



Munich Personal RePEc Archive

The Microfoundations of the Keynesian Wage-Price Spiral

Malikane, Christopher

University of the Witwatersrand, School of Economic and Business
Sciences

29 November 2012

Online at <https://mpra.ub.uni-muenchen.de/42921/>
MPRA Paper No. 42921, posted 01 Dec 2012 01:28 UTC

The Microfoundations of the Keynesian Wage-Price Spiral

Christopher Malikane
Macro-Financial Analysis Group
School of Economic and Business Sciences
University of the Witwatersrand
1 Jan Smuts Avenue
Johannesburg
2050

Abstract

We derive the backward-looking Keynesian wage-price spiral from microfoundations. The optimal price Phillips curve features one lag of price inflation, the lag of the labour share, excess demand pressure, speed-limit effects and supply shocks. The wage Phillips curve features current and lagged price inflation, excess demand pressure up to the second lag, and the lag of nominal wage inflation. We estimate this model for six developed and emerging market economies and find that the model fits the data well. In general, nominal wages are more flexible than prices with respect to demand pressure. The baseline model rejects the inclusion of supply shocks and indexation of wages in developed economies and some emerging markets.

Keywords: microfoundations, wage and price Phillips curves, forward and backward-looking behaviour.

JEL Codes: E12, E24, E31, E32.

1. Introduction

This paper provides the microfoundations for the backward-looking Keynesian wage-price spiral similar to that of Chiarella and Flaschel (2000), Fair (2000, 2008), Flaschel et.al.(2001) and Asada et.al.(2006). Fuhrer (1995) notes that one of the major criticisms of the Phillips curve is its lack of microfoundations. Since then, new Keynesian economists, e.g. Galí and Gertler (1999), formulated a Phillips curve relation from microfoundations thereby providing theoretical grounds for "the return of the Phillips curve" as noted

⁰Email: christopher.malikane@wits.ac.za. Tel: +27-11-717-8109. Fax: +27-11-717-8081.

by Gali (2000). Their optimisation procedure yields a relation in which current inflation is determined by expected future inflation and marginal cost. Further developments of the new Keynesian Phillips produced a hybrid formulation that features both forward and backward-looking inflation terms, under the assumption of price indexation.

Despite its microfoundations, the success of the new Keynesian Phillips curve in describing inflation dynamics remains elusive and the econometrics that support it has been a source of debate. In relation to the econometric aspects, a major problem with the new Keynesian Phillips curve has been that of identification. Bardsen et.al. (2004), Mavroeidis (2004, 2005) and Martins and Gabriel (2009) find that the new Keynesian Phillips curve is weakly identified. Another set of problems relate to the significance and the sign of the marginal cost variable. Rudd and Whelan (2005, 2007) find that the forcing variable exhibits insignificance and sometimes a negative sign in the case of the US. In the case of the Euro-Area, Mazumder (2012) finds an insignificant and negative sign for the forcing variable. Similarly Abbas and Sgro (2012) estimate the new Keynesian equation for Australia and find insignificant and negative signs. In relation to forecast performance Gordon (2011) finds that the new Keynesian model delivers poor results compared to the triangle model, while Fair (2008) finds that the new Keynesian model is the worst performer compared to the triangle model and the wage-price spiral.

In his econometric analysis Fuhrer (1995) finds that the traditional, backward-looking Phillips curve, seems to do well in describing the data. In addition the backward-looking Phillips curve is robust to the Lucas critique. Gordon (2011) also finds that the triangle model with the time-varying NAIRU describes the data very well. Another strand of the Phillips curve literature maintains the relevance of the wage Phillips curve in describing inflation dynamics. Fair (2000, 2008), Chiarella and Flaschel (2000), Flaschel et.al. (2001) and Asada et.al. (2006) argue that reduced-form formulations such as the new Keynesian Phillips curve and triangle model do not provide adequate descriptions of the inflation process. Despite their empirical success, the problem with the backward-looking triangle and wage-price Phillips curves is that they are not based on microfoundations.

The new Keynesian literature has since moved to incorporate wage dynamics into the inflation process. For example, Sbordone (2001), Erceg et.al. (2000),

Huang and Liu (2002), Woodford (2003, Chapter 3), Gertler et.al.(2008) and Gali (2011) provide microfoundations for "the return of the wage Phillips curve". New Keynesian wage-price Phillips curves are now standard in DSGE models e.g. Amato and Laubach (2003), Smets and Wouters (2003), Ambler et.al. (2011), Carlsson and Westermark (2011) and Gali et.al.(2012). In all the new Keynesian formulations the wage Phillips curve assumes a similar form to the price Phillips curve in that the forward-looking term appears on the right hand side. This result has been criticised by Mankiw (2001) and Asada et.al. (2006) for generating counter-factual inflation dynamics. The presence of the forward-looking term on the right hand side of the wage and price Phillips curves thus remains a challenge for the new Keynesian model.

The contribution of this paper is twofold. Firstly, we provide the microfoundations for the backward-looking Keynesian wage-price spiral. As pointed out by Fuhrer (1995), there seems to be consensus in Phillips curve literature that backward-looking Phillips curves lack microfoundations. Our paper provides microfoundations that make the forward-looking term appear on the left hand side, thereby providing theoretical grounds for "the return of the backward-looking Phillips curve". In this sense, this paper responds to the criticism of backward-looking Phillips curves that have been levelled by Gali et.al. (2001). As in Gali (2011) we are able to explicitly provide a structural interpretation of the parameters of our model, thereby providing an optimisation-based competing perspective to the new Keynesian view. Secondly, we estimate the backward-looking wage-price spiral for a set of developed and emerging markets to test if the model fits the data.

The paper is structured as follows: Section 2 derives the Keynesian price and wage Phillips curves from microfoundations. Section 3 provides empirical evidence of the model. Section 4 concludes.

2. Staggered wage-price setting and wage-price Phillips curves

2.1 Staggered price-setting

Assume identical firms that operate in an environment where variations in prices are of such a magnitude that they do not significantly affect the level of demand faced by each firm. We index each firm in this environment by j . Since firms are identical, we assume that the price set by each firm is the

same as the aggregate price level P_t . Further assume, along the lines of Batini et.al. (2005), fixed input requirements in production such that : $X_{it} = \delta_i Y_{jt}$, where X_{it} is the amount of non-labour input i required in production and δ_i is the input requirement coefficient. The representative firm engages in one-period contracts so that, at a point in time t , it produces real output Y_{jt} and contracts to sell a fraction of this output τ_p at price P_t in time $t + 1$. The rest of the output $(1 - \tau_p) Y_{jt}$ and the increment ΔY_{jt+1} will be sold at P_{t+1} .

The fraction τ_p is analogous to the Calvo (1983) probability of no price change and is thus a measure of price rigidity. In our instance the τ_p applies to output produced by firms and not to firms. We can then write the real revenue of the firm at time t as follows:

$$R_{jt}^p = \Delta Y_{jt} + (1 - \tau_p) Y_{jt-1} + \tau_p \left(\frac{P_{t-1}}{P_t} \right) Y_{jt-1} \quad (1)$$

where R_{jt}^p is real revenue. Eq. (1) is derived from noting that $Y_{jt} = \Delta Y_{jt} + Y_{jt-1}$, i.e. current output is incremental output plus past output. Over and above production costs, the firm incurs losses due to lack of price flexibility imposed by the one-period contract. The assumption that there are costs associated with contracts is due to Rotemberg (1982) and is used by new Keynesian economists as an alternative to the Calvo-based derivation of the Phillips curve (see e.g. Batini et.al.(2005), Fuhrer et.al.(2009:16), Ascari et.al.(2011) and Guender (2011)). Assume an amount equal to χ_p of last period output is demanded at the current price but the firm can only sell it at last period's price because of the contract. Then the real loss from the contract is:

$$\Theta_{jt}^p = \tau_p \chi_p \left(\frac{P_t - P_{t-1}}{P_t} \right) Y_{jt-1}, \quad (2)$$

where $\chi_p > 0$. The firm's objective is to maximise discounted expected profits, given the nominal wage and demand Y_{jt} by choosing the optimal price. This problem can be stated as follows:

$$\max_{P_t} \Pi_{jt} = E_t \sum_{k=0}^{\infty} \beta^k \left(R_{jt+k}^p - \frac{W_{t+k} L_{jt+k}}{P_{t+k}} - \frac{Y_{jt+k}}{P_{t+k}} \sum_{i=1}^n \delta_i P_{it+k} - \Theta_{jt+k}^p \right) \quad (3)$$

where Π_{jt} is real aggregate profits of the firm, W_t is the nominal wage, L_{jt} is the level of employment which is determined by demand, given the real wage, P_{it} is the price of input i and n is the number of non-labour inputs and β is the discount factor. The first order condition for price-setting yields the following relationship:

$$-\tau_p (1 + \chi_p) \left(\frac{P_{t-1}}{P_t} \right) Y_{jt-1} + \frac{W_t L_{jt}}{P_t} + \left(\sum_{i=1}^n \frac{\delta_i P_{it}}{P_t} + \mathcal{F}^p E_t \left(\frac{P_t}{P_{t+1}} \right) \right) Y_{jt} = 0, \quad (4)$$

where we have set $\mathcal{F}^p = \beta \tau_p (1 + \chi_p)$ for compactness. Denote the labour share in firm j by S_{jt} , we can then write eq.(4) as follows:

$$E_t (1 + \hat{p}_{t+1})^{-1} Y_{jt} = \frac{1}{\beta} (1 + \hat{p}_t)^{-1} Y_{jt-1} - \frac{Y_{jt}}{\mathcal{F}^p} \left(S_{jt} + \sum_{i=1}^n \delta_i \frac{P_{it}}{P_t} \right), \quad (5)$$

where \hat{p}_t denotes the price inflation rate. Linearising eq.(5) around the steady state and aggregating across firms we obtain the following Phillips curve relation:

$$E_t \hat{p}_{t+1} = a_p \hat{p}_t + a_s \tilde{s}_t + a'_y \tilde{y}_t - a''_y \tilde{y}_{t-1} + \sum_{i=1}^n a_{pi} \tilde{p}_{it}, \quad (6)$$

where \tilde{s}_t is the level deviation of the labour share from the steady state, \tilde{p}_{it} is the level deviation of real input prices from the steady state and \tilde{y}_t is the percentage deviation of output from potential. In order to explicitly express the coefficients in eq.(6) in terms of all the underlying structural parameters, we explicitly write \mathcal{F}^p so that:

$$\begin{aligned}
a_p &= \frac{1}{\beta}, \quad a_s = \frac{(1 + \widehat{p}_0)^2}{\beta \tau_p (1 + \chi_p)}, \quad a_y = \frac{(1 + \widehat{p}_0)}{\beta}, \quad a_{pi} = \frac{(1 + \widehat{p}_0)^2 \delta_i}{\beta \tau_p (1 + \chi_p)} \\
a'_y &= (1 + \widehat{p}_0) + \frac{(1 + \widehat{p}_0)^2}{\beta \tau_p (1 + \chi_p)} + \sum_{i=1}^n a_{pi},
\end{aligned}$$

Eq.(6) is different from the new Keynesian Phillips curve in four respects. Firstly, the forward-looking term is now on the left hand side. Secondly, the output gap appears on the right hand side together with the labour share. Thirdly, eq.(6) exhibits speed-limit effects directly from the optimisation exercise. Fourthly, in contrast to new Keynesian derivations, inflation persistence arises from optimisation in the context of one-period contracts as opposed to persistence being a result purely of rule-of-thumb price setting.

We now turn to the structural interpretation of the parameters. If all output is sold at current prices, i.e. no contracts, the Phillips curve becomes vertical since $\tau_p = 0$. In this instance prices are fully flexible. This result is consistent with the idea that in the long run, where prices are fully flexible, the Phillips curve assumes a vertical shape. This result is analogous to the effect of the Calvo probability parameter on the slope of the new Keynesian Phillips curve (see the coefficient of marginal cost in Galí and Gertler (1999) and Galí (2000) in particular). Furthermore, if there is an increase in χ_p the slope of the Phillips curve would fall. The intuition for this is that current prices do not fully adjust to "excess demand" because part of current output is contracted at previous period prices.

Flowing from this interpretation, it follows that even if firms enter into one period contracts, i.e. $0 < \tau_p < 1$, if in period t the portion of aggregate demand that is due to customers who are willing and able to pay at current prices does not exceed $(1 - \tau_p) Y_{t-1} + \Delta Y_t$, then the contract will not effectively generate price rigidity. This is the case because firms could meet aggregate demand at prevailing market prices. Thus as long as $\chi_p > 0$ the contract will effectively create price rigidity. We therefore interpret the combination $\tau_p (1 + \chi_p)$ as a measure of "effective price rigidity".

Our price Phillips curve features speed-limit effects captured by lagged output gap. The recognition of this term in Phillips curve literature is highlighted by Mehra (2004) and Gordon (2011). Mehra in particular finds that

the inclusion of the change in the output gap in the hybrid specification boosts the significance and size of the backward-looking term in the new Keynesian model. Gordon mentions the role of lagged excess demand terms, with their zigzag signs. Assume that $E_t \widehat{p}_{t+1} = \widehat{p}_{t+1} + \eta_{t+1}$, where $E_t \eta_{t+1} = 0$. Then we can write eq.(4) as:

$$\widehat{p}_t = a_p \widehat{p}_{t-1} + a_s \widetilde{s}_{t-1} + (a'_y - a''_y) \widetilde{y}_{t-1} + a''_y \Delta \widetilde{y}_{t-1} + \sum_{i=1}^n a_{pi} \widetilde{p}_{it-1} - \eta_t \quad (7)$$

Eq.(7) is the *structural Keynesian Price Phillips curve*. It explains the observation by Mehra (2004), although he conducts his analysis within the new Keynesian setup, that the omission of supply shocks may be responsible for the finding that the output gap is irrelevant to inflation dynamics. If supply shocks are omitted, then $\sum_{i=1}^n a_{pi}$ does not appear in a'_y , this biases the output gap parameter downwards. This point is also made by Gordon (2011). If the labour share is not included, the output gap parameter is further biased downwards.

We now consider firms that use a rule-of-thumb to set prices. At each point in time, a fraction of firms ϵ_p sets prices in an optimal way whilst the rest uses some rule-of-thumb. This assumption is similar to the new Keynesian derivation of the hybrid Phillips curve (Gali and Gertler, 1999). By assuming that the aggregate price level is the geometric average of the price set by optimising and rule-of-thumb firms, we can write the inflation rate aggregate price index as:

$$\widehat{p}_t = \epsilon_p \widehat{p}_t^o + (1 - \epsilon_p) v_p \pi_t \quad (8)$$

where \widehat{p}_t^o is the optimal price inflation rate and $\pi_t = \frac{1}{m} \sum_{j=1}^m \widehat{p}_{t-j}$ is the indexing variable used by rule-of-thumb firms and v_p is the indexation parameter. Our indexation rule follows Smets and Wouters (2003) and Christiano et al. (2005) in postulating an autoregressive inflation rule-of-thumb. However

Smets and Wouters, and Christiano et al. assume a restrictive rule for non-optimising firms wherein these firms set prices with consideration only of one-lag inflation. Our formulation follows Gali (2011), who also uses the moving average to construct a smoother indexing variable. Zhang and Clovis (2010) also argue for higher lags for the indexing variable, since they find that with the one-lag rule-of-thumb, estimations of the new Keynesian price Phillips curve generate serially correlated residuals. Thus there is scope for long lags in inflation to enter the Phillips curve as in Gordon (1997, 2011), through rule-of-thumb behaviour. Using eq.(7) for the optimal price inflation rate, we can write the price Phillips curve as follows:

$$\begin{aligned}\widehat{p}_t &= \epsilon_p a_p \widehat{p}_{t-1}^o + (1 - \epsilon_p) v_p \pi_t + \epsilon_p a_s \widetilde{s}_{t-1} + \epsilon_p (a_y' - a_y'') \widetilde{y}_{t-1} + \epsilon_p a_y'' \Delta \widetilde{y}_{t-1} \\ &\quad + \epsilon_p \sum_{i=1}^n a_{pi} \widetilde{p}_{it-1} + \varepsilon_t^p\end{aligned}\quad (9)$$

where $\varepsilon_t^p = -\epsilon_p \eta_t^p$. Note that we can express eq.(9) in terms of nominal unit labour cost by recalling that $\widehat{s}_t = \widehat{w}_t - \widehat{p}_t$ where \widehat{w}_t is nominal unit labour cost inflation and hence $\widetilde{s}_t = \widehat{w}_t - \widehat{p}_t + \widetilde{s}_{t-1}$. Inserting this into eq.(9) we get the following price Phillips curve:

$$\begin{aligned}\widehat{p}_t &= \epsilon_p a_p \widehat{p}_{t-1}^o + (1 - \epsilon_p) v_p \pi_t + \epsilon_p a_s \widetilde{s}_{t-2} + \epsilon_p (a_y' - a_y'') \widetilde{y}_{t-1} \\ &\quad + \epsilon_p a_y'' \Delta \widetilde{y}_{t-1} + \epsilon_p a_s \widehat{p}_{t-1} + \epsilon_p a_s \widehat{w}_{t-1} + \epsilon_p \sum_{i=1}^n \delta_i \widetilde{p}_{it-1} + \varepsilon_t^p\end{aligned}\quad (10)$$

Eq. (10) provides the micro-founded version of the traditional Phillips curve in which rule-of-thumb firms are combined with optimising firms.

2.2 Staggered wage-setting

The derivation of the law of motion for nominal wages follows analogously from the price-setting process. Assume that variations in nominal wages are of such a magnitude that they do not significantly affect the aggregate

demand for labour. Workers engage in one-period contracts so that a fraction of workers τ_w contract to supply labour at period $t + 1$ at nominal wages W_t . The rest of the workers $(1 - \tau_w) L_t$ and newly employed workers ΔL_{t+1} will sell their labour at a nominal wage W_{t+1} . These assumptions are similar to the Calvo-type formulation presented by Gali (2011). Total real earnings by workers R_t^w can be written as follows:

$$R_t^w = \frac{W_t}{P_t} \Delta L_t + (1 - \tau_w) \frac{W_t}{P_t} L_{t-1} + \tau_w \frac{W_{t-1}}{P_t} L_{t-1}, \quad (11)$$

We assume a fraction χ_w of contracted labour is demanded at the current nominal but workers can only sell it at last period's nominal wage. The real loss from the contract is:

$$\Theta_t^w = \tau_w \chi_w \left(\frac{W_t - W_{t-1}}{P_t} \right) L_{t-1} \quad (12)$$

Workers strive to consume above some exogenously determined subsistence level of consumption. Thus if workers consume at subsistence level, their utility is zero and when they consume below the subsistence level, their utility is negative. Let the number of baskets of goods and services that constitute the subsistence level be an exogenously determined amount C_{st}^w . Each basket is purchased at a price P_t . Therefore workers want to maximise the difference between their real earnings and the cost of subsistence consumption. The idea of a subsistence level of consumption in the utility function can be found in the form of the reservation wage in Sen and Dutt (1995) and in Campbell and Viceira (2002:177). The role of the reservation wage in explaining wage dynamics is also analysed by Blanchard and Katz (1999). Workers seek to solve the following problem:

$$\max_{W_t} U_t = \sum_{k=0}^{\infty} \beta^k \frac{\Psi_t^{1-\vartheta_w}}{1 - \vartheta_w} \quad (13)$$

where $\Psi_t = R_t^w - \Theta_t^w - C_{st}^w$, subject to eqs.(11) and (12). The first-order condition to this problem yields the following result:

$$\Psi_t^{-\vartheta_w} \left(\frac{L_t}{P_t} - \tau_w (1 + \chi_w) \frac{L_{t-1}}{P_t} \right) + \tau_w (1 + \chi_w) \beta E_t \frac{L_t}{P_{t+1}} \Psi_{t+1}^{-\vartheta_w} = 0 \quad (14)$$

Multiplying by $\frac{P_t}{L_t}$ both sides, eq.(14) can be further simplified to get the following:

$$\Psi_t'^{-\vartheta_w} \left(1 - \left(\frac{\tau_w (1 + \chi_w)}{1 + \widehat{l}_t} \right) \right) = -\tau_w (1 + \chi_w) \beta E_t \left(\frac{\Psi_{t+1}'^{-\vartheta_w}}{1 + \widehat{p}_{t+1}} \right), \quad (15)$$

where \widehat{l}_t is the growth rate of employment and $\Psi_t' = \frac{\Psi_t}{s_0 Y_0}$. We note from eq.(15) that in the steady state:

$$1 - \left(\frac{\tau_w (1 + \chi_w)}{1 + \widehat{l}_0} \right) = -\frac{\tau_w (1 + \chi_w) \beta}{1 + \widehat{p}_0}, \quad (16)$$

which makes the sign of the left hand side of eq.(16) definitely negative. We further note from this that $\tau_w (1 + \chi_w) > 1$. We linearise Ψ_t' to get the following equation:

$$\begin{aligned} \widetilde{\Psi}_t' &\approx \widetilde{s}_t + \widetilde{y}_t - \frac{1}{1 + \widehat{l}_0} \left(\widetilde{s}_t + \widetilde{y}_t - \frac{\widehat{l}_t}{1 + \widehat{l}_0} \right) + \frac{\tau_w}{1 + \widehat{p}_0} \left(\widetilde{s}_{t-1} + \widetilde{y}_{t-1} - \frac{\widehat{p}_t}{1 + \widehat{p}_0} \right) \\ &+ \frac{1 - \tau_w}{1 + \widehat{l}_0} \left(\widetilde{s}_t + \widetilde{y}_t - \frac{\widehat{l}_t}{1 + \widehat{l}_0} \right) - \frac{\tau_w \chi_w}{1 + \widehat{l}_0} \left(\widetilde{s}_t + \widetilde{y}_t - \frac{\widehat{l}_t}{1 + \widehat{l}_0} \right) \\ &+ \frac{\tau_w \chi_w}{1 + \widehat{p}_0} \left(\widetilde{s}_{t-1} + \widetilde{y}_{t-1} - \frac{\widehat{p}_t}{1 + \widehat{p}_0} \right) - \varkappa_c \widetilde{c}_{st}^w \end{aligned} \quad (17)$$

where $\varkappa_c = \frac{C_{s0}^w}{s_0 Y_0}$. Collecting like terms together and exploiting eq.(16) yields the following:

$$\begin{aligned}\tilde{\Psi}'_t \approx & -\frac{\mathcal{F}^w \beta}{1 + \hat{p}_0} (\tilde{s}_t + \tilde{y}_t) + \frac{\mathcal{F}^w}{(1 + \hat{l}_0)^2} \hat{l}_t + \frac{\mathcal{F}^w}{1 + \hat{p}_0} (\tilde{s}_{t-1} + \tilde{y}_{t-1}) \\ & - \frac{\mathcal{F}^w}{(1 + \hat{p}_0)^2} \hat{p}_t - \varkappa_c \tilde{c}_{st}^w\end{aligned}\quad (18)$$

where for compactness $\mathcal{F}^w = \tau_w (1 + \chi_w) > 1 + \hat{l}_0$, is our measure of effective nominal wage rigidity. Since we assume fixed non-labour input requirements in production, we can write the production function as follows:

$$Y_t = A_t L_t^\alpha \left[\prod_{i=1}^n (\delta_i Y_t)^{\theta_i} \right]^{1-\alpha}, \quad (19)$$

where we have assumed that the capital stock is fixed by normalising it to unity and, as in King et al. (1988), we have set $A_t = 1$ in the steady state. The reduced-form production function can be written follows:

$$Y_t = A'_t L_t^\sigma, \quad (20)$$

where $\phi = (1 - \alpha) \sum_{i=1}^n \theta_i$, $\sigma = \frac{\alpha}{1-\phi}$ and $A'_t = \left(A_t \prod_{i=1}^n \delta_i^{\theta_i} \right)^{\frac{1}{1-\phi}}$. It follows that $\tilde{l}_t = \frac{\tilde{y}_t - \tilde{a}_t}{\sigma}$. By eliminating \tilde{l}_t from eq.(18) and collecting like terms together we obtain:

$$\begin{aligned}\tilde{\Psi}'_t \approx & -\frac{\mathcal{F}^w \beta}{1 + \hat{p}_0} (\tilde{s}_t + \tilde{y}_t) + \frac{\mathcal{F}^w}{\sigma (1 + \hat{l}_0)^2} \Delta \tilde{y}_t + \frac{\mathcal{F}^w}{1 + \hat{p}_0} (\tilde{s}_{t-1} + \tilde{y}_{t-1}) \\ & - \frac{\mathcal{F}^w}{(1 + \hat{p}_0)^2} \hat{p}_t - \zeta_t^w\end{aligned}\quad (21)$$

where $\zeta_t^w = \varkappa_c \tilde{c}_{st}^w + \frac{\mathcal{F}^w}{\sigma(1+\hat{l}_0)^2} \Delta \tilde{a}_t'$. We can therefore express the linearised version of the first-order condition eq.(15) as follows:

$$\tilde{\Psi}'_t + \left(\frac{1 + \hat{p}_0}{\beta \sigma \vartheta_w (1 + \hat{l}_0)^2} \right) (\Delta \tilde{y}_t - \Delta \tilde{a}_t') = E_t \left(\tilde{\Psi}'_{t+1} + \frac{1}{\vartheta_w (1 + \hat{p}_0)} \hat{p}_{t+1} \right), \quad (22)$$

By exploiting eq.(16) and collecting like terms together, the first-order condition yields the following result:

$$E_t \Delta \tilde{s}_{t+1} = E_t \left(b_p \hat{p}_{t+1} + b'_y \Delta \tilde{y}_{t+1} - \xi_{t+1}^w \right) + b'_p \hat{p}_t + b''_y \Delta \tilde{y}_t + b_s \Delta \tilde{s}_t \quad (23)$$

where, by using the fact that $\left(\frac{1+\hat{p}_0}{1+\hat{l}_0} \right) = \left(\frac{\mathcal{F}^w \beta + (1+\hat{p}_0)}{\mathcal{F}^w} \right)$ from eq.(16), we have eliminated \hat{l}_0 from the parameters:

$$\begin{aligned} b_p &= \frac{1}{\vartheta_w \mathcal{F}^w \beta} - \frac{1}{\beta (1 + \hat{p}_0)}, \quad b'_p = \frac{1}{\beta (1 + \hat{p}_0)}, \quad b'_y = \left(\frac{1}{\sigma} + \frac{(1 + \hat{p}_0)}{\beta \sigma \mathcal{F}^w} - 1 \right), \\ b''_y &= \frac{1}{\beta} + \frac{(1 + \hat{p}_0) (\mathcal{F}^w \beta + (1 + \hat{p}_0))}{(\mathcal{F}^w \beta)^2 \sigma \vartheta_w} - \left(\frac{1}{\sigma} + \frac{(1 + \hat{p}_0)}{\beta \sigma \mathcal{F}^w} \right) \\ b_s &= \frac{1}{\beta}, \quad \xi_{t+1}^w = \left(\frac{(1 + \hat{p}_0) (\mathcal{F}^w \beta + (1 + \hat{p}_0))}{\mathcal{F}^w \beta^2 \sigma \vartheta_w} \right) \Delta \tilde{a}_t' + \left(\frac{1 + \hat{p}_0}{\mathcal{F}^w \beta} \right) \Delta \zeta_{t+1}^w, \end{aligned}$$

Recall that the growth rate of the labour share is, by definition, $\hat{s}_t = \hat{w}_t - \hat{p}_t$. Also by definition we know that $\hat{s}_t = \tilde{s}_t - \tilde{s}_{t-1} - \hat{s}_t^*$. Accordingly we can write the deviation of the labour share from trend as: $\tilde{s}_t = \tilde{s}_{t-1} + \hat{w}_t - \hat{p}_t + \hat{s}_t^*$. We can then express eq.(23) as follows:

$$E_t \hat{w}_{t+1} = \left(E_t (1 + b_p) \hat{p}_{t+1} + b'_y \Delta \tilde{y}_{t+1} - \xi_{t+1}^{ww} \right) + (b'_p - b_s) \hat{p}_t + b''_y \Delta \tilde{y}_t + b_s \hat{w}_t + \xi_{t+1}^{ww} \quad (24)$$

where $\xi_{t+1}^{ww} = \left(\xi_{t+1}^w + E_t \widehat{s}_{t+1}^* \right)$. Eq.(24) differs from the new Keynesian wage Phillips curve in that the expectations term is on the left hand side. In addition, future nominal unit labour cost inflation depends on past nominal unit labour cost inflation and lags of changes in the output gap. Taking eq.(24) one step backwards we get the following relationship:

$$\widehat{w}_t = (1 + b_p) \widehat{p}_t + \left(b'_p - b_s \right) \widehat{p}_{t-1} + b'_y \Delta \widetilde{y}_t + b''_y \Delta \widetilde{y}_{t-1} + b_s \widehat{w}_{t-1} + \varepsilon_t^w \quad (25)$$

where $\varepsilon_t^w = \xi_t^{ww} - \eta_t^w$ and η_t^w is the expectational error. Eq.(25) is the *structural Keynesian Wage Phillips Curve*. Nominal unit labour cost inflation depends on a lag of itself, current price inflation, lagged price inflation, the rate-of-change in the output gap, and the lag of the rate-of-change in the output gap. In Gali (2011) nominal wage inflation depends on expected future nominal wage inflation as in all new Keynesian formulations (e.g. Amato and Laubach (2003), Woodford (2003, Chapter 3) among others), lagged inflation due to one-lag indexation and the current unemployment gap. It is the dependence of current inflation to expected future fundamentals that has been the source of criticism of the new Keynesian price Phillips curve (see Asada et.al.(2006) and Rudd and Whelan (2007)). The new Keynesian wage Phillips curve inherits the weaknesses of the new Keynesian price Phillips curve, which are not present in our model.

Our model of the wage-price dynamics further clarifies the role of the nominal wage Phillips curve in traditional Keynesian Phillips curve literature. Gordon (2011) relies on the reduced-form single-equation triangle model on the grounds that wage-wage inertia is not significant and furthermore follows Sims (1987) in dispensing with the wage Phillips curve on the grounds that separate wage and price Phillips curves cannot be identified. On the other hand Fair (2000, 2008) and Flaschel et.al. (2001) among others, insist on the importance of separate wage and price Phillips curves because these improve accuracy in describing the inflation process. In our case, to remove the wage-wage inertia is impossible, since its coefficient is the inverse of the discount factor. Thus, our derivation of the Keynesian wage and price Phillips curves provides structural grounds for the relevance of both curves in describing inflation dynamics.

Suppose $\tau_w = 0$, which means that $\mathcal{F}^w = 0$, then wage Phillips curve becomes vertical. In this instance there are no contracts in the labour market and nominal wages are perfectly flexible. If χ_w increases, i.e. the share of labour that is demanded at current wages but is contracted at previous wages increases, then nominal wages will respond less to the business cycle. The wage Phillips curve becomes flatter because b'_y will fall. Thus the parameters of our wage Phillips curve have consistent structural interpretation similar to that of Gali (2011).

Eq.(25) can be expressed in terms of the unemployment rate. In order to do this we recall from eq.(20) that output is a function of employment. By implication potential output is a function of the labour force. Denote potential output by Y_t^* and the labour force by N_t . Therefore, similar to Gali (2011), the level of employment is simply $(1 - u_t) N_t$, where u_t is the unemployment rate under the assumption of active search. It follows that we can write the output gap as $\tilde{y}_t = \tilde{a}_t - \sigma \tilde{u}_t$, which is the inverse of a simple Okun's law. Therefore we can write the wage Phillips curve as:

$$\hat{w}_t = (1 + b_p) \hat{p}_t + (b'_p - b_s) \hat{p}_{t-1} - \sigma (b'_y \Delta \tilde{u}_t + b''_y \Delta \tilde{u}_{t-1}) + b_s \hat{w}_{t-1} - \xi_t^{ww} \quad (26)$$

where $\eta_t^w = \xi_t^{ww} + \sigma (b'_y \Delta \tilde{a}_t + b''_y \Delta \tilde{a}_{t-1})$. Eq.(26) is similar to the wage Phillips curve in Gali (2011), except that Gali's formulation generates lags of the unemployment by appealing to an empirical AR(2) process that drives the unemployment rate in the case of the US. In our case, this AR(2) process arises from the optimisation process. We now introduce rule-of-thumb workers. At a point in time a fraction of workers ϵ_w sets nominal wages in an optimal way whilst the rest uses the rule-of-thumb. Rule-of-thumb workers set their nominal wages such that the nominal unit labour cost evolves according to:

$$\frac{W_t L_t}{Y_t} = (1 + \pi_t)^{\nu_w} \frac{W_{t-1} L_{t-1}}{Y_{t-1}} \quad (27)$$

where π_t is the moving average of the price inflation rate and $\nu_w > 0$ is the indexing parameter. Assume the aggregate nominal unit labour cost index is a geometric average of the optimal nominal unit labour cost and the one set

by rule-of-thumb firms, where ϵ_w is the fraction of workers who set nominal wages optimally. The optimal nominal unit labour cost evolves according to eq.(26). Consequently, we can write the nominal wage Phillips curve as:

$$\begin{aligned} \widehat{w}_t = & \epsilon_w (1 + b_p) \widehat{p}_t + \epsilon_w (b'_p - b_s) \widehat{p}_{t-1} - \epsilon_w \sigma (b'_y \Delta \widetilde{u}_t + b''_y \Delta \widetilde{u}_{t-1}) \\ & + \epsilon_w b_s \widehat{w}_{t-1}^o + (1 - \epsilon_w) \nu_w \pi_t + \eta_t^{ww}, \end{aligned} \quad (28)$$

where $\eta_t^{ww} = -\epsilon_w \xi_t^{ww} + (1 - \epsilon_w) \varepsilon_t^w$ and \widehat{w}_t^o is the optimal nominal unit labour cost inflation rate. Eqs.(7) and (26) or eqs.(10) and (28) constitute the wage-price spiral similar to the one proposed by Chiarella and Flaschel (2000), Flaschel et.al. (2001) and Asada et.al.(2006). These authors differ slightly with the formulation by Fair (2008) in that they incorporate the unemployment rate in the wage Phillips curve and the rate of capacity utilisation in the price Phillips curve. Fair on the other hand, argues that the wage-price Phillips curves must be specified in log levels and not in terms of inflation rates and that the empirically relevant demand pressure is the unemployment rate. Eqs.(7) and (26) can be easily written in level terms, in line with Fair's suggestion and the production function can be used to replace the output gap in eq.(7) with the unemployment rate.

3. Empirical results

We are now in a position to estimate the parameters of our wage-price spiral and to check how well it fits the data. For purposes of this exercise we consider six developed and six emerging market economies. For the six developed economies we have: the United States, United Kingdom, Canada, Germany, France and Australia. For the six emerging market economies we have: Brazil, Mexico, Poland, Turkey, South Korea and South Africa. Data is drawn from the International Financial Statistics database and where there are gaps, we used the OECD database and country statistical offices. The data is quarterly with a sample from 1975:1–2012:2 for developed economies. For emerging markets the data starts from 1995–2012:2. Inflation is measured using the CPI, supply shocks are measured by consumer prices for energy, food and the import price deflator, all drawn from the OECD database. Real output is measured by real GDP. Percentage deviations from trend are derived using the HP-filter.

One problem with estimation of optimisation-based models is that of "free parameters". Chari et.al.(2009) and Fair (2012) raise this problem in their critical review of new Keynesian models. In Gertler et al. (2008) three parameters are calibrated. Gali (2011) takes the inverse of the Frisch labour supply elasticity as the free parameter. In both these studies, the "zero steady state inflation rate" assumption is imposed. In our case, the discount rate, steady state inflation rate, labour share and the inverse of the Okun co-efficient are the free parameters. Table 1 reports the calibration that we use to estimate the model for each country.

Table 1: Calibrated parameters

	β	\hat{p}_0	S_0	σ^{-1}
Developed Economies				
Australia	1	0.05	0.44	0.54
Canada	1	0.04	1.10**	0.43
Germany	1	0.03	1.10**	0.37
France	1	0.05	1.88**	0.37
United Kingdom	1	0.05	1.04*	0.34
United States	1	0.04	1.04*	0.45
Emerging Markets				
Brazil	1	0.06	0.08**	0.30
Mexico	1	0.08	0.53	0.30
Poland	1	0.06	0.53	0.30
South Africa	1	0.07	0.53	0.31
Korea Republic	1	0.04	0.71	0.24
Turkey	1	0.32	1.57*	0.30

Notes: *Index of the labour share, **Index of real unit labour cost

The challenge with our formulation is that we cannot separately identify the parameters τ_p and χ_p . We thus estimate the measure of effective price rigidity. The calibration in Table 1 allows us us to estimate the following set of parameters $(\epsilon_p, \nu_p, \delta_i, (\tau_p (1 + \chi_p))^{-1})$. In the case of the nominal wage Phillips curve, we recall that $\phi = (1 - \alpha) \sum_{i=1}^n \theta_i$ and $\sigma = \frac{\alpha}{1 - \phi}$. We note that σ is the production function based, structural parameter that is the inverse of the parameter that underlies Okun's Law. To calibrate σ we

follow Ball et al.(2012), who estimate Okun’s Law for a number of countries . In some of the countries, e.g. the US and the UK, indices of the labour share are reported. In such cases we calibrate the parameter to be the mean value of the index over the sample period. The last column of Table 1 presents the calibration for σ . This allows us to estimate the parameter set $(\epsilon_w, \vartheta_w^{-1}, (\tau_w(1 + \chi_w))^{-1}, \nu_w)$.

Note from eq.(25) that prices drive wages contemporaneously, while from eq.(7) wages drive prices with a lag. Given the recursive structure of the price and nominal wage Phillips curves, we estimate the eq.(9) and the output gap version of eq.(28) using non-linear least squares. We also note that eq.(9) features \hat{p}_{t-1}^o on the right hand side and eq.(28) features \hat{w}_{t-1}^o on the right hand side. In order to implement the estimation, we use \hat{p}_{t-1} and \hat{w}_{t-1} as proxies. Lastly we mention that estimates of the nominal wage Phillips curve for South Africa and Poland yielded unreasonably large and insignificant results for ϑ_w^{-1} . We then calibrated this parameter for these countries to be 0.15, in line with other emerging market economies.

Tables 2 and 3 present estimates of the Phillips curves. In both developed and emerging market economies, the baseline price Phillips curve rejects the inclusion of supply shocks. We also find that in general, there is substantial price indexation, with a co-efficient above 0.9. This is substantially higher than the estimate by Smets and Wouters (2003). Our measure of effective price rigidity carries the wrong sign and is not significant in Canada and France. The same result can be observed in the case of Brazil and Mexico. In the case of these Latin American economies, these results may be due to data quality but from the theoretical point of view, in so far as the composite rigidity parameter is not significant, they suggest that prices are rigid in these economies. For the rest of the economies effective price rigidity has the correct sign and is highly significant.

In relation to the nominal wage Phillips curve, we observe that in general wage indexation is rejected in developed economies, except partially for Germany and Australia when the 4-quarter moving average is used. This result is not consistent with the estimates of Smets and Wouters (2003) and Gali (2011). In emerging market economies, wage indexation is rejected in Brazil and Mexico. We also find that the inverse of effective nominal wage rigidity is higher in emerging markets, implying that wages are more flexible in emerging markets than in developed markets. Across developed and emerging

markets, we also find that nominal wages are more flexible than prices. This result is in line with Amato and Laubach (2003) and Flaschel et al.(2007) for the case of the United States.

In terms of the empirical fit of the Phillips curves, we observe that in general they exhibit a high level of R^2 . However, the results also show that there is significant serial correlation among the residuals (see the $\chi^2(4)$ probability in all the regressions). In relation to the nominal wage Phillips curve, this serial correlation can be partially explained to be the result of persistence in subsistence consumption and productivity shocks. Gali (2011) attributes possible serial correlation in the error term of the wage Phillips curve to persistent variations in the markup. In relation to the price Phillips curve, the serial correlation signals that more work still has to be done to ensure that the baseline model accounts for systematic variation in the error term. Overall however, our wage-price Phillips curves fit the data well and are admitted by data from most of the cross section of countries.

Conclusion

Backward-looking wage and price Phillips curves have been severely criticised for their lack of microfoundations. Consequently, their estimated parameters lack structural interpretation. Despite this major weakness, these Phillips curves have been found to be stable across samples (Fuhrer (1995) and Gordon (2011)). In addition, backward-looking Phillips curves have been found to outperform the new Keynesian Phillips curves when it comes to inflation forecasting (Fair (2008), Gordon (2011)). On the other hand, estimates of the micro-founded new Keynesian Phillips curves have been found to yield counter-intuitive signs of the marginal cost or output gap parameter (see Rudd and Whelan (2007), Mazumder (2012) and Abbas and Sgro (2012) among others). In addition estimates of the new Keynesian Phillips curve have been found to suffer from weak identification (Mavroeidis (2004, 2005), Martins and Gabriel (2009)).

In this paper we have derived the backward-looking wage and price Phillips curves from microfoundations. We are thus able to provide a structural interpretation of the parameters of the backward-looking Phillips curves. Interestingly, some of the empirical observations by scholars in the field e.g. Mehra (2004) and Gordon (2011), have a straightforward structural interpretation. For example, these authors observe that the exclusion of supply

shocks in Phillips curve estimations biases the parameter on the demand pressure term downwards. We are able to explain this by the link between output and the input requirements of firms. Secondly, some Phillips curve estimations yield a negative sign on the demand pressure variable. We are able to link this phenomenon to the role played by speed-limit effects in a misspecified Phillips curve.

Our paper therefore contributes to the literature by addressing the long-standing criticism of backward-looking Keynesian Phillips curves. In so doing, it posits an optimisation-based competing perspective on inflation dynamics to the new Keynesian perspective. Estimations of the wage-price Phillips curves show that this model fits the data very well. Across the board, we find wages to be effectively more flexible than prices and we find that nominal wages are more flexible in emerging markets than in developed economies. Furthermore, the nominal wage Phillips curve rejects wage indexation in developed economies and in some emerging markets.

References

- Abbas S., Sgro P.M., 2011. New Keynesian Phillips curve and inflation dynamics in Australia. *Economic Modelling* 28, 2022–2033.
- Amato J., Laubach T., 2003. Estimation and control of an optimization-based model with sticky prices and wages. *Journal of Economic Dynamics and Control* 27, 1181–1215.
- Ambler S., Guay A., Phanouf L., 2012. Endogenous business cycle propagation and the persistence problem: The role of labor-market frictions. *Journal of Economic Dynamics and Control* 36, 47–62. (2012)
- Asada T., Chen P., Chiarella C., Flaschel P., 2006. Keynesian dynamics and the wage-price spiral: A baseline disequilibrium model. *Journal of Macroeconomics* 28, 90–130.
- Ascari G., Castelnuovo E., Rossi L., 2011. Calvo vs Rotemberg in a trend inflation world: An empirical investigation. *Journal of Economic Dynamics and Control* 35, 1852–1867.
- Ball L., Leigh D., Loungani P., 2012. Okun’s law: Fit at 50? Paper presented at 13th Jacques Polak Annual Research Conference. International Monetary Fund, November 8–9.
- Bardsen G., Jansen E.S., Nymoen R., 2004. Econometric evaluation of the new Keynesian Phillips curve. *Oxford Bulletin of Economics and Statistics* 66, Supplement, 0305–9049.

- Batini N., Jackson B., Nickell S., 2005. An open-economy new Keynesian Phillips curve for the UK. *Journal of Monetary Economics* 52, 1061–1071.
- Blanchard O.J., Katz L.F., 1999. Wage dynamics: Reconciling theory and evidence. *American Economic Association Papers and Proceedings* (May), 69–74.
- Calvo G.A., 1983. Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12, 383–398.)
- Campbell J.Y., Viceira L.M., 2002. *Strategic asset allocation*. Oxford, Oxford University Press.
- Carlsson M., Westermark A., 2012. The new Keynesian Phillips curve and staggered price and wage determination in a model with firm-specific labor. *Journal of Economic Dynamics and Control* 35, 579–603.
- Chari V.V., Kehoe P.J., McGrattan E.R., 2009. New Keynesian models: not yet useful for policy analysis. *American Economic Journal Macroeconomics* 1, 242–266.
- Chiarella C., Flaschel P., 2000. High order disequilibrium growth dynamics: Theoretical aspects and numerical features. *Journal of Economic Dynamics and Control* 24, 935–963.
- Christiano L.J., Eichenbaum M., Evans C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113, 1–45.
- Erceg C.J., Henderson D.W., Levin A.T., 2000. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46, 281–313.
- Fair R.C., 2000. Testing the nairu model for the United States. *Review of Economics and Statistics* 82, 64–71.
- Fair R.C., 2008. Testing price equations. *European Economic Review*. 52, 1424–1437.
- Fair R.C., 2012. Has macro progressed? *Journal of Macroeconomics* 34, 2–10.
- Flaschel P., Gong G., Semmler W., 2001. A Keynesian macroeconomic framework for the analysis of monetary policy rules. *Journal of Economic Behaviour and Organization* 46, 101–136.
- Fuhrer J.C., 1995. The Phillips curve is alive and well. *New England Review*. March/April, 41–56.
- Fuhrer J.C., Kodrzycki Y., Little J.S., Olivei G.P., 2009. The Phillips curve in historical context. in Fuhrer J.C. et al. (ed). *Understanding inflation and*

- the implications for monetary policy: A Phillips curve retrospective. MIT Press, Cambridge MA.
- Gali J., Gertler M., 1999. Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics* 44, 195–222.
- Gali J., 2000. The return of the Phillips curve and other recent developments in business cycle theory. *Spanish Economic Review* 2, 1–10.
- Gali J., 2011. The return of the wage Phillips curve. *Journal of the European Economic Association* 9, 436–461.
- Gali J., Smets F., Wouters R., 2011. Unemployment in an estimated new Keynesian model. NBER Working Paper 17084.
- Gertler M., Sala L., Trigari A., 2008. An estimated monetary DSGE model with unemployment and staggered nominal wage bargaining. *Journal of Money, Credit and Banking* 40, 1713–1764.
- Gordon J., 1997. The time-varying nairu and its implications for economic policy. *Journal of Economic Perspectives* 11, 11–32.
- Gordon R.J., 2011. The history of the Phillips curve: Consensus and bifurcation. *Economica* 78, 10–50.
- Guender A., 2011. The timeless perspective vs. discretion: Theory and monetary policy implications for an open economy. *Journal of International Money and Finance* 30, 1638–1658.
- Huang K.X.D., Liu Z., 2002. Staggered price-setting, staggered wage-setting, and business cycle persistence. *Journal of Monetary Economics* 49, 405–433.
- King R.G., Plosser C.I., Rebelo S.T., 1988. Production, growth and business cycles I: The basic neoclassical model. *Journal of Monetary Economics* 21, 195–232.
- Mankiw N.G., 2001. The inexorable and mysterious tradeoff between inflation and unemployment. *Economic Journal* 111, C45–C61.
- Martins L.F., Gabriel V.J., 2009. New Keynesian Phillips curves and potential identification failures: A generalized empirical likelihood analysis. *Journal of Macroeconomics* 31, 561–571.
- Mavroeidis S., 2005. Identification in forward-looking models estimated by GMM, with an Application to the Phillips Curve. *Journal of Money, Credit and Banking* 37, 421–448.
- Mavroeidis S., 2004. Weak identification of forward-looking models in monetary economics. *Oxford Bulletin of Economics and Statistics*. 66, Supplement, 0305–9049.
- Mazumder, S., 2012. The new Keynesian Phillips curve and the cyclicity of marginal cost. *Journal of Macroeconomics* 32, 747–765.

- Mehra Y.P., 2004. The output gap, expected future inflation and inflation dynamics: Another look. *Topics in Macroeconomics* 4, Article 17.
- Rotemberg J., 1982. Monopolistic price adjustment and aggregate output. *Review of Economic Studies* 49, 517–531.
- Rudd J., Whelan K., 2007. Modelling inflation dynamics: A critical review of recent research. *Journal of Money, Credit and Banking* 30 Supplement, 155–170.
- Rudd J., Whelan K., 2005. New tests of the new-Keynesian Phillips curve. *Journal of Monetary Economics* 52, 1167–1181.
- Sbordone A., 2001. An Optimizing model of US wage and price dynamics. Rutgers University Working Paper.
- Sen A., Dutt A., 1995. Wage bargaining, imperfect competition and the markup: Optimizing microfoundations. *Economics Letters* 48, 15–20.
- Sims C.A., 1987. Comment. *Brookings Papers of Economic Activity* 1, 117–120.
- Smets F., Wouters R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro Area. *Journal of the European Economic Association* 1, 1123–1175.
- Woodford M., 2003. *Interest and Prices*. New Jersey, Princeton University Press.
- Zhang C., Clovis J., 2010. The new Keynesian Phillips curve of rational expectations: A serial correlation extension. *Journal of Applied Economics* 13, 159–179.