MPRA
Munich Personal RePEc Archive

Mixed oligopoly with consumer-friendly public firms

Roy Chowdhury, Prabal

Indian Statistical Institute, Delhi Center

July 2007

Online at https://mpra.ub.uni-muenchen.de/4255/
MPRA Paper No. 4255, posted 26 Jul 2007 UTC

Mixed Oligopoly with Consumer-friendly Public Firms

Prabal Roy Chowdhury
Indian Statistical Institute - Delhi Centre

July 26, 2007

Abstract

We consider a mixed oligopoly with a public firm that maximizes the sum of its own profits and consumers' surplus. We characterize the unique pure strategy equilibrium and show that as long as the cost function is not "too concave", privatization reduces welfare. We find that while the first best cannot be implemented using a tax/subsidy policy that is the same for all firms, a budget-balancing policy that involves a tax on the public firm, coupled with subsidies to the private firms, can do so. Further, the optimal tax/subsidy policy is critically dependent on whether there is privatization or not.

JEL Classification: L1, L2, L3.
Keywords: Mixed oligopoly; public firms; subsidy; tax; irrelevance principle; privatization.

‡ Indian Statistical Institute, Delhi Centre, 7 S.J.S. Sansanwal Marg, New Delhi 110016, India.
Phone: +91-11-4149 3930.
Fax: +91-11-4149. 3981
E-mail: prabalrc@isid.ac.in

1 Introduction

One of the major debates in the transitional economies, e.g. India, concerns the privatization of public firms. Clearly, any informed debate needs a framework that incorporates some of the ground realities in transitional economies. In this paper we make a beginning in developing such a framework. As our analysis shows, many of the results in the literature are sensitive to whether these realities are modeled or not.

While there is a growing and large literature on mixed oligopoly that examines the interaction between private and public firms, this literature generally assumes that the public firms maximize social welfare. In transitional economies it may, however, be more reasonable to assume that the public firms are *consumer-friendly,* i.e. they maximize the sum of their own profits and consumers' surplus. Given that many such transitional economies have a long history of public firms being guided by socialist ideals, and the suspicion with which the private sector is viewed under socialism, it seems reasonable to assume that public firms in such economies put very little weight on private sector profits. Even though some of these economies are moving away from their socialist past, the suspicion of the private sector remains.[1]

Further, we allow for the fact that public firms in transitional economies often have soft budget constraints. Thus loss-making public firms are often bailed out with cheap credit, or even direct subsidy. In the Indian context, for example, many public firms are loss-making, sometimes chronically so.[2] This is formalized by assuming that the public firm does not have to meet

[1] In India, for example, privatization of public sector units is facing rough weather. The main argument being that, apart from their own profits, public firms also care for the consumers.

[2] In the Indian context, Fertiliser Corporation was the largest loss-making CPSU (central public sector unit) with a loss of Rs. 1210 crore in 2004-05. The top ten loss-making CPSUs had an aggregate loss of Rs. 6060 crore (Public Enterprises Survey, 2004-2005). In fact, of the 292 CPSUs, about 92 are *chronically* loss-making.

any budget constraint.

We consider a mixed oligopoly with one consumer-friendly public firm and n private firms. We demonstrate that there is a unique pure strategy equilibrium. Further, as long as the cost function is not 'too concave' (to be precise, marginal cost is convex, or linear), privatization reduces welfare, i.e. the welfare level under mixed oligopoly is *higher* compared to that under privatization (i.e. Cournot competition). In particular, this is true for quadratic cost functions. This is interesting since, in a framework with *welfare maximizing* public firms and quadratic costs, De Fraja and Delbono (1989) find that the mixed oligopoly welfare level may be *lower* compared to Cournot competition.

Given that the public firms are consumer-friendly, we than turn to analyzing the consumers' surplus. We find that it is higher compared to that under Cournot competition. Further, the consumers' surplus under mixed oligopoly is greater (respectively less) than the first best level, if the marginal cost function is concave (respectively convex).

The existing mixed oligopoly literature with welfare maximizing public firms finds that a non-discriminating subsidy (which is same across all firms) implements the first best irrespective of whether there is privatization or not. Further, the level of subsidy is the same in both cases. Contributions in this field include, among others, White (1996), Poyago-Theotoky (2001), Myles (2002), Fjell and Heywood (2004) and Kato and Tomaru (2007). This set of results is sometimes referred to as the *irrelevance principle*, since it suggests that privatization of public firms is irrelevant if there is a welfare maximizing tax/subsidy policy in place.

We re-examine the irrelevance principle in a framework with consumer-friendly public firms. To this end we examine if the first best outcome can be implemented through an appropriate tax/subsidy policy. In case the policy is non-discriminating, we find that the first best cannot be implemented. Interestingly, however, the first best can be implemented by levying an ap-

propriate per unit *tax* on the public firm, and providing a per unit *subsidy* to the private firms. These results are in contrast to those under privatization (i.e. Cournot competition) where it is well known that a non-discriminating subsidy can implement the first best.[3] Thus our results suggest that in the presence of consumer-friendly public firms the optimal tax/subsidy policy is critically dependent on whether there is privatization or not, i.e. the irrelevance principle *does not* hold.

In the next section we set up the model and solve for the unique equilibrium. We then compare the welfare level with that under Cournot competition. In section 3, we examine the optimal tax/subsidy policy. Section 4 concludes.

2 The Model

There are n private firms and one consumer-friendly public firm all producing a homogeneous good. The private firms are profit maximizers, whereas the public firm maximizes the sum of its own profits and consumers' surplus.[4] We examine a mixed oligopoly, i.e. a simultaneous move quantity-setting game where every firm maximizes its own objective.

The output of the i-th private firm is denoted q_i and that of the public firm is q_0, so that $Q = q_0 + \sum_1^n q_i$. The inverse demand function is $f(Q)$. Firms are symmetric with the cost function of all firms, including the public firm, being $c(q)$.

Assumption 1. (i) The inverse demand $f : (0, \infty) \to [0, \infty)$ and $\exists \hat{Q},\ 0 < \hat{Q} < \infty$, such that $f(Q) > 0$ if $0 \leq Q < \hat{Q}$, and $f(Q) = 0$ if $Q \geq \hat{Q}$. Further, $f(Q)$ is twice differentiable, decreasing and (weakly) concave, i.e. $f'(Q) < 0$ and $f''(Q) \leq 0$, for all Q such that $\hat{Q} > Q > 0$.

[3] See, e.g. the mixed oligopoly literature with welfare maximizing public firms.

[4] Matsumura (1998) examines a mixed duopoly where the public firm maximizes a weighted sum of its own profits and social welfare. In such a scenario he examines the issue of partial privatization.

(ii) The cost function $c : [0, \infty) \to [0, \infty)$ is twice differentiable, increasing and convex, i.e. $c'(q_i) > 0$ and $c''(q_i) > 0, \forall q_i \geq 0$. Also, $c(0) = c'(0) = 0$.

The assumption that $f''(Q) \leq 0$ implies that the second order conditions for the private firms are satisfied. Assumption 2 below ensures that the corresponding condition holds for the public firm.

Assumption 2. $f'(Q) - (Q - q)f''(Q) - c''(q) < 0, \forall Q \geq q > 0$.

One sufficient condition (given that $f''(Q) \leq 0$), for A2 to hold is that the marginal demand function, $f'(Q)$, is elastic, i.e. $\frac{f'(Q)/Q}{f''(Q)} \geq 1$. A2 also holds if the demand function is linear.

The profit function of the i-th firm is given by

$$\pi_i = q_i f(Q) - c(q_i), \; i = 0, 1, \cdots, n. \tag{1}$$

The payoff of the public firm is the sum of its own profit and the consumers' surplus. Thus its payoff

$$P = q_0 f(Q) - c(q_0) + \int_0^Q f(z)dz - f(Q)Q. \tag{2}$$

We then define the first best outcome, i.e. the output vector maximizing social welfare. Clearly, it involves every firm producing q^*, where q^* solves

$$f((n+1)q^*) = c'(q^*). \tag{3}$$

At this output vector price equals marginal cost for every firm. Given A1, q^* is well defined. Let $Q^* = (n+1)q^*$.

Before proceeding further let us solve for the benchmark case of Cournot competition with private firms. This is of particular interest since Cournot competition can be interpreted as arising out of privatizing the public firm.

Cournot competition: There are $n+1$ profit-maximizing firms who compete over quantity. Clearly, the first order conditions (FOCs) involve

$$f(Q) + q_i f'(Q) = c'(q_i). \tag{4}$$

It is standard to show that there is a unique Cournot equilibrium[5] where all firms produce an output level of q_C (> 0) where q_C is the unique solution to $f((n+1)q) + qf'((n+1)q) = c'(q)$. Let $Q_c = (n+1)q_c$. Clearly, $Q_c < Q^*$.

Mixed oligopoly: Let the equilibrium output vector be denoted by $(q'_0, q'_1, \cdots, q'_n)$, with the aggregate output being denoted by Q'.

We first argue that in any equilibrium all private firms have the same output. Suppose to the contrary, $q'_i > q'_j \geq 0$ for some $i \neq j$. Since $q'_i > 0$, $q'_i f'(Q') + f(Q') - c'(q'_i) = 0$. Hence $q'_j f'(Q') + f(Q') - c'(q'_j) > q'_i f'(Q') + f(Q') - c'(q'_i) = 0$, where the first inequality follows from the fact that $q'_i > q'_j$, $f'(Q) < 0$ and $c''(q) > 0$. Thus the j-th firm has an incentive to increase its output, contradiction. Hence $q'_i = q'$ for all $i \geq 1$.

Further, in equilibrium $q' > 0$. Suppose to the contrary $q' = 0$, so that $Q' = q'_0$. Then, from the public firm's first order condition, the equilibrium q'_0 solves $f(q'_0) = c'(q'_0) > 0$. Since $c'(0) = 0$, for q_i small, $q_i f'(q'_0) + f(q'_0) - c'(q_i) > 0$, so that the i-th private firm has an incentive to increase its output.

Consequently, the first order condition of the private firms is given by

$$f(q_0 + nq) + qf'(q_0 + nq) = c'(q). \tag{5}$$

Let $q_0 = g(q)$, where $g(q)$ solves (5) when a solution exists,[6] otherwise let $g(q) = 0$. From A1, $g(q)$ is clearly decreasing in q. Further, $g(0) = \hat{Q} > 0$ (since $c'(0) = 0$) and there exists $\tilde{q}$ such that $g(q) = 0$ $\forall q \geq \tilde{q}$.[7]

Next consider the first order condition of the public firm:

$$f(Q) + q_0 f'(Q) - c'(q_0) - Qf'(Q) = 0. \tag{6}$$

Using the first order conditions of the private firms and the fact that the

[5] See, e.g. Kolstad and Mathiesen (1987).

[6] From A1, if equation (5) has a solution for q_0, it is unique.

[7] Such a $\tilde{q}$ exists since, for $q = \hat{Q}/n$, the LHS of (5) is less than the RHS, $\forall q_0$.

output level of all firms are symmetric, the above can be re-written as:[8]

$$c'(q) - c'(q_0) - (n+1)qf'(Q) = 0. \tag{7}$$

From (7) we can write $q_0 = h(q)$. From A1, $h(q)$ is well defined and increasing in q. Further, $h(0) = 0$ (since $c'(0) = 0$) and $h(q) > 0$ for all q sufficiently large. Thus, $g(q)$ and $h(q)$ have a unique intersection (see Figure 1). Further, from equation (7), $q_0' > q'$.

Summarizing the above discussion we have

Proposition 1 *The mixed oligopoly game has a unique equilibrium where all private firms produce the same output, say q'. Further, the output level of the public firm exceeds that of the private firms, i.e. q'.*

2.1 Welfare Analysis

To begin with note that while the first best outcome is symmetric, the mixed oligopoly one is not. This immediately implies that the mixed oligopoly outcome is sub-optimal.

We then examine the welfare effects of privatization, i.e. compare the welfare under a mixed oligopoly with that under Cournot competition. We start by comparing the consumers' surplus (i.e. the aggregate output) under the two regimes. Given that the public firms focus on consumers' surplus (apart from own profits of course), this is also of independent interest. Clearly, the Cournot equilibrium is defined by the intersection of $g(q)$ with the 45 degree line (see figure 1). Next recall that the mixed oligopoly involves $q_0' > q'$, so that it lies above the 45 degree line. Hence, from figure 1, $q_0' > q_C > q'$. Next, totally differentiating (5)

$$\frac{dQ}{dq} = \frac{dq_0}{dq} + n = \frac{c''(q) - f'(Q)}{qf''(Q) + f'(Q)} < 0. \tag{8}$$

[8]Note that for q_0 small, the LHS of equation (6) is strictly positive. Thus any equilibrium must involve a positive output for the public firm.

Since $q' < q_C$, from (8) it follows that the aggregate output level is higher under a mixed oligopoly. Given that the public firm's objective function includes consumers' surplus, this is quite intuitive. This immediately implies that the consumers' surplus is higher compared to that under Cournot competition.

Lemma 1 *The aggregate output, and consequently consumers' surplus, under mixed oligopoly exceed that under Cournot competition.*

As an intermediate step to comparing the welfare levels under mixed oligopoly and Cournot competition, we next compare the aggregate output (and hence consumers' surplus) under mixed oligopoly and the first best. We find that the result depends on the whether the marginal cost function is convex, or concave.

Lemma 2 *The aggregate output (and consequently consumers' surplus) under mixed oligopoly is greater (respectively lower) than the first best level if the marginal cost, $c'(q)$, is concave (respectively convex). The two are equal if the marginal cost function is linear.*

Proof. Multiplying equation (5) by n, and adding (6) we have that

$$(n+1)f(Q') - nc'(q') - c'(q'_0) = 0. \tag{9}$$

If $c'(q)$ is convex, then $nc'(q') + c'(q'_0) > (n+1)c'(\frac{Q'}{n+1})$. Hence, from (9),

$$(n+1)f(Q') - (n+1)c'(\frac{Q'}{n+1}) > 0. \tag{10}$$

Next recall that the first best solves

$$f(Q^*) - c'(\frac{Q^*}{n+1}) = 0. \tag{11}$$

Given that $f(Q) - c'(\frac{Q}{n+1})$ is decreasing in Q, from (10) and (11), $Q' < Q^*$. Analogous arguments go through if $c'(q)$ is concave, or linear. ■

Suppose the marginal cost function is either convex, or linear. Then, from Lemma 1 and 2, $Q_c < Q' \leq Q^*$. Given that the first best must be symmetric, the welfare under symmetry can be written as $\int_0^Q f(z)dz - (n+1)c(\frac{Q}{n+1})$, which is concave in Q. Hence welfare is increasing in Q till Q^*. Since $Q_c < Q' \leq Q^*$, this implies that the welfare level is higher under a mixed oligopoly compared to Cournot competition.

Proposition 2 *If the marginal cost function is either convex or linear, then privatization reduces welfare, i.e. welfare under mixed oligopoly exceeds that under Cournot competition.*

Note that Proposition 2 assumes that the cost function should not be "too concave." The intuition behind this restriction is as follows. Under a mixed oligopoly the public firm over-produces compared to Cournot competition with private firms. If, to the contrary, the cost function is "too concave", then there is an incentive to produce too much, even compared to the first best, leading to welfare loss. Observe that Proposition 2 allows for linear marginal cost, i.e. quadratic cost functions. As discussed in the introduction, this result is in contrast to De Fraja and Delbono (1989).

3 Implementing the First Best

We then examine if the first best outcome can be implemented using some optimal subsidy or tax policy.

We call a tax/subsidy schedule to be *non-discriminating* if the same policy is applied to all firms. We first argue that the first best cannot be implemented using such a non-discriminating tax/subsidy schedule.

Proposition 3 *There exists no non-discriminating tax/subsidy policy that can implement the first best.*

Proof. Suppose to the contrary there exists some tax/subsidy policy $s(q)$, such that all firms produce at q^*. Then the FOC of the private firms

involve $f(Q^*) + q^* f'(Q^*) - c'(q^*) + s(q^*) = 0$. Hence, the FOC of the public firm is $-Q^* f'(Q^*) = 0$, which is a contradiction (since $0 < Q^* < \hat{Q}$). ■

Given Proposition 3, it is natural to ask if there is any tax/subsidy policy that can implement the first best. Interestingly, we find that the first best can be implemented through a constant *tax* on the public firm, coupled with a subsidy on the private firms. Further, the optimal policy is *budget balancing.*

Proposition 4 *A constant per unit tax* $t = -(n+1)q^* f'(Q^*)$ *on the public firm, coupled with a constant per unit subsidy of* $s = -q^* f'(Q^*)$ *on the private firms, implements the first best and is budget-balancing.*

Proof. We can mimic the argument in Proposition 1 to show that a unique equilibrium exists. It then remains to show that given that all other firms are producing q^*, it is optimal for every firm to do so. The FOC of the i-th private firm involves

$$f(q_i + nq^*) + q_i f'(q_i + nq^*) - c'(q_i) + s = 0. \tag{12}$$

Clearly, given that $s = -q^* f'(Q^*)$ and (3), $q_i = q^*$ solves the above equation. Next consider the public firm. The first order condition involves

$$c'(q_0) + t - c'(q^*) + (n+1)q^* f'(q_0 + nq^*) = 0. \tag{13}$$

Clearly, $q_0 = q^*$ satisfies the above equation.

Finally, this policy is budget balancing since $t = (n+1)s$. ■

It is well known that under privatization (i.e. Cournot competition) a non-discriminating subsidy can implement the first best (see, among others, Kato and Tomaru (2007)). Thus Propositions 3 and 4, in conjunction with this fact, demonstrate that the nature of the optimal tax/subsidy policy is critically dependent on whether there is privatization or not, i.e. the *irrelevance principle* does not hold.

4 Conclusion

We consider a mixed oligopoly where the public firm maximizes the sum of its own profits and consumers' surplus. Apart from purely theoretical interest,[9] this is of relevance to many transitional economies. We characterize the unique equilibrium, and derive some interesting welfare properties. As long as the cost function is not 'too concave', we find that privatization reduces welfare. Further, we solve for the optimal tax/subsidy policy. Our results suggest that the optimal tax/subsidy policy is critically dependent on whether there is privatization or not.

Thus, in general, many of the results in the mixed oligopoly literature, e.g. the irrelevance principle, seem to depend on how the public firm is modeled, i.e. whether it is welfare maximizing, or consumer-friendly. From a policy perspective, this suggests that one needs to be careful while applying policy prescriptions drawn from the mixed oligopoly literature with welfare-maximizing firms to transitional economies.

[9] De Fraja and Delbono (1989), for example, mention that they do not allow for any agency problem. Our paper can be interpreted as a contribution in that direction, in the sense that we allow public managers to have different objectives from public authority.

5 References

De Fraja, G., Delbono, F., 1989. Alternative strategies of a public enterprise in oligopoly. Oxford Economic Papers 41, 302-311.

Fjell, K., Heywood, J. S., 2004. Mixed oligopoly, subsidization and the order of firm's moves: the relevance of privatization. Economics Letters 83, 411-416.

Kato, K., Tomaru, T., 2007. Mixed oligopoly, privatization, subsidization, and the order of firms' moves: several types of objectives. Economics Letters 96, 287-292.

Kolstad, C., Mathiesen, L., 1987. Necessary and sufficient conditions for uniqueness of a Cournot equilibrium. Review of Economic Studies 54, 681-690.

Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 70, 473-483.

Myles, G., 2002. Mixed oligopoly, subsidization and the order of firm's moves: an irrelevance result for the general case. Economics Bulletin 12, 1-6.

Poyago-Theotoky, J., 2001. Mixed oligopoly, subsidization and the order of firm's moves: an irrelevance result. Economics Bulletin 12, 1-5.

Public Enterprises Survey, 2004-2005. Ministry of Heavy Industries and Public Enterprises, Government of India.

White, M., 1996. Mixed oligopoly, privatization and subsidization. Economics Letters 53, 189-195.

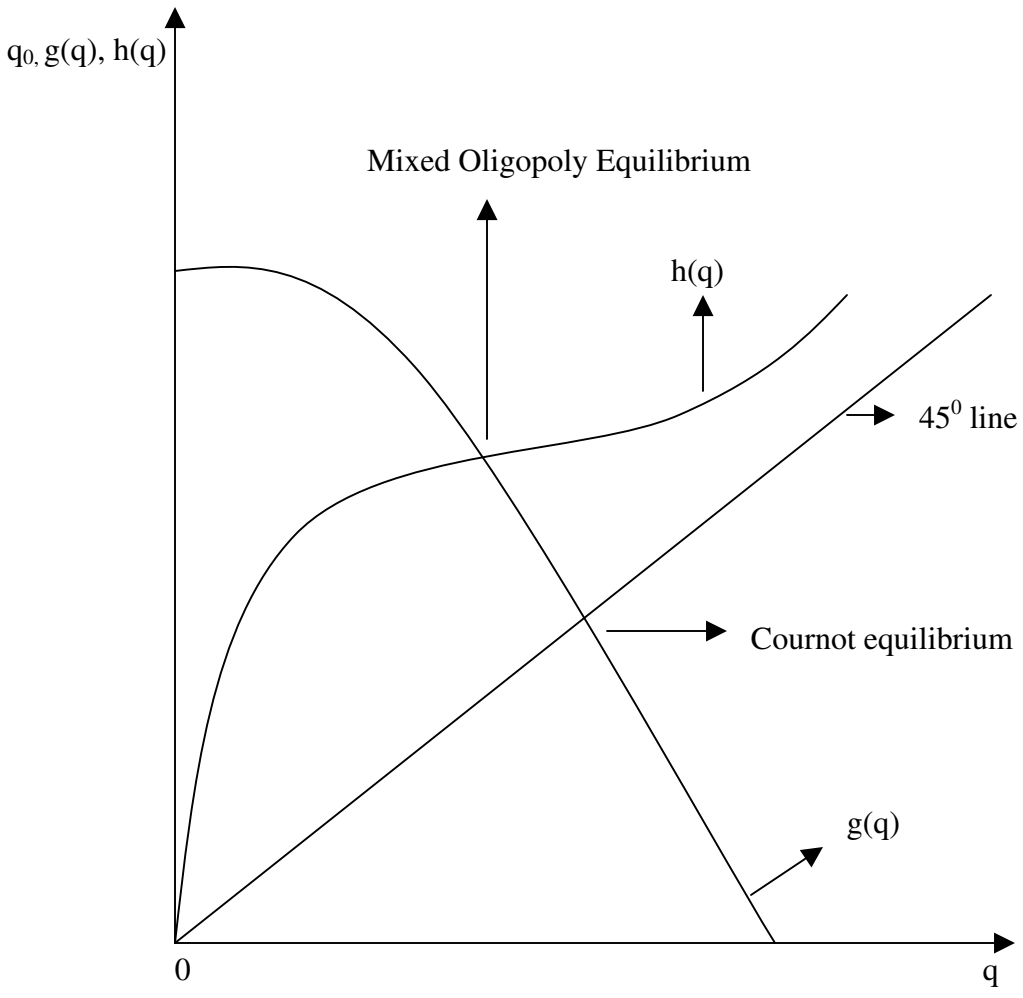

Figure 1: Mixed Oligopoly and Cournot Competition