Market characteristics and chaos dynamics in stock markets: an international comparison

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by

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Market characteristics and chaos dynamics in stock markets: an international comparison

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Abstract

The chaos theory assumes that the returns dynamics are not normally distributed and more complex approaches have to be used to study these time series. In fact, the Fractal Market Hypothesis assumes that the returns dynamics are not independent of the investors' attitudes and represent the result of the interaction of traders who, frequently, adopt different investment styles.

The studies proposed in literature try to identify the best approach to define the fractal dimension using, in particular, data of highly developed financial markets where a more complete set of information is available and the price determination mechanism is more efficient.

A fault found with these approaches is that the results do not allow making out if there is a relationship between fractal dimension and market characteristics and, besides, it is hard to understand which aspects are more relevant in the definition of the fractal market dimension. In fact, previous studies analysed market liquidity for a limited number of countries and no other aspects related to market transactions have been considered.

Using a large sample of world stock indexes, I try to identify the main market characteristics that influence returns dynamics. This study, carried out having recourse to the Rescaled Range Analysis (R/S) approach, shows that markets characteristic, like liquidity, type of admissible orders and so on, influence the R/S capability to study returns dynamics.

1. Introduction

Capital markets are characterized by significant differences in investors' attitudes and expectations that, as a rule, determine strange price dynamics that are unlike those suggested by classical linear models[1].

Even the simplest model of interaction with heterogeneous traders - chartists versus fundamentalists - proposed in literature to explain unexpected dynamics of the stock market[2] seem to fit better if nonlinearity is assumed[3] and, more generally, the higher the complexity of the system being analysed the bigger are the benefits related to nonlinear models.

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[1] Westerhoff F.H. (2005), "Heterogenous traders, price volume signals and complex asset price dynamics", *Discrete Dynamics in Nature and Society*, vol. 1, pp. 19-29.

[2] De Long J.B., Shleifer A., Summers L.H. and Waldman R.J. (1991), "The survival of noise traders in financial markets", *Journal of Business*, vol. 64, pp. 1-19.

[3] Kaizoji T. (2002), "Speculative price dynamics in a heterogeneous agent model", *Nonlinear dynamics, Psychology and Life Science*, vol. 6, pp. 217-229.

Many stock market studies try to identify the best model to predict future performance, but there is not clear evidence of the dominance of one approach with respect to others.[4] In fact, it has been shown that nonlinearity leads to better results with respect to the random walk hypothesis[5] but the choice among different nonlinear approaches is not at all easy and the capability of different approaches to achieve good results is affected by the degree of chaotic dynamics that characterizes the market.

International evidence proves the relevance of chaos dynamics to explain the dynamics of the most actively traded financial instruments, especially in well-organized markets[6]. Quite a few of these works have merely considered a single market and, frequently, paid considerable attention to well-developed economies. There are not many works focusing on undeveloped countries[7] and / or comparing different countries[8] and, therefore, there is no clear evidence of the main reasons for the difference in chaos level in different markets.

This paper analyses the role of the financial market characteristics in the degree of chaotic dynamics using the standard approach proposed in literature in order to evaluate stock markets. It starts with a brief analysis of the literature dealing with chaos in general, its estimation measures and its application to the stock market. (Paragraph 2) The analysis being proposed considers a few major stock markets worldwide and tries to verify if differences in the chaos degree may be explained based on a number of market characteristics. International evidence shows that the role of a few market characteristics is not residual in the selection of the best statistical model to predict future dynamics. (Paragraph 3) The conclusions endeavour to evaluate the impact of these results on the stock market predicting models and the future prospects for the best model to predict stock dynamics in different market scenarios. (Paragraph 4)

2. Chaos theory and stock market

The studies proposed in literature to analyse and predict stock price dynamics assume that, by looking at the past, one may collect useful information to understand the price formation mechanism. The initial approaches proposed in literature, the so-called technical analyses, assume that the price dynamics could be approximated with linear trends and could be analysed using a standard mathematical or graphical approach[9]. The high number of factors that are likely to influence the stock market dynamics makes this assumption incorrect and calls for the definition of more complex approaches that may succeed in studying these multiple relationships[10].

The nonlinear models are a heterogeneous set of econometric approaches that allow higher predictability levels, but not all the approaches may be easily applied to real data[11]. Deterministic chaos represents the best trade-off to establish fixed rules in order to link future dynamics to past

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[5] Hsieh D.A. (1991), "Chaos and non linear dynamics: applications for financial markets", *Journal of Finance*, vol. 46, pp. 1839-1877.

[6] Mucley C. (2004), "Empirical asset return distributions: is chaos the culprit", *Applied Economic Letters*, vol. 11, pp. 81-86.

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results of a time series without imposing excessively simple assumptions[12]. In essence, chaos is a nonlinear deterministic process that looks random[13] because it is the result of an irregular oscillatory process influenced by an initial condition and characterized by an irregular periodicity[14].

The chaos theory assumes that complex dynamics may be explained if they are considered as a combination of more simple trends that are easy to understand:[15] the higher the number of breakdowns, the higher the probability of identifying a few previously known basic profiles[16].
Chaotic trends may be studied considering some significant points that represent attractors or deflectors for the time series being analysed and the periodicity that exists in the relevant data[17].

The next two paragraphs analyse in detail the stock market and try to point out the main approaches suggested in literature to evaluate stock dynamics (paragraph 2.1) and evidence of the effect of market characteristics on chaotic dynamics (paragraph 2.2).

2.1 Estimation procedures for chaotic dynamics

The nonlinear dynamics assumption calls for the definition of a few aspects that are required to understand the rationality of past trends and to define the expected dynamics. The main characteristics may be identified in[18]:

- the type of randomness;
- the fractal dimension;
- the duration of the cycle;
- the relevance of past results.

The first analysis considers the time series noise and attempts to verify whether it may be considered a classical “white noise” or a “chaotic noise”[19]. The test adopted to analyse this aspect is the Brock, Dechert and Scheinkman test (BDS), which tries to ascertain whether a time series may be considered random or it presents a hidden structure[20]. Mathematically:

$$W_{nT}(\varepsilon) = \sqrt{T}\left[C_{n,T}(\varepsilon) - C_{1,T}(\varepsilon)^n\right] / \sigma_{n,T}(\varepsilon)$$

where the statistic represents a ratio between the spread of error terms with respect to the normality assumption $\left(C_{n,T}(\varepsilon) - C_{1,T}(\varepsilon)^n\right)$ and the asymptotic standard error $\left(\sigma_{n,T}(\varepsilon)\right)$[21]. A zero

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[21] Olmeda I. and Perez J. (1995), “Non linear dynamics and chaos in the Spanish stock market”, *Investigaciones Economicas*, vol. 19, pp. 217-248.

value of the statistic is obtained only when the time series' error $\left(C_{n,T}(\varepsilon)\right)$ is IID and in all the other scenarios it is possible (not necessary) to identify chaos dynamics[22].

The fractal dimension represents the number of basis elements (fractals) necessary to define an object that is similar to the trend being analysed[23] and, mathematically, it represents the number of degrees of freedom necessary to define a polynomial function that fits correctly the real dynamics[24]. The higher the complexity of the time series being analysed, the higher the estimated fractal dimension[25].

In nonlinear models, the role of long-term dependence may not be considered by studying the simple covariance or autocovariance and more complex approaches have to be used[26]. One of the most commonly used approaches is the rescaled range analysis (R/S analysis) that tries to check the role of past dynamics considering the maximum and minimum range with respect to the standard deviation[27]. In formulas: [28]

$$H = \lim_{n \to \infty} \frac{RS}{\ln(n)}$$

where the value of H is estimated considering an approximately infinite horizon (n) and the results of an autoregressive estimate of the role of past results (RS). The RS factor is estimated considering residuals of a standard linear model using this formula:

$$RS = \frac{1}{\frac{1}{T}\sum_{t=1}^{T} x(t) - E[x(t)]} \left\{ \max_{0<\tau<T}\left[\sum_{t=1}^{\tau} x(t) - E(x(t))\right] - \min_{0<\tau<T}\left[\sum_{t=1}^{\tau} x(t) - E(x(t))\right]\right\}$$

where the R/S measure is constructed considering the maximum spread observed in the period $\left(\max_{0<\tau<T}\left[\sum_{t=1}^{\tau} x(t) - E(x(t))\right] - \min_{0<\tau<T}\left[\sum_{t=1}^{\tau} x(t) - E(x(t))\right]\right)$ with respect to the classical standard deviation measure $\left(\sum_{t=1}^{\tau} x(t) - E(x(t))\right)$. The index varies from zero to one and measures the role of past performance in predicting future dynamics.[29]

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[24] Kugiumtzis D., Lillekjendlie B. and Christophersen N. (1995), *Chaotic time series. Part 1: Estimation of some invariant properties in state space*, University of Oslo working paper.

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[29] Los C.A. (2004), *Measuring the degree of financial market efficiency*, Kent State University working paper.

The approach being proposed represents a simplified approach to evaluate the degree of chaotic dynamics but, even if some adjustments were proposed in literature[30], there is no clear evidence of the higher forecasting capability of these new approaches[31].

One of the major applications of this approach is related to the possibility of using this statistic also to study the length of cycles that are relevant to a market. This approach assumes the possibility of defining the reversal point considering the point to be the ratio between R/S estimated for different time periods and the number of observations and looking for the period when the natural growing trend of the ratio is interrupted.[32] In formulas:

$$\text{if } H_n = \frac{RS_n}{\ln(n)} < H_{n-1} = \frac{RS_{n-1}}{\ln(n-1)} \quad \Longrightarrow \quad \textit{market cycle duration} = n$$

Results obtained by this test are strictly influenced by the variability of the time series and may call for the definition of a threshold to differentiate wrong signals from inversions.

The relevance of nonlinear trends with respect to randomness is assessed by studying the relevance of previous history on the results. The long-term dependence is considered by comparing the results achieved with the results obtained by the same statistics estimated on the scrambled series. The scrambled series is constructed using a random criterion that allows defining a new time series that is very different from the original time series.[33] After estimating these two time series, the relevance of the fractal dimension is higher if the results achieved are worse for the scrambled series than for the original time series.[34]

All these approaches work on a series of error estimates that could be obtained using different filtering criteria. This characteristic allows applying these models to different scenarios but implies that the results are strictly influenced by the type of data used and by the criteria adopted in filtering the time series.[35]

2.2 The relationship between market characteristics and stock price dynamics

Stock market transactions are characterized by irregular dynamics in prices and volumes that may not be predicted by standard linear forecasting methods[36]. In fact, trends identified by different linear models are not stable over time[37] and a significant increase or decrease in volatility causes the uselessness of previously estimated models[38].

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The lack of predictability of stock market returns demonstrated by classical linear methodologies led to the development of studies that tried to demonstrate the randomness of stock markets[39]. Random approaches are not useful to predict market dynamics and better results may be obtained if the analyst assumes that there is an underlying relationship in the stock price historical trends that cannot be analysed using such simple models as linear approaches[40].

The hypothesis that history is not relevant to predict future stock price dynamics cannot be correct since all the investors define their investment strategy based results obtained in the past. Even if there are differences in the information available[41] and/or it may be assumed that response functions to the same information are different for each investor[42], it may be useful to define models to predict future performance.

The usefulness of the approach being proposed is linked to a few market characteristics that are likely to influence the impact of investors' choices on stock market dynamics. The main aspects identified in literature as a significant explanation of chaos dynamics are:

- asymmetric transaction costs;
- type of orders;
- type of investors;
- transactions volume.

All the factors that are likely to influence the net results obtained by investors may affect the market price dynamics and/or volume of transactions[43]. The portfolio optimization process is more complex for the trade off between transaction costs and volume / opportunity of trading[44] and, as a rule, high transaction costs determine a lower frequency of portfolio re-balances and a lower volume of transactions[45].

Market price dynamics are influenced by investors' choices and constraints in the implementation of the strategies being adopted[46]. All the world markets are electronically based but differences may be identified in the type of orders that may be used[47]. Different types of admitted orders may influence price dynamics because the effectiveness of a few trading strategies is related to the possibility of defining limits to the validity of buying and/or selling orders and to the presence/absence of liquidity providers[48]. The opportunity to define time or price-related conditions

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[48] Seppi D.J. (1997), "Liquidity provisions with limit orders and specialists", *Review of Financial Studies*, vol. 10, pp. 103-150.

for the transactions reduces the impact of randomness on the stock price dynamics[49] and causes different market dynamics[50] because investors can select in which scenario the order will be executed. Markets where this possibility is offered are characterized by a partial independence of investors' strategies from short-term variations[51] and the trends observed seem to be significantly independent by noise and substantially related to investors' strategies and expectations[52]. The decision to define a few conditions for the execution of orders allow making price dynamics independent of a transitory lack of demand or supply for each type of stock and ensuring that the price dynamics reflects in an improved manner the long term expectations of investors.[53]

The role of institutional investors in the market could be relevant because these traders are usually more capable of identifying investment opportunities and defining the best type of order that allows them to achieve the best results.[54] This advantage with respect to individual investors is related to the experience that allows them to predict future dynamics and to reduce the risk exposure that characterizes stock investments.[55] Considering the institutional investors, a special role is played by dealers and/or market makers, traders that are likely to influence the variability of price in each day of trading and to reduce the market risk.[56] Markets with liquidity providers are usually characterized by a more stationary price envelope[57] or, more generally, by a hidden trend that is clearer than in other markets.[58]

Stock price dynamics are influenced by the number of investors that actively trade in the market and a significant variability in the number of investors could influence the proximity of the trading price to the fundamental value[59]. In fact, markets characterized by a low number of traders and/or transactions achieve equilibriums that could be significantly different from the optimal scenario based on the stocks' fundamental value and price dynamics for this type of market could be difficult to forecast. The non-linearity of the stock price dynamics is influenced by the number of transactions relative to each stock.[60] Therefore, with reference to each time period being considered, it may be ascertained that a higher (lower) level of transactions implies a lower (higher) capability of the linear model to explain the market dynamics[61].

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[49] Iori G., Daniels M.G., Famer J.D., Gillemot L., Krishnamurty S. and Smith E. (2003), "An analysis of price impact function in order driven markets", *Phisica A*, vol. 324, pp. 146-151.
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4. The impact of stock market characteristics on chaos theory: empirical evidence

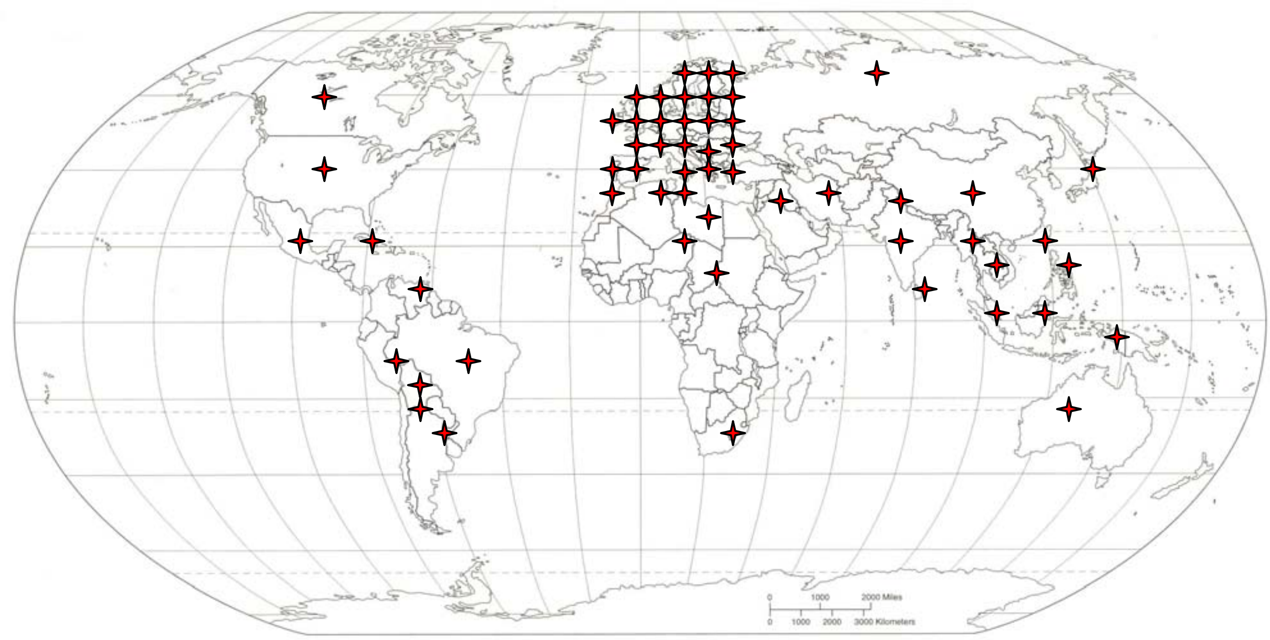
4.1 The sample

The aim of this paper is to study the market efficiency of each country; hence, it stands to reason that the chaos theory principles and methodologies apply directly to the stock market indexes.[62] In fact, the alternative of considering each stock listed in each stock market could produce results that are strictly affected by the criteria adopted in the selection of stocks.

The analysis being proposed tries to inspect some of the main world markets and to study markets that present significant differences in market characteristics. The sample is constructed considering indexes characterized by the availability of data for a sufficient time period to be useful in analyzing chaos dynamics and, based on results presented in literature[63], the indexes considered are those for which data has been available for not less than ten years.[64] (Graph 1)

Graph 1

Sample description



✦ = *Stock market analysed*

The sample is constructed considering almost one index - the most representative - for each country, with a total of fifty indexes. The data are collected daily for a ten-year time period (1996-2005) using the DataStream database.

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[62] Pandey V., Kohers T. and Kohers G. (1998), "Deterministic non linearity in the stock returns of major European equity markets in the United States, *Financial Review*, vol. 33, pp. 45-64.

[63] Jaditz T. and Sayers C. (1993), "Is chaos generic in economic data?", *International Journal of Bifurcations and Chaos*, vol. 3, pp. 745-755.

[64] The list of indexes selected for each country is shown in the appendix.

4.2 The characteristics of world stock markets

The relevance of the previously evidenced variables may only be tested by defining rules that allow collecting data for all or a relevant percentage of the countries being considered. All the assumptions made afterwards represent a simplification of the approach, but they may be considered the best solution based on the data available for the analysis.

The transaction cost at an aggregate level may only be considered in part, because a quote is characterized by a fixed transaction cost that is independent of the type of stock considered and the bid-ask spread that is typical for each stock[65]. Hence, the decision to consider only the mean transaction costs applied to the transaction during the time period being considered.

The trading mechanism may be examined considering market statements and the possibility to define different type of orders. The adoption of an international comparison of strategies in different markets requires the definition of a standard classification that may be applied to all the markets. The choice is to define the most general one that discriminate orders only on the basis of the type of constraint imposed: time, quantity and price.

The relative importance of institutional investors with respect to individuals may hardly be evaluated by comparing the number and/or volume of trades because the activism of these investors is strictly related to available information and expectations, and no data about these aspects is on hand. The only unquestionable datum that may be used to evaluate the potential role of institutional investors is the presence or absence of dealers or market makers established by law.

The study of liquidity considers a standard proxy like the number of trades in the period being analysed. More in detail, the suggested approach studies daily trades and, considering the high variability of volumes related to market anomalies[66], tries to define a classification of stock markets breaking them down in four categories based on the mean amount of trades for all the periods being considered.[67]

The table below summarizes the selected characteristics with respect to the countries considered in the analysis. (Table 1)

[65] Atkins A.B. and Dyl E.A. (1990), “Price reversal, bid ask spreads and market efficiency”, *Journal of Financial and Quantitative Analysis*, vol. 25, pp. 535-547.
[66] Chordia T., Roll R. and Subrahmayam A. (2001), “Market liquidity and trading activity”, *Journal of Finance*, vol. 56, pp. 501-530.
[67] The high heterogeneity of the sample causes the uselessness of a non arbitrary approach to define a threshold with respect to the distribution of the number of trades.

Table 1

Main characteristics of the markets being considered

	Type of orders			Dealers and/or market makers	Mean percentage of transaction costs*	Mean daily number of trades
	Price limited orders	Time limited orders	Quantity limited orders			
Argentina	☐	☐	☐	☑	-	25000-50000
Australia	☑	☑	☑	☑	0.02%	25000- 50000
Austria	☑	☑	☑	☑	-	More than 100000
Belgium	☑	☑	☑	☑	-	25000- 50000
Brazil	☑	☐	☑	☑	-	25000- 50000
Canada	☑	☑	☑	☑	0.02%	More than 100000
Czech Republic	☑	☑	☑	☑	-	25000- 50000
Chile	☑	☑	☐	☐	-	Lower than 25000
China	☐	☐	☐	☑	-	25000- 50000
Egypt	☐	☐	☐	☐	-	Lower than 25000
Finland	☑	☑	☑	☑	0.02%	50000-100000
France	☑	☑	☑	☑	-	25000- 50000
Germany	☑	☑	☐	☑	-	25000- 50000
Hong Kong	☑	☐	☐	☑	0.02%	Lower than 25000
Holland	☑	☑	☑	☑	-	25000- 50000
Iceland	☐	☐	☐	☐	0.02%	25000- 50000
Ireland	☑	☑	☑	☑	0.02%	Lower than 25000
Israel	☑	☐	☐	☑	-	Lower than 25000
Italy	☑	☑	☑	☑	-	50000-100000
Jamaica	☑	☑	☑	☑	-	Lower than 25000
Japan	☑	☑	☑	☑	0.01%	More than 100000
Jordan	☐	☐	☐	☐	0.05%	Lower than 25000
Kenya	☐	☐	☐	☐	-	Lower than 25000
Korea	☐	☐	☐	☐	-	Lower than 25000
Malaysia	☑	☐	☑	☐	0.06%	Lower than 25000
Mauritius	☐	☐	☐	☐	1.25%	Lower than 25000
Mexico	☑	☑	☐	☐	-	Lower than 25000
Morocco	☑	☑	☐	☐	-	Lower than 25000
New Zealand	☑	☑	☑	☑	-	More than 100000
Pakistan	☐	☑	☑	☐	-	Lower than 25000
Peru	☑	☑	☑	☐	0.08%	Lower than 25000
Poland	☑	☑	☑	☑	-	Lower than 25000
Portugal	☑	☑	☑	☑	0.03%	Lower than 25000
Singapore	☐	☐	☐	☑	0.05%	Lower than 25000
Slovakia	☑	☑	☑	☑	0.08%	Lower than 25000
Spain	☑	☑	☑	☑	0.20%	25000-50000
Sri Lanka	☑	☑	☐	☐	0.02%	Lower than 25000
Sweden	☑	☑	☑	☑	-	Lower than 25000
Switzerland	☑	☑	☑	☑	-	Lower than 25000
Thailand	☑	☐	☐	☐	0.02%	Lower than 25000
Hungary	☑	☑	☑	☑	0.03%	25000- 50000
UK	☑	☑	☑	☑	0.03%	More than 100000
USA	☑	☑	☑	☑	0.04%	More than 100000
Venezuela	☐	☐	☐	☐	-	Lower than 25000

*Notes: * Fees presented in the table are the mean fees applied to individual investors*

Source: author's elaboration of official market data

Even if the aggregation of all order types in only three macro categories reduces the variability among the countries, the analysis of the sample allows singling out differences because only 51% of the markets present a complete set of orders. A simple data analysis shows that there is no predominant solution and so it is possible to verify if different choices in type of orders admitted impact on stock price dynamics.

The resulting sample considers markets with different degrees of stability related to the presence or absence of market makers and by different degrees of liquidity. In fact, the available data allows ascertaining that over 35% of the markets have no market maker and, even if it is clear that some of the markets being considered are very small, the big markets are highly heterogeneous with respect to the degree of activism of the investors.

Transaction costs data are more difficult to collect since the decision to delegate to the market the definition of the proper price may have a negative impact on the ability to monitor correctly the amount of the fee applied to market participants.

With reference to all the data taken into consideration, this allows ascertaining the existence of differences among the countries being considered that permit to analyse whether these characteristics may influence the degree of chaotic dynamics of each type of market.

4.3 The model

Markets dynamics are studied considering the stock index value and estimating the daily returns using the standard logarithmic approach. In formulas

$$ {}_j r_t = \ln\left({}_j I_t \big/ {}_j I_{t-1} \right) $$

where ${}_j I_t$ represents the stock market index considered for the country *j*.

The analysis of the chaotic degree considers likely methods of estimation of the hidden basic linear function and tries to verify whether there are results independent of the methodology used and/or methodologies that are better suited. The role of past forecast results in forecasting is evaluated considering very simple approaches that could replicate results obtainable by standard technical analysis tools: moving average and trends. The selected econometric models are: [68]

- $r_t = AR(n) = c + \phi_1 r_{t-1} + ... + \phi_n r_{t-n}$
- $r_t = MA(m) = c + \psi_i \varepsilon_{t-1} + ... + \psi_n \varepsilon_{t-m}$
- $r_t = ARMA(n,m) = c + \phi_1 r_{t-1} + ... + \phi_n r_{t-n} + \psi_i \varepsilon_{t-1} + ... + \psi_n \varepsilon_{t-m}$

The first approach represents the more simple proxy for the noise traders' strategy because it assumes that there is a direct and proportional response of traders to each variation in a stock index $(\phi_i r_{t-i})$.[69] The impact of each variation is related to the number of lags (n) considered in the models.

The Moving Average (MA) approach assumes that the responses are not related to the latest variations but represent the result of the study of the time series dynamics on a longer time

[68] Hamilton J.D. (1995), *Econometria delle serie storiche*, Monduzzi Editore, Bologna, pp. 51-87.
[69] Beja and Goldman (1980), "On the dynamic of prices in disequilibrium", *Journal of Finance,* vol. 35, pp. 235-248.

period (n). The first coefficients represent the sensitiveness to new data and the number of means considered represents an estimate of the degree of smoothing time series variations.

ARIMA models are the most generic approaches used as filter and represent a combination of AR and MA models. These models could be considered useful to predict strategies adopted by more sophisticated technical analysts that are able to consider signals offered by different technical indicators.

The high heterogeneity of the sample does not allow getting to the identification of a model suited for all the markets being considered. Hence, the lag that permits to get to the best result for each market has to be estimated. The decision to use more than one model to estimate hidden trends leads to results that are partially independent of the assumption made in the construction of the hidden model.

All the previously suggested tests about the type of randomness and all the statistics related to the degree of chaotic dynamics need to be estimated with reference to all the filters being proposed and for all the de-trend time series being considered. The cycle lengths identified with the Hurst index are only estimated in respect of the time series showing chaotic dynamics.

4.4 Results

The analysis of the degree of randomness of the series filtered with different criteria is conducted for all the filters proposed above and the results obtained permit to reject the hypothesis of a random dynamics also for all the proposed criteria. The results that have been obtained show that, as a rule, residuals are not identically distributed for a large majority of filters and, in respect of a few countries, this relationship may be verified independently of the selected filter.[70] (Table 2)

[70] The table only shows the results of the worst criteria that may be adopted to construct residual series. Results obtained with other lag and/or time periods are better than those shown in the table and will be available on request.

Table 2

Hurst index and Scrambled Hurst

Countries	BDS			Countries	BDS		
	AR (n) worst filter	MA (n) worst filter	ARIMA (n) worst filter		AR (n) worst filter	MA (n) worst filter	ARIMA (n) worst filter
Argentina	9.87**	26.32***	9.23***	**Kenya**	23.46***	25.52***	26.22***
Australia	8.41**	24.71***	8.54***	**Korea**	6.49**	26.14***	6.30**
Austria	8.38**	25.23***	8.49***	**Malaysia**	16.70***	20.68***	0.16
Belgium	12.24***	22.89***	12.19***	**Mauritius**	8.07**	26.92***	8.85**
Brazil	12.93***	26.24***	12.34***	**Mexico**	9.30**	24.28***	9.07**
Canada	9.30**	24.28***	9.07**	**Morocco**	19.20***	23.79***	18.83***
Czech Republic	8.19**	24.73***	8.25**	**New Zealand**	9.39**	24.82***	9.10**
Chile	12.22***	16.72***	12.45***	**Pakistan**	15.26***	25.92***	15.36***
China	11.06***	28.34***	10.97***	**Peru**	9.48**	23.86***	9.79**
Egypt	15.18***	20.51***	14.82***	**Poland**	11.31***	26.70***	11.04***
Finland	12.69***	24.75***	12.12***	**Portugal**	-9.95**	-0.21	-11.09***
France	9.20***	27.02***	8.92**	**Singapore**	14.35***	25.18***	14.16***
Germany	11.43***	28.01***	11.47***	**Slovakia**	6.74**	26.75***	6.63**
Hong Kong	14.91***	24.95***	-1.02	**Spain**	10.11**	27.97***	10.15***
Holland	13.73***	28.00***	13.39***	**Sri Lanka**	20.24***	21.76***	19.57***
Iceland	12.52***	24.17***	12.39***	**Sweden**	11.15***	24.80***	11.06**
Ireland	8.41**	26.25***	8.35***	**Switzerland**	13.86***	27.15***	13.96***
Israel	3.76*	24.77***	3.58*	**Thailand**	22.75***	26.13***	26.38***
Italy	11.31***	26.70***	11.04***	**Hungary**	12.74***	26.27***	13.39***
Jamaica	-1.06	25.13***	-1.07	**UK**	12.05***	26.47***	-0.21
Japan	-0.03	25.52***	-0.04	**USA**	7.15**	26.52***	7.22**
Jordan	9.81**	21.52***	9.88**	**Venezuela**	13.26***	22.77***	13.19***

Notes: * Statistically significant at 90%
** Statistically significant at 95%
*** Statistically significant at 99%

The analysis of randomness in error time shows that there are quite a few scenarios where the chaos dynamics is likely to explain some of the errors relative to the standard linear model. The degree of chaotic dimension is analyzed looking for the best specification of the three models being proposed and comparing the results with the scrambled one. (Table 3)

The markets being considered seem to point to the presence of chaotic dynamics because results obtained through the AR filter and the ARIMA filter identify a hurst index higher than the random scenario (H=0.5) and bigger than the scrambled one in over 65% of the countries.

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Table 3

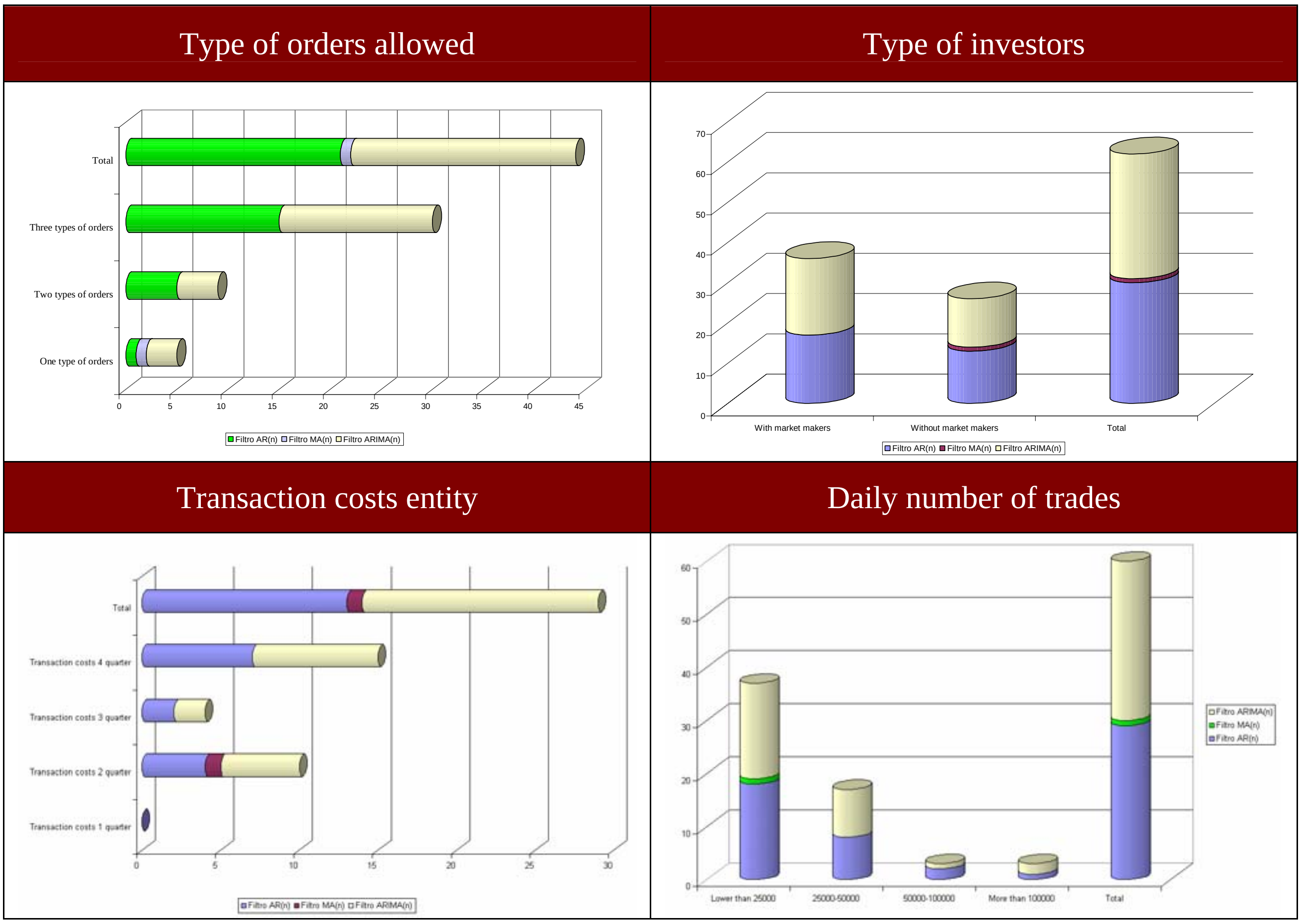
Hurst index and Scrambled Hurst

	AR (n) best filter		MA (n) best filter		ARIMA (n) best filter	
	Hurst	Hurst scrambled	Hurst	Hurst scrambled	Hurst	Hurst scrambled
Argentina	0.56	0.33	0.12	0.43	0.56	0.45
Australia	0.46	0.46	0.10	0.39	0.51	0.48
Austria	0.55	0.43	0.03	0.40	0.55	0.47
Belgium	0.54	0.47	0.10	0.46	0.54	0.48
Brazil	0.46	0.38	0.09	0.42	0.46	0.42
Canada	0.46	0.40	0.02	0.49	0.45	0.45
Czech Republic	0.55	0.35	0.15	0.47	0.54	0.46
Chile	0.53	0.45	0.21	0.42	0.53	0.45
China	0.50	0.38	0.02	0.42	0.49	0.46
Egypt	0.60	0.45	0.16	0.43	0.59	0.44
Finland	0.55	0.42	0.08	0.46	0.54	0.41
France	0.54	0.39	0.09	0.48	0.54	0.43
Germany	0.51	0.43	0.01	0.49	0.52	0.38
Hong Kong	0.40	0.42	0.17	0.53	0.53	0.49
Holland	0.53	0.44	0.09	0.42	0.55	0.40
Iceland	0.61	0.39	0.23	0.50	0.60	0.45
Ireland	0.54	0.49	0.05	0.47	0.49	0.46
Israel	0.49	0.34	0.18	0.47	0.51	0.44
Italy	0.51	0.36	0.01	0.49	0.50	0.45
Jamaica	0.49	0.45	-0.21	0.39	0.49	0.40
Japan	0.48	0.47	-0.07	0.52	0.49	0.49
Jordan	0.64	0.43	0.02	0.36	0.64	0.49
Kenya	0.61	0.40	0.45	0.51	0.60	0.47
Korea	0.55	0.37	0.16	0.43	0.55	0.38
Malaysia	0.51	0.51	0.42	0.51	0.49	0.50
Mauritius	0.60	0.44	0.20	0.41	0.57	0.39
Mexico	0.46	0.41	0.02	0.42	0.45	0.46
Morocco	0.58	0.36	0.13	0.46	0.56	0.49
New Zealand	0.48	0.48	0.16	0.45	0.45	0.48
Pakistan	0.53	0.43	0.04	0.47	0.49	0.41
Peru	0.56	0.41	0.16	0.40	0.56	0.39
Poland	0.49	0.41	0.01	0.37	0.50	0.39
Portugal	1.00	0.45	0.48	0.51	0.98	0.49
Singapore	0.53	0.37	0.18	0.42	0.53	0.40
Slovakia	0.61	0.41	0.19	0.48	0.61	0.47
Spain	0.53	0.49	0.11	0.51	0.52	0.40
Sri Lanka	0.55	0.38	0.36	0.47	0.53	0.38
Sweden	0.53	0.44	0.03	0.41	0.54	0.40
Switzerland	0.51	0.41	0.09	0.45	0.52	0.46
Thailand	0.67	0.45	0.52	0.48	0.67	0.44
Hungary	0.55	0.49	0.12	0.47	0.52	0.34
UK	0.50	0.39	0.02	0.43	0.54	0.42
USA	0.47	0.48	-0.05	0.34	0.48	0.43
Venezuela	0.59	0.42	0.31	0.48	0.55	0.44

More in detail, the markets exhibiting more chaotic characteristics show high order type heterogeneity (more than 65% of these markets seem chaotic) and, in more than a half percentage of cases, chaotic markets are characterized by the presence of a market maker. The degree of chaotic dynamics is also clearer for markets with lower trade volumes, but there seems to be no relationship with the amount of the transaction costs. (Graph 2)

Graph 2

Market characteristics and degree of chaotic dynamics



Looking only at the indexes showing chaotic patterns for almost one of the criteria being proposed, it becomes possible to study the mean duration of the cycles that characterize these markets and to look for other relationships between market characteristics and chaos dynamics. (Table 4)

Table 4

Market cycle estimated with hurst exponent

Country	AR (n)			MA (n)			ARIMA (n)		
	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min
Argentina	16	274	832	16	22	768	1	0	0
Australia	247	960	32	246	1088	32	274	1472	16
Austria	309	1280	54	309	1280	32	6	256	1
Belgium	213	768	16	213	768	32	54	512	1
Chile	163	1152	16	245	1152	32	163	1024	16
Czech Republic	225	960	6	247	960	22	17	768	1
Egypt	353	1280	22	353	1280	32	1	2	1
Finland	154	1024	16	225	1024	32	14	1280	1
France	225	768	54	274	832	32	13	1088	1
Germany	113	448	6	131	768	32	2	71	1
Holland	165	768	16	190	768	16	2	116	1
Hong Kong	68	214	16	123	262	32	47	1035	1
Hungary	122	640	32	122	640	32	3	358	1
Iceland	326	1600	32	247	1280	32	190	1152	12
Ireland	189	768	6	189	768	16	4	114	1
Israel	161	448	16	181	640	16	12	1088	1
Italy	155	320	16	189	640	16	5	418	1
Jordan	309	1216	48	412	1088	128	7	422	1
Kenya	247	1024	6	326	1248	128	190	512	16
Korea	97	224	32	97	256	32	60	1280	1
Mauritius	412	640	96	392	1152	128	24	576	1
Morocco	353	1664	16	238	512	16	16	416	1
Pakistan	618	1472	198	618	1472	198	7	272	1
Peru	154	288	64	154	384	64	353	1280	32
Portugal	823	1280	550	80	1280	16	22	1280	1
Singapore	81	192	16	101	230	32	13	1280	1
Slovakia	443	1664	48	274	1920	30	11	1280	1
Spain	206	768	32	274	1024	54	18	768	1
Sri Lanka	274	832	16	353	896	32	247	832	32
Sweden	142	448	32	189	768	32	189	768	32
Switzerland	274	768	64	309	768	64	7	800	1
Thailand	225	768	54	247	768	54	4	232	1
UK	131	640	32	122	640	32	122	640	32
Venezuela	412	1344	32	494	1344	64	274	768	16

Note: All data are expressed in number of days before the reversion

Cycles estimated with more complex models (ARIMA) vary more frequently during the period analysed with respect to all the countries being considered. A comparison of the cycles of different countries clearly suggests that the liquidity not only affects the degree of chaos but also the type of cyclicality. In fact, as a rule, highly liquid markets have a higher frequency of reversion. This could be considered a direct consequence of the high number of investors that interact in the market.

5. Conclusions

An international comparison shows that the forecasting methods are likely to be significantly influenced by the characteristics of the market being analysed and the interpretation of the results of by each forecasting process could be re-thought based on the evidence suggested in this paper.

In fact, the differences identified in the degree of nonlinearity for different market structures clearly point to the impossibility of assuming that a single methodology is best, irrespective of the market being analysed. In fact, the different degree of nonlinearity implies a different length of the cycles that are relevant for all the forecasting methodologies and are likely to affect to a significant extent the results.

The next step of the analysis could be the search of the best technique to predict the future performance of stock markets that are homogenous in respect of one or more variables identified in this paper.[71] The recourse to more complex approaches, like e-GARCH[72], could prove useful for future developments as they allow studying not only day or monthly dynamics but also the impact of chaos on strategies adopted by investors looking to intraday data.[73]

Another future development of the analysis could be identified in the analysis of chaotic dynamics in the trend of individual stocks.[74] This different approach could be useful because the degree of chaotic dynamics is not necessarily independent of firm characteristics[75] and it could be interesting to analyse which are the main characteristics of stocks that demonstrates a more chaotic dynamic.[76]

The results obtained from a comparison of different markets could be useful to analyse stock markets, even if the world markets tend to achieve a high level of integration. A few differences, like the mean volume of trades and the relative incidence of institutional investors[77], are not likely to disappear with the finalization of the integration process. Hence, it could prove useful to replicate the analysis considering new market characteristics that allow discriminating among different stock markets.

[71] For a review of different methodologies proposed in literature, see Kugiumtzis D., Lillekjendlie B. and Christophersen N. (1995), *Chaotic time series. Part I1: system identification and prediction*, University of Oslo working paper.

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Appendix

Table A 1

Index selected for each country analysed

Paese	Index
Argentina	MERVAL
Austria	AUSTRIA TRADED INDEX
Belgium	BELMID
Brazil	BOVESPA
Canada	S&P/TSX COMPOSITE INDEX
Czech Republic	PX GLOBAL INDEX
Chile	IGPA
Croatia	CROBEX
Denmark	OMX Copenhagen
Estonia	OMX TALLIN
Europe	DJSTOCK 50
Finland	OMX Helsinki
France	CAC 40
Germany	DAX
Greece	ATHEX ALL SHARE
Holland	AMSTERDAM SE ALL SHARES
Hungary	BUX
Iceland	ICEX ALL SHARES
Ireland	ISEQ
Italy	MIB30
Italy	S&P MIB
Jamaica	S&P/IFCF M JAMAICA
Lithuania	NOMURA
Luxembourg	LUXX
Mexico	IPC
Norway	OBX UTBYTTEJUSTERT
Peru	DS Market
Poland	WARSAW GENERAL INDEX
Portugal	PSI GENERAL
Romania	BET
Russia	RTS INTERFAX COMPOSITE
Slovakia	SAX 16
Spain	IBEX 35
Sweden	OMX Stockholm
Switzerland	SWISS index
Turkey	ISE National all shares
USA	S&P 500
Venezuela	VENEZUELA SE General