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Breitmoser, Yves and Bolle, Friedel and Otto, Philipp E.

EUV Frankfurt (Oder)

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# The core with random utility and interdependent preferences: Theory and experimental evidence*


Yves Breitmoser — EUV Frankfurt (Oder)

Friedel Bolle† — EUV Frankfurt (Oder)

Philipp E. Otto — EUV Frankfurt (Oder)


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**Abstract**

Experimental analyses of Shapley-Shubik assignment games revealed that the core prediction is biased. The competing hypotheses are that subjects either have interdependent preferences or a limited understanding of outcomes in alternative matches. To evaluate these hypotheses econometrically, we introduce core concepts with random utility perturbations. The "logit core" converges to a uniform distribution on the original core as noise disappears. With noise, it captures the non-uniform distribution of observations inside and outside the core, and contrary to regression, it predicts robustly out-of-sample. The logit core thus constitutes a conceptual basis for econometric analyses of assignment problems, and by capturing the whole distribution of outcomes, it allows us to extract all information by maximum likelihood methods. Using this approach, we then show that the core's prediction bias results from overstating the subjects' grasp of outcomes in alternative matches, while social preferences are only of minor relevance.



*JEL classification:* C71, C90, D64

*Keywords:* cooperative game, core, random utility, social preferences, laboratory experiment, descriptive adequacy, predictive adequacy


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†Corresponding author. Postfach 1786, 15207 Frankfurt (Oder), Germany, Email: bolle@euv-frankfurt-o.de, Phone/Fax: +3355534 2289/2390

# 1 Introduction

Consider the "assignment game" originally defined by Koopmans and Beckmann (1957) and Shapley and Shubik (1972). Firms and workers generate a surplus when they match. They match one-to-one, i.e. each worker can match with only one firm and vice versa, and firms and workers can transfer payoffs within matches but not between matches. The assignment game is canonically solved using the core. This is particularly convenient, as its core is generally not empty and characterized as the solution set of a simple linear program (Koopmans and Beckmann, 1957), besides satisfying various technically attractive properties.[1] For these reasons, the core solution of the assignment game has become a standard model of labor markets (Crawford and Knoer, 1981; Kelso Jr and Crawford, 1982) and other markets for indivisible goods (Roth, 1985), and overall it is one of the most successful models from cooperative game theory.

The experimental evidence paints a mixed picture, however. As for assignment games, Tenbrunsel et al. (1999) and more recently Otto and Bolle (2011) found that the core actually fits poorly—it is biased systematically and most experimental observations are not in the core, but near the equal split.[2] This is consistent with earlier evidence on the core in voting games, which after the initial findings of Fiorina and Plott (1978) and Berl et al. (1976) revealed that the core is a "poor predictor in games containing a fair alternative" (Eavey and Miller, 1984, p. 570; see also McKelvey and Ordeshook, 1981). The results are also consistent with a bulk of evidence from experimental analyses of non-cooperative games, which shows that standard game-theoretic predictions on "competitive behavior" may fit poorly (see for example ultimatum and dictator games, as surveyed in Camerer, 2003). In non-cooperative games, such deviations are well explained by accounting for interdependent preferences, random utility perturbations, and non-equilibrium models of reasoning. The purpose of the present

[1] In addition, the core of assignment games is known to be a polytope with the form of a 45°-lattice (Quint, 1991a), it satisfies the CoMa property (Hamers et al., 2002), it has been axiomatized (Toda, 2005) and implemented non-cooperatively (Pérez-Castrillo and Sotomayor, 2002; Halaburda, 2010). While some of these properties do not generalize to $m$-sided markets (Quint, 1991b), they tend to generalize to one-sided matching (Quint, 1996) and multiple-partners games (Sotomayor, 1999).

[2] There is further experimental research on assignment games that focuses on the efficiency of matching mechanisms; see Olson and Porter (1994); Nalbantian and Schotter (1995), for mechanisms with transfers, and Kagel and Roth (2000); Chen and Sönmez (2002, 2006); Pais and Pintér (2008), for mechanisms without transfers.

paper is to introduce these extensions into the cooperative concept of the core and to evaluate the source of its prediction bias revisiting the data of Otto and Bolle (2011).

The main hypotheses are that the bias follows from either interdependence of preferences (i.e. fairness concerns, as Eavey and Miller, 1984, suggest) or a limited grasp (or heavy discount) of outcomes in alternative matches (as for example Selten, 1972, suggests). The latter hypothesis loosely relates to the level-$k$ model in non-cooperative games (Stahl and Wilson, 1995; Camerer et al., 2004; Costa-Gomes et al., 2009), according to which level-1 players believe the opponents are non-strategic, level-2 players believe the opponents are level 1, and so on. A cooperative solution corresponding with level-1 reasoning is the *level-1 equal division core* (or level-1 core, for short): Players do not consider payoffs in alternative matches at all and they consider an outcome to be satisfactory ("stable") if all players' payoffs are sufficiently close to the equal split. The next step, the cooperative solution at level 2, is the *level-2 equal-division core* (which Selten, 1972, called "equal-division core"): Level-2 players believe that the level-1 solution (the equal split) would result in any alternative match and consider an outcome to be stable if no pair of players can benefit by forming such an alternative match. In the *core*, finally, all players have full understanding of all alternative matches. In an initial analysis of these concepts (Section 3), we find that the limitation of the level of reasoning explains most the aforementioned systematic deviations from the core. Quantitatively, the level-1 core fits best amongst the considered models, while interdependent preferences appear to be of minor relevance only.

A fundamental issue in any evaluation of core concepts is that only two statistics are available: the hit rate and the relative area covered by the core. Selten (1991) proposes to use their difference as a measure for the core's goodness of fit. Unfortunately, this measure does not use all information that is available, e.g. it cannot distinguish between close hits and clear hits or close misses and wide misses, and it does not lend itself to statistical inference since it is not supported by even asymptotic theory (for further discussion, see Hey, 1998). To identify the source of the prediction bias, we therefore introduce a novel concept, the core for players with random utility perturbations. The "logit core" predicts the actual distribution of outcomes and thus allows us to extract all of the information contained in the data set by maximum likelihood estimation. It relates conceptually to the logit equilibrium (McKelvey and Palfrey, 1995) and converges to a uniform distribution on the core as noise disappears.

We show that allowing for random utility perturbations explains the distribution of observations inside the (level-1) core as well as the occasional occurrence of observations outside of it. To outline the intuition underlying the logit core concept, introducing random utility perturbations into the core yields a measure for the "stochastic stability" of outcomes. Thus, instead of being perfectly stable or unstable, outcomes inside the core are now "stochastically more stable" than outcomes close to its boundary, which in turn are stochastically more stable than outcomes outside the core. The experimental observations are distributed largely proportionally to their stochastic stability.

The low-level logit cores fit the distribution of observations both qualitatively and quantitatively, and we show that they fit highly significantly better than two alternative models with noise, namely a random behavior model, where the outcome is in the core with probability $1-\varepsilon$ and outside of it with probability $\varepsilon$, and a regression model. Finally, we evaluate the predictive adequacy of the logit core, in order to eliminate the possibility of overfitting and to show that the logit core fits robustly (as suggested by e.g. Hey et al., 2010). The analysis reveals that preferences have significant spiteful components after accounting for random utility and that the most adequate concept (both descriptively and predictively) merges the level-1 and level-2 equal division cores. Intuitively, subjects bargain rather spitefully (i.e. competitively), and they are content with a given allocation if it is either close to the equal split (level 1) or if they cannot improve by forming an alternative match with equally split payoffs (level 2).

Overall, the results show that merging a cooperative solution concept (the core) with behavioral concepts such as random utility and limited depth of reasoning qualitatively and robustly explains the main stylized facts in experimental assignment games. Thus, the introduced logit core substantially extends the scope of both cooperative game theory, by opening it toward likelihood-based econometric methods, and behavioral game theory, by showing its applicability to cooperative games.

Section 2 defines assignment games and describes the experimental design. Section 3 defines the cores with interdependent preferences and limited levels of reasoning and briefly discusses their comparative relevance. Section 4 introduces our main conceptual innovation, the logit core. Section 5 evaluates its adequacy econometrically. Section 6 concludes. The supplementary material contains further robustness checks.

## 2 Basic definitions and experimental games

Let $W$ be a finite, non-empty set of "workers" and $F$ be a finite, non-empty set of "firms." The productivity of the potential matches $(w, f) \in W \times F$ between workers and firms is denoted as $C \in \mathbb{R}_+^{W \times F}$, i.e. $C_{w,f}$ is the value if $w$ and $f$ match. The allocation of their value $C_{w,f}$ is to be negotiated between $w$ and $f$. Players that are unmatched obtain zero payoff. The outcome of an assignment game is a payoff profile $(x_i)_{i \in W \cup F}$.

The *core* contains all outcomes where no subset of players can increase their payoffs by rematching. An outcome $(x_i) \in \mathbb{R}_+^N$, $N = W \cup F$, is in the core if (and only if) it is feasible and $x_w + x_f \geq C_{w,f}$ for all $w \in W, f \in F$. All core outcomes are socially efficient, i.e. they maximize the productivity aggregated over all matches. Koopmans and Beckmann (1957) and Shapley and Shubik (1972) show that the core is generally non-empty (in assignment games) and that transfers between matches are neither made nor required to sustain core allocations.[3] Solymosi and Raghavan (2001) provide necessary and sufficient conditions for the core to be stable in the sense of von Neumann-Morgenstern, and these conditions will be satisfied in our experimental games. Driessen (1998) shows that the kernel is included in the core of assignment games, and Núñez and Rafels (2003) obtain a similar result for the $\tau$-value.

Otto and Bolle (2011) implement the $2 \times 2$ assignment games in a laboratory experiment, testing the predictive adequacy of the core. Let the set of workers be denoted as $W = \{W_1, W_2\}$ and the set of firms as $F = \{F_1, F_2\}$. The productivities for all matches in all treatments $T1 \ldots T6$ are provided in Table 1. Note that $C_{1,1} \leq C_{1,2} \leq C_{2,1} < C_{2,2}$ applies in all treatments and that the players may match in either of two ways. The matching $\{(W_1, F_1), (W_2, F_2)\}$ will be called "$A$-matching," and $\{(W_1, F_2), (W_2, F_1)\}$ will be called "$B$-matching." In $T1$ and $T2$, $A$-matching is efficient, in $T4$ and $T5$, $B$-matching is efficient, and in $T3$ and $T6$, both matchings are efficient. In the latter case, the core is degenerate, i.e. it has zero volume in the outcome space. Otherwise, its volume is positive. For each of these efficiency conditions, the productivity matrix is either symmetric ($C_{1,2} = C_{2,1}$) or asymmetric ($C_{1,2} < C_{2,1}$). Thus, the six treatments yield a $3 \times 2$ factorial design to cover all relevant scenarios.

[3]This is not the case for most alternative solution concepts in assignment games. For example, nucleolus, Shapley Value, and Stable Sets (the von Neumann-Morgenstern Solution) of the Assignment Game require the possibility of transfers between matches.

Figure 1: The experimental data in relation to the core. (Note that if *B*-matching is inefficient, then the core predicts *A*-matching, and vice versa.)

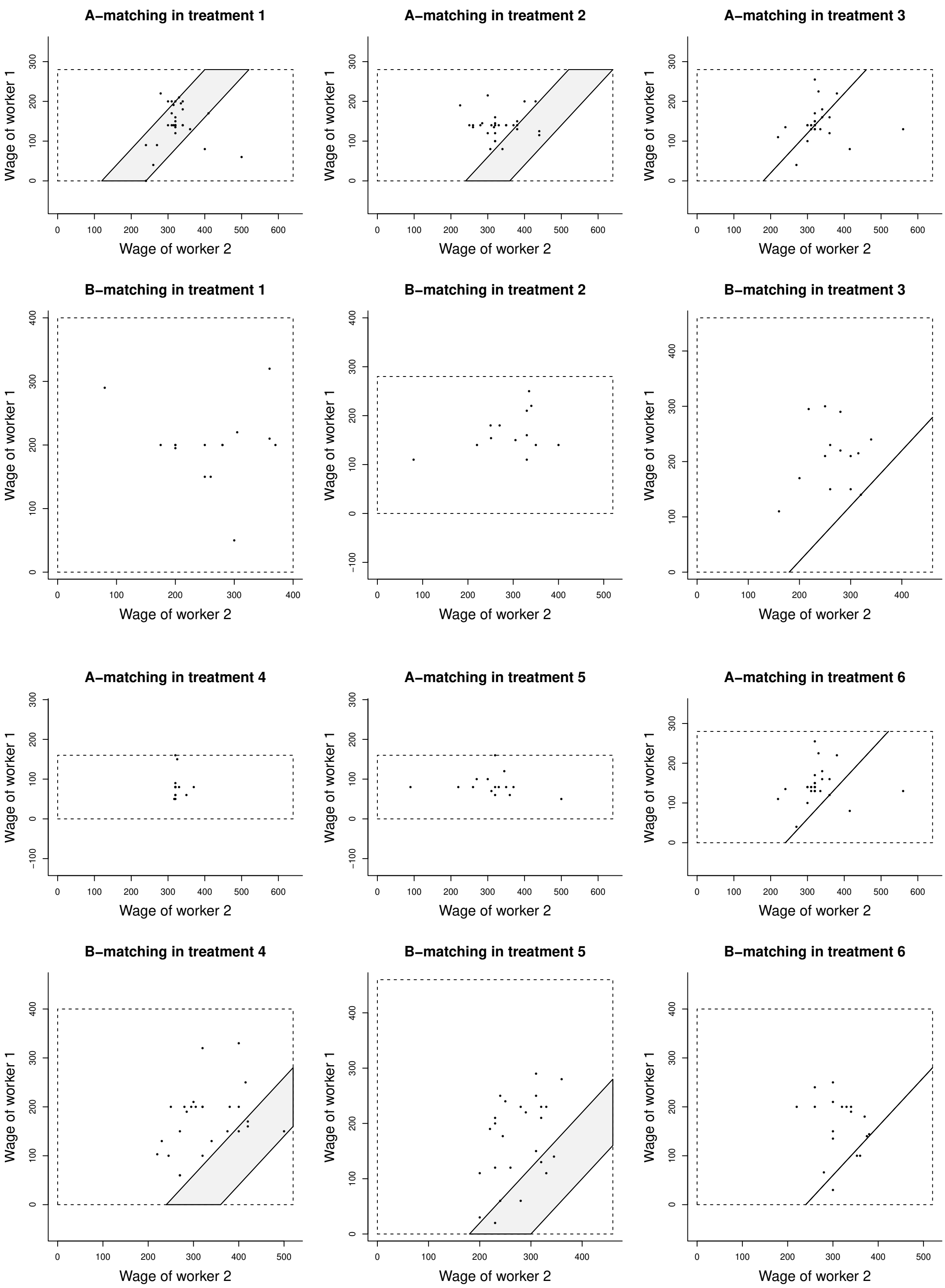


Table 1: Productivities of matches in the six experimental treatments

| | | $T1$ | | $T2$ | | $T3$ | | $T4$ | | $T5$ | | $T6$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C_{1,1}$ | $C_{1,2}$ | <u>280</u> | 400 | <u>280</u> | 280 | 280 | 460 | 160 | <u>400</u> | 160 | <u>460</u> | 280 | 400 |
| $C_{2,1}$ | $C_{2,2}$ | 400 | <u>640</u> | 520 | <u>640</u> | 460 | 640 | <u>520</u> | 640 | <u>460</u> | 640 | 520 | 640 |

*Note:* $C_{w,f}$ is the productivity of the match $(w, f) \in W \times F$. The pairings in the socially efficient matching are underlined if unique. In $T3$ and $T6$, both matchings are efficient.

Experimental logistics, instructions, and basic descriptive statistics are discussed in Otto and Bolle (2011) and reviewed in the supplementary material.[4] The observations that are most relevant for our purpose are summarized in Figure 1. It plots all outcomes in relation to the core in the various treatments and illustrates the main stylized facts mentioned above (see also Tenbrunsel et al., 1999). The core predicts poorly, overall less than 20% of the observations are in the core, inefficient matching can be observed regularly, and overall observations are more egalitarian than predicted by the core.[5] Two possible explanations for these systematic observations are that subjects have social preferences and that the stability requirements of the core are too strong or computationally too complex. This possible sources are investigated in the next section.

# 3 Interdependent preferences or limited depth of reasoning?

In this section, we introduce the three basic core concepts for players with interdependent preferences based on which we seek to understand whether limited depth of reasoning or interdependence of preferences is responsible for the biases observed in Figure 1. We consider preferences that are interdependent in the sense that *i*'s utility may depend on all of the entities that he can explicitly observe in the experiment: the

[4]Briefly, the order of the treatments and the individual allocation to positions was randomized over the sessions. Every subject was allocated to a worker position three times and to a firm position three times. No subject interacted with the same co-participant in more than three of the six games. As Otto and Bolle (2011) verified, there was no indication of reputation building or learning.

[5]In addition, incomplete matching has been observed in 12% of the games. As Otto and Bolle (2011) verified, almost all of these incompletions result from last-second rematching of the provisional partners, i.e. just before the 10-minute time line for the negotiations ended. These incomplete matches are therefore not intended by the unmatched players and as such unexplainable by concepts such as the core. In our analysis, we therefore discard these 12% of the observations.

own payoff, the partner's payoff, and the partner's identity. This requires additional notation. The set of players is $N = W \cup F$, and the set of $i$'s potential partners is $N_i = F \cup \{\emptyset\}$ if $i \in W$ and $N_i = W \cup \{\emptyset\}$ if $i \in F$ ("$\emptyset$" indicates that $i$ remains single). The utility of $i \in N$ is a function $U_i(x_i, x_j, j) : \mathbb{R}^2_+ \times N_i \to \mathbb{R}$.

In order to define the core for such generalized utilities, we have to account for two new phenomena. On the one hand, the stability of an outcome depends on the matching that applies, since utilities depend on the matching. Hence, the definition of "outcome" needs to be extended to also include the matching. Define a matching $m$ as a function $m : N \to N \cup \{\emptyset\}$ satisfying, for all $i \in N$, $m(i) \in N_i$ and $m(i) \neq \emptyset \Rightarrow m(m(i)) = i$. Let $M$ be the set of all these matchings. The set of outcomes can now be defined as

$$\mathbf{X} = \big\{(\mathbf{x}, m) \in \mathbb{R}^N_+ \times M \mid \forall i \in N : m(i) = \emptyset \Rightarrow x_i = 0 \text{ and} \\ \forall w \in W : \ m(w) \neq \emptyset \Rightarrow x_w + x_{m(w)} \leq C_{w,m(w)}\big\}.$$

On the other hand, when defining stability under generalized utilities, we need to explicitly take into account that it may be preferable to be single than to share the surplus generated by a match in a highly asymmetric way. Being single will usually not be stable, but it has to be included as an option. To this end, let $C_{w,\emptyset} = C_{\emptyset,f} = 0$ for all $w, f$ denote the productivity of single players, and let $U_\emptyset = 0$ denote the utility of the dummy player "$\emptyset$" who represents the partner of an unmatched player.

**Definition 3.1** (Core)**.** The core is the set of outcomes $(\mathbf{x}, m) \in \mathbf{X}$ such that, for all $i \in N$, all $j \in N_i$, and all $x' \in [0, C_{i,j}]$,

$$U_i(x_i, x_{m(i)}, m(i)) \geq U_i(x', C_{i,j} - x', j) \quad \text{or} \quad U_j(x_j, x_{m(j)}, m(j)) \geq U_j(C_{i,j} - x', x', i).$$

The literature following Selten (1972) has analyzed a solution concept that weakens the stability requirements of the core. Selten's equal-division core, to which we will refer as *level-2 core*, states that an outcome is stable whenever no pair of players exists who would benefit if they coalesce and share their surplus equally. Thus, the players consider only a specific alternative outcome rather than all possible alternatives.

**Definition 3.2** (Level-2 core)**.** The level-2 core is the set of outcomes $(\mathbf{x}, m) \in \mathbf{X}$ such that, for all $i \in N$, all $j \in N_i$, and $x' = C_{i,j}/2$,

$$U_i(x_i, x_{m(i)}, m(i)) \geq U_i(x', C_{i,j} - x', j) \quad \text{or} \quad U_j(x_j, x_{m(j)}, m(j)) \geq U_j(C_{i,j} - x', x', i).$$

Otto and Bolle (2011) reduce the rationality requirement even further and say that an outcome may already be stable if the allocations are sufficiently close to the equal split in all matches. This *level-1 core* reflects the idea that players may not try to predict possible payoff allocations in alternative matches at all, arguably due to the uncertainty underlying the necessary negotiations.

**Definition 3.3** (Level-1 core)**.** Fix $\gamma > 0$. The level-1 core is the set of outcomes $(\mathbf{x}, m) \in \mathbf{X}$ such that for all $i \in N$, $U_i(x_i, x_{m(i)}, m(i)) \geq U_i(C_{i,m(i)}/2, C_{i,m(i)}/2, m(i)) - \gamma$.

The level-1 core turned out to be most descriptive concept in Otto and Bolle (2011). Two potential objections to this result are that Otto and Bolle's analysis neglected interdependent preferences, while higher-level cores may be more descriptive if we account for those, and that their result may possibly not be robust to changing circumstances. In order to examine robustness, we will distinguish descriptive and predictive adequacy. The *descriptive adequacy* is determined by fitting the parameters to the whole sample and evaluating their fit on this very sample. The *predictive adequacy* is determined by fitting the parameters to the observations from four of the six treatments, evaluating their fit on the observations from the remaining two treatments, and rotating so that all observations are used exactly once in the evaluation stage once.[6]

Since we are also interested in verifying whether the results obtained by maximizing the likelihood of the logit core, as done below, differ qualitatively from those obtained by maximizing Selten's score[7] for the core (as proposed in the literature), we

[6]This approach combines cross validation (Burman, 1989; Zhang, 1993) with non-random holdout samples (Keane and Wolpin, 2007). Exploring predictive adequacy as a measure of robustness has been advocated recently by Hey et al. (2010) and Wilcox (2008, 2011), amongst others.

[7]The "Selten score" (Selten, 1972, 1991) measures the goodness of fit in this case where likelihoods are not available (they are zero whenever a single observation is not in the core), and where sums of squared differences are not available as there is not reliable measure for the distance between $A$-matching and $B$-matching. *Selten's score* of a solution concept is the difference between (i) the relative frequency of observations compatible with the concept and (ii) the share of internally Pareto efficient outcomes compatible with the concept. An outcome $(\mathbf{x}, m) \in \mathbf{X}$ is "internally Pareto efficient" if the players allocate the whole surplus generated within their matches, i.e. if $m(i) \neq \emptyset$ and $x_i + x_{m(i)} = C_{i,m(i)}$ for all $i \in N$.

Table 2: Selten scores (higher is better) for egoistic and altruistic preferences

| | # Parameters | | Descriptive adequacy | | Predictive adequacy | |
|---|---|---|---|---|---|---|
| | Ego | Altr | Egoism | Altruism | Egoism | Altruism |
| Level-1 Core | 1 | 3 | 0.575 | 0.612 | 0.574 | 0.586 |
| Level-2 Core | 0 | 2 | 0.312 | 0.512 | 0.312 | 0.509 |
| Core | 0 | 2 | 0.122 | 0.276 | 0.122 | 0.253 |

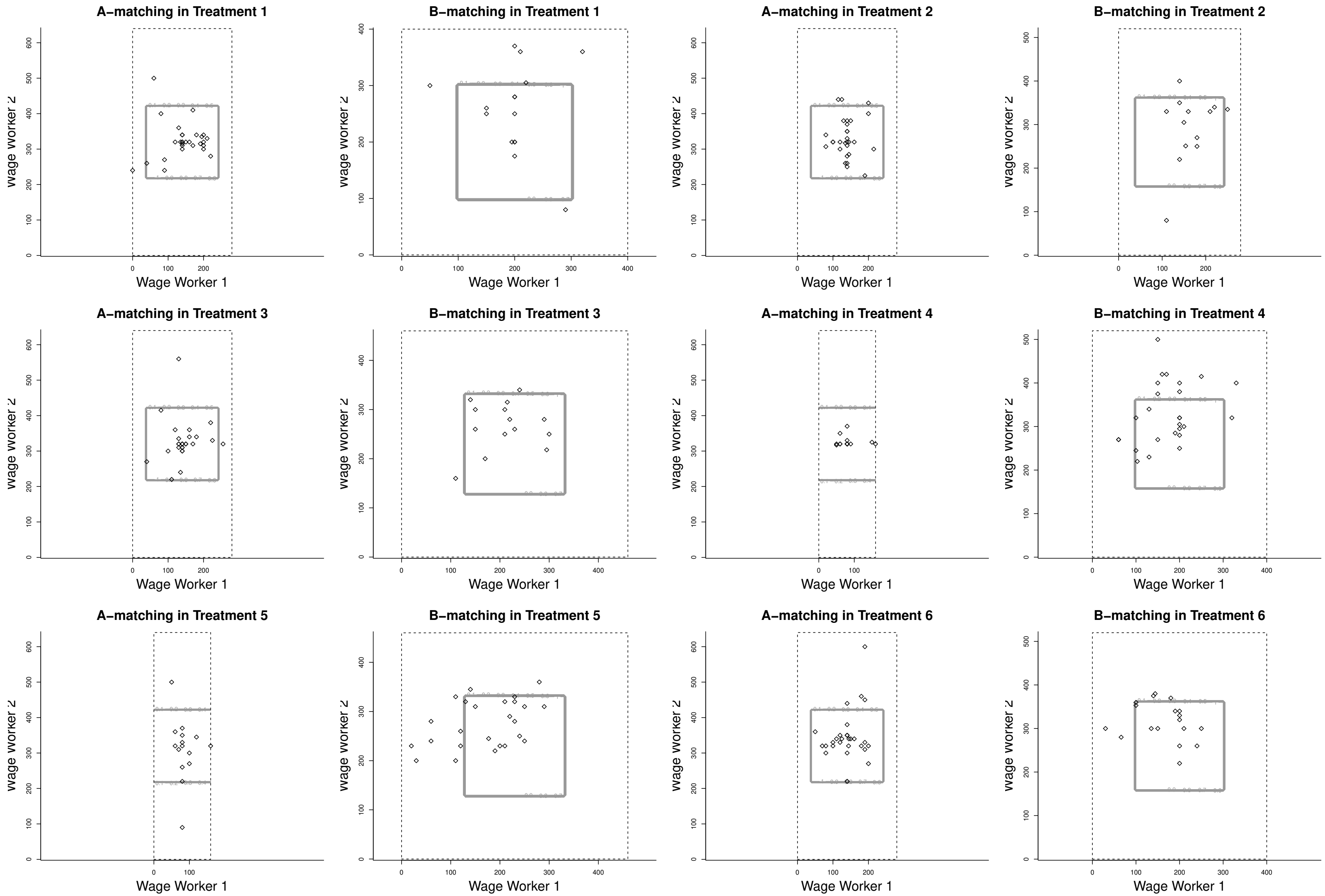
**Note:** Level-1 core, level-2 core, and core are defined in Definitions 3.1–3.3; the Selten score is defined in Footnote 7.

now evaluate the three core variants in terms of Selten's score, allowing for egoistic and altruistic interdependent preferences.[8] The altruistic utility function is defined as $U_i(x_i,x_j,j) = x_i + \alpha x_j + \beta\overline{C}_{ij}$, where $\alpha$ and $\beta$ are free parameters and $\overline{C}_{ij}$ is the productivity (i.e. sum of payoffs) in the other match. In case $i$ is single, i.e. $j = \emptyset$, the utility is $U_i(0,0,\emptyset) = 0 + \beta \cdot \max\{C_{2,1}, C_{2,2}\}$. The model parameters are estimated by maximizing Selten's score jointly over all parameters, using a gradient free algorithm for the initial approach to the maximum, a Newton method to ensure convergence, and various starting values to verify globality of the maximum.

The results are summarized in Table 2. The best fitting concepts are the level-1 core with either egoistic or altruistic preferences and the level-2 core for altruistic preferences. They each contain approximately 80% of the observations but cover only 20%–30% of the outcome space. Their Selten scores do not differ significantly (in Wilcoxon-tests of Selten's scores at the session level, with $p = .02$) and they all fit significantly better than all other models (at $p < .001$). Thus, the bias of the core observed in Figure 1 appears to be primarily due to the limited level of reasoning, i.e. the limited grasp of possible outcomes in alternative matches, while interdependent preferences seem to be of minor relevance at best. Further, as the predictive adequacy reported in Table 2 and the plot in Figure 2 show, the prediction bias is robustly eliminated in the level-1 core. In the remaining two sections, we introduce and analyze random utility cores, in order to account for the fact that the observations are not distributed uniformly on the level-1 core, that the incompatible ones do not seem to be uniformly outside of it, and that observations outside of the core occur in the first place. This will allow us to extract all information contained in the data set and make likelihood-based inference.

[8]Further analysis, which is provided in the supplementary material, shows that inequity aversion and CES utilities do not improve upon linear altruism in this context.

Figure 2: The level-1 core of players with egoistic preferences (with $\gamma = 100$ in all cases)



# 4 Explaining the outcome distribution: The logit core

## Simple bargaining games

Initially, consider a simple bargaining problem. There are two players negotiating the allocation of a cake valued $C > 0$. We define the random utility bargaining problem by adding a random utility component to the outside option. The distribution of the random component is general in the following definition, but it will be logistic in all subsequent applications, i.e. the difference of two i.i.d. extreme-value distributed random variables. This corresponds closely with the approach taken in non-cooperative game theory (see e.g. McKelvey and Palfrey, 1995, Goeree and Holt, 1999, Weizsäcker, 2003, Turocy, 2005). Illustrations follow shortly.

**Definition 4.1** (Random utility bargaining game)**.** The set of players is $N = \{1,2\}$, the set of possible outcomes is $\mathbf{X} = \{\mathbf{x} \in \mathbb{R}^2_+ \mid x_1 + x_2 \le C\}$ for some $C > 0$, and the players' disagreement payoffs are $\underline{x}_1, \underline{x}_2 \in [0,C]$ with $\underline{x}_1 + \underline{x}_2 < C$. For both $i \in N$, utilities are $u_i(\mathbf{x}) = x_i$ for all $\mathbf{x} \in \mathbf{X}$ and $\tilde{u}_i(\underline{\mathbf{x}}) = \underline{x}_i + \varepsilon_i$ for the outside option. The distributions of $\varepsilon_1$ and $\varepsilon_2$ are continuous, stochastically independent, and characterized by the cumulative distribution functions $F_1$ and $F_2$, respectively.

In the unperturbed game, an allocation is "stable" if and only if it is individually rational and Pareto efficient. The core $\mathbf{X}^c \subseteq \mathbf{X}$ is the set of stable allocations.

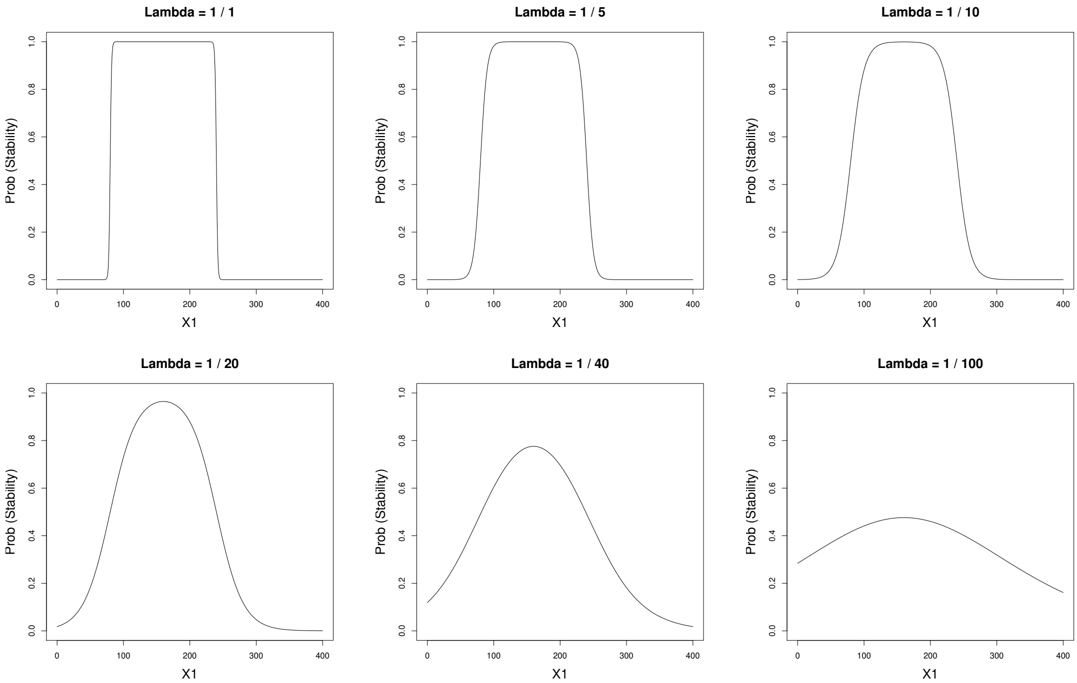
$$\mathbf{X}^C = \big\{\mathbf{x} \in \mathbf{X} \mid x_i \ge \underline{x}_i \,\wedge\, x_j \ge \underline{x}_j \,\wedge\, x_i + x_j = C\big\}$$

If utilities are random, stability becomes a stochastic property, as individual rationality is stochastic. The probability that player $i$ is content with $\mathbf{x}$ is $\Pr(x_i \ge \underline{x}_i + \varepsilon_i)$, and the probability that $\mathbf{x}$ is individually rational (i.e. that both players are content) is

$$\pi(\mathbf{x}) = \Pr(x_i \ge \underline{x}_i + \varepsilon_i) \cdot \Pr(x_j \ge \underline{x}_j + \varepsilon_j). \tag{1}$$

This probability is called the *stochastic stability* of $\mathbf{x}$, and an outcome $\mathbf{x}$ is called *stochastically more stable* than $\mathbf{x}'$ if $\pi(\mathbf{x}) > \pi(\mathbf{x}')$. Figure 3 plots the probabilities that outcome $(x_1, x_2)$ is individually rational conditional on Pareto efficiency (i.e. $x_2 = C - x_1$), assuming logistic perturbations. If the utility perturbations $\varepsilon_i$ have logistic distribution with scale parameter $s = 1/\lambda$, then $i$ is content with probability $1/\big[1 +$

Figure 3: Stochastic stability in bargaining games for varying precisions $\lambda$




**Note:** The cake size is $C = 400$ and the outside options are $(\underline{x}_1, \underline{x}_2) = (120, 240)$. The plotted functions are $\Pr(x_1 \geq \underline{x}_1 + \varepsilon_1 \text{ and } x_2 \geq \underline{x}_2 + \varepsilon_2)$, $\varepsilon_1, \varepsilon_2$ being i.i.d. logistic, as functions of $x_1$ with $x_2 = C - x_1$.

$\exp(\lambda(\underline{x}_i - x_i))\big]$.

This ordering of outcomes generalizes deterministic stability in an intuitive way. As the precision $\lambda$ tends to infinity, stochastic stability converges pointwise to the stability indicator $\mathbf{1}_{\mathbf{x} \in \mathbf{X}^C}$ of the unperturbed game. The stochastically most stable allocation is generally in the interior of the core of the unperturbed game, and if perturbations are identically distributed for the players, the stochastically most stable outcome is the Nash solution. This is easy to see if perturbations are logistic $F_i(r) = 1/\big(1 + \exp(-\lambda_i r)\big)$ for all $r \in \mathbb{R}$ and $i \in N$. In this case, the stochastic stability $\pi(\mathbf{x}) = \big(1 + e^{-\lambda_i(x_i - \underline{x}_i)}\big)^{-1} * \big(1 + e^{-\lambda_j(1 - x_i - \underline{x}_j)}\big)^{-1}$ is maximized if

$$\frac{1 + e^{\lambda_i(x_i - \underline{x}_i)}}{1 + e^{\lambda_j(1 - x_i - \underline{x}_j)}} = \frac{\lambda_i}{\lambda_j}.$$

If errors are i.i.d. logistic, then $\lambda_i = \lambda_j$ and the most stable outcome is the Nash bargaining solution. The following result establishes equivalence between the Nash solution and the stochastically most stable outcome for a general class of distributions.

**Lemma 4.2.** *Assume the random utility components of all players are i.i.d. with cumulative density $F$. If $F$ is symmetric, $F(x) = 1 - F(-x)$, and has quasi-concave density, then the unique maximizer of $\pi(\mathbf{x})$ is $x_i = (\underline{x}_i + C - \underline{x}_j)/2$ and $x_j = (\underline{x}_j + C - \underline{x}_i)/2$.*

*Proof.* The first-order condition for $\max_{x_i} F(x_i - \underline{x}_i) * F(C - x_i - \underline{x}_j)$ yields

$$f(x_i - \underline{x}_i)/f(C - x_i - \underline{x}_j) = F(x_i - \underline{x}_i)/F(C - x_i - \underline{x}_j).$$

The claimed solution implies $x_i - \underline{x}_i = C - x_i - \underline{x}_j =: x'$ and hence satisfies the condition. Next, $\underline{x}_i + \underline{x}_j < C$ implies $x' = (C - \underline{x}_i - \underline{x}_j)/2 > 0$. The second-order condition (for the claimed solution to be a maximum) is $2f'(x')F(x') < 2f(x')f(x')$. By symmetry and quasi-concavity, $x' > 0$ implies $f'(x') \leq 0$; since all other terms are positive, the condition holds. Finally, consider the case $x_i - \underline{x}_i \neq x'$, and without loss, assume $x_i - \underline{x}_i > x'$. Hence, $C - x_i - \underline{x}_j < x'$. By symmetry and quasi-concavity, $f(x_i - \underline{x}_i) \leq f(C - x_i - \underline{x}_j)$, and by monotonicity, $F(x_i - \underline{x}_i) > F(C - x_i - \underline{x}_j)$; hence, the first-order condition is violated, which proves uniqueness. □

Further, outcomes close to the Nash solution are stochastically more stable than distant outcomes, outcomes in the core are stochastically more stable than outcomes outside of it, and outcomes close to the core are stochastically more stable than outcomes distant to the core. These characteristics correspond closely with the intuition that we wish to capture, and for this reason we define a solution concept where the probability that outcome $\mathbf{x}$ results is proportional to its stochastic stability. Specifically, the *random utility core* is the probability density $f_C \in \Delta\big(\mathrm{PF}(\mathbf{X})\big)$ on the Pareto frontier that is proportional to the above measure of stochastic stability.

$$f_C(\mathbf{x}) = \pi(\mathbf{x}) / \int_{\mathrm{PF}(\mathbf{X})} \pi\big(\tilde{\mathbf{x}}\big) d\tilde{\mathbf{x}} \tag{2}$$

As stated, the integration is along the Pareto frontier defined as $\mathrm{PF}(\mathbf{X}) = \{\mathbf{x} \in \mathbf{X} \mid \mathbf{x} \geq \mathbf{x}' \; \forall \mathbf{x}' \in \mathbf{X}\}$. Obviously, Pareto efficiency could be relaxed as well, similar to the way individual rationality has been relaxed. We abstain from doing so here, as the subjects in our experiment as well as those in many previous bargaining experiments manage to allocate the whole cake in almost all cases. The deviations from the core are therefore not due to Pareto inefficiency. The following establishes a simple axiomatic foundation of the random utility core in bargaining games.

**Proposition 4.3.** *For bargaining games (Def. 4.1), the following statements are equivalent.*

1. *$f_C$ satisfies Eq.* (2) *for* $\pi(\mathbf{x}) = F_1(x_1 - \underline{x}_1) * F_2(x_2 - \underline{x}_2)$.
2. *$f_C$ satisfies the following conditions.*

   A1 *Continuity and Pareto efficiency: $f_C$ is the density of a continuous distribution on $PF(\mathbf{X})$.*

   A2 *Proportional stability: $f_C(\mathbf{x})$ is proportional to the probability that all players prefer $\mathbf{x}$ to their outside option, i.e. to* $\Pr\left(u_i(\mathbf{x}) \geq \tilde{u}_i(\underline{\mathbf{x}})\ \forall i\right)$.

*Proof.* 2. $\Rightarrow$ 1.: By the definition of the game, $\varepsilon_1$ and $\varepsilon_2$ are independent, and thus $\Pr\left(u_i(\mathbf{x}) \geq \tilde{u}_i(\underline{\mathbf{x}})\ \forall i\right) = \Pr\left(u_1(\mathbf{x}) \geq \tilde{u}_1(\underline{\mathbf{x}})\right) \cdot \Pr\left(u_2(\mathbf{x}) \geq \tilde{u}_2(\underline{\mathbf{x}})\right) =: \pi(\mathbf{x})$. A2 implies $f_C(\mathbf{x}) = a \cdot \pi(\mathbf{x})$ for some $a > 0$. Finally, since $f_C$ is a density with support only on the Pareto frontier (A1), $a = 1/\int_{\mathrm{PF}(\mathbf{X})} \pi(\mathbf{x}) d\mathbf{x}$. 1. $\Rightarrow$ 2. can be verified easily. □

Note that A2 implies independence of irrelevant alternatives (IIA), i.e. $f_C(\mathbf{x}'|\mathbf{X}') \cdot f_C(\mathbf{x}|\mathbf{X}'') = f_C(\mathbf{x}'|\mathbf{X}'') \cdot f_C(\mathbf{x}|\mathbf{X}')$ for all $\mathbf{x}, \mathbf{x}' \in \mathbf{X}'$ and all measurable $\mathbf{X}' \subseteq \mathbf{X}'' \subseteq \mathrm{PF}(\mathbf{X})$.

## Assignment games

Assignment games generalize bargaining games by endogenizing the outside options. For each player, every feasible partner other than his current one represents an outside option, while the values of these outside options depend on the payoff allocations negotiated in their matches. The more my prospective partners in their current matches make, the less the outside options are worth to me. Aside from taking these changes into account, the above definition of the random utility core generalizes immediately. In particular, we maintain the assumption that the utilities of the outside options are random (e.g. logistic in the logit core).

**Definition 4.4** (Random utility assignment game)**.** For each outcome $(\mathbf{x}, m) \in \mathbf{X}$, the utility of $i \in N$ is $u_i(\mathbf{x}, m) = U_i(x_i, x_{m(i)}, m(i))$. The utility of the blocking coalition $(i, j)$, $j \neq m(i)$, with wages $(x_i, x_j) \in \mathbb{R}_+^2$ is $\tilde{u}_i(x_i, j) = U_i(x_i, x_j, j) + \varepsilon_{i,j}$, and the utility of the outside option is $u_i(x_i, \emptyset) = U_i(0, 0, \emptyset) + \varepsilon_{i,\emptyset}$. The distributions of $\varepsilon_{i,j}$ are continuous and stochastically independent over all $i \in N$ and $j \in N_i$.

As before, we define stochastic stability as the probability that the allocation is stable, i.e. that no pair of players can rematch profitably. In the random utility assignment game, the stochastic stability of outcome $(\mathbf{x}, m)$ therefore is

$$\pi(\mathbf{x},m) = \Pr\big(\forall i \in N, \forall j \in N_i, \forall x' \in [0, C_{i,j}] : \\ u_i(\mathbf{x},m) \geq \tilde{u}_i(x', j) \text{ or } u_j(\mathbf{x},m) \geq \tilde{u}_j(C_{i,j} - x', i)\big), \quad (3)$$

and exploiting all independence assumptions in Def. 4.4, it simplifies to

$$\pi(\mathbf{x},m) = \prod_{i \in N} \prod_{j \in N_i} \int_{\mathbb{R}} \int_{\mathbb{R}} f_{i,j}(\varepsilon_{i,j}) f_{j,i}(\varepsilon_{j,i}) \, \mathbf{1}\big\{\forall x' \in [0, C_{i,j}] : \\ U_i(x_i, x_{m(i)}, m(i)) \geq U_i(x', C_{i,j} - x', j) + \varepsilon_{i,j} \text{ or} \\ U_i(x_j, x_{m(j)}, m(j)) \geq U_j(C_{i,j} - x', x', i) + \varepsilon_{j,i}\big\} d\varepsilon_{i,j} d\varepsilon_{j,i}. \quad (4)$$

Intuitively, at the beginning of the assignment game, the utility perturbations $(\varepsilon_{i,j})$ are drawn. One may think of $\varepsilon_{i,j}$ as a measure of the "chemistry" between $i$ and $j$ (in the eyes of $i$). The players then play the assignment game and evaluate possible allocations by comparing them to the outside options plus the utility perturbations. The stochastic stability is the ex-ante probability that a given allocation will be stable.

The random utility core is again defined as the density of continuous distribution on the Pareto frontier. We assume that players are able to match completely and to allocate the whole surplus generated in their respective match.[9] Let $\text{IPF}(\mathbf{X}')$ denote the set of such allocations within $\mathbf{X}' \subseteq \mathbf{X}$ and let $M^* \subset M$ denote the set of complete matchings. Now, using $\mathbf{X}_m = \{\mathbf{x} \in \mathbb{R}^n \mid (\mathbf{x}, m) \in \mathbf{X}\}$, the random utility core is

$$f_C(\mathbf{x},m) = \pi(\mathbf{x},m) / \sum_{m \in M^*} \int_{\text{IPF}(\mathbf{X}_m)} \pi\big(\tilde{\mathbf{x}}, m\big) d\tilde{\mathbf{x}}. \quad (5)$$

Clearly, the random utility core converges pointwise to the uniform distribution on the core if the utility variances approach zero. The assumption that stochastic stability and outcome density are proportional is particularly simple and it turns out to fit our data well (see below). Alternative assumptions may prove appropriate in alternative

[9]That is, we assume that outcomes satisfy internal Pareto efficiency as defined in Fn. 7. These assumptions can be relaxed straightforwardly, but they reflect the basic observations made in the experiment. In turn, we do not assume "external Pareto efficiency" (i.e. social efficiency), which seems too strong in light of the above observations.

classes of games.

**Proposition 4.5.** *For any assignment game, the following statements are equivalent.*

1. *$f_C$ is the random utility core defined in Eqs.* (3) *and* (5).
2. *$f_C$ satisfies the following conditions.*

   A1 *Continuity and internal Pareto efficiency: $f_C$ is the density of a continuous distribution on the set outcomes satisfying internal Pareto efficiency.*

   A2 *Proportional stability: $f_C(\mathbf{x}, m)$ is proportional to the stochastic stability $\pi(\mathbf{x}, m)$.*

*Proof.* The proof is very similar to that of Prop. 4.3 and therefore skipped. □

Finally, the stochastic stabilities of level-1 and level-2 cores follow straightforwardly. The level-1 core with random utility has the stochastic stability

$$\pi(\mathbf{x}, m) = \Pr\left(\forall i \in N: \; u_i(\mathbf{x}, m) > \tilde{u}_i(C_{i,m(i)}/2, m(i))\right), \tag{6}$$

and the level-2 core has the stochastic stability

$$\pi(\mathbf{x}, m) = \Pr\big(\forall i \in N, \forall j \in N_i, x' = C_{i,j}/2 : \\ \tilde{u}_i(x', j) \leq u_i(\mathbf{x}, m) \text{ or } \tilde{u}_j(C_{i,j} - x', j) \leq u_j(\mathbf{x}, m)\big). \tag{7}$$

# 5 Econometric evaluation of the logit core

In this section, we evaluate both descriptive and predictive adequacy of the logit core revisiting the data of Otto and Bolle (2011). As above, descriptive adequacy measures the goodness of fit to the whole sample after fitting the parameters to the whole sample, and predictive adequacy measures the goodness of fit out of sample, after fitting parameters to a subset of treatments, evaluating the predictions on the remaining treatments, and rotating such that all treatments are used once. We consider the logit core in all three variants (level-1 core, level-2 core, core) and contrast it with regression models and random behavior cores for further robustness checking. In all cases, the likelihood is maximized jointly over all parameters by first gradient-free and second

Newton methods, and a variety of starting values is used to verify globality of the maximum. Table 3 lists absolute values of the log-likelihoods for all models and Figure 4 illustrates the fit of the best-fitting model.[10]

As for the logit cores, see Table 3a, the level-1 core for egoistic preferences and the level-2 core for altruistic preferences do not differ significantly ($p > 0.1$) and the latter fits significantly better than all other concepts ($p < 0.1$).[11] The difference between descriptive and predictive adequacy is insignificant for these concepts, i.e. the result is robust and not due to overfitting. This confirms that subjects do not consider all possible outcomes in alternative matches when they evaluate their current situation. The observation that level-1 and level-2 cores fit the data similarly well is surprising, however. The experiment had been designed to distinguish explicitly between these possibilities. In all treatments, we can distinguish "strong" and "weak" players, based on payoffs from their respective outside options (the alternative match). Thus, either the alternative match is strategically relevant, in which case the level-2 core should fit better than the level-1 core, or income equality is of primary relevance, in which case the level-1 core should fit better. Since both concepts fit similarly, we infer that indeed both motives are of strategic relevance and affect "stability" of outcomes.[12]

In order to investigate this hypothesis more conclusively, consider the following *mixed-level core* nesting both level-1 and level-2 cores. Its stochastic stability index is the weighted mean of the stochastic stabilities $\pi_1$ in the level-1 logit core for egoistic preferences and $\pi_2$ in the level-2 logit core for altruistic preferences (which are the best-fitting concepts out of sample), with weights $\mu \in [0,1]$.

$$\pi_{mix}(\mathbf{x},m) = (1-\mu)\cdot\pi_1(\mathbf{x},m) + \mu\cdot\pi_2(\mathbf{x},m) \tag{8}$$

The level-1 core obtains for $\mu = 0$, the level-2 core obtains for $\mu = 1$, and other values of $\mu$ yield mixtures of these concepts. Table 3b reports the parameter estimates and Ta-

[10]The supplementary material contains all parameter estimates, the results of likelihood-ratio tests (nested and non-nested Vuong tests) conducted to verify significance of differences in goodness of fit, and analyses of alternative models with (amongst others) inequity averse players.

[11]The Vuong (1989) tests for nested/non-nested models (as appropriate) are done using non-parametric tests at the session level. That is, with $(f_s)_{s=1}^{28}$ and $(g_s)_{s=1}^{28}$ denoting the log-likelihoods of two competing models in the 28 independent sessions, we evaluate the null hypothesis $H_0 : \ln(f_i/g_i) = 0$ using Wilcoxon signed-rank tests. All test results are provided as supplementary material.

[12]Note that this is not a matter of social preferences, as we explicitly tested for inequity aversion, as reported in the supplementary material, which did not improve the goodness of fit.

Table 3: Goodness of fit: Overview of all models

(a) Goodness-of-fit $|LL|$ of the random utility (logit) cores

| | # Parameters | | Descriptive adequacy | | Predictive adequacy | |
|---|---|---|---|---|---|---|
| | Ego | Altr | Egoism | Altruism | Egoism | Altruism |
| Level-1 logit core | 1 | 3 | 3003.89 | 2987.35 | 3007.54 | 3013.29 |
| Level-2 logit core | 1 | 3 | 3156.63 | 2979.29 | 3157.7 | 2992.64 |
| Logit core | 1 | 3 | 3157.26 | 2998.09 | 3160.67 | 3013.57 |
| Mixed-level logit core | 3 | 5 | 2982.87 | 2945.37 | 2986.07 | 2956.98 |

(b) Parameter estimates of the mixed-level logit core with altruism

| $\lambda_1$ | $\lambda_2$ | $\alpha$ | $\beta$ | $\mu$ | $LL$ |
|---|---|---|---|---|---|
| 13.5567 | 32.8933 | −0.489 | −0.2537 | 0.0294 | −2945.37 |
| (1.4161) | (3.6258) | (0.053) | (0.0537) | (0.0084) | |

**Note:** $\lambda_1$ and $\lambda_2$ are the precision parameters of the level-1 and level-2 components (resp.) of the mixed core defined in Eq. (8), $\alpha, \beta$ are the altruism coefficients, and $\mu$ is the mixture weight. The standard errors are obtained from the information matrix.

(c) Goodness-of-fit $|LL|$ of the random behavior cores

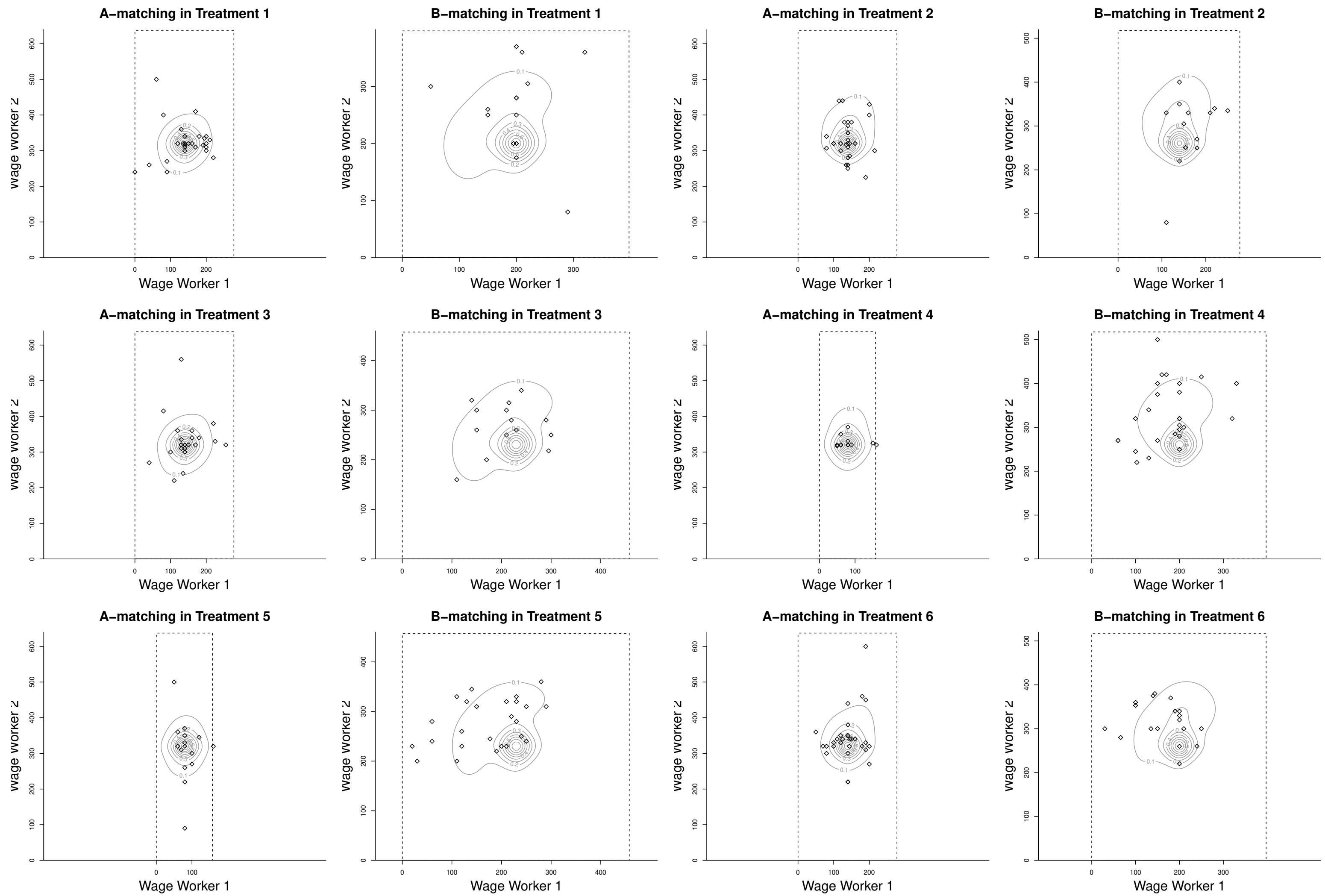
| | # Parameters | | Descriptive adequacy | | Predictive adequacy | |
|---|---|---|---|---|---|---|
| | Ego | Altr | Egoism | Altruism | Egoism | Altruism |
| Level-1 core | 1 | 3 | 3085.07 | 3057.39 | 3085.70 | 3068.21 |
| Level-2 core | 1 | 3 | 3225.18 | 3106.25 | 3225.79 | 3330.29 |
| Core | 1 | 3 | 3253.32 | 3191.34 | 3255.17 | 3208.04 |

(d) Goodness-of-fit of the regression models

| | #Pars | Descriptive $|LL|$ | Predictive $|LL|$ |
|---|---|---|---|
| *Regression on treatment parameters* | | | |
| Restricted coefficients | 14 | 2971.60 | 3052.92 |
| Unrestricted coefficients | 24 | 2964.04 | 3038.54 |
| *Regression on theoretically relevant parameters* | | | |
| Restricted coefficients | 7 | 3006.40 | 3015.56 |
| Unrestricted coefficients | 11 | 2979.82 | 3120.54 |

**Note:** The "treatment parameters" are the three productivities $C_{1,1}$, $C_{1,2}$, $C_{2,1}$ that are not constant across treatments, and the "theoretically relevant parameters" are the efficiency difference between $A$- and $B$-matching, i.e. the difference $(C_{1,1}+C_{2,2})-(C_{1,2}+C_{2,1})$ and the symmetric indicator $I_{C_{1,2}=C_{2,1}}$. In the models with "restricted coefficients", the regression coefficients are equal across $A$ and $B$ matching, in those with "unrestricted coefficients", this restriction is lifted. For example, under the "Regression on treatment parameters" model, the probability of $A$-matching is $\Pr(A) = 1/\big(1+\exp(-I_0 - p_{0,C_{1,1}}C_{1,1} - p_{0,C_{1,2}}C_{1,2} - p_{0,C_{2,1}}C_{2,1})\big)$, wages of workers $i \in \{1,2\}$ are distributed as $f(w_i) = \Pr(A)\cdot f(w_{i|A}) + (1-\Pr(A))\cdot f(w_{i|B})$, with $w_{i|M} \sim \mathcal{N}\big(I_{i|M} + p_{i|M,C_{1,1}}\, C_{1,1} + p_{i|M,C_{1,2}}\, C_{1,2} + p_{i|M,C_{2,1}}\, C_{2,1}, \sigma^2_{i|M}\big)$ and support $\big[0, \overline{w}_{i|M}\big]$, using $(\overline{w}_{1|A}, \overline{w}_{2|A}, \overline{w}_{1|B}, \overline{w}_{2|B}) = (C_{1,1}, C_{2,2}, C_{1,2}, C_{2,1})$. The other models are defined correspondingly (details are provided in the supplementary material).

Figure 4: Contour plots of the relative stochastic stabilities for the mixed-level logit core with altruistic preferences. The iso-lines connect outcomes with the same stochastic stability and hence the same predicted density. Outcomes along the the outmost line (at “0.1”) have 10% of the stochastic stability (and density) of the stochastically most stable outcome in the respective game.



ble 3a reports descriptive and predictive adequacy. It shows that the mixed-level core allowing for interdependent preferences (i.e. spite) improves the goodness of fit substantially, by at least 30 log-likelihood points in relation to all other models, and as we verified in Vuong tests, these differences are highly significant ($p < .01$) in all cases. Although the spite coefficients are highly significant, too, even the mixed-level core with egoistic preferences improves on all other models. We therefore conclude that the subjects' main criterion for stability is equality of incomes within matches, while the potential payoff from the alternative match and the interdependence of preferences are of secondary, but significant relevance.

Comparing these results with those obtained by maximizing the Selten score in Section 3, two observations are noteworthy. On the one hand, the mixed-level core, which is the unambiguously best-fitting model here, cannot be defined without introducing the notion of stochastic stability. Thus, the logit core substantially extends the range of models that can be considered. On the other hand, between the models considered above, the ranking of the three best models actually inverted, toward (1) Level-2 Altruism, (2) Level-1 Egoism, and (3) Level-1 Altruism, with the difference between (1) and (3) even being significant. This follows from using all information contained in the distribution of outcomes, such as the distance from an outside observation to the core, and in particular by smoothening the transition between close misses and close hits. In addition, of course, the maximum likelihood estimates allow for straightforward computation of standard errors as well as model evaluation based on likelihood-ratio tests and information criteria.

Figure 4 shows that the mixed-level core also fits qualitatively. Its Cox-Snell pseudo-$R^2$ is $\tilde{R}^2 = 0.9239$,[13] which confirms the positive visual impression of the plots. The estimated utility parameters are $\alpha = -0.49$ and $\beta = -0.25$ (Table 3b), which confirms that players are spiteful when evaluating alternative outcomes. This indicates competitive bargaining and thus seems to be reasonable.

Finally, we conduct two robustness checks to verify the results. On the one hand, the logit core was defined to capture the distribution of the observations inside the core, as central observations are more frequent than borderline observations, as well as the distribution outside the core, as borderline observations are more frequent than

[13]The Cox-Snell pseudo-$R^2$ is $\tilde{R}^2 = 1 - (L(\text{Baseline})/L(\text{Model}))^{2/N}$ with $L$ being the likelihood function and the Baseline model being the model predicting uniform randomization. Its log-likelihood is $-3276.27$ and the number of observations is $N = 257$.

distant observations. To verify this implication, define a *random behavior core* with uniform noise as follows: Outcomes are distributed either uniformly inside the core, with probability $1/(1+\varepsilon)$, or uniformly outside the core, with probability $\varepsilon/(1+\varepsilon)$. As $\varepsilon$ tends to zero, this random behavior core converges uniformly to the uniform distribution on the core, as does the logit core as noise disappears, and it has the same number of parameters. The only difference is the specification of noise. Its definition applies to level-1 and level-2 cores straightforwardly. Table 3c reports the descriptive and predictive adequacy of all six (basic) variants. It shows that random behavior cores fit uniformly worse than their respective logit core counterparts, and all of these differences are highly significant. In addition to Figure 4 and pseudo-$R^2$ reported above, this confirms that the continuous stochastic stability implied by the logit core fits the outcome distribution well.

On the other hand, the core is a strategic model, and to verify whether this is a fruitful approach toward analyzing negotiation outcomes in the first place, let us now examine non-strategic and reduced-form regression models. As shown above, subjects evaluate outcomes primarily by their difference to the equal split, which suggests that regression models may fit indeed. We investigate this hypothesis on four alternative regression models, in order to be able to address this issue conclusively. Similarly to the logit cores, all regression models must predict the probabilities of $A$ and $B$ matching as well as the distribution of wages $w_1, w_2$. The first two models are standard and regress these variables on the treatment parameters $C_{1,1}$, $C_{1,2}$, $C_{2,1}$ (note that $C_{2,2}$ is held constant in all treatments). The other two models represent the prediction of the best fitting structural model, the mixed-level core, in reduced form. They use the same information, namely the efficiency gain in $A$-matching, the symmetry indicator $I_{C_{1,2}=C_{2,1}}$, and the values of the outside options in each form of matching. For each class of models, we distinguish a parsimonious "restricted" form, where the wage coefficients are constant across $A$ and $B$ matching, and an "unrestricted" form where coefficients are flexible (for further information, see the note to Table 3d).

The goodness-of-fit measures for all models are listed in Table 3d. Three of the four regression models improve upon the level-$k$ logit cores in-sample, which confirms that the basic distribution can be fit using regression. However, these three models overfit drastically and their predictive adequacy is poor—they are significantly worse than *all* random utility models allowing for altruistic preferences. Thus, the in-sample fit merely stems from their vastly extended parameter spaces and is overfit. It does not allow for reliable inference. The single regression model that avoids overfitting is the regression on "strategically relevant parameters" with "restricted coefficients", which has poor descriptive and predictive adequacy, however. Thus, we conclude that the strategic solution concept "logit core" is substantially more adequate in predicting (laboratory) assignment outcomes than at least such "standard" regression models.

# 6 Conclusion

This paper has analyzed cooperative assignment games with the intention of understanding the sources of the prediction bias implied by the core. The competing hypotheses were that subjects have interdependent preferences or limited depth of reasoning. In order to fully capture the distribution of observations, and to extract all information contained therein, we extended the core concept to allow for random utility perturbations—an approach that proved descriptive in many previous experiments on decision theory and non-cooperative game theory. We found that the logit core indeed fits the distribution of outcomes and that subjects deviate from the core primarily due to a severely limited grasp (or due to severe discounting) of possible outcomes in alternative matches. Stable outcomes tend to be close to the equal split, as the "level-1 core" is the main component of the identified model, while alternative matches tend to matter only under the simplified assumption that the equal split will obtain there, and interdependence of preferences is significant but of minor relevance. The identified model nests level-1 and level-2 cores and fits both qualitatively and quantitatively, notably without overfitting.

Both interdependence of preferences and random utility perturbations are novel in analyses of cooperative games. Their adequacy in the present analysis and their widespread use in non-cooperative game theory suggest that further research is warranted. Besides further analyses of random-utility concepts in cooperative bargaining games, further research may also investigate values of games with random utility, and perhaps most interestingly, it may evaluate the descriptive and predictive adequacies of cooperative and non-cooperative models in comparative studies. This may help to map out their respective fields of application and to define new concepts modeling the key insights from both branches.

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