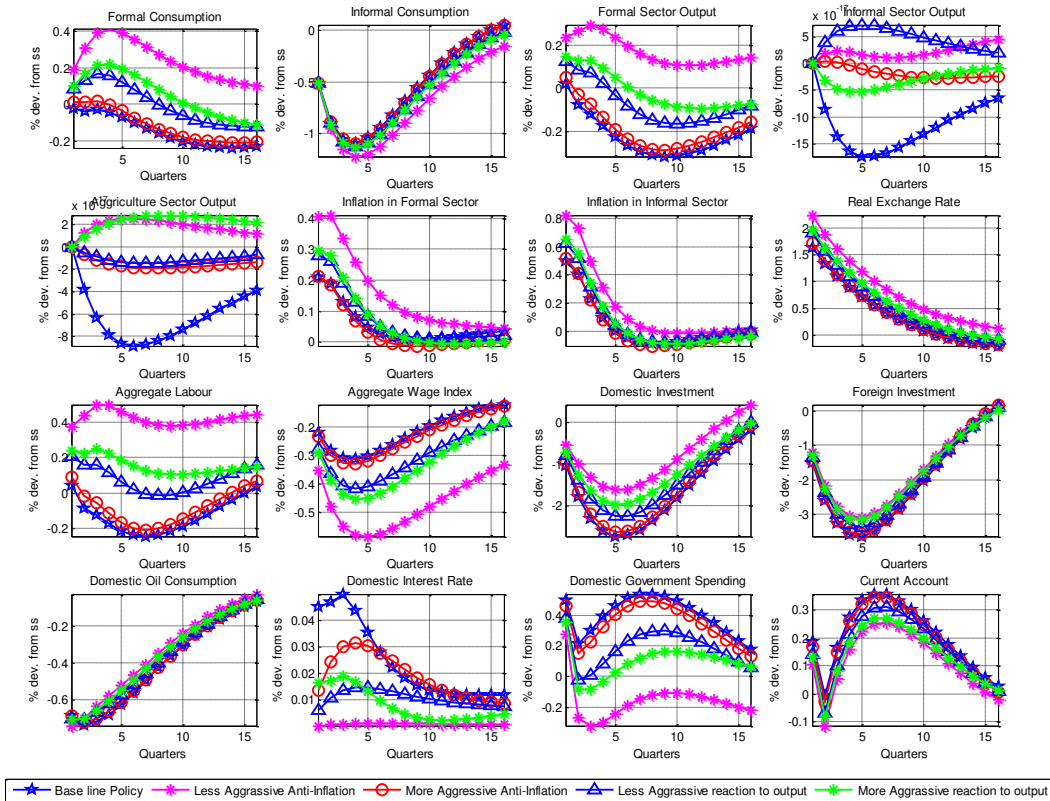## Figure C8: Impulse response to positive foreign interest rate shock

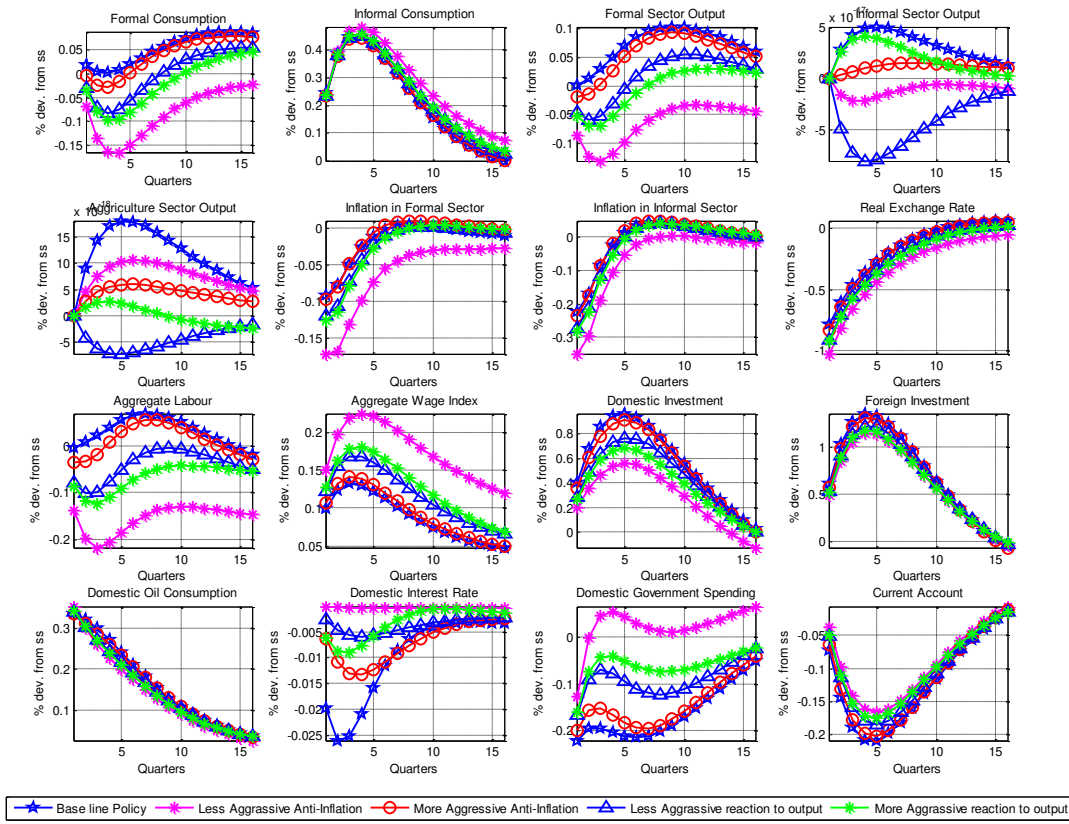# Figure C9: Impulse Response to positive foreign inflation price shock

| Base line Policy | Less Aggrassive Anti-Inflation | More Aggressive Anti-Inflation | Less Aggrassive reaction to output | More Aggressive reaction to output |

Figure C10: Impulse response to transitory negative productivity shock in formal sector

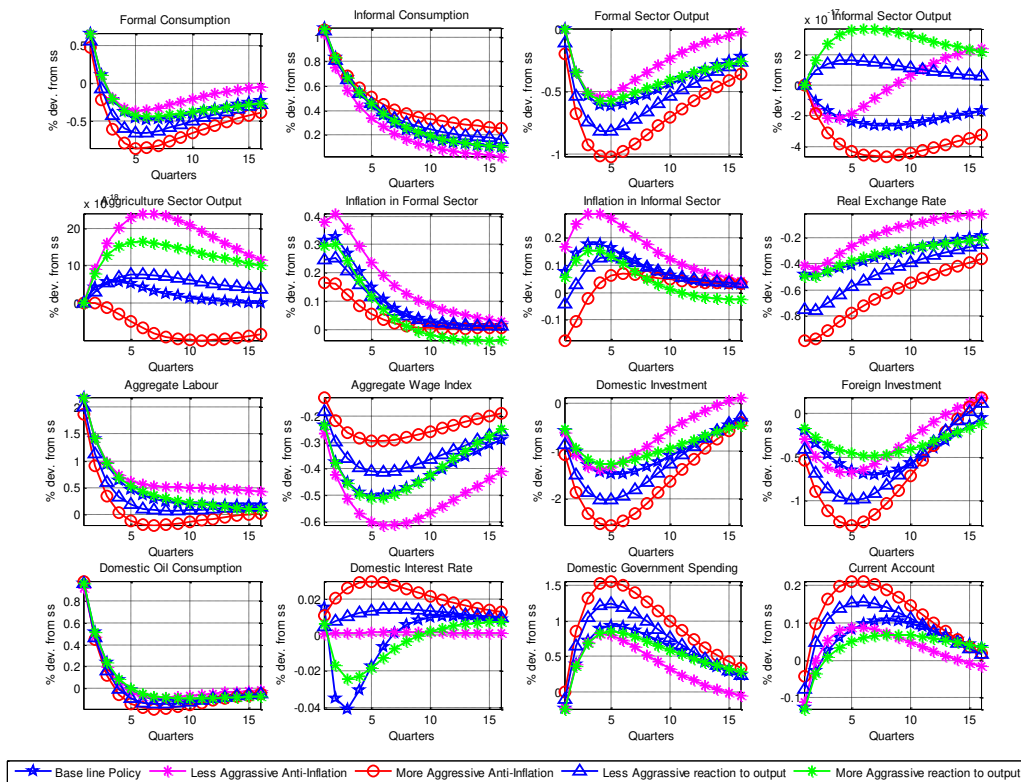| Base line Policy | Less Aggrassive Anti-Inflation | More Aggressive Anti-Inflation | Less Aggrassive reaction to output | More Aggressive reaction to output |

Figure C11: Impulse response to permanent negative productivity shock
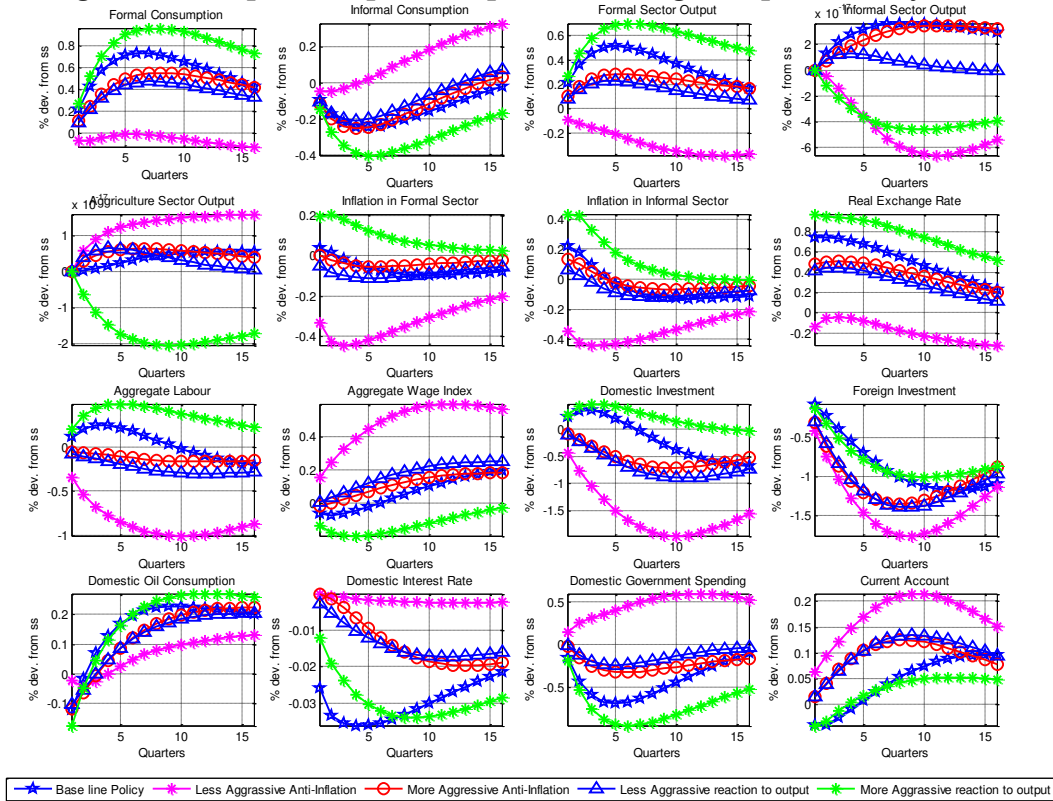

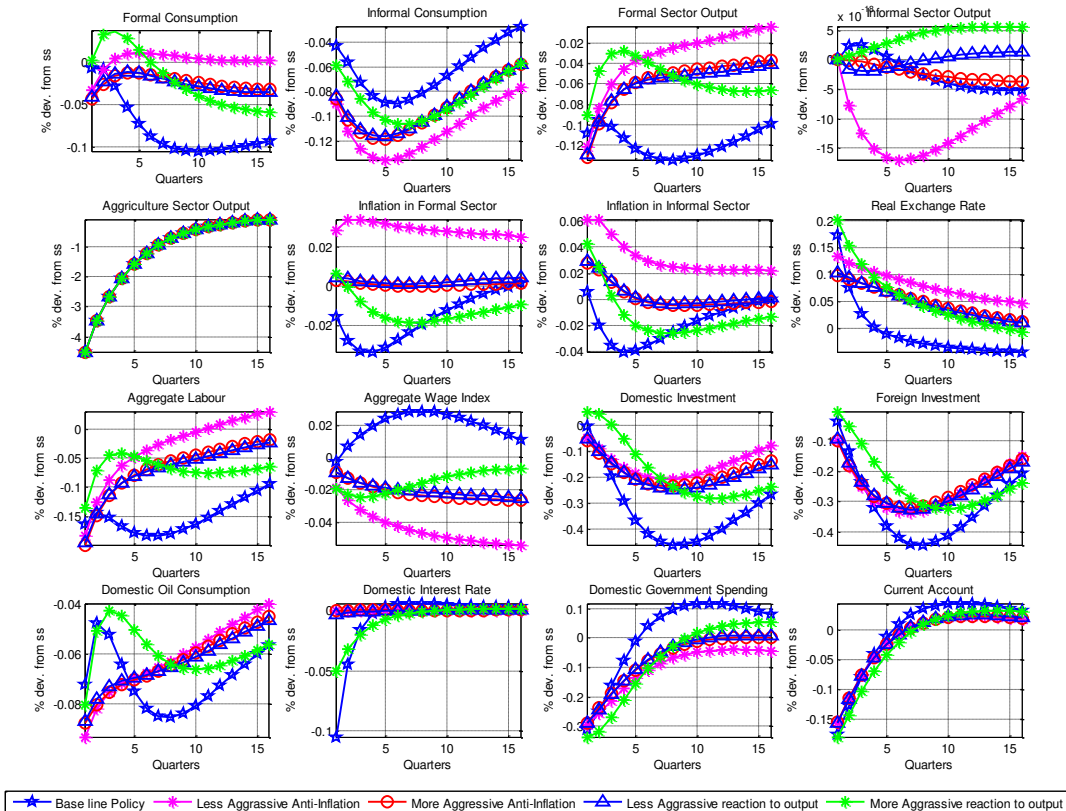**Figure C12: Impulse response to negative agriculture commodity production shock**
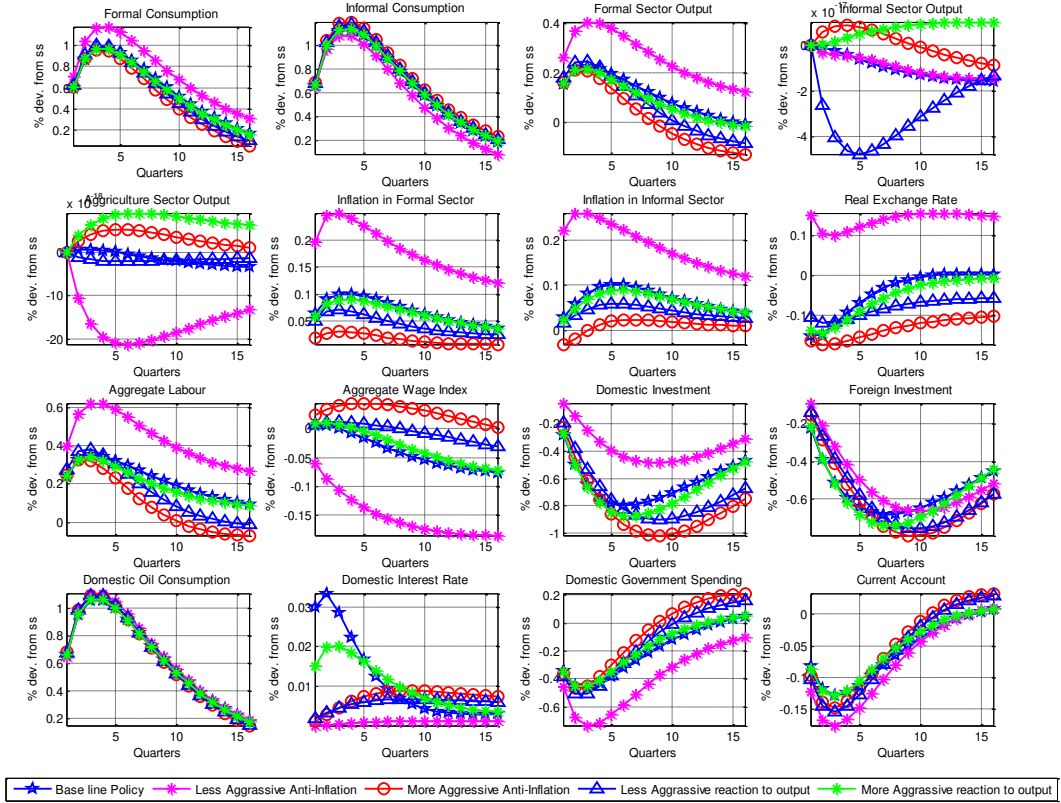
Figure C13: Impulse response to preference shock
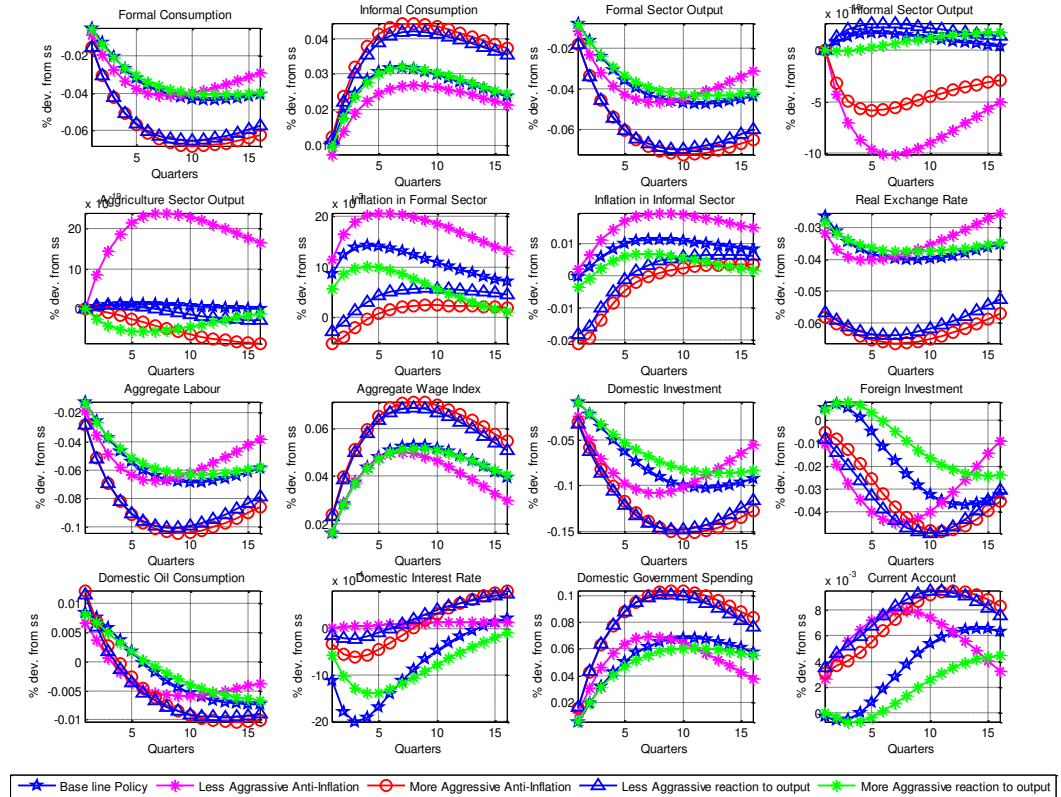

Figure C14: Impulse response to domestic labour supply shock

Figure C15: Impulse response to domestic tight monetary policy shock

Base line Policy — Less Aggrassive Anti-Inflation — More Aggrassive Anti-Inflation — Less Aggrassive reaction to output — More Aggrassive reaction to output
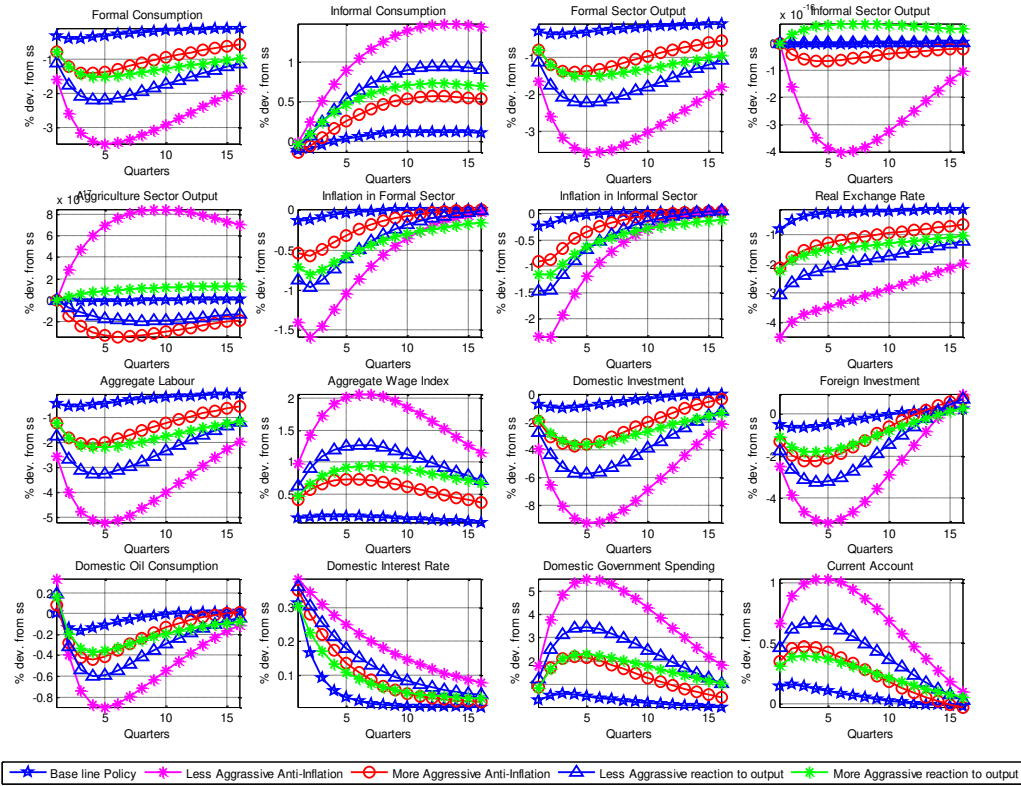
Figure C16: Impulse response to domestic fiscal policy shock

Base line Policy — Less Aggrassive Anti-Inflation — More Aggrassive Anti-Inflation — Less Aggrassive reaction to output — More Aggrassive reaction to output
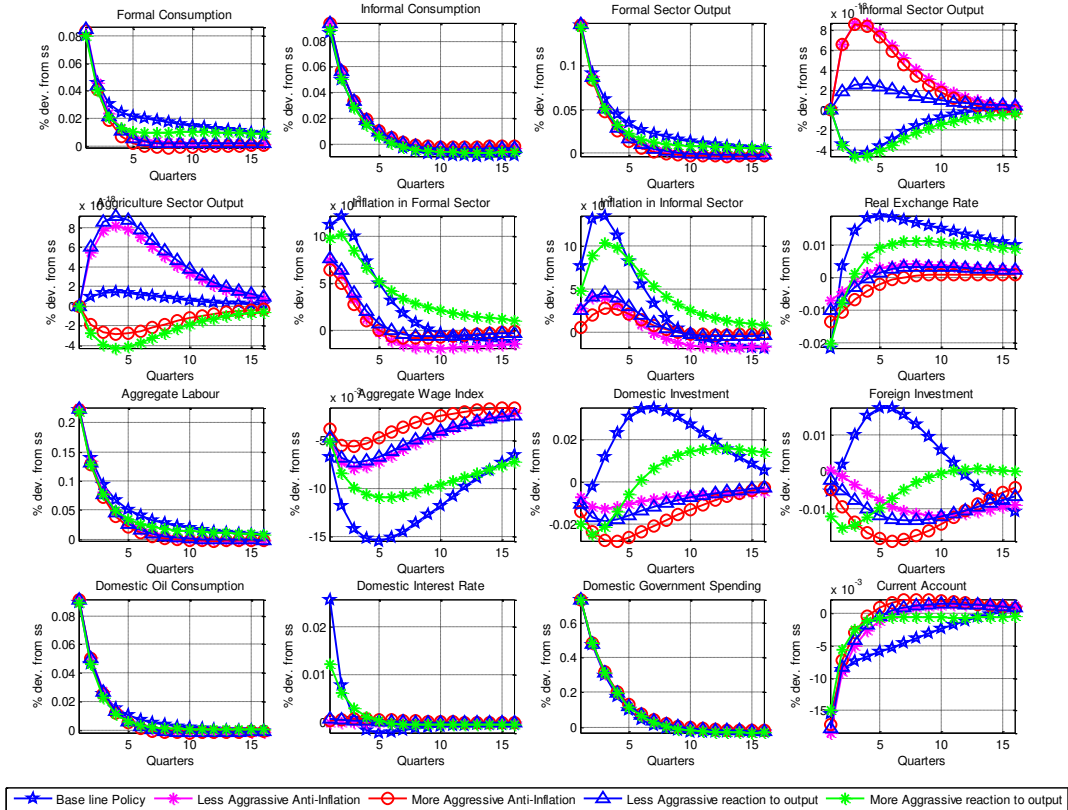
Table C7: Variance Decomposition
(Calibration results from Baseline version of the Model)

Variables\Shocks	S.01	S.02	S.03	S.04	S.05	S.06	S.07	S.08	S.09	S.10	S.11	S.12	S.13	S.14	S.15	S.16	Domestic Contribution	Foreign Contribution
Formal Consumption	9.11	0.68	6.27	5.49	2.67	0.35	3.32	0.13	24.59	0.05	8.00	3.53	9.14	23.42	2.59	0.68	50.1%	49.9%
Informal Consumption	3.77	0.09	7.63	2.49	8.72	1.34	0.27	0.02	10.54	0.01	2.08	55.86	4.62	1.29	1.04	0.22	72.9%	27.1%
Formal sector Output	14.02	1.22	10.25	13.75	4.85	0.45	3.76	0.18	1.43	0.17	18.18	8.06	4.48	10.07	8.43	0.70	47.7%	52.3%
Informal sector Output	3.31	0.09	7.22	2.01	8.32	2.85	2.63	0.02	10.16	0.01	1.60	56.04	4.17	0.80	0.55	0.22	74.1%	25.9%
Agriculture Commodity Output	13.69	0.73	9.87	13.41	4.40	2.85	2.63	0.18	0.94	0.17	17.90	7.65	4.03	9.69	8.03	3.87	47.8%	52.2%
Inflation in Formal Sector	19.43	0.37	0.59	1.34	6.84	1.07	2.36	0.13	5.17	0.03	3.28	48.29	3.24	7.45	0.33	0.10	79.2%	20.8%
Inflation in Informal sector	3.90	0.21	0.92	0.57	11.33	2.07	2.38	0.04	2.05	0.01	3.92	64.21	1.42	6.82	0.10	0.05	76.8%	23.2%
Real Exchange Rate	6.33	0.21	3.27	0.83	24.19	4.67	4.57	0.10	0.24	0.01	9.44	30.05	0.77	15.19	0.04	0.09	51.0%	49.0%
Aggregate Labour	24.76	1.11	1.49	16.90	3.39	0.27	4.57	0.20	2.84	0.24	15.73	10.26	1.22	4.14	12.36	0.49	60.2%	39.8%
Aggregate Wages	14.36	0.10	9.27	0.71	4.51	0.79	0.95	0.18	1.26	0.01	1.06	43.20	17.10	6.22	0.10	0.17	61.3%	38.7%
Domestic Investment	4.81	0.54	8.21	2.74	16.34	1.75	1.35	0.04	1.89	0.00	20.88	25.22	2.33	1.94	11.36	0.59	55.3%	44.7%
Foreign Investment	0.83	0.31	8.01	1.96	17.17	2.03	0.48	0.00	0.98	0.00	12.99	45.27	1.86	3.51	0.01	4.59	65.5%	34.6%
Oil Consumption	0.45	0.04	1.28	0.64	1.20	0.20	0.03	0.00	2.73	0.00	0.08	0.47	92.09	0.45	0.28	0.06	3.9%	96.1%
Domestic Interest Rate	1.82	4.46	0.80	12.82	4.62	0.94	45.69	0.01	1.47	0.24	15.41	0.74	0.34	5.99	0.75	3.91	73.8%	26.3%
Government Consumption	8.16	0.51	32.46	11.78	4.20	0.62	2.23	0.09	1.48	1.03	14.20	11.97	4.30	4.23	0.33	2.42	42.1%	57.9%
Current Account	1.46	0.92	2.93	12.31	13.54	3.90	1.51	0.01	1.20	0.01	21.49	31.16	0.45	1.75	0.40	6.94	64.7%	35.3%

Table Note:

Shock.01	transitory negative productivity shock in formal sector	Shock.09	positive preference shock
Shock.02	negative agriculture commodity production shock	Shock.10	domestic fiscal policy shock
Shock.03	negative foreign commodity price shock	Shock.11	negative investment adjustment cost shock
Shock.04	negative foreign demand shock	Shock.12	negative domestic investment shock
Shock.05	positive foreign interest rate shock	Shock.13	negative foreign investment shock
Shock.06	positive foreign inflation price shock	Shock.14	positive import price shock
Shock.07	domestic tight monetary policy shock	Shock.15	positive international oil price shock
Shock.08	domestic labour supply shock	Shock.16	permanent negative productivity shock

Table C8: Performance of Alternative Monetary Policy Specifications

Variables	Baseline policy[*,1]	Less aggressive anti-inflation policy[*,2]	More aggressive anti-inflation policy[*,3]	Policy with less aggressive reaction to output[*,4]	Policy with more aggressive reaction to output[*,5]
Formal Consumption	5.365	12.698	7.119	8.970	8.353
Informal Consumption	9.316	12.021	9.239	9.876	9.865
Formal sector Output	4.837	12.820	7.221	9.033	7.627
Informal sector Output	4.811	4.653	4.276	5.102	4.418
Agriculture Commodity Output	7.269	7.031	6.964	7.208	7.011
Inflation in Formal Sector	1.376	4.614	1.297	2.269	2.264
Inflation in Informal sector	2.180	5.832	2.199	3.292	3.211
Real Exchange Rate	5.718	14.781	8.826	10.761	9.582
Aggregate Labour	6.071	18.411	8.191	11.637	9.367
Aggregate Wages	4.792	10.067	4.429	6.161	5.832
Domestic Investment	20.219	34.319	24.414	27.225	20.926
Foreign Investment	24.992	30.922	26.923	27.869	24.096
Oil Consumption	17.269	17.234	17.365	17.360	17.283
Domestic Interest Rate	0.554	0.878	0.618	0.699	0.565
Government Consumption	9.414	19.601	12.298	14.616	11.823
Current Account	2.816	4.309	3.192	3.504	2.818
Welfare Loss (Formal Sector)	-22.156	-174.546	-43.848	-210.115	-149.063
Welfare Loss (Informal Sector)	-54.948	-393.238	-55.905	-125.291	-119.230

Table Note:

*/ Corresponding to each policy rule specification, percent standard deviations are given for each variable.

1/ Baseline policy: $\psi_i = 0.63$; $\psi_\pi = 1.21$; $\psi_y = 0.60$ and $\psi_{rer} = 0.05$

2/ Less aggressive anti-inflation policy: $\psi_i = 0.90$; $\psi_\pi = 1.01$; $\psi_y = 0$ and $\psi_{rer} = 0$

3/ More aggressive anti-inflation policy: $\psi_i = 0.90$; $\psi_\pi = 1.65$; $\psi_y = 0$ and $\psi_{rer} = 0$

4/ Policy with less aggressive reaction to output: $\psi_i = 0.90$; $\psi_\pi = 1.21$; $\psi_y = 0.53$ and $\psi_{rer} = 0$

5/ Policy with more aggressive reaction to output: $\psi_i = 0.90$; $\psi_\pi = 1.21$; $\psi_y = 0.95$ and $\psi_{rer} = 0$

Figure C17: Determinacy and E-Stability Plots of Monetary Policy Rule
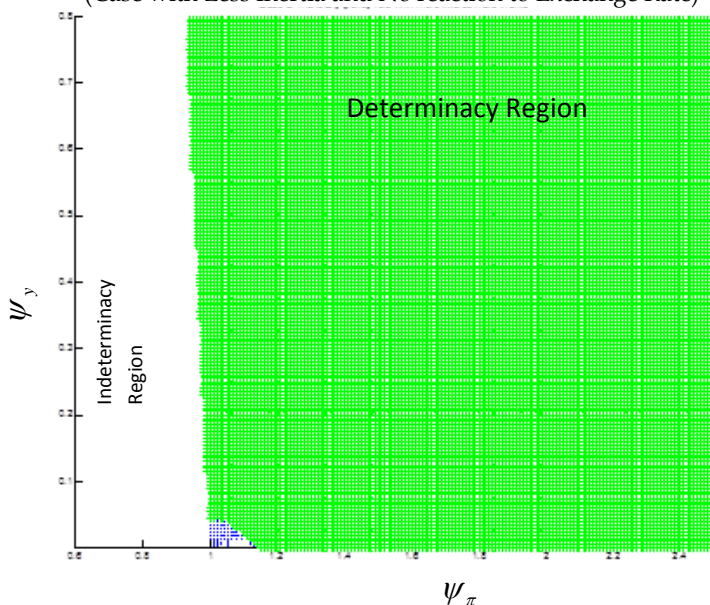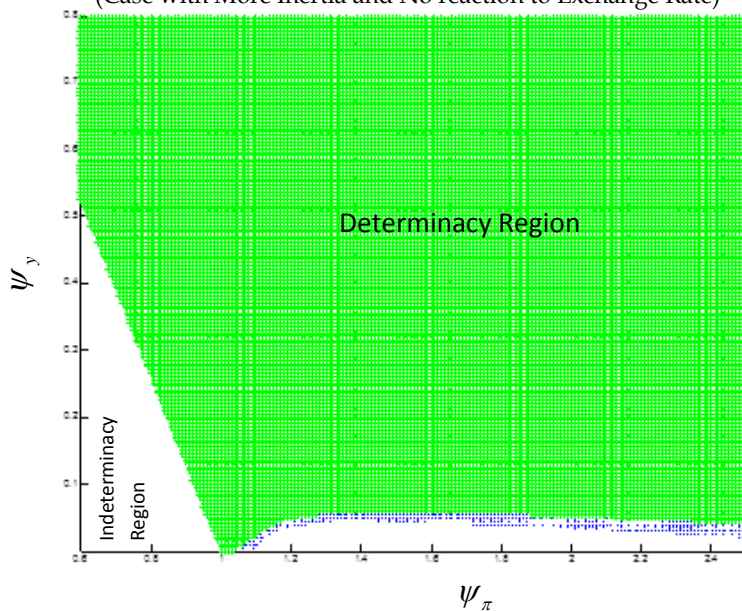(Case with Less Inertia and No reaction to Exchange Rate)

Figure Note:
Monetary policy rule is given as:

$$\tilde{r}_t = \psi_i \tilde{r}_{t-1} + \left(1 - \psi_i\right)\psi_\pi \pi_{C,t} + \left(1 - \psi_i\right)\psi_y \Delta y_{D,t} + + \left(1 - \psi_i\right)\psi_{rer} rer_t + \xi_{m,t}$$

For this case: $\psi_i = 0.63$ and $\psi_{rer} = 0$

Figure C18: Determinacy and E-Stability Plots of Monetary Policy Rule
(Case with More Inertia and No reaction to Exchange Rate)

Figure Note:
Monetary policy rule is given as:

$$\tilde{r}_t = \psi_i \tilde{r}_{t-1} + \left(1 - \psi_i\right)\psi_\pi \pi_{C,t} + \left(1 - \psi_i\right)\psi_y \Delta y_{D,t} + + \left(1 - \psi_i\right)\psi_{rer} rer_t + \xi_{m,t}$$

For this case: $\psi_i = 0.90$ and $\psi_{rer} = 0$

Figure C19: Determinacy and E-Stability Plots of Monetary Policy Rule
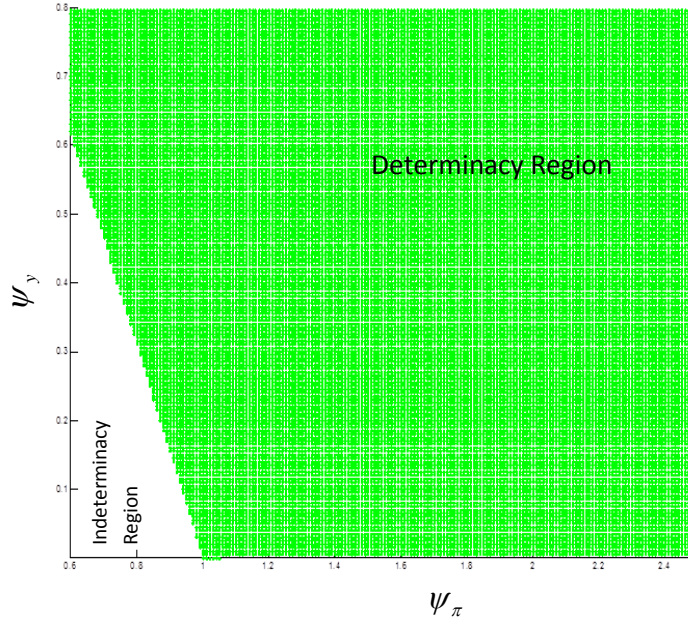(Case with Less Inertia and optimal reaction to Exchange Rate)

Figure Note:
Monetary policy rule is given as:
$$\tilde{r}_t = \psi_i \tilde{r}_{t-1} + \left(1-\psi_i\right)\psi_\pi \pi_{C,t} + \left(1-\psi_i\right)\psi_y \Delta y_{D,t} + + \left(1-\psi_i\right)\psi_{rer} rer_t + \xi_{m,t}$$
For this case: $\psi_i = 0.63$ and $\psi_{rer} = 0.05$

Figure C20: Determinacy and E-Stability Plots of Monetary Policy Rule
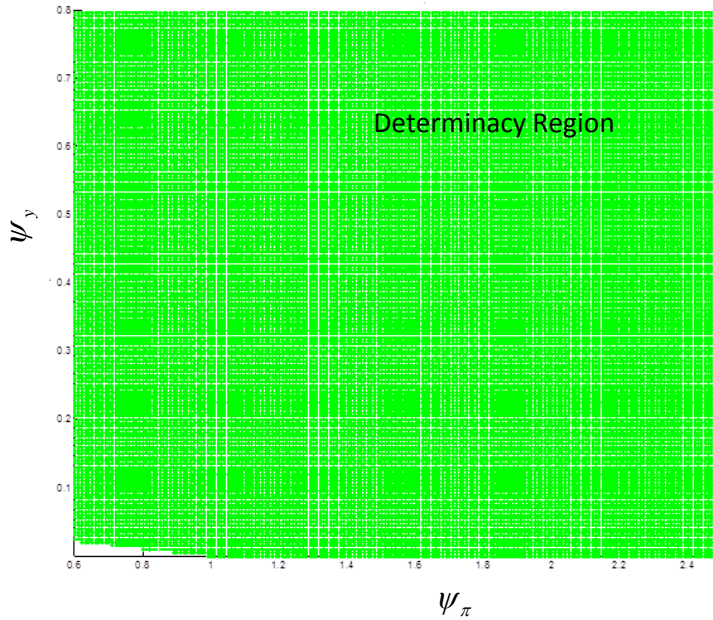(Case with More Inertia and optimal reaction to Exchange Rate)

Figure Note:
Monetary policy rule is given as:
$$\tilde{r}_t = \psi_i \tilde{r}_{t-1} + \left(1-\psi_i\right)\psi_\pi \pi_{C,t} + \left(1-\psi_i\right)\psi_y \Delta y_{D,t} + + \left(1-\psi_i\right)\psi_{rer} rer_t + \xi_{m,t}$$
For this case: $\psi_i = 0.90$ and $\psi_{rer} = 0.05$