Deceptive advertising with rational buyers

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Deceptive Advertising with Rational Buyers*

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Abstract

We study a Bertrand game where two sellers supplying products of different and unverifiable qualities can outwit potential clients through their (costly) deceptive advertising. We characterize a class of pooling equilibria where sellers post the same price regardless of their quality and low quality ones deceive buyers. Although in these equilibria low quality goods are purchased with positive probability, the buyer (expected) utility can be higher than in a fully separating equilibrium. It is also argued that low quality sellers invest more in deceptive advertising the better is their reputation *vis-à-vis* potential clients — i.e., firms that are better trusted by customers, have greater incentives to invest in deceptive advertising when they produce a low quality product. Finally, we characterize the optimal monitoring effort exerted by a regulatory agency who seeks to identify and punish deceptive practices. When the objective of this agency is to maximize consumer surplus, its monitoring effort is larger than under social welfare maximization.

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1 Introduction

The importance of advertising as a competitive tool has long been recognized — see, e.g., Bagwell (2007) among others. Firms usually engage in advertising campaigns to inform potential clients about the existence of their brand and products. Hence, investments in quality promotion and aggressive marketing strategies, often involving comparative claims, allow firms to compete more fiercely by stealing customers one another, triggering price wars that push prices down toward the competitive level. This is particularly so in industries where products are vertically differentiated and firms offering homogeneous goods compete for customers based on quality — e.g., service, optionals, special features, etc.

When the quality of an item on sale can be verified before purchase (the so called *search* goods), these complementary sales channels are unlikely to harm consumers insofar as they reduce search costs and allow to better match tastes with consumption choices — see, e.g., Lewis and Sappington (1994) and Johnson and Myatt (2006) among others. But, very often, product quality is not fully verifiable in real life — e.g., *credence goods.* In these cases advertising might have a dark side, namely low quality firms may use marketing channels to pull the wool over consumers' eyes and induce them into purchases they would have otherwise not undertaken. These practices are known as *misleading* or *deceptive* advertising, which is loosely[1] defined as any explicit (or even implicit) statement that has the *potential* to deceive consumers, meaning that its embedded claim may not realize after the purchase.

This potential danger has been well recognized by antitrust and competition policy authorities all over the world — see, e.g., the FTC Policy Statement on Deception (1982).[2] Essentially, when firms can safely engage in deceptive advertising, the civil law principle stating that customers have a "right to know" what they are purchasing is likely to be violated. In addition, policy makers have grown wary of potential market distortions of deceptive practices as they tend to induce unfair competition.[3] As a result, misleading advertising is nowadays generally sanctioned according to country specific regulations designed to protect consumers and foster competition. The major challenge implementing such policies, however, is that deceptive advertising might be indistinguishable from truthful one.

A large debate among legal experts and policymakers, both at national and international level has emphasized that one key step would be the identification of successful strategies allowing to spot and sanction unlawful advertising practices. In the US, for instance, the FTC regulates unfair and deceptive practices on a case-by-case basis and occasionally with industry-wide regulations. Industry standards

[1]The actual statute defines false advertising as a "means of advertisement other than labeling, which is misleading in a material respect; and in determining whether an advertisement is misleading, there shall be taken into account (among other things) not only representations made or suggested by statement, word, design, device, sound, or any combination thereof, but also the extent to which the advertisement fails to reveal facts material in the light of such representations or material with respect to consequences which may result from the use of the commodity to which the advertisement relates under the conditions prescribed in said advertisement, or under such conditions as are customary or usual." (Wilson, Lee. "The Advertising Law Guide: A Friendly Desktop Reference for Advertising Professionals". Allworth Press, NY, 2000).

[2]Available at: `http://www.ftc.gov/bcp/policystmt/ad-decept.htm`

[3]In fact, while consumer protection laws are formally distinct from anti-trust law, the intimate connection between the two is reflected by the fact that in most countries the authority on customer protection and anti-trust is held by a single Agency.

are also adopted by individual firms, by advertisers' associations, and often by the media — such as newspapers, magazines, radio, television and the Internet. Publishers and broadcasters, for example, have realized that dishonest advertising reflects unfavorably on them as well as the businesses doing the advertising. As a consequence, the National Advertising Division has established a National Advertising Review Board comprised of advertisers, advertising agencies, and the general public to deal with complaints. When the Board receives a complaint from a competing business or consumer, it examines the complaint. If the ad is deemed deceptive, the board puts pressure on advertisers through persuasion, publicity, or, in extreme cases, legal action.

The analysis we offer in this paper is a contribution to this debate. The research questions that will be addressed are the following: Why do firms engage in deceptive advertising? What are the cost-benefit trade-offs that shape misleading advertising decisions? What are their key determinants and welfare effects?

To address these issues we consider a simple environment where two sellers (firms) compete à la Bertrand to attract a representative buyer (consumer). Sellers produce vertically differentiated products whose quality is not observable by the buyer and unverifiable in court. Qualities are (perfectly) negatively correlated, and a low quality seller may choose to deceive the buyer so as to induce him into a wrong purchase. For given price, the buyer's objective is to minimize the danger of a wrong purchase. He knows that one of the two sellers supplies a low quality good and hence uses all the available information to make inference about sellers' qualities.

Within this setting, we first note that equilibria where sellers deceive the buyer are incompatible with separating behavior at the pricing stage, as otherwise the information about qualities would be fully reflected by the equilibrium prices and there would be no scope for deception. Then, we characterize a class of pooling equilibria where sellers post the same price regardless of their type and low quality ones deceive the buyer with some (endogenous) probability. Interestingly, in these equilibria sellers invest more in deceptive advertising the better is their (exogenous) reputation — i.e., the more a firm is trusted by its potential clients, the greater are its incentives to invest in deceptive advertising when it produces a low quality product. Surprisingly, though, even if low quality goods are purchased with positive probability in such equilibria, the *ex-ante* welfare of the buyer can be higher than in a fully separating equilibrium, where the high quality item is purchased with certainty. This is due to a novel pro-competitive effect, which rests upon the buyer's ignorance about sellers' product quality in a pooling outcome: since the low quality seller can induce the buyer to purchase its item through deceptive advertising, the high quality seller has less market power than in a separating equilibrium. As a consequence, pooling prices might be lower than separating ones. This is particularly so if the difference between the two quality levels is large enough, in which case the beneficial effect of ignorance on the high quality seller monopoly power is magnified.

Interestingly, this prediction, is very different than the findings of Heidhues, Kőszegi, and Murooka (2012) who also analyze market for deceptive products but in a context with naive consumers. In contrast to them, our deceptive equilibria are pro-competitive *vis-à-vis* the corresponding no deception market outcome: in our model the possibility of deception allows low quality sellers to stay on the market alongside high quality ones and this generates a downward pressure on prices. Therefore, our analysis highlights a novel tension between the goals of enhancing competition and that of protecting the quality of market public information. This suggests that authorities seeking to maximize consumer surplus may have conflicting statutory objectives: increasing competition may lead worsen transparency *vis-à-vis* final buyers (and vice-versa).

Nevertheless, from an efficiency point of view — i.e., total welfare maximization — we find that pooling equilibria are unambiguously worse than separating ones. This is because the beneficial effect on the buyer utility stemming from a reduced market power of the high quality sellers washes out when aggregating the players' (expected) utilities/profits. And, pooling equilibria are inefficient as they involve socially wasteful misleading advertising. Finally, with no surprise, low quality sellers unambiguously gain from coordinating on a pooling equilibrium, whereas high quality ones prefer separating ones.

Building on these results, we then analyze a simple policy that involves the enforcement of a (costly) monitoring technology aimed at detecting misleading and deceptive conducts. More precisely, we assume that a benevolent planner (e.g., a regulatory agency) commits to check the truthfulness of sellers' ads randomly, and if a lie is discovered a mandated full reimbursement of the price paid by the cheated buyer is enforced. In this setting, we find that a planner concerned with total welfare maximization prefers more intense monitoring than one that cares about customers only. This is because misleading advertising is decreasing with respect to monitoring intensity, and the buyer might enjoy to be cheated and then reimbursed rather than paying a positive price for a high quality item. We also show that, thanks to this effect, the impact of an increase of the pooling price on the optimal monitoring intensity is non-monotone, suggesting the counter intuitive conclusion that in response to higher prices, consumers may benefit from lower protection.

The rest of the paper is organized as follows. Section 2 relates our work to the growing literature on deceptive practices. Section 3 lies down the model. Section 4 describes the properties of equilibria with "full coverage" — i.e., when the ads reach the buyer with probability 1, and provides the characterization analysis of pooling and separating equilibria. Section 5 addresses selection and welfare issues and performs the relevant comparative statics analysis. In Section 6 we perform the policy analysis and discuss the behavior of a benevolent social planner that invests in a detection technology to identify and sanction deceiving firms. Section 7 concludes. All proofs are in the appendix.

2 Related literature

Our paper shares common features with the growing literature on deceptive practices. A seminal contribution to this debate is that of Anderson and Renault (2009) who study a model where competing firms disclose horizontally differentiated attributes (valued differently by heterogeneous consumers), assuming that product qualities are known.[4] In their paper product qualities are a device to indicate large or

[4]See also Anderson and Renault (2006) for a similar disclosure approach with a monopolistic firm.

small firms in terms of their equilibrium market shares with a strong/high quality firm having a larger share than a weak/low quality firm. If market sizes are very different (product qualities are sufficiently different), the equilibrium of the disclosure game has only the weak firm disclosing horizontal attributes and the strong one not. If comparative advertising is allowed, then the weak firm will disclose the horizontal attributes of both products (and so it is truly comparative). The key difference between our model and theirs is that we focus on deceptive practices while in their model information disclosure is always truthful.

Barigozzi, Garella, and Peitz (2009), instead, study an entry game where the entrant's quality is uncertain and comparative advertising — i.e., comparing quality of one's own product to that of a rival's — empowers the latter to file for court intervention if it believes the comparison to be false or misleading. They show that comparative advertising can be a credible signal of high quality in instances where generic advertising is also viable. In contrast to them, in our model there is no way to credibly signal high quality because firms cannot file for court intervention — i.e., we do not allow for explicit comparative claims. Moreover, in our model the quality of both firms is unknown, while in their set up the incumbent quality is common knowledge. These differences allow us to analyze the welfare implications of deceptive advertising from a more symmetric perspective relative to that of the entry perspective taken in their paper. Finally, while in their paper the entrant can signal its quality neither with price alone nor with price and generic advertising in our model buyers also rely on prices when updating their beliefs.

Hattori and Higashida (2012) study the welfare effects of persuasive advertising in a duopoly model where horizontally differentiated firms that compete by setting prices, engage in deceptive advertising. They characterize the welfare effects of alternative forms of regulatory policies, by emphasizing (among other things) the positive effects of market interventions aimed at educating consumers. By contrast, our objective is to study a context where customers are Bayes rational and the scope for deceptive advertising has a direct impact on prices through the informative content of the ads sent by firms to potential clients.

Heidhues, Kőszegi, and Murooka (2012) studies conditions facilitating profitable deception in a competitive market for potentially deceptive products. Firms selling homogenous products simultaneously set a transparent "up-front price" and an "additional price", and decide whether to shroud the additional price from naive consumers. The major difference between our models is that our consumers are rational while theirs are naive. As a result they find that deceptive equilibria where firms shroud prices have anti-competitive features whereby firms want rivals to obtain sufficient rents from deception so as to resist the temptation of unshrouding and breaking the equilibrium engaging in full price competition. To the contrary, our deceptive equilibria are pro-competitive *vis-à-vis* the corresponding no deception market outcome: in our model the possibility of deception allows low quality producers to stay on the market alongside high quality ones and this generates a downward pressure on prices.

The work on *de-marketing* by Miklós-Thal and Zhang (2013) borrows attribution theory from psychology and builds a model where sellers may prefer to under-advertise their high quality product in equilibrium. By so doing, sellers "confuse" potential future buyers as to the true cause of possibly low sales, which may be attributed to the low profile marketing choice (observed) rather than to the low quality of the item on sale (unobserved). Our work shares common features with Miklós-Thal and Zhang (2013) in that buyers are Bayes-rational and discount information in a similar fashion. However, a fundamental difference between our and their model regards the way advertising affects preferences: while in their analysis advertising increases demand by arousing potential buyers' curiosity — a reduced form of persuasive advertising — in our setting advertising persuades customers only through changes in probabilities leaving unaltered their attitude towards learning an item's quality — i.e., demand is enhanced only through subjective probability appraisal.

Wang (2011) builds a model where a monopolist offers information about its product features and then buyers may search for additional product information in order to better evaluate it before purchasing. He focuses on the amount of information offered through ads and searched by consumers showing that the equilibrium price is non linear in the search cost and that information disclosure and acquisition strategies may or may not coexist. The equilibrium price non linearity follows from the fact that, for medium search costs, the monopolist hides information and lowers strategically the price to reduce the incentive for the buyer to search additional information. In so doing he maximizes the surplus he can extract from buyers. The pattern is reverted when search costs are too low or too high and the price looses the strategic effect just described. In this model advertising can be not fully informative but it cannot be misleading, which is instead a key feature of our analysis.

Of course, stemming from Chamberlin (1933) a large body of IO literature has cast advertising choices within the traditional oligopoly framework — see, e.g., Bagwell (2007) for an excellent survey. These models highlight a number of important features of advertising choices, such as their informative[5] and persuasive[6] role, the link between advertising and market structure[7] and welfare properties, but they are all rather silent on the potential danger of deceptive practices, which is instead the novel aspect emphasized in our analysis.

Finally, at a more abstract level, it is worth mentioning that our analysis also relates to the mechanism design literature on informed principals. Martimort and Moreira (2010) is the first paper to analyze an abstract common agency game where two privately informed principals deal with a common uninformed agent. In a more complex environment, they show that when two principals directly communicate with a (common) agent, both separating and pooling outcomes can emerge. Our equilibrium characterization is somewhat coherent with this contribution even if in our setting sellers are able to communicate with the buyer only through ads — i.e., there is no *direct* communication — and lies (deceptive ads) are costly — i.e., they may fail to reach the target. A further key difference between our set-up Martimort and Moreira (2010) is that we only assume linear prices while they allow for fully non-linear contracts. This

[5]See, e.g., Bagwell and Riordan (1991), Grossman and Shapiro (1984), Orzach, Overgaard, and Tauman (2002) and Wernerfelt (1994) for more on informative advertising.

[6]See, e.g., Banerjee and Bandyopadhyay (2003), Becker and Murphy (1993) and Dixit and Norman (1978).

[7]See, e.g., Bagwell and Ramey (1988), Bagwell and Ramey (1991), Chu (1992) and Galeotti and Moraga-González (2008).

restriction is compelling given the scope of our application and allows to derive simple policy predictions that would be otherwise difficult to obtain.

3 The model

Players. Two sellers supply identical products with different qualities to a single (representative) buyer. Sellers (indexed by $i = 1, 2$) compete by setting prices and each supplies a product of quality $q_i \in \{l, h\}$, where l stays for low and h for high. The buyer purchases one unit of product from either seller. The utility from consuming a item of quality q_i is $\theta(q_i)$ — i.e., the utility of consuming a low (resp. high) quality item is $\theta(l) = \theta_l$ (resp. $\theta(h) = \theta_h$), where $\theta_h - \theta_l = 1$ with no loss of insights. His net utility from consumption is

$$u(\theta(q_i), p_i) = \theta(q_i) - p_i,$$

when buying from seller i supplying quality q_i at a price p_i. No consumption entails zero utility.[8]

Communication. Prices are observed before purchase, while product qualities are advertised through informative ads. We assume that, whenever the quality of a good is *ex-post* different from the one advertised *ex-ante*, it is impossible to claim the difference — i.e., sellers supply *credence goods* (see, e.g., Leland (1979), Cabral (2000) and Wernerfelt (1988) among many others). Hence, there is scope for deceptive advertising.

Each seller advertises its product by sending a number of ads stating that it is of high quality. The higher the number of ads, the larger the probability they reach the buyer. We define the probability of reaching the buyer as 'coverage'.[9] Specifically, if seller i produces a high quality product, it advertises truthfully and its coverage is $\rho_i \in [0, 1]$ — i.e., its ads reach the buyer with probability ρ_i. If seller i is of low quality, its ads deceive the buyer and its coverage is $\mu_i \in [0, 1]$ — i.e., its ads reach the buyer with probability μ_i. Deceptive advertising has a cost $c(\mu_i) = \frac{\mu_i^2}{2\psi}$ (with $\psi > 0$) while truthful advertising is costless. The justification of this assumption (a version of which is also made in Miklós-Thal and Zhang (2013)) is that making something inherently of bad quality appear of good quality is more costly than simply declare the good quality of a high quality good. This assumption is harmless if the advertising costs are so low that the high quality seller will always choose a full coverage advertising campaign. If not, the model becomes more complicated without providing any new significant insight.

Uncertainty. Qualities are perfectly negatively correlated — i.e., $q_1 = l$ whenever $q_2 = h$ and *vice versa* — and this is common knowledge to all players. Hence, each seller is aware of both own and the

[8] The hypothesis of a single representative buyer is made only for tractability and to isolate the pure effects of deceptive advertising from those stemming from product segmentation that would arise with a downward sloping demand function. Our results would not change if there was a continuum of buyers and firms can perfectly discriminate among them. See, however, Grossman and Shapiro (1984) and Galeotti and Moraga-González (2008) for models with advertising, differentiated products and segmentation.

[9] This is sometimes referred to as 'advertising rate' in the literature.

competitor's product quality, while the buyer knows that there is only one good quality product but cannot tell which. The interpretation of this assumption is that product qualities can always be ranked: it is known in the market that some products are better than others, but the winner is not obvious outright. The sellers know which product is best out of their insider/specialist information, while the buyer only has a prior about that.[10]

The buyer prior belief about seller i's quality is $\tau_i = \Pr(q_i = h) = \Pr(q_{-i} = l)$, with $\tau_i = 1 - \tau_{-i}$ for each $i = 1, 2$.

Timing. The timing of the game is as follows:

t=0 The state $\mathbf{q} = (q_1, q_2)$ realizes and sellers learn it.

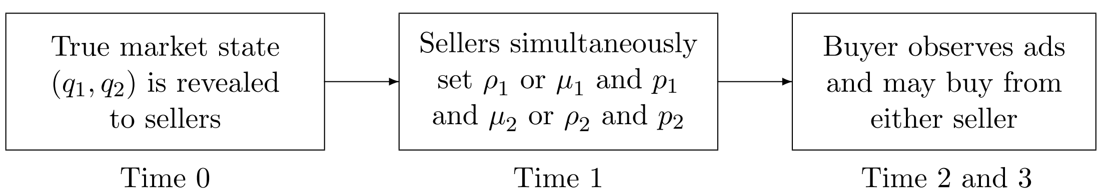
t=1 Sellers *simultaneously* and *independently* choose coverages and prices.

t=2 The buyer receives ads (if any) and observes the posted prices.

t=3 He decides which store to patronize. Trade (if any) takes place.

Figure (1) below illustrates the time-line of the game

Figure 1: Timing of the advertising game



Equilibrium concept and strategies. Since the game is sequential and involves asymmetric information, the equilibrium concept is *weak* Perfect Bayesian Equilibrium (see, e.g., Mas-Colell, Whinston, and Green (1995)). Let $a_i = (\rho_i, \mu_i, p_i)$ be seller i's action profile: where $\rho_i \in [0,1]$ and $\mu_i \in [0,1]$ are, respectively, its truthful and deceptive ads coverage, and p_i the posted price. Actions are taken simultaneously by sellers at Time 1. Hence, a strategy profile $\sigma_i(.)$ for seller i is a mapping of the types' realization $\mathbf{q} \in \{(h,l),(l,h)\} \equiv \mathbf{Q}$ into an action profile a_i. The buyer's action space is {buy 1, buy 2, not buy}. With no loss of generality we will focus on equilibria where trade occurs with certainty — i.e., Pr(not buy) $= 0$ — and denote $\alpha_i = \Pr(\text{buy } i) = 1 - \Pr(\text{buy } j)$.

[10] Of course, some products may have many features so that they perform well along some of them and do poorly along other dimensions. In such cases — which may not be infrequent for sufficiently complex products — it may well be the case that qualities rank the same, which would be of no interest for our purposes.

4 Full coverage and positive deception equilibria

In this section we characterize the class of equilibria where the buyer receives a good signal from the high quality seller with certainty — i.e., $\rho_i = 1$. We will label these outcomes as *full coverage* equilibria. However, within this class of equilibria, we are interested in those with positive deception. That is, outcomes of the game where low-quality sellers invest in deceptive advertising ($\mu_i > 0$ but typically $\mu_i < 1$). In so doing, we will first describe how sellers' adverting choices affect the buyer's behavior and then turn to the equilibrium characterization.

Consider first Stage 2: the buyer observes the posted prices and at most one ad from each seller. Using this information he updates his beliefs on products' qualities consistently (whenever possible) with equilibrium strategies. Let $\boldsymbol{\sigma}(.)=\{\sigma_i(.)\}_{i=1,2}$ be sellers' strategy profile. Let $\mathbf{s} = (s_1, s_2)$ be the vector of messages received by the buyer with $s_i \in S_i \equiv \{h, \varnothing\}$ and $\varnothing$ standing for no information received. A strategy profile $\boldsymbol{\sigma}$ induces a signals' probability distribution $\eta(\mathbf{s}|\boldsymbol{\sigma}) : S_1 \times S_2 \to [0, 1]$. Given the strategy profile $\boldsymbol{\sigma}$, denote by $\beta_{\boldsymbol{\sigma}}(q_i|\mathbf{s}, \mathbf{p})$ the buyer's beliefs about seller i's quality q_i upon observing the vector of prices $\mathbf{p}$ and the profile of ads $\mathbf{s}$. Slightly abusing notation, the Bayes rule then implies

$$\beta_{\boldsymbol{\sigma}}(q_i|\mathbf{s}, \mathbf{p}) = \frac{\sum_{q_j} \Pr(q_i|\mathbf{s}, \mathbf{p}(q_i, q_j))\, \eta(\mathbf{s}|\boldsymbol{\sigma}(q_i, q_j))}{\sum_{\mathbf{q}} \Pr(q_i|\mathbf{s}, \mathbf{p}(\mathbf{q}))\, \eta(\mathbf{s}|\boldsymbol{\sigma}(\mathbf{q}))}, \tag{1}$$

where $\Pr(\mathbf{q}|\mathbf{s}, \mathbf{p})$ is the probability assigned by the buyer to state $\mathbf{q}$ given sellers' pricing rule $\mathbf{p}(\mathbf{q})$ and the profile of realized ads $\mathbf{s}$.

Equation 1 nests two updating processes: one based on information conveyed by prices, the other based on signals and coverage strategies. In fact, whenever equilibrium prices are different conditionally on sellers' types, they convey full information about quality — i.e., $\Pr(\mathbf{q}|\mathbf{s}, \mathbf{p}(.)) \in \{0, 1\}$ and, consequently, $\beta_{\boldsymbol{\sigma}}(q_i|\mathbf{s}, \mathbf{p}) \in \{0, 1\}$. In this case, sellers *separate* so that ads have no informative value. On the other hand, when both sellers charge the same price regardless of their type, no information can be inferred from prices. Hence, ads are the only information available to the buyer to make his consumption decision — i.e., $\Pr(q_i = h|\mathbf{p}) = \tau_i$ while $\beta_\sigma(q_i|\mathbf{s}, \boldsymbol{\sigma})$ depends non trivially on equilibrium coverage decisions through $\eta(\mathbf{s}|\boldsymbol{\sigma})$.[11]

As a result, given a pair of signals $\mathbf{s}$ and prices $\mathbf{p}$ and a profile of strategies $\boldsymbol{\sigma}$, the buyer will patronize seller i (resp. j) if and only if

$$\begin{aligned} \sum_{q_i} \beta_{\boldsymbol{\sigma}}(q_i|\mathbf{s}, \mathbf{p})\, \theta(q_i) - p_i \quad &> \quad \sum_{q_j} \beta_{\boldsymbol{\sigma}}(q_j|\mathbf{s}, \mathbf{p})\, \theta(q_j) - p_j \quad (\text{resp. } <) \quad \Rightarrow \\ \alpha_i(\mathbf{s}, \mathbf{p}) \quad &= \quad 1 \quad (\text{resp. } \alpha_i(\mathbf{s}, \mathbf{p}) = 0). \end{aligned}$$

When indifferent he randomizes — i.e., he buys from seller i with probability $\alpha_i(\mathbf{s}, \mathbf{p}) \in (0, 1)$.

[11] In other words, prices don't convey more information than the one already incorporated into prior beliefs.

Seller i's optimization program when it supplies a low quality item is

$$\max_{p_i \geq 0, \mu_i \in [0,1]} \left\{ \sum_{s_i} \Pr(s_i)\, \alpha_i(s_i, s_j = h, \mathbf{p})\, p_i - c(\mu_i) \right\},$$

where $\Pr(s_i = h) = 1 - \Pr(s_i = \varnothing) = \mu_i$ for each $i = 1, 2$. Notice that, in a full coverage equilibrium the buyer always gets a signal $s_j = h$ from the high quality seller. Therefore, if he does not receive an ad from seller i he can immediately infer that i is of low quality, due to perfect (negative) correlation of sellers' qualities.

Seller i's optimization program when it supplies a high quality item is

$$\max_{p_i \geq 0} \left\{ \sum_{s_j} \Pr(s_j)\, \alpha_i(s_i = h, s_j, \mathbf{p})\, p_i \right\}.$$

That is, high quality sellers simply maximize revenues since truthful advertising is costless.

Building on this characterization, we are now ready to prove the following preliminary lemma that is helpful to understand the nature of the problem at hand. In particular, it shows that equilibria where sellers engage in deceptive advertising must involve pooling along the price dimension.

Lemma 1. *A separating equilibrium where prices reveal qualities — i.e., such that sellers of different qualities post different prices — features no deceptive advertising.*

The economic intuition of this result is straightforward. In a separating equilibrium sellers perfectly reveal their quality through prices. Hence, there is no scope for deception since misleading advertising is costly. As a result, deceptive advertising can only emerge in a pooling equilibrium — i.e., in an outcome of the game where prices do not convey information about qualities. In what follows we will therefore focus on this class of equilibria. To do so, however, we have to specify *off-equilibrium* beliefs — i.e., the buyer's beliefs about sellers' qualities whenever he observes a price different from the expected (pooling) one.

A1 *Whenever a seller posts an off-equilibrium price, the buyer assigns to this seller probability* 1 *of selling a low quality product.*

This assumption is standard in the literature and is for instance imposed also in Miklós-Thal and Zhang (2013). Later on in the analysis we will argue that this restriction is indeed appealing as it satfies a common refinement in signaling games — i.e., Divinity (D1) introduced by Banks and Sobel (1987). Finally, to rule out corner solutions where low quality sellers deceive the buyer with probability 1 we assume throughout:

A2 *The equilibrium deception level of the low quality sellers is always smaller than 1 — i.e.,* $\mu_i < 1$.

In terms of the primitives of the model, this requires

$$\psi < \min\left\{1, \frac{2}{2\theta_l + 1}\right\}. \tag{A2}$$

Ruling out equilibria with full coverage by both sellers makes the buyer's updating process non-trivial. In fact, with corner solutions, no information is exchanged in equilibrium beyond the one incorporated in prior beliefs.

4.1 Positive deception (pooling) equilibria

In this section we characterize the set of pooling equilibria with positive deception under the off-equilibrium beliefs specified in **A1**. Consider a candidate equilibrium where sellers post the same price (say p^*) regardless of their quality. Moreover, let μ_i^* be seller i's investment in deceptive advertising and denote by α_i^* the buyer's equilibrium purchasing rule.

Buyer's behavior. To begin with, we consider the buyer's optimization and updating behavior. In a pooling equilibrium, the buyer observes one good signal (h) and either a nil ($\varnothing$) or another good signal.

Hence, the bad quality seller is surely detected when its ad misses the target — i.e., when it is not received by the buyer. As a consequence

$$\begin{aligned} \beta_{\boldsymbol{\sigma}^*}\left(q_i = h | s_i = h, s_j = \varnothing, \boldsymbol{\sigma}^*\right) &= 1, \\ \beta_{\boldsymbol{\sigma}^*}\left(q_i = h | s_i = \varnothing, s_j = h, \boldsymbol{\sigma}^*\right) &= 0, \end{aligned}$$

where $\boldsymbol{\sigma}^* = (\boldsymbol{\mu}^*, \mathbf{p}^*)$. When, instead, the buyer observes a good signal from both sellers, he realizes that one of them is deceiving him, but he is uncertain about the identity of this seller. As a consequence, he must update beliefs on qualities based on equilibrium strategies — i.e., applying (1)

$$\begin{aligned} \beta_{\boldsymbol{\sigma}^*}\left(q_1 = h | s_1 = h, s_2 = h, \boldsymbol{\sigma}^*\right) &= \frac{\tau_1 \mu_2^*}{\tau_1 \mu_2^* + \tau_2 \mu_1^*}, \\ \beta_{\boldsymbol{\sigma}^*}\left(q_2 = h | s_1 = h, s_2 = h, \boldsymbol{\sigma}^*\right) &= \frac{\tau_2 \mu_1^*}{\tau_2 \mu_1^* + \tau_1 \mu_2^*}. \end{aligned}$$

In order to characterize the buyer's purchasing rule it is useful to introduce a simple but important preliminary result. In a pooling equilibrium the buyer's optimal purchasing decision only depends on the inference he is able to draw from the received ads because sellers charge the same price regardless of their types. Hence, he is indifferent between buying from either sellers only when his posteriors are the same — i.e., when the deception ratio μ_1^*/μ_2^* equals the relative 'trust' that (behind the veil of ignorance) the buyer avails to the two sellers τ_1/τ_2.

Lemma 2. *Assume* $\tau_1\mu_2^* = \tau_2\mu_1^*$, *then*

$$\alpha_i^*\left(s_i = h, s_{-i} = h, \mathbf{p}^*\right) = \tau_i \quad \forall i = 1, 2.$$

The intuition for this result is as follows: if deceptive choices are such that the buyer's posteriors are the same, then there is not enough evidence to unravel sellers' qualities and the purchasing rule must be based only on priors.

Summuing up, for any given (pooling) price p^* that satisfies the buyer's participation constraint, upon receiving two good signals, the equilibrium purchasing rule is

$$\alpha_i^*\left(s_i = h, s_{-i} = h, \mathbf{p}^*\right) = \begin{cases} 1 & \Leftrightarrow \quad \tau_i\mu_{-i}^* > \tau_{-i}\mu_i^* \\ \alpha_i^* \in (0,1) & \Leftrightarrow \quad \tau_i\mu_{-i}^* = \tau_{-i}\mu_i^* \\ 0 & \Leftrightarrow \quad \tau_i\mu_{-i}^* < \tau_{-i}\mu_i^* \end{cases}.$$

Finally, given sellers' equilibrium choices, the buyer must have an incentive to buy at the prevailing pooling price — i.e.,

$$\sum_{q_1}\beta_{\boldsymbol{\sigma}^*}\left(q_1|\mathbf{s},\boldsymbol{\sigma}^*\right)\theta\left(q_1\right) - p^* = \sum_{q_2}\beta_{\boldsymbol{\sigma}^*}\left(q_2|\mathbf{s},\boldsymbol{\sigma}^*\right)\theta\left(q_2\right) - p^* = \tfrac{\theta_l+\theta_h}{2} - p^* \geq 0 \quad \Leftrightarrow \quad p^* \leq \theta_l + \tfrac{1}{2}, \quad (2)$$

where $\frac{\theta_l+\theta_h}{2} = \theta_l + \frac{1}{2}$ is the maximal price that the buyer is willing to pay when he perceives the two sellers as being equally likely to supply a high quality item — i.e., the expected quality when no information can be learned from prices and ads. Clearly, this price is higher the closer is the low quality item to the high quality one.

Equilibrium deception choice. Given the buyer's equilibrium behavior characterized above, we will now study how low quality sellers chose their equilibrium deception level. For any given (candidate equilibrium) pooling price p^*, the optimization program of seller i when it is of low quality is

$$\max_{\mu_i\in[0,1]}\left\{\mu_i\alpha_i^*\left(s_i = h, s_j = h, \boldsymbol{\sigma}^*\right)p^* - c\left(\mu_i\right)\right\},$$

whose first-order condition (necessary and sufficient) for an optimum yields

$$p^*\alpha_i^*\left(s_i = h, s_j = h, \boldsymbol{\sigma}^*\right) - \frac{\mu_i^*}{\psi} = 0 \quad \Rightarrow \quad \mu_i^* = \psi p^*\alpha_i^*\left(s_i = h, s_j = h, \boldsymbol{\sigma}^*\right). \quad (3)$$

Essentially, a low quality seller invests more in deceptive advertising the higher its per ad expected sales revenue — i.e., the larger the probability of making a sale, $\alpha_i^*(.)$, and the higher the pooling price, p^* — and the lower the marginal cost of deceiving the buyer — i.e., the higher the parameter ψ.

Equilibrium pooling prices. Having analyzed the buyer's equilibrium behavior and the sellers' equilibrium deception choices, we can now characterize the set of equilibrium pooling prices.

An equilibrium price $p^* \geq 0$ needs to satisfy two intuitive requirements: (i) it must meet the buyer's participation constraint in equation (2); (ii) it must not be vulnerable to deviations by low as well as high quality sellers — i.e., no seller has to gain from posting a price different than p^* and setting a deception level different from μ_i^*.

First, notice that seller i cannot gain from investing a deception level different from μ_i^* unless it does not plan to deviate at the pricing stage too. This follows from the optimality condition (3) and the fact that coverage decisions are unobservable — i.e., changing only the investment level in deceptive advertising has no strategic impact on the behavior of the buyer and the rival, but it only alters the own expected revenues. Hence, a potentially profitable deviation must necessarily involve a price change, which under **A1** triggers the off-equilibrium belief that the deviant seller is perceived by the buyer as the low quality one. As a consequence, a successful price deviation involves no deception to begin with. Let $\alpha_i(p_i)$ be the buyer's choice rule when he observes the off-equilibrium price p_i, and denote by

$$\pi_i^d(p^*|l) \equiv \max_{p_i}\{\alpha_i(p_i)\,p_i : \alpha_i(p_i) = 1 \Leftrightarrow p_i < p^* - 1\} = p^* - 1,$$

be the expected deviation profit of seller i. That is, the expected profit that seller i makes when charging a price p_i low enough to attract the buyer — i.e., such that the rival makes no sales even if it is perceived by the buyer as a high quality seller. Recalling that $\theta_h - \theta_l = 1$, this requires $\theta_l - p_i > \theta_h - p^*$ if and only if $p_i < p^* - 1$. Clearly, this expected profit is increasing in p^* since the incentive to undercut the rival is larger when the (candidate) equilibrium price is higher.

By the same token, denote by

$$\pi_i^*(p^*|l) \equiv \max_{\mu_i}\left\{\alpha_i^*(s_i = h, s_j = h, \boldsymbol{\sigma}^*)\,\mu_i p^* - \frac{\mu_i^2}{2\psi}\right\} = \frac{\psi\tau_i^2 p^{*2}}{2}, \tag{4}$$

the equilibrium expected profit of a low quality seller. Notice that the above expression is increasing and convex in p^* as opposed to (4.1) that is linear in p^*. This is for two reasons. When the equilibrium price p^* increases, there is not only an expansion of the low quality seller's revenue, but there is also an expansion of the net benefits from deceptive advertising. Hence, an increase in p^* not only increases sales revenues but, *ceteris paribus*, it also increases the incentive to invest in misleading advertising.

Recalling that $\alpha_i^* = \tau_i$ by Lemma 2, the relevant no deviation constraint for seller i is

$$\pi_i^d(p^*|l) \leq \pi_i^*(p^*|l) \quad \Leftrightarrow \quad p^* - 1 \leq \frac{\psi\tau_i^2 p^{*2}}{2}.$$

Hence, low quality sellers have no incentive to deviate if and only if

$$p^* - 1 - \tfrac{\psi p^{*2}\underline{\tau}^2}{2} \leq 0. \tag{5}$$

where, hereafter, $\underline{\tau} \equiv \min_{i=1,2}\tau_i < \frac{1}{2}$ will denote the lowest prior assigned by the buyer to either seller.

The set of solutions of inequality (5), hereafter $\mathcal{P}'(l)$, is defined as

$$p^* \in \mathcal{P}'(l) \equiv \left\{p^* : p^* \in \left[0, \underline{p}^*(l)\right] \cup \left[\overline{p}^*(l), +\infty\right]\right\},$$

where it is easy to verify that

$$1 < \underline{p}^*(l) \equiv \tfrac{1-\sqrt{1-2\psi\underline{\tau}^2}}{\psi\underline{\tau}^2} \leq \overline{p}^*(l) \equiv \tfrac{1+\sqrt{1-2\psi\underline{\tau}^2}}{\psi\underline{\tau}^2}.$$

The structure of the set $\mathcal{P}'(l)$ reflects the idea that relatively small pooling prices p^* cannot be easily undercut by a low quality seller because this would yield a too low revenue to the deviant. Nevertheless, a relatively large pooling price may also be not vulnerable to a price deviation by a low quality seller. This is because when sellers charge a high price in equilibrium, the seller producing a low quality item invests relatively more in deceptive advertising, which (other things being equal) increases its selling probability, whereby making a deviation from this outcome unprofitable.

Of course, the buyer must find it convenient to buy the item at equilibrium. Hence, the intersection between the set of prices that satisfy participation constraint (2) and $\mathcal{P}'(l)$ defines the set of pooling prices that are not vulnerable to deviations by low quality sellers and that secure trade at equilibrium — i.e.,

$$\mathcal{P}^*(l) \equiv \mathcal{P}'(l) \cap \left\{p^* : p^* \leq \theta_l + \tfrac{1}{2}\right\}.$$

Consider now seller i's deviation when it produces the high quality item. Of course, seller i has no incentive to reduce its coverage because truthful advertising is costless. Hence, a profitable deviation of a high quality seller must only involve a price deviation. Let

$$\pi_i^d(p^*|h) \equiv \max_{p_i} \left\{\alpha_i(p_i)\, p_i : \alpha_i(p_i) = 1 \Leftrightarrow p_i < p^* - 1\right\} = p^* - 1,$$

be seller i's maximal (expected) profit from deviation when the rival charges p^* and the buyer's belief structure follows **A1**. Notice that this expression is the same as that obtained for a low cost seller. This is because, under **A1**, deviant sellers are perceived by the buyer as low quality ones.

Similarly, let

$$\pi_i^*(p^*|h) \equiv \left[(1-\mu_j^*) + \mu_j^* \alpha_i^*(s_i = h, s_j = h, \boldsymbol{\sigma}^*)\right] p^* = \left[1 - \psi(1-\tau_i)^2 p^*\right] p^* \qquad (6)$$

be seller i's equilibrium expected profit. Notice that $\pi_i^*(p^*|h)$ is strictly concave in p^*. This is because an increase in the equilibrium pooling price expands the revenue of a high quality seller, and it also increases the investment of the low quality seller in misleading advertising, which in turn increases the likelihood of a tie — i.e., the probability that (ex post) the buyer has not enough information to infer sellers' qualities, and thus bases his consumption choice only on his priors. Hence, the no deviation condition for seller i when it is of high quality is

$$\pi_i^d\left(p^*|h\right) \leq \pi_i^*\left(p^*|h\right) \quad \forall i \quad \Leftrightarrow \quad p^* \leq p^*\left(h\right) \equiv \min_{i=1,2} \frac{1}{1-\tau_i}\sqrt{\frac{1}{\psi}}$$

This constraint requires an upper-bound on the equilibrium pooling price. Since the equilibrium deception level of the low quality seller is increasing in the pooling price, a too high equilibrium price would reduce so much the probability with which a high quality seller makes a sale to trigger a price deviation by this seller. Summing up, the set of pooling prices that are not vulnerable to deviations by high quality sellers and that meet the buyer's participation constraint is

$$\mathcal{P}^*(h) \equiv \left\{p^* : p^* \leq \min\left\{\theta_l + \tfrac{1}{2}, p^*\left(h\right)\right\}\right\},$$

where it is easy to verify that $\underline{p}^*(l) \leq p^*\left(h\right) \leq \overline{p}^*(l)$.

Equipped with the above characterization, we can now state the main proposition of the section which illustrates the key features of pooling equilibria in our game.

Proposition 1. *Under **A1** and **A2**, there exists a continuum of pooling equilibria with the following features:*

- *Every price $p^* \in \mathcal{P}^* \equiv [0, p^*_{\max}]$ can be supported in equilibrium. Moreover, $p^*_{\max} > 0$ is such that*

$$p^*_{\max} = \begin{cases} \underline{p}^*(l) & \Leftrightarrow \quad \theta_l \geq \theta^* \\ \theta_l + \frac{1}{2} & \Leftrightarrow \quad \theta_l < \theta^* \end{cases},$$

where

$$\theta^* \equiv \frac{1-\sqrt{1-2\psi\underline{\tau}^2}}{\psi\underline{\tau}^2} - \frac{1}{2},$$

with θ^* being increasing in ψ and $\underline{\tau}$.

- *For any price $p^* \in \mathcal{P}^*$ the equilibrium deceptive advertising μ_i^* is:*

$$\mu_i^* = \psi\tau_i p^* < 1 \quad \forall i = 1,2 \quad \Rightarrow \quad \frac{\mu_1^*}{\mu_2^*} = \frac{\tau_1}{1-\tau_1},$$

and implies the following posteriors when the buyer receives two positive ads:

$$\beta_{\boldsymbol{\sigma}^*}\left(q_1 = h|s_1 = h, s_2 = h, \boldsymbol{\sigma}^*\right) = \beta_{\boldsymbol{\sigma}^*}\left(q_2 = h|s_1 = h, s_2 = h, \boldsymbol{\sigma}^*\right) = \tfrac{1}{2}.$$

- *The buyer's equilibrium strategy $\alpha_i^*\left(\mathbf{s}\right)$ is such that:*

$$\alpha_i^*\left(\mathbf{s}\right) = \begin{cases} 0 & \Leftrightarrow \quad s_i = \varnothing,\ s_{-i} = h \\ \tau_i & \Leftrightarrow \quad s_i = h,\ s_{-i} = h \\ 1 & \Leftrightarrow \quad s_i = h,\ s_{-i} = \varnothing \end{cases}.$$

This result echoes the findings of Martimort and Moreira (2010) showing that common agency games with informed principals may feature pooling equilibria where the informed principals offer the same allocations regardless of their types — i.e., what they call uninformative equilibria. In our simpler model, however, since communication can occur also via advertising, even such pooling allocations may entail some communication (i.e., learning by the common buyer).

Proposition 1 has also some interesting comparative statics implications on the sellers' deception behavior. First, for any price $p^* \in int\mathcal{P}^*$, a seller engages in more deceptive advertising the more it is trusted *ex-ante* by the buyer — i.e., the higher buyer's prior on its quality. Essentially, when the buyer is confident enough that a seller's product is of high quality, he tends to buy more frequently from it, yet the seller deceives him more. In fact, since the buyer has just his prior beliefs to guide his consumption choice, a highly trusted low quality seller behaves opportunistically by exploiting such a 'reputational' advantage. Second, the tendency to deceive the buyer is increasing both in ψ and in p^*. In words, an increase in ψ implies a decrease in the marginal cost of deception, which (*ceteris paribus*) fosters deception. The same holds for an increase in the pooling price p^*, which increases the surplus that can be extracted from the buyer if he is induced to believe that the product is of high quality.

Moreover, in a positive deception equilibrium the buyer behaves according to a 50-50 posterior on sellers' quality when he receives two (positive) ads. In fact, were this not true, the buyer would strictly prefer to buy from one seller, which would (in turn) make the competitor's costly investment in deception useless. Hence, being indifferent, the buyer may buy equivalently from either seller. However, it turns out that the only mixed strategy of the blind buyer sustaining the equilibrium is the most intuitive: mixing between the sellers according to the prior information he has about their quality — i.e., his *bias*.

Finally, it is also interesting to notice that whether the maximal pooling price is pinned down by the sellers' incentive compatibility constraints or by the buyer's participation constraint is determined only by the level of θ_l (since we have normalized $\theta_h - \theta_l = 1$). On the one hand, when θ_l is sufficiently large (higher than θ^*), sellers' do not gain much from deceptive advertising, the buyer is happy to buy even at a relatively large price, so that incentive constraints bound from above the maximal price that can be charged in a pooling equilibrium. On the other hand, when θ_l is sufficiently low (smaller than θ^*) there is a large potential damage from deception to the buyer, so that the maximal price that can be charged in equilibrium is on the buyer's participation constraint.

In the next result we study how this maximal price responds to changes of the parameters of the model.

Corollary 1. *If $p^*_{\max} = \underline{p}^*(l)$, then $p^*_{\max}$ is increasing in both $\underline{\tau}$ and ψ.*

The intuition for this result is straightforward. A higher ψ makes deception more profitable since it reduces its (marginal) cost. Similarly, a larger $\underline{\tau}$ makes low quality sellers more eager to deceive the buyer since it expands the chance of a sale by a low quality seller.

A remark on equilibrium refinements. We conclude the section by arguing that the equilibrium outcomes sustained by the off-equilibrium beliefs specified in **A1** and characterized in Proposition 1 survive to the weakest version of Divinity (D1) introduced by Banks and Sobel (1987).[12] To this purpose, consider a price deviation $p \neq p^*$, and assume that the buyer's strategy is to buy from the deviant seller with probability α. The equilibrium price p^* satisfies D1 if and only if $\alpha_i^l < \alpha_i^h$, where α_i^h is the buyer's strategy that makes seller i indifferent between deviating and sticking to the equilibrium price when he is of high quality — i.e.,

$$\alpha_i^h p = \left[\tau_i \mu_j^* + 1 - \mu_j^*\right] p^*.$$

By the same token, α_i^l is the buyer's strategy that makes seller i indifferent between deviating and sticking to the equilibrium price when he is of low quality — i.e.,

$$\alpha_i^l p = \tau_i \mu_i^* p^*.$$

It can be easily verified that $\alpha_i^h > \alpha_i^l$ for every i regardless of p^* so that whenever a high quality seller can gain from deviation, the low quality one does so too. Hence, all pooling equilibria characterized above meet D1. Essentially, for any given off-equilibrium behavior of the buyer, a high quality seller has less incentive to deviate than a low quality one because his equilibrium sale probability is higher.

5 Selection, welfare and comparative statics

The multiplicity of equilibria characterized in Proposition 1 opens the door to selection issues. How do sellers select equilibria? What is the impact of the underlying parameters on the buyer surplus and total welfare?

It is reasonable to consider the equilibrium that features the highest pooling price, $p^*_{\max}$. In fact, such equilibrium can be easily calculated by each seller and it is a Pareto dominant outcome for both of them, making coordination relatively easy. Hence, focusing on the maximal pooling price equilibrium ($p^*_{\max}$), in the next proposition we perform some relevant comparative statics.

Proposition 2. *If sellers coordinate on the pooling equilibrium with the highest price:*

- $\pi_i^* \left(p^*_{\max}|h\right) > \pi_i^* \left(p^*_{\max}|l\right)$.
- $\pi_i^* \left(p^*_{\max}|q_i\right)$ *increases with* τ_i *(and decreases with* τ_j*) for any* $q_i \in \{l, h\}$.
- $\pi_i^* \left(p^*_{\max}|l\right) > \pi_j^* \left(p^*_{\max}|l\right)$ *and* $\pi_i^* \left(p^*_{\max}|h\right) > \pi_j^* \left(p^*_{\max}|h\right)$ *if and only if* $\tau_i > \tau_j$.

The first result is intuitive: a seller enjoys a higher expected profit whenever its product is of high quality, independently of the trust accorded by the buyer (as reflected by τ_i). This is because, regardless of the buyer's prior beliefs, deceptive ads are costly yet necessary in order to attract demand when a seller supplies a low quality item. On the other hand, a seller producing a high quality item reaches the buyer with certainty at no cost.

[12] Of course, if our equilibria satisfy D1 they also satisfy the Cho and Kreps (1987) intuitive criterion.

The second result is more subtle. The reason why seller i profit increases with τ_i regardless of its quality is as follows. If seller i produces a low quality good, an increase in τ_i expands its incentive to deceive the buyer, whereby increasing the probability of making a sale, thus expanding own profits. When, instead, seller i is of high quality, an increase of τ_i reduces τ_j since qualities are negatively correlated, which (*ceteris paribus*) reduces seller j's incentive to deceive the buyer, thus increasing seller i's probability of making the sale.

The third result clearly states that, keeping quality constant, being trusted more by the buyer boosts profits. Consider first $q = h$. Then $\tau_i > \tau_j$ implies that when seller i has high quality seller j does little deceptive advertising, which in turn expands seller i's profits. When, instead, seller i has low quality it invests more in deceptive advertising, whereby reducing seller j's profits. Next, consider $q = l$. Then $\tau_i > \tau_j$ implies that when seller i has low quality exploits more the buyer's trust and attracts more demand trough deception than seller j does when it has low quality, resulting in a higher profit for i than for j.

In this equilibrium one may wonder how the buyer surplus responds to changes of the underlying parameters. Since in our model the buyer buys only one unit of product, his (expected) utility when he has observed an identical message from both sellers — and hence his posteriors are 1/2 — and pays the pooling price $p^*_{\max}$ is

$$v\left(s_i = h, s_j = h\right) = \tfrac{\theta_l + \theta_h}{2} - p^*_{\max}.$$

This is because in this case his posterior assigns equal probabilities to both sellers selling a high quality good. When, instead, the buyer observes a single ad, he infers with certainty the high quality seller and buys from it, with a resulting utility

$$v\left(s_i = h, s_j = \varnothing\right) = v\left(s_i = \varnothing, s_j = h\right) = \underbrace{1 + \theta_l}_{=\theta_h} - p^*_{\max}.$$

Hence, the (unconditional) expected utility of the buyer before receiving ads in a pooling equilibrium where both sellers charge the price $p^*_{\max}$ is

$$\begin{aligned}\mathrm{E}_{\mathbf{s}}[v(s_i, s_j)] = \tau_i \left[(1 - \mu^*_j) v\left(s_i = h, s_j = \varnothing\right) + \mu^*_j v\left(s_i = h, s_j = h\right)\right] + \\ + \tau_j \left[\left(1 - \mu^*_i\right) v\left(s_i = \varnothing, s_j = h\right) + \mu^*_i v\left(s_i = h, s_j = h\right)\right].\end{aligned}$$

Setting $\mu^*_i = \psi \tau_i p^*_{\max}$ and $\tau_j = 1 - \tau_i$, this expression simplifies to

$$\mathrm{E}_{\mathbf{s}}[v(s_i, s_j)] = 1 + \theta_l - \left[1 + \tau_i \left(1 - \tau_i\right) \psi\right] p^*_{\max}. \tag{7}$$

The economic interpretation of this expression is straightforward. The buyer unconditional expected utility equals the difference between the highest possible quality $\theta_h = 1 + \theta_l$ and the term $\left[1 + \tau_i \left(1 - \tau_i\right) \psi\right] p^*_{\max}$, which reflects the sellers' incentive to deceive the buyer when they coordinate on the maximal pooling price $p^*_{\max}$.

Indeed, the scalar $1+\tau_i(1-\tau_i)\psi$ is greater than 1 (as it would be in the monopoly case with given quality) and it increases in ψ and in $\left|\tau_i - \frac{1}{2}\right|$. When there is no uncertainty about sellers' quality — i.e., $\tau_i = 0$ or $\tau_i = 1$ — equation (7) simply implies that the buyer's unconditional expected utility equals the difference between the high quality item utility and the price. If instead $\tau_i \in (0,1)$ an additional loss appears: $\tau_i(1-\tau_i)\psi p_{\max}$. The term $\tau_i(1-\tau_i)$ is the variance of the Bernoulli distribution with probability τ_i over a seller's quality and reflects the a priori uncertainty over the quality of either seller. Clearly, the smaller the distance between τ_i and $\frac{1}{2}$, the more the buyer is uncertain as to which seller is the high quality one, which again increases the *ex-ante* probability of a wrong purchase.

Clearly, the larger ψ is, the lower is the (marginal) cost of deception and the higher the low quality seller's coverage, implying a larger probability of a wrong purchase.

In the next proposition we study how these parameters affect the buyer surplus.

Proposition 3. *If sellers coordinate on the pooling equilibrium with highest price:*

- *The expected buyer surplus,* $\mathrm{E}_{\mathbf{s}}[v(s_i, s_j)]$*, is decreasing in* $\underline{\tau}$ *and increasing in* θ_l *if* $p^*_{\max} = \underline{p}^*(l)$*.*
- *The expected buyer surplus,* $\mathrm{E}_{\mathbf{s}}[v(s_i, s_j)]$*, is decreasing in* $\underline{\tau}$ *and decreasing in* θ_l *if* $p^*_{\max} = \theta_l + \frac{1}{2}$*.*

The reason why the buyer surplus is unambiguously decreasing in $\underline{\tau}$, the lowest between the two τ_i, is as follows. As $\underline{\tau}$ gets closer to $\frac{1}{2}$ the *ex-ante* uncertainty about sellers' qualities grows larger. This makes the *ex-ante* probability of making a wrong purchase higher, whereby reducing the buyer's expected utility, as discussed above. Moreover, when $p^*_{\max} = \underline{p}^*(l)$, sellers in equilibrium exploit the increased uncertainty by raising the maximum price ($p^*_{\max}$ increases with $\underline{\tau}$), extracting a higher fraction of the buyer's surplus.

The reason why the buyer surplus increases with θ_l when the equilibrium price $p^*_{\max}$ is pinned down by the incentive compatibility constraint of the low quality seller — see equation (5) — hinges on the simple fact that $\underline{p}^*(l)$ does not depend on θ_l, while the expected quality is increasing with it. When, instead, the equilibrium price is determined by the buyer's participation constraint — see equation (2) — a higher minimum quality has a detrimental impact on the buyer's expected welfare. This is because a higher θ_l expands the buyer's willingness to pay, which induces sellers to profitably charge a higher price $p^*_{\max}$ and deceive him more. The combination of these two effects more than compensates the positive effect of an increased θ_l on the average quality the buyer expects to consume, making him worse off.

Having studied buyer surplus and sellers' profits, we conclude by analyzing the impact on total welfare of the model parameters. As with buyer surplus, we evaluate total welfare from the perspective of a social planner (or a public Authority) who is unaware of the true qualities and forms expectations on them using the same prior information available to the buyer — i.e., τ_1 and τ_2. While the expressions for profits of the low and high quality seller are given in equations (4) and (6) respectively, the *ex-ante* expected total welfare (ETW hereafter) is

$$\text{ETW} = \text{E}_{\mathbf{s}}[v(s_i, s_j)] + \tau_i \left[\pi_i^* \left(p_{\max}^*|h\right) + \pi_j^* \left(p_{\max}^*|l\right)\right] + (1-\tau_i) \left[\pi_i^* \left(p_{\max}^*|l\right) + \pi_j^* \left(p_{\max}^*|h\right)\right],$$

which, using (4), (6) and (7), simplifies to

$$\text{ETW} = 1 + \theta_l - \psi\tau_i (1-\tau_i) \frac{2 + p_{\max}^*}{2} p_{\max}^*,$$

As before, the comparative statics performed below applies to the case where sellers coordinate on the highest pooling price, $p_{\max}^*$.

Proposition 4. *If sellers coordinate on the pooling equilibrium with highest price, the Expected Total Welfare is:*

- *increasing in* $\left|\tau_i - \frac{1}{2}\right|$.
- *decreasing in* ψ.
- *increasing in* θ_l *when* $p_{\max}^* = \underline{p}^*(l)$ *or, when* $p_{\max}^* = \theta_l + \frac{1}{2}$, *if*

$$\theta_l \leq \theta_w \equiv \frac{1}{\psi\tau_i (1-\tau_i)} - \frac{3}{2}.$$

Otherwise it is decreasing.

Proposition 4 has several interesting features. A change in τ_i has multiple effects on total welfare. First, whenever the distance between τ_i and $\frac{1}{2}$ increases, the *ex-ante* uncertainty decreases, reducing the loss in efficiency caused by asymmetric information. In other words, the higher the distance from $\frac{1}{2}$, the lower is the low quality sellers' incentive to deceive the buyer, which is also more likely to consume a low quality product. Thus, as τ_i moves apart from $\frac{1}{2}$ the ETW tends to be larger. Moreover, there is another effect induced by a change of τ_i, if the maximal pooling price equals $\underline{p}^*(l)$. This effect does not influence total welfare, but it alters the price causing a transfer between the sellers and the buyer with no net effect. In particular, recalling that $\underline{p}^*(l)$ increases with $\underline{\tau}$, the transfer benefits the sellers (resp. the buyer) if $\tau_i < \frac{1}{2}$ (resp. $>$).

An increase in ψ amounts to lower deception costs, thereby inducing more deception and a waste of resources which is individually rational for a bad quality seller, yet negative from a social welfare perspective.

The result on θ_l is better understood by considering each case separately. First, when $p_{\max}^* = \underline{p}^*(l)$, while not altering the maximum price, an increase in θ_l simply increases the overall gains from trade. By contrast, when $p_{\max}^* = \theta_l + \frac{1}{2}$, the impact of θ_l on total welfare is inverted-U shaped. Recalling that the effect of a change in θ_l on the equilibrium price washes out when considering total welfare maximization, two main forces contribute to explain the result. As before, an increase in θ_l tends to increase welfare since it makes the buyer better-off when buying a low quality product. Nevertheless, a higher θ_l also has a negative impact on welfare since it tends to increase the equilibrium (maximal) pooling price, which (*ceteris paribus*), makes low quality sellers more willing to invest in wasteful deceptive advertising. When θ_l is small the former effect outweighs the latter, whose magnitude is proportional to the equilibrium price (the opposite obtains when θ_l is large).

5.1 Separating vs pooling equilibria

In this section we study more closely separating equilibria. In this class of equilibria prices fully reveal quality and beliefs are trivial. Thus, the advertising decisions are redundant and set at zero whenever costly. In other words, there is no scope for deception when prices signal quality. When advertising is costless, instead, it may be set arbitrarily. For consistency with the pooling analysis developed so far, and for comparison purposes, in what follows we will focus on full coverage separating equilibria defined as follows:

Definition. *A full coverage separating equilibrium is a separating equilibrium in which the high quality seller i sets $\rho_i = 1$ — i.e., it fully covers the market with truthful quality ads.*

Hence, Bertrand competition with different qualities and zero production costs implies (by standard arguments) that in a separating equilibrium with truthful advertising the buyer buys from the seller with high quality at price $p^*(h) = \theta_h - \theta_l = 1$, while the low quality seller prices at $p^*(l) = 0$. Thus, if it exists, a separating equilibrium necessarily requires $\mathbf{p}^* = (1, 0)$. Of course, $\mathbf{p}^*$ must not be vulnerable to deviations by either seller.

The high quality seller clearly cannot gain from deviating. It makes a positive profit at a separating equilibrium while, because of **A1**, it would make zero profits following any conceivable deviation. Hence, the only meaningful departure from equilibrium behavior is for the low quality seller, which may wish to match the high quality price $p^*(h)$ inducing a pooling equilibrium and setting the corresponding optimal deception. Such a deviation would make the buyer uncertain as to who's who between the sellers and would allow the low quality seller to win his purchase with some probability. This deviation is viable as long as the separating price of the high quality seller is such that there exists a pooling equilibrium with an identical price $p^* = p^*(h)$. If so, the separating equilibrium would be destroyed by the very existence of the pooling equilibrium. The conditions under which a full coverage separating equilibrium exists are given in the following lemma:

Lemma 3. *A necessary and sufficient condition for the existence of a full coverage separating equilibrium is $\theta_l < \frac{1}{2}$. In this parameter region the equilibrium price induced by the separating behavior is higher than the maximal pooling price.*

Hence, for $\theta_l < \frac{1}{2}$ the game features a multiplicity of equilibria with a unique separating outcome and potentially many pooling ones. Notice further that Lemma 3 means that full information transmission might not be achieved in equilibrium as long as the difference between qualities is lower than twice the lowest quality. That is, a very large quality premium is necessary for a high quality seller to successfully differentiate in equilibrium: as qualities get closer (i.e., θ_l grows larger) the incentive to deceive the buyer becomes so strong that separation cannot be achieved.

Given the multiplicity of equilibria emerging in the parameter region where $\theta_l < \frac{1}{2}$, a natural normative exercise is the comparison of the welfare properties of the two classes of equilibria characterized above. Consider a pooling equilibrium with price $p^* \in [0, p^*_{\max}]$ and the separating equilibrium with price $\mathbf{p}^* = (1, 0)$. Can we rank these equilibria according to a welfare criterion? Which one of these outcomes is the one that makes the buyer better-off?

Proposition 5. *Assume that $\theta_l < \frac{1}{2}$, so that the outcome of the game can yield either the separating or a pooling equilibrium. Then:*

- *The expected total welfare is unambiguously larger in a separating equilibrium.*
- *Low quality sellers unambiguously gain from coordinating on a pooling equilibrium.*
- *High quality sellers instead prefer to coordinate on the separating equilibrium.*
- *The buyer is unambiguously better-off in a pooling equilibrium if*

$$\theta_l \leq \tilde{\theta} \equiv \frac{1}{2} - \frac{\tau_i (1 - \tau_i) \psi}{1 + \tau_i (1 - \tau_i) \psi},$$

  with $\tilde{\theta}$ being decreasing in ψ and decreasing (resp. increasing) in τ_i for $\tau_i < \frac{1}{2}$ (resp. $>$). If, instead, $\theta_l > \tilde{\theta}$, the buyer is better-off in a pooling equilibrium with price p^* if and only if

$$p^* \leq \frac{1}{1 + \tau_i (1 - \tau_i) \psi}.$$

To understand why total welfare is unambiguously higher in the separating equilibrium, consider the most competitive pooling where both sellers charge the competitive price $p^* = 0$ so that there is no deception at equilibrium. It can easily be shown that total welfare in the separating and in the pooling equilibria is the same. In fact, because $p^* = 0$, the low quality seller has no incentive to deceive the buyer, so that trade is fully efficient. However, since a larger p^* makes low quality sellers more willing to invest in deceptive ads, *ceteris paribus* an increase of p^* reduces welfare as deception is costly.

Also, the low quality seller gains from the possibility of luring the buyer into buying its low quality good. This makes low quality sellers happier in the pooling equilibrium since they would not sell in a separating equilibrium. The reason why high quality sellers prefer to coordinate on the separating equilibrium is that in this type of outcome they sell with certainty at a price higher than the maximal pooling one as stated in Lemma 3.

Interestingly, the buyer may prefer to be deceived in a pooling equilibrium rather than knowing the quality purchased in the separating equilibrium. The reason why the buyer may prefer such an outcome rests on the multiplicity implied by such class of equilibria. Suppose that sellers coordinate on a pooling equilibrium that features a price p^* close to zero. Then, seller i's incentive to deceive the buyer is weak (i.e., μ_i^* is small) and the negative effect of deception is negligible. Of course, for θ_l small enough the quality premium that must be paid to a high quality seller in the separating equilibrium is high, whereby making the buyer unambiguously better-off in the pooling outcome.

6 Authority sanctioning and welfare implications

As illustrated in the previous section, the effect of deceptive advertising on buyer surplus is ambiguous. A natural question from a policy perspective — and in particular from the viewpoint of buyer protection — is therefore whether it is possible to reduce the overall inefficiency caused by deceptive behavior and/or improve buyer surplus when pooling equilibria are likely to endanger the buyer.

Hence, suppose that a buyer who has purchased a low quality good advertised as high quality is given the opportunity to file a complaint with a buyer protection Authority and, with probability γ, he obtains a refund of the full price.[13] In the US, for instance, the FTC has the power to require that advertisers prove their claims.[14] The agency also requires that this information be available to any buyer who asks for it. If the FTC determines that an advertisement is deceptive, it can stop the ad and order the sponsor to issue corrections. Some companies are fined for their illegal acts — see, e.g., the FTC Policy Statement on Deception (1982).

The parameter γ can be interpreted as the Authority's deterrence power, reflecting the chances it detects deception and triggers a full reimbursement. Hastak and Mazis (2011), for instance, discuss thoroughly a type of deceptive advertising which is certainly perceived by buyers but not easily verified by a court. In particular, an Authority may well face cases where the misleading content of an ad is neatly perceived yet not necessarily self evident in probation. This provides a natural justification of the assumption on the Authority's probabilistic detection capability.

Under this assumption, using the approach developed in the previous section, the optimal coverage for a low quality seller in a pooling equilibrium becomes

$$\hat{\mu}_i = \psi \tau_i (1 - \gamma) \hat{p},$$

where $\hat{p}$ denotes the equilibrium price for any policy such that $\gamma > 0$.[15] The new outcome is intuitive: the higher the chance it is forced to a reimbursement, γ, the lower the amount of deception a seller

[13] The effect of a punishment harsher than full reimbursement is not considered simply because this type of schemes may violate firms *ex-post* limited liability constraints, resulting in enforcement problems.

[14] The Advertising Substantiation Program has been issued by the FTC in April 1979 and it has been discussed thoroughly by Sauer and Leffler (1990).

[15] Of course, $\hat{p} = p^*$ for $\gamma = 0$.

indulges on for a given price.

Following the same reasoning of Proposition 1, we introduce the following result:

Proposition 6. *Under* ***A1*** *and* ***A2****, there exist a continuum of pooling equilibria with the following features:*

- *Every price* $\hat{p} \in \hat{\mathcal{P}} \equiv [0, \hat{p}_{\max}]$*, with* $\hat{p}_{\max} > 0$*, can be supported in the pooling equilibrium. Moreover*

$$\hat{p}_{\max} = \begin{cases} \underline{p}(l) & \Leftrightarrow \quad \theta_l \geq \hat{\theta} \\ \theta_l + \frac{1}{2} & \Leftrightarrow \quad \theta_l < \hat{\theta} \end{cases},$$

  where now

$$\underline{p}\,(l) = \frac{1-\sqrt{1-2\psi\underline{\tau}^2(1-\gamma)^2}}{\psi\underline{\tau}^2(1-\gamma)^2}, \qquad \hat{\theta} \equiv \underline{p}\,(l) - \frac{1}{2},$$

  with $\underline{\hat{p}}\,(l)$ *being increasing in* ψ *and* $\underline{\tau}$ *and decreasing in* γ*.*

- *For any equilibrium price* $\hat{p} \in \hat{\mathcal{P}}$ *the equilibrium deceptive advertising* $\hat{\mu}_i$ *is such that*

$$\hat{\mu}_i = \psi\tau_i\hat{p}\,(1-\gamma) < 1 \quad \forall i = 1, 2 \quad \Rightarrow \quad \frac{\hat{\mu}_1}{\hat{\mu}_2} = \frac{\tau_1}{1-\tau_1},$$

  implying the following posteriors when the buyer receives two ads

$$\hat{\beta}\,(q_1 = h | s_1 = h, s_2 = h, \hat{\boldsymbol{\sigma}}) = \hat{\beta}\,(q_2 = h | s_1 = h, s_2 = h, \hat{\boldsymbol{\sigma}}) = \tfrac{1}{2}.$$

- *The buyer's equilibrium strategy* $\alpha_i^*(\mathbf{s})$ *is such that*

$$\hat{\alpha}_i\,(\mathbf{s}) = \begin{cases} 0 & \Leftrightarrow \quad (s_i = \varnothing, s_{-i} = h) \\ \tau_i & \Leftrightarrow \quad (s_i = h, s_{-i} = h) \\ 1 & \Leftrightarrow \quad (s_i = h, s_{-i} = \varnothing) \end{cases}.$$

The result stated in Proposition 6 features the same type of properties of that stated in Proposition 1. Note, however, that due to the reimbursement chance γ, the convenience of deception is diminished and misleading ads are reduced accordingly.

To grasp some additional intuition beyond what we have already learnt from Proposition 6, let's state the following additional result which parallels Corollary 1:

Corollary 2. *If* $\hat{p}_{\max} = \underline{p}(l)$*, then* $\hat{p}_{\max}$ *is decreasing in* γ *and increasing in both* $\underline{\tau}$ *and* ψ*.*

The intuition is straightforward. The maximal price that sellers can charge in a pooling equilibrium with positive deception is decreasing in the Authority's monitoring intensity γ simply because this weakens the incentive to deviate and reduces the gap between the pooling and the separating outcomes.

We now proceed to characterize full coverage separating equilibria in this new environment. Indeed, it turns out that the possibility of a refund doesn't change the equilibrium characterization. This is not surprising, since in a separating equilibrium the buyer will always buy the high quality good — i.e., the refund will never actually take place. Therefore, Lemma 3 applies practically unchanged. In particular, pooling equilibria do exist alongside with the separating outcome whenever the quality gain is substantive relative to the minimum quality ($\theta_l < \frac{1}{2}$).

However, because the pooling allocation depends on γ, what clearly changes relative to the analysis without the Authority — i.e., Proposition 5 — is the welfare distribution and the equilibrium efficiency in terms of welfare loss . An increase of γ has multiple effects on welfare. First, it reduces the level of deception chosen by the low quality seller lowering the associated cost, which in turn causes a net increase in sellers' aggregate welfare — and a welfare transfer from the bad to the good seller. Second, a higher γ increases the buyer's protection from fraud and this *protection* effect is stronger the higher the *ex-ante* uncertainty about quality — i.e., the closer the priors are to $\frac{1}{2}$. Notice, however, that the larger is γ the higher are the chances that the buyer gets reimbursed when deceived but the lower the chances he is actually deceived as the low quality seller airs less and less ads. It can be shown that the net effect on buyer's welfare is positive.

We are now able to tackle the natural question motivating this section: What is the optimal policy set by an Authority? Or, in other words, what is the optimal deterrence γ that a benevolent planner would want to implement? To this end we shall assume that the Authority faces an enforcement cost $c(\gamma)$ which is increasing, convex and satisfies standard Inada conditions — i.e., $c'(\gamma) > 0$, $c''(\gamma) > 0$, $c'(1) = +\infty$ and $c'(0) = 0$.[16]

Proposition 7. *Given the cost function above, for any price $\hat{p}$ that is expected to be charged in a pooling equilibrium and any prior τ_i, the optimal policy $\hat{\gamma}$ has the following characteristics:*

- *The policy that maximizes Buyer Surplus $\hat{\gamma}_b$ is such that*

$$\psi\tau_i(1-\tau_i)\,\hat{p}\,(1+\hat{p}\,(1-2\hat{\gamma}_b)) = c'(\hat{\gamma}_b).$$

  Notice that $\hat{\gamma}_b$ is increasing in ψ and decreasing in the distance $\left|\tau_i - \frac{1}{2}\right|$. The effect of $\hat{p}$ on $\hat{\gamma}_b$ depends on the level of $\hat{p}$: $\hat{\gamma}_b$ is increasing (resp. decreasing) in $\hat{p}$ when $\hat{p}$ is small enough (resp. large) and $c(\cdot)$ is not too steep.

- *The policy that maximizes Expected Total Welfare $\hat{\gamma}_w$ is such that*

$$\psi\tau_i(1-\tau_i)\,\hat{p}\,(1+\hat{p}\,(1-\hat{\gamma}_w)) = c'(\hat{\gamma}_w),$$

  which is increasing in ψ, decreasing in the distance $\left|\tau_i - \frac{1}{2}\right|$ and unambiguously increasing in $\hat{p}$.

An authority wishes to implement a higher level of protection when the cost of deception is low (ψ large) and/or incentives to deceive the buyer is high, which is the case when the level of *ex-ante*

[16] A cost function that satisfies these assumptions altogether is $c(\gamma) = \frac{\gamma^2}{1-\gamma}$.

uncertainty is high (τ_i close to $\frac{1}{2}$). The first result can be interpreted as the usual result in law and economics (see, e.g., Becker (1968) which relates the intensity of monitoring of the criminal activity and the harshness of the punishment). In fact the cost of deception increases (in expected terms) if sellers are heavily fined for deception. Moreover, the welfare maximizing protection degree is larger the higher the pooling price: in fact, besides a welfare transfer from the buyer to the sellers, an increase in the pooling price fosters more wasteful deception, which reduces the overall welfare because of its dead-weight cost.

A less intuitive result is that, when maximizing buyer surplus, the Authority may set lower optimal protection as the pooling price grows large enough. Indeed, as the price increases, *ceteris paribus* deception increases. This in turn makes protection more valuable as the buyers obtains higher reimbursement more often. However, the benefit of an increase in buyer protection is mitigated by the fact that equilibrium deception is optimally reduced by the low quality seller as γ grows larger, and it is discovered more often. When the price is large enough, the combination of these effects makes the expected reimbursement increment (implied by a larger γ) relatively smaller than the cost increase. So, at some point, the protection level that maximizes buyer surplus must be decreasing in the equilibrium price $\hat{p}$.

Notice that the effect just described is not present when the Authority maximizes the Expected Total Welfare. In this case the reimbursement has a neutral effect on welfare as it plays a simple redistributive role. Hence,

Corollary 3. *Buyer protection requires less enforcement than welfare maximization:* $\hat{\gamma}_b < \hat{\gamma}_w$.

This result offers the surprising prediction that the more an Authority cares about buyers, the less it should protect them from deceptive advertising.

7 Concluding remarks

We studied a simple Bertrand game where two sellers supplying products of different and unverifiable qualities can outwit buyers through their (costly) deceptive advertising. The core contribution of the paper is the characterization of a class of pooling equilibria where low quality sellers deceive buyers that are Bayes rational and make their purchase decision on the basis of the available information. It turns out that, although in these outcomes low quality goods are purchased with positive probability, buyer surplus can be higher than in a fully separating equilibrium thanks to a novel pro-competitive effect due to buyers' ignorance about sellers' product quality. It is also argued that low quality sellers invest more in deceptive advertising the better is their reputation *vis-à-vis* buyers — i.e., the more a seller is trusted by buyers, the more pronounced are its incentives to invest in deceptive advertising when it produces a low quality product.

Finally, as a normative exercise, we have also analyzed a simple policy that involves the enforcement of a (costly) monitoring technology aimed at detecting misleading and deceptive conducts. We found that total welfare maximization requires a more intense monitoring technology than buyer welfare. This is because misleading advertising is decreasing with respect to the intensity of the monitoring technology, and buyers might enjoy to be cheated and then reimbursed rather than paying a greater price for a high quality item. We have also shown that, thanks to this effect, the impact of an increase of the pooling price on the optimal monitoring intensity is non-monotone, suggesting that in response to higher prices buyers may benefit from lower protection.

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Appendix

Proof of Lemma 1. In a separating equilibrium all the information about quality is transmitted by prices and ads have no informative content. Hence, the coverage of a low quality seller at a separating equilibrium is optimally set at zero because deceptive ads are costly. ■

Proof of Lemma 2. Consider a full coverage equilibrium featuring $\tau_1\mu_2^* = \tau_2\mu_1^*$. Having observed the same price and a high signal from both sellers, the buyer uses (1) to update his prior information to a posterior which equals $\beta^*(q_i|\mathbf{s}, \boldsymbol{\sigma}^*) = \frac{1}{2}$ for each seller $i = 1, 2$. Thus, he is *ex-post* indifferent between sellers and randomizes buying from i with probability $\alpha_i(\mathbf{s}, \boldsymbol{\sigma}^*)$. However, the only mixed strategy which makes the equilibrium sustainable is $\alpha_i^*(\mathbf{s}, \boldsymbol{\sigma}^*) = \tau_i$, as it will be clear in what follows.

In fact, say seller i has low quality and the pooling equilibrium features price p^*. Then, if seller i's coverage is $\mu_i \in (0, 1)$, with probability μ_i the buyer observes seller i's signal as well as that of seller j and buys from i with probability α_i^*. Thus, seller i's expected demand is $\alpha_i^*\mu_i$ and the corresponding expected profit is

$$\pi_i^*(p^*|l) = \alpha_i^*\mu_i p^* - \tfrac{\mu_i^2}{2\psi},$$

which yields an optimal coverage equal to $\mu_i^* = \psi p^* \alpha_i^*$. Finally, condition $\tau_i\mu_{-i}^* = \tau_{-i}\mu_i^*$ implies $\tau_i\alpha_{-i}^* = \tau_{-i}\alpha_i^*$ and thus $\alpha_i^* = \tau_i$. ■

Proof of Proposition 1. Consider full coverage equilibria — i.e., $\rho_i = \rho_j = 1$. Let p^* be the price that is expected to be charged in the candidate (pooling) equilibrium. The set of equilibrium pooling prices as well as the equilibrium buyer's strategy are stated in the Proposition and the formal arguments that allow to derive them have been already illustrated in the text, and will be omitted here for brevity.

In what follows we characterize the equilibrium level of deception. buyer's posteriors upon receipt of a high quality ad from both sellers, depend on both sellers' equilibrium deception strategies. Applying (1) we have

$$\beta^*(q_i = h|s_i = h, s_j = h, \boldsymbol{\sigma}^*) \equiv \beta_i^* = \tfrac{\tau_i\mu_j^*}{\tau_i\mu_j^* + \tau_j\mu_i^*},$$

where it should be noticed that

$$\beta_i^* \gtreqless \beta_j^* \quad \Leftrightarrow \quad \tau_i\mu_j^* \gtreqless \tau_j\mu_i^*.$$

The buyer buys from i if $\beta_i^* > \beta_j^*$ — provided $\beta_i^*\theta_h + (1 - \beta_i^*)\theta_l > p^*$ — and vice versa. If $\beta_i^* = \beta_j^*$ he mixes buying from i with probability $\alpha_i^* = \tau_i$ as proven in Lemma 2.

We now argue that the equality of posteriors $\beta_i^* = \beta_j^*$ is a necessary condition for the existence of a positive deception pooling equilibrium. Notice first that μ_i^* is the behavior strategy of seller i when his type is $q_i = l$ — i.e., the optimal level of deception he would set behind the veil of ignorance — before learning his type.

Suppose now that $q_i = h$ and $q_j = l$ and that, before learning their types, sellers i and j have set $\tau_i\mu_j^* > \tau_j\mu_i^*$, implying $\beta_i^* > \beta_j^*$. Then, this cannot be an equilibrium. In fact, *ex-post*seller j will sell no item while spending resources in deceptive advertising. But this implies that seller j is not optimizing and has an incentive to reduce deception: a contradiction.

Suppose now that $q_i = l$ and $q_j = h$ and that, before learning their types, sellers i and j have set $\tau_i\mu_j^* < \tau_j\mu_i^*$, implying $\beta_i^* < \beta_j^*$. Then, this cannot be an equilibrium. In fact, *ex-post* i will sell no item while spending resources in deceptive advertising. But this implies that seller i is not optimizing and has an incentive to reduce deception. Because in a *weak* Perfect Bayesian equilibrium strategies must be optimal for each possible type of every player, we conclude that a necessary condition for the existence of a pooling equilibrium with positive deception is

$$\beta_i^* = \beta_j^* \quad \Leftrightarrow \quad \tau_i \mu_j^* = \tau_j \mu_i^*.$$

The characterization of the equilibrium deception levels has been derived in equation (3), and will be thus omitted here. To complete the proof it remains to show that under assumption **A2** the equilibrium deception coverage μ_i^* is interior if $\mu_i^* = \tau_i \psi p_{\max}^* < 1$ for $i = 1, 2$. It is easy to verify that a sufficient condition for this to be true is

$$(1 - \underline{\tau})\, \psi p_{\max}^* < 1,$$

which always holds under **A2**. ■

Proof of Corollary 1. The result follows from simple algebra and given the parameter restrictions. ■

Proof of Proposition 2. Given a pooling price $p_{\max}^*$, the expected profits of each seller conditional on its product quality are

$$\pi_i^*(p_{\max}^*|l) = \frac{\tau_i^2 \psi p_{\max}^{*2}}{2}, \qquad \pi_i^*(p_{\max}^*|h) = \left[1 - (1 - \tau_i)^2 \psi p_{\max}^*\right] p_{\max}^*.$$

Throughout the proof we shall keep in mind the two cases $\tau_i \in \{\underline{\tau}, 1 - \underline{\tau}\}$ and that, depending on the level of θ_l, the corresponding $p_{\max}^*$ is either $p_{\max}^* = \theta_l + \frac{1}{2}$ or $p_{\max}^* = \underline{p}^*(l)$, which can be written as

$$p_{\max}^* = \begin{cases} \frac{1 - \sqrt{1 - 2\psi \tau_i^2}}{\psi \tau_i^2} & \text{if} \quad \tau_i = \underline{\tau} < \frac{1}{2} \\ \frac{1 - \sqrt{1 - 2\psi (1 - \tau_i)^2}}{\psi (1 - \tau_i)^2} & \text{if} \quad \tau_i = 1 - \underline{\tau} > \frac{1}{2} \end{cases}.$$

Step 1. We start by proving that

$$\pi_i^*(p_{\max}^*|h) > \pi_i^*(p_{\max}^*|l).$$

The argument is by contradiction. Suppose that the above inequality is reversed. Then, it can be verfied that it holds if and only if

$$p_{\max}^* \geq \frac{2}{\psi\left[\tau_i^2 + 2(1 - \tau_i)^2\right]}. \tag{A1}$$

Two cases must be considered depending on the sign of $\tau_i - 1/2$.

Case 1. $\tau_i = \underline{\tau} \leq 1/2$.

When θ_l is large, (A1) requires

$$p_{\max}^* = \frac{1 - \sqrt{1 - 2\psi \tau_i^2}}{\psi \tau_i^2} \geq \frac{2}{\psi\left(\tau_i^2 + 2(1 - \tau_i)^2\right)} \quad \Leftrightarrow \quad 1 - \frac{2\tau_i^2}{\tau_i^2 + 2(1 - \tau_i)^2} \geq \sqrt{1 - 2\psi \tau_i^2},$$

where, since $\tau_i < 1/2$ and $\psi < 1$ (by assumption **A2)** both sides of the last inequality are between 0 and 1. Hence,

$$\left[1 - \frac{2\tau_i^2}{\tau_i^2 + 2(1 - \tau_i)^2}\right]^2 \geq 1 - 2\psi \tau_i^2 \quad \Leftrightarrow \quad \psi \geq \frac{4(1 - \tau_i)^2}{(2 - \tau_i(4 - 3\tau_i))^2} > 1,$$

which violates **A2**, which provides the desired contradiction.

By contrast, when θ_l is small the equilibrium characterization implies that

$$p^*_{\max} = \theta_l + \tfrac{1}{2} < \tfrac{1-\sqrt{1-2\psi\tau_i^2}}{\psi\tau_i^2} < \tfrac{2}{\psi\left[\tau_i^2+2(1-\tau_i)^2\right]},$$

where the last inequality has been proven in the previous point. Hence the result.

Case 2. $\tau_i = 1-\underline{\tau} \geq \frac{1}{2}$.

Suppose, again by contradiction, that $\pi_i^*(p^*_{\max}|l) > \pi_i^*(p^*_{\max}|h)$. In this case inequality (A1) holds if and only if

$$\tfrac{1-\sqrt{1-2\psi(1-\tau_i)^2}}{\psi(1-\tau_i)^2} \geq \tfrac{2}{\psi\left[\tau_i^2+2(1-\tau_i)^2\right]} \quad \Leftrightarrow \quad \tfrac{\tau_i^2}{\tau_i^2+2(1-\tau_i)^2} \geq \sqrt{1-2\psi\left(1-\tau_i\right)^2},$$

where both sides of the second inequality are between 0 and 1. Hence,

$$\left[\tfrac{\tau_i^2}{\tau_i^2+2(1-\tau_i)^2}\right]^2 \geq 1-2\psi\left(1-\tau_i\right)^2 \quad \Leftrightarrow \quad \psi \geq \tfrac{2-4\tau_i(1-\tau_i)}{\left[\tau_i^2+2(1-\tau_i)^2\right]^2} > 1,$$

which clearly violates **A2**. Thus (A1) is not satisfied, which yields the desired contradiction.

Step 2. Let's now prove that $\pi_i^*(p^*_{\max}|q)$ increases with τ_i (and decreases with τ_j) for any $q \in \{l,h\}$.

Notice first that, whenever $\pi_i^*(p^*_{\max}|q)$ increases with τ_i, it necessarily decreases with τ_j because $\tau_i = 1-\tau_j$. Second, notice than when $p^*_{\max} = \theta_l + \frac{1}{2}$ the proof is trivial. So, consider $p^*_{\max} = \underline{p}^*(l)$. Differentiating with respect to τ_i we have

$$\frac{\partial \pi_i^*(p^*_{\max}|l)}{\partial \tau_i} = \psi\tau_i p^*_{\max}\left[p^*_{\max} + \tau_i\frac{\partial p^*_{\max}}{\partial \tau_i}\right], \tag{A2}$$

$$\frac{\partial \pi_i^*(p^*_{\max}|h)}{\partial \tau_i} = \frac{\partial p^*_{\max}}{\partial \tau_i} + 2\psi\left(1-\tau_i\right)p^*_{\max}\left[p^*_{\max} - \left(1-\tau_i\right)\frac{\partial p^*_{\max}}{\partial \tau_i}\right]. \tag{A3}$$

Again, we have two cases to analyze depending on the sign of $\tau_i - 1/2$. Before proceeding it should noticed that

$$\frac{\partial p^*_{\max}}{\partial \tau_i} = \begin{cases} \frac{2-2\psi\tau_i^2-2\sqrt{1-2\psi\tau_i^2}}{\psi\tau_i^3\sqrt{1-2\psi\tau_i^2}} > 0 & \text{if} \quad \tau_i = \underline{\tau} < \frac{1}{2} \\ \frac{2\psi(1-\tau_i)^2-2+2\sqrt{1-2\psi(1-\tau_i)^2}}{\psi(1-\tau_i)^3\sqrt{1-2\psi(1-\tau_i)^2}} < 0 & \text{if} \quad \tau_i = 1-\underline{\tau} > \frac{1}{2} \end{cases}.$$

Let's proceed then case by case.

Case 1. $\tau_i = \underline{\tau} \leq 1/2$.

In this region of parameters $p^*_{\max}$ increases with τ_i. Hence, it follows immediately from (A2) that $\pi_i^*(p^*_{\max}|l)$ is increasing in τ_i. Moreover,

$$\frac{\partial \pi_i^*(p^*_{\max}|h)}{\partial \tau_i} = \frac{2\left[1-\psi\tau_i^2-\sqrt{1-2\psi\tau_i^2}\right](\tau_i(6-\tau_i)-4)+4\psi\tau_i^2(1-\tau_i)\left[1-\sqrt{1-2\psi\tau_i^2}\right]}{\psi\tau_i^5\sqrt{1-2\psi\tau_i^2}} \tag{A4}$$

Notice first that the denominator is certainly positive. The numerator is negative if and only if

$$\frac{\psi\left[1-\sqrt{1-2\psi\tau_i^2}\right]}{1-\psi\tau_i^2-\sqrt{1-2\psi\tau_i^2}} < \frac{4-\tau_i(6-\tau_i)}{2\tau_i^2(1-\tau_i)},$$

where it can be verified that left-hand side of this inequality is increasing in ψ. Thus, a necessary condition for $\pi_i^*(p_{\max}^*|h)$ to be decreasing in τ_i for at least some $\tau_i < 1/2$, is

$$\lim_{\psi\to 1} \frac{\psi\left[1-\sqrt{1-2\psi\tau_i^2}\right]}{1-\psi\tau_i^2-\sqrt{1-2\psi\tau_i^2}} = \frac{1-\sqrt{1-2\tau_i^2}}{1-\tau_i^2-\sqrt{1-2\tau_i^2}} < \frac{4-\tau_i(6-\tau_i)}{2\tau_i^2(1-\tau_i)}.$$

But, this turns out to be impossible for all $\tau_i < 1/2$. Hence, (A4) cannot be negative and we must conclude that $\pi_i^*(p_{\max}^*|h)$ is increasing in τ_i.

Case 2. $\tau_i = 1-\underline{\tau} \geq 1/2$.

Differentiating with respect to τ_i it follws that

$$\frac{\partial\pi_i^*(p_{\max}^*|l)}{\partial\tau_i} = \tau_i\left[1-\sqrt{1-2\psi\left(1-\tau_i\right)^2}\right]\cdot\frac{2\psi(1-\tau_i)^2-(1+\tau_i)\left[1-\sqrt{1-2\psi(1-\tau_i)^2}\right]}{\psi(1-\tau_i)^5\sqrt{1-2\psi(1-\tau_i)^2}}.$$

We shall now prove that (7) cannot be negative. In fact, the first multiplier and the denominator of the second multiplier are positive and the sign depends on the numerator. Using $\tau_i > 1/2$ and $\psi \leq 1$, it is easy to verify that the sign of (7) is negative if and only if

$$\sqrt{1-2\psi\left(1-\tau_i\right)^2} < 1-\frac{2\psi(1-\tau_i)^2}{1+\tau_i} \quad\Leftrightarrow\quad \psi > \frac{1+\tau_i}{2(1-\tau_i)} \geq \frac{3}{2},$$

which clearly violates **A2**. Hence, the sign of (7) is positive and $\pi_i^*\left(p_{\max}^*|\theta_l\right)$ increases with τ_i.

Second, we prove that $\pi_i^*(p_{\max}^*|h)$ is increasing in τ_i. In this case the expression in (A3) specifies to

$$\frac{\partial\pi_i^*(p_{\max}^*|h)}{\partial\tau_i} = \frac{2-2\psi(1-\tau_i)^2-2\sqrt{1-2\psi(1-\tau_i)^2}}{\psi(1-\tau_i)^3\sqrt{1-2\psi(1-\tau_i)^2}}. \tag{A5}$$

Again, proceeding by contradiction, suppose that (A5) is negative. This is true if and only if

$$1-\psi\left(1-\tau_i\right)^2 < \sqrt{1-2\psi\left(1-\tau_i\right)^2} \quad\Leftrightarrow\quad -\psi^2\left(1-\tau_i\right)^4 > 0,$$

which is clearly impossible. Hence, the sign of (A5) is positive and so $\pi_i^*(p_{\max}^*|h)$ is increasing in τ_i.

Step 3. It trivially follows from $\tau_i < \tau_j$ that

$$\pi_i^*\left(p_{\max}^*|l\right) = \frac{\tau_i^2 p_{\max}^{*2}}{2} < \frac{(1-\tau_i)^2 p_{\max}^{*2}}{2} = \pi_j^*\left(p_{\max}^*|l\right),$$

$$\pi_i^*\left(p_{\max}^*|h\right) = \left[1-\left(1-\tau_i\right)^2 p_{\max}^*\right]p_{\max}^* < \left[1-\tau_i^2 p_{\max}^*\right]p_{\max}^* = \pi_j^*\left(p_{\max}^*|h\right),$$

which concludes the proof. ■

Proof of Proposition 3. The proof is straightforward for $p_{\max}^* = \theta_l + \frac{1}{2}$. Consider then $p_{\max}^* = \underline{p}^*(l)$. Proceeding again by contradiction, notice that

$$\frac{\partial \mathrm{E}_{\mathbf{s}}[v(s_i,s_j)]}{\partial\underline{\tau}} = \frac{(2+\psi\underline{\tau})\left[1-\sqrt{1-2\psi\underline{\tau}^2}\right]-2\psi\underline{\tau}^2\left[1+\psi\underline{\tau}^2\right]}{\psi\underline{\tau}^3\sqrt{1-2\psi\underline{\tau}^2}} > 0 \quad\Leftrightarrow\quad -\frac{2(1-\underline{\tau})+\psi\underline{\tau}\left[1-2\underline{\tau}+2\underline{\tau}^2\left(2+\psi\underline{\tau}^2\right)\right]}{(2+\psi\underline{\tau})^2} > 0,$$

which is clearly impossible. Hence, $\mathrm{E}_{\mathbf{s}}[v(s_i, s_j)]$ must be decreasing in $\underline{\tau}$. The comparative statics with respect to θ_l is obvious. ■

Proof of Proposition 4. The Expected Total Welfare is

$$\mathrm{ETW} = 1 + \theta_l - \psi\tau_i\left(1-\tau_i\right)\tfrac{2+p^*_{\max}}{2}p^*_{\max}.$$

Let's proceed point by point.

Point 1. Suppose that $p^*_{\max} = \theta_l + \frac{1}{2}$. The ETW is clearly increasing in the distance between τ_i and $\frac{1}{2}$. When, instead, $p^*_{\max} = \underline{p}^*(l)$, we have two different scenarios to analyze: $\tau_i = \underline{\tau}$ and $\tau_i = 1-\underline{\tau}$. We will prove that ETW is decreasing with respect to τ_i in the first case — i.e., when $\tau_i < 1/2$ and an increase in τ_i reduces the distance to $\frac{1}{2}$ — and increasing in the second — i.e., when $\tau_i > 1/2$ and an increase in τ_i increases the distance to $1/2$.

First, take $\tau_i = \underline{\tau}$. Differentiating with respect ot τ_i

$$\tfrac{\partial \mathrm{ETW}}{\partial \tau_i} = -\psi\left[\left(1-2\tau_i\right)\tfrac{2+p^*_{\max}}{2}p^*_{\max} + \tau_i\left(1-\tau_i\right)\left(1+p^*_{\max}\right)\tfrac{\partial p^*_{\max}}{\partial \tau_i}\right] < 0,$$

because $\tau_i < 1/2$ and $p^*_{\max}$ is increasing in τ_i, where the latter inequality follows, by contradiction, from

$$\tfrac{\partial p^*_{\max}}{\partial \tau_i} = \tfrac{2-2\psi\tau_i^2-2\sqrt{1-2\psi\tau_i^2}}{\psi\tau_i^3\sqrt{1-2\psi\tau_i^2}} < 0 \quad\Leftrightarrow\quad -\psi^2\tau_i^4 > 0,$$

which is clearly impossible.

Next, take $\tau_i = 1-\underline{\tau}$. In this case, because $p^*_{\max}$ is decreasing in τ_i. Hence, we have to calculate the whole derivative and argue that

$$\tfrac{\partial \mathrm{ETW}}{\partial \tau_i} = \left(1+2\tau_i\right)\frac{1-\psi(1-\tau_i)^2\left[\frac{2\psi(1-\tau_i)^3}{1+2\tau_i}+1\right]-\sqrt{1-2\psi(1-\tau_i)^2}}{\psi(1-\tau_i)^4\sqrt{1-2\psi(1-\tau_i)^2}} > 0. \tag{A6}$$

Suppose, by contradiction, that ETW is non-increasing in τ_i. Then, the numerator of the second multiplier in (A6) must be negative, which can be true if and only if

$$\sqrt{1-2\psi\left(1-\tau_i\right)^2} > 1-\psi\left(1-\tau_i\right)^2\left[1+\tfrac{2\psi(1-\tau_i)^3}{1+2\tau_i}\right]. \tag{A7}$$

First, we show that the right-hand side of inequality (A7) is positive. Let

$$f\left(\psi,\tau_i\right) \equiv \psi\left(1-\tau_i\right)^2\left[1+\tfrac{2\psi(1-\tau_i)^3}{1+2\tau_i}\right].$$

Notice that $f\left(\psi,\tau_i\right)$ is increasing in ψ. Let's thus maximize it for $\psi = 1$. Notice that, for all $\tau_i \geq \frac{1}{2}$

$$\tfrac{\partial f(1,\tau_i)}{\partial \tau_i} = -2\left(1-\tau_i\right)\tfrac{9\tau_i^3+(1-\tau_i)\left[8-\tau_i+8\tau_i^3\right]}{(2\tau_i+1)^2} < 0.$$

Thus $f\left(1,\tau_i\right)$ has a maximum at $\tau_i = \frac{1}{2}$, and it can be readily shown that $f(1,1/2) < 1$. Then the inequality in (A7) is preserved taking squares on both sides. Hence, (A7) holds if and only if

$$1-2\psi\left(1-\tau_i\right)^2 > \left[1-\psi\left(1-\tau_i\right)^2\left(1+\frac{2\psi(1-\tau_i)^3}{1+2\tau_i}\right)\right]^2 \quad \Leftrightarrow$$
$$3\left(1-2\tau_i\right)\left(1+2\tau_i\right)-4\psi\left(1-\tau_i\right)^3\left[1+2\tau_i+\psi\left(1-\tau_i\right)^3\right]>0,$$

which is impossible since $\tau_i > 1/2$. This provides the desired contradiction.

Point 2. When $p^*_{\max} = \theta_l + \frac{1}{2}$ the ETW is obviously decreasing in ψ. When $p^*_{\max} = \underline{p}^*_l(l)$ we have that

$$\frac{\partial \text{ETW}}{\partial \psi} = -\tau_i\left(1-\tau_i\right)\left[\frac{2+p^*_{\max}}{2}p^*_{\max} + \psi\left(1+p^*_{\max}\right)\frac{\partial p^*_{\max}}{\partial \psi}\right] < 0,$$

because $p^*_{\max}$ is increasing in τ_i, which, in turn, follows by contradiction from the observation that

$$\frac{\partial p^*_{\max}}{\partial \tau_i} = \frac{1-\psi\tau_i^2-\sqrt{1-2\psi\tau_i^2}}{\psi^2\tau_i^2\sqrt{1-2\psi\tau_i^2}} < 0 \quad \Leftrightarrow \quad -\psi^2\tau_i^4 > 0.$$

Point 3. Showing that ETW is increasing in θ_l when when $p^*_{\max} = \underline{p}^*(l)$ is trivial because $\underline{p}^*(l)$ does not depend on θ_l. When, instead, $p^*_{\max} = \theta_l + \frac{1}{2}$ one can immediately show that

$$\frac{\partial \text{ETW}}{\partial \theta_l} = 1-\psi\tau_i\left(1-\tau_i\right)\left[\theta_l+\tfrac{3}{2}\right] \gtreqless 0 \quad \Leftrightarrow \quad \theta_l \lesseqgtr \theta_w \equiv \frac{1}{\psi\tau_i(1-\tau_i)} - \frac{3}{2}. \ \blacksquare$$

Proof of Lemma 3. A full coverage separating equilibrium, as argued in the text, may exist only if $\overline{p}^* > p^*_{\max}$. In other words, a necessary condition for the existence of such an equilibrium is that the high quality good price $\overline{p}^*$ $(=1)$ at a separating equilibrium is higher that the maximal pooling price $p^*_{\max}$, because this discourages the low quality seller from mimicking the high quality seller inducing a pooling equilibrium. Depending on $p^*_{\max}$, the condition $\overline{p}^* > p^*_{\max}$ specifies into the following two cases:

(i) If $p^*_{\max} = \underline{p}^*(l)$, a separating equilibrium never exists: in fact its existence would require

$$1 > \frac{1-\sqrt{1-2\psi\underline{\tau}^2}}{\psi\underline{\tau}^2},$$

which is never true.

(ii) If $p^*_{\max} = \theta_l + \frac{1}{2}$, existence of a separating equilibrium requires $1 > \theta_l + \frac{1}{2}$ or $\theta_l < \frac{1}{2}$.

Thus, separating equilibria exist along with pooling equilibria only when $\theta_l < \frac{1}{2}$ (and $p^*_{\max} = \theta_l + \frac{1}{2}$) while there are only pooling equilibria for $\theta_l > \frac{1}{2}$. $\blacksquare$

Proof of Proposition 5. For the proof it is useful the following Table 1 below

Table 1	Separating	Pooling
$\pi^*_j\left(p^*\vert h\right)$	1	$\left(1-\psi\tau_i^2p^*\right)p^*$
$\pi^*_i\left(p^*\vert l\right)$	0	$\frac{\psi\tau_i^2p^{*2}}{2}$
$\text{E}_s[v(s_i,s_j)]$	θ_l	$1+\theta_l-\left[1+\tau_i\left(1-\tau_i\right)\psi\right]p^*$
ETW	$1+\theta_l$	$1+\theta_l-\psi\tau_i\left(1-\tau_i\right)p^*\frac{2+p^*}{2}$

where profits of the low (i) and high (j) quality seller, Buyer Surplus and Expected Total Welfare are reported for both the separating and the pooling equilibrium. Profits are calculated *ex-post* while Buyer Surplus and Expected Total Welfare are calculated *ex-ante*.

The result on Expected Total Welfare (first point) follows immediately, as well as the statement on which equilibrium is preferred by the low quality seller (second point). The third point follows from the observation that, because $\theta_l < \frac{1}{2}$, $p^* \leq p^*_{\max} = \theta_l + \frac{1}{2} < 1$.

The result on Buyer Surplus follows from subtracting $\mathrm{E}_s[v(s_i, s_j)]$ under the two equilibrium types with the pooling price set to its upper limit of $p^*_{\max} = \theta_l + \frac{1}{2}$ and solving the inequality for θ_l. This yields the sufficient condition

$$\theta_l \leq \tilde{\theta} \equiv \tfrac{1}{2} - \tfrac{\tau_i(1-\tau_i)\psi}{1+\tau_i(1-\tau_i)\psi}.$$

To obtain the necessary and sufficient condition valid for all $p^* \leq \theta_l + \frac{1}{2}$, it suffices to subtract $\mathrm{E}_s[v(s_i, s_j)]$ under the two equilibrium types and solve the inequality for p^*, which yields

$$p^* < \tfrac{1}{1+\tau_i(1-\tau_i)\psi}. \quad \blacksquare$$

Proof of Proposition 6. First, when there is an authortiy that exerts a detection effort γ, the optimal deception in a pooling equilibrium is

$$\hat{\mu}_i = \psi\tau_i\,(1-\gamma)\,\hat{p}$$

where we have already taken into consideration the fact that the buyer's strategy entails $\hat{\alpha}_i(s_i = h, s_{-i} = h) = \tau_i$ and corner solutions are ruled out — sufficient conditions will be provided at the end of the proof. The proof of the above statement follows the very same lines of reasoning as of that of Proposition (1), once recognizing that the expected demand of a low quality seller is $\hat{\alpha}_i\mu_i\,(1-\gamma)\,\hat{p} - \left(\mu_i^2/2\psi\right)$. We omit the remaining part of the proof for brevity. Thus, for a low quality seller, deviation and equilibrium profits are now

$$\pi_i\,(\hat{p}|l) \equiv \max_{p_i}\left\{\alpha_i\,(p_i)\,p_i : \alpha_i\,(p_i) = 1 \Leftrightarrow p_i < \hat{p} - 1\right\} = \hat{p} - 1,$$

$$\hat{\pi}_i\,(\hat{p}|l) \equiv \max_{\mu_i}\left\{\hat{\alpha}_i\,(s_i = h, s_j = h, \hat{\boldsymbol{\sigma}})\,\mu_i\hat{p} - \tfrac{\mu_i^2}{2\psi}\right\} = \tfrac{\tau_i^2\psi((1-\gamma)\hat{p})^2}{2}.$$

A low quality seller does not deviate when

$$\pi_i\,(\hat{p}|l) \leq \hat{\pi}_i\,(\hat{p}|l) \quad \Leftrightarrow \quad \hat{p} - 1 \leq \tfrac{\tau_i^2\psi((1-\gamma)\hat{p})^2}{2}.$$

Letting

$$\underline{p}\,(l) \equiv \tfrac{1-\sqrt{1-2\psi\tau_i^2(1-\gamma)^2}}{\psi\tau_i^2(1-\gamma)^2}, \qquad \overline{p}\,(l) \equiv \tfrac{1+\sqrt{1-2\psi\tau_i^2(1-\gamma)^2}}{\psi\tau_i^2(1-\gamma)^2},$$

with $1 < \underline{p}\,(l) \leq \overline{p}\,(l)$. We characterize the set of prices making a low quality seller willing to play equilibrium strategies

$$\hat{p} \in \hat{\mathcal{P}}'(l) \equiv \left\{\hat{p} : \hat{p} \in \left[0, \underline{p}(l)\right] \cup \left[\overline{p}\,(l)\,, +\infty\right]\right\}.$$

As mentioned above, we are interested in positive deception pooling equilibria which are acceptable to the buyer

$$\hat{\mathcal{P}}(l) \equiv \hat{\mathcal{P}}'(l) \cap \left\{\hat{p} : \hat{p} \leq \theta_l + \tfrac{1}{2}\right\},$$

where the second set represents the participation constraint of the buyer. Notice that the latter is unchanged because the buyer's posteriors in equilibrium are, again, equal to 1/2.

Let's now move to the incentives of a high quality seller. Its profits under deviation and in equilibrium are, respectively

$$\pi_i\left(\hat{p}|h\right) \equiv \max_{p_i}\left\{\alpha_i\left(p_i\right)p_i : \alpha_i\left(p_i\right)=1 \Leftrightarrow p_i<\hat{p}-1\right\}=\hat{p}-1,$$

$$\hat{\pi}_i\left(\hat{p}|h\right) \equiv \left[\left(1-\hat{\mu}_j\right)+\hat{\mu}_j\hat{\alpha}_i\left(s_i=h,s_j=h,\hat{\boldsymbol{\sigma}}\right)\right]\hat{p}=\left[1-\psi\left(1-\gamma\right)\left(1-\tau_i\right)^2\hat{p}\right]\hat{p},$$

which yield the following no deviation condition

$$\pi_i\left(\hat{p}|h\right)\leq\hat{\pi}_i\left(\hat{p}|h\right) \quad \Leftrightarrow \quad \hat{p}-1\leq\left[1-\psi\left(1-\gamma\right)\left(1-\tau_i\right)^2\hat{p}\right]\hat{p}.$$

Letting

$$\underline{p}\left(h\right)\equiv-\tfrac{1}{\left(1-\tau_i\right)\sqrt{\psi\left(1-\gamma\right)}}, \qquad \overline{p}\left(h\right)\equiv\tfrac{1}{\left(1-\tau_i\right)\sqrt{\psi\left(1-\gamma\right)}},$$

with

$$\underline{p}\left(h\right)<0<\overline{p}\left(h\right),$$

we have

$$\hat{\mathcal{P}}(h)\equiv\left\{\hat{p}:\hat{p}\leq\min\left\{\overline{p}\left(h\right),\theta_l+\tfrac{1}{2}\right\}\right\}.$$

where, again, the participation constraint of the buyer has already been incorporated. We shall note that $\overline{p}\left(h\right)$ is increasing in γ. Notice also that it can be shown that $\underline{p}\left(l\right)<\overline{p}\left(h\right)<\overline{p}\left(l\right)$ for $\psi\leq 1$ and $\underline{\tau}<1/2$. Considering the above characterization, the result stated in the proposition follows.

Finally, it is easy to show that under **A2** there are no corner solutions. ■

Proof of Corollary 2. The result follows from simple algebra and given the parameter restrictions. ■

Proof of Proposition 7. Consider $\theta_l<\frac{1}{2}$ and let τ_i be the generic prior on seller i's quality. Take a pooling price equal to $\hat{p}$ and proceed point by point.

The policy which maximizes buyer's expected utility in a pooling equilibrium is

$$\hat{\gamma}_b\equiv\arg\max_{\gamma\in[0,1]}\left\{1+\theta_l-\hat{p}-\left(1-\gamma\hat{p}\right)\psi\tau_i\left(1-\tau_i\right)\hat{p}\left(1-\gamma\right)-c\left(\gamma\right)\right\},$$

where the maximand is buyer surplus minus the enforcemtn cost $c\left(\gamma\right)$. Given the properties of $c(.)$, the problem is concave and is solved by the unique value $\hat{\gamma}_b$ such that

$$\psi\tau_i\left(1-\tau_i\right)\hat{p}\left(1+\hat{p}\left(1-2\hat{\gamma}_b\right)\right)=c'\left(\hat{\gamma}_b\right). \tag{A8}$$

Notice that $\hat{\gamma}_b$ is increasing in ψ and decreasing in the distance of τ_i from $\frac{1}{2}$ because $c''\left(.\right)>0$. As for the effect of $\hat{p}$ on $\hat{\gamma}_b$, applying the Implicit Function Theorem to (A8), we obtain

$$\tfrac{\partial\hat{\gamma}_b}{\partial\hat{p}}=\tfrac{\psi\tau_i\left(1-\tau_i\right)\left(1+2\hat{p}\left(1-2\hat{\gamma}_b\right)\right)}{2\psi\tau_i\left(1-\tau_i\right)\hat{p}^2+c''\left(\hat{\gamma}_b\right)}\gtreqless 0 \quad \Leftrightarrow \quad \hat{\gamma}_b\lesseqgtr\tilde{\gamma},$$

where

$$\tilde{\gamma}\equiv\tfrac{1+2\hat{p}}{4\hat{p}}. \tag{A9}$$

To determine how $\hat{\gamma}_b$ vaires with respect to $\hat{p}$, substitue $\tilde{\gamma}$ into the first-order condition (A8), so to obtain

$$\tfrac{\psi\tau_i(1-\tau_i)}{2}\hat{p} = c'\left(\tilde{\gamma}\right). \tag{A10}$$

The left-hand side of (A10) is increasing in $\hat{p}$, while the right-hand side is decreasing. Assume that

$$\tfrac{\psi\tau_i(1-\tau_i)\hat{p}_{\max}}{2} > c'\left(\tilde{\gamma}\right).$$

Thus, if $c\left(.\right)$ is not too steep, there exists $\tilde{p} \in (0, p^*_{\max})$ such that the first-order condtion is 0 when evaluated at $(\tilde{\gamma}, \tilde{p})$. Notice that $\tilde{p}$ is increasing in ψ and decreasing in the distance of the priors to $\frac{1}{2}$. Suppose now that $\hat{p} < \tilde{p}$. Thus, from (A10) we know the first-order condition has negative sign at $\tilde{\gamma}$ which directly implies that $\hat{\gamma}_b < \tilde{\gamma}$ and, from (A9), that $\hat{\gamma}_b$ is increasing in $\hat{p}$. By contrast, when $\tilde{p} > \tilde{p}$ equation (A10) implies the first-order condition has positive sign at $\tilde{\gamma}$ which in turn implies that $\hat{\gamma}_b > \tilde{\gamma}$ must hold and, from (A9), $\hat{\gamma}_b$ is decreasing in $\tilde{p}$.

The policy maximizing the total welfare in a pooling equilibrium is set by the Authority before knowing sellers' qualities. Thus it maximizes the *ex-ante* welfare. Formally, it solves the following program

$$\hat{\gamma}_w \equiv \underset{\gamma\in[0,1]}{\arg\max} \left\{1 + \theta_l - \psi\tau_i\left(1-\tau_i\right)\hat{p}\left(1-\gamma\right)\tfrac{2+\hat{p}(1-\gamma)}{2} - c\left(\gamma\right)\right\}.$$

It can be easily shown that ETW is strictly concave. Hence, the solution γ^*_w solves the following necessary and sufficient first-order condition

$$\psi\tau_i\left(1-\tau_i\right)\hat{p}\left(1+\hat{p}\left(1-\hat{\gamma}_w\right)\right) = c'\left(\hat{\gamma}_w\right).$$

Notice that $\hat{\gamma}_w$ is increasing in ψ, decreasing in the distance of τ_i from $\frac{1}{2}$ and increasing in $\hat{p}$ because $c''\left(.\right) > 0$. In addition, it can be noticed that $\hat{\gamma}_w > \hat{\gamma}_b$. ■