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Formal and informal markets: A strategic and evolutionary perspective*

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Abstract

We investigate the coexistence of formal and informal markets. In formal markets, we assume sellers can publicly advertise their prices and locations, whereas in informal markets, sellers need to trade through bilateral bargaining so as to remain anonymous from the taxing authority. We consider two models. As a benchmark, we first only allow sellers to switch between markets, which enables us to derive some analytical results that show the existence of a stable equilibrium where formal and informal markets coexist. We also establish that some sellers will migrate from the formal market to the informal market if the formal market's advantage in quality assurance erodes, or the government imposes higher taxes and regulations in the formal market, or the risk of crime and/or confiscation decreases in the informal market, or the number of buyers in the informal market increases. Some sellers will migrate from the informal market to the formal market whenever the opposite changes occur. We then allow both sellers and buyers to switch between markets. In this model, we illustrate that if the net costs of trading for sellers in the formal sector and buyers in the informal sector have opposite signs, then there is a unique locally stable equilibrium where formal and informal markets coexist.

1 Introduction

While the definition of informal economies is subject to some disagreement, there is never any debate that sellers in these markets strive to remain anonymous from taxing and regu-

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lating authorities.[1] This simple and basic fact that sellers operate in the informal economy to avoid detection by authorities has very important implications on the type of trading protocols that these sellers can use to attract buyers. This is an aspect that has not received much attention by the literature. In this paper we explore the two most common trading protocols used in these markets, and study their implications for the coexistence of formal and informal activity while explicitly taking into account the strategic behavior of agents. We then also incorporate other distinguishing features which deliver different trading costs between formal and informal markets, such as taxes, regulation and the provision of quality assurance in the formal market, and the risk of crime and/or confiscation in the informal market as well as the relative market tightness in these markets.

Bargaining was the predominant trading protocol until 1820s. The use of posted prices by sellers is a relatively recent phenomenon. Its ascent and eventual widespread use date back to 1823, when Alexander Stewart introduced posted prices in his New York City 'Marble Dry Goods Palace', which quickly grew to become the largest store in the U.S.[2] The main differences between these two trading protocols are their implied informational requirements. It is important to note that informative advertising is crucial for price posting to be effective. In order to attract buyers, price-posting sellers need to send informative signals describing their product, price, and —more importantly —their location. Such detailed information is observed by competitors, potential buyers, and the taxing authority as well.[3] Hence, firms that post prices cannot avoid the taxing authority's attention. Clearly, the public observability of price posting is incompatible with informal activity, as the latter requires a degree of anonymity. Thus, in order to remain hidden from the taxing authority, informal sellers need to use a trading protocol that requires minimal public information about their whereabouts. Bargaining offers such a possibility for informal sellers. Hence, once sellers and buyers meet in decentralized markets, they simply bargain with each other. Since there is no credible and effective public commitment to any prior price announcement, for both parties there is always the possibility to renegotiate.

Given the crucial role of informational requirements associated with the different trading protocols, in this paper we explore the consequences of having price posting and bargaining in different markets. In particular, we study the coexistence of formal and informal activity as an equilibrium outcome. Agents can move between markets depending on their relative payoffs. In the informal sector, sellers and buyers split the surplus via bargaining. In the formal sector, firms post prices publicly, while each buyer chooses which seller to visit. Sellers producing in formal markets must be registered, and must pay their taxes. Further, given that formal sellers cannot escape regulatory authorities and/or courts, these firms can credibly provide quality assurance to their customers.[4] In contrast, informal sellers cannot credibly provide any quality assurance to their customers, as it would be prohibitively

[1] See Feige (1989, 1994) for more on this definition. For more on informal markets, see De Soto (1989) and Portes, Castells, and Benton (1989), among others.

[2] Other famous merchants followed his lead soon (Scull and Fuller, 1967). Macy's advertisements from the 1850s stated that prior to the use of posted prices by Macy's, "there was no regular price for anything", while with posted prices "even a child can trade with us" (Scull and Fuller, 1967, p. 83).

[3] See Bagwell (2007) for more on the evolution of advertising.

[4] This is consistent with anti-lemon laws enforcing certain money-back guarantees in these markets.

costly for their customers to get such quality assurance enforced against these firms via courts and/or regulatory authorities.

To better understand the implications of these different characteristics, we first study the consequences of having different trading protocols, and only allow sellers to switch between markets. Within this simplified environment, Theorem 1 states the conditions under which there is a stable equilibrium where formal and informal markets coexist. Theorem 1 also provides some comparative statics describing how the equilibrium responds to changes in some relevant parameters. We find that some sellers will migrate from the formal to the informal market if the formal market's advantage in quality assurance erodes, or the government imposes higher taxes and regulations in the formal market, or the risk of crime and/or confiscation decreases in the informal market, or the number of buyers in the informal market increases. Conversely, sellers will migrate from the informal market to the formal market whenever the opposite changes occur in these parameters.

Next, we relax the immobility of buyers, and consider an environment where both sellers and buyers can switch between formal and informal markets. In this richer environment, Result 2 summarizes numerical results and qualitative analysis which show that, for a broad range of parameter values, there is a stable equilibrium where formal and informal markets coexist, both of nontrivial size. When the net lump-sum cost for a seller in the formal sector relative to a seller in the informal sector and the net lump-sum cost for a buyer in the formal sector relative to a buyer in the informal sector have opposite signs, then there is only a locally stable coexistence of formal and informal markets. If these net lump-sum costs are both negative, then there is only a locally stable pure formal market equilibrium; if they are both positive, then there is only a locally stable pure informal market equilibrium.

The remainder of the paper is organized as follows. Section 2 reviews prior literature. Section 3 lays out our general modelling assumptions and presents the benchmark model where only sellers are mobile. Section 4 describes the more complex model, where both sellers and buyers can switch between markets. Appendix A contains proofs. Appendix B is an alphabetical index of notation.

2 Literature Review

To estimate the size of the informal economy, the existing literature has resorted to surveys, and has also analyzed discrepancies in data from multiple sources, such as wages paid versus taxes raised, data from household expenditure surveys versus retail trade surveys, and expenditure data versus income reported by the taxing authorities.[5]

Much less attention has been paid to the theoretical foundations of the coexistence of formal and informal activity. Existing equilibrium analyses that study the coexistence of informal and formal markets share the assumption that these activities are different in nature. These differences are such that either the goods being produced are assumed to be different, as in Aruoba (2010), or differences in enforceability of contracts in formal and informal markets as in Quintin (2008) or the technologies used to produce the goods or the

[5]See Schneider and Enste (2000) for a thorough review of this literature.

means of payment required to obtain the goods are assumed to be different as in Koreshkova (2006), Antunes and Cavalcanti (2007) , Amaral and Quintin (2006) and D'Erasmo and Boedo (2012).

The literature has so far not explored the importance of different trading protocols and their implied informational requirements for the coexistence of formal and informal activity. An exception is that of Gomis-Porqueras et al. (2012), who consider an environment that produces a homogenous good in different markets that use different trading protocols. Agents in decentralized markets bargain, and can evade taxes by deciding what fraction of the trade is to be made visible to the taxing authority. In contrast to the present paper, not all activity in bargaining markets is informal in Gomis-Porqueras et al. (2012).

3 Benchmark Model

Consider an economy with a large number of buyers and sellers. These agents trade a single perishable commodity. At any given point in time, capacity-constrained sellers produce at most one indivisible unit of the perishable good. Since these goods are perishable, it is not welfare maximizing for sellers to accumulate an inventory of unsold goods. Thus sellers "produce on demand" as is assumed in the price-posting literature (see Burdett et al. (2001)).Buyers consume the good after purchase. The perishable commodity can be purchased in both formal and informal markets. These markets differ along several dimensions. In this section we consider government taxes and regulations, and provision of quality assurance in formal markets, while crime and/or confiscation risk is faced by sellers when trading in informal markets.

Let us assume a very large population of buyers and a very large population of sellers.[6] Let b be the ratio of buyers to sellers in the whole economy; i.e. there are b buyers for every seller. Each population of agents is split between formal and informal markets. Let b^{fo} be the ratio of buyers in the formal market, versus the total number of sellers in *both* markets. Likewise let b^{in} be the fraction of agents in the informal market, versus the total number of sellers in both markets. Thus we have $b^{\text{fo}} + b^{\text{in}} = b$. Let s^{fo} be the fraction of sellers in the formal market, and let s^{in} be the fraction of sellers in the informal market; this implies $s^{\text{fo}} + s^{\text{in}} = 1$. Throughout the rest of the paper we are interested in characterizing the properties of b^{fo}, b^{in}, s^{fo}, and s^{in} that are observed in equilibrium.

In this section, to isolate the effects of having different trading protocols across markets, we suppose that only *sellers* can switch between markets. In other words, we will suppose that b^{fo} and b^{in} are exogenously fixed, while only s^{fo} and s^{in} are endogenous.[7] To see why this simplification is plausible, at least for a short term or medium term model, note that, while sellers' predominant factor in deciding where to locate their business is to be close to buyers, for most buyers accessibility to sellers is not of first order importance. Households take into account other factors when making their location choices. These include access to the workplace and schools and quality of the neighbourhood and commuting costs, and other

[6] We will mainly consider the limit as both populations become infinite.

[7] Later (in Section 4), we will assume that both buyers and sellers can move between markets; thus, b^{fo}, b^{in}, s^{fo}, and s^{in} will all be endogenously determined.

factors, such as social class barriers.[8] Thus, if one takes into account these other factors, then the buyer's decision on where to buy goods is less affected by the sellers' location. In such a situation we can think of buyers being streamed into one market or the other on the basis of exogenous factors like physical location or education, whereas the seller's location is endogenous and strategic.

3.1 Preferences

Buyers and sellers have quasilinear utilities. A buyer in the formal market obtains a value of $v^{\text{fo}}_{\text{bu}}$ when she consumes a unit of the good. On the other hand, a buyer in the informal market obtains a value of $v^{\text{in}}_{\text{bu}}$ when she consumes the good.[9] Thus, if the formal (informal) market price of the good is p^{fo} (p^{in}), then the formal (informal) buyer obtains a total payoff of $v^{\text{fo}}_{\text{bu}} - p^{\text{fo}}$ ($v^{\text{in}}_{\text{bu}} - p^{\text{in}}$).

Sellers in the formal market incur a cost of $c^{\text{fo}}_{\text{se}}$ per unit of good produced, which includes both labor and input costs. Thus the total payoff of formal sellers is $p^{\text{fo}} - c^{\text{fo}}_{\text{se}}$. On the other hand, sellers in the informal market have a unit cost of production $c^{\text{in}}_{\text{se}}$.[10] Thus the total payoff of informal sellers is $p^{\text{in}} - c^{\text{in}}_{\text{se}}$.

In order for trades to occur in the both markets, individual rationality for both the buyers and sellers needs to hold, which requires $c^{\text{fo}}_{\text{se}} \leq p^{\text{fo}} \leq v^{\text{fo}}_{\text{bu}}$ and $c^{\text{in}}_{\text{se}} \leq p^{\text{in}} \leq v^{\text{in}}_{\text{bu}}$. Let $g^{\text{fo}} := v^{\text{fo}}_{\text{bu}} - c^{\text{fo}}_{\text{se}}$ be the measure of the total "gains from trade" in the formal market, while $g^{\text{in}} := v^{\text{in}}_{\text{bu}} - c^{\text{in}}_{\text{se}}$ is the total "gains from trade" in the informal market. Without loss of generality, we can normalize the buyer and seller's utility functions so that $g^{\text{fo}} = 1$.[11] The total payoffs that a buyer and seller receive in each market depend on the specific trading protocol that agents face when trading in each market. If agents do not trade, then each buyer and each seller obtains a zero payoff.

3.2 Quality Assurance

An important distinguishing feature of formal markets relative to informal ones —and one which has not previously been emphasized by the literature —is the provision of quality assurance. This can take many forms such as free repair/replacement, a full money-back guarantee, on-site customer service, twenty-four hour telephone customer assistance, and/or cash compensation for unsatisfactory product performance. Since formal sellers are monitored by government authorities, and can enter into binding contracts, they can credibly provide such quality assurance to their customers. Furthermore, because they must incur costs to repair or replace defective merchandise, formal sellers have also financial incentives to detect and eliminate defective products before they reach the market.

[8] For example, illiterate buyers will find it much more difficult to participate in the formal market as fine print regarding the sale and quality assurance conditions do not convey any additional information for them.

[9] In Section 3.2, we will explain why, in general, $v^{\text{in}}_{\text{bu}} < v^{\text{fo}}_{\text{bu}}$.

[10] In Section 3.2, we will explain why, in general, $c^{\text{in}}_{\text{se}} < c^{\text{fo}}_{\text{se}}$.

[11] In Section 3.2, we will see that, in general, $g^{\text{in}} < g^{\text{fo}}$.

In contrast, informal sellers *cannot* offer quality assurance, as they are unregistered with the authorities, so that their service contracts and warranties would be prohibitively costly for their customers to enforce against them via courts and/or regulatory authorities. This also means they are less likely to incur the cost of ensuring quality during production. Thus, goods are more likely to be defective in the informal market, and they come without quality assurance or warranties. We suppose that, when a good is defective, it provides less utility to the buyer. Thus, if $v^{\text{in}}_{\text{bu}}$ is the expected utility to the buyer of a unit of the good purchased in the informal market, then $v^{\text{in}}_{\text{bu}} < v^{\text{fo}}_{\text{bu}}$

Let $c^{\text{in}}_{\text{se}}$ be the per unit cost of producing a unit of the good without any quality assurance, which is the unit cost sellers producing for the informal market. Let q be the average cost of providing "quality assurance" to the buyer. Thus the unit cost of production for formal sellers is $c^{\text{fo}}_{\text{se}} = c^{\text{in}}_{\text{se}} + q$ (this implicitly assumes that all sellers have the same technology to produce the good). To simplify exposition, we assume that formal sellers pass all of the the quality assurance cost q onto the buyer.

Let the total value for the buyer of purchasing the good in the formal market be given by $v^{\text{fo}}_{\text{bu}} = v^{\text{in}}_{\text{bu}} + \alpha(q)$, where q is the money the formal seller spends on quality assurance per unit sold and $\alpha(q)$ is the benefit that each formal buyer receives from this assurance. We assume that α is increasing, differentiable, and concave, with $\alpha(0) = 0$.[12]

Lemma A *If $\alpha(\cdot)$ is increasing, differentiable, and concave, and formal sellers provide the efficient level of quality assurance, then $g^{\text{in}} \leq g^{\text{fo}}$.*

The trading protocols employed in the formal and informal markets are used as mechanisms for dividing the gains from trade between the buyer and the seller. Thus, the fact that $g^{\text{in}} \leq g^{\text{fo}}$ implies that there is generally a larger surplus to be divided in the formal market so both buyer and seller can potentially be better off. This is one of the reasons that the formal market is attractive in the first place. It also implies that the government can tax a fraction of up to $T_0 = g^{\text{fo}} - g^{\text{in}}$ of the surplus in the formal market without driving participants into the informal market. As we shall see below, in fact the government can safely impose taxes much higher than T_0 without undermining the formal market.

3.3 Taxation and theft

Suppose the government taxes a fraction $T^{\text{fo}}_{\text{se}}$ of the profits of each seller in the formal market. This may take the form of income tax, or a value added tax on the sale of the goods, as they imply the same effect. Also, the costs of complying with some regulations may be directly proportional to the amount of goods sold. Finally, formal sellers may be exposed to legal liability from customers, which will also be directly proportional to the amount of goods sold. All these proportional costs can be incorporated into $T^{\text{fo}}_{\text{se}}$.[13]

[12]Concavity is a very reasonable assumption in this context, because α is generally bounded above: $\lim_{q\to\infty} \alpha(q) = v^* - v^{\text{in}}_{\text{bu}}$, where v^* is the value of consuming a "perfect" commodity, with zero probability of defects.

[13]Note that any taxes which are nominally paid by the *buyer* in the formal market (e.g. sales tax) can just as easily be interpreted as taxes paid by the *seller* (i.e. we suppose they are already factored into the posted price), and thus, incorporated into $T^{\text{fo}}_{\text{se}}$.

In the informal sector, sellers pay no taxes. But each informal seller faces a risk that her money will be stolen. This effectively functions as a form of "income tax" on informal sector earnings. Let $T_{\mathrm{se}}^{\mathrm{in}}$ represent this effective "tax rate". Also, some police or criminals may demand bribes or "protection fees" which are proportional to the seller's earnings. All these costs can be incorporated into $T_{\mathrm{se}}^{\mathrm{in}}$.

In the next sections we study how the coexistence of formal and informal market activity is affected by increases or decreases in $T_{\mathrm{se}}^{\mathrm{fo}}$ and $T_{\mathrm{se}}^{\mathrm{inf}}$.

3.4 Matching and Trading

Formal and informal markets have different matching technologies and trading protocols as both markets require different *degrees* of public information. At a given point in time, the proportion of buyers trading in formal and informal markets are fixed. Below we provide more details about these markets' respective matching technologies and trading protocols.

3.4.1 Formal Market

In the formal market, each seller has a fixed location; i.e. a store. In order to attract buyers, formal sellers advertise their posted price and location. The location and the price of each seller are common knowledge. In order to capture these market features, we use the directed search framework of Burdett et al. (2001). Each buyer in the formal market can visit one seller per period, and buyers' visits of sellers are not coordinated. Buyers are more likely to visit the seller with the lowest posted price. But since buyers are not coordinated, they may face more competition at these locations. If multiple buyers choose to visit the same seller, then only one of the buyers can purchase the good, while the rest of buyers receive a payoff of 0. On the other hand, if *no* buyers visit a seller, then he cannot sell his good, so he receives a payoff of 0.

Since all buyers are *ex-ante* identical, we focus on the mixed-strategy equilibrium in which buyers use the same mixed strategy to choose which seller to visit.[14] Likewise, since all sellers are also *ex-ante* identical, they all use the same pricing strategy. The next theorem summarizes the main results of Burdett et al. (2001).

Theorem 0. *Let m be the total number of sellers in the formal market, and let B_f be the ratio of buyers to sellers in the formal market (so there are $B_f\,m$ buyers). There is a unique symmetric Nash equilibrium of the formal market game with the following characteristics. All sellers post an identical price of p. All buyers use the same mixed strategy: they randomly visit all sellers with equal probability. Let Φ be the probability that any given seller sells his product (i.e. is visited by at least one buyer), and let Ω be the probability that any given buyer purchases the good. Then p, Φ, and Ω are entirely determined by B_f and m. Furthermore, if we let $m\to\infty$ while holding B_f fixed, then we get:*

[14] This approach is common in the price posting literature.

$$p(B_f) := \lim_{m\to\infty} p(B_f,m) = c^{\rm fo}_{\rm se} + u^{\rm fo}_{\rm se}(B_f), \tag{1}$$

$$\textit{where} \quad u^{\rm fo}_{\rm se}(B_f,) := 1 - \frac{B_f}{e^{B_f}-1}. \tag{2}$$

$$\textit{Also,} \quad P^{\rm fo}_{\rm se}(B_f) := \lim_{m\to\infty} \Phi(B_f,m) = 1 - e^{-B_f}, \tag{3}$$

$$\textit{and} \quad P^{\rm fo}_{\rm bu}(B_f) := \lim_{m\to\infty} \Omega(B_f,m) = \frac{P^{\rm fo}_{\rm se}(B_f)}{B_f}. \tag{4}$$

Intuitively, $P^{\rm fo}_{\rm se}(B_f)$ ($P^{\rm fo}_{\rm bu}(B_f)$) is the probability that any particular seller sells his goods (the probability that any particular buyer obtains the goods) during each round of participation in the infinite-population formal market. If a seller makes a sale in the infinite formal market ($m\to\infty$), then his pre-tax payoff is $u^{\rm fo}_{\rm se}(B_f)$; otherwise his payoff is 0. Thus, the seller's pre-tax *expected* payoff in the formal market game is $U^{\rm fo}_{\rm se}(B_f) := P^{\rm fo}_{\rm se}(B_f) \cdot u^{\rm fo}_{\rm se}(B_f)$.

Recall that $b^{\rm fo}$ is the ratio of buyers in the formal market, versus the total number of sellers in *both* markets, while $s^{\rm fo}$ is the fraction of sellers in the formal market, versus the total number of sellers in both markets. As a result, we have that $B_f = b^{\rm fo}/s^{\rm fo}$. Given that the proportional tax rate paid by formal sellers is $T^{\rm fo}_{\rm se}$, the seller's expected *after-tax* payoff in the formal market is given by:

$$\begin{aligned} \widetilde{U}^{\rm fo}_{\rm se}(s^{\rm fo}) &:= (1-T^{\rm fo}_{\rm se})\, U^{\rm in}_{\rm se}\left(\frac{b^{\rm fo}}{s^{\rm fo}}\right) \\ &= (1-T^{\rm fo}_{\rm se})\left(1-\exp\left(\frac{-b^{\rm fo}}{s^{\rm fo}}\right)\right)\left(1-\frac{b^{\rm fo}/s^{\rm fo}}{\exp(b^{\rm fo}/s^{\rm fo})-1}\right). \end{aligned} \tag{5}$$

Note that we write $\widetilde{U}^{\rm fo}_{\rm se}$ as a function of $s^{\rm fo}$ only, because in this model, $b^{\rm fo}$ is fixed.

3.4.2 Informal Market

Informal sellers cannot publicly advertise prices nor locations, because they are trying to avoid government detection. As in the formal sector, each informal buyer can only visit *one* seller per period, and buyers' visits of sellers are not coordinated. The matching probabilities are again given by the directed search model of Burdett et al. (2001). Thus, if $B_i := b^{\rm in}/s^{\rm in}$ is the ratio of buyers to sellers in the informal market, then equation (3) in Theorem 0 implies that the probability that any given informal seller makes a sale during any given period is given by $P^{\rm in}_{\rm se}(B_i) = 1 - e^{-B_i}$. Substituting $B_i := b^{\rm in}/s^{\rm in}$, we get the following matching probability for sellers:

$$P^{\rm in}_{\rm se}\left(\frac{b^{\rm in}}{s^{\rm in}}\right) = 1 - \exp\left(\frac{-b^{\rm in}}{s^{\rm in}}\right). \tag{6}$$

Instead of trading at publicly posted prices, the informal seller and buyer negotiate a price thereby splitting the total informal market surplus, $g^{\rm in}$. Here we assume that the informal seller receives a fraction $\eta(B_i) \in [0,1]$ of the surplus, while the informal buyer receives the remaining fraction $1-\eta(B_i)$.[15] Thus, we have $u^{\text{in}}_{\text{se}} := \eta(B_i)\, g^{\text{in}}$. Then the resulting pre-theft expected payoff for sellers in the informal market is given by:[16]

$$U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) \;:=\; u^{\text{in}}_{\text{se}} \cdot P^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) \;=\; g^{\text{in}}\,\eta\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right) P^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{s^{\text{in}}}\right). \tag{7}$$

Clearly, the higher the ratio of buyers to sellers in the informal market, the stronger each seller's negotiating position becomes, and the better each seller will do in bilateral bargaining. In the limit when there are infinitely many buyers for every seller, the sellers will capture *all* of the surplus in the informal market. Thus, we suppose that the seller's bargaining power η is an increasing function, such that:[17]

$$\lim_{B_i \to \infty} \eta(B_i) \;=\; 1.$$

Recall that b^{in} is a constant, while $s^{\text{in}} = 1 - s^{\text{fo}}$; thus, we can regard the informal seller's utility as a function of s^{fo} only. Recall that $T^{\text{in}}_{\text{se}}$ is the seller's expected losses due to monetary theft in the informal sector. Thus, the relevant expect payoff for the informal sellers is:

$$\widetilde{U}^{\text{in}}_{\text{se}}(s^{\text{fo}}) \;:=\; (1-T^{\text{in}}_{\text{se}})\, U^{\text{in}}_{\text{se}}\left(\frac{b^{\text{in}}}{1-s^{\text{fo}}}\right) \;=\; (1-T^{\text{in}}_{\text{se}})\, g^{\text{in}}\,\eta\left(\frac{b^{\text{in}}}{1-s^{\text{fo}}}\right)\left(1-\exp\left(\frac{-b^{\text{in}}}{1-s^{\text{fo}}}\right)\right). \tag{8}$$

3.5 Equilibrium

Given a fixed fraction of buyers participating in the formal and informal market (b^{fo} and b^{in} respectively), a seller will find the formal market more attractive than the informal market if and only if the corresponding expected payoff is higher. Sellers will slowly migrate between the two markets until the payoff from both markets is the same. Thus, we say the economy is in *equilibrium* if and only if s^{fo} and s^{in} are such that

$$\widetilde{U}^{\text{fo}}_{\text{se}}(s^*) \quad=\quad \widetilde{U}^{\text{in}}_{\text{se}}(s^*) \tag{9}$$

where $s^* \in [0,1]$ corresponds to the equilibrium fraction of formal sellers.

There are at least three formal frameworks that lead to the equilibrium represented by equation (9), which we now describe.

Uncorrelated, symmetric mixed Nash equilibrium. We can think of a static environment, where each of the sellers plays a mixed strategy, randomly choosing whether to participate in the formal or informal market. Sellers cannot coordinate, so there is no correlation between their strategies. All sellers are *ex-ante* identical, so they play identical strategies, given by the probability vector $(s^{\text{fo}}, s^{\text{in}})$. Equation (9) is then equivalent to saying that this profile of mixed strategies is a Nash equilibrium, as in Camera and Delacroix (2004) or Michelacci and Suarez (2006).

[15] Later, in Section 4.2, we will present one possible model of this surplus division process, but there is no need to commit to a specific model here.

[16] The expected utility of *buyers* in the informal market is irrelevant to the dynamics of this model, because we have assumed that they are immobile.

[17] It would also be reasonable to assume $\lim_{B_i \searrow 0} \eta(B_i) = 0$. But this is unnecessary for our analysis.

Best response dynamics. The equilibrium described in equation (9) can also be interpreted in terms of agents playing an infinitely repeated game. During each round, each seller can decide whether to participate in the formal or informal market. Sellers migrate from one market to the other at a rate which is proportional to the payoff differential between the two markets. Let $s^{\mathrm{fo}}(t)$, and $s^{\mathrm{in}}(t)$ represent the populations of formal/informal sellers at time t, and define

$$\dot{s}^{\mathrm{fo}}(t) := s^{\mathrm{fo}}(t+1) - s^{\mathrm{fo}}(t) \qquad \text{and} \qquad \dot{s}^{\mathrm{in}}(t) := s^{\mathrm{in}}(t+1) - s^{\mathrm{in}}(t).$$

Then we have that

$$\dot{s}^{\mathrm{fo}}(t) \quad = \quad -\dot{s}^{\mathrm{in}}(t) \quad = \quad \lambda_{\mathrm{se}}\left(\widetilde{U}^{\mathrm{fo}}_{\mathrm{se}}\left(\frac{b^{\mathrm{fo}}(t)}{s^{\mathrm{fo}}(t)}\right) - \widetilde{U}^{\mathrm{in}}_{\mathrm{se}}\left(\frac{s^{\mathrm{in}}(t)}{b^{\mathrm{in}}(t)}\right)\right), \tag{10}$$

where $\lambda_{\mathrm{se}} : \mathbb{R} \longrightarrow \mathbb{R}$ is a strictly increasing function which modulates the speed of adjustment, with $\lambda_{\mathrm{se}}(0) = 0$. Typically, λ_{se} is multiplication by a positive constant.[18] We can also consider a continuous-time version of this model, where $\dot{s}^{\mathrm{fo}}(t)$ and $\dot{s}^{\mathrm{in}}(t)$ represent the *derivatives* of the functions $s^{\mathrm{fo}}(t)$ and $s^{\mathrm{in}}(t)$ at time t. In either case, equation (17) is the necessary and sufficient condition for a population distribution $(s^{\mathrm{fo}}, s^{\mathrm{in}})$ to be a fixed point of the dynamics.

Replicator/Imitation dynamics. As in best response dynamics, suppose there is an infinite sequence of time periods, with trade occurring in each market during each time period. But instead of migrating between markets in response to higher payoffs, agents learn by *imitating* other agents. The more agents choose a particular strategy, and the better they are doing relative to the average payoff, the more likely it is that other agents will imitate their behavior.

Alternatively, we can interpret the same model in terms of successive generations of agents. During each time period, some agents produce one or more children, and some agents die. Children remain in the same market as their parents.[19] The net reproductive rate (births minus deaths) of each market type is determined by how much the payoff for that market exceeds the population average payoff. To be precise, the population average payoff for sellers at time t is given by:

$$s^{\mathrm{fo}}(t)\, U^{\mathrm{fo}}_{\mathrm{se}}\left(\frac{b^{\mathrm{fo}}(t)}{s^{\mathrm{fo}}(t)}\right) + s^{\mathrm{in}}(t)\, U^{\mathrm{in}}_{\mathrm{se}}\left(\frac{s^{\mathrm{in}}(t)}{b^{\mathrm{in}}(t)}\right),$$

so the reproductive rate of the *formal* sellers will be:

$$\rho(t) = (1 - s^{\mathrm{fo}}(t))\, U^{\mathrm{fo}}_{\mathrm{se}}\left(\frac{b^{\mathrm{fo}}(t)}{s^{\mathrm{fo}}(t)}\right) - s^{\mathrm{in}}(t)\, U^{\mathrm{in}}_{\mathrm{se}}\left(\frac{s^{\mathrm{in}}(t)}{b^{\mathrm{in}}(t)}\right).$$

[18] If λ_{se} was an *odd* function, then the dynamical system would converge to equilibrium just as quickly from either direction. Thus, the informal market would show a symmetric response to tax increases and tax decreases, as found by Christopoulos (2003). However, λ_{se} might *not* be odd. For example, it might cost more for a seller to switch from the informal market to the formal market than vice versa (e.g. because of the need to acquire licenses, rent a retail location, etc.); this would be reflected by having $|\lambda_{\mathrm{se}}(r)| < |\lambda_{\mathrm{se}}(-r)|$ for any given $r > 0$. This is consistent with empirical findings by Giles et al. (2001) and Wang et al. (2012).

[19] Note that, in this interpretation, it is not accurate to view the model as a repeated game, since individual agents only live for one period.

The population of formal sellers will grow (or shrink) exponentially at this rate. Formally, we have $\dot{s}^{\mathrm{fo}}(t) = \lambda_{\mathrm{se}}\,\rho(t)\cdot s^{\mathrm{fo}}(t)$, where $\lambda_{\mathrm{se}} > 0$ is some constant. This leads to the following dynamical equation

$$\dot{s}^{\mathrm{fo}}(t) = -\dot{s}^{\mathrm{in}}(t) \;=\; \lambda_{\mathrm{se}}\, s^{\mathrm{fo}}(t)\, s^{\mathrm{in}}(t) \left(\widetilde{U}^{\mathrm{fo}}_{\mathrm{se}}\left(\frac{b^{\mathrm{fo}}(t)}{s^{\mathrm{fo}}(t)}\right) - \widetilde{U}^{\mathrm{in}}_{\mathrm{se}}\left(\frac{s^{\mathrm{in}}(t)}{b^{\mathrm{in}}(t)}\right)\right), \tag{11}$$

where $\lambda_{\mathrm{se}} > 0$ is a constant. Again, this dynamical equation has both a discrete-time and a continuous-time interpretation. In either case, equation (9) is a necessary and sufficient condition for a population distribution $(s^{\mathrm{fo}}, s^{\mathrm{in}})$ to be a "nontrivial" fixed point of the dynamics. Here, "nontrivial" refers to the fact that the replicator dynamics always have "trivial" fixed points where either $b^{\mathrm{fo}} = 0$ or $b^{\mathrm{in}} = 0$ and either $s^{\mathrm{fo}} = 0$ or $s^{\mathrm{in}} = 0$. However, unless these "pure population" equilibria arise from a solution to equation (9), they are generally unstable to small perturbations. Thus, a pure population of this type will be destabilized as soon as even *one* of the reproducing agents produces a "mutant" child of the opposite type. Thus, we can safely ignore these trivial equilibria, and focus only on the equilibria described by equation (9).

3.6 Existence and properties of equilibrium

Let us define $\widetilde{U}(s^{\mathrm{fo}}) := \widetilde{U}^{\mathrm{fo}}_{\mathrm{se}}(s^{\mathrm{fo}}) - \widetilde{U}^{\mathrm{in}}_{\mathrm{se}}(s^{\mathrm{fo}})$ which represents the net gain of the formal market over the informal market for sellers. Equilibrium equation (9) is equivalent to $\widetilde{U}(s^*) = 0$. We say an equilibrium s^* is *locally stable* if $\widetilde{U}'(s^*) < 0$; this implies local stability under either best response dynamics or replicator dynamics. The equilibrium (9) is a *mixed market* (non-trivial) equilibrium if $0 < s^* < 1$. The main result of this section is the following.

Theorem 1 *If we satisfy the following conditions*

$$1 - \frac{b^{\mathrm{fo}} + 1}{\exp(b^{\mathrm{fo}})} \quad < \quad \frac{(1 - T^{\mathrm{in}}_{\mathrm{se}})\, g^{\mathrm{in}}}{(1 - T^{\mathrm{fo}}_{\mathrm{se}})} \quad < \quad \frac{1}{\eta(b^{\mathrm{in}})\,\left(1 - \exp(-b^{\mathrm{in}})\right)},$$

*then there is a locally stable mixed market equilibrium $s^* \in (0,1)$. Furthermore, if we treat s^* as a function of the parameters g^{in}, $T^{\mathrm{fo}}_{\mathrm{se}}$, $T^{\mathrm{in}}_{\mathrm{se}}$, b^{in} and b^{fo}, then **(i)** s^* is decreasing as a function of g^{in}, $T^{\mathrm{fo}}_{\mathrm{se}}$, and b^{in}; and **(ii)** s^* is increasing as a function of $T^{\mathrm{in}}_{\mathrm{se}}$ and b^{fo}.*

Thus, for a broad range of parameters of the model, formal and informal markets of nontrivial size will coexist in a stable equilibrium. Furthermore, some sellers will migrate from the formal market to the informal market if the formal market's advantage in quality assurance erodes (g^{in} increases relative to g^{fo}), or the government imposes higher taxes and regulations ($T^{\mathrm{fo}}_{\mathrm{se}}$ increases), or more buyers migrate to the informal market (b^{in} increases). Conversely, sellers will migrate from the informal market back to the formal market whenever the opposite changes occur in these parameters. Likewise, sellers will migrate to the formal market if the risk of crime and/or confiscation increases in the informal market (i.e. $T^{\mathrm{in}}_{\mathrm{se}}$ increases) or if buyers migrate to the formal market (b^{fo} increases).[20]

[20]If the total population of buyers is constant, then b^{fo} and b^{in} are two sides of the same coin. But formally, we can decouple the movements of b^{fo} and b^{in}; this allows us to, for example, consider a scenario where the total population of buyers increases, but all the new buyers go to the formal market.

4 Mobile Buyers and Sellers

Now we will relax the immobility of buyers, and consider an environment where both sellers and buyers can switch among formal and informal markets. In other words, we treat $b^{\rm in}$ and $b^{\rm fo}$ as endogenous variables, in addition to $s^{\rm in}$ and $s^{\rm fo}$. As in Section 3, we also consider factors other than the trading protocols which make formal and informal different. These include quality assurance and government taxes and regulations in formal markets, as well as crime and/or confiscation risk in informal markets.

4.1 Formal markets

As in Section 3.4.1, we suppose that the behaviour and the payoffs of buyers and sellers in the formal market are described by the model of Burdett et al. (2001), as summarized in Theorem 0. Recall that $B_f = b^{\rm fo}/s^{\rm fo}$ is the ratio of buyers to sellers in the formal market. Then a formal seller's pre-tax expected utility is again given by $U^{\rm fo}_{\rm se}(B_f) := P^{\rm fo}_{\rm se}(B_f) \cdot u^{\rm fo}_{\rm se}(B_f)$, where $P^{\rm fo}_{\rm se}(B_f)$ and $u^{\rm fo}_{\rm se}(B_f)$ are defined in equations (2) and (3).

Meanwhile, let $p(B_f)$ be the formal market equilibrium price from equation (1). If a formal market buyer makes a purchase, then her payoff will be given by:

$$\begin{aligned} u^{\rm fo}_{\rm bu}(B_f) \quad := \quad v^{\rm fo}_{\rm bu} - p(B_f) \quad &= \quad v^{\rm fo}_{\rm bu} - c^{\rm fo}_{\rm se} - u^{\rm fo}_{\rm se}(B_f) \\ = \quad g^{\rm fo} - u^{\rm fo}_{\rm se}(B_f) \quad &= \quad 1 - u^{\rm fo}_{\rm se}(B_f) \quad = \quad \frac{B_f}{e^{B_f} - 1}. \end{aligned} \tag{12}$$

If a buyer doesn't make a purchase because she competed against other buyers, then her payoff is zero. Thus, her *expected* payoff for participating in the infinite formal market game is $U^{\rm fo}_{\rm bu}(B_f) := P^{\rm fo}_{\rm bu}(B_f) \cdot u^{\rm fo}_{\rm bu}(B_f)$, where $P^{\rm fo}_{\rm bu}(B_f)$ is defined in equation (4).

4.2 Informal Markets

In this new model, informal buyers are the ones who have fixed locations (home or workplace), and the sellers are the ones who visit them. This new feature tries to capture the door to door selling strategy used by informal sellers in some developing countries. As in the previous section, the matching probabilities and tie breaking rule are given by the directed search model of Burdett et al. (2001). Here the roles of buyers and sellers reversed relative to Burdett et al. (2001). Thus, if $S_i := s^{\rm in}/b^{\rm in}$ is the ratio of sellers to buyers in the informal market, then the probability for a given buyer to be visited by at least one seller during any given period is obtained by replacing B_f with S_i in equation (3), to obtain:

$$P^{\rm in}_{\rm bu}(S_i) \quad = \quad 1 - e^{-S_i}. \tag{13}$$

Likewise, the probability of a given seller making a sale to the one buyer he visits is given by:

$$P^{\rm in}_{\rm se}(S_i) \quad = \quad \frac{P^{\rm in}_{\rm bu}(S_i)}{S_i}. \tag{14}$$

In section 3.4.2, we assumed that the informal seller and buyer negotiated to split the surplus according to proportions $(\eta, 1-\eta)$, where η depended on the ratio of buyers to sellers in this market.[21] Now, however, we model the negotiation process more explicitly, via the Nash bargaining model. Because of our assumptions about the utility functions of the buyer and seller, the (bargaining) set of feasible utility allocations is the convex hull of the points $(0,0)$, $(g^{\text{in}},0)$, and $(0,g^{\text{in}})$, where g^{in} is the total gains from trade to be divided in the informal market. Thus, the Pareto frontier is the diagonal line from $(g^{\text{in}},0)$ to $(0,g^{\text{in}})$. Hence, the Nash bargaining solution, the egalitarian bargaining solution, and the Kalai-Smorodinsky bargaining solution all yield the same outcome. Thus it does not matter which bargaining solution we use.

The bargaining outcome will be determined by the "outside options" available to buyers and sellers. Formally, let $U^{\text{in}}_{\text{se}}$ be the expected payoff for a seller participating in the informal market, and let $U^{\text{in}}_{\text{bu}}$ be the expected payoff for a buyer participating in the informal market. These payoffs depend on the informal seller/buyer ratio $S_i := s^{\text{in}}/b^{\text{in}}$. Let $\delta \in (0,1)$ be a discount factor. If bargaining breaks down, then both parties must re-enter the informal market during the next period. Then, in the Nash bargaining framework, the outside option for the seller is $\delta\, U^{\text{in}}_{\text{se}}(S_i)$, while the outside option for the buyer is $\delta\, U^{\text{in}}_{\text{bu}}(S_i)$. The Nash bargaining solution thus awards the seller a payoff of $u^{\text{in}}_{\text{se}}(S_i)$ and the buyer a payoff of $u^{\text{in}}_{\text{se}}(S_i)$, where

$$\begin{aligned} u^{\text{in}}_{\text{se}}(S_i) &= \frac{\delta\, U^{\text{in}}_{\text{se}}(S_i) + g^{\text{in}} - \delta\, U^{\text{in}}_{\text{bu}}(S_i)}{2}; \\ \text{and} \quad u^{\text{in}}_{\text{bu}}(S_i) &= \frac{\delta\, U^{\text{in}}_{\text{bu}}(S_i) + g^{\text{in}} - \delta\, U^{\text{in}}_{\text{se}}(S_i)}{2}. \end{aligned} \tag{15}$$

However, we also have that $U^{\text{in}}_{\text{se}}(S_i) = P^{\text{in}}_{\text{se}}(S_i)\, u^{\text{in}}_{\text{se}}(S_i)$ and $U^{\text{in}}_{\text{bu}}(S_i) = P^{\text{in}}_{\text{bu}}(S_i)\, u^{\text{in}}_{\text{bu}}(S_i)$. If we now substitute these expressions into (15), we obtain a pair of linear equations for $u^{\text{in}}_{\text{se}}(S_i)$ and $u^{\text{in}}_{\text{bu}}(S_i)$. Solving these equations yields the following buyers' payoff

$$\begin{aligned} u^{\text{in}}_{\text{se}}(S_i) &= g^{\text{in}}\, \frac{\delta\, P^{\text{in}}_{\text{bu}}(S_i) - 1}{\delta\, P^{\text{in}}_{\text{bu}}(S_i) + \delta\, P^{\text{in}}_{\text{se}}(S_i) - 2}; \\ u^{\text{in}}_{\text{bu}}(S_i) &= g^{\text{in}}\, \frac{\delta\, P^{\text{in}}_{\text{se}}(S_i) - 1}{\delta\, P^{\text{in}}_{\text{bu}}(S_i) + \delta\, P^{\text{in}}_{\text{se}}(S_i) - 2}; \end{aligned} \tag{16}$$

that describe the new payoffs for the buyer and seller trading in the informal market.[22]

[21] It was not necessary to be any more specific about the negotiation process in order to obtain Theorem 1.

[22] If $\delta = 1$, then the bargaining outcome (16) can be seen as a particular case of the abstract surplus-division model considered in Section 3.4.2. To see this, let $B_i := 1/S_i$, and let $\eta(B_i) := u^{\text{in}}_{\text{se}}(S_i)/g^{\text{in}}$, where $u^{\text{in}}_{\text{se}}(S_i)$ is defined as in Eq.(16). Then η satisfies the conditions proposed in Section 3: it is an increasing function of B_i (because $u^{\text{in}}_{\text{se}}$ is a decreasing function of S_i, and the limit (**??**) holds because $u^{\text{in}}_{\text{se}}(0) = g^{\text{in}}$.

4.3 Asymmetric costs

As discussed in Section 3.3, T_{se}^{fo} is the effective tax rate paid by sellers in the formal market. In this Section we also consider other exogenous costs that are incurred when participating in these markets. During each that period a seller participates in the formal market, we assume that she incurs a lump-sum cost of L_{se}^{fo} dollars. This cost represents the combined cost of renting (or purchasing) retail space, paying for licensing fees and following government regulations (e.g. fire safety codes). Note that these costs, unlike the proportional tax T_{se}^{fo}, do not depend on earnings. Meanwhile, formal buyers incur a lump-sum cost of L_{bu}^{fo} dollars. This represents the costs of transportation to the formal market area of the city, the "shoe-leather costs" of visiting various merchants, etc.

Recall from Section 3.3 that each seller also faces a risk that her money will be stolen. This possibility can be represented as an implied "tax rate" of T_{se}^{in} on her earnings. Now we further suppose that each informal seller also incurs a lump-sum cost of L_{se}^{in} dollars. This cost represents the costs associated with being an itinerant merchant, bribing corrupt officials, and paying non government protection services as well as the expected cost of having her merchandise confiscated as there are no records for these items to have any legal claim.[23] Similarly, informal buyers incur a lump-sum cost of L_{bu}^{fo} dollars which reflects the opportunity cost of waiting around for sellers to arrive.

Let us define $L_{se} := L_{se}^{fo} - L_{se}^{in}$ and $L_{bu} := L_{bu}^{fo} - L_{bu}^{in}$. Intuitively, L_{se} represents the *net* lump-sum cost for sellers in the formal sector; likewise, L_{bu} represents the net lump-sum cost for buyers in the formal sector.[24] For modelling purposes, it is equivalent to suppose that informal buyers and sellers face *no* lump sums, whereas formal buyers and sellers face lump sums of L_{bu} and L_{se} respectively.

Let $R_{se}^{fo} := 1 - T_{se}^{fo}$ and $R_{se}^{in} := 1 - T_{se}^{in}$ representing the "residual" earnings rate of sellers in the formal and informal markets after losses due to taxes, theft, etc. Let $R_{se} := R_{se}^{fo}/R_{se}^{in}$; this is effectively the "net" residual earnings rate for formal sellers, if we normalize the informal sellers' residual earnings rate to 1. For modelling purposes, it is equivalent to suppose that that informal sellers capture *all* their earnings, while formal sellers only capture a proportion R_{se}. This is equivalent to supposing that informal sellers face *no* risk of theft, while formal sellers pay an *effective* tax rate of $T_{ax} := 1 - R_{se}$. Note that if expected losses due to theft in the informal market are higher that the formal tax rate, then we will have $R_{se} > 1$, which implies that $T_{ax} < 0$.

4.4 Equilibrium

Having specified all differential costs of trading in formal and informal markets, we can now analyze the corresponding equilibrium for this new environment. As in Section 3, a seller will find the formal market more attractive than the informal market if and only if $(1 - T_{ax})\, U_{se}^{fo}(b^{fo}/s^{fo}) - L_{se} > U_{se}^{in}(s^{in}/b^{in})$. Likewise, a buyer will find the formal market more

[23]Note that the informal seller pays the same bribes or protection fees whether or not her business is successful.

[24]These net lump sums could be negative, if the costs in the informal sector are higher than the formal sector.

attractive than the informal market if and only if $U^{\rm fo}_{\rm bu}(b^{\rm fo}/s^{\rm fo}) - L_{\rm bu} > U^{\rm in}_{\rm bu}(s^{\rm in}/b^{\rm in})$. Thus, an *equilibrium* exists if and only if $b^{\rm fo}$, $b^{\rm in}$, $s^{\rm fo}$, and $s^{\rm in}$ satisfies the following conditions:

$$(1 - T_{ax})\, U^{\rm fo}_{\rm se}\left(\frac{b^{\rm fo}}{s^{\rm fo}}\right) - L_{\rm se} \;=\; U^{\rm in}_{\rm se}\left(\frac{s^{\rm in}}{b^{\rm in}}\right) \quad \text{and} \quad U^{\rm fo}_{\rm bu}\left(\frac{b^{\rm fo}}{s^{\rm fo}}\right) - L_{\rm bu} \;=\; U^{\rm in}_{\rm bu}\left(\frac{s^{\rm in}}{b^{\rm in}}\right). \tag{17}$$

Since $b^{\rm fo} + b^{\rm in} = b$ and $s^{\rm fo} + s^{\rm in} = 1$, we only need to solve for two endogenous variables, namely $b^{\rm in} \in [0, b]$ and $s^{\rm in} \in [0, 1]$. We can then rewrite equation (17) as follows:

$$(1 - T_{ax})\, U^{\rm fo}_{\rm se}\left(\frac{b - b^{\rm in}}{1 - s^{\rm in}}\right) - L_{\rm se} \;=\; U^{\rm in}_{\rm se}\left(\frac{s^{\rm in}}{b^{\rm in}}\right) \text{ and } U^{\rm fo}_{\rm bu}\left(\frac{b - b^{\rm in}}{1 - s^{\rm in}}\right) - L_{\rm bu} \;=\; U^{\rm in}_{\rm bu}\left(\frac{s^{\rm in}}{b^{\rm in}}\right), \tag{18}$$

which is fully characterized by an ordered pair $(b^{\rm in}, s^{\rm in})$.

As in the model of Section 3, we can interpret this equilibrium in three ways. First, it can be understood as describing a symmetric mixed-strategy Nash equilibrium of a game with a large population of identical buyers and another large population of identical sellers. All sellers are *ex ante* identical, so they play identical strategies, given by the probability vector $(s^{\rm fo}, s^{\rm in})$. All buyers are *ex ante* identical, so they play identical strategies, given by the probability vector $(b^{\rm fo}, b^{\rm in})/b$. Equation (17) is then equivalent to saying that this profile of mixed strategies is a Nash equilibrium.

Second, we can suppose that the populations of buyers and sellers in the formal and informal markets evolve over time according to best response dynamics. This yields the dynamical equations:

$$\begin{aligned} \dot{s}^{\rm fo}(t) \;&=\; -\dot{s}^{\rm in}(t) \;=\; \lambda_{\rm se}\left((1 - T_{ax})\, U^{\rm fo}_{\rm se}\left(\frac{b^{\rm fo}(t)}{s^{\rm fo}(t)}\right) - L_{\rm se} - U^{\rm in}_{\rm se}\left(\frac{s^{\rm in}(t)}{b^{\rm in}(t)}\right)\right) \\ \text{and} & \\ \dot{b}^{\rm fo}(t) \;&=\; -\dot{b}^{\rm in}(t) \;=\; \lambda_{\rm bu}\left(U^{\rm fo}_{\rm bu}\left(\frac{b^{\rm fo}(t)}{s^{\rm fo}(t)}\right) - L_{\rm bu} - U^{\rm in}_{\rm bu}\left(\frac{s^{\rm in}(t)}{b^{\rm in}(t)}\right)\right). \end{aligned} \tag{19}$$

Here, $\lambda_{\rm bu} : \mathbb{R} \longrightarrow \mathbb{R}$ and $\lambda_{\rm se} : \mathbb{R} \longrightarrow \mathbb{R}$ are strictly increasing functions which modulate the speed of adjustment, with $\lambda_{\rm bu}(0) = 0 = \lambda_{\rm se}(0)$.[25] We can also consider a continuous-time version of this model, where $\dot{b}^{\rm fo}(t)$, $\dot{b}^{\rm in}(t)$, $\dot{s}^{\rm fo}(t)$, and $\dot{s}^{\rm in}(t)$ represent the derivatives at time t. In either case, equation (17) is the necessary and sufficient condition for a population distribution $(b^{\rm fo}, b^{\rm in}, s^{\rm fo}, s^{\rm in})$ to be a fixed point of the dynamics (19).

Finally, we could suppose that the buyer/seller populations evolve according to replicator/imitation dynamics. This yields dynamical equations:

$$\begin{aligned} \dot{s}^{\rm fo}(t) = -\dot{s}^{\rm in}(t) \;&=\; \lambda_{\rm se}\, s^{\rm fo}(t)\, s^{\rm in}(t)\, \left((1 - T_{ax})\, U^{\rm fo}_{\rm se}\left(\frac{b^{\rm fo}(t)}{s^{\rm fo}(t)}\right) - L_{\rm se} - U^{\rm in}_{\rm se}\left(\frac{s^{\rm in}(t)}{b^{\rm in}(t)}\right)\right) \\ \text{and} & \\ \dot{b}^{\rm fo}(t) = -\dot{b}^{\rm in}(t) \;&=\; \lambda_{\rm bu}\, b^{\rm fo}(t)\, b^{\rm in}(t)\, \left(U^{\rm fo}_{\rm bu}\left(\frac{b^{\rm fo}(t)}{s^{\rm fo}(t)}\right) - L_{\rm bu} - U^{\rm in}_{\rm bu}\left(\frac{s^{\rm in}(t)}{b^{\rm in}(t)}\right)\right), \end{aligned} \tag{20}$$

[25] Typically, $\lambda_{\rm bu}$ and $\lambda_{\rm se}$ are are just multiplication by some positive constant.

where $\lambda_{\rm se} > 0$ and $\lambda_{\rm bu} > 0$ are constants. Again, this dynamical equation has both a discrete-time and a continuous-time interpretation. In either case, equation (17) is a necessary and sufficient condition for a population distribution $(b^{\rm fo}, b^{\rm in}, s^{\rm fo}, s^{\rm in})$ to be a "nontrivial" fixed point of the dynamics (20). Here, "nontrivial" refers to the fact that the replicator dynamics always has "trivial" fixed points where either $b^{\rm fo} = 0$ or $b^{\rm in} = 0$ and either $s^{\rm fo} = 0$ or $s^{\rm in} = 0$.

An equilibrium (b^*, s^*) is *locally stable* if it is an attracting fixed point under the best-response dynamics described by the dynamical equations (19). In other words, there exists some neighbourhood $\mathcal{U}$ around (b^*, s^*) such that, for any $(b^{\rm in}, s^{\rm in})$ in $\mathcal{U}$, the forward-time orbit of $(b^{\rm in}, s^{\rm in})$ under (19) converges to (b^*, s^*). If $0 < b^* < b$ and $0 < s^* < 1$, then this also implies that (b^*, s^*) is an attracting fixed point under the replicator dynamics described by the dynamical equations (20).[26]

Graphically, it is easy to identify a locally stable equilibrium. To this end, let us rewrite (10) in the more abstract form as follows:

$$\dot{b}^{\rm in} = \beta(b^{\rm in}, s^{\rm in})$$

$$\dot{s}^{\rm in} = \sigma(b^{\rm in}, s^{\rm in})$$

where β and σ are the functions appearing on the right hand side of the best responses in (19). Then an equilibrium is simply an intersection of the two *isoclines* $\mathcal{B} := \{(b^{\rm in}, s^{\rm in});\ \beta(b^{\rm in}, s^{\rm in}) = 0\}$ and $\mathcal{S} := \{(b^{\rm in}, s^{\rm in});\ \sigma(b^{\rm in}, s^{\rm in}) = 0\}$. Typically, $\mathcal{B}$ and $\mathcal{S}$ are smooth curves in the rectangular domain $[0, b] \times [0, 1]$ given by:

$$\begin{aligned} \mathcal{S}(T_{ax}, L_{\rm se}) &:= \left\{(s^{\rm in}, b^{\rm in}) \in [0,1] \times [0,b]\ ;\ (1 - T_{ax})\, U^{\rm fo}_{\rm se}\left(\frac{b - b^{\rm in}}{1 - s^{\rm in}}\right) - L_{\rm se} = U^{\rm in}_{\rm se}\left(\frac{s^{\rm in}}{b^{\rm in}}\right)\right\}, \\ \text{and} \quad \mathcal{B}(L_{\rm bu}) &:= \left\{(s^{\rm in}, b^{\rm in}) \in [0,1] \times [0,b]\ ;\ U^{\rm fo}_{\rm bu}\left(\frac{b - b^{\rm in}}{1 - s^{\rm in}}\right) - L_{\rm bu} = U^{\rm in}_{\rm bu}\left(\frac{s^{\rm in}}{b^{\rm in}}\right)\right\}. \end{aligned} \tag{21}$$

The equilibrium (b^*, s^*) is locally stable if the following conditions are met in a neighbourhood of (b^*, s^*):

(i) The absolute slope of $\mathcal{B}$ at (b^*, s^*) is larger than the absolute slope of $\mathcal{S}$ at this point.[27]

(ii) β is positive to the left of $\mathcal{B}$, and negative to the right of $\mathcal{B}$.

(iii) σ is positive below $\mathcal{S}$, and negative above $\mathcal{S}$.

If the population of informal buyers unilaterally dips below (above) b^*, then Condition (ii) says that the payoff for informal buyers will be higher (lower) than the payoff for formal buyers, causing buyers to migrate into (out of) the informal market until $b^{\rm in} = b^*$. Likewise,

[26]Note that the vector field determined by (20) is obtained by multiplying the vector field defined by (19) by a scalar function which is positive everywhere in $(0, b) \times (0, 1)$. Thus, a stable fixed point for (19) is also a stable fixed point for (20).

[27]Heuristically, this means we can think of $\mathcal{B}$ as a roughly "vertical" curve near (b^*, s^*), whereas $\mathcal{S}$ is roughly "horizontal" near (b^*, s^*).

if the population of informal sellers unilaterally dips below (above) s^*, then Condition (iii) says that the payoff for informal sellers will be higher (lower) than the payoff for formal sellers, causing sellers to migrate into (out of) the informal market, until $s^{\text{in}} = s^*$. Thus, a stable equilibrium is such that any point to the left (right) of $\mathcal{B}$ will move in a rightward (leftwards) direction and any point below (above) $\mathcal{S}$ will move upwards (downwards).

We say there is a *pure formal market equilibrium* if the point $(b^{\text{in}}, s^{\text{in}}) = (0,0)$ satisfies the equilibrium condition given by (18). We say there is a *pure informal market equilibrium* if the point $(b^{\text{in}}, s^{\text{in}}) = (b,1)$ satisfies equation (18). Finally a *mixed-market equilibrium* is a point $(b^*, s^*) \in (0,b) \times (0,1)$ which satisfies equation (18). To establish the robust coexistence of formal and informal markets, we must show that *there exists a locally stable mixed-market equilibrium.*

4.5 Coexistence of formal and informal markets

In the next section, we examine different scenarios and their implications for the resulting equilibria. Given the complexity of the model, no closed form solutions exist, so that numerical analysis are required to characterize the equilibrium.

4.5.1 No taxes, no quality assurance

To isolate the implications of the trading protocol, we will first consider an environment with $T_{ax} = 0$ and $g^{\text{in}} = g^{\text{fo}}$. In other words, we initially suppose that the formal market has no quality assurance advantage, and that neither market has a tax advantage. This would occur, for example, if the tax rate in the formal market exactly matched the rate of theft in the informal market, and if products had zero probability of defects or if $\alpha(q) = q$ for all q.

When $T_{ax} = 0$ and $L_{\text{bu}} = L_{\text{se}} = 0$, the two curves $\mathcal{S}(0,0)$ and $\mathcal{B}(0)$ characterizing the stability of the equilibrium are very close to diagonals. Heuristically, this means that buyers and sellers are both essentially indifferent between the two markets, as long as

$$\frac{b^{\text{in}}}{s^{\text{in}}} \;=\; b \;=\; \frac{b^{\text{fo}}}{s^{\text{fo}}}. \tag{22}$$

Numerical methods suggest that, in this case, buyers and sellers exhibit a very weak preference for an all-formal market equilibrium. But the difference in payoff between the all-formal market equilibrium and other points on the diagonal (22) is so small that all points on this diagonal could be regarded as "quasi-equilibria". However, if $L_{\text{bu}} \neq 0$ and $L_{\text{se}} \neq 0$, then the picture becomes much clearer.

Result 2.

(a) *If L_{bu} and L_{se} have opposite signs, and $|L_{\text{bu}}|$ and $|L_{\text{se}}|$ are large enough, then there is a locally stable mixed market equilibrium.*

(b) *If $L_{\text{bu}} < 0$ and $L_{\text{se}} < 0$, and $|L_{\text{bu}}|$ and $|L_{\text{se}}|$ are large enough, then there is no mixed market equilibrium. Instead, there is a locally stable pure formal market equilibrium.*

(c) *If $L_{bu} > 0$ and $L_{se} > 0$, and $|L_{bu}|$ and $|L_{se}|$ are large enough, then there is no mixed market equilibrium. Instead, there is a locally stable pure informal market equilibrium.*

In all three cases, the equilibrium appears to be unique.

Result 2(a) covers two cases. We have $L_{bu} > 0$ and $L_{se} < 0$ if, for example, buyers face entrance fees or transportation costs to enter the formal sector, while sellers must pay bribes or protection money to participate in the informal sector. On the other hand, we have $L_{bu} < 0$ and $L_{se} > 0$ if, for example, sellers must pay license fees in the formal sector, while buyers experience some inconvenience in the informal sector. In either case, Result 2(a) says there will be a robust equilibrium where formal and informal markets coexist.

To understand why Result 2 is true, first observe that an equilibrium (18) is any crossing point of the isocline $\mathcal{B}(L_{bu})$ (from equation (21)) and the isocline

$$\mathcal{S}(0, L_{se}) \quad := \quad \left\{(s^{in}, b^{in}) \in [0,1] \times [0,b] \; ; \; U_{se}^{fo}\left(\frac{b - b^{in}}{1 - s^{in}}\right) - L_{se} = U_{se}^{in}\left(\frac{s^{in}}{b^{in}}\right)\right\}.$$

Let us now define the functions $\beta, \sigma : [0,b] \times [0,1] \longrightarrow \mathbb{R}$ by setting

$$\begin{aligned}
\sigma(b^{in}, s^{in}) &:= \quad U_{se}^{in}\left(\frac{s^{in}}{b^{in}}\right) - U_{se}^{fo}\left(\frac{b - b^{in}}{1 - s^{in}}\right) \\
\beta(b^{in}, s^{in}) &:= \quad U_{bu}^{in}\left(\frac{s^{in}}{b^{in}}\right) - U_{bu}^{fo}\left(\frac{b - b^{in}}{1 - s^{in}}\right),
\end{aligned}$$

for all $b^{in} \in [0,b]$ and $s^{in} \in [0,1]$. Suppose $L_{se} = 0$; then σ measures how relatively attractive the informal market is for sellers. If $\sigma(b^{in}, s^{in})$ is positive (negative), then sellers will move into (out of) the informal market, so s^{in} will increase (decrease). Likewise, suppose $L_{bu} = 0$; then β measures how relatively attractive the informal market is for buyers. If $\beta(b^{in}, s^{in})$ is positive (negative), then buyers will move into (out of) the informal market, so b^{in} will increase (decrease). The isocontours of β are the isoclines $\mathcal{B}(L_{bu})$ for various choices of L_{bu}. The isocontours of σ are the isoclines $\mathcal{S}(0, L_{se})$ for various choices of L_{se}. These isocontours cross if and only if the gradient vector field $\nabla\sigma$ is *not* parallel to the gradient vector field $\nabla\beta$. So this is what we must demonstrate to prove Result 2.

If the two gradient vector fields *were* parallel, then we would have

$$\phi(b^{in}, s^{in}) \quad := \quad \frac{\nabla\sigma(b^{in}, s^{in}) \bullet \nabla\beta(b^{in}, s^{in})}{\|\nabla\sigma(b^{in}, s^{in})\| \cdot \|\nabla\beta(b^{in}, s^{in})\|} \quad = \quad \pm 1, \tag{23}$$

for all $b^{in} \in [0,b]$ and $s^{in} \in [0,1]$. The explicit formula for the function ϕ defined in (23) is extremely complicated, and must be manipulated using a symbolic computation package like *Maple* or *Mathematica*. However, using such a package, it is easy to check that $\phi(b^{in}, s^{in}) \neq \pm 1$, for any choice of (b^{in}, s^{in}) which is not close to the diagonal line $\{(b^{in}, s^{in});\ b^{in}/s^{in} = b/s\}$. Indeed, using a symbolic computation package, one can verify that

$$\lim_{\epsilon \to 0} \phi(b - \epsilon, \epsilon) \quad = \quad 0.$$

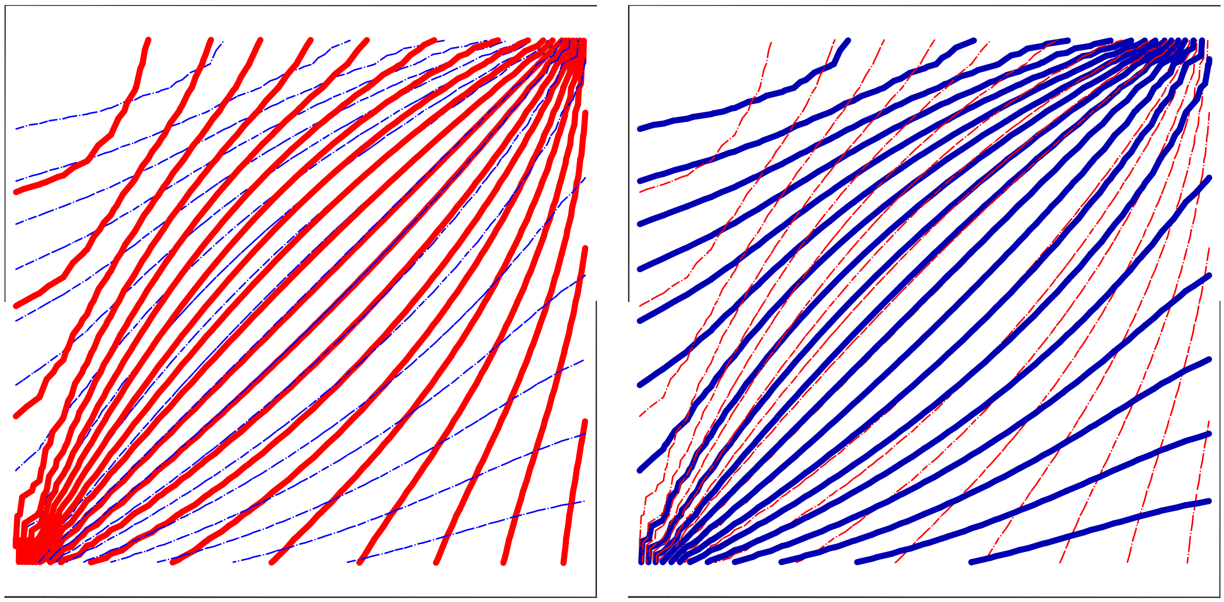


Figure 1: In these figures, the horizontal axis represents b^{in}, on a scale from 0 to b, while the vertical axis represents s^{in}, on a scale from 0 to 1. *Left.* The curves $\mathcal{B}(L_{\text{bu}})$, for various values of L_{bu}. *Right.* The curves $\mathcal{S}(0, L_{\text{se}})$, for various values of L_{se}. (Here we suppose $\delta = 0.99$ and $g^{\text{in}} = g^{\text{fo}}$.)

In other words, if $(b^{\text{in}}, s^{\text{in}})$ is close to $(b, 0)$ (the southeast corner of the domain $[0, b] \times [0, s]$), then the gradient vectors $\nabla\sigma(b^{\text{in}}, s^{\text{in}})$ and $\nabla\beta(b^{\text{in}}, s^{\text{in}})$ are not only non-parallel, but nearly *orthogonal*, meaning that the isocontours $\mathcal{S}$ and $\mathcal{B}$ cross at right angles.[28]

In Figure 1, σ is decreasing as s^{in} increases or as b^{in} decreases, and the isocontour along the diagonal corresponds to $\sigma = 0$. Thus, $\sigma < 0$ for points in the northwest half of the picture (above the diagonal), while $\sigma > 0$ in the southeast half (below the diagonal). Likewise, β is decreasing as s^{in} decreases or as b^{in} increases, and the isocontour along the diagonal corresponds to $\beta = 0$. Thus, $\beta > 0$ above the diagonal, while $\beta < 0$ below the diagonal. Thus, the crossings above the diagonal correspond to the case $L_{\text{bu}} < 0 < L_{\text{se}}$, whereas the crossings below the diagonal correspond to the case $L_{\text{se}} < 0 < L_{\text{bu}}$, as described in Result 2(a).

Any crossing of the curves $\mathcal{B}(L_{\text{bu}})$ and $\mathcal{S}(0, L_{\text{se}})$ will determine an equilibrium (18) of the economy. However, not all such equilibria are locally stable. If the slope of $\mathcal{S}(0, L_{\text{se}})$ is *less* than the slope of $\mathcal{B}(L_{\text{bu}})$ when they cross, then it is easy to check that conditions (i)-(iii) from Section 4.4 are satisfied, so that the equilibrium is locally stable. For example, Figure 2 shows the curves $\mathcal{B}(-0.5)$ and $\mathcal{S}(0, 0.5)$ intersecting in a locally stable equilibrium. Figure 3 shows the curves $\mathcal{B}(0.4)$ and $\mathcal{S}(0, -0.4)$ intersecting in a locally stable equilibrium.

Thus, there is a stable equilibrium with a mixture of formal and informal markets whenever the buyers and sellers face lump-sum costs in different markets. However, if both buyers and sellers face lump-sum costs in the *same* market, then the curves do *not* cross.

[28] Unfortunately, it is not possible to obtain a similar asymptotic result for $(b^{\text{in}}, s^{\text{in}})$ is close to $(0, 1)$, because ϕ has a singularity there.

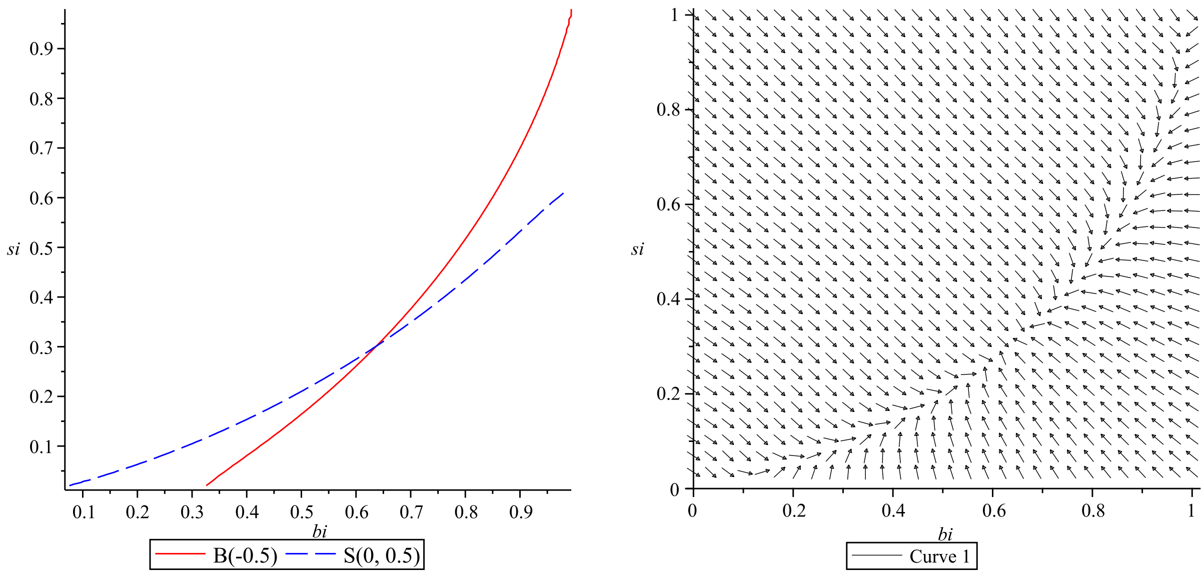


Figure 2: An illustration of Result 2(a). *Left.* An equilibrium defined by the crossing of $\mathcal{S}(0, 0.5)$ (dashed) and $\mathcal{B}(-0.5)$ (solid). *Right.* This equilibrium is locally stable under the best response dynamics given by the dynamical equations (19). (Here we suppose $\delta = 0.99$ and $g^{\text{in}} = g^{\text{fo}}$.)

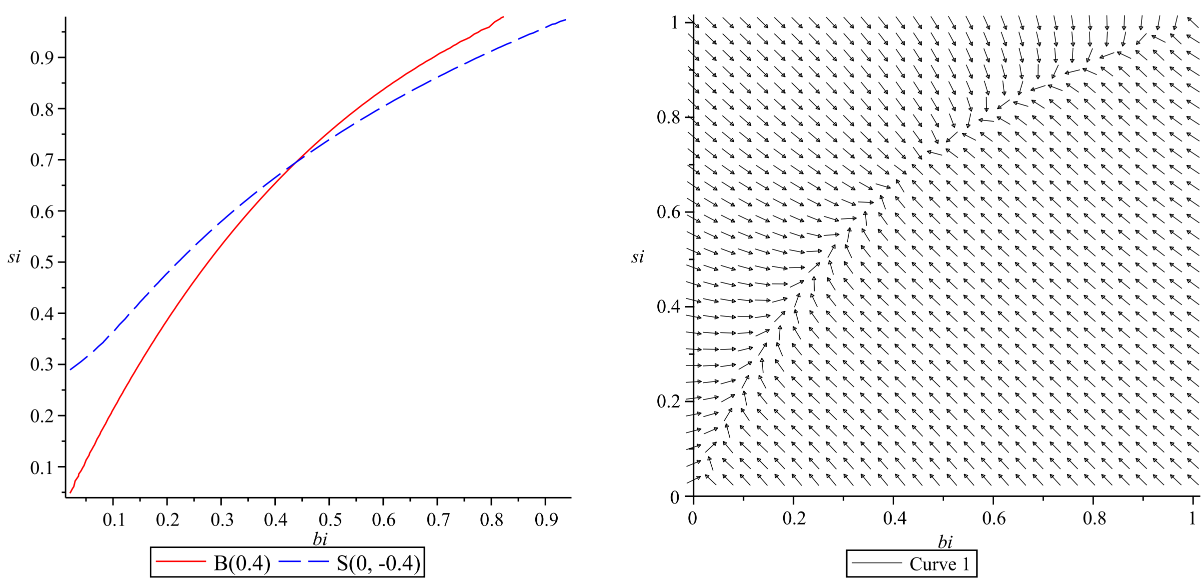


Figure 3: Another illustration of Result 2(a). *Left.* An equilibrium defined by the crossing of $\mathcal{S}(0, -0.4)$ (dashed) and $\mathcal{B}(0.4)$ (solid). *Right.* This equilibrium is locally stable under the best response dynamics given by the dynamical equations (19). (Here we suppose $\delta = 0.99$ and $g^{\text{in}} = g^{\text{fo}}$.)

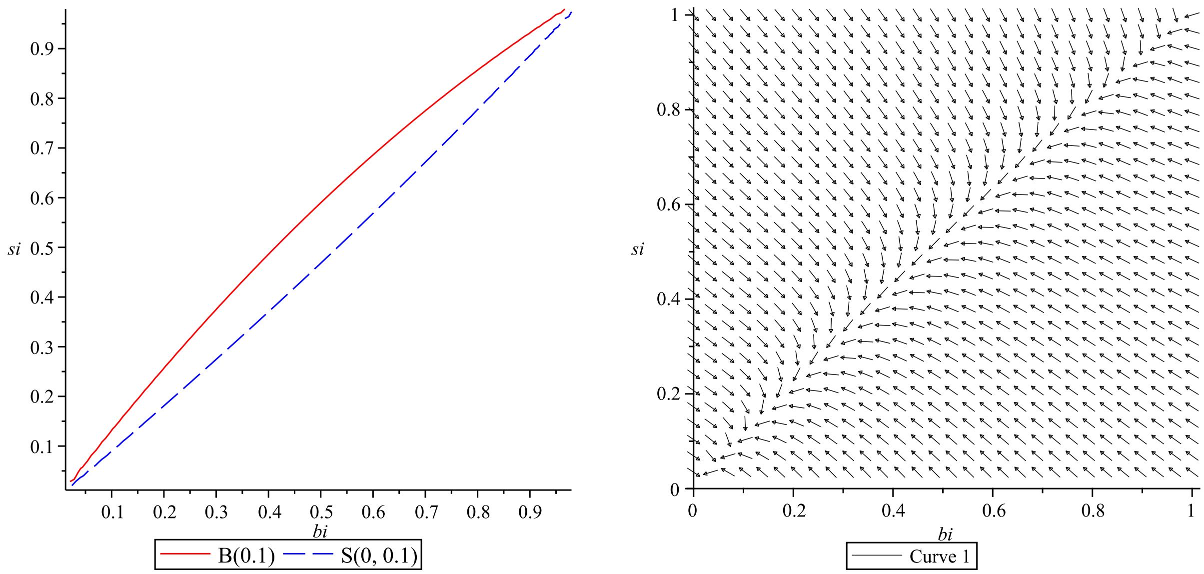


Figure 4: An illustration of Result 2(b). *Left.* The curves $\mathcal{S}(0, 0.1)$ (dashed) and $\mathcal{B}(0.1)$ (solid) do not cross. *Right.* $\mathcal{S}(0, 0.1)$ is below $\mathcal{B}(0.1)$, so under the best response dynamics (19), the system evolves to the pure formal market equilibrium $(0, 0)$.

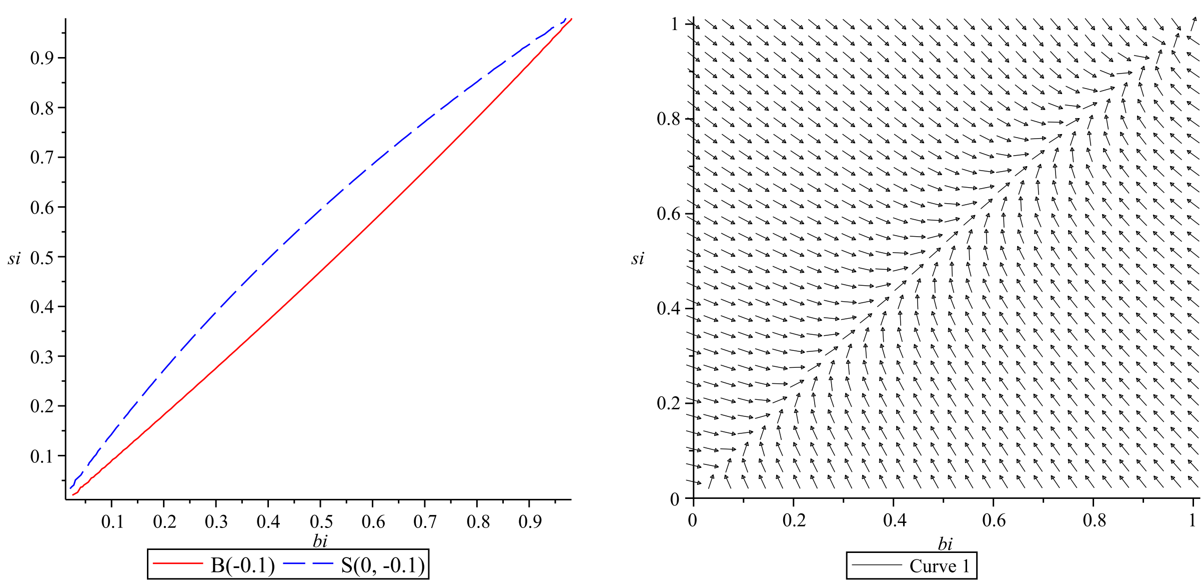
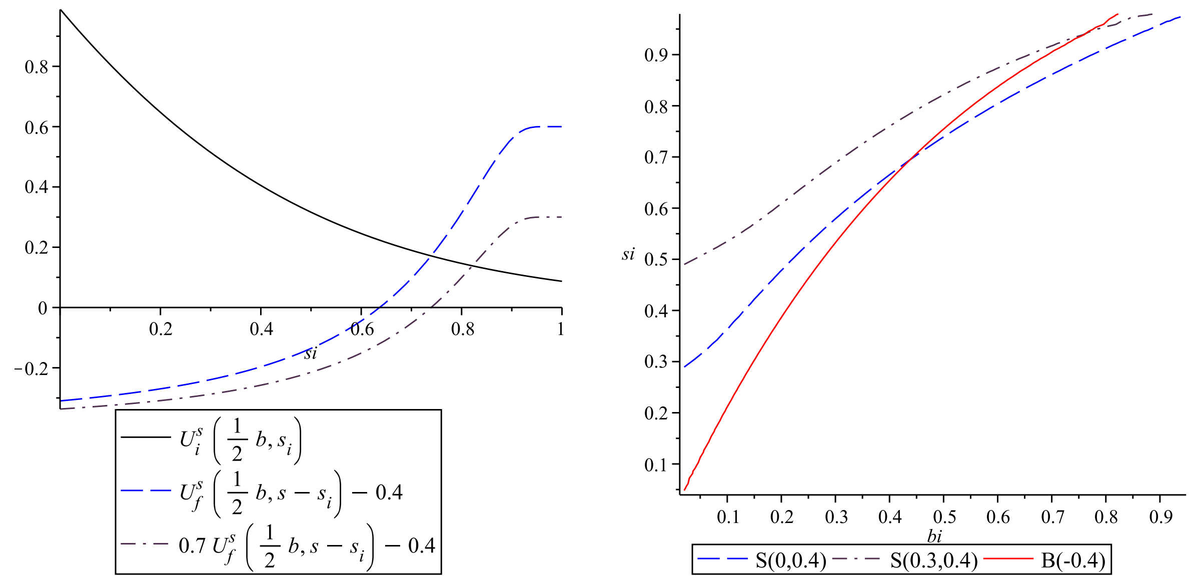


Figure 5: An illustration of Result 2(c). *Left.* The curves $\mathcal{S}(0, -0.1)$ (dashed) and $\mathcal{B}(-0.1)$ (solid) do not cross. *Right.* $\mathcal{S}(0, -0.1)$ is above $\mathcal{B}(-0.1)$, so under the best response dynamics (19), the system evolves to the pure informal market equilibrium $(b, 1)$.



(a) (b)

Figure 6: (a) For any fixed level of b^{in}, an increase in the net tax level T_{ax} will increase the equilibrium level of s^{in}. (In this picture we have fixed $b^{\mathrm{in}} = b/2$.) (b) Thus, increasing the net tax from $T_{ax} = 0$ to $T_{ax} = 0.3$ shifts the curve $\mathcal{S}(T_{ax}, 0.4)$ upwards, so that the intersection with $\mathcal{B}(-0.4)$ moves to the northeast; in other words, both buyers and sellers migrate from the formal to the informal market. In this picture we have $L_{\mathrm{se}} := 0.4$ and $L_{\mathrm{bu}} = -0.4$, but we would get a similar picture for any $L_{\mathrm{bu}} < 0 < L_{\mathrm{se}}$.

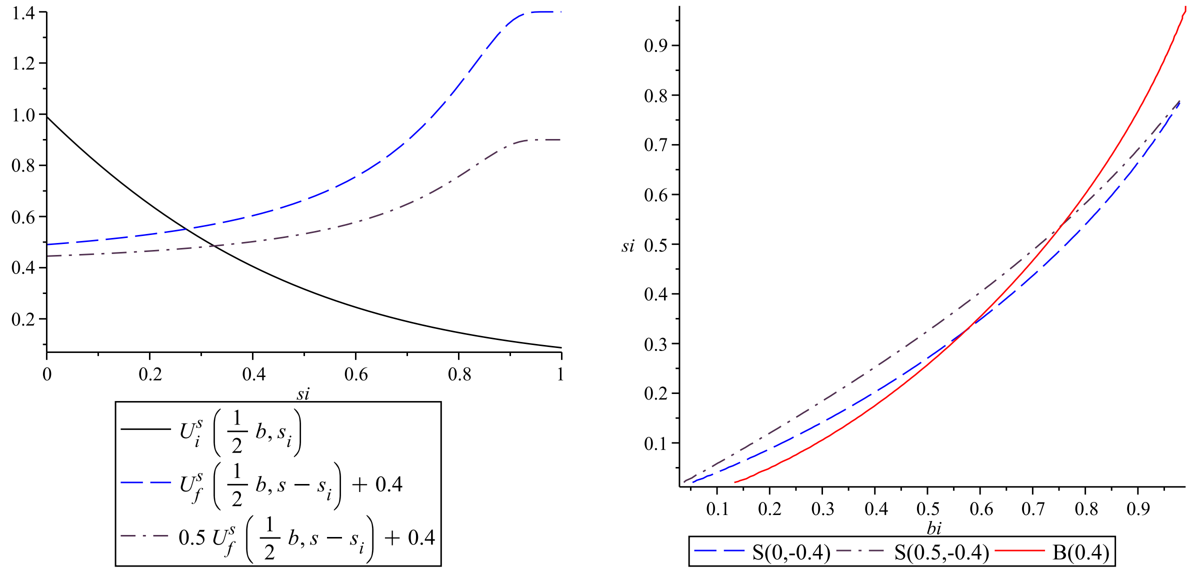
In this case, the dynamics cause all buyers and sellers to migrate to the market *without* the lump-sum costs. If $\mathcal{S}(0, L_{\mathrm{se}})$ is always below $\mathcal{B}(L_{\mathrm{bu}})$, then all buyers and sellers migrate to the formal market, as described by Result 2(b). If $\mathcal{S}(0, L_{\mathrm{se}})$ is always above $\mathcal{B}(L_{\mathrm{bu}})$, then all buyers and sellers migrate to the informal market, as described by Result 2(c). Figures 4 and 5 illustrate this fact.

4.5.2 Crime and Taxation

So far we have considered the case when the tax rate in the formal sector is exactly equal to the crime rate in the informal sector, so that $T_{ax} = 0$. *Ceteris paribus*, raising tax rates in the formal sector (or lowering crime rates in the informal sector) will cause some buyers and sellers to migrate from the formal to the informal market. To see this, consider the isocline

$$\mathcal{S}(T_{ax}, L_{\mathrm{se}}) \quad := \quad \left\{ (s^{\mathrm{in}}, b^{\mathrm{in}}) \in [0,1] \times [0,b] \;;\; (1 - T_{ax})\, U^{\mathrm{fo}}_{\mathrm{se}}\left(\frac{b - b^{\mathrm{in}}}{1 - s^{\mathrm{in}}}\right) - L_{\mathrm{se}} = U^{\mathrm{in}}_{\mathrm{se}}\left(\frac{s^{\mathrm{in}}}{b^{\mathrm{in}}}\right) \right\}$$

for any net tax level T_{ax}. We claim that increasing T_{ax} will cause this curve to shift upwards. For example, consider Figure 6(a). Here, we have fixed $b^{\mathrm{in}} = b/2$, and we suppose $L_{\mathrm{se}} = 0.4$. The downward sloping curve is $U^{\mathrm{in}}_{\mathrm{se}}(b/2, s^{\mathrm{in}})$ —the payoff for informal sellers, as a function of s^{in}. The upward sloping curves are the payoffs for formal sellers, as a function of s^{in}. The dashed curve is $U^{\mathrm{fo}}_{\mathrm{se}}(b/2, s - s^{\mathrm{in}}) - 0.4$; this is the payoff with $T_{ax} = 0$ (i.e. no net taxation).

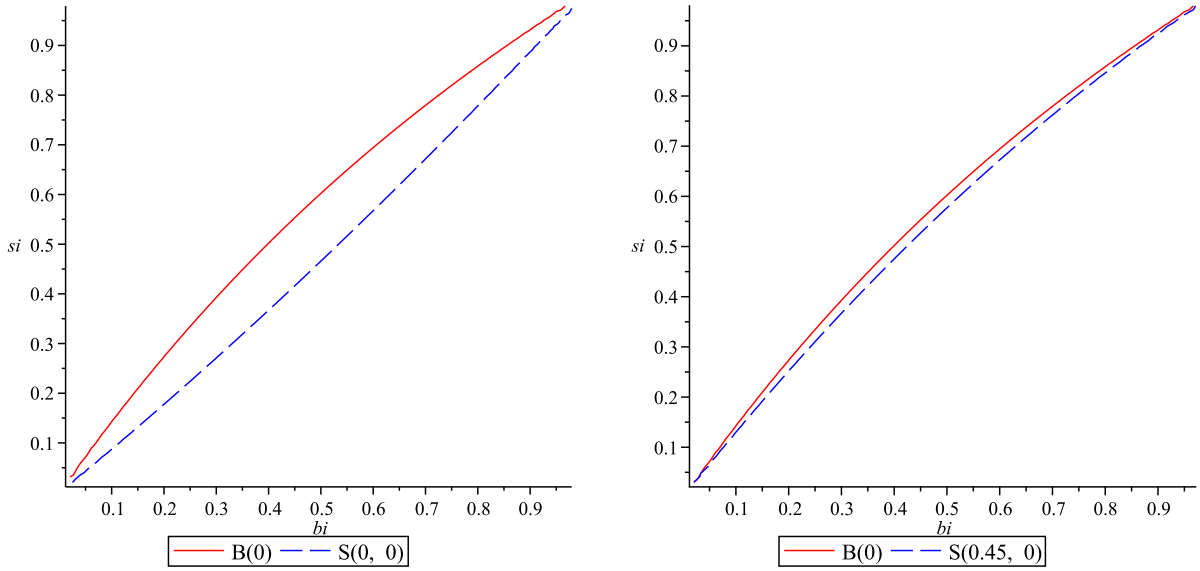


(a) (b)

Figure 7: Same interpretation as Figure 6, only now $L_{\mathrm{se}} = -0.4$ while $L_{\mathrm{bu}} := 0.4$, and we compare the net tax levels $T_{ax} = 0$ and $T_{ax} = 0.5$ We would get a similar picture for any $L_{\mathrm{bu}} > 0 > L_{\mathrm{se}}$. Note that the effect of taxation is smaller in this case than in Figure 6.

The dot-dashed curve is $0.7 \cdot U_{\mathrm{se}}^{\mathrm{fo}}(b/2, s - s^{\mathrm{in}}) - 0.4$; this is the payoff with $T_{ax} = 0.3$ (i.e. a net taxation rate of 30%). Note how the intersection with $U_{\mathrm{se}}^{\mathrm{in}}(b/2, s^{\mathrm{in}})$ shifts to the right as we increase T_{ax}, indicating that equilibrium occurs at a higher value of s^{in} (i.e. more sellers enter the informal market). By repeating this argument for every value of b^{in}, we can see that the curve $\mathcal{S}(0.3, 0.4)$ must be *above* the curve $\mathcal{S}(0, 0.4)$. Since $\mathcal{B}$ slopes upwards, the intersection of $\mathcal{S}(0.3, 0.4)$ with the curve $\mathcal{B}(L_{\mathrm{bu}})$ will thus be *northeast* of the intersection of $\mathcal{S}(0, 0.4)$ with the curve $\mathcal{B}(L_{\mathrm{bu}})$, as shown in Figure 6(b). In other words, higher formal taxes will cause a larger fraction of both buyers and sellers to migrate to the informal sector. However, as long as the net tax is small enough, the new equilibrium is still a mixed-market type.

Figure 6 showed the case when $L_{\mathrm{se}} > 0 > L_{\mathrm{bu}}$ —in other words, sellers must pay net lump-sum costs to enter the formal sector (e.g. the costs of retail space and licenses), while buyers pay a net lump-sum costs to enter the informal sector (e.g. inconvenience). Figure 7 shows the opposite case, when $L_{\mathrm{se}} < 0 < L_{\mathrm{bu}}$ —in other words, sellers must pay a net lump-sum costs to enter the informal sector (e.g. due to bribes and shoe-leather costs), while buyers pay a net fee to enter the informal sector (e.g. due to transportation costs). The impact of taxation is similar. In Figure 7(a), we again fix $b^{\mathrm{in}} = b/2$, but we now suppose $L_{\mathrm{se}} = -0.4$. The downward sloping curve is again $U_{\mathrm{se}}^{\mathrm{in}}(b/2, s^{\mathrm{in}})$ —the payoff for informal sellers, as a function of s^{in}. The upward sloping curves are the payoffs for formal sellers, as a function of s^{in}. The dashed curve is $U_{\mathrm{se}}^{\mathrm{fo}}(b/2, s - s^{\mathrm{in}}) + 0.4$; this is the payoff with no net taxation. The dot-dashed curve is $0.5 \cdot U_{\mathrm{se}}^{\mathrm{fo}}(b/2, s - s^{\mathrm{in}}) + 0.4$; this is the payoff with $T_{ax} = 0.5$ —a net taxation rate of 50%. Again, the intersection with $U_{\mathrm{se}}^{\mathrm{in}}(b/2, s^{\mathrm{in}})$ shifts



(a) (b)

Figure 8: (a) If $g^{\text{in}} = 0.9\, g^{\text{fo}}$, then all buyers and sellers migrate to the formal market because of its quality assurance advantage. (b) The formal market remains the only stable equilibrium, even if the government imposes 45% taxation.

to the right, indicating that equilibrium occurs at a higher value of s^{in} (i.e. more sellers in the informal market). By repeating this argument for every value of b^{in}, we can see that the curve $\mathcal{S}(0.5, -0.4)$ must be *above* the curve $\mathcal{S}(0, -0.4)$. Since $\mathcal{B}$ slopes upwards, the intersection of $\mathcal{S}(0.3, -0.4)$ with the curve $\mathcal{B}(L_{\text{bu}})$ will again be northeast of the the intersection of $\mathcal{S}(0, 0.4)$ with the curve $\mathcal{B}(L_{\text{bu}})$, as shown in Figure 7(b).

Note that the impact of taxation is stronger in Figure 6 than in Figure 7, despite the fact that the net tax increase in Figure 7 was $T_{ax} = 0.5$, whereas in Figure 6 it was only $T_{ax} = 0.3$. In other words, the effect of taxation depends on the relative lump-sum entry costs sellers and buyers face in the formal market: taxation causes a stronger effect in a situation when the sellers (but not buyers) must pay a net positive fee to enter the formal market, whereas taxation causes a weaker effect when it is buyers (but not sellers) who must pay a net fee to enter the formal market.

We have considered the effects of a formal sector tax increase (or informal sector crime decrease), but the analysis of the opposite change is exactly analogous. If formal sector taxes *decrease* (or if informal crime risks *increase*), then the seller's payoff curves in Figures 6 and 7 will shift to the *left*, causing the market equilibrium to shift to a lower level of informal market participation.

4.5.3 Quality assurance versus Taxation

So far all of our discussion has assumed that $g^{\text{in}} = g^{\text{fo}}$. In other words, we suppose that the formal market has no advantage over the informal market due to quality assurance. This would be the case, for example, if the quality assurance technology has constant returns to scale in Lemma A. If $g^{\text{in}} < g^{\text{fo}}$, then the "quality assurance advantage" of the formal market creates a rent which the government can tax. For example, suppose $g^{\text{in}} = 0.9\, g^{\text{fo}}$. Figure 8(a) shows a market with no net taxation and no lump-sum costs (i.e. $T_{ax} = L_{\text{se}} = L_{\text{bu}} = 0$). We see that $\mathcal{S}(0,0)$ is always below $\mathcal{B}(0)$, so all buyers and sellers migrate to the formal market. Figure 8(b) shows a market with no lump-sum costs (i.e. $L_{\text{se}} = L_{\text{bu}} = 0$), but with a 45% net tax rate on the formal sector (i.e. $T_{ax} = 0.45$), we see that $\mathcal{S}(0.45, 0)$ is *still* below $\mathcal{B}(0)$, so that all buyers and sellers remain in the formal market. Thus, if the formal market has even a small quality assurance advantage, then it can withstand a large amount of government taxation.

5 Conclusion

A fundamental feature of informal markets is that their sellers strive to remain anonymous from government authorities. This property has important implications on the type of trading protocols that these sellers can use to attract buyers in these markets. This is an aspect that has not received much attention by the literature and that we have explored.

In this paper we consider an environment with formal and informal markets where capacity-constrained sellers produce at most one indivisible unit of the perishable good. In formal markets, sellers can publicly post their prices and locations which buyers are directed to, while in informal markets sellers, who cannot post their prices or locations, have to meet and bargain with buyers in an undirected - or random - way as this reduces their chances of being observed by government authorities. Within this environment, we study the existence of formal and informal activity while explicitly taking into account the strategic behavior of agents. We also incorporate various factors that affect buyers' and sellers' switching decisions between formal and informal markets —factors such as taxes, regulations and quality assurance in the formal market, versus the risk of crime and/or confiscation in the informal market.

In our benchmark model we first assume that only sellers can switch between markets so that the number of buyers in formal and informal markets is always fixed. When the trading protocol is the only distinguishing feature between these markets, we can then analytically show that formal and informal markets of nontrivial size coexist in a stable equilibrium. This finding is robust to richer environments where formal markets are able to provide quality assurances, pay taxes and informal sellers face the risk that their sales proceeds can be stolen.

Once we relax the immobility of buyers, we have a less tractable environment and analytical solutions are not possible. Nevertheless, this richer environment provides very interesting dynamics in terms of general lump-sum costs and benefits of buyers and sellers in both formal and informal markets.[29]

[29]To be more specific, this richer environment suggests that the effect of taxation depends on the relative lump-sum entry costs sellers and buyers face in the formal market (taxation causes a stronger effect in a situation when the sellers —but not buyers —must pay a net positive fee to enter the formal market, whereas taxation causes a weaker effect when it is buyers —but not sellers —who must pay a net fee to

This paper shows that an important aspect in understanding the coexistence of formal and informal markets is to examine the different trading protocols in these markets. By doing so we are able to explore the strategic decisions of buyers and sellers change as the differential costs of participating in these markets are taken into account.

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enter the formal market). This richer environment also allows asymmetric responses to tax increases and tax decreases, in that it might cost more for a seller to switch from the informal market to the formal market than vice versa (e.g. because of the need to acquire licenses, rent a retail location, etc.).

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Appendix A: Proofs

Proof of Lemma A. The efficient value q^* of investment in quality assurance is the value such that $\alpha'^{*}) = 1$ (i.e. such that one additional cent spent on quality assurance increases the buyer's expected utility by exactly one cent). Since α' is nonincreasing (by concavity), we have $\alpha'(q) \geq 1$ for all $q \in [0, q^*]$. Thus, since $\alpha(0) = 0$, the Fundamental Theorem of Calculus implies that $\alpha(q^*) \geq q^*$ (i.e. the benefit of quality assurance outweighs its cost). Thus,

$$\frac{g^{\mathrm{fo}}}{g^{\mathrm{in}}} \quad = \quad \frac{v_{\mathrm{bu}}^{\mathrm{fo}} - c_{\mathrm{se}}^{\mathrm{fo}}}{v_{\mathrm{bu}}^{\mathrm{in}} - c_{\mathrm{se}}^{\mathrm{in}}} \quad = \quad \frac{v_{\mathrm{bu}}^{\mathrm{in}} + \alpha(q) - c_{\mathrm{se}}^{\mathrm{in}} - q}{v_{\mathrm{bu}}^{\mathrm{in}} - c_{\mathrm{se}}^{\mathrm{in}}} \quad = \quad 1 + \frac{\alpha(q) - q}{v_{\mathrm{bu}}^{\mathrm{in}} - c_{\mathrm{se}}^{\mathrm{in}}} \quad \geq \quad 1,$$

because $\alpha(q) \geq q$. Thus, $g^{\mathrm{in}} \leq g^{\mathrm{fo}}$. □

Proof of Theorem 1. We must show that the interval $[0, 1]$ contains a zero for the function $\widetilde{U}$. By inspecting formulae (5) and (8), we see that, for all $s^{\mathrm{fo}} \in [0, 1]$, we have

$$\begin{aligned}
\widetilde{U}(s^{\mathrm{fo}}) &= \widetilde{U}_{\mathrm{se}}^{\mathrm{fo}}(s^{\mathrm{fo}}) - \widetilde{U}_{\mathrm{se}}^{\mathrm{in}}(s^{\mathrm{fo}}) \quad = \quad (1 - T_{\mathrm{se}}^{\mathrm{fo}})\, \widehat{U}(s^{\mathrm{fo}}), \\
\text{with} \quad \widehat{U}(s^{\mathrm{fo}}) &:= \widehat{U}_1(s^{\mathrm{fo}}) - K\, \widehat{U}_2(s^{\mathrm{fo}}), \\
\text{where} \quad K &:= \frac{(1 - T_{\mathrm{se}}^{\mathrm{in}})\, g^{\mathrm{in}}}{(1 - T_{\mathrm{se}}^{\mathrm{fo}})}, \\
\text{while} \quad \widehat{U}_1(s^{\mathrm{fo}}) &:= \left[1 - \exp\left(\frac{-b^{\mathrm{fo}}}{s^{\mathrm{fo}}}\right)\right] \cdot \left(1 - \frac{b^{\mathrm{fo}}/s^{\mathrm{fo}}}{\exp(b^{\mathrm{fo}}/s^{\mathrm{fo}}) - 1}\right), \quad \text{by Eq.(5),} \\
\text{and} \quad \widehat{U}_2(s^{\mathrm{fo}}) &:= \eta\left(\frac{b^{\mathrm{in}}}{1 - s^{\mathrm{fo}}}\right) \cdot \left[1 - \exp\left(\frac{-b^{\mathrm{in}}}{1 - s^{\mathrm{fo}}}\right)\right] \qquad \text{by Eq.(8).}
\end{aligned} \tag{24}$$

Clearly, it will be sufficient to find a zero for $\widehat{U}$ instead. From equation (**??**), simple computations yield:

$$\lim_{s\searrow 0} \widehat{U}(s) \;=\; 1 - K\,\eta(b^{\text{in}})\,(1-\exp(-b^{\text{in}})) \quad \text{and} \quad \lim_{s\nearrow 1} \widehat{U}(s) \;=\; 1 - \frac{1+b^{\text{fo}}}{\exp(b^{\text{fo}})} - K.$$

From here, it is easy to check that

$$\begin{aligned} \left(K < \frac{1}{\eta(b^{\text{in}})\,(1-\exp(-b^{\text{in}}))}\right) &\implies \left(\lim_{s\searrow 0} \widehat{U}(s) > 0\right) \\ \text{and} \quad \left(K > 1 - \frac{b^{\text{fo}}+1}{\exp(b^{\text{fo}})}\right) &\implies \left(\lim_{s\nearrow 1} \widehat{U}(s) < 0\right). \end{aligned} \tag{25}$$

But $\widehat{U}$ is continuous on $[0,1]$. Thus, if K satisfies both the conditions in (25), then the Intermediate Value Theorem implies that $\widehat{U}(s^*) = 0$ for some $s^* \in (0,1)$. Furthermore, $\widehat{U}$ is going from positive values (near 0) to negative values (near 1), so $\widehat{U}$ must be decreasing near s^*; hence s^* is a stable equilibrium.

Now, it is easy to check that the function $\widehat{U}_2$ is positive everywhere on $[0,1]$. Thus, if K increases, then the graph of $\widehat{U}$ will move downwards everywhere. Since $\widehat{U}$ is decreasing near s^*, a downwards movement of the graph will cause s^* to move to the left in the interval $[0,1]$. In other words, s^* will decrease when K increases. By inspection of formula (24), K is increasing with g^{in} and $T_{\text{se}}^{\text{fo}}$, while it is decreasing with $T_{\text{se}}^{\text{in}}$. Thus, s^* is *decreasing* with g^{in} and $T_{\text{se}}^{\text{fo}}$, and increasing with $T_{\text{se}}^{\text{in}}$.

Meanwhile, $\widehat{U}_1$ is clearly increasing as a function of b^{fo}, and independent of b^{in}. On the other hand, $\widehat{U}_2$ is independent of b^{fo}, but increasing as a function of b^{in} (because η is an increasing function, by hypothesis). Thus, $\widehat{U}$ is *decreasing* as a function of b^{in} (because K is positive by inspection of formula (24)). Thus, if we increase b^{fo}, then the graph of $\widehat{U}$ is will move upwards (and hence, s^* will move to the right), whereas if we increase b^{in}, then the graph of $\widehat{U}$ will move downwards (hence, s^* will move to the left). Thus, s^* is an increasing function of b^{fo}, and a decreasing function of b^{in}. $\square$

Appendix B: Alphabetical index of notation

$\alpha(q)$ Benefit (to the formal buyers) of quality assurance (e.g. warranties, free repair service, etc.)

b^{in} Ratio of buyers in the informal market, relative to population of sellers in *both* markets.

b^{fo} Ratio of buyers in the formal market, relative to population of sellers in *both* markets.

$b = b^{\text{in}} + b^{\text{fo}}$. Overall ratio of buyers to sellers in the whole economy.

$B_f := b^{\text{fo}}/s^{\text{fo}}$. Ratio of buyers to sellers in formal market.

$B_i := b^{\text{in}}/s^{\text{in}}$. Ratio of buyers to sellers in the informal market.

$c_{\text{se}}^{\text{in}}$ Cost of production in the informal market.

$c^{\text{fo}}_{\text{se}}$ Cost of production in the formal market. (Includes quality assurance, but not taxes or regulatory compliance.)

δ Discount factor (in section 4).

η Bargaining strength of informal sellers (in section 3).

g^{in} $:= v^{\text{in}}_{\text{bu}} - c^{\text{in}}_{\text{se}}$. The gains from trade in the informal market.

g^{fo} $:= v^{\text{fo}}_{\text{bu}} - c^{\text{fo}}_{\text{se}}$. The gains from trade in the formal market.

$L^{\text{in}}_{\text{bu}}$ Lump sum costs for informal buyers (e.g. inconvenience).

$L^{\text{fo}}_{\text{bu}}$ Lump sum costs for formal buyers (e.g. transportation and shoe leather costs).

$L^{\text{in}}_{\text{se}}$ Lump sum costs for informal sellers (e.g. crime risk, bribery, protection money, shoe leather).

$L^{\text{fo}}_{\text{se}}$ Lump sum costs for formal sellers (e.g. regulatory compliance, license fees, rent).

L_{bu} "Net" lump sum costs for formal buyers.

L_{se} "Net" lump sum costs for formal sellers.

$P^{\text{in}}_{\text{bu}}$ Match probability for informal buyers.

$P^{\text{fo}}_{\text{bu}}$ Match probability for formal buyers.

$P^{\text{in}}_{\text{se}}$ Match probability for informal sellers.

$P^{\text{fo}}_{\text{se}}$ Match probability for formal sellers.

q Expenditure on quality assurance technology by formal sellers.

$R^{\text{in}}_{\text{se}}$ $= 1 - T^{\text{in}}_{\text{se}}$, the residual earnings rate for informal sellers.

$R^{\text{fo}}_{\text{se}}$ $= 1 - T^{\text{fo}}_{\text{se}}$, the residual earnings rate for formal sellers.

R_{se} $= R^{\text{fo}}_{\text{se}}/R^{\text{in}}_{\text{se}}$, the "net" residual earnings rate for formal sellers.

s^{in} Proportion of sellers in the informal market.

s^{fo} Proportion of sellers in the formal market.

S_i $:= s^{\text{in}}/b^{\text{in}}$. Ratio of sellers to buyers in the informal market.

t Time (in dynamical interpretation of model).

$T^{\text{in}}_{\text{se}}$ Expected costs of monetary crime for informal sellers.

$T^{\text{fo}}_{\text{se}}$ Taxes and unit regulatory costs for formal sellers.

T_{ax} "Net" tax burden for formal sellers.

$u_{\text{bu}}^{\text{in}}$ Utility of a purchase for informal buyers.

$u_{\text{bu}}^{\text{fo}}$ Utility of a purchase for formal buyers.

$u_{\text{se}}^{\text{in}}$ Utility of a sale for informal sellers.

$u_{\text{se}}^{\text{fo}}$ Utility of a sale for formal sellers.

$U_{\text{bu}}^{\text{in}}$ $= P_{\text{bu}}^{\text{in}} u_{\text{bu}}^{\text{in}}$, the expected utility of informal buyers.

$U_{\text{bu}}^{\text{fo}}$ $= P_{\text{bu}}^{\text{fo}} u_{\text{bu}}^{\text{fo}}$, the expected utility of formal buyers.

$U_{\text{se}}^{\text{in}}$ $= P_{\text{se}}^{\text{in}} u_{\text{se}}^{\text{in}}$, the expected utility of informal sellers.

$U_{\text{se}}^{\text{fo}}$ $= P_{\text{se}}^{\text{fo}} u_{\text{se}}^{\text{fo}}$, the expected utility of formal sellers.

$v_{\text{bu}}^{\text{in}}$ Value of merchandise to informal buyer.

$v_{\text{bu}}^{\text{fo}}$ Value of merchandise to formal buyer.