BSWithJump Model And Pricing Of Quanto CDS With FX Devaluation Risk

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Abstract

We present a new model for pricing quanto CDS where the FX could be strongly dependent on the credit reference. The model assumes lognormal hazard rate and deterministic FX local volatility where the FX spot can jump at time of default of the credit reference. We present the model, the calibration algorithm, and the quanto CDS pricing.

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Introduction

In this paper, we present a new model for pricing quanto CDS where the FX is strongly correlated to the credit name. Example: CDS on BRAZIL state with USD as natural currency but paid in real (BRAZIL currency). We would like the model to take into account the FX devaluation risk at time of default. By adding the jump of the FX at time of default, the FX dynamic is more realistic than the simple BS model and the model is producing richer term structures of quanto CDS curves. The jump size will allow us to control the short dated quanto CDS while the volatility of the intensity and the correlation between the intensity and the FX will control the term structure of quanto CDS. The jump parameter is very simple to mark by the traders as it represents one minus the ratio of local currency CDS to domestic currency (typically USD) CDS for short dated maturities.

The first section describes the dynamic of the emerging market FX where we assume constant jump size and stochastic intensity. The second section describes the pricing of FX options within the model. The third section describes the calibration of LN model (lognormal credit intensity) and the calibration of the FX volatility to the term structure of ATM volatilities. In the fourth section, we give some examples of how quanto CDS depends on the various model parameters. In the last section, we give a conclusion and present possible extensions of BSWithJump model.

1. BSWithJump model

1.1 Definitions and notations

N_t is a non-homogeneous Poisson process with intensity λ_t.

τ is the first time where the Poisson process jump or default time.

$S_t^{d/loc}$ is the FX spot where d is domestic currency (typically a G7) and loc is the foreign currency (typically: an emerging market currency).

In all the paper, we assume that the interest rates are deterministic.

We denote $B^d\left(0,T\right)$, and $B^{loc}\left(0,T\right)$ respectively the domestic zero coupon and local currency zero coupon respectively with maturity T.

1.2 Modelling the default intensity: LN model

We suppose that the intensity follows a lognormal process with constant volatility and constant mean reversion.

$$\lambda_t^d = \Gamma^d\left(t\right)e^{\sigma^\lambda e^{-\kappa t}\int_0^t e^{\kappa s}dW_s^\lambda} = \Gamma^d\left(t\right)e^{\sigma_t^\lambda Z_t^\lambda}$$

In practice, we always set the mean reversion to zero.

For simplicity we note $\lambda_t^d = \lambda_t$ and $\Gamma_t^d = \Gamma_t$

1.3 Modelling Emerging market FX with jump at default time

We assume that the FX spot the dynamic:

$$\frac{dS_t^{d/loc}}{S_{t^-}^{d/loc}} = \left(r_t^d - r_t^{loc}\right)dt + \sigma_t^{fx} dW_t^{fx} - J^{d/loc}\left(dN_{t\wedge\tau} - \lambda_t 1_{\{\tau>t\}} dt\right)$$

$$\lambda_t = \Gamma(t) e^{\sigma_t^\lambda Z_t^\lambda}$$

$$d\left\langle W_t^{fx}, W_t^\lambda \right\rangle = \rho dt$$

The process Z_t^λ is a Gaussian process described in the previous section.
$J^{d/loc}$ is a constant between 0 and 1.
In case of deterministic credit, the spot process follows a lognormal dynamic before and after the default time.
The FX spot **can jump only once**: at time of default.
We suppose that the interest rates are deterministic.
Let us calculate the dynamic of $\ln S_t^{d/loc}$

$$d\ln S_t^{d/loc} = \left(r_t^d - r_t^{loc} - \frac{\left(\sigma_t^{fx}\right)^2}{2} + J^{d/loc}\lambda_t 1_{\{\tau>t\}} \right) dt + \sigma_t^{fx} dW_t^{d/loc} + \ln\left(1 - J^{d/loc}\right) dN_{t\wedge\tau}$$

It follows that the stock process is given by:

$$\begin{aligned} S_t^{d/loc} &= X_t^{d/loc} \exp\left(\ln\left(1 - J^{d/loc}\right)\int_0^t dN_{s\wedge\tau} + J^{d/loc}\int_0^t \lambda_s 1_{\{\tau>s\}} ds\right) \\ &= X_t^{d/loc}\left[\left(1 - J^{d/loc}\right)^{N_{t\wedge\tau}} \exp\left(J^{d/loc}\int_0^{t\wedge\tau} \lambda_s ds\right)\right] \\ &= X_t^{d/loc} Z_t^{d/loc} \end{aligned}$$

Where

$$X_t^{d/loc} = S_0^{d/loc} \frac{B^{loc}(0,t)}{B^d(0,t)} \exp\left(\int_0^t \sigma_u^{fx} dW_u^{fx} - \frac{1}{2}\int_0^t \left(\sigma_u^{fx}\right)^2 du\right)$$

The stock process is the product of the forward a continuous martingale and a discontinuous martingale.

2 Pricing FX call options

In order to price a call option within this model we need to separate the calculation into two cases: default before maturity and no default before maturity.

$$\begin{aligned} C(T,K) &= B^d(0,T) E\left(\left(S_T^{d/loc} - K\right)_+ \left(1_{\{\tau\le T\}} + 1_{\{\tau>T\}}\right)\right) \\ &= B^d(0,T)\int_0^T E\left[\left(S_T^{d/loc} 1_{\{\tau=u\}} - K\right)_+ dQ^d(\tau = u)\right] \\ &\quad + B^d(0,T) E\left(\left(S_T^{d/loc} - K\right)_+ 1_{\{\tau>T\}}\right) \end{aligned}$$

The FX spot conditionally on default occurring at u where $u \le T$ is:

$$S_T^{d/loc} 1_{\{\tau=u\}} = S_0^{d/loc} X_T^{d/loc} (1 - J) \exp\left(J\int_0^u \lambda_s ds\right) 1_{\{\tau=u\}}$$

It follows that

$$C(T,K)=B^d(0,T)\int_0^T E^{Q^d}\left(\lambda_u e^{-\int_0^u \lambda_s ds}\left((1-J)X_T^{d/loc}e^{J\int_0^u \lambda_s ds}-K\right)_+\right)du$$

$$+B^d(0,T)E^{Q^d}\left(e^{-\int_0^T \lambda_s ds}\left(X_T^{d/loc}e^{J\int_0^T \lambda_s ds}-K\right)_+\right)$$

$$=C_{def}(T,K)+C_{sur}(T,K)$$

C_{def} is the default part of the call price

C_{sur} is the survival part of the call price.

2.1 Simplification of the default part

The default part of the call price makes the calibration of the FX volatility using forward PDEs more difficult because it depends on the whole path of the intensity and the survival probabilities. We will work out C_{def} to make it depends only on the terminal values of markovian processes.

We perform a change in C_{def}

$$v=e^{-\int_0^u \lambda_s ds}$$

$$dv=-\lambda_u e^{-\int_0^u \lambda_s ds}$$

The expression of C_{def} becomes:

$$C_{def}(T,K)=B^d(0,T)E^{Q^d}\left((1-J)X_T^{d/loc}\int_{e^{-\int_0^T \lambda_s ds}}^1\left(\frac{1}{v^J}-\frac{1}{k^J}\right)_+dv\right)$$

$$k=\left(\frac{(1-J)X_T^{d/loc}}{K}\right)^{\frac{1}{J}}$$

We define the process $Y_T^{d/loc}=X_t^{d/loc}e^{J\int_0^t \lambda_s ds}$ and the function $A(x)=\int_0^x\left(\frac{1}{v^J}-\frac{1}{k^J}\right)_+dv$

It follows that:

$$C_{def}(T,K)=B^d(0,T)E^{Q^d}\left((1-J)X_T^{d/loc}A(1)\right)-B^d(0,T)E^{Q^d}\left((1-J)X_T^{d/loc}A\left(e^{-\int_0^T \lambda_s ds}\right)\right)$$

$$=D_1-D_2$$

Let's calculate A(x):

$$A(x)=\int_0^x\left(\frac{1}{v^J}-\frac{1}{k^J}\right)_+dv=\left(1_{\{k>x\}}+1_{\{k<x\}}\right)\int_0^x\left(\frac{1}{v^J}-\frac{1}{k^J}\right)_+dv$$

$$=1_{\{k>x\}}\int_0^x\left(\frac{1}{v^J}-\frac{1}{k^J}\right)dv+1_{\{k<x\}}\int_0^k\left(\frac{1}{v^J}-\frac{1}{k^J}\right)dv$$

$$=1_{\{k>x\}}\left(\frac{x^{1-J}}{1-J}-\frac{x}{k^J}\right)+1_{\{k<x\}}\left(\frac{k^{1-J}}{1-J}-k^{1-J}\right)$$

By replacing x by 1 in A, we find D_1

$$D_1 = B^d\left(0,T\right)E^{Q^d}\left(\left[X_T^{d/loc} - K\right]1_{\left\{(1-J)X_T^{d/loc} > K\right\}}\right)$$
$$+B^d\left(0,T\right)JE^{Q^d}\left(\left(\frac{\left(1-J\right)X_T^{d/loc}}{K}\right)^{\frac{1-J}{J}}1_{\left\{(1-J)X_T^{d/loc} < K\right\}}\right)$$

By replacing x by $e^{-\int_0^T \lambda_s ds}$ in A, we find D_2

$$D_2 = B^d\left(0,T\right)E^{Q^d}\left(e^{-\int_0^T \lambda_s ds}\left[Y_T^{d/loc} - K\right]1_{\left\{(1-J)Y_T^{d/loc} > K\right\}}\right)$$
$$+B^d\left(0,T\right)JE^{Q^d}\left(e^{-\int_0^T \lambda_s ds}\left(\frac{\left(1-J\right)Y_T^{d/loc}}{K}\right)^{\frac{1-J}{J}}1_{\left\{(1-J)Y_T^{d/loc} < K\right\}}\right)$$

Grouping our terms, we derive the following expression for the call price:

$$C\left(T,K\right) = B^d\left(0,T\right)E^{Q^d}\left(1_{\left\{(1-J)X_T^{d/loc} > K\right\}}\left(X_T^{d/loc} - K\right)\right)$$
$$+B^d\left(0,T\right)JE^{Q^d}\left(1_{\left\{(1-J)X_T^{d/loc} < K\right\}}X_T^{d/loc}\left(\frac{\left(1-J\right)X_T^{d/loc}}{K}\right)^{\frac{1-J}{J}}\right)$$
$$-B^d\left(0,T\right)E^{Q^d}\left(e^{-\int_0^T \lambda_s ds}1_{\left\{(1-J)Y_T^{d/loc} > K\right\}}\left(Y_T^{d/loc} - K\right)\right)$$
$$-B^d\left(0,T\right)JE^{Q^d}\left(e^{-\int_0^T \lambda_s ds}1_{\left\{(1-J)Y_T^{d/loc} < K\right\}}Y_T^{d/loc}\left(\frac{\left(1-J\right)Y_T^{d/loc}}{K}\right)^{\frac{1-J}{J}}\right)$$
$$+B^d\left(0,T\right)E^{Q^d}\left(e^{-\int_0^T \lambda_s ds}\left(Y_T^{d/loc} - K\right)_+\right)$$

We group the call option terms in two parts where the first part can be calculated using a closed form solution.

$$C_{cf}\left(T,K\right) = B^d\left(0,T\right)E^{Q^d}\left(f\left(X_T^{d/loc}\right)\right)$$
$$C_{ncf}\left(T,K\right) = B^d\left(0,T\right)E^{Q^d}\left(e^{-\int_0^T \lambda_s ds}g\left(Y_T^{d/loc}\right)\right)$$

The first part of the call option formula depends only on the terminal value of the lognormal process $X_T^{d/loc}$ and the survival value of a function of the terminal value $Y_T^{d/loc}$. This form will allow us to calibrate the FX volatility more easily than the original formula.

2.2 Pricing of FX options in case of deterministic credit

In case of deterministic credit, the FX option price is given by a closed form solution. In the previous section, we have proved that a call option price is the sum of five terms:

$$C(T,K) = C_1(T,K) + C_2(T,K) - C_3(T,K) - C_4(T,K) + C_5(T,K)$$

2.2.1 Calculation of $C_1(T,K)$ and $C_3(T,K)$

$C_1(T,K)$ is given by:

$$\begin{aligned}
C_1(T,K) &= B^d(0,T) E^{Q^d}\left(1_{\left\{(1-J)X_T^{d/loc}>K\right\}}\left(X_T^{d/loc}-K\right)\right) \\
&= B^d(0,T) F_T^{d/loc} E^{Q^d}\left(1_{\left\{(1-J)X_T^{d/loc}>K\right\}} M_T^{d/loc}\right) - B^d(0,T) K Q^d\left((1-J)X_T^{d/loc}>K\right) \\
&= B^d(0,T) F_T^{d/loc} Q^M\left((1-J)F_T^{d/loc} M_T^{d/loc}>K\right) - B^d(0,T) K Q^d\left((1-J)X_T^{d/loc}>K\right) \\
&= B^{loc}(0,T) S_0^{d/loc} N(d_1) - B^d(0,T) K N(d_2)
\end{aligned}$$

$$d_1 = \frac{\ln\left(\frac{F_T^{d/loc}(1-J)}{K}\right)}{\Sigma_T} + \frac{1}{2}\Sigma_T$$

$$d_2 = d_1 - \Sigma_T$$

$C_3(T,K)$ is similar to $C_1(T,K)$, the only difference is the forwards are different:

$$\begin{aligned}
C_3(T,K) &= B^d(0,T) E^{Q^d}\left(e^{-\int_0^T \lambda_s ds} 1_{\left\{(1-J)Y_T^{d/loc}>K\right\}}\left(Y_T^{d/loc}-K\right)\right) \\
&= B^{loc}(0,T) S_0^{d/loc} e^{-(1-J)\int_0^T \lambda_s ds} N(g_1) - B^d(0,T) e^{-\int_0^T \lambda_s ds} K N(g_2)
\end{aligned}$$

$$g_1 = \frac{\ln\left(\frac{F_T^{d/loc} e^{J\int_0^T \lambda_s ds}(1-J)}{K}\right)}{\Sigma_T} + \frac{1}{2}\Sigma_T$$

$$g_2 = d_1 - \Sigma_T$$

2.2.2 Calculation of $C_2(T,K)$ and $C_4(T,K)$

$C_2(T,K)$ is given by:

$$C_2(T,K) = B^d(0,T)JF_T^{d/loc}\left(\frac{(1-J)F_T^{d/loc}}{K}\right)^{\frac{1-J}{J}} E^{Q^d}\left(1_{\left\{(1-J)X_T^{d/loc}<K\right\}}\left(M_T^{d/loc}\right)^{\frac{1}{J}}\right)$$

$$= B^d(0,T)JF_T^{d/loc}\left(\frac{(1-J)F_T^{d/loc}}{K}\right)^{\frac{1-J}{J}} \exp\left(\frac{1-J}{2J^2}(\Sigma_T)^2\right)$$

$$*E^{Q^d}\left(1_{\left\{(1-J)X_T^{d/loc}<K\right\}}\exp\left(\frac{1}{J}\int_0^T\sigma_u dW_u - \frac{1}{2J^2}(\Sigma_T)^2\right)\right)$$

$$= B^d(0,T)JF_T^{d/loc}\left(\frac{(1-J)F_T^{d/loc}}{K}\right)^{\frac{1-J}{J}} \exp\left(\frac{1-J}{2J^2}(\Sigma_T)^2\right)Q^{M,J}\left((1-J)X_T^{d/loc}<K\right)$$

$$= B^d(0,T)JF_T^{d/loc}\left(\frac{(1-J)F_T^{d/loc}}{K}\right)^{\frac{1-J}{J}} \exp\left(\frac{1-J}{2J^2}(\Sigma_T)^2\right)N(d_3)$$

Where d_3 is given by:

$$d_3 = \frac{1}{\Sigma_T}\ln\left(\frac{(1-J)F_T^{d/loc}\exp\left(-\frac{1}{J}(\Sigma_T)^2\right)}{K}\right)+\frac{1}{2}\Sigma_T$$

$$= \frac{1}{\Sigma_T}\ln\left(\frac{(1-J)F_T^{d/loc}}{K}\right)+\left(\frac{1}{2}-\frac{1}{J}\right)\Sigma_T$$

Similarly to $C_2(T,K)$, the quantity $C_4(T,K)$

$$C_4(T,K) = B^d(0,T)e^{-\int_0^T\lambda_s ds}JF_T^{d/loc}\left(\frac{(1-J)F_T^{d/loc}e^{J\int_0^T\lambda_s ds}}{K}\right)^{\frac{1-J}{J}} E^{Q^d}\left(1_{\left\{(1-J)Y_T^{d/loc}<K\right\}}\left(M_T^{d/loc}\right)^{\frac{1}{J}}\right)$$

$$= B^d(0,T)e^{-\int_0^T\lambda_s ds}JF_T^{d/loc}\left(\frac{(1-J)F_T^{d/loc}e^{J\int_0^T\lambda_s ds}}{K}\right)^{\frac{1-J}{J}} \exp\left(\frac{1-J}{2J^2}(\Sigma_T)^2\right)N(g_3)$$

Where g_3 is given by:

$$g_3 = \frac{1}{\Sigma_T}\ln\left(\frac{(1-J)F_T^{d/loc}e^{J\int_0^T\lambda_s ds}}{K}\right)+\left(\frac{1}{2}-\frac{1}{J}\right)\Sigma_T$$

2.2.3 Calculation of $C_5(T,K)$

The quantity $C_5(T,K)$ is a BS type formula and given by:

$$C_5(T,K) = B^d(0,T)E^{Q^d}\left(e^{-\int_0^T \lambda_s ds}\left(Y_T^{d/loc}-K\right)_+\right)$$

$$= B^{loc}(0,T)e^{-(1-J)\int_0^T \lambda_s ds} S_0^{d/loc} N(f_1) - B^d(0,T)e^{-\int_0^T \lambda_s ds} KN(f_2)$$

Where f_1 and f_2 are given by:

$$f_1 = \frac{1}{\Sigma_T}\ln\left(\frac{F_T^{d/loc}e^{J\int_0^T \lambda_s ds}}{K}\right)+\frac{1}{2}\Sigma_T$$

$$f_2 = f_1 - \Sigma_T$$

3 Model Calibration

3.1 LN Calibration

The calibration of the model consists on calibrating the function $\Gamma(t)$ to the term structure of survival probabilities.

We define the green function $G(T,K) = E\left(e^{-\int_0^T \lambda_s ds}\delta\left(Z_T^\lambda - K\right)\right)$. We know that this green function is solution of the Fokker-plank equation

$$\frac{\partial G}{\partial T} - \frac{\left(\sigma_T^\lambda\right)^2}{2}\frac{\partial^2 G}{\partial K^2} + \lambda_T G = 0$$

$$G(0,K) = \delta(K)$$

Given a fine schedule (example: weekly) $t_0 = 0, t_1, .., t_n = T^f$, and the green function at time t_i, we look for $\Gamma_{i,i+1}$ (value of the function $\Gamma(t)$ between t_i and t_{i+1}) that will verify the equation:

$$Q(0,t_{i+1}) = E\left(e^{-\int_0^{t_{i+1}} \lambda_s ds}\right) = E\left(\frac{e^{-\int_0^{t_i} \lambda_s ds}}{1+(t_{i+1}-t_i)\Gamma_{i,i+1}e^{\sigma_{t_i} Z_{t_i}}}\right) = \int_{-\infty}^{\infty} \frac{G(t_i,K)}{1+(t_{i+1}-t_i)\Gamma_{i,i+1}e^{\sigma_{t_i} K}}dK$$

Once we calculate $\Gamma_{i,i+1}$ using a root finder algorithm (Newton for example), we calculate the green function at time t_{i+1} by propagating the forward PDE from t_i to t_{i+1}. We repeat these two steps until we calibrate the survival probability up to the final maturity T^f.

We use Cranck-Nicholson PDE scheme for the forward PDE. We have two numerical parameters that allow us to control the calibration accuracy: the number of states and the number of steps. We recommend using 401 for the number of states and the maximum of 401 and 52* T^f (weekly steps) for the number of steps, where T^f is the last calibration maturity. These parameters ensure a very accurate calibration, even for extreme CDS curves and model parameters.

3.2 Calibration of BSWithJump to ATM FX options

We assume that we are given a lognormal intensity model calibrated to survival probabilities. In this section, we will describe the calibration of FX volatility to ATM implied volatilities.

In the previous section, we derived the following expression for the call option price:

$$C(T,K) = B^d(0,T)E^{Q^d}\left(g\left(X_T^{d/loc}\right)\right) + B^d(0,T)E^{Q^d}\left(e^{-\int_0^T \lambda_s ds} g\left(Y_T^{d/loc}\right)\right)$$

We will focus only on the calculation of the second term as the first one can easily be calculated using a numerical integration since $X_T^{d/loc}$ is a lognormal process.

We define the process $Z_T^Y = \ln\left(\frac{B^d(0,T)}{S_0^{d/loc} B^{loc}(0,T)} Y_T^{d/loc}\right)$

The dynamic of Z_T^Y is given by

$$dZ_T^Y = \left(J\lambda_t - \frac{1}{2}\left(\sigma_t^{fx}\right)^2\right)dt + \sigma_t^{fx} dW_t^{fx}$$

We define the green function

$$G\left(T,K^\lambda,K^Y\right) = E^{Q^d}\left(e^{-\int_0^T \lambda_s ds}\delta\left(Z_T^Y - K^Y\right)\delta\left(Z_T^\lambda - K^\lambda\right)\right)$$

$\delta(.)$ is the Dirac function.

The green function G is solution to of the Fokker-Plank equation:

$$\frac{\partial G}{\partial T} + \left(J\lambda_T - \frac{1}{2}\left(\sigma_T^{fx}\right)^2\right)\frac{\partial G}{\partial K^Y} - \frac{1}{2}\left(\sigma_T^\lambda\right)^2 \frac{\partial^2 G}{\partial\left(K^\lambda\right)^2} - \frac{1}{2}\left(\sigma_u^{fx}\right)^2 \frac{\partial^2 G}{\partial\left(K^Y\right)^2} - \rho\sigma_T^{fx}\sigma_T^\lambda \frac{\partial^2 G}{\partial K^\lambda \partial K^Y} + \lambda_T G = 0$$

$$G\left(0,K^\lambda,K^Y\right) = \delta\left(K^\lambda\right)\delta\left(K^Y\right)$$

The call price at maturity *T* can be calculated easily if we know the survival joint pdf of the FX spot and the intensity at *T*.

We suppose that the volatility function σ_T^{fx} is piece wise constant function therefore; we calibrate it using a root finder algorithm by propagating (forward) the green function from today to *T*.

To ensure a good calibration of the model to short-term FX options and long-term FX options, we use two forward PDEs: the first one to calibrate the short dated FX option up to maturity T_1 and a second one to calibrate the FX options from T_1 to the last calibration maturity T_2 if $T_2 > T_1$.

4 Pricing quanto survival probabilities and quanto CDS

4.1 Pricing Quanto survival probabilities

Let us calculate the local currency survival probability or the quanto survival probability:

$$\begin{aligned}
Q^{loc}(0,T) &= E^{Q^d}\left(\frac{B^d(0,T)S_T^{d/loc}}{S_0 B^{loc}(0,T)} 1_{\{\tau>T\}}\right) \\
&= E^{Q^d}\left(M_T^{d/loc}\exp\left((1-J)N_{\tau\wedge T} + J\int_0^T \lambda_u 1_{\{\tau>u\}}du\right)1_{\{\tau>T\}}\right) \\
&= E^{Q^d}\left(M_T^{d/loc}\exp\left(-\int_0^T \lambda_u du\right)\exp\left(J\int_0^T \lambda_u du\right)\right) \\
&= E^{Q^d}\left(M_T^{d/loc}\exp\left(-(1-J)\int_0^T \lambda_u du\right)\right)
\end{aligned}$$

$M_T^{d/loc}$ is an exponential martingale $M_T^{d/loc} = \exp\left(\int_0^T \sigma_u^{fx} dW_u^{fx} - \frac{1}{2}\int_0^T \left(\sigma_u^{fx}\right)^2 du\right)$

We can see that the quanto survival probability is similar to the quanto survival probability with no BS model except that the intensity is multiplied by a coefficient $1-J$. By doing a change of numeraire, we conclude that the quanto survival probability is:

$$Q^{loc}\left(0,T\right) = E^{Q^M}\left(\exp\left(-\left(1-J\right)\int_0^T \lambda_u du\right)\right)$$

The intensity is lognormal under the domestic measure and stays lognormal under the new measure with the same volatility and mean reversion but different Γ_T^{loc} function.

The intensity of default under the local currency is a LN model with a Γ_T^{loc} function given by the formula:

$$\Gamma_T^{loc} = \left(1-J\right)\Gamma_T \exp\left(\rho\sigma^\lambda e^{-\kappa T}\int_0^T e^{\kappa u}\sigma_u^{fx} du\right)$$

The term structure of quanto survival probability can be easily calculated using the same forward PDE on the green function defined in the LN calibration section.

If the correlation between the FX and credit is 0 we can see that the ratio local currency CDS to the domestic CDS is approximately $\left(1-J\right)$. This is true for very short dated maturities even if the correlation is not 0.

4.2 *Pricing quanto CDS*

Once we calibrate the model, we can price the quanto survival probabilities using a forward PDE as explained in previous section. The pricing of quanto CDS is the straightforward given the quanto survival probabilities as we suppose deterministic interest rates.

The local currency quanto CDS are not liquid but we can get some quotations from brokers. The local currency CDS is usually quoted as percentage of the USD denominated CDS.

4.3 *Example: Quanto CDS*

We will show via an example how the quanto CDS depends on different model parameters. We chose an arbitrary Mexican corporate CDS, which is quoted in USD. We would like to see how the local currency (MXN) CDS depends on various model parameters.

We represent all the results as the ratio of quanto CDS to the USD CDS.

4.3.1 Market Data

The CDS is given by (quotation currency: USD) is

Mat	1y	3y	3y	5y	7y	10y
CDS	111	131	147	177	187	197

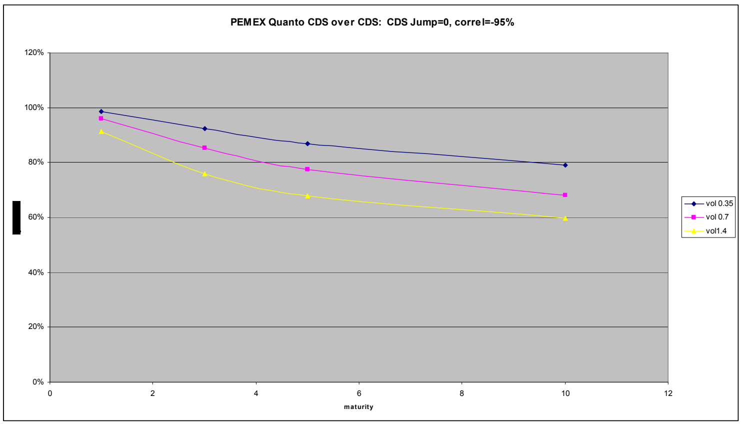
The recover is 40%

The USDMXN ATM volatility is given by:

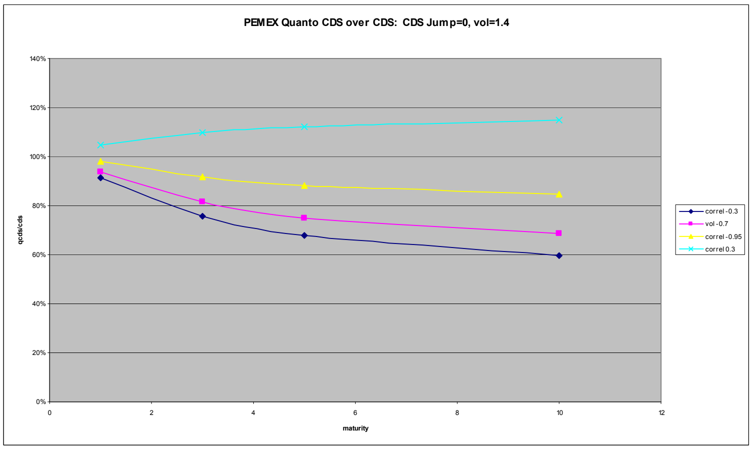
Mat	1w	2w	1m	2m	3m	6m	9m	1y	2y	3y	4y	5y	7y	10y
volatilities	14.0%	14.5%	16.0%	16.0%	16.0%	16.0%	16.0%	16.0%	16.0%	16.0%	16.3%	16.5%	16.5%	16.5%

4.3.2 Impact of credit volatility and FX-credit correlation

We suppose that the jump size is 0 (no jump). Below we show two graphs:
The first one shows how the ratio qcds/cds depends on the credit volatility and the second one how it depends on the correlation FX-credit.



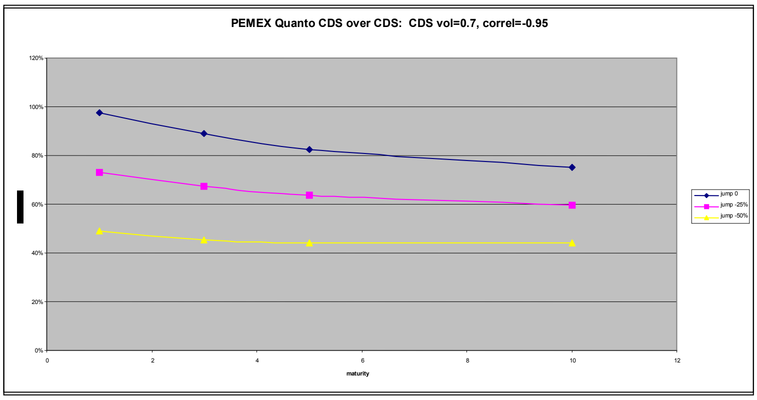
We can see from the first graph that higher credit volatility gives us a lower qcds/cds if the correlation is negative and higher if the correlation is positive.



We can see from the second graph that given credit volatility a higher correlation (absolute value) gives us a lower qcds/cds if the correlation is negative and higher if the correlation is positive.
We can see from these two graphs that the long-term qcds/cds vary in a relatively large range when the credit volatility and the FX-credit correlation vary. However, the short-term ratio stays always close to 100%.

4.3.3 Impact of the Jump size

We can see from the graph below that the model can generate various values for the ratio qcds/cds in the short-term and long-term when we vary the jump size, the credit volatility, and the FX-credit volatility. The jump size controls the overall level of qcds/cds while the credit volatility and the correlation FX-credit controls the term structure of qcds/cds.



5 Conclusion and possible extensions of the model

In this paper, we have presented a new model that take into account the FX devaluation risk. We have presented how to calibrate the model using forward PDE and, the pricing of quanto CDS in this framework. The model could be used even if the FX is not strongly linked to the credit reference (In this case, we can set the jump size to 0). In this paper, we have specified a constant jump size, but all the formulas and calibrations stay almost the same if we use a random jump size. We preferred to use a constant jump because the quanto CDS (linear payoff on the FX) does not depends a lot on the variance of the jump size. This model is convenient for pricing linear FX structures which knock out at time of default (like quanto CDS) but in order to price more exotic structures, we need to add a local volatility and/or stochastic volatility component in order to calibrate the FX smile and not only the ATM volatility. This would be the subject of coming research. Another interesting extension of the model is pricing of quanto FTD where we need to take into account the different dependencies of the constituents of the FTD basket and the FX. Hence, we need to specify a different jump size to each credit name.

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